An approach to neutrosophic soft rough set and its properties

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Abstract

In this paper a new approach is being introduced to study roughness through neutrosophic soft sets. This new model is called neutrosophic right neighborhood .The concept of neutrosophic soft rough set approximations will be defined, properties of suggested approximations are deduced and proved and then some of neutrosophic soft rough set concepts will be defined along with several propositions and illustrative examples. Finally, we illustrate that classical rough sets model can be viewed as a special case of the suggested model in this paper.

Keywords: neutrosophic set, neutrosophic soft rough set approximations, neutrosophic soft set, rough set approximations, soft set.

1. Introduction

Set theory is a basic branch of a classical mathematics, which requires that all input data must be precise. However, most real life problems in biology, engineering, economics, environmental science, social science, medical science and many other fields, involve imprecise data. In order to describe and extract the useful information hidden in uncertain data, scientists and engineers have become interested in modeling vagueness. In recent years, many theories based on uncertainty have been proposed, such as fuzzy set theory [1], intuitionistic fuzzy set theory [2], vague set theory [3] and theory of interval mathematics [4].

Pawlak [5] initiated the concept of rough set theory as a new approach towards soft computing finding a wide application. It manages the vagueness in data system and has been successfully used to discover the hidden patterns in it, based on what is already known. In Pawlak's work, any vague concept can be replaced by a pair of precise sets called lower and upper approximations, based on an equivalence relation. But, almost real life applications cannot be solved by using equivalence relations. Hence, many generalized models of traditional one, have been proposed to solve this problem. These models based on similarity relation [6], preference relation [7], tolerance relation [8] , dominance relation [9], arbitrary binary relation [10,11], coverings[12,13], different neighborhood operators [14,15], and using uncertain function [16]. There are many research on new developments of rough set and its applications such as near approximations in topological spaces [17], generalized near rough probability in topological spaces [18], topological characterizations of covering for special covering- based upper approximation operators [19], generalized fuzzy rough approximation operators determined by fuzzy implicators [20], mathematical innovations of a modern topology in medical events [21], on some topological properties of pessimistic multigranular rough sets [22], on rough approximations of groups [23], rough approximation-based random model for quarry location, stone materials transportation problem [24], neighborhood rough sets based multi-label classification for automatic image annotation[25] and rough sets determined by tolerances[26].

Soft set was initiated by Molodtsov [27], for dealing with uncertainties. The soft set is a set associated with a set of parameters and has been applied in several directions. Maji et al. [28] discussed the application of soft set theory in a decision making problem and extended classical soft sets to fuzzy soft sets [29]. Yang et al. [30] proposed interval-valued fuzzy soft sets. Chen et al. [31] introduced a new definition of soft set parametrization reduction. Recently, others have developed the classical soft set theory and applied them for solving some real problems in many papers such as temporal analysis of infectious diseases: influenza [32], soft sets and soft groups [33], soft semi rings [34], soft decision making for patients suspected in influenza [35], applications of soft sets in ideal theory of BCK/BCI-algebras [36].

Feng et al. [37] introduced the soft rough set model and proved its properties. Smarandache [38] proposed the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data. Maji [39] introduced neutrosophic soft set which can be viewed as a new path of thinking for engineers, mathematicians, computer scientists and others. This thinking is further extended to the application of neutrosophic set theory in decision making problems such as trapezoidal neutrosophic set and its application to multiple attribute decision-making [40], the generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making [41], and multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean [42].

We will merge the concept of soft rough set and neutrosophic soft set as an attempt to introduce the concept of neutrosophic soft rough set approximations. Properties of suggested approximations are deduced and proved. Neutrosophic soft rough relations will be defined, along with several propositions and illustrative examples.

2. Preliminaries

In this section we recall some definitions and properties regarding rough set, neutrosophic set, soft set and neutrosophic soft set theories required in this paper.

The main idea of rough set theory comes from Pawlak's work [5]. In his work, any vague concept is replaced by a pair of precise concepts called lower and upper approximations. Suppose we are given a set of objects U, called the universe and E is an equivalence relation, representing our knowledge about the elements of U. The space (U, E) is called Pawlak approximation space. To characterize any vague concept $X \subseteq U$, with respect to E, we will need the basic concepts of rough set theory, the following definitions and proposition are given as follows.

Definition 1. [5] An equivalence class of an element $x \in U$, determined by the equivalence relation E is

$$[x]_E = \{x^{'} \in U : E(x) = E(x^{'})\}.$$

Definition 2. [5] Lower, upper and boundary approximations of a subset $X \subseteq U$ are defined as

$$\underline{E}(X) = \cup\{[x]_E : [x]_E \subseteq X\},$$

$$\overline{E}(X) = \cup\{[x]_E : [x]_E \cap X \neq \phi\},$$

$$BND_E(X) = \overline{E}(X) - \underline{E}(X).$$

Definition 3. [5] Let$A = (U, E)$ be an approximation space and let $X \subset U$.
By the accuracy of approximation of X in A we mean the number

$$\alpha_E(X) = \frac{|\ \underline{E}(X)\ |}{|\ \overline{E}(X)\ |}, \overline{E}(X) \neq \emptyset.$$

Obviously, $0 \leq \alpha_E(X) \leq 1$. If $\overline{E}(X) = \underline{E}(X)$, then X is crisp (exact) set, with respect to E, otherwise X is rough set.

Properties of Pawlak's approximations are listed in the following proposition.

Proposition 1. [5] For every $X, Y \subset U$ and every approximation space $A = (U, E)$ the following properties hold:

1. $\underline{E}(X) \subseteq X \subseteq \overline{E}(X)$.
2. $\underline{E}(\phi) = \phi = \overline{E}(\phi)$ and $\underline{E}(U) = U = \overline{E}(U)$.
3. $\overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y)$.
4. $\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$.
5. $X \subseteq Y$, then $\underline{E}(X) \subseteq \underline{E}(Y)$ and $\overline{E}(X) \subseteq \overline{E}(Y)$.
6. $\underline{E}(X \cup Y) \supseteq \underline{E}(X) \cup \underline{E}(Y)$.
7. $\overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y)$.
8. $\underline{E}(X^c) = [\overline{E}(X)]^c$, where X^c is the complement of X.
9. $\overline{E}(X^c) = [\underline{E}(X)]^c$.
10. $\underline{E}(\underline{E}(X)) = \overline{E}(\underline{E}(X)) = \underline{E}(X)$.
11. $\overline{E}(\overline{E}(X)) = \underline{E}(\overline{E}(X)) = \overline{E}(X)$.

Definition 4. [43] An information system is a quadruple $IS = (U, A, V, f)$, where U is a non-empty finite set of objects, A is a non-empty finite set of attributes, $V = \cup\{V_e, e \in A\}$, V_e is the value set of attribute e, and $f : U \times A \to V$, is called an information (knowledge) function.

Definition 5. [27] Let U be an initial universe set, E be a set of parameters, $A \subseteq E$ and let $P(U)$ denotes the power set of U. Then, a pair $S = (F, A)$ is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of S.

Smarandache defined the neutrosophic set as follows.

Definition 6. [38] A neutrosophic set A on the universe of discourse U is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}, \text{where}$$
$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}, \text{where}$$
$$T, I, F : U \longrightarrow]^{-}0; 1^{+}[.$$

Maji defined the neutrosophic soft set as follows.

Definition 7. [39] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$, and let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by

$$F : A \longrightarrow P(U).$$

3. Neutrosophic soft rough set approximations(NSR-set approximations).

In this section, NSR-lower and NSR-upper approximations are introduced and their properties are deduced, proved and illustrated by several examples.

For illustration of the meaning of neutrosophic soft set, we consider the following example.

Example 1. Let U be a set of cars under consideration and E is the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence involving generalized neutrosophic words. Consider E = {beautiful, cheap, expensive, wide, modern, in good repair, costly, comfortable}. In this case, to define a neutrosophic soft set means to point out beautiful cars, cheap cars and so on. Suppose that, there are five cars in the universe U, given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where $A \subset E$, and each e_i is a specific criterion for cars: e_1 stands for (beautiful), e_2 stands for (cheap), e_3 stands for (modern), e_4 stands for (comfortable).

A neutrosophic soft set can be represented in the form of a table as shown in Table 1. In this table, the entries are c_{ij} corresponding to the car h_i and the parameter e_j, where c_{ij} = (true membership value of h_i, indeterminacy-membership value of h_i, falsity membership value of h_i) in $F(e_j)$. Table 1, represents the neutrosophic soft set (F, A) as follows.

U	e_1	e_2	e_3	e_4
h_1	(0.6, 0.6, 0.2)	(0.8, 0.4, 0.3)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
h_2	(0.4, 0.6, 0.6)	(0.6, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.6, 0.6)
h_3	(0.6, 0.4, 0.2)	(0.8, 0.1, 0.3)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
h_4	(0.6, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.5, 0.2, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.2)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table 1. Tabular representation of neutrosophic soft set.

Definition 8. Let (G, A) be a neutrosophic soft set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to $e \in A$ is defined as follows

$$h_e = \{h_i \in U : T_e(h_i) \geq T_e(h) \textit{ and } I_e(h_i) \geq I_e(h) \textit{ and } F_e(h_i) \leq F_e(h)\}.$$

Definition 9. Let (G, A) be a neutrosophic soft set on a universe U. The family of all neutrosophic right neighborhoods is defined as follows

$$\xi = \{h_e : h \in U, e \in A\}.$$

The following example illustrates the meaning of neutrosophic right neighborhoods.

Example 2. Based on the information in Example 1, we can deduce the following statements.

$$h_{1e_1} = h_{1e_2} = h_{1e_3} = h_{1e_4} = \{h_1\},$$
$$h_{2e_1} = h_{2e_3} = \{h_1, h_2\},\ h_{2e_2} = \{h_1, h_2, h_4, h_5\},\ h_{2e_4} = \{h_1, h_2, h_3\},$$
$$h_{3e_1} = h_{3e_4} = \{h_1, h_3\},\ h_{3e_2} = \{h_1, h_3, h_4, h_5\},\ h_{3e_3} = \{h_1, h_3, h_5\},$$
$$h_{4e_1} = \{h_1, h_3, h_4\},\ h_{4e_2} = \{h_4, h_5\},\ h_{4e_3} = U,\ h_{4e_4} = \{h_1, h_2, h_3, h_4\},$$
$$h_{5e_1} = h_{5e_2} = h_{5e_4} = \{h_5\},\ h_{5e_3} = \{h_1, h_5\}.$$

It follows that, $\xi = \{\ \{h_1\}, \{h_5\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_1, h_5\}, \{h_4, h_5\}, \{h_1, h_2, h_3\}, \{h_1, h_3, h_4\}, \{h_1, h_3, h_5\}, \{h_1, h_2, h_3, h_4\}, \{h_1, h_2, h_4, h_5\}, \{h_1, h_3, h_4, h_5\}, U\ \}$.

Proposition 2. Let (G, A) be a neutrosophic soft set on a universe U, ξ is the family of all neutrosophic right neighborhoods on it, and let $R_e : U \to \xi, R_e(h) = h_e$. Then, the following statements are satisfied

1. R_e is reflexive relation.
2. R_e is transitive relation.

Proof.

Let $\langle h_1, T_e(h_1), I_e(h_1), F_e(h_1)\rangle$, $\langle h_2, T_e(h_2), I_e(h_2), F_e(h_2)\rangle$ and $\langle h_3, T_e(h_3), I_e(h_3), F_e(h_3)\rangle \in (G, A)$. Then,

1. Obviously, for all $i = 1, 2, 3$, $T_e(h_i) \geq T_e(h_i)$, $I_e(h_i) \geq I_e(h_i)$, $F_e(h_i) \leq F_e(h_i)$ Hence, for every $e \in A$, $h_i \in h_{ie}$ and $h_i R_e h_i$ and thus R_e is reflexive relation.

2. Let $h_1 R_e h_2$ and $h_2 R_e h_3$. Then, $h_2 \in h_{1e}$ and $h_3 \in h_{2e}$. Hence, $T_e(h_2) \geq T_e(h_1)$, $I_e(h_2) \geq I_e(h_1)$, $F_e(h_2) \leq F_e(h_1)$, $T_e(h_3) \geq T_e(h_2)$, $I_e(h_3) \geq I_e(h_2)$ and $F_e(h_3) \leq F_e(h_2)$. Consequently, we have $T_e(h_3) \geq T_e(h_1)$, $I_e(h_3) \geq I_e(h_1)$ and $F_e(h_3) \leq F_e(h_1)$. It follows that, $h_3 \in h_{1e}$ and $h_1 R_e h_3$ and thus R_e is transitive relation.

Note that R_e in Proposition 2 may not necessarily be symmetric relation,as illustrated by the following example.

Example 3. From Example 2, we have, $h_{1e_1} = \{h_1\}$ and $h_{3e_1} = \{h_1, h_3\}$. Hence, $(h_3, h_1) \in R_{e_1}$ but $(h_1, h_3) \notin R_{e_1}$. Thus, R_e isn't symmetric relation.

Neutrosophic soft lower and upper approximations are defined as follows.

Definition 10. Let (G, A) be a neutrosophic soft set on a universe U, and let ξ be the family of all neutrosophic right neighborhoods. The neutrosophic soft lower and neutrosophic soft upper approximations of any subset X based on ξ, respectively, are

$$NR_* X = \cup\{Y \in \xi : Y \subseteq X\},$$

$$NR^* X = \cup\{Y \in \xi : Y \cap X \neq \emptyset\}.$$

We refer to $NR_* X$ and $NR^* X$ as neutrosophic soft rough approximations of X (NSR-set approximations) with respect to A.

Remark 1. For any considered set X in a neutrosophic soft set (G, A), the sets $Pos_{NR} X = NR_* X$, $Neg_{NR} X = [NR^* X]^c$, $b_{NR} X = NR^* X - NR_* X$, are called the NSR-positive, NSR-negative and NSR-boundary regions of a considered set X, respectively. The meaning of $Pos_{NR} X$ is the set of all elements, which are surely belonging to X, $Neg_{NR} X$ is the set of all elements, which are surely not belonging to X and $b_{NR} X$ is the elements of X, which are not determined by (G, A). Consequently, NSR-boundary region of any considered set is the initial problem of any real life application.

Properties of neutrosophic soft rough approximations are concluded in the following proposition.

Proposition 3. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then the following properties hold.

1. $NR_* X \subseteq X \subseteq NR^* X$.
2. $NR_* \emptyset = NR^* \emptyset = \emptyset$.
3. $NR_* U = NR^* U = U$.
4. $X \subseteq Z \Rightarrow NR_* X \subseteq NR_* Z$.
5. $X \subseteq Z \Rightarrow NR^* X \subseteq NR^* Z$.
6. $NR_* (X \cap Z) \subseteq NR_* X \cap NR_* Z$.
7. $NR_* (X \cup Z) \supseteq NR_* X \cup NR_* Z$.
8. $NR^* (X \cap Z) \subseteq NR^* X \cap NR^* Z$.
9. $NR^* (X \cup Z) = NR^* X \cup NR^* Z$.

Proof.

1. From Definition 10, we can deduce that, $NR_* X \subseteq X$. Also, let $h \in X$, but R_e, defined in Proposition 2, is reflexive relation. For all $e \in A$, there exists h_e such that, $h \in h_e$, and there exists $Y \in \xi$ such that, $Y \cap X \neq \emptyset$. Hence, $h \in NR^* X$. Thus, $NR_* X \subseteq X \subseteq NR^* X$.

2. Proof of 2, follows directly, from Definition 10.

3. From property 1, we have $U \subseteq NR^* U$. Since U is the universe set, $NR^* U = U$. From Definition 10, we have $NR_* U = \cup\{Y \in \xi : Y \subseteq U\}$, but for all $h \in U$, there exists $h_e \in \xi$ such that, $h \in h_e \subseteq U$. Hence, $NR_* U = U$. Thus, $NR_* U = NR^* U = U$.

4. Let $X \subseteq Z$ and $h \in NR_* X$.There exists $Y \in \xi$ such that, $h \in Y \subseteq X$. But $X \subseteq Z$, thus $h \in Y \subseteq Z$. Hence, $h \in NR_* Z$. Consequently, $NR_* X \subseteq NR_* Z$.

5. Let $X \subseteq Z$ and $h \in NR^{*}X$. There exists $Y \in \xi$ such that, $h \in Y$, $Y \cap X \neq \emptyset$. But $X \subseteq Z$, thus $Y \cap Z \neq \emptyset$. Hence, $h \in NR^{*}Z$. Thus, $NR^{*}X \subseteq NR^{*}Z$.

6. Let $h \in NR_{*}(X \cap Z) = \cup\{Y \in \xi : Y \subseteq (X \cap Z)\}$. There exists $Y \in \xi$ such that, $h \in Y \subseteq (X \cap Z)$, $h \in Y \subseteq X$ and $h \in Y \subseteq Z$. Consequently, $h \in NR_{*}X$ and $h \in NR_{*}Z$, implying $h \in NR_{*}X \cap NR_{*}Z$. Thus, $NR_{*}(X \cap Z) \subseteq NR_{*}X \cap NR_{*}Z$.

7. Let $h \notin NR_{*}(X \cup Z) = \cup\{Y \in \xi : Y \subseteq X \cup Z\}$. For all $e \in X$, $h \in Y$, we have $Y \nsubseteq X \cup Z$, thus for all $e \in A$, $h \in Y$, we have $Y \nsubseteq X$ and $Y \nsubseteq Z$. Consequently, $h \notin NR_{*}X$ and $h \notin NR_{*}Z$, implying $h \notin NR_{*}X \cup NR_{*}Z$. Thus, $NR_{*}(X \cup Z) \supseteq NR_{*}X \cup NR_{*}Z$.

8. Let $h \in NR^{*}(X \cap Z) = \cup\{Y \in \xi : Y \cap (X \cap Z) \neq \emptyset\}$. There exists $Y \in \xi$ such that, $h \in Y$, $Y\cap$ $(X \cap Z) \neq \emptyset$, $Y \cap X \neq \emptyset$ and $Y \cap Z \neq \emptyset$. Consequently, $h \in NR^{*}X$ and $h \in NR^{*}Z$, implying $h \in NR^{*}X \cap NR^{*}Z$. Thus, $NR^{*}(X \cap Z) \subseteq NR^{*}X \cap NR^{*}Z$.

9. Let $h \notin NR^{*}(X \cup Z) = \cup\{Y \in \xi : Y \cap (X \cup Z) \neq \emptyset\}$. For all $e \in A$, $h \in Y$, we have $Y \cap (X \cup Z) = \emptyset$. For all $e \in A$, $h \in Y$, we have $Y \cap X = \emptyset$ and $Y \cap Z = \emptyset$. Consequently, $h \notin NR^{*}X$ and $h \notin NR^{*}Z$, implying $h \notin NR^{*}X \cup NR^{*}Z$. Therefore, $NR^{*}(X \cup Z) \supseteq NR^{*}X \cup NR^{*}Z$. Also, let $h \in NR^{*}(X \cup Z)$ $= \cup\{Y \in \xi : Y \cap (X \cup Z) \neq \emptyset\}$, and thus, there exists $Y \in \xi$ such that, $h \in Y$, $Y \cap (X \cup Z) \neq \emptyset$. It follows that, $Y \cap X \neq \emptyset$ or $Y \cap Z \neq \emptyset$. Consequently, $h \in NR^{*}X$ or $h \in NR^{*}Z$. Hence, $h \in NR^{*}X \cup NR^{*}Z$, and $NR^{*}X \cup NR^{*}Z \supseteq NR^{*}(X \cup Z)$. Thus, $NR^{*}X \cup NR^{*}Z = NR^{*}(X \cup Z)$.

The following example illustrates that the converse of property 1 in Proposition 3 doesn't hold.

Example 4. From Example 1, if $X = \{h_1, h_4\}$, then $NR_{*}X = \{h_1\}$ and $NR^{*}X = U$. Hence, $NR_{*}X \neq X$ and $X \neq NR^{*}X$.

The following example illustrates that the converse of property 4 in Proposition 3.2 doesn't hold.

Example 5. From Example 1, if $X = \{h_2\}$ and $Z = \{h_1, h_2\}$, then $NR_{*}X = \emptyset$, $NR_{*}Z = \{h_1, h_2\}$. Thus, $NR_{*}X \neq NR_{*}Z$.

The following example illustrates that the converse of property 5 in Proposition 3 doesn't hold.

Example 6. According to Example 1. Let $A = \{e_1\}$, then, $\xi = \{\{h_1\}, \{h_5\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_1, h_3, h_4\}\}$. If $X = \{h_2\}$ and $Z = \{h_1, h_2\}$, then, $NR^{*}X = \{h_1, h_2\}$ and $NR^{*}Z = \{h_1, h_2, h_3, h_4\}$. Hence, $NR^{*}X \neq NR^{*}Z$.

The following example illustrates that the converse of property 6 in Proposition 3 doesn't hold.

Example 7. From Example 1, if $X = \{h_1, h_3, h_4\}$ and $Z = \{h_1, h_4, h_5\}$, then $NR_* X = \{h_1, h_3, h_4\}$, $NR_* Z = \{h_1, h_4, h_5\}$ and $NR_*(X \cap Z) = \{h_1\}$. Hence, $NR_*(X \cap Z) \neq NR_* X \cap NR_* Z$.

The following example illustrates that the converse of property 7 in Proposition 3 doesn't hold.

Example 8. From Example 1, if $X = \{h_1\}$ and $Z = \{h_2\}$, then $NR_* X = \{h_1\}$, $NR_* Z = \emptyset$ and $NR_*(X \cup Z) = \{h_1, h_2\}$. Hence, $NR_*(X \cup Z) \neq NR_* X \cup NR_* Z$.

The following example illustrates that the converse of property 8 in Proposition 3 doesn't hold.

Example 9. From Example 6, if $X = \{h_2, h_5\}$ and $Z = \{h_1, h_3, h_5\}$, then $NR^* X = \{h_1, h_2, h_5\}$, $NR^* Z = U$ and $NR^*(X \cap Z) = \{h_5\}$. Hence, $NR^*(X \cap Z) \neq NR^* X \cap NR^* Z$.

Proposition 4. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then the following properties hold.

1. $NR_*\ NR_* X = NR_* X$.
2. $NR^*\ NR^* X \supseteq NR^* X$.
3. $NR_*\ NR^* X = NR^* X$.
4. $NR^*\ NR_* X \supseteq NR_* X$.
5. $NR_* X^c \supseteq [NR^* X]^c$.
6. $NR^* X^c \supseteq [NR_* X]^c$.

Proof.

1. Let $W = NR_* X$ and $h \in W = \cup\{Y \in \xi : Y \subseteq X\}$. Then, for some $e \in A$, $h \in Y \subseteq W$. So, $h \in NR_* W$. Hence, $W \subseteq NR_* W$. Thus, $NR_* X \subseteq NR_*\ NR_* X$. Also, from property 1 of Proposition 3, we have $NR_* X \subseteq X$ and by using property 4 of Proposition 3, we get $NR_*\ NR_* X \subseteq NR_* X$. Consequently, $NR_*\ NR_* X = NR_* X$.

2. Let $W = NR^* X$. By using property 1 of Proposition 3, we have $W \subseteq NR^* W$. Thus, $NR^* NR^* X \supseteq NR^* X$.

3. Let $W = NR^* X$. By using property 1 of Proposition 3, we have $NR_* W \subseteq W$. Also, let $h \in W = \cup\{Y \in \xi : Y \cap X \neq \emptyset\}$, hence there exists $Y \in \xi$ such that, $h \in Y \subseteq W$. It follows that, $h \in NR_* W$. Consequently, $W \subseteq NR_* W$, then $W = NR_* W$, but $W = NR^* X$. Thus, $NR_* NR^* X = NR^* X$.

4. Let $W = NR_* X$. By using property 1 of Proposition 3, we have $W \subseteq NR^* W$. Thus, $NR^* NR_* X \supseteq NR_* X$.

5. Let $h \notin NR_* X^c$. Then, for all $Y \in \xi$ such that $h \in Y$, we have $Y \not\subset X^c$ and $Y \cap X^c = \emptyset$. It follows that $Y \cap X \neq \emptyset$, hence $h \in NR^* X$ and $h \notin [NR^* X]^c$. Thus, $NR_* X^c \supseteq [NR^* X]^c$.

6. From property 5 of Proposition 4, we have $NR_* X^c \supseteq [NR^* X]^c$.
Thus, $NR_* X \supseteq [NR^* X^c]^c$ implying $NR^* X^c \supseteq [NR_* X]^c$.

The following example illustrates that the converse of property 2 in Proposition 4 doesn't hold.

Example 10. From Example 6, if $X = \{h_2\}$, then $NR^* X = \{h_1, h_2\}$ and NR^* $NR^* X = \{h_1, h_2, h_3, h_4\}$. Hence NR^* $NR^* X \neq NR^* X$.

The following example illustrates that the converse of property 4 in Proposition 4 doesn't hold.

Example 11. From Example 6, if $X = \{h_1, h_4\}$, then $NR_* X = \{h_1\}$ and NR^* $NR_* X = \{h_1, h_2, h_3, h_4\}$. Hence, NR^* $NR_* X \neq NR_* X$.

The following example illustrates that the converse of property 5 in Proposition 4 doesn't hold.

Example 12. From Example 6, if $X = \{h_3\}$, then $NR_* X^c = \{h_1, h_2, h_5\}$ and $[NR^* X]^c = \{h_2, h_5\}$. Hence, $NR_* X^c \neq [NR^* X]^c$.

The following example illustrates that the converse of property 6 in Proposition 4 doesn't hold.

Example 13. From Example 6, if $X = \{h_1, h_2, h_4, h_5\}$, then $[NR_* X]^c = \{h_3, h_4\}$ and $NR^* X^c = \{h_1, h_3, h_4\}$. Hence, $[NR_* X]^c \neq NR^* X^c$.

Proposition 5. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then,

$$NR_*(X - Z) \subseteq NR_* X - NR_* Z.$$

Proof.

Let $u \in NR_*(X - Z) = \cup\{Y \in \xi : Y \subseteq (X - Z)\}$. There exists $Y \in \xi$ such that, $u \in Y \subseteq (X - Z)$, $u \in Y \subseteq X$ and $u \in Y \nsubseteq Z$. Consequently, $u \in NR_* X$ and $u \notin NR_* Z$, thus $u \in NR_* X - NR_* Z$. Therefore, $NR_*(X - Z) \subseteq NR_* X - NR_* Z$.

The following example illustrates that the converse of proposition 5 doesn't hold.

Example 14. From Example 1, if $X = \{h_1, h_3, h_5\}$ and $Z = \{h_1, h_5\}$, then $NR_* X = \{h_1, h_3, h_5\}$, $NR_* Z = \{h_1, p_5\}$, $NR_*(X - Z) = \emptyset$ and $NR_* X - NR_* Z = \{h_3\}$. Hence, $NR_* X - NR_* Z \neq NR_*(X - Z)$.

Proposition 6. Let (G, A) be a neutrosophic soft set on a universe U, and let $X, Z \subseteq U$. Then, the following property holds.

$$NR^*(X - Z) \neq NR^* X - NR^* Z.$$

The following example illustrates Proposition 6.

Example 15. From Example 6, if $X = \{h_1, h_3, h_5\}$ and $Z = \{h_1, h_5\}$, then $NR^* X = U$, $NR^* Z = U$, $NR^*(X - Z) = \{h_1, h_3, h_4\}$ and $NR^* X - NR^* Z = \emptyset$. Hence, $NR^* X - NR^* Z \neq NR^*(X - Z)$.

Remark 2. A comparison between traditional rough and neutrosophic soft rough approaches, by using their properties, is concluded in Table 2, as follows.

Traditional rough properties	Neutrosophic soft rough properties
$\underline{E}(X \cap Z) = \underline{E}(X) \cap \underline{E}(Z)$	$NR_*(X \cap Z) \subseteq NR_* X \cap NR_* Z$
$\overline{E}(\overline{E}(X)) = \overline{E}(X)$	$NR^* NR^* X \supseteq NR^* X$
$\overline{E}(\underline{E}(X)) = \underline{E}(X)$	$NR^* NR_* X \supseteq NR_* X$
$\underline{E}(X^c) = [\overline{E}(X)]^c$	$NR_* X^c \supseteq [NR^* X]^c$
$\overline{E}(X^c) = [\underline{E}(X)]^c$	$NR^* X^c \supseteq [NR_* X]^c$

Table 2. Comparison between traditional rough and neutrosophic soft rough properties.

4. Neutrosophic soft rough set concepts (NSR-set concepts).

In this section, some of neutrosophic soft rough concepts are defined as a generalization of rough concepts.

Neutrosophic soft rough (NSR) definability of any subset $X \subseteq U$, is defined as follows

Definition 11. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X \subseteq U$. A subset $X \subseteq U$, is called

1. NSR-definable (NSR-exact) set, if $NR_* X = NR^* X = X$.
2. Internally NSR-definable set, if $NR_* X = X$ and $NR^* X \neq X$.
3. Externally NSR-definable set, if $NR_* X \neq X$ and $NR^* X = X$.
4. NSR-rough set, if $NR_* X \neq X$ and $NR^* X \neq X$.

The following example illustrates Definition 11.

Example 16. From Example 6, we can deduce that, $\{h_1, h_2, h_3, h_4\}$ is NSR-definable set, whereas $\{h_1\}$, $\{h_5\}$, $\{h_1, h_2\}$, $\{h_1, h_3\}$, $\{h_1, h_5\}$, $\{h_1, h_3, h_4\}$, $\{h_1, h_3, h_5\}$, $\{h_1, h_2, h_3, h_5\}$, $\{h_1, h_3, h_4, h_5\}$ are internally NSR-definable sets, while the rest of the subsets of U are NSR-rough sets.

We can determine the degree of NSR-crispness (exactness) of any subset $X \subseteq U$, by using NSR_P-accuracy measure denoted by $C_{NSR} X$, which is defined as follows.

Definition 12. Let (G, A) be a neutrosophic soft set on a unverse U and let $X \subseteq U$. Then,

$$C_{NSR} X = \frac{NR_* X}{NR^* X}, \; X \neq \phi.$$

Remark 3. Let (G, A) be a neutrosophic soft set on a unverse U. A subset $X \subseteq U$ is NSR-definable, if and only if, $C_{NSR} X = 1$.

Neutrosophic soft rough (NSR)-membership relations are defined as follows.

Definition 13. Let (G, A) be a neutrosophic soft set on a unverse U, and let $x \in U$, $X \subseteq U$. NSR-membership relations, denoted by $\underline{\in}_{NSR}$ and $\overline{\in}_{NSR}$, are defined as follows.

$$x \underline{\in}_{NSR} X, \text{ if } x \in NR_* X,$$

$$x \overline{\in}_{NSR} X, \text{ if } x \in NR^* X.$$

Proposition 7. Let (G, A) be a neutrosophic soft set on a unverse U, and let $x \in U$, $X \subseteq U$. Then,

1. $x \underline{\in}_{NSR} X \longrightarrow x \in X$.
2. $x \overline{\notin}_{NSR} X \longrightarrow x \notin X$.

Proof. Proof of 1 and 2 follows directly from Definition 13.

The following example illustrates that the converse of Proposition 7 doesn't hold.

Example 17. In Example 1, if $X = \{h_2, h_5\}$, then $NR_* X = \{h_5\}$ and $NR^* X = U$. Hence $h_2 \underline{\notin}_{NSR} X$, although $h_2 \in X$, and $h_3 \notin X$, although $h_3 \overline{\in}_{NSR} X$.

Neutrosophic soft rough (NSR)-inclusion relations are defined as follows.

Definition 14. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. NSR-inclusion relations, denoted by $\underset{\rightarrow}{\subset}_{NSR}$ and $\overset{\rightarrow}{\subset}_{NSR}$, are defined as follows

$$X \underset{\rightarrow}{\subset}_{NSR} Z, \text{ if } NR_* X \subseteq NR_* Z,$$

$$X \overset{\rightarrow}{\subset}_{NSR} Z, \text{ if } NR^* X \subseteq NR^* Z.$$

Proposition 8. Let (G, A) be a neutrosophic soft set on a unverse U and let $X, Z \subseteq U$. Then,

$$X \subseteq Z \longrightarrow X \underset{\rightarrow}{\subset}_{NSR} Z \wedge X \overset{\rightarrow}{\subset}_{NSR} Z.$$

Proof. From Proposition 3, we get the proof directly.

The following example illustrates that the inverse of Proposition 8 doesn't hold.

Example 18. In Example 6, if $X = \{h_1, h_4\}$ and $Z = \{h_1, h_5\}$, then $NR_* X = \{h_1\}$, $NR_* Z = \{h_1, h_5\}$, $NR^* X = \{h_1, h_2, h_3, h_4\}$ and $NR^* Z = U$. Hence, $X \underset{\rightarrow}{\subset}_{NSR} Z$ and $X \overset{\rightarrow}{\subset}_{NSR} Z$, although $X \nsubseteq Z$.

Neutrosophic soft rough (NSR)-equality relations are defined as follows.

Definition 15. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. NSR-equality relations are defined as follows

$$X \overline{\sim}_{NSR} Z, \text{ if } \; NR_* X = NR_* Z,$$

$$X \simeq_{NSR} Z, \text{ if } \; NR^* X = NR^* Z,$$

$$X \approx_{NSR} Z, \text{ if } \; X \eqsim_{NSR} Z \text{ and } X \simeq_{NSR} Z.$$

The following example illustrates Definition 15.

Example 19. In Example 6, suppose $X_1 = \{h_2\}$, $X_2 = \{h_3\}$, $X_3 = \{h_1, h_2\}$, $X_4 = \{h_1, h_4\}$, $X_5 = \{h_3, h_5\}$ and $X_6 = \{h_4, h_5\}$. Then, $NR_* X_1 = NR_* X_2 = \emptyset$, $NR^* X_3 = NR^* X_4 = \{h_1, h_2, h_3, h_4\}$, $NR_* X_5 = NR_* X_6 = \{h_5\}$ and $NR^* X_5 = NR^* X_6 = \{h_1, h_3, h_4, h_5\}$. Consequently, $X_1 \eqsim_{NSR} X_2$, $X_3 \simeq_{NSR} X_4$ and $X_5 \approx_{NSR} X_6$.

Proposition 9. Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then,

1. $X \eqsim_{NSR} NR_* X$.
2. $X = Y \longrightarrow X \approx_{NSR} Z$.
3. $X \subseteq Z$, $Z \eqsim_{NSR} \emptyset \longrightarrow X \eqsim_{NSR} \emptyset$.
4. $X \subseteq Z$, $X \eqsim_{NSR} U \longrightarrow Z \eqsim_{NSR} U$.
5. $X \subseteq Z$, $Z \simeq_{NSR} \emptyset \longrightarrow X \simeq_{NSR} \emptyset$.
6. $X \subseteq Z$, $X \simeq_{NSR} U \longrightarrow Z \simeq_{NSR} U$.

Proof. The proof can be obtained directly from Propositions 3 and 4.

Remark 4. Let (G, A) be a neutrosophic soft set on a unverse U, and let $h \in U$, $X \subseteq U$. If we consider the case where $T_e(h_i) > 0.5$, then $e(h) = 1$, otherwise $e(h) = 0$, and the neutrosophic right neighborhood of an element h is replaced by the following equivalence class $[h] = \{h_i \in U : e(h_i) = e(h),\ e \in A\}$. It follows that the neutrosophic soft rough set approximations will become Pawlak's rough set approximations (proposed lower and upper approximations will be $NR_* X = \{h \in U : [h] \subseteq X\}$ and $NR^* X = \{h \in U : [h] \cap X \neq \emptyset\}$). Thus all properties of traditional rough set approximations will be satisfied, hence Pawlak's approach to rough sets is a special case of the proposed approach in this paper.

5. Conclusion

We have defined the notion of neutrosophic soft rough set approximations by using a new neighborhood named neutrosophic right neighborhood. Several properties of neutrosophic soft rough sets have been defined and propositions and illustrative examples have been presented. Finally, it has been shown that the proposal model is a generalization of Pawlak's model whereby Pawlak's approach to rough sets can be viewed as a special case of neutrosophic soft approach to rough sets. Our future work is to extend this model by using topological and bitopological structures so as to be able to apply it to many practical problems in economics, engineering, and medical science.

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