An Algorithm for Medical Magnetic Resonance Image Non-Local Means Denoising

STEFAN KOROLIJA, EVA TUBA and MILAN TUBA
John Naisbitt University
Faculty of Computer Science
Bulevar umetnosti 29, 11070 Belgrade
SERBIA
korolija91@yahoo.com, etuba@acm.org, tuba@ieee.org

Abstract: Digital images and digital image processing were widely researched in the past decades and special place in this field have medical images. Magnetic resonance images are a very important class of medical images and their enhancement is very significant for diagnostic process. In this paper we presented an algorithm for improving magnetic resonance images of the brain. Common degradation of the magnetic resonance images is caused by the noise. We tested adjusted non-local means filter for removing random noise in the magnetic resonance images of the brain. Several evolution metrics were used to prove the quality of the proposed method.

Key–Words: Magnetic resonance images, magnetic resonance noise, non-local means filter, magnetic resonance images denoising

1 Introduction

Digital images were widely used during recent years. Many different fields use applications that include digital images and some image processing, including quality control [1], astronomy [2], meteorology [3], etc. Image processing can be at the lower lever (denoising [4], contrast enhancement [5], etc.) or higher level (segmentation [6], thresholding [7], [8], [9], shape recognition that includes optical character recognition [10], [11], [12], face recognition [13], skin detection [14], lip detection [15], etc.)

One of the areas in which digital image processing is very necessary is medicine since digital images are very important for medical diagnosis. In the field of medical imaging, techniques for digital image processing began to be used first in late 1960's and early 1970's. There are several types of medical images that are used such as magnetic resonance imaging (MRI), ultrasound imaging (US) and computed tomography (CT).

Magnetic resonance is used in medicine radiology to diagnose and report the diseases like tumors or cancer. Also it is used for treatment monitoring. This method does not use damaging radiation which makes this method very popular. Magnetic resonance imaging uses a magnetic field and pulses of radio wave energy in order to make pictures of organs and structures inside the human body. Reading MRI can give different information about organs. Some of that information can be seen using other medical imaging methods such as X-ray, computed tomography and others, but MRI may also present some problems that cannot be seen using other methods.

For making MRI images, the area of the body that need to be studied is placed inside a special machine that creates strong magnetic field. That magnetic field makes protons in the body to align with the field. Radio-frequency pulses through the patient, which stimulates protons to spin out and strain around the magnetic field. After turning off the radio-frequency field the MRI sensors detect released energy of the protons. Time necessary for the protons to realign with the magnetic field and the amount of energy that was released are changing based on the environment as well as on the chemical nature of the molecules. Pictures from an MRI scan are digital images that can be saved and stored on a computer for further study. The images also can be reviewed remotely, such as in a clinic or an operating room. In some cases, contrast material may be used during the MRI scan to show certain structures more clearly. Advantage of the MRI is that it generates high resolution images of soft tissues that are found in the human body. Most complex soft organ in human body is the brain.

For medical image processing noise is one of the major problems which undesirably corrupts medical images. Procedure of image denoising in image processing has a role to remove a noise from image, while retaining its quality. Noise removal is applied to var-

ious medical images enhancements. There are different types of noise that appear in digital images. Some types include Gaussian noise, salt and pepper noise, speckle noise, Rician noise, fractional Brownian motion noise, etc. Gaussian noise often appears in natural images, speckle noise is observed in ultrasound images, Rician noise affects magnetic resonance images while random noise can appear in any type of images. Model of the noise depends on its source [16]. In digital images is very difficult to remove the noise that has a low frequency because it is difficult to distinguish low frequency noise from the real signal [17]. Generation of noise can arise because of poor instruments in image processing or interface. Noise on digital images may be obtained by compression, error in transmission or some other factors. MRI images are corrupted by various types of noise [18].

For the mentioned types of noise, denoising techniques should consider the image quality. Better image quality contributes to better diagnosis of the disease. In the diagnosis of a tumor an important role plays accurate detection and location of tumors [19]. The cells that reproduce uncontrollably result in a brain tumor. To diagnose this disease the most common method used is magnetic resonance imaging.

In this paper we proposed an algorithm for removing random noise from MRI brain images. We proposed a non-local means filter and we evaluated results using several metrics.

The remainder of this paper is divided into five sections. Section 2 provides literature overview of techniques and methods used for magnetic resonance image denoising. That section describes different types of transformations used for denoising. Section 3 describes various types of noise that can be found in magnetic resonance images and their characteristics. Section 4 presents the proposed algorithm to eliminate random noise from magnetic resonance images of brain which is based on non-local means filter. Section 5 presents the results obtained during the processing of magnetic resonance images with the proposed algorithm and evaluation metrics. Evaluation metrics were used for comparison of some calculation results. At the end in 6 conclusion is given.

2 Literature Review

There are different methods and filters for noise reduction in magnetic resonance images. One of the filters which removes noise like Rician noise is based on Wiener filter [20]. This filter uses neutrosophic set which is applied in image domain. The image is transformed to neutrosophic set domain which uses three sets: True, Indeterminacy and False. Wiener filter method is used to remove noise for True and False sets to decrease Indeterminacy. Magnetic resonance images can be found from database Brainweb that contains images affected by the Rician noise. Performance of the Wiener filter could be compared with some other filters, like a non-local means filter or anisotropic diffusion filter.

Magnetic resonance images may be affected by random noise which limits accuracy of measurements. Removing the noise of this type from an image can be done through a non-local means filter which has its own parameters. For this method it is necessary to find the optimal parameters for different levels of noise so that the filter be adaptable to the characteristics of the noise in the magnetic resonance images. This technique was successfully tested by Manjon et al. [21].

In the magnetic resonance images main source of noise is thermal noise. Image reconstruction is performed by inverse discrete Fourier transformation. Noise which was reconstructed is complex white Gaussian noise. Signal magnitude is used for computerized analysis and diagnosis. The method that can be used over noisy MRI images is bilateral filter in underestimated wavelet domain [22]. The wavelet transform allows to ensure the presence of coefficients that are noisy. Bilateral filter is applied to the transformed coefficients and it removes noisy coefficients. Compared with classical wavelet domain denoising, reconstructive MRI data will give higher peak to signal ratio.

Using magnetic resonance imaging the brain tumor can be extracted by applying a mathematical morphological reconstruction. MRI images of the brain are affected by noise pulses. This method is explained by Sharma and Meghrajani [23]. In the preprocessing the magnetic resonance image global threshold technique is applied. On the processed image mathematical morphological reconstruction operation was used in order to segment the brain tumor. Proposed algorithm was adjusted to segment non-uniform intensity regions of brain tumor. Salt and pepper noise was removed by mathematical morphological operator.

Denoising based on wavelet transform has the possibility to improve the magnetic resonance imaging. Usually, uniform spatial distribution of the noise is required which is not the case in the images obtained with parallel MRI. Delakis et al. in [24] proposed a new algorithm for filtering parallel magnetic resonance images. This algorithm takes out edges from the original image and than generates a noise map from wavelet coefficients. With the aim to save the spatial resolution, at locations of edges noise map was set to zero. Directional analysis was used to calculate noise in the area where edges have a low con-

trast. The performance of this algorithm was compared with other methods and the results showed that proposed algorithm is comparable with them.

Magnetic resonance images can be affected by fractional Brownian motion noise. To reduce this kind of the noise from the brain MRI images methods that use wavelet-based thresholding techniques can be used. Some of these techniques of thresholding are: visu shrink, sure shrink and Bayes shrink. In [18] Rajeswaran and Gokilavani compared mentioned techniques. For comparing performance evaluation metrics were used. Evaluation metrics included calculation for: peak signal to noise ratio (PSNR), absolute error, fractal dimension, image enhancement factor (IEF), structural content, structural similarity index metric (SSIM), average difference (AD) and maximum difference (MD). These metric are also used in this paper.

For denoising magnetic resonance images discrete wavelet transform algorithm can also be used. In [25] technique based on discrete wavelet transform and wavelet thresholding at different level for removing random noise was proposed. Different wavelet families for denoising magnetic resonance images of the brain were used such as Haar transform, DB2, DB4, Sym2, Sym4 and others. Evaluation metrics were used for comparing the results. Quality of magnetic resonance image denoised with the Haar wavelet transform were better in visual terms.

Magnetic resonance imaging is used in various fields of medicine to determine a disease such as cancer or tumor. Noise corrupts medical images and contributes to the fact that brings bad diagnosis of the diseases. Therefore, the methods of removing noise and keeping important signal in the best possible condition are very important for further medical research of illness that occurred in a patient.

3 Magnetic Resonance Image Noise

MRI technique is often used in the diagnostics of tumors of different parts of the body. With this technique a high quality image of the human body should be obtained that reveals a possible disease. The patient is scanned using an MRI machine while the MRI images are generated via computer [26].

In this paper we consider MRI images of the brain. MRI images can contain some kind of degradation such as noise. In the MRI images of the brain there are several various types of noise which can be present. Some MRI images can be affected by Gaussian noise, Rician noise, fractional Brownian motion noise, speckle noise, random noise and others.

Technique for generation of fractional Brownian motion noise was explained and implemented in [18] by Rajeswaran and Gokilavani. Fractional Brownian motion is non-stationary stochastic process and it represents continuous Gaussian process that has mean equal to zero. Parameter of this noise is Hurst parameter H $0 < H < 1$, and it determines the kind of the process. Special case is for $H = 1/2$ [27]. Fractional Brownian motion is defined by the following equation:

$$B_H(t) = \frac{1}{\Gamma(H+1/2)} \int_0^t (t-s)^{H-1/2} dB(s) \quad (1)$$

where $B(t)$ represents standard Brownian motion and $H \in (0,1)$ represents Hurst parameter. This equation gives poor results for applications with fractional Brownian motion because of its over-emphasizing of the origin [28]. Instead of Eq. 1 Weyl's integral was introduced:

$$B_H(t) = B_H(0) + \frac{1}{\Gamma(H+1/2)} * \quad (2)$$
$$\Big[\int_{-\infty}^{0} ((t-s)^{H-1/2} - (-s)^{H-1/2}) dB(s) + \int_0^t (t-s)^{H-1/2} dB(s) \Big]$$

Another equation for fractional Brownian motion represented with time-frequency and dual-frequency were described by Oigard et al. in [29].

Second common type of noise in MRI images is Gaussian noise. Noise is distributed evenly over the image, at each pixel of the image random value from Gaussian distribution was added. Gaussian distribution of noise is implemented using the following equation:

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(g-m)^2}{2\sigma^2}} \quad (3)$$

where g is gray level, m is average or mean of the function and σ is the standard deviation.

Rician noise can be obtained from complex Gaussian noise. Rician noise is another noise that can corrupt magnetic resonance images. This noise had probability density function for intensity x given by the following equation:

$$p(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+A^2}{2\sigma^2}} I_0(\frac{xA}{\sigma^2}) \quad (4)$$

Rician noise dependents on the signal in magnetic resonance image, which is not zero-mean. Distribution of Rician noise is closer to Gaussian in bright

regions. Denoising the Rician noise with Wiener filter was implemented by Nowak in [30]. This method refers errors between observed data and intensities of the magnetic resonance images. Magnetic resonance images which had Rician noise could also be denoised using wave atom shrinkage [31].

Multiplicative noise also known as speckle noise, appears in different imaging systems as well as in magnetic resonance images. Speckle noise in medicine images often appeare in images that can provide useful diagnostic information about the disease in the human body. This noise is caused by errors in data transmission. Speckle noise can be based on gamma distribution and in that case it can be defined by the following equation:

$$F(g) = \frac{g^a}{(a-1)!a^\alpha}\, e^{-\frac{g^2}{a}} \tag{5}$$

As it was mentioned before, magnetic resonance imaging techniques can be an effective way to determine the diagnosis of the patient. Magnetic resonance images can be damaged by random noise. Random noise limits the image analysis when processing on computers. It is difficult to carry out an assessment based on MRI that is affected with this kind of noise. Denoising technique makes it possible to eliminate the noise in order to obtain a more certain diagnosis of the patient. Denoising technique uses a filter through which the noise is removed. Methods of filtering can have a defect, such that high-frequency signals are eliminated from the component causing blurred edges in magnetic resonance images. Removing the random noise in the magnetic resonance images using a filter is explained in the next section.

4 Proposed Algorithm

The proposed algorithm for removing noise from images of magnetic resonance technique is non-local means (NL-means) which is based on non-local averaging of all the pixels in images. Non-local means filter for denoising takes a mean of all pixels contained in the image. The measurement is performed between the pixels and the extent of their similarity to the marked pixel. Compared to the local means algorithms, non-local means algorithm gives clearer filter results so less detail is lost in images. Non-local means algorithm was proposed by Buades et al. [32].

Non-local means filter is an efficient method for denoising magnetic resonance image, because it keeps the borders of tissue in the right way. This type of filter has its limitations, because the calculation of similarity weight is exercised over the whole space in the neighborhood. The impact of noise in magnetic resonance imaging significantly affects the accuracy of similarity weight. Non-local algorithm calculates pixel similarity weight of the entire neighborhood. The accuracy similarity weights depend on the level of the noise intensity.

Non-local means algorithm is based on a process of averaging to incorporate all pixes in the image. In the filter processing, the process of averaging may be restricted to $M \times M$ window matrix that includes only some pixels, so that the window matrix $M \times M$ is smaller than the dimensions of the entire image. Value of centered pixel of window matrix is calculated as weighted average of pixels that belong to that window. In our proposed method we used window of the size 3×3 biased by the empirically determined weighted mean of the larger 9×9 window. Non-local means algorithm is based on the definition of the concept of similarity in the local context intensity in the neighborhood of each pixel rather than the intensity which is related only to the pixel itself. Non-local means algorithm is defined by the following equation:

$$u(p) = \frac{1}{C(p)} \int_\Omega v(q) f(p,q) dq \tag{6}$$

where Ω is the area of the image, p and q are two points within the image, $u(p)$ is filtered value of the image at point p while $v(q)$ is unfiltered value of the image at point q. Weighting function is $f(p,q)$. The integral is evaluated over $\forall\ q \in \Omega$. $C(p)$ presents a normalizing factor, defined by following equation:

$$C(p) = \int_\Omega f(p,q) dq \tag{7}$$

For non-local means method which is used to remove the noise of the magnetic resonance images, there are some criteria for testing the performance of this algorithm [33], [18]. There are different types of evaluation metrics for testing the performance such as MSE (mean square error), PSNR (peak to signal ratio), NK (normalized cross correlation), AD (average difference), SC (structural content), MD (maximum difference), NAE (normalized absolute error) and IEF (image enhancement factor). This metrics were presented and used in [18].

The mathematical formula for calculating mean square error (MSE) is defined by the following equation (smaller is better):

$$MSE = \frac{1}{N * N} \sum_{i=1}^{N} \sum_{j=1}^{N} (x^*_{i,j} - x_{i,j})^2 \tag{8}$$

where $x^*_{i,j}$ represents pixels of the original image, $x_{i,j}$

represents pixels of the restored image and N is the dimension of the image.

For peak to signal noise ratio (PSNR) mathematical equation is presented by (larger is better):

$$PSNR = 10 \log \frac{65025}{MSE} \tag{9}$$

The equation for normalized cross correlation (NK) is presented by (closer to 1 is better):

$$NK = \frac{\sum_{i,j}^{N} \sum_{i,j}^{N} x_{i,j}^{*} x_{i,j}}{\sum_{i,j}^{N} \sum_{i,j}^{N} (x_{i,j}^{*})^2} \tag{10}$$

The equation for average difference (AD) is presented by (smaller is better):

$$AD = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} (x_{i,j}^{*} - x_{i,j})}{N * N} \tag{11}$$

Structural content (SC) is defined by the following equation (closer to 1 is better):

$$SC = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} x_{i,j}^{2}}{\sum_{i=1}^{N} \sum_{j=1}^{N} (x_{i,j}^{*})^2} \tag{12}$$

Next metrics that can be used for quality estimation of denoising image techniques is maximum difference (MD). Maximum difference can be calculated by the following equation (smaller is better):

$$MD = \max(|x_{i,j}^{*} - x_{i,j}|) \tag{13}$$

Normalized absolute error (NAE) is defined by the following expression (smaller is better):

$$NAE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} |x_{i,j}^{*} - x_{i,j}|}{\sum_{i=1}^{N} \sum_{j=1}^{N} x_{i,j}^{*}} \tag{14}$$

Last metric that was used in this paper is image enhancement factor (IEF). This metric is presented by the following expression (larger is better):

$$IEF = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} |x_{i,j}^{noise} - x_{i,j}^{*}|}{\sum_{i=1}^{N} \sum_{j=1}^{N} |x_{i,j} - x_{i,j}^{*}|} \tag{15}$$

where $x_{i,j}^{noise}$ is image with noise, $x_{i,j}^{*}$ is original image and $x_{i,j}$ is denoised image.

All this formulas are used for the estimation of the results of denoising the magnetic resonance images using the proposed method.

5 Experimental Results

In this paper experiments for magnetic resonance images denoising were implemented using the following system: Intel ® Core™ i7-3770K CPU at 4GHz, 8GB RAM, Windows 10 Professional OS. Proposed algorithm has been implemented in the $Matlab$ version R2015a. Magnetic resonance images used for testing of the proposed method are from dataset of brain MRI and they can be found free for download at [34]. All test images are downloaded images from web-based medical image depository and all images are 256 gray scale images of the size 256×256. Five axial, T2-wighted brain MRI slices are considered. The images are in .png format. The original images are shown in Fig. 1.

Random noise was generated and inserted into the mentioned magnetic resonance images. At each pixel a random value from the range $[-15, 15]$ from uniform distribution was added. After that proposed nonlocal means filter algorithm was used to remove random noise from images.

Graphical result of denoising image Slice 022 is shown in Fig. 2. Fig. 2(a) shows original image, Fig. 2(b) represents the magnetic resonance image of brain affected by random noise and finally Fig. 2(c) shows the image where the noise is removed by the proposed method.

Table 1: Calculation of evaluation metrics

Evaluation metrics	Slice 22		Slice 32		Slice 42		Slice 52		Slice 62	
	with noise	denoised	with noise	denoised	with noise	denoised	with noise	denoised	with noise	denoised
MSE	39.2341	23.4604	43.7980	31.7724	40.3466	37.9051	38.7754	36.7468	38.4724	38.0105
PSNR	32.1942	34.4275	31.7163	33.1103	32.0727	32.3438	32.2452	32.4786	32.2793	32.3318
NK	1.0345	0.9929	1.0320	0.9915	1.0308	0.9918	1.0327	0.9919	1.0303	0.9930
AD	3.7775	2.6506	3.7648	2.9381	3.7539	3.1320	3.7016	3.1905	3.7062	3.2856
SC	0.9266	1.0082	0.9313	1.0099	0.9351	1.0092	0.9323	1.0095	0.9377	1.0082
MD	84	74	168	80	132	99	104	59	128	81
NAE	0.0909	0.0710	0.0856	0.0726	0.0789	0.0711	0.0710	0.0657	0.0637	0.0602
IEF	-	1.6697	-	1.3749	-	1.0684	-	1.0662	-	1.0161

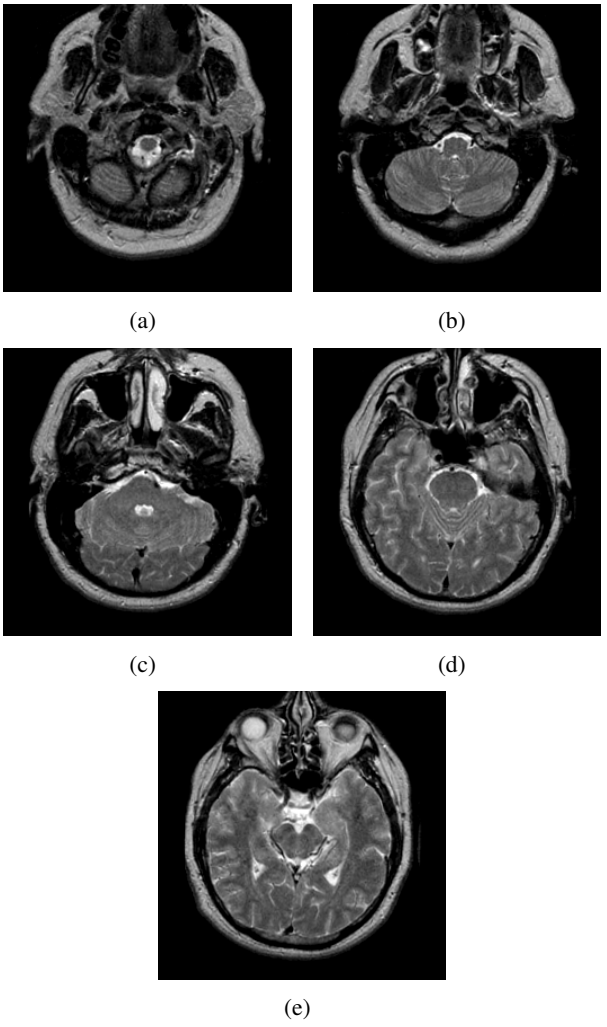

Figure 1: Original magnetic resonance images (a) Slice 22, (b) Slice 32, (c) Slice 42, (d) Slice 52, (e) Slice 62

In the Table 1 the calculation of evaluation metrics MSE, PSNR, IEF, NK, AD, SC, MD and NAE are presented.

6 Conclusion

An algorithm with the adjusted non-local means filter was constructed for removal of the random noise from magnetic resonance images. In our proposed method we used window of the size 3×3 biased by the empirically determined weighted mean of the larger 9×9 window. The proposed algorithm was tested on different magnetic resonance images of brain from the standard database [34]. For the quality of results different measures of quality evaluation metrics were used that included MSE (mean square error), PSNR (peak to signal ratio), NK (normalized cross correlation), AD (average difference), SC (structural content), MD

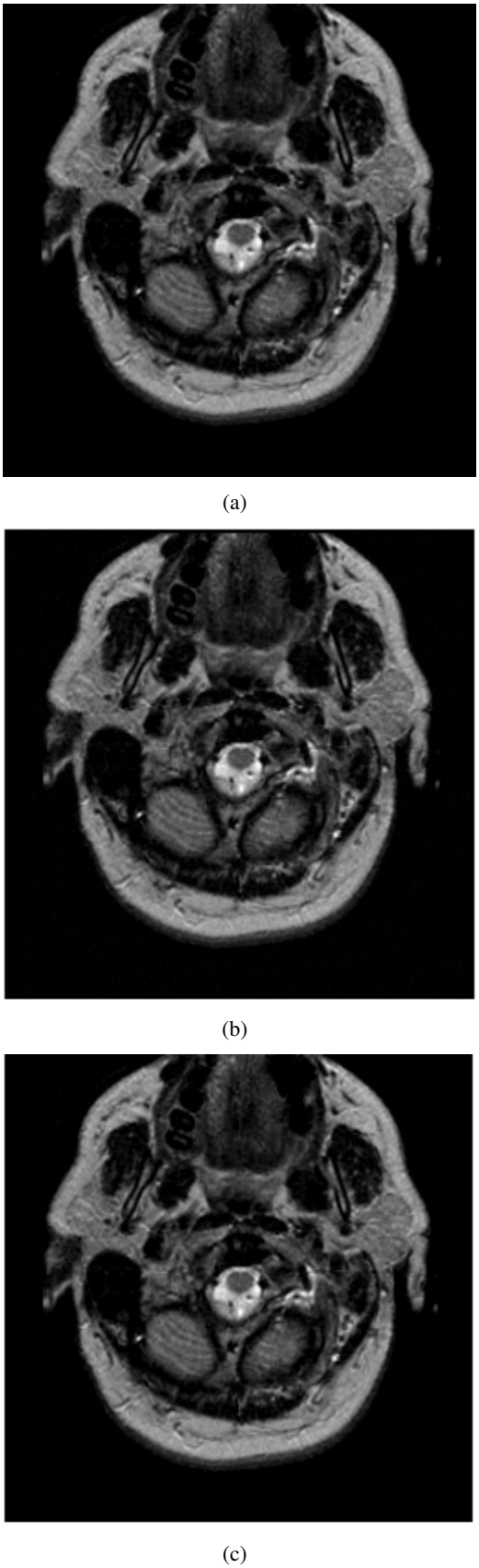

Figure 2: Slice 22 (a) Original MRI, (b) MRI with noise, (c) Denoised MRI

(maximum difference), NAE (normalized absolute error) and IEF (image enhancement factor). In all cases results were satisfactory. Future research can include different types of means filters like weighted median filter and testing can be carried on images of other human organs.

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