An Alternative Combination Rule for Evidential Reasoning

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***Abstract*—In order to have a normal behavior in combination of bodies of evidence, this paper proposes a new combination rule. This rule includes the cardinality of focal set elements in conjunctive operation and the conflict redistribution to all steps. Based on the focal set cardinalities, the conflict redistribution is assigned using factors. The weighted factors are computed from the original and the conjunctive masses assigned to each focal element. This strategy forces the conflict redistribution in favor of the more committed hypothesis. Our method is evaluated and compared with some numerical examples reported in the literature. As result, this rule redistributes the conflict in favor of the more committed hypothesis and gives intuitive interpretation for combining multiple information sources with coherent results.**

***Keywords*—Evidential Reasoning, Belief functions, Evidence theory, Combination rule, Conflict redistribution.**

I. Introduction

The Dempster-Shafer Theory (DST) is quite popular because it seems well-suited for representing and managing uncertainty in a wide range of engineering problems [1], [2]. Ever since the publication of Zadeh's example in 1979, Dempster's rule has been criticized intensively especially in the presence of highly conflicting beliefs because such beliefs are considered to be responsible of the counterintuitive results produced by this rule [3]. Therefore, many attempts to modify the original Dempester's combination rule have been made in order to overcome this conflict. These modifications differ in the way of modeling the conflict redistribution, used to combine evidences. These modifications can be classified into two classes. The first class of methods attempt to overcome the counterintuitive behaviors by introducing some modifications to the Dempster's rule [4]–[9]. The main idea behind these modifications is to transfer total or partial conflicting masses proportionally to empty or non-empty sets according to some combination results. The second class of methods are based on the same fundamental principle as Dempster's rule and, in addition, they use a correction to original bodies of evidence [10]–[14]. These methods are based on arithmetic average, a distance measure or cross merging of the evidences to be combined. However, these classes of rules have some weaknesses. The first class favors the conflicting sources with the bigger evidences and the second class can solve conflict by replacing the evidences, but with inappropriately small weighted evidence values.

In addition, another abnormal behavior has not been pointed out in the literature. This abnormal behavior concerns the criticism of Dempster's rule behavior in the absence of conflicting beliefs. So this rule always seems to be inadequate and gives unexpected combination results of evidences. Note that the major preceding modifications have the same behavior in this situation because they are based only on the conflict redistribution. We believe that the result of the combination of belief mass assignments on different sets without their cardinalities does not fit with the objective of uncertainty reasoning over evidences accumulation. Since the combination results cannot reflect the difference between bodies of evidence, we cannot make this difference in decision making step using the maximum pignistic probability criteria.

Admittedly, the Dempster's rule modification problem has received considerable research attention but this point has not been addressed enough. And this work focuses on this particular issue and proposes an alternative combination rule called Combination Rule of Evidences with Cardinalities (CREC). The proposed rule can be classified into the first class where the conjunctive rule and the conflict proportionalization use both the sources information and their cardinalities. However, the total conflict is proportionally redistributed on focal set elements as in Dempster's rule combination with respect to some constraints. These constraints, which represent the weighting factors, are the weighted sum computed from the basic probability assignments (bpas) of the sources information and their combination according to their cardinalities. The proposed alternative combination rule is evaluated and compared with some numerical examples reported in the literature. As a result, this approach produces the association relationship between evidences, providing coherent results when the conflict between sources becomes either absent or present.

The rest of this paper is organized as follows. We first introduce the background of DS theory in Section II. Section III provides a description of Dempster's rule combination modification. The proposed combination rule for multiple sources is presented in Section IV. Section V compares the CREC rule with other similar works. Finally, Section VI concludes the paper.

II. Brief Outline of the Belief Function Theory

The belief function theory (also known as Dempster-Shafer theory) initiated from Dempster's work and further extended by Shafer [15]. This theory has attractive properties which provide significantly richer information in fusion area. Based on Shafer's model, the frame of discernment is a set of mutually exclusive and exhaustive hypotheses about problem domains. From a frame of discernment Θ correspondingly, 2^{Θ} is the power set of Θ, then a basic belief assignment (bba) or proper mass is defined as a mapping $m(\cdot)$:

$$m(\emptyset) = 0 \text{ and } \sum_{X \in 2^{\Theta}} m(X) = 1 \quad (1)$$

The Dempster's combination rule is the normalized conjunctive operation which aims to aggregate evidences from multiple independent sources defined within the same frame of discernment. Based on Shafer's model of the frame Θ, Dempster's rule for two sources is defined by the following equations:

$$m_{DS}(X) = \frac{m_{12}^c(X)}{1 - m_{12}^c(\emptyset)} \quad (2)$$

$$m_{12}^c(X) = \sum_{\substack{X_1, X_2 \in 2^{\Theta} \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2) \quad (3)$$

$$m_{12}^c(\emptyset) = \sum_{\substack{X_1, X_2 \in 2^{\Theta} \\ X_1 \cap X_2 = \emptyset}} m_1(X_1) m_2(X_2) \quad (4)$$

where $m_{12}^c(X)$ and $m_{12}^c(\emptyset)$ represent the conventional conjunctive consensus operator and the conflict of the combination between the two sources respectively.

From a given bba m, the decision functions are defined as: The belief function $Bel(X)$, which measures the belief that hypothesis X is true and it is given by :

$$Bel(X) = \sum_{Y \subseteq X} m(Y) \quad (5)$$

The plausibility function $Pl(X)$ can be interpreted as a measure of the total belief that hypothesis X can be true and it is given by the following formula:

$$Pl(X) = \sum_{X \cap Y \neq \emptyset} m(Y) \quad (6)$$

The pignistic probability transformation is generally considered as a good basis for a decision rule [16], [17]. It is defined for all $X \in 2^{\Theta}$, with $X \neq \emptyset$; by:

$$BetP(X) = \sum_{\substack{Y \in 2^{\Theta} \\ Y \neq \emptyset}} \frac{|X \cap Y|}{|Y|} \frac{m(Y)}{1 - m(\emptyset)} \quad (7)$$

However, when fusing highly conflicting data using Dempsters' combination rule, one can obtain counterintuitive results [7], [8], [18]–[21]. So, Numerous alternative combination rules have been proposed to avoid this abnormal behavior.

III. Related Work

Rules for combining conflicting evidence have received considerable attention for more than two decades. Therefore, many solutions proposed alternatives to Dempster's rule of combinations in order to overcome the non-intuitive combination results. The counterintuitive outcome is a direct result of managing evidential conflict which is imputed to the combination rule or the involved evidences. The management of this potential conflict has spurred multiple modifications to Dempster's combination rule in order to guarantee that counterintuitive results will not occur. These multiple modifications can be classified into two classes.

The approaches in the first class attempt to overcome the counterintuitive behaviors by modifying the Dempster's combination rule to solve the problem of how to redistribute and manage evidential conflict. These approaches have a totally different basis compared to Dempster's rule. Yager [4] proposed a modified rule which removes the normalizing process of Dempster's rule. This is justified by the fact that the conflict cannot provide any useful information. But there are also defects in the proposed combination rule because it assigns the conflicting mass assignments to the universal set of the frame of discernment (degree of ignorance) and also it is not associative. An alternative approach proposed in [5] is tounify combination rules, by using aspects of both Dempster's and Yager's combination rules according to the evidential conflict value. Dubois and Prade rule [22] is based on disjunctive operator and assigns the conflicting mass assignment to the union of focal element, compelling us to make a choice of possible combinations. But this disjunctive rule usually causes degradations to data specificity. Smets [23] proposed a modified method, which is equivalent to the non-normalized version of Dempster's rule. This method assigns the conflicting mass assignments to an empty set, by relying on the close-world assumption. Lefevre et *al.* [6] have proposed a unified formulation for rules of combinations based on the reallocation (convex combination) of the conflicting masses using the conjunctive consensus. Many propositional conflict redistribution rules emerged in order to overcome the non-intuitive combination in Dempster-Shafer theory in highly conflicting evidence [7]. The idea behind these rules is to transfer total or partial conflicting masses proportionally to non-empty set and the partial ignorance involved in the model according to some constraints. Dezert and Smarandache [7] proposed several proportional conflict redistribution methods (PCR1-5) to redistribute the partial or total conflicting mass on the focal elements implied in the local conflict. The most efficient method is the PCR5 rule and the generalized rule PCR6, proposed thereafter in [24]. The authors have proved that these rules do not yield counterintuitive results in highly conflicting situations, but these rules are not associative and the complexity for their implementation is higher when compared to the complexity of Dempster's rule [8]. An alternative combination mechanism called combination by compromise as a consensus generator has been proposed in [11].

The second category represents the approaches that are based on the corrective strategies when using Dempster's rule. Murphy proposed a combination rule [10] based on arithmetic average of belief functions associated with the evidences to be combined. This method is a commutative but not associative trade-off rule and solves the disappearing ignorance problem caused by the operation of intersection in Dempster's rule. However, this method does not consider the association relationship between the evidences, which is not reasonable in some situations. Thereafter, Deng et *al.* [12] used the same idea introduced in [25], [26] and proposed a more effective definition of evidential conflict based on evidential distance measure. However, the previous methods based on distance measure are applicable to a finite frame of discernment and can improve the reliability of the combination results compared to the first category of methods. Another approach called Weight Evidential Rule has been proposed in [14] to construct a new Weighted Belief Distribution to be combined. This rule completes and enhances Dempster's rule by identifying,through a novel reliability perturbation analysis,the manner to combine multiple pieces of independent evidence that are each fully reliable and conflicting.

These two classes of rules have some disadvantages. The first class of rules assume that the sources are conflicting and favors the conflicting sources with the bigger evidences in conflict redistribution step. And when it is not the case, these rules have a similar behavior as Dempster's rule. The problem of the second class of rules can be linked to some extent by replacing the evidences with inappropriately small weighted evidence values computed using the distance functions. So, these distances do not appear to be as well justified and do not use all the information of the evidence bodies lie on [18]. In addition, these rules do not prevent the masses of the focal elements containing a low certain information (partial or total ignorance) in the combination process. This is due to the fact that Dempster's rule and its derivates implicitly assume that all possible pair of focal elements are equally confirmed by the combined evidence [19].

IV. The proposed Combination Rule

A. Motivation

In this subsection, we point out some counterintuitive examples showing the abnormal behavior of the above classes of combination rules in the absence or the presence of conflicting beliefs. So that, these rules seem to be inadequate and give counterintuitive combination results of evidences if the evidences of the partial or total ignorance are null. Let us consider an example for unmanned aircraft vehicle surveillance system and suppose two sensors have made an Unmanned Aircraft Vehicle (UAV) observation , whose signature ensures that it is a v_1, a v_2, a v_3 or a v_4: $\Theta = \{v_1, v_2, v_3, v_4\}$.

Example 1: (Absence of the conflict) In the first situation, each sensor has made an observation with the following bba's:

$$m_1(v_1) = 0.5 \qquad m_1(v_2 \cup v_3) = 0.5$$
$$m_2(v_1 \cup v_2) = 0.5 \qquad m_2(v_1 \cup v_2 \cup v_4) = 0.5$$

Applying the rules of the first class, we obtained the same result as Dempster's rule of combination :

$$m^c_{12}(v_1) = 0.5, \quad m^c_{12}(v_2) = 0.5$$

with the conventional conflicting mass $m^c_{12}(\emptyset) = 0$.

As can be seen, the UAV $\{v_2\}$ seems to share three times a mass of 0.5, while $\{v_1\}$ only has to share twice a mass 0.5 with $\{v_2\}$ and $\{v_2, v_4\}$ and is once assigned 0.5 individually. Using the rules of the first class in this situation, the evidences of the partial ignorance are null ($m^c_{12}(v_2 \cup v_3) = 0$ and $m^c_{12}(v_1 \cup v_2 \cup v_4) = 0$). These rules are not suitable to allow decision making. Thus, after computing the maximum of the belief, the plausibility or the pignistic probability to provide a decision from the remaining combination result, we obtained a belief that the unmanned aircraft vehicle is likely to be v_1 or v_2: ($Bel(v_1) = Bel(v_2) = 0.5$, $Pl(v_1) = Pl(v_2) = 0.5$ or $BetP(v_1) = BetP(v_2) = 0.5$). Hence this result seems counterintuitive because $m_1(v_1)$ is more precise than $m_1(v_2 \cup v_3)$ and the result does not reinforce the idea that the unmanned aircraft vehicle is v_1. The reason is that m_1 has the certainty of 0.5 for $\{v_1\}$ (50 % of certainty) and m_2 has the certainty of 0.5 for the focal sets $\{v_1, v_2\}$ and $\{v_1, v_2, v_3\}$ which includes $\{v_1\}$. So these last certainties might give some additional supports in the favor of $\{v_1\}$, while that in combination step, the combined evidence for $\{v_1\}$ must be larger than the certainty of m_1. Moreover, no certainty is given to $\{v_2\}$. Nevertheless, certainties of $\{v_2, v_3\}$, $\{v_1, v_3\}$ and $\{v_1, v_2, v_3\}$ which both include $\{v_2\}$ are assigned to $\{v_2\}$. In addition, using the pignistic probability, the first class of rules cannot makes decision because the belief on $\{v_1, v_2\}$ and $\{v_1, v_2, v_3\}$ are equal to zero. However, the second class of rules can make decision based on the maximum of the pignistic probability.

Example 2: (Presence of the conflict) Let us consider now an observation with the following bba's:

$$m_1(v_1) = 0.8 \qquad m_1(\Theta) = 0.2$$
$$m_2(v_1) = 0.1 \qquad m_2(v_3) = 0.9$$
$$m_3(v_1) = 0.4 \qquad m_3(\Theta) = 0.6$$

Applying the PCR6 rule [8] we obtain the following result:

$$m_{123|PCR6}(v_1) = 0.434,\ m_{123|PCR6}(v_3) = 0.4437,$$
$$m_{123|PCR6}(\Theta) = 0.1223$$

Clearly a proportionalization process of PCR6 rule does not redistribute well the quantity of the conflicting mass to the focal set element $\{v_1\}$ because it does not take into account the mass fusion of the set elements $\{v_1\}$ for all observations. Hence the focal element $\{v_3\}$ absorbs almost all of the conflicting mass (the majority) due to the higher value of its mass (0.9) in the conflict redistribution process despite this single mass that confirms $\{v_3\}$. This means that we are sure that it is v_3. However, this result is counterintuitive because it does not reinforce the idea that the UAV is v_1. The reason is that most of the mass is assigned to the UAV $\{v_1\}$ by the three sensors (initial confidence): $m_1(v_1) = 0.8$ $m_2(v_1) = 0.1$ and $m_3(v_1) = 0.4$. So the PCR6 rule fails in conflict redistribution if there is an absorbing focal element.

Finally, we see that these behaviors are not accordant with the objective of reasoning over evidences accumulation since the results cannot reflect the difference between bodies of evidence in the first combination step. So the results cannot tie the decision based on the criteria as maximum of the belief, the plausibility or the pignistic probability if the combined evidences of the partial or total ignorance are null.

B. Definitions

To overcome the drawbacks of the preceding rules, in this study we describe an attractive rule which integrate the cardinalities in the combination step called CREC (Combination Rule of Evidence with Cardinality). Our rule is different from the preceding rules but has the same purpose. The general idea of the proposed combination rule is to separate the term of non-normalized conjunctive rule into two terms with respect to the cardinality of focal set elements. The first term corresponds to the effective conjunctive operator of combination between focal sets (the part in agreement). The second term represents the hidden conflict. More precisely, a part of the mass obtained by the non-normalized conjunctive operator is assigned to the conflict according to the cardinality atomicity of the focal sets involved in this conjunctive combination. This idea is formalized by the following definitions.

For the sake of clarity, we first give some necessary definitions before describing our combination rule.

Definition 1: (Cardinality Atomicity) Let Θ be a frame of discernment and let X_i, $X_j \in 2^\Theta$ be focal sets. We define the cardinality atomicity of the focal set X_i with respect to X_j as:

$$ca(X_i \uparrow X_j) = \frac{|X_i \cap X_j|}{|X_j|}$$

This cardinality definition represents the number of parts of the focal set X_i in X_j.

Definition 2: (Effective Conjunctive rule) Let Θ be a frame of discernment. Then we define the effective conjunctive rule (denoted by $m^{\mathrm{Ec}}(X)$) associated to a focal set X as:

$$m^{\mathrm{Ec}}(X) = \sum_{\substack{X_1, X_2, \cdots, X_s \in 2^\Theta \\ X_1 \cap X_2 \cap \cdots \cap X_s = X}} \prod_{i=1}^{s} ca(X \uparrow X_i) m_i(X_i) \tag{8}$$

The effective conjunctive operator allows the computation of the first term (the part in agreement between the intersection of focal sets) which corresponds to the combination between focal sets evidence according to the split of the belief mass assigned to the union of the singleton focal sets.

Definition 3: (Hidden Conflict) Let Θ be a frame of discernment. Then we define the hidden conflict (denoted by $\overline{m}^{\mathrm{Ec}}(X)$) associated to a focal set X as:

$$\overline{m}^{\mathrm{Ec}}(X) = \sum_{\substack{X_1, X_2, \cdots, X_s \in 2^\Theta \\ X_1 \cap X_2 \cap \cdots \cap X_s = X}} (1 - \prod_{i=1}^{s} ca(X \uparrow X_i)) \prod_{i=1}^{s} m_i(X_i) \tag{9}$$

The hidden conflict represents the second term and it is considered as a conflict arisen by the conjunctive combination of the different intersections between focal sets.

To clear up these two concepts, let us assume $m_1(\{A\}) = 0.5$, $m_2(\{A, B\}) = 0.5$ and $m_3(\{A, B, C\}) = 0.5$. The combination of the two beliefs m_1 and m_2 is different from the one of the two beliefs m_1 and m_3. Using the above definitions, we obtained

$$\begin{aligned} m_{12}^{\mathrm{Ec}}(A) &= ca(\{A\} \uparrow \{A, B\}) \cdot m_1(\{A\}) \cdot m_2(\{A, B\}) \\ &= \frac{1}{2} \cdot 0.5 \cdot 0.5 = 0.125 \\ \overline{m}_{12}^{\mathrm{Ec}}(A) &= (1 - ca(\{A\} \uparrow \{A, B\})) \cdot m_1(\{A\}) \cdot m_2(\{A, B\}) \\ &= \frac{1}{2} \cdot 0.5 \cdot 0.5 = 0.125 \end{aligned}$$

$$\begin{aligned} m_{13}^{\mathrm{Ec}}(A) &= ca(\{A\} \uparrow \{A, B, C\}) \cdot m_1(\{A\}) \cdot m_3(\{A, B, C\}) \\ &= \frac{1}{3} \cdot 0.5 \cdot 0.5 = 0.083 \\ \overline{m}_{13}^{\mathrm{Ec}}(A) &= (1 - ca(\{A\} \uparrow \{A, B, C\})) \cdot m_1(\{A\}) \cdot m_3(\{A, B, C\}) \\ &= \frac{2}{3} \cdot 0.5 \cdot 0.5 = 0.167 \end{aligned}$$

It can be seen that the effective conjunctive combination of the beliefs in both situations gives some additional supports to $\{A\}$ but with different certainties (50% from the belief of $\{A, B\}$ and 33% from the belief of $\{A, B, C\}$). However, the hidden conflict in the first situation is smaller than the one in the second situation. This can be justified by the fact that most of the mass is assigned to the belief of $\{B, C\}$ in the mass $m_3(\{A, B, C\})$. Hence we can distinguish between the combination of these beliefs. In addition, it is easy to verify that the sum of the effective conjunctive combination and the hidden conflict represents the non-normalized conjunctive rule result ($m_{12}^{c} = 0.25$).

To summarize, the mass m_2 is more precise than the mass m_3 to compute the certainty of the focal element $\{A\}$. For this reason, we give more credit to assumptions supported by the source of information which has a lower cardinality.

Hence the above definitions of the effective conjunctive rule and the hidden conflict yield the next result which describes a computation for non-normalized conjunctive.

Proposition 1: (Non-normalized Conjunctive Rule) Let Θ be a frame of discernment and let $X \in 2^\Theta$ be a focal set. Then non-normalized conjunctive rule, given for mass functions defined on 2^Θ is given by:

$$m^{\mathrm{c}}(X) = m^{\mathrm{Ec}}(X) + \overline{m}^{\mathrm{Ec}}(X) \tag{10}$$

Proof 1: Proof is omitted for its simplicity.

C. Principle of CREC

Most suggested combination rules are based on the conflict redistribution and differ on the way they transfer this conflicting mass. In our rule, the global conflicting mass is distributed proportionally to all non-empty sets according to some weighting factors. These factors are computed with respect to the cardinality of the original evidences and those obtained by the effective conjunctive rule. This strategy forces the conflict redistribution in the favor of the most committed hypothesis. The proposed rule mainly consists of four steps: (1) we apply the effective conjunctive rule with respect to the cardinalities; (2) we compute a total conflicting mass which represents the conventional and the hidden conflicts; (3) Thereafter, we calculate the weighting factors using the masses of the sources information and those obtained from the effective conjunctive rule. (4) Finally, we redistribute the total conflicting mass proportionally according to these weighting factors. These steps are formalized by the following equations (11,12,13).

The formulation of our rule allows the redistribution of the global conflict in a better way when compared to the preceding classes of rules according to the coherence of the responses obtained from the effective conjunctive operator. Hence the

CREC rule formula for multiple sources ($s \geq 2$) is ($\forall X \neq \emptyset) \in 2^{\Theta}$

$$m_{1,2,\cdots,s|CREC}(X) = m^{\text{Ec}}(X) + w_{1,2,\cdots,s}(X) \cdot G^c_{1,2,\cdots,s} \quad (11)$$

where $w_{1,2,\cdots,s}(X)$ and $G^c_{1,2,\cdots,s}$ are the weighted factors and the global conflict defined by equations (eq.12 and eq.13) respectively.

$$w_{1,2,\cdots,s}(X) = \frac{1}{\lambda} \cdot \left(\sum_{i=1}^{s} m_i(X) + \sum_{\substack{Y \in 2^{\Theta} \\ Y \neq X}} c(Y) \cdot ca(X \uparrow Y) \cdot m(Y) \right) \quad (12)$$

where λ is the normalization term of the weighting factors so that

$$\sum_{X \in 2^{\Theta}} w_{1,2,\cdots,s}(X) = 1$$

and

$$c(Y) = \begin{cases} 1 & \text{if } Y \text{ is involved in } m^{Ec}(X) \\ 0 & \text{otherwise;} \end{cases}$$

A non-empty set Y is considered to be involved in effective conjunctive rule of the focal set X if $Y \cap X = X$ and $m^{Ec}(X) \neq 0$.

$$G^c = m^c_{1,\cdots,s}(\emptyset) + \sum_{X \in 2^{\Theta}} \overline{m}^{\text{Ec}}_{1,\cdots,s}(X) \quad (13)$$

where $m^c_{1,\cdots,s}(\emptyset)$ is the conventional conflict. Thus, the global conflict represents the conventional and the hidden conflicts.

Proposition 2: (Proper Mass) The mass $m_{1,2,\cdots,s|CREC}(\cdot)$ is a proper mass.

Proof 2: A proper mass means that a mass $m_{1,2,\cdots,s|CREC}(\cdot)$ must satisfy all requirements given by Eq. 1. When X is an empty set, the value of $m(X)$ is equal to 0. By definition, the mass $m_{1,2,\cdots,s|CREC}(X) = m^{\text{Ec}}(X) + w_{1,2,\cdots,s}(X) \cdot G^c_{1,2,\cdots,s}$. Hence, $\sum_{X \in 2^{\Theta}} m_{1,2,\cdots,s|CREC}(X) = \sum_{X \in 2^{\Theta}} m^{\text{Ec}}(X) + \sum_{X \in 2^{\Theta}} \left[w_{1,2,\cdots,s}(X) \cdot G^c_{1,2,\cdots,s}\right]$. In addition, we have $\sum_{X \in 2^{\Theta}} w_{1,2,\cdots,s}(X) = 1$ due to the normalization process. So the term $\sum_{X \in 2^{\Theta}} \left[w_{1,2,\cdots,s}(X) \cdot G^c_{1,2,\cdots,s}\right] = G^c_{1,2,\cdots,s}$. From Eq. 13, we get $\sum_{X \in 2^{\Theta}} m_{1,2,\cdots,s|CREC}(X) = \sum_{X \in 2^{\Theta}} m^{\text{Ec}}(X) + m^c_{1,\cdots,s}(\phi) + \sum_{X \in 2^{\Theta}} \overline{m}^{\text{EC}}_{1,\cdots,s}(X)$. According to Proposition 1, we get $\sum_{X \in 2^{\Theta}} m^c(X) + m^c_{1,\cdots,s}(\phi)$ which is equals to1 (Orthogonal sum). Therefore, $m_{1,2,\cdots,s|CREC}(\cdot)$ is a proper mass. □

Now, let us describe in details how this rule operates through Example 1 (i.e., UAV example 1). The proposed rule starts by computing the effective conjunction and the hidden conflict masses. So we obtained the following results:

with the global conflicting mass:

$$\begin{aligned} G^c &= m^c_{1,2}(\phi) + \sum_{X \in 2^{\Theta}} \overline{m}^{\text{EC}}_{1,2}(X) \\ &= 0 + (0.292 + 0.396) = 0.688 \end{aligned}$$

The weighting factors are given by

$$\begin{aligned} w_{1,2}(v_1) &= 0.2972 \\ w_{1,2}(v_2) &= 0.2162 \\ w_{1,2}(v_1 \cup v_2) &= 0.1622 \\ w_{1,2}(v_2 \cup v_3) &= 0.1622 \\ w_{1,2}(v_1 \cup v_2 \cup v_4) &= 0.1622 \end{aligned}$$

After computing all weighting factors, we compute the final beliefs using Eq. (11). The result is therefore given by

$$\begin{aligned} m_{1,2|CREC}(v_1) &= m^{\text{Ec}}(v_1) + w_{1,2}(v_1) \cdot G^c_{1,2} \\ &= 0.208 + 0.2972 \cdot 0.688 = 0.412 \\ m_{1,2|CREC}(v_2) &= m^{\text{Ec}}(v_2) + w_{1,2}(v_2) \cdot G^c_{1,2} \\ &= 0.1042 + 0.2162 \cdot 0.688 = 0.252 \\ m_{1,2|CREC}(v_1 \cup v_2) &= m^{\text{Ec}}(v_1 \cup v_2) + w_{1,2}(v_1 \cup v_2) \cdot G^c_{1,2} \\ &= 0.112 \\ m_{1,2|CREC}(v_2 \cup v_3) &= m^{\text{Ec}}(v_2 \cup v_3) + w_{1,2}(v_2 \cup v_3) \cdot G^c_{1,2} \\ &= 0.112 \\ m_{1,2|CREC}(v_1 \cup v_2 \cup v_4) &= m^{\text{Ec}}(v_1 \cup v_2 \cup v_4) + w_{1,2}(v_1 \cup v_2 \cup v_4) \cdot G^c_{1,2} \\ &= 0.112 \end{aligned}$$

Clearly, from this result, we can observe that the effective combined mass and the weighted factor of the focal set $\{v_1\}$ are bigger than those of $\{v_2\}$. This is due to the fact that the effective combined mass $m^{Ec}_{12}(v_1)$ is computed with respect to the masses of the focal sets $\{v_1\}, \{v_1 \cup v_2\}, \{v_1 \cup v_2, \cup v_4\}$. Whereas the combined mass $m^{Ec}_{12}(v_2)$ is computed from the masses of focal sets $\{v_1 \cup v_2\}$, $\{v_2 \cup v_3\}$ and $\{v_1 \cup v_2, \cup v_4\}$. As a result, our rule effectively exploits only the little certain information of the masses, that support the focal element $\{v_3\}$ with a higher cardinality.

In the same way, the combination results for examples 2 are:
$m_{1,2|CREC}(v_1) = 0.467$, $m_{1,2|CREC}(v_3) = 0.3211$, $m_{1,2|CREC}(v_1 \cup v_2 \cup v_3 \cup v_4) = 0.2119$.

Note that the idea of our rule does not correspond to the principle of insufficient reason [17]: a mass assigned to the union of n atomic sets is split equally among these n sets because we save a strength of the belief functions which states that the mass assigned to a set is not split between its elements but remains with all the elements (the complete masses of sets are used in effective conjunction operator).

Reconsider Example 1 and let us suppose now that the mass to the sets $\{v_2, v_3\}$, $\{v_1, v_2\}$ and $\{v_1, v_2, v_4\}$ are split between their elements. This split gives the following bba's:

$$\begin{aligned} m_1(v_1) &= 0.5 & m_2(v_1) &= 0.4167 \\ m_1(v_2) &= 0.25 & m_2(v_2) &= 0.4167 \\ m_1(v_3) &= 0.25 & m_2(v_3) &= 0 \\ m_1(v_4) &= 0 & m_2(v_4) &= 0.1666 \end{aligned}$$

Applying Dempster's, Murphy's, PCR6 and CREC rules, we obtained the different combined mass results as the one

$$
\begin{aligned}
m_{12}^{\mathrm{Ec}}(v_1) &= ca(\{v_1\} \uparrow \{v_1,v_2\}) \cdot m_1(\{v_1\}) \cdot m_2(\{v_1,v_2\}) + ca(\{v_1\} \uparrow \{v_1,v_2,v_4\}) \cdot m_1(\{v_1\}) \cdot m_2(\{v_1,v_2,v_4\}) \\
&= \tfrac{1}{2} \cdot 0.5 \cdot 0.5 + \tfrac{1}{3} \cdot 0.5 \cdot 0.5 = 0.125 + 0.0825 = 0.208 \\
\overline{m}_{12}^{\mathrm{Ec}}(v_1) &= (1 - ca(\{v_1\} \uparrow \{v_1,v_2\})) \cdot m_1(\{v_1\}) \cdot m_2(\{v_1,v_2\}) + (1 - ca(\{v_1\} \uparrow \{v_1,v_2,v_4\})) \cdot m_1(\{v_1\}) \cdot m_2(\{v_1,v_2,v_4\}) \\
&= \tfrac{1}{2} \cdot 0.5 \cdot 0.5 + \tfrac{2}{3} \cdot 0.5 \cdot 0.5 = 0.0.125 + 0.167 = 0.292 \\
m_{12}^{\mathrm{Ec}}(v_2) &= ca(\{v_2\} \uparrow \{v_2,v_3\}) \cdot ca(\{v_2\} \uparrow \{v_1,v_2\}) \cdot m_1(\{v_2,v_3\}) \cdot m_2(\{v_1,v_2\}) \\
&\quad + ca(\{v_2\} \uparrow \{v_2,v_3\}) \cdot ca(\{v_2\} \uparrow \{v_1,v_2,v_4\}) \cdot m_1(\{v_2,v_3\}) \cdot m_2(\{v_1,v_2,v_4\}) = \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 0.5 \cdot 0.5 + \tfrac{1}{2} \cdot \tfrac{1}{3} \cdot 0.5 \cdot 0.5 \\
&= 0.0625 + 0.042 = 0.1042 \\
\overline{m}_{12}^{\mathrm{Ec}}(v_2) &= (1 - ca(\{v_2\} \uparrow \{v_2,v_3\}) \cdot ca(\{v_2\} \uparrow \{v_1,v_2\}) \cdot m_1(\{v_2,v_3\}) \cdot m_2(\{v_1,v_2\}) \\
&\quad + (1 - ca(\{v_2\} \uparrow \{v_2,v_3\}) \cdot ca(\{v_2\} \uparrow \{v_1,v_2,v_4\})) \cdot m_1(\{v_2,v_3\}) \cdot m_2(\{v_1,v_2,v_4\}) = \tfrac{3}{4} \cdot 0.5 \cdot 0.5 + \tfrac{5}{6} \cdot 0.5 \cdot 0.5 \\
&= 0.1875 + 0.2083 = 0.396
\end{aligned}
$$

TABLE I. COMBINED RESULT FOR SPLITED MASS

Rule	Combined mass			
	v_1	v_2	v_3	v_4
Dempster's	0.666	0.333	0	0
PCR6	0.515	0.328	0.103	0.054
Murphy's	0.611	0.323	0.045	0.0201
CREC	0.523	0.333	0.086	0.058

given by the CREC rule without split of the sets as reported in Table I.

This shows that the CREC rule can make the difference between beliefs in combination steps and the results satisfy the requirements of combination where other rules draw the same combined masses without showing this difference. Thus our rule refines combination results according to the cardinality of the focal sets, yielding a more intuitive final result.

V. Numerical Simulation Results

The first part of this section describes the analysis of some well-known examples discussed in the previous published methods. The second part is reserved to the comparison of the CREC rule and some ones of each class.

A. The CREC Rule Behavior for Well-known examples

In this subsection, we are going to show the behavior of our rule to combine belief functions in Shafer's model. These well-known examples are the counter-examples to Dempster's rule.

Example 3: (Zadeh's Paradox) Let us have $\Theta = \{v_1, v_2, v_3\}$ and two beliefs assignments $m_1(v_1) = 0.9$; $m_1(v_2) = 0.1$ and $m_2(v_2) = 0.1$; $m_2(v_3) = 0.9$. Using the CREC formula for 2 sources ($s = 2$) described in section IV, we obtain $m_{12}^{\mathrm{Ec}}(v_1) = m_{12}^{\mathrm{Ec}}(v_3) = 0.4455$ and $m_{12}^{\mathrm{Ec}}(v_2) = 0.1090$. This result is intuitive and it is similar to the one obtained in all class of combination rules published in literature. A low belief proposition v_2 acquires the lowest value after combination, whereas high belief value of v_1 and v_3 dos not vanish into zero as in Dempster's rule. Thus the CREC rule does not give the total certainty to a minority opinion.

Example 4: (Certainty Convergence) [11] Suppose that a body of evidence gives $m_1(v_1) = 0.5$; $m_1(v_2) = 0.5$ and the other does $m_2(v_1) = 0.5$; $m_2(v_1 \cup v_2) = 0.5$. Applying the CREC rule, one has $m_{12}^{\mathrm{Ec}}(v_1) = 0.625$, $m_{12}^{\mathrm{Ec}}(v_2) = 0.275$ and $m_{12}^{\mathrm{Ec}}(v_1 \cup v_2) = 0.1$. From the result, it can be seen that the CREC rule has a normal behavior and somewhat reflects the certainty convergence to the focal element $\{v_1\}$ because both bodies of evidence have certainty of 0.5 for this focal element.

Example 5: (Loss of Majority Opinion) This example is also adopted from [11]. Let us consider the Shafer's model on $\Theta = \{v_1, v_2, v_3\}$ and three beliefs assignments $m_1(v_1) = 0.9$, $m_1(v_1 \cup v_3) = 0.1$; $m_2(v_1 \cup v_2) = 0.8$; $m_2(v_1 \cup v_3) = 0.2$ and $m_3(v_2) = 0.5$; $m_3(v_3) = 0.5$. Combining these masses using Dempster's rule gives $m_{123}(v_1) = 0$, due to the fact that the third belief function does not have the v_1 in its focal elements. Hence with the CREC rule, one has $m_{12}^{\mathrm{Ec}}(v_1) = 0.285$, $m_{12}^{\mathrm{Ec}}(v_2) = 0.1583$, $m_{12}^{\mathrm{Ec}}(v_3) = 0.2083$, $m_{12}^{\mathrm{Ec}}(v_1 \cup v_2) = 0.2533$, $m_{12}^{\mathrm{Ec}}(v_1 \cup v_3) = 0.095$. As can be seen, our rule does not exclude the elements $\{v_1, v_2\}$ which are supported by many bodies of evidence with null ones. So the majority opinion is reflected since $m_{12}^{\mathrm{Ec}}(v_1 \cup v_3) < m_{12}^{\mathrm{Ec}}(v_2) < m_{12}^{\mathrm{Ec}}(v_3) < m_{12}^{\mathrm{Ec}}(v_1 \cup v_2) < m_{12}^{\mathrm{Ec}}(v_1)$. In addition, this result indicates that small values of the obtained combined masses are preferable because the plausibility of the focal element $\{v_3\}$ is not too weak when compared to the results obtained from Murphy's rule ($m_{123|Murphy}(v_3) = 0.0477$).

Example 6: (Unearned Certainty) In this example, we combine two beliefs assignments: $m_1(v_1) = 0.5$, $m_1(v_2 \cup v_3) = 0.5$ and $m_2(v_1 \cup v_2) = 0.5$; $m_2(v_3) = 0.5$. From these bodies of evidence, the combined mass can be obtained as: $m_{12}^{\mathrm{Ec}}(v_1) = 0.2969$, $m_{12}^{\mathrm{Ec}}(v_2) = 0.1771$, $m_{12}^{\mathrm{Ec}}(v_3) = 0.2969$, $m_{12}^{\mathrm{Ec}}(v_1 \cup v_2) = 0.1146$, $m_{12}^{\mathrm{Ec}}(v_2 \cup v_3) = 0.1146$. In this case, the combined mass $m_{12}^{\mathrm{Ec}}(v_1) = 0.2969$ (resp. $m_{12}^{\mathrm{Ec}}(v_3) = 0.2969$) seems to be naturally intuitive, because both $m_1(v_1) = 0.5$ and $m_2(v_1 \cup v_2) = 0.5$ give some additional supports to $\{v_1\}$ (resp. $\{v_3\}$) in respect to its weighed factor $w_{12}(v_1) = 0.25$ (resp.$w_{12}(v_3) = 0.25$). However, the combined mass result $m_{12}^{\mathrm{Ec}}(v_2) = 0.1771$ is obtained by effective conjunctive operator and its weighed factor $w_{12}(v_2) = 0.167$. So, the part conflict redistribution for the focal set $\{v_2\}$ is weaker than the part of the focal set element $\{v_1\}$ (resp. $\{v_3\}$).

Example 7: (Experts Problem) [8] In this example, three experts are able to express their opinion only in the form of partial or total ignorance. Let us $\Theta = \{v_1, v_2, v_3\}$ and the opinions on Θ are: $m_1(v_1 \cup v_2) = 0.7$, $m_1(\Theta) = 0.3$, $m_2(v_1 \cup v_3) = 0.6$, $m_2(\Theta) = 0.4$, $m_3(v_2 \cup v_3) = 0.5$, $m_3(\Theta) = 0.3$. The application of the CREC rule gives the following results:

$m_{12}^{\mathrm{Ec}}(v_1) = 0.0175$, $m_{12}^{\mathrm{Ec}}(v_2) = 0.0117$, $m_{12}^{\mathrm{Ec}}(v_3) = 0.0075$, $m_{12}^{\mathrm{Ec}}(v_1 \cup v_2) = 0.2811$, $m_{12}^{\mathrm{Ec}}(v_1 \cup v_3) = 0.2308$, $m_{12}^{\mathrm{Ec}}(v_2 \cup v_3) = 0.1894$ and $m_{12}^{\mathrm{Ec}}(\Theta) = 0.2620$. These results are more in accordance with the conclusion that the beliefs obtained for the singleton focal sets are lower focal sets are lower than the masses of the union sets. The reason is that the CREC rule uses the split of evidence based on the cardinality instead of the total evidence. Contrary, in Dempster's rule and the two classes, the conjunctive operator favors the singleton sets compared to the union ones in this case.

Example 8: (True and False) [27] In this example, we have total conflict between two beliefs. Let us $\Theta = \{v_1, v_2\}$ and the opinion on Θ are: $m_1(v_1) = 1,\ m_1(v_2) = 0$ and $m_2(v_1) = 0,\ m_2(v_2) = 1$. The CREC rule yields the following result: $m_{12}^{\mathrm{Ec}}(v_1) = 0.5$ and $m_{12}^{\mathrm{Ec}}(v_2) = 0.5$. This result is reasonable. However, Dempster's rule is not applicable and the major rules of the preceding classes produce the same result except for the disjunctive rule [22] which leads to the vacuous belief assignment ($m(\Theta) = 1$).

Example 9: (Vacuous belief assignment) Unfortunately, our rule does not preserve the vacuous belief assignment (Disappearing Ignorance: $m(\Theta = 1)$) when some sources become totally ignorant. The disappearing ignorance property in Dempster's rule is an advantage in combination process because only the evidences without ignorance are combined. However, some authors [10], [19], [20] have criticized this property caused by conjunctive operation and specified that the counterintuitive results of Dempster's rule is caused by this ignorance. We presume that the disappearing ignorance is not justified in all situations of beliefs combination especially if the belief assignment of some focal sets are represented only by vacuous belief assignment of one source (they are missing in other sources). As illustration, let us consider again our UAV's example and the following two bba's:

$$m_1(v_1) = 0.5 \qquad m_1(v_2 \cup v_4) = 0.5 \qquad m_2(\Theta) = 1$$

For this example, the application of the first class rules yield the first belief assignments ($m_1(\cdot)$) which neglects completely the UAV's belief supporting $\{v_3\}$ ($m_{12}^{c}(\Theta) = 0$). However, the CREC rule produces: $m_{12}^{\mathrm{Ec}}(v_1) = 0.295$, $m_{12}^{\mathrm{Ec}}(v_2 \cup v_4\}) = 0.477$, and $m_{12}^{\mathrm{Ec}}(\Theta) = 0.227$. As can be seen our rule cannot suppress or neglect the weakest belief committed to the UAV v_3 which can be supported by the second totally ignorant source ($m_2(\Theta) = 1$).

B. Comparison of Combination Rules

Let us compare the CREC rule to one of the preceding class of rules for three counterintuitive examples. Hence we suggested for comparison Dempster's rule, PCR6 (class 1), and Murphy's rule (class 2) in order to show the behavior of these rules.

First, reconsider $\Theta = \{v_1, v_2, v_3, v_4\}$ provided in Example 1 with the following bba's:

$$\begin{aligned} &m_1(v_1) = 0.6 && m_2(v_1) = 0 \\ &m_1(v_2) = 0 && m_2(v_2) = 0.6 \\ &m_1(v_1 \cup v_2) = 0 && m_2(v_1 \cup v_2) = 0.4 \\ &m_1(v_1 \cup v_2 \cup v_3 \cup v_4) = 0.4 && m_2(v_1 \cup v_2 \cup v_3 \cup v_4) = 0 \end{aligned}$$

TABLE II. Comparative results for decision making

Rule	Criteria					
	$Bel(\cdot)$		$Pl(\cdot)$		$BetP(\cdot)$	
UAV	v_1	v_2	v_1	v_2	v_1	v_2
Dempster's	0.375	0.375	0.625	0.625	0.5	0.5
PCR6	0.420	0.420	0.580	0.580	0.5	0.5
Murphy's	0.402	0.402	0.598	0.598	0.488	0.488
CREC	0.357	0.267	0.733	0.643	0.515	0.426

TABLE III. Comparative results for decision making with the novel observation

Rule	Criteria					
	$Bel(\cdot)$		$Pl(\cdot)$		$BetP(\cdot)$	
UAV	v_1	v_2	v_1	v_2	v_1	v_2
Dempster's	0.375	0.375	0.625	0.625	0.5	0.5
PCR6	0.420	0.420	0.580	0.580	0.5	0.5
Murphy's	0.402	0.402	0.598	0.598	0.491	0.491
CREC	0.345	0.271	0.729	0.655	0.513	0.440

On the given numerical example, the results obtained are reported in Table II. As can be seen, Dempster's rule and a rule of each class cannot provide decision using the maximum of one of the three criteria: credibility, plausibility, and pignistic probability. The reason is that these criteria provide ambiguity due to the fact that the same decision value is put on the two UAVs: v_1 and v_2. However, using our rule, these criteria allow to make decision which gives different values on all the focal elements: v_1 and v_2.

Second, let us suppose that the first sensor of the previous example has made an UAV's observation with the following bba's: $m_1(v_1) = 0.6$, $m_1(v_1 \cup v_2 \cup v_3) = 0.2$, $m_1(v_1 \cup v_2 \cup v_3 \cup v_4) = 0.2$

Table III illustrates the decision making results for the novel observation. As can be seen, the previous decision ambiguity between the two UAVs is still present with a belief on v_1 equals to v_2 one for all criteria. Note that the plausibility in Murphy's approach has been changed. Contrary, most of the decision results based on the CREC rule are close to the truth and they do not present ambiguity with the highest decision value on v_1 for all criteria. Moreover, the belief on the UAV v_2 is higher than the one in the preceding case because the belief of the focal set $\{v_1 \cup v_2 \cup v_3\}$ gives some additional supports to $\{v_2\}$. This is achieved by the effective conjunctive and the conflict redistribution which is based on the cardinality of the two focal sets $m_1(v_1 \cup v_2 \cup v_3)$ and $m_1(v_1 \cup v_2 \cup v_3 \cup v_4)$.

Finally, reconsider $\Theta = \{v_1, v_2, v_3, v_4\}$ provided in Example 1 with the following bba's:

$$\begin{aligned} &m_1(v_1) = 0.5 && m_1(v_2 \cup v_3 \cup v_4) = 0.5 \\ &m_2(v_2) = 0.5 && m_2(v_1 \cup v_4) = 0.5 \end{aligned}$$

TABLE IV. Comparison of the combined mass results

	Focal element		
Rules	v_1	v_2	v_4
Dempster's	0.333	0.333	0.333
Murphy's	0.3	0.3	0.2
PCR6	0.375	0.375	0.25
CREC	0.324	0.259	0.152

TABLE V. ACTIVITY RECOGNITION ACCURACY USING DIFFERENT COMBINATION RULES.

Combination rule	Recognition rate
Murphy	0.6813
CREC	0.6541
PCR6	0.6534
Dempster-Shafer	0.5441

Table IV presents the combined bba's results obtained by Dempster's rule and one of the preceding two classes. It can be seen that Dempster's rule still indicates that the UVA is likely to be v_1, v_2 or v_4. The results obtained by Murphy's rule are quite similar to those of PCR6 rule which produces equal results for the focal elements v_1 and v_2. In fact, these results are produced by proportionalisation of the beliefs allocated by PCR6 rule to conflicts on these elements. However, in CREC rule, the combined masses are completely different because the mass of belief committed to each focal element is supported by the belief on the focal elements having different cardinalities.

The proposed CREC rule is evaluated with the same temporal extensions for activity recognition used in [9] in the same conditions. Table V shows that the proposed rule outperforms all other combination rules except Murphy's rule.

VI. CONCLUSION

In this paper, we proposed a new way to combine belief functions in evidential reasoning. The main idea of the proposed combination rule is to separate the term of non-normalized conjunctive rule in two terms with respect to the cardinality of focal set elements. The first term corresponds to the effective conjunctive operator of combination between focal sets according to the split of the belief mass assigned to the union of the singleton focal sets (the part in agreement). The second term corresponds to the part of the belief mass assigned to the rest of the union of the singleton focal sets (the hidden conflict). With this idea, combining evidences becomes possible even in the absence of conflicting beliefs, and this by applying the proposed rule. As a result, it has been proven that our rule does not provide counterintuitive results and it can be considered as an interesting compromise between the Dempster's rule and the rules of the first class. In addition, this work describes several examples and also presents some comparisons with other combination rules published in literature in order to show the useful and the normal behavior of our combination rule. We believe that the CREC rule behavior is accordant with the objective of evidential reasoning over evidences accumulation where the results reflect really the difference between bodies of evidence.

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