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A Note on Square Neutrosophic Fuzzy Matrices

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Abstract. In this article, we shall define the addition and multiplication of two neutrosophic fuzzy matrices. Thereafter, some properties of addition and multiplication of these matrices are also put forward.

Keywords: Neutrosophic fuzzy matrice, Neutrosophic Set.

1 Introduction

Neutrosophic sets theory was proposed by Florentin Smarandache [1] in 1999, where each element had three associated defining functions, namely the membership function (T), the non-membership (F) function and the indeterminacy function (I) defined on the universe of discourse X, the three functions are completely independent. The theory has been found extensive application in various field [2,3,4,5,6,7,8,9,10,11] for dealing with indeterminate and inconsistent information in real world.Neutrosophic set is a part of neutrosophy which studied the origin, nature and scope of neutralities, as well as their interactions with ideational spectra. The neutrosophic set generalized the concept of classical fuzzy set [12, 13], interval-valued fuzzy set, intuitionistic fuzzy set [14, 15], and so on.

Also as we know, matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [17] Thomason, introduced the fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In 2004, W. B. V. Kandasamy and F. Smarandache introduced fuzzy relational maps and neutrosophic relational maps.

Our aim ,In this paper is to propose another type of fuzzy neutrosophic matrices ,called “square neutrosophic fuzzy matrices”, whose entries is of the form a+Ib (neutrosophic number) , where a,b are the elements of [0,1] and I is an indeterminate such that I^n=I, n being a positive integer. In this study we will focus on square neutrosophic fuzzy matrices.The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set, neutrosophic matrices, Fuzzy integral neutrosophic matrices and fuzzy matrix. Section 3 presents a new type of fuzzy neutrosophic matrices and investigated some properties such as addition and multiplication. Conclusions appear in the last section.

2 Preliminaries

In this section we recall some concept such as , neutrosophic set, neutrosophic matrices and fuzzy neutrosophic matrices proposed by W. B. V. Kandasamy and F. Smarandache in their books [16] , and also the concept of fuzzy matrix .

Definition 2.1 (Neutrosophic Sets).[1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions T, I, F : U→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or

Falsehood) of the element x ∈ U to the set A with the condition.

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of $]^{-}0, 1^{+}[$ we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2 (Neutrosophic matrix) [16].

Let $M_{m \times n} = \{(a_{ij}) / a_{ij} \in K(I)\}$, where K (I), is a neutrosophic field. We call $M_{m \times n}$ to be the neutrosophic matrix.

Example 1: Let Q(I) = ⟨Q ∪ I ⟩be the neutrosophic field

$$M_{3 \times 3} = \begin{pmatrix} 0 & 1 & 0 \\ -2 & 4I & 2 \\ 3I & 1 & -I \end{pmatrix}$$

$M_{3 \times 3}$ denotes the neutrosophic matrix, with entries from rationals and the indeterminacy.

Definition 2.3 (Fuzzy integral neutrosophic matrices)

Let N = [0, 1] ∪ I where I is the indeterminacy. The m ×n matrices $\mathbf{M}_{m \times n} = \{(a_{ij}) / a_{ij} \in [0,1] \cup I\}$ is called the fuzzy integral neutrosophic matrices. Clearly the class of m ×n matrices is contained in the class of fuzzy integral neutrosophic matrices.

An integral fuzzy neutrosophic row vector is a **1 × n** integral fuzzy neutrosophic matrix, Similarly an integral fuzzy neutrosophic column vector is a m ×1 integral fuzzy neutrosophic matrix.

Example 2 : Let $A_{3 \times 3} = \begin{pmatrix} 0 & 1 & 0.3 \\ 0.9 & I & 0.2 \\ I & 1 & I \end{pmatrix}$

A is a 3 ×3 integral fuzzy neutrosophic matrix.

Definition 2.5 (Fuzzy neutrosophic matrix) [16]

Let $\mathbf{N_s}$= [0, 1] ∪ nI / n ∈ (0, 1]}; we call the set $\mathbf{N_s}$ to be the fuzzy neutrosophic set. Let $\mathbf{N_s}$ be the fuzzy neutrosophic set. $\mathbf{M}_{m \times n} = \{(a_{ij}) / a_{ij} \in \mathbf{N_s}\}$ we call the matrices with entries from $\mathbf{N_s}$ to be the fuzzy neutrosophic matrices.

Example 3: Let $\mathbf{N_s}$= [0,1] ∪{nI/ n ∈ (0,1]} be the set

$$P = \begin{pmatrix} 0 & 0.2I & I \\ I & 0.01I & 0 \\ 0.31I & 0.53I & 0.1 \end{pmatrix}$$

is a 3 ×3 fuzzy neutrosophic matrix

Definition 2.6 (Fuzzy matrix) [17]

A fuzzy matrix is a matrix which has its elements from the interval [0, 1], called the unit fuzzy interval. A m x n fuzzy matrix for which m = n (i.e the number of rows is equal to the number of columns) and whose elements belong to the unit interval [0, 1] is called a fuzzy square matrix of order n. A fuzzy square matrix of order two is expressed in the following way

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where the entries a,b,c,d all belongs to the interval [0,1].

3 Some Properties of Square Neutrosophic Fuzzy Matrices

In this section ,we define a new type of fuzzy neutrosophic set and define some operations on this neutrosophic fuzzy matrice.

3.1 .Definition (Neutrosophic Fuzzy Matrices)

Let A be a neutrosophic fuzzy matrices, whose entries is of the form a+Ib (neutrosophic number) , where a,b are the elements of [0,1] and I is an indeterminate such that I^n=I, n being a positive integer.

$$A = \begin{pmatrix} a_1 + I\,b_1 & a_2 + I\,b_2 \\ a_3 + I\,b_3 & a_4 + I\,b_4 \end{pmatrix}$$

3.2.Arithmetic with Square Neutrosophic Fuzzy Matrices

In this section we shall define the addition and multiplication of neutrosophic fuzzy matrices along with some properties associated with such matrices.

3.2.1. Addition Operation of two Neutrosophic Fuzzy Matrices

Let us consider two neutrosophic fuzzy matrices as

A=$\begin{pmatrix} a_1 + I\,b_1 & a_2 + I\,b_2 \\ a_3 + I\,b_3 & a_4 + I\,b_4 \end{pmatrix}$and B=$\begin{pmatrix} c_1 + I\,d_1 & c_2 + I\,d_2 \\ c_3 + I\,d_3 & c_4 + I\,d_4 \end{pmatrix}$

Then we would like to define the addition of these two matrices as

$$A + B = [C_{ij}]$$

Where

$C_{11} = \max(a_{1,}c_1) + I\max(b_1, d_1)$

$C_{12} = \max(a_{2,}c_2) + I\max(b_2, d_2)$

$C_{21} = \max(a_{3,}c_3) + I\max(b_3, d_3)$

$\mathbf{C_{22} = \max(a_{4,}c_4) + I\max(b_4, d_4)}$

It is noted that the matrices defined by our way is reduced to fuzzy neutrosophic matrix when a =

Properties 1

The following properties can be found to hold in cases of neutrosophic fuzzy matrix multiplication

(i) A+B = B+A

(ii) (A+B)+C = A+(B+C)

3.2.2 Multiplication Operation of Neutrosophic Fuzzy Matrices

Let us consider two neutrosophic fuzzy matrices as $A = [a_{ij} + Ib_{ij}]$ and $B = [c_{ij} + Id_{ij}]$. Then we shall define the multiplication of these two neutrosophic fuzzy matrices as

$AB = [\max\min(a_{ij}, c_{ji}) + I\max\min(b_{ij}, d_{ji})]$. It can be defined in the following way:

If the above mentioned neutrosophic fuzzy matrices are considered then we can define the product of the above matrices as

A B = $[D_{ij}]$, where

$D_{11} = [\max\min\{(a_{1,}c_1), (a_{2,}c_3)\} + I\max\min\{(b_1, d_1), (b_2, d_3)\}]$

$D_{12} = [\max\min\{(a_{1,}c_2), (a_{2,}c_4)\} + I\max\min\{(b_1, d_2), (b_2, d_4)\}]$

$D_{21} = [\max\min\{(a_{3,}c_1), (a_{4,}c_3)\} + I\max\min\{(b_3, d_1), (b_4, d_3)\}]$

$D_{22} = [\max\min\{(a_{3,}c_2), (a_{4,}c_4)\} + I\max\min\{(b_3, d_2), (b_4, d_4)\}]$

It is important to mention here that if the multiplication of two neutrosophic fuzzy matrices is defined in the above way then the following properties can be observed to hold:

Properties

(i) $AB \neq BA$

(ii) A(B+C)=AB+AC

2.4.1 Numerical Example

Let us consider three neutrosophic fuzzy matrices as

A=$\begin{pmatrix} 0.1 + I\,0.3 & 0.4 + I\,0.1 \\ 0.2 + I\,0.4 & 0.1 + I\,0.7 \end{pmatrix}$

B=$\begin{pmatrix} 0.2 + I\,0.3 & 0.5 + I\,0.4 \\ 0.3 + I\,0.8 & 0.9 + I\,0.1 \end{pmatrix}$

C= $\begin{pmatrix} 0.4 + I\,0.3 & 0.4 + I\,0.3 \\ 0.2 + I\,0.7 & 0.2 + I\,0.4 \end{pmatrix}$

B+C = $\begin{pmatrix} 0.4 + I\,0.6 & 0.5 + I\,0.4 \\ 0.6 + I\,0.8 & 0.9 + I\,0.2 \end{pmatrix}$

A (B +C)= $\begin{pmatrix} 0.1 + I\,0.3 & 0.4 + I\,0.1 \\ 0.2 + I\,0.4 & 0.1 + I\,0.7 \end{pmatrix} \begin{pmatrix} 0.4 + I\,0.6 & 0.5 + I\,0.4 \\ 0.6 + I\,0.8 & 0.9 + I\,0.2 \end{pmatrix}$

Let us take

A (B +C) = $\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, where

A_{11} =max{min(0.1, 0.4), min(0.4,0.6)}+I max{min(0.3, 0.6), min(0.1, 0.8)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

A_{12} = max{min(0.1, 0.5), min(0.4,0.9)}+I max{min(0.3, 0.4), min(0.1, 0.2)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

A_{21} = max{min(0.2, 0.4), min(0.1,0.6)}+I max{min(0.4, 0.6), min(0.7, 0.8)}

= max(0.2, 0.1) + I max (0.4, 0.7)

= 0.2 + I 0.7

A_{22} = max{min(0.2, 0.5), min(0.1,0.9)}+I max{min(0.4, 0.4), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.4, 0.2)

= 0.2 + I 0.4

Therefore we have

A (B + C)= $\begin{pmatrix} 0.4 + \mathrm{I}\ 0.3 & 0.4 + \mathrm{I}\ 0.3 \\ 0.2 + \mathrm{I}\ 0.7 & 0.2 + \mathrm{I}\ 0.4 \end{pmatrix}$

Now we shall see what happens to AB+BC

Then let us calculate AB

A B = $\begin{pmatrix} 0.1 + \mathrm{I}\, 0.3 & 0.4 + \mathrm{I}\, 0.1 \\ 0.2 + \mathrm{I}\, 0.4 & 0.1 + \mathrm{I}\, 0.7 \end{pmatrix} \begin{pmatrix} 0.2 + \mathrm{I}\, 0.3 & 0.5 + \mathrm{I}\, 0.4 \\ 0.3 + \mathrm{I}\, 0.8 & 0.9 + \mathrm{I}\, 0.1 \end{pmatrix}$

Let is now consider

A B= $\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$, where

C_{11} = max{min(0.1, 0.2), min(0.4,0.3)}+I max{min(0.3, 0.3), min(0.1, 0.8)}

= max(0.1, 0.3) + I max (0.3, 0.1)

= 0.3 + I 0.3

C_{12} = max{min(0.1, 0.5), min(0.4,0.9)}+I max{min(0.3, 0.4), min(0.1, 0.1)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

C_{21} = max{min(0.2, 0.2), min(0.1,0.3)}+I max{min(0.4, 0.3), min(0.7, 0.8)}

= max(0.2, 0.1) + I max (0.3, 0.7)

= 0.2 + I 0.7

C_{22} = max{min(0.2, 0.5), min(0.1,0.9)}+I max{min(0.4, 0.4), min(0.7, 0.1)}

= max(0.2, 0.1) + I max (0.4, 0.1)

= 0.2 + I 0.4

Let us consider A C= $\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$, where

E_{11} = max{min(0.1, 0.4), min(0.4,0.6)}+I max{min(0.3, 0.6), min(0.1, 0.2)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

E_{12} = max{min(0.1, 0.5), min(0.4,0.3)}+I max{min(0.3, 0.3), min(0.1, 0.2)}

= max(0.1, 0.3) + I max (0.3, 0.1)

= 0.3 + I 0.3

E_{21} = max{min(0.2, 0.4), min(0.1,0.6)}+I max{min(0.4, 0.6), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.4, 0.2)

= 0.2 + I 0.2

E_{22} = max{min(0.2, 0.5), min(0.1,0.3)}+I max{min(0.4, 0.3), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.3, 0.2)

= 0.2 + I 0.3

Thus we have

$C_{11} + E_{11}$= (0.3 + I 0.3) + (0.4 + I 0.3)

= 0.4 + I 0.3

$C_{12} + E_{12}$= (0.4+ I 0.3) + (0.3 + I 0.3)

= 0.4 + I 0.3

$C_{21} + E_{21}$= (0.2+ I 0.7) + (0.2 + I 0.2)

= 0.2+ I 0.7

$C_{22} + E_{22}$= (0.2+ I 0.4) + (0.2 + I 0.3)

$=0.2+ I\ 0.4$

Thus, we get, A B + A C= $\begin{pmatrix} 0.4+I\ 0.3 & 0.3+I\ 0.3 \\ 0.2+I\ 0.7 & 0.2+I\ 0.4 \end{pmatrix}$

From the above results, it can be established that

A (B+C) = AB + AC

4. Conclusions

According the newly defined addition and multiplication operation of neutrosophic fuzzy matrices, it can be seen that some of the properties of arithmetic operation of these matrices are analogous to the classical matrices. Further some future works are necessary to deal with some more properties and operations of such kind of matrices.

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