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An Extended TOPSIS Method for Multiple Attribute Decision Making based on Interval Neutrosophic Uncertain Linguistic Variables

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Abstract: The interval neutrosophic uncertain linguistic variables can easily express the indeterminate and inconsistent information in real world, and TOPSIS is a very effective decision making method more and more extensive applications. In this paper, we will extend the TOPSIS method to deal with the interval neutrosophic uncertain linguistic information, and propose an extended TOPSIS method to solve the multiple attribute decision making problems in which the attribute value takes the form of the interval neutrosophic uncertain linguistic variables and attribute weight is unknown. Firstly, the operational rules and properties for the interval neutrosophic variables are introduced. Then the distance between two interval neutrosophic uncertain linguistic variables is proposed and the attribute weight is calculated by the maximizing deviation method, and the closeness coefficients to the ideal solution for each alternatives. Finally, an illustrative example is given to illustrate the decision making steps and the effectiveness of the proposed method.

Keywords: The interval neutrosophic linguistic, multiple attribute decision making, TOPSIS, maximizing deviation method

I-Introduction

F. Smarandache [7] proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership function. The concept of neutrosophic set is generalization of classic set, fuzzy set [25], intuitionistic fuzzy set [22], interval intuitionistic fuzzy set [23,24] and so on. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. From scientific or engineering point of view, the neutrosophic set and set-theoretic view, operators need to be specified .Otherwise, it will be difficult to apply in the real applications. Therefore, H. Wang et al [8] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Furthermore, H. Wang et al.[9] proposed the set theoretic operations on an instance of neutrosophic set called interval valued neutrosophic set (IVNS) which is more flexible and practical than NS. The works on neutrosophic set (NS) and interval valued neutrosophic set (IVNS), in theories and application have been progressing rapidly (e.g, [1,2,4,6,7,8,9,10,11,12,13,14,15,16,17, ,18,19,20,21,27,28,29,30,31,32,33,35,36,37,38,39,40,41,42 ,43,44,45,46,47,48,53].

Multiple attribute decision making (MADM) problem are of importance in most kinds of fields such as engineering, economics, and management. In many situations decision makers have incomplete , indeterminate and inconsistent information about alternatives with respect to attributes. It is well known that the conventional and fuzzy or intuitionistic fuzzy decision making analysis [26, 50, 51,] using different techniques tools have been found to be inadequate to handle indeterminate an inconsistent data. So, Recently, neutrosophic multicriteria decision making problems have been proposed to deal with such situation.

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method, initially introduced by C. L. Hwang and Yoon [3], is a widely used method for dealing with MADM problems, which focuses on choosing the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). The traditional TOPSIS is only used to solve the decision making problems with crisp numbers, and many extended TOPSIS were proposed to deal with fuzzy information. Z. Yue [55] extended TOPSIS to deal with interval numbers, G. Lee et al.[5] extend TOPSIS to deal wit fuzzy numbers, P. D. Liu and Su [34], Y. Q. Wei and Liu [49] extended TOPSIS to linguistic information environments, Recently, Z. Zhang and C. Wu [53] proposed the single valued neutrosophic or interval neutrosophic TOPSIS method to calculate the relative closeness coefficient of each alternative to the single valued neutrosophic or interval neutrosophic positive ideal solution, based on which the considered alternatives are ranked and then the most desirable one is selected. P. Biswas et al. [32] introduced single –valued neutrosophic multiple attribute decision making problem with incompletely known or completely unknown attribute weight information based on modified GRA.

Based on the linguistic variable and the concept of interval neutrosophic sets, J. Ye [19] defined interval neutrosophic linguistic variable, as well as its operation principles, and developed some new aggregation operators for the interval neutrosophic linguistic information, including interval neutrosophic linguistic arithmetic weighted average (INLAWA) operator, linguistic geometric weighted average(INLGWA) operator and discuss some properties. Furthermore, he proposed the decision making method for multiple attribute decision making (MADM) problems with an illustrated example to show the process of decision making and the effectiveness of the proposed method. In order to process incomplete, indeterminate and inconsistent information more efficiency and precisely J. Ye [20] further proposed the interval neutrosophic uncertain linguistic variables by combining uncertain linguistic variables and interval neutrosophic sets, and proposed the operational rules, score function , accuracy functions ,and certainty function of interval neutrosophic uncertain linguistic variables. Then the interval neutrosophic uncertain linguistic weighted arithmetic averaging (INULWAA) and the interval neutrosophic uncertain linguistic weighted arithmetic averaging (INULWGA) operator are developed, and a multiple attribute decision method with interval neutrosphic uncertain linguistic information was developed.

To do so, the remainder of this paper is set out as follows. Section 2 briefly recall some basic concepts of neutrosphic sets, single valued neutrosophic sets (SVNSs), interval neutrosophic sets(INSs), interval neutrosophic linguistic variables and interval neutrosophic uncertain linguistic variables. In section 3, we develop an extended TOPSIS method for the interval neutrosophic uncertain linguistic variables, In section 4, we give an application example to show the decision making steps, In section 5, a comparison with existing methods are presented. Finally, section 6 concludes the paper.

II-Preliminaries

In the following, we shall introduce some basic concepts related to uncertain linguistic variables, single valued neutrosophic set, interval neutrosophic sets, interval neutrosophic uncertain linguistic sets, and interval neutrosophic uncertain linguistic set.

2.1 Neutrosophic sets

Definition 2.1 [7]

Let U be a universe of discourse then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X \}$,

Where the functions $T_A(x), I_A(x), F_A(x): U \to]^-0,1+[$define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+. \quad \textbf{(1)}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

2.2 Single valued Neutrosophic Sets

Definition 2.2 [8]

Let X be an universe of discourse, then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X \}$,

where the functions $T_A(x), I_A(x), F_A(x) : U \to [0,1]$define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ **(2)**

Definition 2.3 [8]

A single valued neutrosophic set A is contained in another single valued neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$. **(3)**

2.3 Interval Neutrosophic Sets

Definition 2.4[9]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq [0, 1]$.

For two IVNS, A_{IVNS}= {<x, $[T_A^L(x), T_A^U(x)]$, $[I_A^L(x), I_A^U(x)]$, $[F_A^L(x), F_A^U(x)]$ > | x ∈ X **} (4)**

And B_{IVNS}= {<x, $[T_B^L(x), T_B^U(x)]$, $[I_B^L(x), I_B^U(x)]$, $[F_B^L(x), F_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1) $A_{IVNS} \subseteq B_{IVNS}$ If and only if $T_A^L(x) \le T_B^L(x)$, $T_A^U(x) \le T_B^U(x)$, $I_A^L(x) \ge I_B^L(x)$, $I_A^U(x) \ge I_B^U(x)$, $F_A^L(x) \ge F_B^L(x)$, $F_A^U(x) \ge F_B^U(x)$

(2) $A_{IVNS} = B_{IVNS}$ if and only if , $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any x ∈ X

The complement of A_{IVNS} is denoted by A^o_{IVNS} and is defined by

A^o_{IVNS}= {<x, $[F_A^L(x), F_A^U(x)]$>, $[1 - I_A^U(x), 1 - I_A^L(x)]$, $[T_A^L(x), T_A^U(x)]$ | x ∈ X }

A∩B ={ <x , [min($T_A^L(x)$,$T_B^L(x)$), min($T_A^U(x)$,$T_B^U(x)$)], [max($I_A^L(x)$,$I_B^L(x)$), max($I_A^U(x)$,$I_B^U(x)$], [max($F_A^L(x)$,$F_B^L(x)$), max(F(x),$F_B^U(x)$)] >: x ∈ X }

A∪B ={ <x , [max($T_A^L(x)$,$T_B^L(x)$), max($T_A^U(x)$,$T_B^U(x)$)], [min($I_A^L(x)$,$I_B^L(x)$), min($I_A^U(x)$,$I_B^U(x)$], [min($F_A^L(x)$,$F_B^L(x)$), min($F_A^U(x)$,$F_B^U(x)$)] >: x ∈ X }

2.4 Uncertain linguistic variable.

A linguistic set is defined as a finite and completely ordered discreet term set,

$S = (s_0, s_1, \dots, s_{l-1})$, where l is the odd value. For example, when l=7, the linguistic term set S can be defined as follows: S={s_0(extremely low); s_1(very low); s_2(low); s_3(medium); s_4(high); s_5(very high); s_6(extermley high)}

Definition 2.5. Suppose $\tilde{s} = [s_a, s_b]$, where $s_a, s_b \in \tilde{S}$ with a ≤ b are the lower limit and the upper limit of S, respectively. Then $\tilde{s}$ is called an uncertain linguitic varaible.

Definition 2.6. Suppose $\tilde{s}_1 = [s_{a_1}, s_{b_1}]$ and $\tilde{s}_2 = [s_{a_2}, s_{b_2}]$ are two uncertain linguistic variable ,then the distance between $\tilde{s}_1$ and $\tilde{s}_2$ is defined as follows.

$d(\tilde{s}_1, \tilde{s}_2) = \frac{1}{2(l-1)} (|a_2 - a_1| + |b_2 - b_1|)$ **(5)**

2.5 Interval neutrosophic linguistic set

Based on interval neutrosophic set and linguistic variables, J. Ye [18] presented the extension form of the linguistic set, i.e, interval neutroosphic linguistic set, which is shown as follows:

Definition 2.7 :[19] An interval neutrosophic linguistic set A in X can be defined as

A ={<x, $s_{\theta(x)}$, (T_A(x), I_A(x), F_A(x))>| x ∈ X} **(6)**

Where $s_{\theta(x)} \in \hat{s}$, $T_A(x) = [T_A^L(x), T_A^U(x)] \subseteq [0.1]$, $I_A(x) = [I_A^L(x), I_A^U(x)] \subseteq [0.1]$, and $F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0.1]$ with the condition $0 \le T_A^U(x) + I_A^U(x) + F_A^U(x) \le 3$ for any x ∈ X. The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree, the indeterminacy –membership degree, and the falsity-membership degree with interval values of the element x in X to the linguistic variable $s_{\theta(x)}$.

2.6 Interval neutrosophic uncertain linguistic set.

Based on interval neutrosophic set and uncertain linguistic variables, J.Ye [20] presented the extension form of the uncertain linguistic set, i.e, interval neutrosphic uncertain linguistic set, which is shown as follows:

Definition 2.8 :[20] An interval neutrosophic uncertain linguistic set A in X can be defined as

A ={<x,[$s_{\theta(x)}, s_{\rho(x)}$], ($T_A$(x), I_A(x), F_A(x))>| x ∈ X} **(7)**

Where $s_{\theta(x)} \in \hat{s}$, $T_A(x) = [T_A^L(x), T_A^U(x)] \subseteq [0.1]$, $I_A(x) = [I_A^L(x), I_A^U(x)] \subseteq [0.1]$, and $F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0.1]$ with the condition $0 \le T_A^U(x) + I_A^U(x) + F_A^U(x) \le 3$ for any x ∈ X. The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree, the indeterminacy–membership degree, and the falsity-membership degree with interval values of the element x in X to the uncertain linguistic variable [$s_{\theta(x)}, s_{\rho(x)}$].

Definition 2.9 Let $\tilde{a}_1$=< [$s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}$], ([$T^L(\tilde{a}_1)$,$T^U(\tilde{a}_1)$], [$I^L(\tilde{a}_1)$,$I^U(\tilde{a}_1)$], [$F^L(\tilde{a}_1)$,$F^U(\tilde{a}_1)$])> and $\tilde{a}_2$={<x, [$s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}$], ([$T^L(\tilde{a}_2)$,$T^U(\tilde{a}_2)$], [$I^L(\tilde{a}_2)$,$I^U(\tilde{a}_2)$], [$F^L(\tilde{a}_2)$,$F^U(\tilde{a}_2)$])>

be two INULVs and λ ≥ 0, then the operational laws of INULVs are defined as follows:

$\tilde{a}_1 \oplus \tilde{a}_2$ =< $[s_{\theta(\tilde{a}_1)+\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)+\rho(\tilde{a}_2)}]$, ([$T^L(\tilde{a}_1)$+ $T^L(\tilde{a}_2)$- $T^L(\tilde{a}_1)$ $T^L(\tilde{a}_2)$,$T^U(\tilde{a}_1)$+ $T^U(\tilde{a}_2)$- $T^U(\tilde{a}_1)$ $T^U(\tilde{a}_2)$], [$I^L(\tilde{a}_1)$ $I^L(\tilde{a}_2)$,$I^U(\tilde{a}_1)$ $I^U(\tilde{a}_2)$], [$F^L(\tilde{a}_1)$ $F(\tilde{a}_2)$,$F^U(\tilde{a}_1)$ $F^L(\tilde{a}_2)$])> **(8)**

$\tilde{a}_1 \otimes \tilde{a}_2$ =< $[s_{\theta(\tilde{a}_1)\times\theta(\tilde{a}_2)}]$, ([$T^L(\tilde{a}_1)$ $T^L(\tilde{a}_2)$, $T^U(\tilde{a}_1)$ $T^U(\tilde{a}_2)$], [$I^L(\tilde{a}_1)$+ $I^L(\tilde{a}_2)$ - $I^L(\tilde{a}_1)$ $I^L(\tilde{a}_2)$, $I^U(\tilde{a}_1)$+ $I^U(\tilde{a}_2)$- $I^U(\tilde{a}_1)$ $I^U(\tilde{a}_2)$], [$F^L(\tilde{a}_1)$+ $F^L(\tilde{a}_2)$ - $F^L(\tilde{a}_1)$ $F(\tilde{a}_2)$, $F^U(\tilde{a}_1)$+ $F^U(\tilde{a}_2)$ - $F^U(\tilde{a}_1)$ $F^U(\tilde{a}_2)$])> **(9)**

$\lambda\tilde{a}_1$=<$[s_{\lambda\theta(\tilde{a}_1)}, s_{\lambda\rho(\tilde{a}_1)}]$,([1-$(1-T^L(\tilde{a}_1))^\lambda$,1-$(1-T^U(\tilde{a}_1))^\lambda$], [$(I^L(\tilde{a}_1))^\lambda$,$(I^U(\tilde{a}_1))^\lambda$], [$(F^L(\tilde{a}_1))^\lambda$,$(F^U(\tilde{a}_1))^\lambda$]>

(10)

$\tilde{a}_1^\lambda$=< $[s_{\theta^\lambda(\tilde{a}_1)}, s_{\rho^\lambda(\tilde{a}_1)}]$, ([$(T^L(\tilde{a}_1))^\lambda$,$(T^U(\tilde{a}_1))^\lambda$], [1-$(1-I^L(\tilde{a}_1))^\lambda$, 1-$(1-I^U(\tilde{a}_1))^\lambda$], [1-$(1-F^L(\tilde{a}_1))^\lambda$, 1-$(1-F^U(\tilde{a}_1))^\lambda$]> **(11)**

Obviously, the above operational results are still INULVs.

III. The Extended TOPSIS for the Interval Neutrosophic Uncertain Linguistic Variables

A. The description of decision making problems with interval neutrosphic uncertain linguistic information.

For the MADM problems with interval neutrosophic uncertain variables, there are m alternatives A= $(A_1, A_2,\dots, A_m)$ which can be evaluated by n attributes C=$(C_1, C_2,\dots, C_n)$ and the weight of attributes A_i is w_i, and meets the conditions $0 \le w_i \le 1$, $\sum_{j=1}^n w_j$=1.Suppose z_{ij} (i=1, 2,…, n; j=1, 2,…, m) is the evaluation values of alternative A_i with respect to attribute C_j

And it can be represented by interval neutrosophic uncertain linguistic variable z_{ij}= <$[x_{ij}^L, x_{ij}^U]$,([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U])>, where $[x_{ij}^L, x_{ij}^U]$ is the uncertain linguistic variable, and $x_{ij}^L, x_{ij}^U \in$ S, S =$(s_0, s_1,\dots, s_{l-1})$, T_{ij}^L, T_{ij}^U, I_{ij}^L, I_{ij}^U and F_{ij}^L, $F_{ij}^U \in$ [0, 1] and $0 \le T_{ij}^U + I_{ij}^U + F_{ij}^U \le 3$. Suppose attribute weight vector W=$(w_1, w_2,\dots w_n)$ is completely unknown, according to these condition, we can rank the alternatives $(A_1, A_2,\dots, A_m)$

B. Obtain the attribute weight vector by the maximizing deviation.

In order to obtain the attribute weight vector, we firstly define the distance between two interval neutrosophic uncertain variables.

Definition 3.1

Let $\tilde{s}_1$ = <$[s_{a_1}, s_{b_1}]$,([T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U])>, $\tilde{s}_2$ = <$[s_{a_2}, s_{b_2}]$,([T_B^L, T_B^U], [I_B^L, I_B^U], [F_B^L, F_B^U])> and $\tilde{s}_3$ = <$[s_{a_3}, s_{b_3}]$,([T_C^L, T_C^U], [I_C^L, I_C^U], [F_C^L, F_C^U])>, be any three interval neutrosophic uncertain linguistic variables, and $\tilde{S}$ be the set of linguistic variables, f is a map, and $f: \tilde{S} \times \tilde{S} \longrightarrow$ R. If d$(\tilde{s}_1, \tilde{s}_2)$ meets the following conditions

(1) $0 \le d_{INULV}(\tilde{s}_1, \tilde{s}_2) \le 1$, $d_{INULV}(\tilde{s}_1, \tilde{s}_1)$= 0

(2) $d_{INULV}(\tilde{s}_1, \tilde{s}_2)$ = $d_{INULV}(\tilde{s}_2, \tilde{s}_1)$

(3) $d_{IVNS}(\tilde{s}_1, \tilde{s}_2)$ + $d_{INULV}(\tilde{s}_2, \tilde{s}_3) \ge d_{INULV}(\tilde{s}_1, \tilde{s}_3)$

then $d_{INULV}(\tilde{s}_1, \tilde{s}_2)$ is called the distance between two interval neutrosophic uncertain linguistic variables $\tilde{s}_1$

Definition 3.2:

Let $\tilde{s}_1$ = <$[s_{a_1}, s_{b_1}]$,([T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U])>, and $\tilde{s}_2$ = <$[s_{a_2}, s_{b_2}]$,([T_B^L, T_B^U], [I_B^L, I_B^U], [F_B^L, F_B^U])>, be any two interval neutrosophic uncertain linguistic variables, then the Hamming distance between $\tilde{s}_1$ and $\tilde{s}_2$ can be defined as follows.

$$d_{INULV}(\tilde{s}_1, \tilde{s}_2) = \frac{1}{12(l-1)} (|a_1 \times T_A^L - a_2 \times T_B^L| + |a_1 \times T_A^U - a_2 \times T_B^U| + |a_1 \times I_A^L - a_2 \times I_B^L| + |a_1 \times I_A^U - a_2 \times I_B^U| + |a_1 \times F_A^L - a_2 \times F_B^L| + |a_1 \times F_A^U - a_2 \times F_B^U| + |b_1 \times T_A^L - b_2 \times T_B^L| + |b_1 \times T_A^U - b_2 \times T_B^U| + |b_1 \times I_A^L - b_2 \times I_B^L| + |b_1 \times I_A^U - b_2 \times I_B^U| + |b_1 \times F_A^L - b_2 \times F_B^L| + |b_1 \times F_A^U - b_2 \times F_B^U|) \quad (12)$$

In order to illustrate the effectiveness of definition 3.2, the distance defined above must meet the three conditions in definition 3.1

Proof

Obviously, the distance defined in (12) can meets the conditions (1) and (2) in definition 3.1

In the following, we will prove that the distance defined in (12) can also meet the condition (3) in definition 3.1

For any one interval neutrosophic uncertain linguistic variable $\tilde{s}_3$ = <$[s_{a_3}, s_{b_3}]$,([T_C^L, T_C^U], [I_C^L, I_C^U], [F_C^L, F_C^U])>,

$$d_{IVNS}(\tilde{s}_1, \tilde{s}_3) = \frac{1}{12(l-1)} (|a_1 \times T_A^L - a_3 \times T_C^L| + |a_1 \times T_A^U - a_3 \times T_C^U| + |a_1 \times I_A^L - a_3 \times I_C^L| + |a_1 \times I_A^U - a_3 \times I_C^U| + |a_1 \times F_A^L - a_3 \times F_C^L| + |a_1 \times F_A^U - a_3 \times F_C^U| + |b_1 \times T_A^L - b_3 \times T_C^L| + |b_1 \times T_A^U - b_3 \times T_C^U| + |b_1 \times I_A^L - b_3 \times I_C^L| + |b_1 \times I_A^U - b_3 \times I_C^U| + |b_1 \times F_A^L - b_3 \times F_C^L| + |b_1 \times F_A^U - b_3 \times F_C^U|)$$

$= \frac{1}{12(l-1)}$ $(|a_1 \times T_A^L - a_2 \times T_B^L + a_2 \times T_B^L - a_3 \times T_C^L|+|a_1 \times T_A^U - a_2 \times T_B^U + a_2 \times T_B^U - a_3 \times T_C^U|+|a_1 \times I_A^L - a_2 \times I_B^L + a_2 \times I_B^L - a_3 \times I_C^L|+|a_1 \times I_A^U - a_2 \times I_B^U + a_2 \times I_B^U - a_3 \times I_C^U|$
$+|a_1 \times F_A^L - a_2 \times F_B^L + a_2 \times F_B^L - a_3 \times F_C^L|+|a_1 \times F_A^U - a_2 \times F_B^U + a_2 \times F_B^U - a_3 \times F_C^U|$

$+|b_1 \times T_A^L - b_2 \times T_B^L + b_2 \times T_B^L - b_3 \times T_C^L|+|b_1 \times T_A^U - b_2 \times T_B^U + b_2 \times T_B^U - b_3 \times T_C^U|+|b_1 \times I_A^L - b_2 \times I_B^L + b_2 \times I_B^L - b_3 \times I_C^L|+|b_1 \times I_A^U - b_2 \times I_B^U + b_2 \times I_B^U - b_3 \times I_C^U|$
$+|b_1 \times F_A^L - b_2 \times F_B^L + b_2 \times F_B^L - a_3 \times F_C^L|+|b_1 \times F_A^U - b_2 \times F_B^U + b_2 \times F_B^U - b_3 \times F_C^U|$

And
$\frac{1}{12(l-1)}$ $(|a_1 \times T_A^L - a_2 \times T_B^L|+|a_2 \times T_B^L - a_3 \times T_C^L|+|a_1 \times T_A^U - a_2 \times T_B^U|+|a_2 \times T_B^U - a_3 \times T_C^U|+|a_1 \times I_A^L - a_2 \times I_B^L|+|a_2 \times I_B^L - a_3 \times I_C^L|+|a_1 \times I_A^U - a_2 \times I_B^U|+|a_2 \times I_B^U - a_3 \times I_C^U|+$

$|a_1 \times F_A^L - a_2 \times F_B^L|+|a_2 \times F_B^L - a_3 \times F_C^L|+|a_1 \times F_A^U - a_2 \times F_B^U|+|a_2 \times F_B^U - a_3 \times F_C^U|+$

$+|b_2 \times T_B^L - b_3 \times T_C^L|+|b_1 \times T_A^U - b_2 \times T_B^U|+|b_2 \times T_B^U - b_3 \times T_C^U|+|b_1 \times I_A^L - b_2 \times I_B^L|+|b_2 \times I_B^L - b_3 \times I_C^L|+|b_1 \times I_A^U - b_2 \times I_B^U|+|b_2 \times I_B^U - b_3 \times I_C^U|+|b_1 \times F_A^L - b_2 \times F_B^L|+|b_2 \times F_B^L - b_3 \times F_C^L|+|b_1 \times F_A^U - b_2 \times F_B^U|+|b_2 \times F_B^U - b_3 \times F_C^U|)$

$= \frac{1}{12(l-1)}$ $(|a_1 \times T_A^L - a_2 \times T_B^L|+|a_1 \times T_A^U - a_2 \times T_B^U|+|a_1 \times I_A^L - a_2 \times I_B^L|+|a_1 \times I_A^U - a_2 \times I_B^U|+|a_1 \times F_A^L - a_2 \times F_B^L|$
$+|a_1 \times F_A^U - a_2 \times F_B^U|+|b_1 \times T_A^L - b_2 \times T_B^L|+|b_1 \times T_A^U - b_2 \times T_B^U|+|b_1 \times I_A^L - b_2 \times I_B^L|+|b_1 \times I_A^U - b_2 \times I_B^U| +|b_1 \times F_A^L - b_2 \times F_B^L|+|b_1 \times F_A^U - b_2 \times F_B^U|+$
$|a_2 \times T_B^L - a_3 \times T_C^L|+|a_2 \times T_B^U - a_3 \times T_C^U|+|a_2 \times I_B^L - a_3 \times I_C^L|+|a_2 \times I_B^U - a_3 \times I_C^U|+|a_2 \times F_B^L - a_3 \times F_C^L|+|a_2 \times F_B^U - a_3 \times F_C^U|+|b_2 \times T_B^L - b_3 \times T_C^L|+|b_2 \times T_B^U - b_3 \times T_C^U|+|b_2 \times I_B^L - b_3 \times I_C^L|+|b_2 \times I_B^U - b_3 \times I_C^U|+|b_2 \times F_B^L - b_3 \times F_C^L|+|b_2 \times F_B^U - b_3 \times F_C^U|)$

$= \frac{1}{12(l-1)}$ $(|a_1 \times T_A^L - a_2 \times T_B^L|+|a_1 \times T_A^U - a_2 \times T_B^U|+|a_1 \times I_A^L - a_2 \times I_B^L|+|a_1 \times I_A^U - a_2 \times I_B^U|+|a_1 \times F_A^L - a_2 \times F_B^L|$
$+|a_1 \times F_A^U - a_2 \times F_B^U|+|b_1 \times T_A^L - b_2 \times T_B^L|+|b_1 \times T_A^U - b_2 \times T_B^U|+|b_1 \times I_A^L - b_2 \times I_B^L|+|b_1 \times I_A^U - b_2 \times I_B^U| +|b_1 \times F_A^L - b_2 \times F_B^L|+|b_1 \times F_A^U - b_2 \times F_B^U|)+$
$\frac{1}{12(l-1)}$ $(|a_2 \times T_B^L - a_3 \times T_C^L|+|a_2 \times T_B^U - a_3 \times T_C^U|+|a_2 \times I_B^L - a_3 \times I_C^L|+|a_2 \times I_B^U - a_3 \times I_C^U|+|a_2 \times F_B^L - a_3 \times F_C^L|+|a_2 \times F_B^U - a_3 \times F_C^U|+|b_2 \times T_B^L - b_3 \times T_C^L|+|b_2 \times T_B^U - b_3 \times T_C^U|+|b_2 \times I_B^L - b_3 \times I_C^L|+|b_2 \times I_B^U - b_3 \times I_C^U|+|b_2 \times F_B^L - b_3 \times F_C^L|+|b_2 \times F_B^U - b_3 \times F_C^U|)$

$=d_{INULV}(\tilde{s}_1, \tilde{s}_2) + d_{INULV}(\tilde{s}_2, \tilde{s}_3)$
So , $d_{INULV}(\tilde{s}_1, \tilde{s}_2) + d_{INULV}(\tilde{s}_2, \tilde{s}_3) \geq d_{INULV}(\tilde{s}_1, \tilde{s}_3)$

Especially, when T_A^L=T_A^U, I_A^L=I_A^U, F_A^L=F_A^U,and T_B^L=T_B^U, I_B^L=I_B^U, and F_B^L=F_B^U the interval neutrosophic uncertain linguistic variables $\tilde{s}_1, \tilde{s}_2$ can be reduced to single valued uncertain linguistic variables. So the single valued neutrosophic uncertain linguistic variables are the special case of the interval neutrosophic uncertain linguistic variables.

Because the attribute weight is fully unknown, we can obtain the attribute weight vector by the maximizing deviation method. Its main idea can be described as follows. If all attribute values z_{ij} (j=1, 2,…, n) in the attribute C_j have a small difference for all alternatives, it shows that the attribute C_j has a small importance in ranking all alternatives, and it can be assigned a small attribute weight, especially, if all attribute values z_{ij} (j=1, 2,…,n) in the attribute C_j are equal, then the attribute C_j has no effect on sorting, and we can set zero to the weight of attribute C_j. On the contrary, if all attribute values z_{ij} (j=1, 2,…, n) in the attribute C_j have a big difference, the attribute C_j will have a big importance in ranking all alternatives, and its weight can be assigned a big value.
Here, based on the maximizing deviation method, we construct an optimization model to determine the optimal relative weights of criteria under interval neutrosophic uncertain linguistic environment. For the criterion $C_i \in$ C, we can use the distance $d(z_{ij}, z_{kj})$ to represent the deviation between attribute values z_{ij} and z_{kj}, and D_{ij} $=\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j$ can present the weighted deviation sum for the alternative A_i to all alternatives, then

$D_j \ (w_j)$=$\sum_{i=1}^{m} D_{ij}(w_j)$= $\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j$ presents the weighted deviation sum for all alternatives, $D\ (w_j)$= $\sum_{j=1}^{n} D_j(w_j)$= $\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j$, presents total weighted deviations for all alternatives with respect to all attributes.

Based on the above analysis, we can construct a non linear programming model to select the weight vector w by maximizing D (w),as follow:

$$\begin{cases} \text{Max } \mathrm{D}(w_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j \\ s.t \ \sum_{j=1}^{n} w_j^2 \ , w_j \in [0\ ,1], j = 1,2, \dots, n \end{cases} \quad (13)$$

Then we can build Lagrange multiplier function, and get

$$\mathrm{L}(w_j,\lambda) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j + \lambda\, (\sum_{j=1}^{n} w_j^2 \text{-}1)$$

$$\text{Let} \begin{cases} \frac{\partial \mathrm{L}(w_j,\lambda)}{\partial w_j} = \sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj})\, w_j + 2\lambda w_j = 0 \\ \frac{\partial \mathrm{L}(w_j,\lambda)}{\partial w_j} = \sum_{j=1}^{n} w_j^2 - 1 = 0 \end{cases}$$

We can get

$$\begin{cases} 2\lambda = \sqrt{\sum_{\mathrm{j}}^{\mathrm{n}}(\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij}, z_{kj}))^2} \\ w_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij},z_{kj})}{\sqrt{\sum_{\mathrm{j}}^{\mathrm{n}}(\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij},z_{kj}))^2}} \end{cases} \quad \textbf{(14)}$$

Then we can get the normalized attribute weight, and have

$$w_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij},z_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(z_{ij},z_{kj})} \quad \textbf{(15)}$$

C. The Extended TOPSIS Method for the Interval Neutrosophic Uncertain linguistic Information.

The standard TOPSIS method can only process the real numbers, and cannot deal with the interval neutrosophic uncertain linguistic information. In the following, we will extend TOPSIS to process the interval neutrosophic uncertain linguistic variables. The steps are shown as follows

(1) Normalize the decision matrix

Considering the benefit or cost type of the attribute values, we can give the normalized matrix R=(r_{ij}), where r_{ij}=<[r_{ij}^L , r_{ij}^U],],([$\dot{T}_{ij}^L$, $\dot{T}_{ij}^U$], [$\dot{I}_{ij}^L$, $\dot{I}_{ij}^U$], [$\dot{F}_{ij}^L$, $\dot{F}_{ij}^U$])>,The normalization can be made shown as follows.

(i) For benefit type,

$$\begin{cases} r_{ij}^L = x_{ij}^L, r_{ij}^U = x_{ij}^U \quad \text{for } (1 \le \mathrm{i} \le \mathrm{m},\ 1 \le \mathrm{j} \le \mathrm{n}) \\ \dot{T}_{ij}^L = T_{ij}^L,\ \dot{T}_{ij}^U = T_{ij}^U,\ \dot{I}_{ij}^L = I_{ij}^L,\ \dot{I}_{ij}^U = I_{ij}^U,\quad \dot{F}_{ij}^L = F_{ij}^L, \dot{F}_{ij}^U = F_{ij}^U \end{cases} \quad \textbf{(16)}$$

(ii) For cost type,

$$\begin{cases} r_{ij}^L = \mathrm{neg}(x_{ij}^U), r_{ij}^U = \mathrm{neg}(\ x_{ij}^L\ ) \quad \text{for } (1 \le \mathrm{i} \le \mathrm{m},\ 1 \le \mathrm{j} \le \mathrm{n}) \\ \dot{T}_{ij}^L = T_{ij}^L,\ \dot{T}_{ij}^U = T_{ij}^U,\ \dot{I}_{ij}^L = I_{ij}^L,\ \dot{I}_{ij}^U = I_{ij}^U,\quad \dot{F}_{ij}^L = F_{ij}^L, \dot{F}_{ij}^U = F_{ij}^U \end{cases} \quad \textbf{(17)}$$

(2) Construct the weighted normalize matrix

$\mathrm{Y}=[y_{ij}]_{m\times n}$

$$\begin{bmatrix} < [y_{11}^L\ , y_{11}^U],], ([\ \ddot{T}_{11}^L, \ddot{T}_{11}^U], [\ \ddot{I}_{11}^L, \ddot{I}_{11}^U], [\ \ddot{F}_{11}^L, \ddot{F}_{11}^U]) > & \cdots & < [y_{11}^L\ , y_{11}^U],], ([\ \ddot{T}_{1n}^L, \ddot{T}_{1n}^U], [\ \ddot{I}_{1n}^L, \ddot{I}_{1n}^U], [\ \ddot{F}_{1n}^L, \ddot{F}_{1n}^U]) > \\ < [y_{21}^L\ , y_{21}^U],], ([\ \ddot{T}_{21}^L, \ddot{T}_{21}^U], [\ \ddot{I}_{21}^L, \ddot{I}_{21}^U], [\ \ddot{F}_{21}^L, \ddot{F}_{21}^U]) > & \cdots. & < [y_{2n}^L\ , y_{2n}^U],], ([\ \ddot{T}_{2n}^L, \ddot{T}_{2n}^U], [\ \ddot{I}_{2n}^L, \ddot{I}_{2n}^U], [\ \ddot{F}_{2n}^L, \ddot{F}_{2n}^U]) > \\ \cdots & & \cdots\ \cdots \\ < [y_{mn}^L\ , y_{mn}^U],], ([\ \ddot{T}_{mn}^L, \ddot{T}_{mn}^U], [\ \ddot{I}_{mn}^L, \ddot{I}_{mn}^U], [\ \ddot{F}_{mn}^L, \ddot{F}_{mn}^U]) > & \cdots & < [y_{mn}^L\ , y_{mn}^U],], ([\ \ddot{T}_{mn}^L, \ddot{T}_{mn}^U], [\ \ddot{I}_{mn}^L, \ddot{I}_{mn}^U], [\ \ddot{F}_{mn}^L, \ddot{F}_{mn}^U]) > \end{bmatrix}$$

Where

$$\begin{cases} y_{ij}^L = w_j r_{ij}^L, y_{ij}^U = w_j r_{ij}^U \\ \ddot{T}_{ij}^L = 1 - (1 - \dot{T}_{ij}^L)^{w_j}, \ddot{T}_{ij}^U = 1 - (1 - \dot{T}_{ij}^U)^{w_j}, \ddot{I}_{ij}^L = (\dot{I}_{ij}^L)^{w_j}, \ddot{I}_{ij}^U = (\dot{I}_{ij}^U)^{w_j},\ \ddot{F}_{ij}^L = (\dot{F}_{ij}^L)^{w_j}, \ddot{F}_{ij}^U = (\dot{F}_{ij}^U)^{w_j} \end{cases} \quad \textbf{(18)}$$

(3) Identify, the sets of the positive ideal solution Y^+= ($y_1^+, y_2^+, \dots, y_m^+$) and the negative ideal solution Y^-= ($y_1^-, y_2^-, \dots, y_m^-$) , then we can get

$$Y^+ = (y_1^+, y_2^+, \dots, y_m^+) = (< [y_1^{L+}\ , y_1^{U+}], ([\ \ddot{T}_1^{L+}, \ddot{T}_1^{U+}], [\ \ddot{I}_1^{L+}, \ddot{I}_1^{U+}], [\ \ddot{F}_1^{L+}, \ddot{F}_1^{U+}]) >, < [y_2^{L+}\ , y_2^{U+}], ([\ \ddot{T}_2^{L+}, \ddot{T}_2^{U+}], [\ \ddot{I}_2^{L+}, \ddot{I}_2^{U+}], [\ \ddot{F}_2^{L+}, \ddot{F}_2^{U+}]) >, \dots, < [y_n^{L+}\ , y_n^{U+}], ([\ \ddot{T}_n^{L+}, \ddot{T}_n^{U+}], [\ \ddot{I}_n^{L+}, \ddot{I}_n^{U+}], [\ \ddot{F}_n^{L+}, \ddot{F}_n^{U+}]) > \quad (19)$$

$Y^{-}= (y_1^-, y_2^-,\ldots, y_m^-)=$
$)=(< [\boldsymbol{y_1^{L-}}, \boldsymbol{y_1^{U-}}], ([\ddot{\boldsymbol{T}}_1^{L-}, \ddot{\boldsymbol{T}}_1^{U-}], [\ddot{\boldsymbol{I}}_1^{L-}, \ddot{\boldsymbol{I}}_1^{U-}], [\ddot{\boldsymbol{F}}_1^{L-}, \ddot{\boldsymbol{F}}_1^{U-}]) >, < [\boldsymbol{y_2^{L-}}, \boldsymbol{y_2^{U-}}], ([\ddot{\boldsymbol{T}}_2^{L-}, \ddot{\boldsymbol{T}}_2^{U-}], [\ddot{\boldsymbol{I}}_2^{L-}, \ddot{\boldsymbol{I}}_2^{U-}], [\ddot{\boldsymbol{F}}_2^{L-}, \ddot{\boldsymbol{F}}_2^{U-}]) >,\ldots, <$
$[\boldsymbol{y_n^{L-}}, \boldsymbol{y_n^{U-}}], ([\ddot{\boldsymbol{T}}_n^{L-}, \ddot{\boldsymbol{T}}_n^{U-}], [\ddot{\boldsymbol{I}}_n^{L-}, \ddot{\boldsymbol{I}}_n^{U-}], [\ddot{\boldsymbol{F}}_n^{L-}, \ddot{\boldsymbol{F}}_n^{U-}]) >$ **(20)**

Where

$$\begin{cases} \boldsymbol{y_j^{L+}} = \mathbf{max}_i(\boldsymbol{y_{ij}^L}), \boldsymbol{y_j^{U+}} = \mathbf{max}_i(\boldsymbol{y_{ij}^U}), \\ \ddot{\boldsymbol{T}}_j^{L+} = \mathbf{max}_i(\ddot{\boldsymbol{T}}_{ij}^L), \ddot{\boldsymbol{T}}_j^{U+} = \mathbf{max}_i(\ddot{\boldsymbol{T}}_{ij}^U), \ddot{\boldsymbol{I}}_j^{L+} = \mathbf{min}_i(\ddot{\boldsymbol{I}}_{ij}^L), \ddot{\boldsymbol{I}}_j^{U+} = \mathbf{min}_i(\ddot{\boldsymbol{I}}_{ij}^U), \ddot{\boldsymbol{F}}_j^{L+} = \mathbf{min}_i(\ddot{\boldsymbol{F}}_{ij}^L), \ddot{\boldsymbol{F}}_j^{U+} = \mathbf{min}_i(\ddot{\boldsymbol{F}}_{ij}^U), \\ \boldsymbol{y_j^{L-}} = \mathbf{min}_i(\boldsymbol{y_{ij}^L}), \boldsymbol{y_j^{U-}} = \mathbf{min}_i(\boldsymbol{y_{ij}^U}), \\ \ddot{\boldsymbol{T}}_j^{L-} = \mathbf{min}_i(\ddot{\boldsymbol{T}}_{ij}^L), \ddot{\boldsymbol{T}}_j^{U-} = \mathbf{min}_i(\ddot{\boldsymbol{T}}_{ij}^U), \ddot{\boldsymbol{I}}_j^{L-} = \mathbf{max}_i(\ddot{\boldsymbol{I}}_{ij}^L), \ddot{\boldsymbol{I}}_j^{U-} = \mathbf{max}_i(\ddot{\boldsymbol{I}}_{ij}^U), \ddot{\boldsymbol{F}}_j^{L-} = \mathbf{max}_i(\ddot{\boldsymbol{F}}_{ij}^L), \ddot{\boldsymbol{F}}_j^{U-} = \mathbf{max}_i(\ddot{\boldsymbol{F}}_{ij}^U), \end{cases}$$
(21)

(4) Obtain the distance between each alternative and the positive ideal solution, and between each alternative and the negative ideal solution, then we can get

$$D^{+}= (d_1^+, d_2^+,\ldots, d_m^+)$$
$$D^{-}= (d_1^-, d_2^-,\ldots, d_m^-) \qquad \textbf{(22)}$$

Where,

$$\begin{cases} d_i^+ = \left[\sum_{j=1}^{n}\left(d(y_{ij}, y_j^+)\right)^2\right]^{\frac{1}{2}} \\ d_i^- = \left[\sum_{j=1}^{n}\left(d(y_{ij}, y_j^-)\right)^2\right]^{\frac{1}{2}} \end{cases} \qquad \textbf{(23)}$$

Where , $d(y_{ij}, y_j^+)$is the distance between the interval valued neutrosophic uncertain linguistic variables y_{ij} and y_j^+ and $d(y_{ij}, y_j^-)$ is the distance between the interval valued neutrosophic uncertain linguistic variables y_{ij} and y_j^- which can be calculated by (12)

(5) Obtain the closeness coefficients of each alternative to the ideal solution, and then we can get

$$cc_i=\frac{d_i^+}{d_i^+ + d_i^-} \quad \text{(i=1,2,…,m)} \qquad \textbf{(24)}$$

(6) Rank the alternatives

According to the closeness coefficient above, we can choose an alternative with minimum cc_i or rank alternatives according to cc_i in ascending order

IV. An illustrative example

In this part, we give an illustrative example adapted from J. Ye [20] for the extended TOPSIS method to multiple attribute decision making problems in which the attribute values are the interval neutrosophic uncertain linguistic variables.

Suppose that an investment company, wants to invest a sum of money in the best option. To invest the money, there is a panel with four possible alternatives: (1) A_1 is car company; (2) A_2 is food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investement company must take a decision according to the three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is a the environmental impact. The weight vector of the attributes is ω= $(0.35, 0.25, 0.4)^{\mathrm{T}}$.The expert evaluates the four possible alternatives of $\mathrm{A_i}$ (i=1,2,3,4) with respect to the three attributes of $\mathrm{C_j}$ (i=1,2,3), where the evaluation information is expressed by the form of INULV values under the linguistic term set S={s_0=extremely poor, s_1=very poor, s_2= poor, s_3= medium, s_4= good, s_5= very good, s_6= extermely good}.

The evaluation information of an alternative $\mathrm{A_i}$ (i=1, 2, 3) with respect to an attribute $\mathrm{C_j}$ (j=1, 2, 3) can be given by the expert. For example, the INUL value of an alternative $\mathrm{A_1}$ with respect to an attribute $\mathrm{C_1}$ is given as <[s_4, s_5], ([0.4, 0.5],[0.2, 0.3], [0.3, 0.4])> by the expert, which indicates that the mark of the alternative $\mathrm{A_1}$ with respect to the attribute $\mathrm{C_1}$ is about the uncertain linguistic value [s_4, s_5,] with the satisfaction degree interval [0.4 ,0.5], indeterminacy degree interval [0.2, 0.3], and dissatisfaction degree interval [0.3, 0.4]. similarly, the four possible alternatives with respect to the three attributes can be evaluated by the expert, thus we can obtain the following interval neutrosophic uncertain linguistic decision matrix:

$(R)_{\mathrm{m\times n}}=$

$$\begin{bmatrix} < ([s_4, s_5], ([0.4, 0.5], [0.2, 0.3], [0.3, 0.4]) > & < ([s_5, s_6], ([0.4, 0.6], [0.1, 0.2], [0.2, 0.4]) > & < ([s_4, s_5], ([0.2, 0.3], [0.1, 0.2], [0.5, 0.6]) > \\ < ([s_5, s_6], ([0.5, 0.7], [0.1, 0.2], [0.2, 0.3]) > & < ([s_4, s_5], ([0.6, 0.7], [0.1, 0.2], [0.2, 0.3]) > & < ([s_4, s_5], ([0.5, 0.7], [0.2, 0.2], [0.1, 0.2]) > \\ < ([s_5, s_6], ([0.3, 0.5], [0.1, 0.2], [0.3, 0.4]) > & < ([s_5, s_6], ([0.5, 0.6], [0.1, 0.3], [0.3, 0.4]) > & < ([s_4, s_4], ([0.5, 0.6], [0.1, 0.3], [0.1, 0.3]) > \\ < ([s_3, s_4], ([0.7, 0.8], [0.0, 0.1], [0.1, 0.2]) > & < ([s_3, s_4], ([0.5, 0.7], [0.1, 0.2], [0.2, 0.3]) > & < ([s_5, s_6], ([0.3, 0.4], [0.1, 0.2], [0.1, 0.2]) > \end{bmatrix}$$

A. Decision steps

To get the best an alternatives, the following steps are involved:

Step 1: Normalization
Because the attributes are all the benefit types, we don’t need the normalization of the decision matrix X
Step 2: Determine the attribute weight vector W, by formula (24), we can get

w_1= 0.337 , w_2= 0.244 , w_3=0.379

Step 3: Construct the weighted normalized matrix, by formula (18), we can get

$$Y = \begin{bmatrix} <([s_{1.508}, s_{1.885}],([0.175,0.229],[0.545,0.635],[0.635,0.708])> & <([s_{1.225}, s_{1.467}],([0.117,0.201],[0.570,0.675],[0.675,0.800])> & <([s_{1.508}, s_{1.885}],([0.081,0.126],[0.420,0.545],[0.77,0.825])> \\ <([s_{1.885}, s_{2.262}],([0.229,0.365],[0.42,0.545],[0.545,0.635])> & <([s_{0.98},, s_{1.225}],([0.201,0.255],[0.570,0.675],[0.675,0.745])> & <([s_{1.508}, s_{1.885}],([0.231,0.365],[0.545,0.545],[0.420,0.545])> \\ <([s_{1.885}, s_{2.262}],([0.125,0.23],[0.42,0.545],[0.635,0.708])> & <([s_{0.98}, s_{1.225}],([0.156,0.201],[0.570,0.745],[0.745,0.800])> & <([s_{1.508}, s_{1.508}],([0.231,0.292],[0.420,0.635],[0.420,0.635])> \\ <([s_{1.131}, s_{1.508}],([0.364,0.455],[0.0,0.42],[0.42,0.545])> & <([s_{0.735}, s_{0.98}],([0.156,0.255],[0.570,0.674],[0.675,0.745])> & <([s_{1.885}, s_{2.262}],([0.126,0.175],[0.420,0.545],[0.420,0.545])> \end{bmatrix}$$

Step 4: Identify the sets of the positive ideal solution $Y^{+}= (y_1^{+}, y_2^{+}, y_3^{+})$ and the negative ideal solution $Y^{-}= (y_1^{-}, y_2^{-}, y_3^{-})$, by formulas (19)- (21), we can get then we can get

$$Y^{+}= (<([s_{1.885}, s_{2.262}],([0.365,0.455],[0,0.42],[0.42,0.545])>, <([s_{1.225}, s_{1.47}],([0.201,0.255],[0.569,0.674],[0.674,0.745])>, <([s_{1.885}, s_{2.262}],([0.230,0.365],[0.420,0.545],[0.420,0.545])>)$$

$$Y^{-}=(<([s_{1.131}, s_{1.508}],([0.126,0.230],[0.545,0.635],[0.635,0.708])>, <([s_{0.735}, s_{0.98}],([0.117,0.201],[0.569,0.745],[0.745,0.799])>, <([s_{1.508}, s_{1.508}],([0.081,0.126],[0.545,0.635],[0.770,0.825])>)$$

Step 5: Obtain the distance between each alternative and the positive ideal solution, and between each alternative and the negative ideal solution, by formulas (22)-(23), we can get
D^{+}= (0.402, 0.065, 0.089, 0.066)

D^{-}= (0.052, 0.073, 0.080, 0.065)

Step 6: Calculate the closeness coefficients of each alternative to the ideal solution, by formula (24) and then we can get

cc_i = (0.885, 0.472, 0.527, 0.503)
Step 7: Rank the alternatives
According to the closeness coefficient above, we can choose an alternative with minimum to cc_i in ascending order. We can get

$$A_2 \geq A_4 \geq A_3 \geq A_1$$

So, the most desirable alternative is A_2

V-Comparison analysis with the existing interval neutrosophic uncertain linguistic multicriteria decision making method.

Recently, J. Ye [20] developed a new method for solving the MCDM problems with interval neutrosophic uncertain linguistic information. In this section, we will perform a comparison analysis between our new method and the existing method, and then highlight the advantages of the new method over the existing method.

(1) Compared with method proposed proposed by J. Ye [20], the method in this paper can solve the MADM problems with unknown weight, and rank the alternatives by the closeness coefficients. However, the method proposed by J. Ye [20] cannot deal with the unknown weight It can be seen that the result of the proposed method is same to the method proposed in [20].

(2) Compared with other extended TOPSIS method
Because the interval neutrosophic uncertain linguistic variables are the generalization of interval neutrosophic linguistic variables (INLV), interval neutrosophic variables (INV),and intuitionistic uncertain linguistic variable. Obviously, the extended TOPSIS method proposed by J. Ye [19], Z. Wei [54], Z. Zhang and C. Wu [3], are the special cases of the proposed method in this paper.
In a word, the method proposed in this paper is more generalized. At the same time, it is also simple and easy to use.

VI-**Conclusion**

In real decision making, there is great deal of qualitative information which can be expressed by uncertain linguistic variables. The interval neutrosophic uncertain linguistic variables were produced by combining the uncertain linguistic variables and interval neutrosophic set, and could easily express the indeterminate and inconsistent information in real world. TOPSIS had been proved to be a very effective decision making method and has been achieved more and more extensive applications. However, the standard TOPSIS method can only process the real numbers. In this paper, we extended TOPSIS method to deal with the interval neutrosophic uncertain linguistic variables information, and proposed an extended TOPSIS method with respect to the MADM problems in which the attribute values take the form of the interval neutrosophic and attribute weight unknown. Firstly, the operational rules and properties for the interval neutrosophic uncertain linguistic variables were presented. Then the distance between two interval neutrosophic uncertain linguistic variables was proposed and the attribute weight was calculated by the maximizing deviation method, and the closeness coefficient to the ideal solution for each alternative used to rank the alternatives. Finally, an illustrative example was given to illustrate the decision making steps, and compared with the existing method and proved the effectiveness of the proposed method. However, we hope that the concept presented here will create new avenue of research in current neutrosophic decision making area.

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