Manuscript

A Reasoning Method in Conditional Evidential Networks based on Dezert-Smarandache Model

Abstract

Aiming to solving the problem that the evidence information based on Dezert-Smarandache (DSm) model can not be fused effectively in Conditional Evidential Network based on Smets/DS model (ENC), a reasoning method in Conditional Evidential Network based on DSm model is proposed. First, the conditional reasoning formular in Conditional Evidential Network based on DSm model is proved and the reasoning algorithm is proposed. Then, the hyper-power set of overlapping hypotheses is refined to a new power set of exhaustive and exclusive hypotheses, and the conditional belief functions of hyper-power set are obtained by using the Generalized Bayesian Theorem (GBT) and the Disjunctive Rule of Combination (DRC) in the new power set. Thirdly, the algorithm procedure in Conditional Evidential Network based on Smets/DS model or DSm model is given. Finally, through the air target situation assessment examples, the effectiveness of the proposed method is verified.

Keywords: Conditional Evidential Network; Dezert-Smarandache model; Information fusion; Multi-source haterogeneous information; Air target situation assessment

1. Introduction

In the current research fields of information fusion, multi-source fusion problems in the same discernment frame have been widely studied. Most research focus on how to carry on effective fusion for highly confict evidence[1-3]. However, due to the factors that information environment becomes more and more complex and technology of diversiform sensors develops rapidly, multi-source information fusion only in the same discernment frame can not meet the requirments of intelligent information and dicision systems and it also can not provide favorable support to situation assessment of information fusion in high levels. Therefore, how to solve the evidence information fusion problems in different discernment frames of multi heterogeneous sensors, is gradually paid attention of scholars [4-12].

For solving the uncertain evidence fusion problems in different frames, Shenoy proposed Value Network (VN) theory[13,14]. VN transforms a prior knowledge of complex questions to the hierarchical network structure which consist of multiple variable nodes and relation nodes. Each variable node represents belief measures of the focal elements in each discernment frame, and relation nodes represent joint belief functions of relevant focal elements in different discernment frames. Evidence information in different discernment frames is transformed to the evidence in the same discernment frame by marginalization and extension. However, knowledge representation and reasoning by joint belief functions causes large storage space and multiplications. Aiming to solve this problem, Xu and Smets[4,5] proposed ENC which replaces the joint belief functions by conditional belief functions. In ENCs, any computations involving two connected variables X and Y are processed on the space Θ_X or Θ_Y, while in the network with joint beliefs, such computations are always done on the product space $\Theta_X \times \Theta_Y$. Thus the computations in an ENC needs fewer set-compatisons and multiplications than networks with joint belief functions.

ENC is based on Smets model or DS model, which requires power sets of exhaustive and exclusive hypotheses. However, for a wide class of fusion problems the intrinsic nature of hypotheses can be only vague and imprecise in such a way that precise refinement is just impossible to obtain in reality so that the exclusive elements θ_i cannot be properly identified and precisely separated. For resolving this problem, DSm model[15] proposed by French scientist Dr. Jean Dezert and American mathematician Florentin considers Θ only as a frame of exhaustive elements $\theta_i, i = 1,\cdots,n$ which can potentially overlap and a hybrid DSm model is also constructed when the integrity constraints are explicitly and formally introduced into the free DSm mode. However, ENC is not fit for fusion problems of haterogeneous evidence information in DSm model. Aiming at realizing effective information fusion and conditional reasoning in DSm model, a reasoning method in Conditional Evidential Network based on DSm model is proposed.

In section 2, belief function theory, ENC, DSm model and DSm rules are introduced briefly. In section 3, firstly, the conditional reasoning formular in Conditional Evidential Network based on DSm model is proved and the reasoning algorithm is proposed; secondly the problem that the evidence information in DSm model can not be fused effectively in ENC is proved; thirdly, the method of calculating conditional belief functions of focal elements in DSm model is given by GBT and DRC through the refinement of the the hyper-power set in DSm model to the power set with exhaustive and exclusive hypotheses. In section 4, the algorithm procedure of the reasoning method in Conditional Evidential Network based on Smets/DS model or DSm model is given. In section 5, examples of the use of reasoning method in Conditional Evidential Network based on DSm model for air target situation assessment are presented.

2. Basic theory

2.1. belief function theory

Let Θ be a finite nonempty set called the discernment frame[16,17]. The mapping $bel: 2^{\Theta} \to [0,1]$ is an belief function if and only if there exists a basic belief assignment (bba) $m: 2^{\Theta} \to [0,1]$ such that:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

$$bel(A) = \sum_{B \subseteq A, B \neq \varnothing} m(B), bel(\varnothing) = 0 \tag{2}$$

Those subsets A such that $m(A) > 0$ are called the focal elements. The value $bel(A)$ quantifies the strength of the belief that the event A occurs. The value $m(A)$ represents the part of belief that supports the fact that A occurs and can't support any more specific event.

Given a belief function or bba, a plausibility function $pl: 2^{\Theta} \to [0,1]$ can be defined as follows:

$$pl(A) = bel(\Theta) - bel(\overline{A}) = \sum_{B \cap A \neq \varnothing} m(B) \text{ , and } pl(\varnothing) = 0 \tag{3}$$

Suppose bel quantifies our belief about the discernment frame Θ and we learn that $\overline{A} \subseteq \Theta$ is false. The resulting conditional belief function $bel(.|A)$ is obtained through the unnormalized rule of conditioning:

$$m(B|A) = \begin{cases} \sum_{X \subseteq \overline{A}} m(B \cup X) & if\ B \subseteq A \subseteq \Theta \\ 0 & otherwise \end{cases} \tag{4}$$

$$bel(B|A) = bel(B \cup \overline{A}) - bel(\overline{A}), \forall B \subseteq \Theta \tag{5}$$

$$pl(B|A) = pl(A \cap B), \forall B \subseteq \Theta \tag{6}$$

Belief function theory[18] is based on the same discernment frame, but in the reality, complex questions are often based on multi variables in different frames. For dealing with fusion problems with multi variables in different frames, the product spaces of the frames of the variables they include are defined as follow:

Let $\mathrm{U} = \{X, Y, Z, \cdots\}$ be a finite set of variables. $\Theta_X = \{x_1, \cdots, x_n\}$ is a frame of variable X , $\Theta_Y = \{y_1, \cdots, y_n\}$ is a frame of variable Y accordingly. The product space of the frames $\Theta_X \times \Theta_Y$ of the variables X and Y is defined: $\Theta_X \times \Theta_Y = \{(x_i, y_i) : x_i \in \Theta_X, y_i \in \Theta_Y\}$

For studying the fusion problem in product space, concepts of projection, extension and marginalization[4,5] are introduced:

Projection of configurations simply means dropping the extra coordinates. If X and Y are sets of variables, $Y \subseteq X$, and x_i is a configuration of Θ_X , then let $x_i^{\downarrow Y}$ denote the projection of x_i on Θ_Y . Then $x^{\downarrow Y}$ is a configuration of Θ_Y . If x is a nonempty subset of Θ_X , then the projection of x on Y , denoted by $x^{\downarrow Y}$, is obtained by $x^{\downarrow Y} = \{x_i^{\downarrow Y} | x_i \in x\}$. If y is a subset of Θ_Y , then the cylindrical extension. of y to X , denoted by $y^{\uparrow X}$, is $y \times \Theta_{X-Y}$

Then if X and Y are sets of variables, Θ_X is a frame of variable X , Θ_Y is a frame of variable Y accordingly. Suppose m_X is a bba on X and m_Y is a bba on Y , the conjunctive combination of m_X and m_Y is defined by

$$m_X \oplus m_Y = m_X^{\uparrow X \times Y} \oplus m_Y^{\uparrow X \times Y} \tag{7}$$

Suppose m is a bba on B and $A \subseteq B \subseteq U, A \neq \varnothing$. The marginal of m for A , denoted by $m^{\downarrow A}$, is the bba on A defined by

$$m^{\downarrow A}(a) = \sum_{b \subseteq \Theta_B, B^{\downarrow A} = a} m(b) \qquad for\ all \quad a \subseteq \Theta_A \tag{8}$$

Ballooning extension

Let X and Y be two disjoint subsets of U , and let $bel_X(.|y_i)$ be a conditional belief function defined on Θ_X for $y_i \in \Theta_Y$. The ballooning extension of the conditional belief function, denoted $m^{X \uparrow XY}$, is the belief function defined on $\Theta_X \times \Theta_Y$ whose bba satisfies[4]:

$$m^{X\uparrow X\times Y}(A)=\begin{cases} m_X(x\mid y_j) & if\ A=(x,y_j)\bigcup(X,\overline{y_j}) \\ 0 & \text{otherwise}\end{cases} \tag{9}$$

2.2. *ENC*

2.2.1. graph structure of ENC

ENC[4] is a directed graph with conditional belief functions for knowledge representation and reasoning, as shown in Figure 1. In ENC, each node represents a variable in the domain of knowledge, and each edge represents a conditional relation between the two nodes it connects. For example, nodes *X*,*Y* and *Z* mean that the knowledge domain is $\mathrm{U}=\{X,Y,Z\}$, and the edges(*X*,*Z*) and (*Y*,*Z*) mean that we have $\{bel_Z(.\mid x_i):x_i\in\Theta_X\}$ and $\{bel_Z(.\mid y_i):y_i\in\Theta_Y\}$

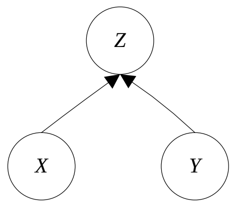

Fig. 1. An example of ENC

2.2.2. The generalized bayesian theorem (GBT)[5]

Given $bel_X(x\mid y_i)=1,\forall y_i\in y$ and $\forall y_i\in\Theta_y,\forall x_i\in\Theta_x$, then

$$bel_Y(y\mid x)=K\bullet(\prod_{y_i\in \overline{y}} bel_X(\overline{x}\mid y_i)-\prod_{y_i\in y} bel_X(\overline{x}\mid y_i)) \tag{10}$$

$$pl_Y(y\mid x)=K\bullet(1-\prod_{y_i\in y}(1-pl_X(x\mid y_i))) \tag{11}$$

$$K^{-1}=1-\prod_{y_i\in\Theta_y} bel_X(\overline{x}\mid y_i)=1-\prod_{y_i\in\Theta_y}(1-pl_X(x\mid y_i)) \tag{12}$$

2.2.3. The disjunctive rule of combination (DRC)[5]

Given $bel_X(x\mid y_i)=1,\forall y_i\in y$ and $\forall y_i\in\Theta_y,\forall x_i\in\Theta_x$, then

$$bel_X(x\mid y)=\prod_{y_i\in y} bel_X(x\mid y_i) \tag{13}$$

$$pl_X(x\mid y)=1-\prod_{y_i\in y}(1-pl_X(x\mid y_i)) \tag{14}$$

$$m_X(x\mid y)=\sum_{(\bigcup_{i:y_i\in y} x_i)=x}\ \prod_{i:y_i\in y} m_X(x_i\mid y_i) \tag{15}$$

2.2.4. Propagating beliefs in an ENC[6]

Inference in ENC in the condition of non-empty aprior mass assignments of evidence is based on the following formular[19].

Assume $x_i\in\Theta_X$, $y_j\in\Theta_Y$, then $\forall x\in\Theta_X$

$$\begin{aligned} m(x) &= \sum_{y\subseteq\Theta_Y} m_X(x\mid y)m_Y(y) \\ bel(x) &= \sum_{y\subseteq\Theta_Y} bel_X(x\mid y)m_Y(y) \\ pl(x) &= \sum_{y\subseteq\Theta_Y} pl_X(x\mid y)m_Y(y) \end{aligned} \tag{16}$$

Consider ENC has two nodes X and Y defined on Θ_X and Θ_Y, respectively. Suppose that there exists some a priori information over Θ_X given by mass function m_0^X and some a prior information over Θ_Y given by m_0^Y. We assume that we also have conditional mass functions $\{m_Y(.|x_i): x_i \in \Theta_X\}$.

For each node in the network, the marginal is computed by combining all the messages received from its neighbours and its own prior information. if we want to compute m^X of the node X, which is the parent of Y, we combin its prior mass function with the message coming from Y,

$$m^X = m_0^X \oplus m_{Y\to X} \tag{17}$$

where $m_{Y\to X}$ is a mass function on X representing the message coming from Y, and is computed by

$$\forall x \subseteq \Theta_X, m_{Y\to X}(x) = \sum_{y\subseteq\Theta_Y} m_0^Y(y) m_X(x|y) \tag{18}$$

$m_X(x|y)$ is given by GBT.

On the other hand, if we want to compute m^Y of the node Y, which is the child of X, we combin its prior mass function with the message coming from X,

$$m^Y = m_0^Y \oplus m_{X\to Y} \tag{19}$$

where $m_{X\to Y}$ is a mass function on Y representing the message coming from X, and is computed by

$$\forall y \subseteq \Theta_Y, m_{X\to Y}(y) = \sum_{x\subseteq\Theta_X} m_0^X(x) m_Y(y|x) \tag{20}$$

$m_Y(y|x)$ is given by DRC.

2.3. DSm model and DSm rules

Consider a finite frame $\Theta = \{\theta_1, \cdots, \theta_n\}$ of the fusion problem under consideration. We abandon Shafer's model 2^Θ by assuming here that the fuzzy/vague/relative nature of elements of Θ can be non-exclusive. Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$, if all multiple focal element is not empty, D^Θ is a free-DSm model[15] as shown in Fig 2. In Fig 2, "i" denotes the part of the diagram which belongs to θ_i only, "ij" denotes the part of the diagram which belongs to θ_i and θ_j only, "ijk" denotes the part of the diagram which belongs to θ_i and θ_j and θ_k only, etc.

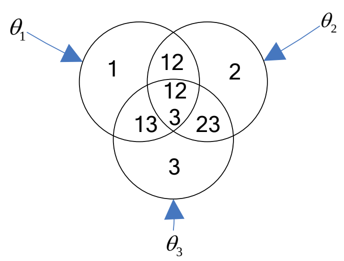

Fig. 2 Venn Diagram for free-DSm model

A hybrid DSm model[15] is defined from the free-DSm model by introducing some integrity constraints on some elements. Indeed, in some fusion problems, some elements θ_i and θ_j of Θ can be fully discernable because they are truly exclusive while other elements cannot be refined into finer exclusive elements. Hybrid DSm models are not only fit for this situation. The other situations is introduced in [15]. Suppose the frame of the fusion problem as show in Fig.2. Let $\theta_1 \cap \theta_3 = \varnothing$, the hybrid DSm model D^{Θ} is shown in Fig 3.

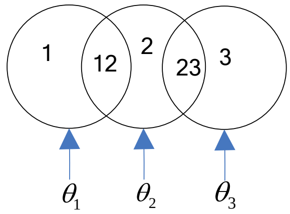

Fig. 3 Venn Diagram for hybrid DSm model

The DSm rules of combination based on the chosen DSm model for $k \geq 2$ independent sources of information is defined for all $A \in D^{\Theta}$, D^{Θ} is the hyper-power set based on DSm model[15]

$$m_{u(\Theta)}(A) = \phi(A)[\sum_{\substack{X_1, X_2, \cdots, X_k \in D^{\Theta} \\ (X_1 \cap X_2 \cap \cdots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i) + \sum_{\substack{X_1, X_2, \cdots, X_k \in \phi \\ [u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k) = A] \vee [(u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k) \in \phi) \wedge (A = I_t)]}} \prod_{i=1}^{k} m_i(X_i)$$
$$+ \sum_{\substack{X_1, X_2, \cdots, X_k \in D^{\Theta} \\ (X_1 \cup X_2 \cup \cdots \cup X_k) = A \\ (X_1 \cap X_2 \cap \cdots \cap X_k) \in \phi}} \prod_{i=1}^{k} m_i(X_i)] \tag{21}$$

where $\phi(A)$ is the characteristic non-emptiness function of a set A, i.e. $\phi(A) = 1$ if $A \notin \varnothing$ and $\phi(A) = 0$, otherwise. For in this short space, we omit the detailed introduction of DSm rules (21), please refer to reference [15] if necessary.

3. A reasoning method in Conditional Evidential Network based on DSm model

3.1. The reasoning formular in Conditional Evidential Network based on DSm model

Theorem 1 Assume D^X and D^Y is the hyper-power set in DSm model and $x \in D^X$, $y_i \in D^Y$, then $\forall x \in D^X$

$$m_{D^X}(x) = \sum_{y_i \in D^Y} m_{D^Y}(x \mid y_i) m_{D^Y}(y_i) \tag{22}$$

$$bel_{D^X}(x) = \sum_{y_i \in G^Y} bel_{D^X}(x \mid y_i) m_{D^Y}(y_i) \tag{23}$$

$$pl_{D^X}(x) = \sum_{y_i \in D^Y} pl_{D^X}(x \mid y_i) m_{D^Y}(y_i) \tag{24}$$

First we prove $m_{D^X}(x) = \sum_{y_i \in D^Y} m_{D^Y}(x \mid y_i) m_{D^Y}(y_i)$.

Proof: As the focal elements of evidence in DSm model can be refined to the union of the focal elements which is exclsive in DS model, denoted by $x = x_i' \cup \cdots \cup x_j' : x \in D^X, x_i' \cup \cdots \cup x_j' \in 2^X$, $y_i = y_i' \cup \cdots \cup y_j' : y_i \in D^Y\ y_i \cup \cdots \cup y_j \in' {}^{Y}$. So the mass assignments functions $m_{D^X}(x \mid y_i)$ in DSm model can be transformed to $\{m_{2^X}(x_i' \cup \cdots \cup x_j' \mid y_i' \cup \cdots \cup y_j') : x_i' \cup \cdots \cup x_j' \in 2^X, y_i' \cup \cdots \cup y_j' \in 2^Y\}$ and the prior mass assignments funtions of evidence $m_{D^Y}(y_i)$ can be transformed to $\forall y_i \in D^Y, m_{D^Y}(y_i) = m_{2^Y}(y_i' \cup \cdots \cup y_j')$.

As the a prior mass assignments funtions of evidence $m_{D^Y}(y_i)$ in DSm model is complete, through the equivalent transformation mass assignments funtions of evidence $m_{2^Y}(y_i'\cup\cdots\cup y_j')$ is also complete, denoted by $\sum_{y_i'\cup\cdots\cup y_j'=y_i} m_{2^Y}(y_i'\cup\cdots\cup y_j')=1$. So $\forall y_i'\cup\cdots\cup y_j'=y_i, y_i'\cup\cdots\cup y_j'$ can be the focal elements in power set in DS model of the evidence and $\forall y_i'\cup\cdots\cup y_j'=y_i, m_{2^Y}(y_i'\cup\cdots\cup y_j')$ can be the mass assignments funtion sin power set in DS model of the evidence.

Then, from formular (16), we can obtain

$$m_{D^X}(x_i'\cup\cdots\cup x_j')=\sum_{y_i'\cup\cdots\cup y_j'\in D^Y} m_X(x_i'\cup\cdots\cup x_j' \mid y_i'\cup\cdots\cup y_j')m_Y(y_i'\cup\cdots\cup y_j')$$

So $m_{D^X}(x)=\sum_{y_i\in D^Y} m_{D^Y}(x\mid y_i)m_{D^Y}(y_i)$. QED.

As the proof of Equation (23) and (24) are similar with Equation (22), we omit it.

3.2. The reasoning algorithm in Conditional Evidential Network based on DSm model

Based on the reasoning formular proved in section 3.1, the reasoning algorithm in Conditional Evidential Network based on DSm model is given.

Consider the evidential network based on DSm model has two nodes X and Y defined in hyper-power sets D^X and D^Y, respectively. Suppose that there exists some a priori information over D^X given by mass function m_0^X and some a prior information over D_Y given by m_0^Y. We assume that we have obtained conditional mass assignments functions of focal elements in hyper-power set of DSm model, denoted by $\{m_{D^Y}(y\mid x): x\in D^X\}$ and $\{m_{D^X}(x\mid y): y\in D^Y\}$.

If we want to compute m^X of the node X, which is the parent of Y, we combin its prior mass function with the message coming from Y,

$$m^X = m_0^X \underset{\mathrm{DSmT}}{\oplus} m_{Y\to X} \tag{25}$$

Where $\underset{\mathrm{DSmT}}{\oplus}$ represents DSm rules and $m_{Y\to X}$ is a mass function on X representing the message coming from Y, and is computed by

$$\forall x\in D^X, m_{Y\to X}(x)=\sum_{y\in D^Y} m_0^Y(y)m_{D^X}(x\mid y) \tag{26}$$

If we want to compute m^Y of the node Y, which is the child of X, we combin its prior mass function with the message coming from X,

$$m^Y = m_0^Y \underset{\mathrm{DSmT}}{\oplus} m_{X\to Y} \tag{27}$$

where $\underset{\mathrm{DSmT}}{\oplus}$ represents DSm rules and $m_{X\to Y}$ is a mass function on Y representing the message coming from X, and is computed by

$$\forall y\in D^Y, m_{X\to Y}(y)=\sum_{x\in D^X} m_0^X(x)m_{D^Y}(y\mid x) \tag{28}$$

3.3. The paradox of formulas of ENC directly applied to the evidence based on DSm model

Consider a frame $\Theta=\{\theta_1,\theta_2\}$ of the fusion problem under consideration, and its hyper-power set of free DSm model is $D^\Theta=\{\theta_1,\theta_2,\theta_1\cap\theta_2\}$ and its power set of DS model is $2^\Theta=\{\theta_1',\theta_2',\theta_1\cap\theta_2\}$, where $\theta_1=\theta_1'\cup(\theta_1\cap\theta_2),\theta_2=\theta_2'\cup(\theta_1\cap\theta_2)$. Suppose A is an event in another frame. Conditional plausibility functions of the focal elements in the power set 2^Θ is known, denoted by $pl(A|\theta_1'),pl(A|\theta_2'),pl(A|\theta_1\cap\theta_2)$. Suppose the mass assignments functions of evidence in D^Θ is $m(\theta_1),m(\theta_2),m(\theta_1\cap\theta_2)$.

First, we apply ENC formulas directly to the evidence based on DSm model for calculating plausibility of A , denoted by $pl_{\mathrm{DS}}(A)$.

$$pl_{\mathrm{DS}}(A)=pl(A|\theta_1')\bullet m(\theta_1)+pl(A|\theta_2')\bullet m(\theta_2)+pl(A|\theta_1\cap\theta_2)\bullet m(\theta_1\cap\theta_2) \tag{29}$$

As D^Θ can be refined to $\{\theta_1',\theta_2',\theta_1\cap\theta_2\}$ and $\theta_1=\theta_1'\cup(\theta_1\cap\theta_2),\theta_2=\theta_2'\cup(\theta_1\cap\theta_2)$.

Then apply reasoning formulas (24) proved in 3.1 to the hyper-power set of D^Θ for calculating plausibility of A , denoted by $pl_{\mathrm{DSm}}(A)$.

$$\begin{aligned}pl_{\mathrm{DSm}}(A)&=pl(A|\theta_1)\bullet m(\theta_1)+pl(A|\theta_2)\bullet m(\theta_2)+pl(A|\theta_1\cap\theta_2)\bullet m(\theta_1\cap\theta_2)\\&=pl(A|\theta_1'\cup(\theta_1\cap\theta_2))\bullet m(\theta_1)+pl(A|\theta_2'\cup(\theta_1\cap\theta_2))\bullet m(\theta_2)+pl(A|\theta_1\cap\theta_2)\bullet m(\theta_1\cap\theta_2)\end{aligned} \tag{30}$$

Compare equation (29)and equation(30).

Assume $pl_{\mathrm{DS}}(A)=pl_{DSm}(A)$, then $pl(A|\theta_1'\cup(\theta_1\cap\theta_2))=pl(A|\theta_1')$ and $pl(A|\theta_2'\cup(\theta_1\cap\theta_2))=pl(A|\theta_2')$.

However, $pl(A|\theta_1'\cup(\theta_1\cap\theta_2))=pl_{A\times\Theta}(A\cap(\theta_1'\cup(\theta_1\cap\theta_2)))>pl_{A\times\Theta}(A\cap\theta_1')=pl(A|\theta_1')$.

Similarly, $pl(A|\theta_2'\cup(\theta_1\cap\theta_2))=pl_{A\times\Theta}(A\cap(\theta_2'\cup(\theta_1\cap\theta_2)))>pl_{A\times\Theta}(A\cap\theta_2')=pl(A|\theta_2')$.

So the assumption of $pl_{\mathrm{DS}}(A)=pl_{DSm}(A)$ is not tenable. Moreover, $pl_{\mathrm{DSm}}(A)>pl_{\mathrm{DS}}(A)$.

3.4. The refinement of hyper- power sets in DSm model

Based on the section 3.3, we can find that if there is the evidence based on DSm model in evidential networks, the original reasoning method ENC can not succeed in obtaining the correct reasoning results. So, the reasoning formular(22-24) in the section 3.1 and the reasoning algorithm in 3.2 is necessary to the reasoning of evidence based on DSm model. From the formular(22-24), we can find that how to calculate the conditional belief functions of non-exclusive focal elements of hyper-power sets based on DSm model is the key to the whole algorithm. However, the refinement of hyper-power sets of evidence based on DSm model, which can transfer the non-exclusive focal elements to the union of the exclusive focal elements in power sets, is the precondition of calculating the conditional belief functions of non-exclusive focal elements.

In this section, the process procedure of the refinement of hyper-power sets of evidence based on DSm model is given as follows:

1) Find the minimum focal element in the hyper-power set of the evidence, that is, find the multiple focal element which has intersection of the the maximum number of focal elements, i.e., let the multiple focal element which has intersection of *k* number of focal elements be $\underbrace{\theta_i\cap\cdots\cap\theta_j}_{k}$, then find the multiple focal element which has maximum number *k*.

2) Find next higher–level focal element of the minimum focal element, denoted by $\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}$. The refinement of this focal element is $\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1} = (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$, where $(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}')$ is the exclusive focal element in power set.

3) Find next higher–level focal element of each focal element $\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}$ in 2), denoted by $\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-2}$. The refinement of this focal element is $\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-2} = (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-2}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$, where $(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-2}'), (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}')$ is the exclusive focal element in power set.

4) Find each next higher–level focal element of the former level of focal element in order as the step 3), untill the refinement of the singleton focal elements is performed as $\theta_i = \theta_i' \cup (\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$.

Finally, the refinement of hyper-power set to the power-set is $\{\theta_i', (\underbrace{\theta_i \cap \theta_j}_{2}'), \cdots, (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}'), (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})\}$. So, the mass assignments funciton of the evidence $m(\theta_i), m(\theta_i \cap \theta_j), \cdots, m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}), m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$ based on DSm model can be transformed to $m(\theta_i' \cup (\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$, $m((\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})), \cdots, m((\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})), m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$. For any event x_i, the conditional assignments funciton of each focal element $m_X(x_i \mid \theta_i), m_X(x_i \mid \underbrace{\theta_i \cap \theta_j}_{2}), \cdots, m_X(x_i \mid \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}), m_X(x_i \mid \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$ based on DSm model can be transformed to $m_X(x_i \mid \theta_i' \cup (\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})), m_X(x_i \mid (\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})), \cdots,$ $m_X(x_i \mid (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})), m_X(x_i \mid \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$.

3.5. The method of calculating the conditional assignments functions of focal elements in hyper- power sets

Assume that the conditional belief functions of focal elements in power sets is known from ENC. For any event $x, x = \cup x_i$, we have $m_X(x_i \mid \theta_i'), m_X(x_i \mid \underbrace{\theta_i \cap \theta_j}_{2}'), \cdots, m_X(x_i \mid \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}'), m\ (x_i \mid \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})$. In this section, the method of calculating the conditional assignments functions of focal elements in hyper-power sets $\{\theta_i, (\underbrace{\theta_i \cap \theta_j}_{2}), \cdots, (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}), (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})\}$ is given by GBT and DRC theory.

3.5.1 Calculating the forward conditional assignments functions of focal elements in hyper-power sets by DRC

Through the refinement of the hyper-power sets in the section 3.4 and DRC theory, the forward conditional assignments functions of focal elements in hyper-power sets can be calculated as follows:

$$m_X(x|\theta_i) = m_X(x|\theta_i' \cup (\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

$$= \sum_{\cup x_i = x} (m_X(x_i|\theta_i')\bullet m_X(x_i|\underbrace{\theta_i \cap \theta_j}_{2}')\bullet\cdots\bullet m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}')\bullet m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

$$m_X(x|\underbrace{\theta_i \cap \theta_j}_{2}) = m_X(x|(\underbrace{\theta_i \cap \theta_j}_{2}') \cup \cdots \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

$$= \sum_{\cup x_i = x} (m_X(x_i|\underbrace{\theta_i \cap \theta_j}_{2}')\bullet\cdots\bullet m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}')\bullet m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

$$\vdots$$

$$m_X(x|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}) = m_X(x|(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}') \cup (\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})) = \sum_{\cup x_i = x} (m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}')\bullet m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

$$m_X(x|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}) = \sum_{\cup x_i = x} (m_X(x_i|\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}))$$

3.5.2 Calculating the backward conditional assignments functions of focal elements in hyper-power sets by GBT

Through the refinement of the hyper-power sets in the section 3.4, GBT and the relation between plausibility functions and assignments functions, the backward conditional assignments functions of focal elements in hyper-power sets can be calculated as follows:

1) Calculate the backward conditional plausibility functions of focal elements in power sets based on DS model by GBT.

$$pl_\Theta(\theta'|x) = K\bullet(1 - \prod_{\theta_i' \in \theta'} (1 - pl_X(x|\theta_i')))$$

$$K^{-1} = 1 - \prod_{\theta_i' \in D^\Theta} (1 - pl_X(x|\theta_i'))$$

2) Calculate the backward conditional plausibility functions of focal elements in hyper-power sets based on DSm model by the refinement and the relation between plausibility functions and assignments functions.

$$m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x) = bel(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x) = 1 - pl_{\Theta}_{\theta' = (D^\Theta - \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k})}(\theta'|x)$$

$$m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}|x) = bel(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

$$= 1 - pl_{\Theta}_{\theta' = (D^\Theta - \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1})}(\theta'|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

$$\vdots$$

$$m(\theta_i \cap \theta_j|x) = bel(\theta_i \cap \theta_j|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3} \subset \theta_i \cap \theta_j} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3}|x) - \cdots - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

$$= 1 - pl_{\Theta}_{\theta' = (D^\Theta - \theta_i \cap \theta_j)}(\theta'|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3} \subset \theta_i \cap \theta_j} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3}|x) - \cdots - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

$$m(\theta_i|x) = bel(\theta_i|x) - \sum_{\theta_i \subset \theta_i \cap \theta_j} m(\theta_i \cap \theta_j|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3} \subset \theta_i \cap \theta_j} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3}|x) - \cdots - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

$$= 1 - pl_{\Theta}_{\theta' = (D^\Theta - \theta_i)}(\theta'|x) - \sum_{\theta_i \subset \theta_i \cap \theta_j} m(\theta_i \cap \theta_j|x) - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3} \subset \theta_i \cap \theta_j} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{3}|x) - \cdots - \sum_{\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k} \subset \underbrace{\theta_i \cap \cdots \cap \theta_j}_{k-1}} m(\underbrace{\theta_i \cap \cdots \cap \theta_j}_{k}|x)$$

4. Algorithm procedure of the reasoning method in Conditional Evidential Network based on DSm model

According to the section 3, algorithm procedure of the reasoning method in Conditional Evidential Network based on DSm model is given as shown in fig.4.

1) First judge whether multi-source evidence fusion is needed in some node of evidential network. If there is muliti-source evidence, DSm rules is performed to obtain the fusion results of this node.

2) Then, judge which model the focal elements of the evidence are based on. If the focal elements of the evidence are not based on DSm model, the method of ENC is applied directly to get the reasoning results of the adjacent node of this node and return the step 1) till the reasoning of the adjacent node is not needed.

3) If the focal elements of the evidence are based on DSm model, the method of calculating the conditional assignments functions of focal elements in hyper-power sets in section 3.5 is performed, and then the reasoning algorithm in Conditional Evidential Network based on DSm model in section 3.2 is carried on to obatin the reasoning results of the adjacent node of this node. Next, return the step 1) till the reasoning of the adjacent node is not needed.

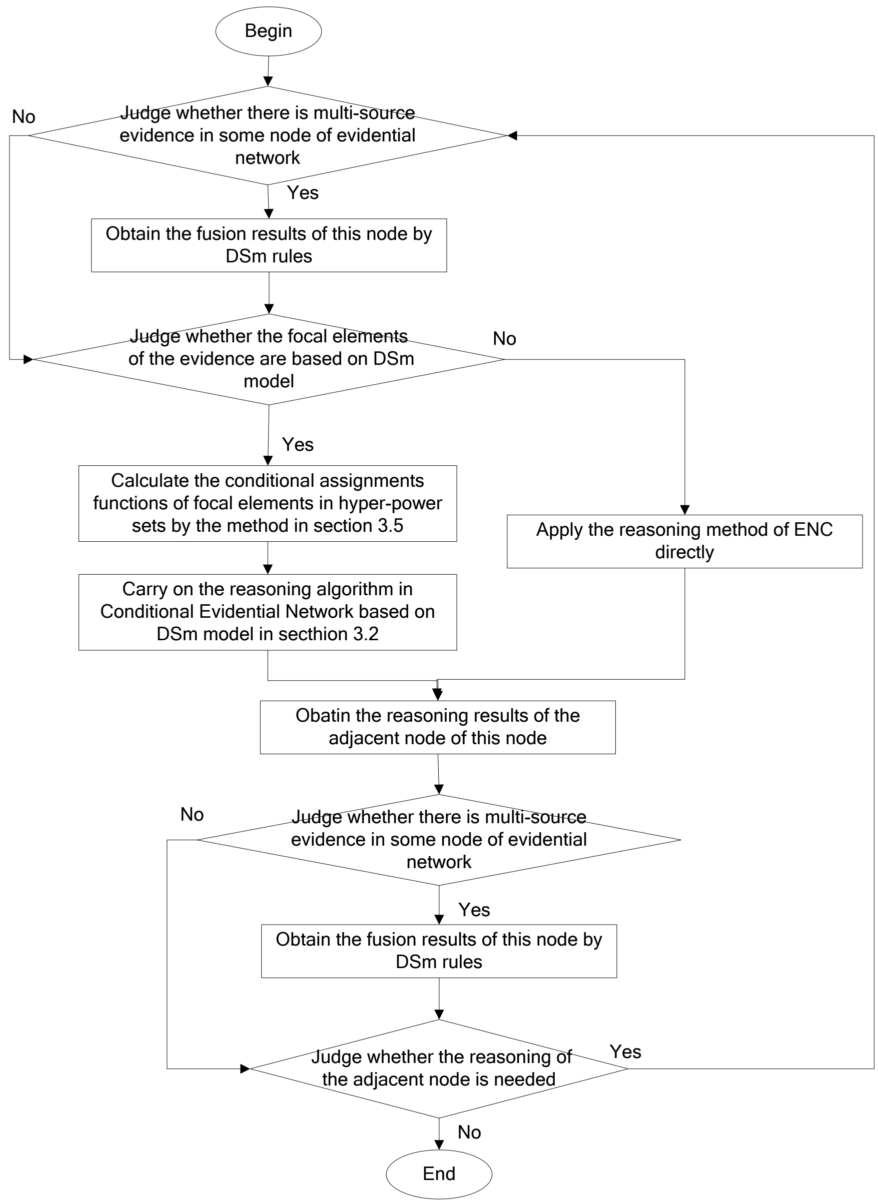

Fig. 4 Algorithm procedure of the reasoning method in Conditional Evidential Network based on DSm model

5. The air target situation assessment examples in DSm model

Through the research on the air target situation assessment, we find it's difficult to require the evidence of the types of air target to be exclusive as some types of air target have multi type's characters and the sources can not distinguish the type of air target which has good stealthy performance. So the source's evidence of types of air target is possible based on DSm model. In this section, we apply the reasoning method in Conditional Evidential Networks based on DSm model to the air target situation assessment. Through the air target situation assessment examples in different cases, the effectiveness of the proposed method is verified.

An evidential network model of air target situation assessment is shown in Fig.5. Suppose the exclusive discernment frame of 'air target situation level' is {1, 2, 3}, the exclusive discernment frame of 'types of air target' is {patrol figter, attack figter, bomb figter} and the exclusive discernment frame of 'distance of air target' is {far, middle, near}.

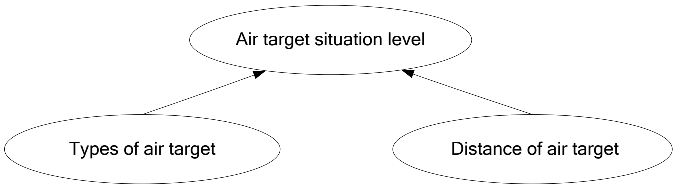

Fig.5 An evidential network model of air target situation assessment

5.1 Air target situation assessment example in the case of no other evidence in the network

Assume that the non-exclusive discernment frame of 'types of air target' is {patrol figter', attack figter, bomb figter'} in evidence, where $\text{attack figter} = \text{patrol figter}' \cap \text{bomb fighter}'$. It's difficult for the source to distinguish the types of figter at all times as the stealthy performance and the noise interference of the fighter or the parameter is not known for the new types of figters. From the expert knowledge, we genarate a prior conditional assignments functions based on DS model as show in tabel 1.

Table 1 A prior conditional assignments functions of 'types of air target' based on DS model

Types of air target / Situation level	patrol figter	attack figter	bomb figter
1	0.1	0.2	0.7
2	0.1	0.5	0.2
3	0.5	0.2	0.1
1,2	0.1	0.05	0
2,3	0.2	0.05	0
13	0	0	0
1,2,3	0	0	0

Suppose in a moment there is an evidence based on DSm model of 'types of the air target', denoted by $m(\text{patrol figter}') = 0.5, m(\text{attack fighter}) = 0.1, m(\text{bomb figter}') = 0.4$ and there is no other evidence in the network in this moment.

In order to simplify the calculation, we use the conditional plausibility functions for reasoning. First, transform the a prior conditional assignments functions based on DS model to the conditional plausibility functions as shown in tabel 2.

Table 2 The conditional plausibility functions of 'types of air target' based on DS model

Situation level \ Types of air target	patrol figter	attack figter	bomb figter
1	0.2	0.25	0.7
2	0.4	0.6	0.2
3	0.7	0.25	0.1

1) First, the reasoning method of ENC is applied directly to the evidence.

The reasoning results are obtained as follows:

$$pl(\text{situation level}=1)=0.5\times0.2+0.1\times0.25+0.4\times0.7=0.405$$
$$pl(\text{situation level}=2)=0.5\times0.4+0.1\times0.6+0.4\times0.2=0.34$$
$$pl(\text{situation level}=3)=0.5\times0.7+0.1\times0.25+0.4\times0.1=0.415$$

As $pl(\text{situation level}=3)>pl(\text{situation level}=1)>pl(\text{situation level}=2)$, the dicision can be maken as $(\text{situation level}=3)\succ(\text{situation level}=1)\succ(\text{situation level}=2)$.

2) Then, we obtain the dicision results by the method in this paper.

(1) Calculate conditional plausibility functions of the focal elements of the evidence in 'types of air target' based on DSm model as shown in tabel 3.

Table 3 The conditional plausibility functions of 'types of air target' based on DSm model

Situation level \ Types of air target	patrol figter'	attack figter	bomb figter'
1	0.4	0.25	0.775
2	0.76	0.6	0.68
3	0.775	0.25	0.325

(2) The reasoning results are obtained as follows:

$$pl(\text{situation level}=1)=0.5\times0.4+0.1\times0.25+0.4\times0.775=0.535$$
$$pl(\text{situation level}=2)=0.5\times0.76+0.1\times0.6+0.4\times0.68=0.712$$
$$pl(\text{situation level}=3)=0.5\times0.775+0.1\times0.25+0.4\times0.325=0.5425$$

(3) As $pl(\text{situation level}=2)>pl(\text{situation level}=3)>pl(\text{situation level}=2)$, the dicision can be maken as $(\text{situation level}=2)\succ(\text{situation level}=3)\succ(\text{situation level}=1)$.

Through the comparison of the decision results by the method in this paper against the results by ENC, we find the cause of the different results is that ENC ignores the focal elements of evidence are based on DSm model. However, this ignorance may result in serious concequences.

5.2 Air target situation assessment example in the case of dynatic discernment frame

Suppose the same evidential network and the same prior conditional plausibility functions of the types of air target as 5.1. Assume in a moment there is an evidence of 'types of the incoming air target', denoted by $m(\text{patrol figter}) = 0.5, m(\text{attack fighter}) = 0.1, m(\text{bomb figter'}) = 0.4$ where partrol figter is the exclusive focal element as the improvement of the sources or the other intelligence informaiton and the focal elements of this evidence are based on hybrid DSm model instead of free DSm model.

1) Calculate conditional plausibility functions of the focal elements of the evidence in 'types of air target' based on hybrid DSm model as shown in tabel 4.

Table 4 The conditional plausibility functions of 'types of air target' based on hybrid DSm model

Types of air target / Situation level	patrol figter	attack figter	bomb figter'
1	0.2	0.25	0.775
2	0.4	0.6	0.68
3	0.7	0.25	0.325

2) The reasoning results are obtained as follows:

$$pl(\text{situation level} = 1) = 0.5\times 0.2 + 0.1\times 0.25 + 0.4\times 0.775 = 0.435$$
$$pl(\text{situation level} = 2) = 0.5\times 0.4 + 0.1\times 0.6 + 0.4\times 0.68 = 0.532$$
$$pl(\text{situation level} = 3) = 0.5\times 0.7 + 0.1\times 0.25 + 0.4\times 0.325 = 0.505$$

Through the comparison of the reasoning results in this section against the results by the method in this paper in section 5.1, we find the plausibility belief functions of situation level changes obviously. Though situation level=2 still has the maximum plausibility belief functions in this section, comparing with section 5.1, it has reduces obviously and been close to situation level=3. The comparison of the results shows that if we ignore the dynatic changes of discernment frame of evidence, the wrong decision results may also be maken.

5.3 Air target situation assessment example in the case of multi-source evidence fusion in evidential networks

Assume the same evidential network and the same prior conditional belief functions of 'types of air target' as 5.1. In this example, a prior conditional assignments functions of 'distance of air target' are given as show in tabel 5, which is the exclusive discernment frame based on DS model.

Table 5 A prior conditional assignments functions of 'distance of air target'

Types of air target / Situation level	far	middle	near
1	0.1	0.3	0.6
2	0.2	0.5	0.3
3	0.4	0.2	0.1
1,2	0.1	0	0
2,3	0.2	0	0

Suppose in a moment there is an evidence of 'type of air target' based on DSm model as section 5.1, denoted by $m(\text{patrol figter'}) = 0.5, m(\text{attack fighter}) = 0.1, m(\text{bomb figter'}) = 0.4$, and an evidence of 'distance of air target' based on DS model, denoted by $m(\text{far}) = 0.2, m(\text{middle}) = 0.3, m(\text{near}) = 0.5$

1) In order to realize effective multi-source evidence fusion, we use the conditional assignments functions for reasoning. Calculate conditional assignments functions of the focal elements of the evidence in 'types of air target' based on DSm model by the method in section 3.5.1 as shown in tabel 6.

Table 6 Conditional assignments functions of 'types of air target' based on DSm model

Types of air target / Situation level	patrol figter'	attack figter	bomb figter'
1	0.02	0.2	0.14
2	0.05	0.5	0.1
3	0.1	0.2	0.02
1,2	0.155	0.05	0.435
2,3	0.45	0.05	0.105
1,3	0.12	0	0.16
1,2,3	0.105	0	0.04

2) As there is only one evidence based on DSm model in 'types of air target', the reasoning algorithm in Conditional Evidential Network based on DSm model in section 3.2 is performed to obtain the reasoning results from this node to 'air target situation level' as follows:

$$
\begin{aligned}
&m(\text{situation level} = 1) = 0.5\times 0.02 + 0.1\times 0.2 + 0.4\times 0.14 = 0.086\\
&m(\text{situation level} = 2) = 0.5\times 0.05 + 0.1\times 0.5 + 0.4\times 0.1 = 0.115\\
&m(\text{situation level} = 3) = 0.5\times 0.1 + 0.1\times 0.2 + 0.4\times 0.02 = 0.078\\
&m(\text{situation level} = 1,2) = 0.5\times 0.155 + 0.1\times 0.05 + 0.4\times 0.435 = 0.2565\\
&m(\text{situation level} = 2,3) = 0.5\times 0.45 + 0.1\times 0.05 + 0.4\times 0.105 = 0.272\\
&m(\text{situation level} = 1,3) = 0.5\times 0.12 + 0.1\times 0 + 0.4\times 0.16 = 0.124\\
&m(\text{situation level} = 1,2,3) = 0.5\times 0.105 + 0.1\times 0 + 0.4\times 0.04 = 0.0685
\end{aligned}
$$

3) As there is only one evidence based on DS model in 'distance of air target', the reasoning method of ENC is ap-

plied directly to obtain the reasoning results from this node to ‘air target situation level’ as follows:

$$m(\text{situation level}=1)=0.2\times 0.1+0.3\times 0.3+0.5\times 0.6=0.41$$
$$m(\text{situation level}=2)=0.2\times 0.2+0.3\times 0.5+0.5\times 0.3=0.34$$
$$m(\text{situation level}=3)=0.2\times 0.4+0.3\times 0.2+0.5\times 0.1=0.19$$
$$m(\text{situation level}=1,2)=0.2\times 0.1+0.3\times 0+0.5\times 0=0.02$$
$$m(\text{situation level}=2,3)=0.2\times 0.2+0.3\times 0+0.5\times 0=0.04$$

4) As there is multi-source evidence from the adjacent nodes in ‘air target situation level’, DSm rules is performed to obtain the fusion results of this node as follows:

$$m(\text{situation level}=1)=0.086\times 0.41+0.086\times 0.02+0.2565\times 0.41+0.124\times 0.41+0.124\times 0.02$$
$$+0.0685\times 0.41=0.22355$$
$$m(\text{situation level}=2)=0.115\times 0.34+0.115\times 0.02+0.115\times 0.04+0.2565\times 0.34+0.2565\times 0.04$$
$$+0.272\times 0.34+0.272\times 0.02+0.0685\times 0.34=0.26468$$
$$m(\text{situation level}=3)=0.078\times 0.19+0.078\times 0.04+0.272\times 0.19+0.124\times 0.19+0.124\times 0.04$$
$$+0.0685\times 0.19=0.111155$$
$$m(\text{situation level}=1,2)=0.086\times 0.34+0.115\times 0.41+0.2565\times 0.02+0.0685\times 0.02=0.08289$$
$$m(\text{situation level}=2,3)=0.115\times 0.19+0.078\times 0.34+0.272\times 0.04+0.0685\times 0.04=0.06199$$
$$m(\text{situation level}=1,3)=0.086\times 0.19+0.078\times 0.41=0.04832$$
$$m(\text{situation level}=1,2,3)=0.086\times 0.04+0.078\times 0.02+0.2565\times 0.19+0.272\times 0.41$$
$$+0.124\times 0.34=0.207415$$

5) Calculate plausibility belief functions from mass assignments functions of ‘air target situation level’ as follows:

$pl(\text{situation level}=1)=0.562175$, $pl(\text{situation level}=2)=0.616975$, $pl(\text{situation level}=3)=0.42888$. So we draw the conclusion that $(\text{situation level}=2)\succ(\text{situation level}=3)\succ pl(\text{situation level}=1)$.

This example shows that the reasoning method in Conditional Evidential Network based on DSm model can deal with multi-source haterogeneous evidence information in evidential networks effectively in despite of DSm model or DS model and draw reasonable decision conclusions.

6. Conclusions

A reasoning method in Conditional Evidential Network based on DSm model is proposed in this paper. The method improves ENC to dealing with haterogeneous evidence not in Smets/DS model. According to the dynatic changes of information model of the evidence in evidential networks, the method calculate new conditional belief functions based on different models by using DRC and GBT, which makes effective and efficient fusion and reasoning of the haterogeneous evidence in different discernment frames based on free DSm model, hybrid DSm model or Smets/DS model. Examples show that the method proposed in this paper can be successfully applied to the field of air target situation assessment and has a profound theoretical significance and engineering practical value.

However, we think that it is important to look at fast approximate evidence fusion methods because if there is multi-source evidence in the network, the computation complexity of mass assignments functions fusion grows exponentially with the increasing focal numbers of the evidence.

References

[1] Hu Lifang, Guan Xin, Deng Yong, Han Deqiang, He You. Measuring conflict functions in generalized power space. *J Chinese Journal of Aeronautics* 2011; 24(1): 65-73.
[2] Hu Lifang, He You, Guan Xin, Han Deqiang, Deng Yong. A new probabilistic transformation in generalized power space. *J Chinese Journal of Aeronautics* 2011; 24(4): 449-460.
[3] He You, Hu Lifang, Guan Xin, Deng Yong, Han Deqiang. New method for measuring the degree of conflict among general basic probability assignments. *J Science China*(*Information sciences*) 2012; 55:312-321.
[4] Hong Xu, Philippe Smets. Reasoning in evidential networks with conditional belief functions. *J International Journal of Approximate Reasoning* 1996; 14: 155-185.
[5] Philippe Smets. Belief functions: the disjunctive rule of combination and the generalized bayesian theorem. *J International Journal of Approximate Reasoning* 1993; 9: 1-35.
[6] Boutheina Ben Yaghlane, Khaled Mellouli. Inference in directed evidential networks based on the tranferable belief mode. *J International Journal of Approximate Reasoning* 2008; 48: 399-418.
[7] Felipe Aguirre, Mohamed Sallak, Frederic Vanderhaegen, Denis Berdjag. An evidential network approach to support uncertain multiviewpoint abductive reasoning. *J Information Sciences* 2013; 253: 110-125.
[8] Wafa Laamari, Boutheina Ben Yaghlane, Christophe Simon. On the complexity of the Graphical Representation and the Belief Inference in the dynamic dirrected evidential networks with conditional belief functions. *SUM 2012, LNAI 7520*, 206-218, 2012.
[9] Boutheina Ben Yaghlane, Philippe Smets, Khaled Mellouli. Directed evidential networks with conditional belief functions. *ECSQARU 2003*, *LNAI 2711*, 291-305, 2003.
[10] Wafa Laamari, Boutheina Ben Yaghlane, Christophe Simon. Comparing evidential graphical models for imprecise reliability. *SUM 2010*, *LNAI 6379*, 191-204, 2010.
[11] Wafa Laamari, Boutheina Ben Yaghlane, Christophe Simon. On the use of a mixed binary join tree for exact inference in dynamic directed evidential networks with conditional belief fuctions. *KSEM 2013*, *LNAI 8041*, 310-324, 2013.
[12] Wafa Laamari, Boutheina Ben Yaghlane, Christophe Simon. New propagation algorithm in dynamic directed evidential networks with conditional belief functions. *IUKM 2013*, *LNAI* 8032, 50-64, 2013.
[13] Shenoy P P, Shafer G. Valuation-based Systems for discrete optimaization. *Proceedings of the Sixth Conference Annual Conference on Uncertianty in Artificial Intelligence*. New York: 385-400.
[14] Shenoy P P. *Valuation-based systems: A framework for managing uncertianty in expert systems*. New York: 1992.
[15] Smarandache F, Dezert J. *Advances and Applications of DSmT for Information Fusion*. American Research Press, Rehoboth, USA, Vol.1, Vol.2, Vol.3, 2004/2006/2009.
[16] Shafer, G.. *A mathematical theory of evidence*. Princeton U.P.,1976.
[17] Philippe Smets. *Belief functions, in Nonstandard Logics for Automated Reasoning*. Academic, London, 253-286, 1988.
[18] Smets Ph., Kennes R. The transferable belief model. *J Artficial Intelligence* 1994; 66: 191-234.
[19] Dubois, D., Prade, H.. On the unicity of Dempster rule of combination. *J Intelligent Systems*; 1986, 1: 133-142.