An multi-criteria decision-making approach based on Choquet integral-based TOPSIS with simplified neutrosophic sets

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Abstract: In this paper, the new approach for multi-criteria decision-making (MCDM) problems is developed based on Choquet integral in the context of simplified neutrosophic environment, where the truth-membership degree, indeterminacy-membership degree and falsity-membership degree for each element are singleton subsets in [0,1]. Firstly, the novel operations of simplified neutrosophic numbers (SNNs) and relational properties are discussed, and the comparison method and distance of SNNs are presented as well. Then two aggregation operators for SNNs, namely the simplified neutrosophic Choquet integral weighted averaging operator and the simplified neutrosophic Choquet integral weighted geometric operator, are defined. The properties among two aggregation operators are further discussed in detail. In addition, based on aggregation operators and TOPSIS (technique for order preference by similarity to ideal solution), a novel approach is developed to handle MCDM problems. Finally, one practical example is provided to illustrate the practicality and effectiveness of the proposed approach. And the comparison analysis is also presented based on the same example.

Keywords: Multi-criteria decision-making; simplified neutrosophic sets; Choquet integral; Aggregation operators

1. Introduction

In many cases, it is difficult for decision-makers to definitely express preference in solving MCDM problems with inaccurate, uncertain or incomplete information. Under these circumstances, Zadeh's fuzzy sets (FSs) [1], where the membership degree is represented by a real number between zero and one, are regarded as an important tool to solve MCDM problems [2, 3], fuzzy logic and approximate reasoning [4], and pattern recognition [5].

However, FSs can not handle some cases that the membership degree is hard to be defined by one specific value. In order to overcome the lack of knowledge of non-membership degrees, Atanassov [6] introduced intuitionistic fuzzy sets (IFSs), the extension of Zadeh's FSs. In addition, Gau and Buehrer [7] defined vague sets. Later, Bustince [8] pointed out that the vague sets and IFSs are mathematically equivalent objects. At present, IFSs have been widely applied in solving MCDM problems [9-16], neural networks [17, 18], medical diagnosis [19], color region extraction [20, 21], and market prediction [22]. IFSs take into account the membership degree, the non-membership degree and the degree of hesitation simultaneously. So it is more flexible and practical in addressing fuzziness and uncertainty than the traditional FSs. Moreover, in some actual cases, the membership degree, the non-membership degree and the hesitation degree of an element in IFSs may be not a specific number. Hence, it was extended to the interval-valued intuitionistic fuzzy sets (IVIFSs) [23]. To handle the situations that people are hesitant to express their preference over objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra and Narukawa [24, 25]. Zhu et al. [26, 27] proposed dual HFSs and outlined their operations and properties. Furthermore, Chen et al. [28] proposed interval-valued hesitant fuzzy sets (IVHFSs) and applied it to MCDM problems. Then Farhadinia [29] discussed the correlation for dual IVHFSs and Peng et al. [30] introduced a MCDM approach with hesitant interval-valued intuitionistic fuzzy sets (HIVIFSs), which is an extension of dual IVHFSs.

Although the FSs theory has been developed and generalized, it can not deal with all sorts of uncertainties in different real problems. Some types of uncertainties, such as the indeterminate information and inconsistent information, can not be handled. For example, when we ask the opinion of an expert about a certain statement, he or she may say the possibility that the statement is true is 0.5, the one that the statement is false is 0.6 and the degree that he or she is not sure is 0.2 [31]. This issue is beyond the scope of the FSs and IFSs. Therefore, some new theories are required.

Smarandache [32, 33] proposed neutrosophic logic and neutrosophic sets (NSs). Rivieccio [34] pointed out that an NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and it lies in $]0^-, 1^+[$, the non-standard unit interval. Obviously, it is the extension of the standard interval $[0, 1]$ of IFSs. And the uncertainty presented here, i.e. indeterminacy factor, is dependent on of truth and falsity values while the incorporated uncertainty is dependent of the degree of belongingness and degree of non-belongingness of IFSs [35]. And the aforementioned example of NSs can be expressed as x(0.5, 0.2, 0.6). However, without specific description, NSs are difficult to apply in real-life situations. Hence, a single-valued neutrosophic sets (SVNSs) was proposed, which is an instance of NSs [31, 35]. Majumdar et al. [35] introduced a measure of entropy of SVNSs. Furthermore, the correlation and correlation coefficient of SVNSs as well as a decision-making method using SVNSs were presented [36]. In addition, Ye [37] also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval [0,1], and proposed an MCDM method using the aggregation operators of SNSs. Wang et al. [38] and Lupiáñez [39] proposed the concept of interval neutrosophic sets (INSs) and gave the set-theoretic operators of INSs. Broumi and Smarandache [40] discussed the correlation coefficient of INSs. Zhang et al. [41] developed the MCDM method based on aggregation operators under interval neutrosophic environment. Furthermore, Ye [42, 43] proposed the similarity measures between SVNSs and INSs based on the relationship between similarity measures and distances. However, in some cases, the SNSs operations provided by Ye [37] may be unreasonable. For instance, the sum of any element and the maximum value should be equal to the maximum one, while it does not hold using the operations [37]. The similarity measures and distances of SVNSs based on those operations also may be incredible. Based on the operations in Ye [37], Peng et al. [44, 45] developed some aggregation operators and outranking relations of SNSs, and applied them to MCDM and MCGDM problems.

However, in those decision-making methods mentioned above, most of the criteria are assumed to be independent of one another. However in real life decision-making problems, the criteria of the problems are often interdependent or interactive. This phenomenon is referred to as correlated criteria in this paper. The Choquet integral [46] is a powerful tool for solving MCDM and MCGDM problems with correlated criteria and has been widely used for this purpose [47-54]. For example, Yager [47] extended the idea of order induced aggregation to the Choquet aggregation and introduced the induced Choquet ordered averaging (I-COA) operator. Meyer and Roubens [48] proposed the fuzzy extension of the Choquet integral and applied it to MCDM problems. Yu et al. [49] used the Choquet integral to propose a hesitant fuzzy aggregation operator and applied it to MCDM problems within a hesitant fuzzy environment. Tan and Chen [50] introduced the intuitionistic fuzzy Choquet integral operator. Tan [51] defined the Choquet integral-based Hamming distance between interval-valued intuitionistic fuzzy values and applied it to MCGDM problems.

Bustince et al. [52] proposed a new MCDM method for interval-valued fuzzy preference relation, which was based on the definition of interval-valued Choquet integrals. Wei et al. [53] developed a generalized triangular fuzzy correlated averaging (GTFCA) operator based on the Choquet integral and OWA operator. Finally, Wang et al. [54] developed some Choquet integral aggregation operators with interval 2-tuple linguistic information and applied them to MCGDM problems.

However, TOPSIS (technique for order preference by similarity to ideal solution), which developed by Hwang and Yoon [55], also plays an important role in solving MCGDM problems and successfully applied in many fields [56-60], whilst the Choquet integral has a critical role in handing MCGDM problems with correlated criteria. Therefore, developing a method of combining these two methods in order to solve simplified neutrosophic MCGDM problems with correlated criteria is seen as a valuable research topic. In this paper, the novel operations and comparison method of SNSs are developed, and the distance of SNSs is proposed. Two aggregation operators are defined based on Choquet integral, and the corresponding properties are discussed. Furthermore, an approach for MCGDM problems with SNSs is developed, which could overcome the drawbacks as we discussed earlier.

The paper is structured as follows. Section 2 contains the definition and the operations of SNSs. In Section 3, some novel operations, comparison method and distance of SNS are defined. In Section 4, we develop two aggregation operators based on Choquet integral, and discuss some properties as well. In Section 5 an approach of MCGDM problems with SNSs is developed. One worked example appear in Section 6. In Section 7 is the conclusion.

2. Preliminaries

In this section, fuzzy measure, the Choquet integral and the definition of HFSs are reviewed. Some operations and comparison laws of HFSs, which will be utilized in the latter analysis, are also presented.

2.1 Fuzzy measure and the Choquet integral

Let $X=\{x_1,x_2,\ldots,x_n\}$ be the set of the criteria, $P(X)$ be the power set of X, then the fuzzy measure μ is defined as follows.

Definition 1 [61]. A fuzzy measure μ on the set X is a set function $\mu: P(X)\to[0,1]$ and satisfies the following axioms:

(1) $\mu(\phi)=0$, $\mu(X)=1$;

(2) if $B_1 \subseteq B_2 \subseteq X$, then $\mu(B_1) \le \mu(B_2)$;

(3) $\mu(B_1 \cup B_2) = \mu(B_1) + \mu(B_2) + \rho\mu(B_1)\mu(B_2)$, for $\forall\ B_1, B_2 \subseteq X$, $B_1 \cap B_2 = \phi$, where $\rho \in (-1, +\infty)$.

In Definition 1, if $\rho = 0$, then the third condition is reduced to the additive measure:

for $\forall B_1, B_2 \subseteq X$, and $B_1 \cap B_2 = \phi$, $\mu(B_1 \cup B_2) = \mu(B_1) + \mu(B_2)$.

If the elements of B_i are independent, then

$$\text{for } \forall B_i \subseteq X,\ \mu(B_i) = \sum_{x_i \in B_i} \mu(x_i). \tag{1}$$

In Definition 1, if $\rho = 0$, then the fuzzy measure is a probability measure and the elements are independent; if $-1 < \rho < 0$, then a redundant relation exists among elements; if $\rho > 0$, then a complementary relation exists among elements.

Definition 2 [46]. Let μ be a fuzzy measure on $(X, P(X))$, $f : X \to [0, +\infty)$, then the Choquet integral on f with respect to μ can be defined as follows:

$$\int_X f d\mu = \int_0^{+\infty} \mu(\{x : f(x) > t\}) dt,$$

where $\{x : f(x) > t\} \in P(X)$ for $\forall\ t \in R^+$. If $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set, then the discrete Choquet integral can be described as:

$$\int_X f d\mu = \sum_{i=1}^{n} f(x_{\sigma(i)})\left(\mu(B_{\sigma(i)}) - \mu(B_{\sigma(i+1)})\right), \tag{2}$$

or

$$\int_X f d\mu = \sum_{i=1}^{n}\left(f(x_{\sigma(i)}) - f(x_{\sigma(i-1)})\right)\mu(B_{\sigma(i)}). \tag{3}$$

Where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, such that $0 \le f(x_{\sigma(1)}) \le f(x_{\sigma(2)}) \le \ldots \le f(x_{\sigma(n)})$, $f(x_{\sigma(0)}) = 0$, $B_{\sigma(i)} = \{x_{\sigma(i)}, x_{\sigma(i+1)}, \ldots, x_{\sigma(n)}\}$ and $\mu(B_{\sigma(n+1)}) = 0$.

Example 1. Let $X = \{x_1, x_2, x_3\}$, $x_1 < x_2 < x_3$, and $f(x) = 2^x$, then $f(x_1) < f(x_2) < f(x_3)$, so $\sigma(1) = 1$, $\sigma(2) = 2$, $\sigma(3) = 3$, $A_1 = \{x_1, x_2, x_3\}$, $A_2 = \{x_2, x_3\}$, $A_3 = \{x_3\}$. Suppose $\mu(x_1) = 0.3$, $\mu(x_2) = 0.25$,

$\mu(x_3)=0.37$, $\mu(x_1,x_2)=0.52$, $\mu(x_1,x_3)=0.65$, $\mu(x_2,x_3)=0.45$, $\mu(x_1,x_2,x_3)=1$; if they are calculated by using Eq. (3), then the following is obtained:

$$\int_X f d\mu = \left(f(x_1)-f(x_{\sigma(0)})\right)\mu(B_1)+\left(f(x_2)-f(x_1)\right)\mu(B_2)+\left(f(x_3)-f(x_2)\right)\mu(B_3)$$

$$=\left(2^{x_1}-0\right)\times 1+\left(2^{x_2}-2^{x_1}\right)\times 0.45+\left(2^{x_3}-2^{x_2}\right)\times 0.37 .$$

If $x_1=1,\ x_2=2,\ x_3=3$, then we have $\int_X f d\mu = 4.38$.

2.2 NSs and SNSs

In this section, the definitions of NSs and SNSs are introduced for the latter analysis.

Definition 3 [32]. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x)+\sup I_A(x)+\sup F_A(x) \le 3^+$.

Definition 4 [32]. An NS A_1 is contained in the another NS A_2, denoted by $A_1 \subseteq A_2$, if and only if $\inf T_{A_1}(x) \le \inf T_{A_2}(x)$, $\sup T_{A_1}(x) \le \sup T_{A_2}(x)$, $\inf I_{A_1}(x) \ge \inf I_{A_2}(x)$, $\sup I_{A_1}(x) \ge \sup I_{A_2}(x)$, $\inf F_{A_1}(x) \ge \inf F_{A_2}(x)$ and $\sup F_{A_1}(x) \ge \sup F_{A_2}(x)$ for any $x \in X$.

Since it is difficult to apply NSs to practical problems, Ye [37] reduced NSs of nonstandard intervals into the SNSs of standard intervals that will preserve the operations of the NSs.

Definition 5 [37]. Let X be a space of points (objects), with a generic element in X denoted by x. An NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, and $F_A(x): X \to [0,1]$. Then, a simplification of A is denoted by

$$A=\left\{\left\langle x, T_A(x), I_A(x), F_A(x)\right\rangle \mid x \in X\right\} \qquad (4)$$

which is called an SNS. It is a subclass of NSs. For convenience, the SNSs is denoted by the simplified symbol $A=\{\langle T_A(x), I_A(x), F_A(x)\rangle\}$. The set of all SNSs is represented as SNSS.

The operations of SNSs are also defined by Ye [37].

Definition 6 [37]. Let A, A_1 and A_2 be three SNSs. For any $x \in X$, the following operations are true.

(1) $A_1 + A_2$
$$=\left\langle T_{A_1}(x)+T_{A_2}(x)-T_{A_1}(x)\cdot T_{A_2}(x), I_{A_1}(x)+I_{A_2}(x)-I_{A_1}(x)\cdot I_{A_2}(x), F_{A_1}(x)+F_{A_2}(x)-F_{A_1}(x)\cdot F_{A_2}(x)\right\rangle;$$

(2) $A_1 \cdot A_2 = \left\langle T_{A_1}(x)\cdot T_{A_2}(x), I_{A_1}(x)\cdot I_{A_2}(x), F_{A_1}(x)\cdot F_{A_2}(x)\right\rangle$;

(3) $\lambda \cdot A = \left\langle 1-\left(1-T_A(x)\right)^{\lambda}, 1-\left(1-I_A(x)\right)^{\lambda}, 1-\left(1-F_A(x)\right)^{\lambda}\right\rangle, \lambda > 0$;

(4) $A^{\lambda} = \left\langle T_A^{\lambda}(x), I_A^{\lambda}(x), F_A^{\lambda}(x)\right\rangle, \lambda > 0$.

There are some limitations related to Definition 6 and these are now outlined.

(1) In some situations, operations, such as $A_1 + A_2$ and $A_1 \cdot A_2$, might be impractical. This can be demonstrated in the example below.

Example 2. Let $A_1 = \langle 0.5, 0.5, 0.5\rangle$ and $A_2 = \langle 1, 0, 0\rangle$ be two SNSs. Clearly, $A_2 = \langle 1, 0, 0\rangle$ is the larger of these SNSs. Theoretically, the sum of any number and the maximum number should be equal to the maximum one. However, according to Definition 6, $A_1 + A_2 = \langle 1, 0.5, 0.5\rangle \neq A_2$, therefore the operation "+" cannot be accepted. Similar contradictions exist in other operations of Definition 6, and thus those defined above are incorrect.

(2) The correlation coefficient of SNSs [36], which is based on the operations of Definition 6, cannot be accepted in some special cases.

Example 3. Let $A_1 = \langle 0.8, 0, 0\rangle$ and $A_2 = \langle 0.7, 0, 0\rangle$ be two SNSs, and $A = \langle 1, 0, 0\rangle$ be the largest one of the SNSs. According to the correlation coefficient of SNSs [36], $W\left(A_1, A\right) = W_2\left(A_2, A\right) = 1$ can be obtained, but this does not indicate which one is the best. However, it is clear that A_1 is superior to A_2.

(3) In addition, the cross-entropy measure for SNSs [42], which is based on the operations of Definition 6, cannot be accepted in special cases.

Example 4. Let $A_1 = \langle 0.1, 0, 0\rangle$ and $A_2 = \langle 0.9, 0, 0\rangle$ be two SNSs, and $A = \langle 1, 0, 0\rangle$ be the largest one of the

SNSs. According to the cross-entropy measure for SNSs [42], $S_1(A_1, A) = S_2(A_2, A) = 1$ can be obtained, which indicates that A_1 is equal to A_2. Yet it is not possible to discern which one is the best. Since $T_{A_2}(x) > T_{A_1}(x)$, $I_{A_2}(x) > I_{A_1}(x)$ and $F_{A_2}(x) > F_{A_1}(x)$ for any x in X, it is clear that A_2 is superior to A_1.

(4) If $I_{A_1}(x) = I_{A_2}(x)$ for any x in X, then A_1 and A_2 are both reduced to two IFSs. However, the operations presented in Definition 6 are not in accordance with the laws of two IFSs [9-22].

Definition 7 [37]. Let $X = \{x_1, x_2, \ldots, x_n\}$, A_1 and A_2 be two SNSs, then A_1 is contained in A_2 i.e. $A_1 \subseteq A_2$ if and only if $T_{A_1}(x) \le T_{A_2}(x)$, $I_{A_1}(x) \ge I_{A_2}(x)$ and $F_{A_1}(x) \ge F_{A_2}(x)$ for any $x \in X$.

Obviously, if the equal is not accepted, then we have $A_1 \subset A_2$.

3. The novel operations, comparison method and distance of SNNs

Subsequently, the novel operations, the comparison method and distance of SNSs are defined.

Definition 8. Let A, A_1 and A_2 be three SNNs. Then the operations of SNNs can be defined as follows:

(1) $\lambda_{*e} A = \left\langle \dfrac{(1+T_A)^\lambda - (1-T_A)^\lambda}{(1+T_A)^\lambda + (1-T_A)^\lambda}, \dfrac{2\cdot(I_A)^\lambda}{(2-I_A)^\lambda + (I_A)^\lambda}, \dfrac{2\cdot(F_A)^\lambda}{(2-F_A)^\lambda + (F_A)^\lambda} \right\rangle, \lambda > 0$;

(2) $(A)^{\wedge_e \lambda} = \left\langle \dfrac{2\cdot(T_A)^\lambda}{(2-T_A)^\lambda + (T_A)^\lambda}, \dfrac{(1+I_A)^\lambda - (1-I_A)^\lambda}{(1+I_A)^\lambda + (1-I_A)^\lambda}, \dfrac{(1+F_A)^\lambda - (1-F_A)^\lambda}{(1+F_A)^\lambda + (1-F_A)^\lambda} \right\rangle, \lambda > 0$;

(3) $A_1 \oplus_e A_2 = \left\langle \dfrac{T_{A_1} + T_{A_2}}{1 + T_{A_1} \cdot T_{A_2}}, \dfrac{I_{A_1} \cdot I_{A_2}}{1 + (1-I_{A_1})\cdot(1-I_{A_2})}, \dfrac{F_{A_1} \cdot F_{A_2}}{1 + (1-F_{A_1})\cdot(1-F_{A_2})} \right\rangle$;

(4) $A_1 \otimes_e A_2 = \left\langle \dfrac{T_{A_1} \cdot T_{A_2}}{1 + (1-T_{A_1})\cdot(1-T_{A_2})}, \dfrac{I_{A_1} + I_{A_2}}{1 + I_{A_1} \cdot I_{A_2}}, \dfrac{F_{A_1} + F_{A_2}}{1 + F_{A_1} \cdot F_{A_2}} \right\rangle$.

Theorem 1. Let A_1, A_2 and A_3 be three SNNs, then the following equations are true.

(1) $A_1 \oplus A_2 = A_2 \oplus A_1$;

(2) $A_1 \otimes A_2 = A_2 \otimes A_1$;

(3) $\lambda(A \oplus B) = \lambda A \oplus \lambda B, \lambda > 0$;

(4) $(A\otimes B)^{\lambda} = A^{\lambda} \oplus B^{\lambda}, \lambda > 0$;

(5) $\lambda_1 A \oplus \lambda_2 A = (\lambda_1 + \lambda_2) A, \lambda_1 > 0, \lambda_2 > 0$;

(6) $A^{\lambda_1} \otimes A^{\lambda_2} = A^{(\lambda_1+\lambda_2)}, \lambda_1 > 0, \lambda_2 > 0$;

(7) $(A\oplus B)\oplus C = A\oplus(B\oplus C)$;

(8) $(A\otimes B)\otimes C = A\otimes(B\otimes C)$.

Example 5. Let $A_1 = \langle 0.6, 0.1, 0.2\rangle$ and $A_2 = \langle 0.5, 0.3, 0.4\rangle$ be two SNNs, and $\lambda = 2$, then we have following results.

(1) $2\cdot A_1 = \left\langle 1-(1-0.6)^2, 0.1^2, 0.2^2\right\rangle = \langle 0.84, 0.01, 0.04\rangle$;

(2) $A_1{}^2 = \left\langle 0.6^2, 1-(1-0.1)^2, 1-(1-0.2)^2\right\rangle = \langle 0.36, 0.19, 0.36\rangle$;

(3) $A_1 \oplus A_2 = \langle 0.6+0.5-0.6\times 0.5, 0.1\times 0.3, 0.2\times 0.4\rangle = \langle 0.80, 0.03, 0.08\rangle$;

(4) $A_1 \otimes A_2 = \langle 0.6\times 0.5, 0.1+0.3-0.1\times 0.3, 0.2+0.4-0.2\times 0.4\rangle = \langle 0.30, 0.37, 0.52\rangle$.

Definition 9. The complement of an SNN A is denoted by A^C, which defined by $A^C = \langle 1-T_A, 1-I_A, 1-F_A\rangle$ for any $x\in X$.

Definition 10. Let A_1 and A_2 be two SNNs, then $A_1 = A_2$ if and only if $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

Based on the score function and accuracy function of IFNs (Xu 2007, 2008, 2010; Yager 2009), the score function, accuracy function and certainty function of an SNN are defined as follows.

Definition 11. Let $A = \langle T_A, I_A, F_A\rangle$ be an SNN, and then the score function $s(A)$, accuracy function $a(A)$ and certainty function $c(A)$ of an SNN are defined as follows:

(1) $s(A) = (T_A + 1 - I_A + 1 - F_A)/3$;

(2) $a(A) = T_A - F_A$;

(3) $c(A) = T_A$.

The score function is an important index in ranking SNNs. For an SNN *A*, the bigger the truth-membership T_A is, the greater the SNN will be; furthermore, the smaller the indeterminacy-membership I_A is, the greater the SNN will be; similarly, the smaller the false-membership F_A is, the greater the SNN will be. For the accuracy function, the bigger the difference between truth and falsity, the more affirmative the statement is. As for the certainty function, the certainty of any SNN positively depends on the value of truth- membership T_A.

On the basis of Definition 11, the method for comparing SNNs can be defined as follows.

Definition 12. Let A_1 and A_2 be two SNNs. The comparison method can be defined as follows:

(1) If $s(A_1) > s(A_2)$, then A_1 is greater than A_2, denoted by $A_1 > A_2$;

(2) If $s(A_1) = s(A_2)$ and $a(A_1) > a(A_2)$, then A_1 is greater than A_2, denoted by $A_1 > A_2$;

(3) If $s(A_1) = s(A_2)$, $a(A_1) = a(A_2)$ and $c(A_1) > c(A_2)$, then A_1 is greater than A_2, denoted by $A_1 > A_2$;

(4) If $s(A_1) = s(A_2)$, $a(A_1) = a(A_2)$ and $c(A_1) = c(A_2)$, then A_1 is equal to A_2, denoted by $A_1 = A_2$.

Example 6. Based on Example 3 and Definition 11, $s(A_1) = \frac{0.8+1-0+1-0}{3} = \frac{2.8}{3}$ and $s(A_2) = \frac{0.7+1-0+1-0}{3} = \frac{2.7}{3}$ can be obtained. According to Definition 12, $s(A_1) > s(A_2)$, therefore $A_1 > A_2$ i.e. A_1 is greater than A_2, which avoids the drawbacks discussed in Example 3.

Example 6. Based on Example 4 and Definition 11, $s(A_1) < s(A_2)$, then $A_2 > A_1$, i.e., A_2 is greater than A_1, which also avoids the shortcomings discussed in Example 4.

Definition 13. Let $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ and $\tilde{A}_j = \langle \tilde{T}_{\tilde{A}_j}, \tilde{I}_{\tilde{A}_j}, \tilde{F}_{\tilde{A}_j} \rangle (j = 1,2,\ldots,n)$ be two collections of SNNs, then the generalized simplified netrosophic normalized distance between A_j and $\tilde{A}_j$ can be defined as follows:

$$d_{gsnn}(A_j, \tilde{A}_j) = \left(\frac{1}{3n} \sum_{j=1}^{n} w_j \left(\left| T_{A_j} - \tilde{T}_{A_j} \right|^{\lambda} + \left| I_{A_j} - \tilde{I}_{A_j} \right|^{\lambda} + \left| F_{A_j} - \tilde{F}_{A_j} \right|^{\lambda} \right) \right)^{1/\lambda}. \quad (5)$$

If $\lambda = 1$, then the generalized weighted simplified netrosophic normalized distance is reduced to the weighted simplified neutrosophic normalized Hamming distance:

$$d_{gsnn}(A_j, \tilde{A}_j) = \frac{1}{3n} \sum_{j=1}^{n} w_j \left(\left| T_{A_j} - \tilde{T}_{A_j} \right| + \left| I_{A_j} - \tilde{I}_{A_j} \right| + \left| F_{A_j} - \tilde{F}_{A_j} \right| \right). \quad (6)$$

If $\lambda = 2$, then the generalized weighted simplified netrosophic normalized distance is reduced to the weighted simplified neutrosophic normalized Euclidean distance:

$$d_{gsnn}(A_j,\tilde{A}_j)=\left(\frac{1}{3n}\sum_{j=1}^{n}w_j\left(\left|T_{A_j}-\tilde{T}_{A_j}\right|^2+\left|I_{A_j}-\tilde{I}_{A_j}\right|^2+\left|F_{A_j}-\tilde{F}_{A_j}\right|^2\right)\right)^{\frac{1}{2}}. \qquad (7)$$

4. Generalized simplified neutrosophic operators based on Choquet integral

In this section, the aggregation operators of SNNs are introduced, the corresponding properties are discussed as well.

Definition 14. Let $A_j=\left\langle T_{A_j},I_{A_j},F_{A_j}\right\rangle$ $(j=1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. Based on fuzzy measure, a simplified neutrosophic Choquet integral weighted averaging (SNCIWA) operator of dimension n is a mapping SNCIWA: $\mathrm{SNN}^n\rightarrow\mathrm{SNN}$ such that

$$\begin{aligned}&SNCIWA_\mu\left(A_1,A_2,\ldots,A_n\right)\\&=\left(\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)\right)A_{\sigma(1)}\oplus\left(\mu\left(B_{\sigma(2)}\right)-\mu\left(B_{\sigma(3)}\right)\right)A_{\sigma(2)}\oplus\ldots\oplus\left(\mu\left(B_{\sigma(n)}\right)-\mu\left(B_{\sigma(n+1)}\right)\right)A_{\sigma(n)}.\end{aligned} \qquad (8)$$

Here $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, and such that $A_{\sigma(1)}\le A_{\sigma(2)}\le\ldots\le A_{\sigma(n)}$. $B_{\sigma(i)}=(\sigma(i),\ldots\sigma(n))$, and $B_{\sigma(n+1)}=\varnothing$.

Theorem 2. Let $A_j=\left\langle T_{A_j},I_{A_j},F_{A_j}\right\rangle$ $(j=1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. Then their aggregated result using the *SNCIWA* operator is also an SNN, and

$$SNCIWA_\mu\left(A_1,A_2,\ldots,A_n\right)=\left\langle\frac{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}},\right.$$
$$\frac{2\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}, \qquad (9)$$
$$\left.\frac{2\prod_{i=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}\right\rangle.$$

Here $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, and such that $A_{\sigma(1)}\le A_{\sigma(2)}\le\ldots\le A_{\sigma(n)}$. $B_{\sigma(j)}=(\sigma(j),\ldots\sigma(n))$, and $B_{\sigma(n+1)}=\varnothing$.

Proof. For simplicity, let $w_{\sigma(j)}=\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)$ in the process of proof. By using the mathematical induction on n:

(1) If $n=2$, based on the operations (1) and (3) in Definition 8,

$$\frac{\dfrac{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}+\dfrac{\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}{1+\dfrac{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\cdot\dfrac{\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}$$

$$=\frac{\left[\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\right]\cdot\left[\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}\right]+\left[\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}\right]\cdot\left[\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\right]}{\left[\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\right]\cdot\left[\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}\right]+\left[\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\right]\cdot\left[\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}\right]}$$

$$=\frac{2\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-2\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{2\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+2\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}$$

$$=\frac{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}$$

and

$$\frac{\dfrac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\cdot\dfrac{2\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(2)}}+\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}{\left(2-\dfrac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\right)\cdot\left(2-\dfrac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\right)+\dfrac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\cdot\dfrac{2\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}$$

$$=\frac{4\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{2\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(2-I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}$$

$$=\frac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(2-I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}.$$

Similarly,

$$\frac{\dfrac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\cdot\dfrac{2\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(2)}}+\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}{\left(2-\dfrac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\right)\cdot\left(2-\dfrac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\right)+\dfrac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}}\cdot\dfrac{2\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}}$$

$$=\frac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(2-F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}.$$

So,

$$SNCIWA_{\mu}\left(A_1,A_2\right)=\left\langle\frac{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}-\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(1+T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1+T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(1-T_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(1-T_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}},\right.$$
$$\frac{2\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(2-I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(I_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(I_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}},$$
$$\left.\frac{2\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}{\left(2-F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(2-F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}+\left(F_{A_{\sigma(1)}}\right)^{w_{\sigma(1)}}\cdot\left(F_{A_{\sigma(2)}}\right)^{w_{\sigma(2)}}}\right\rangle.$$

(2) If Eq. (9) holds for $n=k$, then

$$SNCIWA_{\mu}\left(A_1,A_2,\ldots,A_k\right)=\left\langle\frac{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}-\prod_{j=1}^{k}\left(1-T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}+\prod_{j=1}^{k}\left(1-T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}},\right.$$
$$\frac{2\prod_{j=1}^{k}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k}\left(2-I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}+\prod_{j=1}^{k}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}},$$
$$\left.\frac{2\prod_{i=1}^{k}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k}\left(2-F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}+\prod_{j=1}^{k}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}\right\rangle.$$

If $n=k+1$, by the operations (1) and (3) in Definition 8,

$$\frac{\dfrac{\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}-\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}+\dfrac{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}-\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}{1+\dfrac{\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}-\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\cdot\dfrac{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}-\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}$$

$$=\frac{2\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}-2\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{2\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+2\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}$$

$$=\frac{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}-\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\prod_{j=1}^{k}\left(1+T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1+T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\prod_{j=1}^{k}\left(1-T_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}\cdot\left(1-T_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}},$$

$$\frac{\dfrac{2\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\cdot\dfrac{2\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}{\left(2-\dfrac{2\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\right)\cdot\left(\dfrac{2\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}\right)+\dfrac{2\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\cdot\dfrac{2\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(I_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}$$

$$=\frac{4\prod_{j=1}^{k}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}\cdot\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{2\prod_{j=1}^{k}\left(2-I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}\cdot\left(2-I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+2\prod_{j=1}^{k}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}\cdot\left(I_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}$$

$$=\frac{2\prod_{j=1}^{k+1}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k+1}\left(2-I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}+\prod_{j=1}^{k+1}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}.$$

Similarly,

$$\frac{\dfrac{2\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\cdot\dfrac{2\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}{\left(2-\dfrac{2\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\right)\cdot\left(\dfrac{2\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}\right)+\dfrac{2\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}{\left(2-F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}+\left(F_{A_{\sigma(k+1)}}\right)^{w_{\sigma(k+1)}}}\cdot\dfrac{2\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}{\prod_{j=1}^{k}\left(2-F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}+\prod_{j=1}^{k}\left(F_{A_{\sigma(k)}}\right)^{w_{\sigma(k)}}}}$$

$$=\frac{2\prod_{j=1}^{k+1}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k+1}\left(2-F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}+\prod_{j=1}^{k+1}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}.$$

So,

$$SNCIWA_{\mu}\left(A_1, A_2, \ldots, A_k, A_{k+1}\right) = \left\langle \frac{\prod_{j=1}^{k+1}\left(1+T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}} - \prod_{j=1}^{k+1}\left(1-T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k+1}\left(1+T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}} + \prod_{j=1}^{k+1}\left(1-T_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}, \frac{2\prod_{j=1}^{k+1}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k+1}\left(2-I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}} + \prod_{j=1}^{k+1}\left(I_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}, \frac{2\prod_{i=1}^{k+1}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}}{\prod_{j=1}^{k+1}\left(2-F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}} + \prod_{j=1}^{k+1}\left(F_{A_{\sigma(j)}}\right)^{w_{\sigma(j)}}} \right\rangle.$$

i.e., Eq. (9) holds for $n = k+1$. Thus, Eq. (9) holds for all n, then

$$SNCIWA_{\mu}\left(A_1, A_2, \ldots, A_n\right) = \left\langle \frac{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} - \prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} + \prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}, \frac{2\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} + \prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}, \frac{2\prod_{i=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} + \prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \right\rangle.$$

The proof is complete.

Now some special cases of the SNCIWA operator is considered in the following.

Let $A_j = \left\langle T_{A_j}, I_{A_j}, F_{A_j} \right\rangle$ $\left(j = 1, 2, \ldots, n\right)$ be a collection of SNNs, and μ be a fuzzy measure on X, then followings is true.

(1) $\forall B \in P\left(X\right)$, $\mu\left(B\right) = 1$, then

$$SNCIWA_{\mu}\left(A_1, A_2, \ldots, A_n\right) = \max\left\{A_1, A_2, \ldots, A_n\right\} = A_{\sigma(n)};$$

(2) If $\mu\left(B\right) = 0$ for any $B \in P\left(X\right)$ and $B \neq X$, then

$$SNCIWA_{\mu}\left(A_1, A_2, \ldots, A_n\right) = \min\left\{A_1, A_2, \ldots, A_n\right\} = A_{\sigma(1)};$$

(3) $\forall B_1, B_2 \in P(X)$, $|B_1| = |B_2|$, if $\mu(B_1) = \mu(B_2)$ and $\mu(B_{\sigma(i)}) = \frac{n-i+1}{n} (i = 1, 2, \ldots, n)$, then

$$SNCIWA_\mu(A_1, A_2, \ldots, A_n) = \left\langle \frac{\prod_{j=1}^{n}\left(1+T_{A_j}\right)^{\frac{1}{n}} - \prod_{j=1}^{n}\left(1-T_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(1+T_{A_j}\right)^{\frac{1}{n}} + \prod_{j=1}^{n}\left(1-T_{A_j}\right)^{\frac{1}{n}}}, \frac{2\prod_{j=1}^{n}\left(I_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(2-I_{A_j}\right)^{\frac{1}{n}} + \prod_{j=1}^{n}\left(I_{A_j}\right)^{\frac{1}{n}}}, \frac{2\prod_{i=1}^{n}\left(F_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(2-F_{A_j}\right)^{\frac{1}{n}} + \prod_{j=1}^{n}\left(F_{A_j}\right)^{\frac{1}{n}}} \right\rangle; \tag{10}$$

(4) If $\mu\left(x_{\sigma(j)}\right) = \mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right)$ $(j = 1, 2, \ldots, n)$ .Thus, the SNCIWA operator is reduced to the following simplified neutrosophic weighted averaging operator:

$$SNWA_w(A_1, A_2, \ldots, A_n) = \left\langle \frac{\prod_{j=1}^{n}\left(1+T_{A_j}\right)^{\mu(x_j)} - \prod_{j=1}^{n}\left(1-T_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(1+T_{A_j}\right)^{\mu(x_j)} + \prod_{j=1}^{n}\left(1-T_{A_j}\right)^{\mu(x_j)}}, \frac{2\prod_{j=1}^{n}\left(I_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(2-I_{A_j}\right)^{\mu(x_j)} + \prod_{j=1}^{n}\left(I_{A_j}\right)^{\mu(x_j)}}, \frac{2\prod_{j=1}^{n}\left(F_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(2-F_{A_j}\right)^{\mu(x_j)} + \prod_{j=1}^{n}\left(F_{A_j}\right)^{\mu(x_j)}} \right\rangle; \tag{11}$$

(5) If $\mu(B) = \sum_{j=1}^{\|B\|} w_j$, for any $B \subseteq X$, where $\| B \|$ is the number of the elements in B, then

$$w_j = \mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right), \ j = 1, 2, \cdots, n,$$

Here $w = (w_1, w_2, \ldots, w_n)$, $w_i \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$. Thus the SNCIWA operator is reduced to the following simplified neutrosophic ordered weighted averaging operator:

$$SNOWA_w(A_1, A_2, \ldots, A_n) = \left\langle \frac{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{w_j} - \prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{w_j} + \prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{w_j}}, \frac{2\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(2-I_{A_{\sigma(j)}}\right)^{w_j} + \prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{w_j}}, \frac{2\prod_{i=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(2-F_{A_{\sigma(j)}}\right)^{w_j} + \prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{w_j}} \right\rangle, \tag{12}$$

which was introduced by Peng et al. [44].

Proposition 1. Let $A_j = \left\langle T_{A_j}, I_{A_j}, F_{A_j} \right\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of SNNs, and μ be a fuzzy measure on X . If all $A_j\,(j = 1, 2, \ldots, n)$ are equal, i.e., $A_j = A = \langle T_A, I_A, F_A \rangle$ $(j = 1, 2, \ldots, n)$, then $SNCIWA_\mu(A_1, A_2, \ldots, A_n) = A$.

Proof. Based on Theorem 2, if $A_j = A = \langle T_A, I_A, F_A \rangle$ $(j = 1, 2, \ldots, n)$, then

$$SNCIWA_{\mu}\left(A_1,A_2,\ldots,A_n\right)=\left\langle \frac{\left(1+T_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}-\left(1-T_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}{\left(1+T_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}+\left(1-T_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}},\right.$$

$$\frac{2\left(I_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}{\left(2-I_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}+\left(I_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}},$$

$$\left.\frac{2\left(F_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}{\left(2-F_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}+\left(F_A\right)^{\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}\right\rangle.$$

Since $\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)=1$.

So

$$SNCIWA_{w}\left(A_1,A_2,\ldots,A_n\right)=\left\langle \frac{\left(1+T_A\right)-\left(1-T_A\right)}{\left(1+T_A\right)+\left(1-T_A\right)},\frac{2\left(I_A\right)}{\left(2-I_A\right)+\left(I_A\right)},\frac{2\left(F_A\right)}{\left(2-F_A\right)+\left(F_A\right)}\right\rangle$$

$$=\left\langle T_A,I_A,F_A\right\rangle=A.$$

Proposition 2. Let $A_j=\left\langle T_{A_j},I_{A_j},F_{A_j}\right\rangle$ $(j=1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. If $A_j^*=<T_{A_j^*},I_{A_j^*},F_{A_j^*}>$ and $A_j\subseteq A_j^*$ $(j=1,2,\ldots,n)$, then $SNCIWA_{\mu}\left(A_1,A_2,\ldots,A_n\right)$ $\subseteq SNCIWA_{\mu}\left(A_1^*,A_2^*,\ldots,A_n^*\right)$.

Proof. If $A_j\subseteq A_j^*$ $(j=1,2,\ldots,n)$, then $A_{\sigma(j)}\subseteq A_{\sigma(j)}^*$, i.e., $T_{A_{\sigma(j)}}\leq T_{A_{\sigma(j)}^*}$, $I_{A_{\sigma(j)}}\geq I_{A_{\sigma(j)}^*}$ and $F_{A_{\sigma(j)}}\geq F_{A_{\sigma(j)}^*}$.

Let $f(x)=\frac{1-x}{1+x}$, $x\in[0,1]$; then it is a decreasing function. If $T_{A_{\sigma(j)}}\leq T_{A_{\sigma(j)}^*}$ $(j=1,2,\ldots,n)$, then $f\left(T_{A_{\sigma(j)}^*}\right)\leq f\left(T_{A_{\sigma(j)}}\right)(j=1,2,\ldots,n)$, i.e., $\frac{1-T_{A_{\sigma(j)}^*}}{1+T_{A_{\sigma(j)}^*}}\leq\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}$ $(j=1,2,\ldots,n)$. Since $B_{\sigma(j+1)}\subseteq B_{\sigma(j)}$, then $\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\geq 0$ and $\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)=1$. So

$$\left(\frac{1-T_{A_{\sigma(j)}^*}}{1+T_{A_{\sigma(j)}^*}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\leq\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}^*}}{1+T_{A_{\sigma(j)}^*}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\leq\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$1+\prod_{j=1}^{n}\left(\frac{1-T_{A^*_{\sigma(j)}}}{1+T_{A^*_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\leq 1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$1+\prod_{j=1}^{n}\left(\frac{1-T_{A^*_{\sigma(j)}}}{1+T_{A^*_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\leq 1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\frac{1}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}\leq\frac{1}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A^*_{\sigma(j)}}}{1+T_{A^*_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}},$$

$$\frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}\leq\frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A^*_{\sigma(j)}}}{1+T_{A^*_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}},$$

$$\frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}-1\leq\frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A^*_{\sigma(j)}}}{1+T_{A^*_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}-1,$$

i.e.,

$$\frac{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}\leq\frac{\prod_{j=1}^{n}\left(1+T_{A^*_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-T_{A^*_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{A^*_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-T_{A^*_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}.$$

Let $g(y)=\frac{2-y}{y}$, $y\in(0,1]$; it is a decreasing function on [0,1]. If $I_{A_{\sigma(j)}}\geq I_{A^*_{\sigma(j)}}$ $\left(j=1,2,\ldots,n\right)$, then

$g\left(I_{A_{\sigma(j)}}\right)\geq g\left(I_{A^*_{\sigma(j)}}\right)$, i.e., $\frac{2-I^*_{A_{\sigma(j)}}}{I^*_{A_{\sigma(j)}}}\geq\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}$ $\left(j=1,2,\ldots,n\right)$. Since $\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\geq 0$ $\left(j=1,2,\ldots,n\right)$,

$$\left(\frac{2-I^*_{A_{\sigma(j)}}}{I^*_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\geq\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}.$$ Thus

$$\prod_{j=1}^{n}\left(\frac{2-I^*_{A_{\sigma(j)}}}{I^*_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}\geq\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\prod_{j=1}^{n}\left(\frac{2-I^*_{A_{\sigma(j)}}}{I^*_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1\geq\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1,$$

$$\frac{1}{\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1} \geq \frac{1}{\prod_{j=1}^{n}\left(\frac{2-I^{*}_{A_{\sigma(j)}}}{I^{*}_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1},$$

$$\frac{2}{\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1} \geq \frac{2}{\prod_{j=1}^{n}\left(\frac{2-I^{*}_{A_{\sigma(j)}}}{I^{*}_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1},$$

i.e.,

$$\frac{2\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \geq \frac{2\prod_{j=1}^{n}\left(I_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-I_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{(j)}\right)-\mu\left(B_{(j+1)}\right)}+\prod_{j=1}^{n}\left(I_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}.$$

Similarly, we have

$$\frac{2\prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{(j)}\right)-\mu\left(B_{(j+1)}\right)}+\prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \geq \frac{2\prod_{j=1}^{n}\left(F_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-F_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(F_{A^{*}_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}.$$

According to Definition 7, $SNCIWA_{\mu}(A_1, A_2, \ldots, A_n) \subseteq SNCIWA_{\mu}(A_1^{*}, A_2^{*}, \ldots, A_n^{*})$ can be obtained.

Proposition 3. Let $A_j = \left\langle T_{A_j}, I_{A_j}, F_{A_j} \right\rangle$ $\left(j = 1, 2, \ldots, n\right)$ be a collection of SNNs, and μ be a fuzzy measure on X. If $A^{-} = <\min_j T_{A_j}, \max_j I_{A_j}, \max_j F_{A_j}>$ and $A^{+} = <\max_j T_{A_j}, \min_j I_{A_j}, \min_j F_{A_j}>$ $\left(j = 1, 2, \ldots, n\right)$, then $A^{-} \subseteq SNCIWA_{\mu}\left(A_1, A_2, \ldots, A_n\right) \subseteq A^{+}$.

Proof. Let $f\left(x\right) = \frac{1-x}{1+x}$ and $x \in [0,1]$. Then it is a decreasing function. Since $\min_j T_{A_j} \leq T_{A_{\sigma(j)}} \leq \max_j T_{A_j}$, so $f\left(\max_j T_{A_j}\right) \leq f\left(T_{A_{\sigma(j)}}\right) \leq f\left(\min_j T_{A_j}\right)$, i.e., $\frac{1-\max_j T_{A_j}}{1+\max_j T_{A_j}} \leq \frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}} \leq \frac{1-\min_j T_{A_j}}{1+\min_j T_{A_j}} \left(j = 1, 2, \ldots, n\right)$. Because $B_{\sigma(j+1)} \subseteq B_{\sigma(j)}$, then $\mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right) \geq 0$ and $\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right)\right) = 1$ $\left(j = 1, 2, \ldots, n\right)$. So

$$\left(\frac{1-\max_j T_{A_j}}{1+\max_j T_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \left(\frac{1-\min_j T_{A_j}}{1+\min_j T_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\prod_{j=1}^{n}\left(\frac{1-\max_j T_{A_j}}{1+\max_j T_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \prod_{j=1}^{n}\left(\frac{1-\min_j T_{A_j}}{1+\min_j T_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\left(\frac{1-\max_j T_{A_j}}{1+\max_j T_{A_j}}\right)^{\sum_{j=1}^{n}\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \left(\frac{1-\min_j T_{A_j}}{1+\min_j T_{A_j}}\right)^{\sum_{j=1}^{n}\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\frac{1-\max_j T_{A_j}}{1+\max_j T_{A_j}} \le \prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \frac{1-\min_j T_{A_j}}{1+\min_j T_{A_j}},$$

$$\frac{2}{1+\max_j T_{A_j}} \le 1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \le \frac{2}{1+\min_j T_{A_j}},$$

$$\frac{1+\min_j T_{A_j}}{2} \le \frac{1}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \le \frac{1+\max_j T_{A_j}}{2},$$

$$1+\min_j T_{A_j} \le \frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \le 1+\max_j T_{A_j},$$

$$\min_j T_{A_j} \le \frac{2}{1+\prod_{j=1}^{n}\left(\frac{1-T_{A_{\sigma(j)}}}{1+T_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}-1 \le \max_j T_{A_j},$$

i.e.,

$$\min_j T_{A_j} \le \frac{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \le \max_j T_{A_j}.$$

Let $g(y)=\frac{2-y}{y}$, $y\in(0,1]$; it is a decreasing function on [0,1]. Since $\min_j I_{A_j} \le I_{A_{\sigma(j)}} \le \max_j I_{A_j}$ $(j=1,2,\ldots,n)$, then $g\left(\max_j I_{A_j}\right)\le g\left(I_{A_{\sigma(j)}}\right)\le g\left(\min_j I_{A_j}\right)$, i.e., $\frac{2-\max_j I_{A_j}}{\max_j I_{A_j}} \le \frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}} \le \frac{2-\min_j I_{A_j}}{\min_j I_{A_j}}$ $(j=1,2,\ldots,n)$. Since $\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\ge 0$ and

$\sum_{j=1}^{n}\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)=1\ (j=1,2,\ldots,n)$, so

$$\left(\frac{2-\max_j I_{A_j}}{\max_j I_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \left(\frac{2-\min_j I_{A_j}}{\min_j I_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\prod_{j=1}^{n}\left(\frac{2-\max_j I_{A_j}}{\max_j I_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \prod_{j=1}^{n}\left(\frac{2-\min_j I_{A_j}}{\min_j I_{A_j}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\left(\frac{2-\max_j I_{A_j}}{\max_j I_{A_j}}\right)^{\sum_{j=1}^{n}\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \left(\frac{2-\min_j I_{A_j}}{\min_j I_{A_j}}\right)^{\sum_{j=1}^{n}\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)},$$

$$\frac{2-\max_j I_{A_j}}{\max_j I_{A_j}} \leq \prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)} \leq \frac{2-\min_j I_{A_j}}{\min_j I_{A_j}},$$

$$\frac{2}{\max_j I_{A_j}} \leq \prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1 \leq \frac{2}{\min_j I_{A_j}},$$

$$\frac{\min_j I_{A_j}}{2} \leq \frac{1}{\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1} \leq \frac{\max_j I_{A_j}}{2},\quad \min_j I_{A_j} \leq \frac{2}{\prod_{j=1}^{n}\left(\frac{2-I_{A_{\sigma(j)}}}{I_{A_{\sigma(j)}}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+1} \leq \max_j I_{A_j},$$

Thus, $$\min_j I_{A_j} \leq \frac{2\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \leq \max_j I_{A_j}.$$

Similarly, $$\min_j F_{A_j} \leq \frac{2\prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \leq \max_j F_{A_j}.$$

According to Definition 7, $A^- \subseteq SNCIWA_\mu\left(A_1, A_2, \ldots, A_n\right) \subseteq A^+$.

Definition 15. Let $A_j=\left\langle T_{A_j}, I_{A_j}, F_{A_j}\right\rangle$ $(j=1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. Based on fuzzy measure, a simplified neutrosophic Choquet integral weighted geometric (SNCIWG) operator of dimension n is a mapping SNCIWG: $\mathrm{SNN}^n \rightarrow \mathrm{SNN}$ such that

$$SNCIWG_{\mu}\left(A_1,A_2,\ldots,A_n\right)=\left(A_{\sigma(1)}\right)^{\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)}\otimes\left(A_{\sigma(2)}\right)^{\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)}\otimes\ldots\otimes\left(A_{\sigma(n)}\right)^{\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)}. \quad (13)$$

Here $\left(\sigma(1),\sigma(2),\ldots,\sigma(n)\right)$ is a permutation of $\left(1,2,\ldots,n\right)$, and such that $A_{\sigma(1)}\le A_{\sigma(2)}\le\ldots\le A_{\sigma(n)}$. $B_{\sigma(i)}=\left(\sigma(i),\ldots\sigma(n)\right)$, and $B_{\sigma(n+1)}=\varnothing$.

Theorem 3. Let $A_j=\left\langle T_{A_j},I_{A_j},F_{A_j}\right\rangle$ $\left(j=1,2,\ldots,n\right)$ be a collection of SNNs, and μ be a fuzzy measure on X .Then their aggregated result using the *SNCIWG* operator is also an SNN, and

$$SNCIWG_{\mu}\left(A_1,A_2,\ldots,A_n\right)=\left\langle \frac{2\prod_{j=1}^{n}\left(T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(2-T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(T_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}, \right.$$
$$\frac{\prod_{j=1}^{n}\left(1+I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-I_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}, \quad (14)$$
$$\left. \frac{\prod_{j=1}^{n}\left(1+F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}-\prod_{j=1}^{n}\left(1-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}}{\prod_{j=1}^{n}\left(1+F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{j=1}^{n}\left(1-F_{A_{\sigma(j)}}\right)^{\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)}} \right\rangle.$$

Here $\left(\sigma(1),\sigma(2),\ldots,\sigma(n)\right)$ is a permutation of $\left(1,2,\ldots,n\right)$, and such that $A_{\sigma(1)}\le A_{\sigma(2)}\le\ldots\le A_{\sigma(n)}$. $B_{\sigma(j)}=\left(\sigma(j),\ldots\sigma(n)\right)$, and $B_{\sigma(n+1)}=\varnothing$.

Proof. Theorem 3 can be proved by the mathematical induction method, and the process is omitted here.

Now let's consider some special cases of the SNCIWG operator in the following.

Let $A_j=\left\langle T_{A_j},I_{A_j},F_{A_j}\right\rangle$ $\left(j=1,2,\ldots,n\right)$ be a collection of SNNs, and μ be a fuzzy measure on X, then followings is true.

(1) $\forall B\in P\left(X\right)$, $\mu\left(B\right)=1$, then

$$SNCIWG_{\mu}\left(A_1,A_2,\ldots,A_n\right)=\max\left\{A_1,A_2,\ldots,A_n\right\}=A_{\sigma(n)};$$

(2) If $\mu\left(B\right)=0$ for any $B\in P\left(X\right)$ and $B\ne X$, then

$$SNCIWG_{\mu}\left(A_1,A_2,\ldots,A_n\right)=\min\left\{A_1,A_2,\ldots,A_n\right\}=A_{\sigma(1)};$$

(3) $\forall B_1,B_2\in P\left(X\right)$, $|B_1|=|B_2|$, if $\mu(B_1)=\mu(B_2)$ and $\mu\left(B_{\sigma(j)}\right)=\dfrac{n-j+1}{n}\left(j=1,2,\ldots,n\right)$, then

$SNCIWG_{\mu}\left(A_1, A_2, \ldots, A_n\right)$

$$=\left\langle \frac{2\prod_{j=1}^{n}\left(T_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(2-T_{A_j}\right)^{\frac{1}{n}}+\prod_{j=1}^{n}\left(T_{A_j}\right)^{\frac{1}{n}}}, \frac{\prod_{j=1}^{n}\left(1+I_{A_j}\right)^{\frac{1}{n}}-\prod_{j=1}^{n}\left(1-I_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(1+I_{A_j}\right)^{\frac{1}{n}}+\prod_{j=1}^{n}\left(1-I_{A_j}\right)^{\frac{1}{n}}}, \frac{\prod_{j=1}^{n}\left(1+F_{A_j}\right)^{\frac{1}{n}}-\prod_{j=1}^{n}\left(1-F_{A_j}\right)^{\frac{1}{n}}}{\prod_{j=1}^{n}\left(1+F_{A_j}\right)^{\frac{1}{n}}+\prod_{j=1}^{n}\left(1-F_{A_j}\right)^{\frac{1}{n}}} \right\rangle; \tag{15}$$

(4) If $\mu\left(x_{\sigma(j)}\right)=\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)$ and $j=1,2,\ldots n$ .Thus, the SNCIWG operator is reduced to the following simplified neutrosophic geometric averaging operator:

$SNCIWG_{\mu}\left(A_1, A_2, \ldots, A_n\right)$

$$=\left\langle \frac{2\prod_{j=1}^{n}\left(T_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(2-T_{A_j}\right)^{\mu(x_j)}+\prod_{j=1}^{n}\left(T_{A_j}\right)^{\mu(x_j)}}, \frac{\prod_{j=1}^{n}\left(1+I_{A_j}\right)^{\mu(x_j)}-\prod_{j=1}^{n}\left(1-I_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(1+I_{A_j}\right)^{\mu(x_j)}+\prod_{j=1}^{n}\left(1-I_{A_j}\right)^{\mu(x_j)}}, \frac{\prod_{j=1}^{n}\left(1+F_{A_j}\right)^{\mu(x_j)}-\prod_{j=1}^{n}\left(1-F_{A_j}\right)^{\mu(x_j)}}{\prod_{j=1}^{n}\left(1+F_{A_j}\right)^{\mu(x_j)}+\prod_{j=1}^{n}\left(1-F_{A_j}\right)^{\mu(x_j)}} \right\rangle; \tag{16}$$

(5) If $\mu\left(B\right)=\sum_{j=1}^{\|B\|} w_j$, for any $B \subseteq X$, where $\| B \|$ is the number of the elements in B, then

$$w_j=\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right),\ j=1,2,\cdots,n,$$

Here $w=\left(w_1, w_2, \ldots, w_n\right)$, $w_i \geq 0$ $\left(j=1,2,\ldots,n\right)$ and $\sum_{i=1}^{n} w_i=1$. Thus the SNCIWG operator is reduced to the following simplified neutrosophic ordered geometric averaging operator:

$SNOWG_{w}\left(A_1, A_2, \ldots, A_n\right)$

$$=\left\langle \frac{2\prod_{j=1}^{n}\left(T_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(2-T_{A_{\sigma(j)}}\right)^{w_j}+\prod_{j=1}^{n}\left(T_{A_{\sigma(j)}}\right)^{w_j}}, \frac{\prod_{j=1}^{n}\left(1+I_{A_{\sigma(j)}}\right)^{w_j}-\prod_{j=1}^{n}\left(1-I_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+I_{A_{\sigma(j)}}\right)^{w_j}+\prod_{j=1}^{n}\left(1-I_{A_{\sigma(j)}}\right)^{w_j}}, \frac{\prod_{j=1}^{n}\left(1+F_{A_{\sigma(j)}}\right)^{w_j}-\prod_{j=1}^{n}\left(1-F_{A_{\sigma(j)}}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+F_{A_{\sigma(j)}}\right)^{w_j}+\prod_{j=1}^{n}\left(1-F_{A_{\sigma(j)}}\right)^{w_j}} \right\rangle, \tag{17}$$

which was introduced by Peng et al. [44].

Proposition 4. Let $A_j=\left\langle T_{A_j}, I_{A_j}, F_{A_j}\right\rangle$ $\left(j=1,2,\ldots,n\right)$ be a collection of SNNs, and μ be a fuzzy measure on X . If all $A_j\left(j=1,2,...,n\right)$ are equal, i.e., $A_j=A=<T_A, I_A, F_A>$ $\left(j=1,2,\ldots,n\right)$, then $SNCIWG_{\mu}\left(A_1, A_2, \ldots, A_n\right)=A$.

Proof. The proof is omitted here.

Proposition 5. Let $A_j = \left\langle T_{A_j}, I_{A_j}, F_{A_j} \right\rangle$ $(j = 1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. If $A_j^* = < T_{A_j^*}, I_{A_j^*}, F_{A_j^*} >$ and $A_j \subseteq A_j^*$ $(j = 1,2,\ldots,n)$, then $SNCIWG_\mu(A_1, A_2, \ldots, A_n)$ $\subseteq SNCIWG_\mu(A_1^*, A_2^*, \ldots, A_n^*)$.

Proof. The proof is omitted here.

Proposition 6. Let $A_j = \left\langle T_{A_j}, I_{A_j}, F_{A_j} \right\rangle$ $(j = 1,2,\ldots,n)$ be a collection of SNNs, and μ be a fuzzy measure on X. If $A^- = < \min_j T_{A_j}, \max_j I_{A_j}, \max_j F_{A_j} >$ and $A^+ = < \max_j T_{A_j}, \min_j I_{A_j}, \min_j F_{A_j} >$ $(j = 1,2,\ldots,n)$, then $A^- \subseteq SNCIWG_\mu(A_1, A_2, \ldots, A_n) \subseteq A^+$.

Proof. The proof is omitted here.

5. Choquet integral-based TOPSIS approach of MCGDM with simplified neutrosophic information

Assume there are *n* alternatives $A = \{a_1, a_2, \ldots, a_n\}$ and *m* criteria $C = \{c_1, c_2, \ldots, c_m\}$, and the weight vector of criteria is $w = (w_1, w_2, \ldots, w_m)$, where $w_j \geq 0 (j = 1,2,\ldots,m)$, $\sum_{j=1}^{m} w_j = 1$. Suppose that there are k decision-makers $D = \{d_1, d_2, \ldots, d_k\}$, whose corresponding weight is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k)$. Let $R^k = \left(a_{ij}^k\right)_{n \times m}$ be the simplified neutrosophic decision matrix, where $a_{ij}^k = \langle T_{a_{ij}^k}, I_{a_{ij}^k}, F_{a_{ij}^k} \rangle$ is the value of a criterion, denoted by SNNs, where $T_{a_{ij}^k}$ indicates the truth-membership function that alternative a_i satisfies criterion c_j for the *k*-th decision-maker, $I_{a_{ij}^k}$ indicates the indeterminacy-membership function that alternative a_i satisfies criterion c_j for the *k*-th decision-maker and $F_{a_{ij}^k}$ indicates the falsity-membership function that alternative a_i satisfies criterion c_j for the *k*-th decision-maker. This method is an integration of SNSs and aggregation operators to solve MCGDM problems mentioned above.

The method is an integration of SNSs and the TOPSIS method to handle MCGDM problems mentioned above. In general, there are benefit criteria and cost criteria in MCGDM problems. The cost-type criterion values can be transformed into benefit-type criterion values as follows:

$$b_{ij} = \begin{cases} a_{ij}, & \text{for benefit criterion } c_j \\ \left(a_{ij}\right)^c, & \text{for cost criterion } c_j \end{cases}, \quad (i = 1,2,\ldots,n;\ j = 1,2,\ldots,m). \tag{18}$$

Here $\left(a_{ij}\right)^c$ is the complement of a_{ij} as defined in Definition 7.

In the following, a procedure to rank and select the most desirable alternative(s) is given.

Step 1. Transform the decision matrix.

For each criterion can be divided into two types, including benefit-type which means the lager the better, and cost-type which means the smaller the better. For the benefit-type criteria, nothing is done; for the cost-type criteria, the criterion values can be transformed. We can transform the SNS decision matrix $R^k=\left(a_{ij}^k\right)_{n\times m}$ into a normalized SNS decision matrix $\overline{R}^k=\left(b_{ij}^k\right)_{n\times m}$ based on Eq. (18).

Step 2. Confirm the fuzzy measures and expert sets of D.

Based on the fuzzy measures and expert sets of D, the weight of criteria can be obtained as follows:

$$w_{\sigma(j)}=\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right), i=1,2,\ldots,m.$$

Here $\left(\sigma(1),\sigma(2),\ldots,\sigma(n)\right)$ is a permutation of $\left(1,2,\ldots,n\right)$.

Step 3. Aggregate all the decision-makers' values to get the collective simplified neutrosophic decision matrix.

Utilize the SNCIWA operator and SNCIWG operator to aggregate the SNNs of each decision-maker, and we can get the collective simplified neutrosophic decision matrix $\tilde{R}=\left(\tilde{b}_{ij}\right)_{n\times m}$.

Where

$$\tilde{b}_{ij}=SNCIWA_{\mu}\left(b_{ij}^1,b_{ij}^2,\ldots,b_{ij}^k\right)=\left\langle \frac{\prod_{r=1}^{k}\left(1+T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}-\prod_{r=1}^{k}\left(1-T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{j=1}^{n}\left(1+T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}+\prod_{r=1}^{k}\left(1-T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}, \right.$$
$$\frac{2\prod_{r=1}^{k}\left(I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{r=1}^{k}\left(2-I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(j+1)}\right)}+\prod_{r=1}^{k}\left(I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}, \quad , \tag{19}$$
$$\left.\frac{2\prod_{r=1}^{k}\left(F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{r=1}^{k}\left(2-F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}+\prod_{r=1}^{k}\left(F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}\right\rangle.$$

or

$$\tilde{b}_{ij} = SNCIWG_{\mu}\left(b_{ij}^{1}, b_{ij}^{2}, \ldots, b_{ij}^{k}\right) = \left\langle \frac{2\prod_{r=1}^{k}\left(T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{r=1}^{k}\left(2-T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(j+1)}\right)} + \prod_{r=1}^{k}\left(T_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}, \right.$$

$$\frac{\prod_{r=1}^{k}\left(1+I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)} - \prod_{r=1}^{k}\left(1-I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{r=1}^{k}\left(1+I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)} + \prod_{r=1}^{k}\left(1-I_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}, \quad . \tag{20}$$

$$\left. \frac{\prod_{r=1}^{k}\left(1+F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)} - \prod_{r=1}^{k}\left(1-F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}}{\prod_{r=1}^{k}\left(1+F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)} + \prod_{r=1}^{k}\left(1-F_{b_{ij}^{\sigma(r)}}\right)^{\mu\left(B_{\sigma(r)}\right)-\mu\left(B_{\sigma(r+1)}\right)}} \right\rangle.$$

Here $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, and such that $b_{ij}^{\sigma(1)} \leq b_{ij}^{\sigma(2)} \leq \ldots \leq b_{ij}^{\sigma(k)}$. $B_{\sigma(r)} = (\sigma(r), \ldots, \sigma(k))$, and $B_{\sigma(k+1)} = \varnothing$.

Step 4. Confirm the simplified neutrosophic positive-ideal solution and the negative-ideal solution.

The simplified neutrosophic positive-ideal solution $\tilde{b}_{ij}^{+}$ and the negative-ideal solution $\tilde{b}_{ij}^{-}$ can be determined as follows:

$$\tilde{b}_{j}^{+} = \left\{\left\langle c_j, \left(\max_i T_{\tilde{b}_{ij}}\right), \left(\min_i T_{\tilde{b}_{ij}}\right), \left(\min_i T_{\tilde{b}_{ij}}\right)\right\rangle\right\} \quad \text{and} \quad \tilde{b}_{j}^{-} = \left\{\left\langle c_j, \left(\min_i T_{\tilde{b}_{ij}}\right), \left(\max_i T_{\tilde{b}_{ij}}\right), \left(\max_i T_{\tilde{b}_{ij}}\right)\right\rangle\right\} \quad (j = 1, 2, \ldots, n). \tag{21}$$

Here $\tilde{b}^{+} = \left(\tilde{b}_{1}^{+}, \tilde{b}_{2}^{+}, \ldots, \tilde{b}_{m}^{+}\right)$ and $\tilde{b}^{-} = \left(\tilde{b}_{1}^{-}, \tilde{b}_{2}^{-}, \ldots, \tilde{b}_{m}^{-}\right)$.

Step 5. Confirm the fuzzy measures of criteria of C and criteria sets of C.

Based on the fuzzy measures and expert sets of C, the weight of criteria can be obtained as follows:

$$\tilde{w}_{\sigma(j)} = \mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right), i = 1, 2, \ldots, m.$$

Here $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$.

Step 6. Calculate the distance.

According to Definition 13, the distance between the alternative $a_i = \left(\tilde{b}_{i1}, \tilde{b}_{i2}, \ldots, \tilde{b}_{im}\right)(i = 1, 2, \ldots, n)$ and the simplified neutrosophic positive-ideal solution $\tilde{b}^{+}$ and the distance between the alternative

$a_i = \left(\tilde{b}_{i1}, \tilde{b}_{i2}, \ldots, \tilde{b}_{im}\right)\left(i = 1, 2, \ldots, n\right)$ and the simplified neutrosophic positive-ideal solution $\tilde{b}^-$ can be calculated, respectively:

$$d_{gsnn}^i\left(a_i, \tilde{b}^+\right) = \frac{1}{3}\sum_{j=1}^{m} \tilde{d}_{i\sigma(j)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right)\left(\mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right)\right). \tag{22}$$

Where $\tilde{d}_{i\sigma(j)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) = \left|T_{\tilde{b}_{ij}} - T_{\tilde{b}_j^+}\right| + \left|I_{\tilde{b}_{ij}} - I_{\tilde{b}_j^+}\right| + \left|F_{\tilde{b}_{ij}} - F_{\tilde{b}_j^+}\right|$ and $d_{i\sigma(1)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) \le d_{i\sigma(2)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) \le \ldots \le d_{i\sigma(m)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right)$, $B_{\sigma(j)} = \left(c_{\sigma(j)}, c_{\sigma(j+1)}, \ldots, c_{\sigma(m)}\right)$ and $B_{\sigma(m+1)} = \varnothing$.

$$d_i\left(a_i, \tilde{b}^-\right) = \frac{1}{3}\sum_{j=1}^{m} d_{i\sigma(j)}\left(\tilde{b}_{ij}, \tilde{b}_j^-\right)\left(\mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right)\right). \tag{23}$$

Where $d_{i\sigma(j)}\left(\tilde{b}_{ij}, \tilde{b}_j^-\right) = \left(\left|T_{\tilde{b}_{ij}} - T_{\tilde{b}_j^-}\right| + \left|I_{\tilde{b}_{ij}} - I_{\tilde{b}_j^-}\right| + \left|F_{\tilde{b}_{ij}} - F_{\tilde{b}_j^-}\right|\right)$ and $d_{i\sigma(1)}\left(\tilde{b}_{ij}, \tilde{b}_j^-\right) \le d_{i\sigma(2)}\left(\tilde{b}_{ij}, \tilde{b}_j^-\right) \le \ldots \le d_{i\sigma(m)}\left(\tilde{b}_{ij}, \tilde{b}_j^-\right)$, $B_{\sigma(j)} = \left(c_{\sigma(j)}, c_{\sigma(j+1)}, \ldots, c_{\sigma(m)}\right)$ and $B_{\sigma(m+1)} = \varnothing$.

Step 7. Calculate the closeness coefficient of each alternative.

Based on Step 6, the closeness coefficient of each alternative can be obtained as follows:

$$G\left(a_i\right) = \frac{d_i\left(a_i, \tilde{b}^+\right)}{d_i\left(a_i, \tilde{b}^+\right) + d_i\left(a_i, \tilde{b}^-\right)}\left(i = 1, 2, \ldots, n\right). \tag{24}$$

Step 8: Rank the alternatives.

According to the closeness coefficients $G\left(a_i\right)$, the smaller the value $G\left(a_i\right)$, the better the alternative a_i $\left(i = 1, 2, \ldots, n\right)$.

6. Illustrative examples (adapted from [62])

In this section, an example for the MCDM problem with simplified neutrosophic information is used as the demonstration of the application of the proposed decision-making method, as well as the comparison analysis.

ABC Nonferrous Metals Holding Group Co. Ltd. is a large state-owned company whose main business is producing and selling nonferrous metals. It is also the largest manufacturer of multi-species nonferrous metals in China, with the exception of aluminum. In order to expand its main business, the company is always engaged in overseas investment and a department which consists of executive managers and three experts in the field has been established specifically to make decisions on global mineral investment. Recently, the company has decided to select a pool of alternatives from several foreign countries based on preliminary surveys. In this survey, the focus is on the first step in finding suitable candidate countries. Four countries (alternatives) are taken into consideration, which are denoted by a_1, a_2, a_3 and a_4. During the assessment, four factors including c_1: politics and policy (such as the support of government); c_2: infrastructure (such as railway and highway facilities) are considered according to previous investment examples from the department; c_3: resources (such as the suitability of the minerals and their exploration); c_4: economy (such as development vitality and the stability). The decision-makers can provide their evaluations about the project a_i under the criterion c_j in the form of SNNs $a_{ij}^k = \langle T_{a_{ij}^k}, I_{a_{ij}^k}, F_{a_{ij}^k} \rangle$ $\left(k = 1,2,3; i, j = 1,2,3,4\right)$, which represents their degrees of satisfaction, indeterminacy and dissatisfaction regarding an alternative by using the concept of "excellent" against each criterion. The simplified netrosophic decision matrix $R^k = \left(a_{ij}^k\right)_{n\times m}$ can be found as follows:

$$R^1 = \begin{pmatrix} \langle 0.4,0.1,0.2\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.7,0.1,0.2\rangle & \langle 0.6,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.7,0.2,0.2\rangle \\ \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.4,0.2,0.2\rangle & \langle 0.5,0.1,0.3\rangle \\ \langle 0.6,0.3,0.1\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.7,0.1,0.2\rangle \end{pmatrix},$$

$$R^2 = \begin{pmatrix} \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.4,0.1,0.3\rangle & \langle 0.7,0.2,0.1\rangle \\ \langle 0.5,0.2,0.2\rangle & \langle 0.6,0.2,0.1\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.5,0.2,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.7,0.3,0.2\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.8,0.2,0.2\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.5,0.2,0.1\rangle \end{pmatrix},$$

$$R^3 = \begin{pmatrix} \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.7,0.3,0.2\rangle & \langle 0.6,0.1,0.2\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.7,0.2,0.1\rangle \\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.7,0.1,0.3\rangle \\ \langle 0.6,0.0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.2,0.1\rangle & \langle 0.8,0.2,0.1\rangle \end{pmatrix}.$$

6.1 An illustration of the proposed approach

The procedures of obtaining the optimal alternative, by using the developed method, are shown as following.

Step 1. Normalize the data in Table 1. Because all the criteria are of maximizing type and have the same measurement unit, there is no need for normalization and $\tilde{R} = (\tilde{a}_{ij})_{4\times 4} = (a_{ij})_{4\times 4}$.

Step 2. Determine the fuzzy measure.

Determine the fuzzy measure of expert of K and expert sets of $K=\{k_1,k_2,k_3\}$. Suppose that

$\mu(k_1)=0.5,\ \mu(k_2)=0.3,\ \mu(k_3)=0.2,\ \mu(k_1,k_2)=0.9,\ \mu(k_1,k_3)=0.8,\ \mu(k_1,k_2,k_3)=1$.

Step 3. Aggregate all the decision-makers' values to get the collective simplified neutrosophic deision matrix.

Utilize the SNCIWA operator to aggregate the SNNs of each decision-maker. According to Eq. (20), the collective simplified neutrosophic decision matrix can be obtained as follows:

$$\tilde{R}=\begin{pmatrix} \langle 0.4656,0.1073,0.2085\rangle & \langle 0.4905,0.2000,0.1483\rangle & \langle 0.4356,0.1658,0.3121\rangle & \langle 0.6324,0.1631,0.1631\rangle \\ \langle 0.6360,0.1152,0.2000\rangle & \langle 0.5717,0.1631,0.1931\rangle & \langle 0.4614,0.2359,0.2065\rangle & \langle 0.6818,0.2000,0.1747\rangle \\ \langle 0.4218,0.1325,0.2187\rangle & \langle 0.5000,0.1523,0.1758\rangle & \langle 0.4414,0.1702,0.2085\rangle & \langle 0.6292,0.1573,0.2558\rangle \\ \langle 0.5817,0,0.1152\rangle & \langle 0.6395,0.1931,0.2000\rangle & \langle 0.5213,0.1325,0.1747\rangle & \langle 0.6911,0.1325,0.1523\rangle \end{pmatrix}.$$

Take $\tilde{b}_{11}$ for example, based on Definition 11, the detail compute process are as follows:

$s(b_{11}^1)=0.7000,\ s(b_{11}^2)=0.7667$ and $s(b_{11}^3)=0.6333$.

Then, $s(b_{11}^3)<s(b_{11}^1)<s(b_{11}^2)$. So $b_{11}^3<b_{11}^1<b_{11}^2$, $b_{11}^{\sigma(1)}=b_{11}^3, b_{11}^{\sigma(2)}=b_{11}^1$ and $b_{11}^{\sigma(3)}=b_{11}^2$.

Thus, $\mu(B_{\sigma(1)})-\mu(B_{\sigma(2)})=\mu(k_1,k_2,k_3)-\mu(k_1,k_2)=1-0.9=0.1$;

$\mu(B_{\sigma(2)})-\mu(B_{\sigma(3)})=\mu(k_1,k_2)-\mu(k_2)=0.9-0.3=0.6$;

$\mu(B_{\sigma(3)})-\mu(B_{\sigma(4)})=\mu(k_2)=0.3$.

So

$$\begin{aligned}\tilde{b}_{11}&=SNCIWA_\mu(b_{11}^1,b_{11}^2,b_{11}^3)\\ &=SNCIWA_\mu(\langle 0.4,0.1,0.2\rangle,\langle 0.6,0.1,0.2\rangle,\langle 0.4,0.2,0.3\rangle)\\ &=\Bigg\langle \frac{(1+0.4)^{0.1}\times(1+0.4)^{0.6}\times(1+0.6)^{0.3}-(1-0.4)^{0.1}\times(1-0.4)^{0.6}\times(1-0.6)^{0.3}}{(1+0.4)^{0.1}\times(1+0.4)^{0.6}\times(1+0.6)^{0.3}+(1-0.4)^{0.1}\times(1-0.4)^{0.6}\times(1-0.6)^{0.3}},\\ &\qquad \frac{2\times 0.2^{0.1}\times 0.1^{0.6}\times 0.1^{0.3}}{(2-0.2)^{0.1}\times(2-0.1)^{0.6}\times(2-0.1)^{0.3}+0.2^{0.1}\times 0.1^{0.6}\times 0.1^{0.3}},\\ &\qquad \frac{2\times 0.3^{0.1}\times 0.2^{0.6}\times 0.2^{0.3}}{(2-0.3)^{0.1}\times(2-0.2)^{0.6}\times(2-0.2)^{0.3}+0.3^{0.1}\times 0.2^{0.6}\times 0.2^{0.3}}\Bigg\rangle\\ &=\langle 0.4656,0.1073,0.2085\rangle.\end{aligned}$$

Step 4. Confirm the simplified neutrosophic positive-ideal solution and the negative-ideal solution.

Based on the collective simplified neutrosophic decision matrix $\tilde{R}$ and Eq. (21), the following result can be true.

$$\tilde{b}_1^+ = \left\langle \max\left\{0.4656, 0.6360, 0.4218, 0.5817\right\}, \min\left\{0.1073, 0.1152, 0.1325, 0\right\}, \min\left\{0.2085, 0.2, 0.2187, 0.1152\right\}\right\rangle$$
$$= \langle 0.6360, 0, 0.1152\rangle;$$

$$\tilde{b}_2^+ = \langle 0.6395, 0.1523, 0.1483\rangle;$$

$$\tilde{b}_3^+ = \langle 0.5213, 0.1325, 0.1747\rangle;$$

$$\tilde{b}_4^+ = \langle 0.6911, 0.1325, 0.1523\rangle.$$

Similarly,

$$\tilde{b}_1^- = \langle 0.4218, 0.1325, 0.2187\rangle;$$

$$\tilde{b}_2^- = \langle 0.4905, 0.2000, 0.2000\rangle;$$

$$\tilde{b}_3^- = \langle 0.4356, 0.2359, 0.3121\rangle;$$

$$\tilde{b}_4^- = \langle 0.6292, 0.2000, 0.2558\rangle.$$

Step 5. Confirm the fuzzy measures of criteria of C and the criteria sets of C.

Based on the fuzzy measures of criteria of C and the criteria sets of $C = \left\{c_1, c_2, c_3, c_4\right\}$, suppose that

$\mu(c_1) = 0.30$, $\mu(c_2) = 0.25$, $\mu(c_3) = 0.37$, $\mu(c_4) = 0.20$, $\mu(c_1, c_2) = 0.52$, $\mu(c_1, c_3) = 0.65$, $\mu(c_1, c_4) = 0.50$, $\mu(c_2, c_3) = 0.45$, $\mu(c_2, c_4) = 0.34$, $\mu(c_3, c_4) = 0.42$, $\mu(c_1, c_2, c_3) = 0.85$, $\mu(c_1, c_2, c_4) = 0.68$, $\mu(c_2, c_3, c_4) = 0.57$, $\mu(c_1, c_3, c_4) = 0.76$, $\mu(c_1, c_2, c_3, c_4) = 1$.

Step 6. Calculate the distance.

Based on Eq. (22),

$$\tilde{d}_{11}\left(\tilde{b}_{11}, \tilde{b}_1^+\right)$$
$$= \tilde{d}_{11}\left(\langle 0.4656, 0.1073, 0.2085\rangle, \langle 0.6360, 0, 0.1152\rangle\right)$$
$$= \left|0.4656 - 0.6360\right| + \left|0.1073 - 0\right| + \left|0.2085 - 0.1152\right|$$
$$= 0.3710;$$

$$\tilde{d}_{12}\left(\tilde{b}_{12}, \tilde{b}_2^+\right) = 0.1967; \tilde{d}_{13}\left(\tilde{b}_{13}, \tilde{b}_3^+\right) = 0.2564; \tilde{d}_{14}\left(\tilde{b}_{14}, \tilde{b}_4^+\right) = 0.1001.$$

Since $\tilde{d}_{14}\left(\tilde{b}_{14}, \tilde{b}_4^+\right) < \tilde{d}_{12}\left(\tilde{b}_{12}, \tilde{b}_2^+\right) < \tilde{d}_{13}\left(\tilde{b}_{13}, \tilde{b}_3^+\right) < \tilde{d}_{11}\left(\tilde{b}_{11}, \tilde{b}_1^+\right)$ and

$$\tilde{d}_{1\sigma(1)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) = \tilde{d}_{14}\left(\tilde{b}_{14}, \tilde{b}_4^+\right), \tilde{d}_{1\sigma(2)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) = \tilde{d}_{12}\left(\tilde{b}_{12}, \tilde{b}_2^+\right), \tilde{d}_{1\sigma(3)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) = \tilde{d}_{13}\left(\tilde{b}_{13}, \tilde{b}_3^+\right), \tilde{d}_{1\sigma(4)}\left(\tilde{b}_{ij}, \tilde{b}_j^+\right) = \tilde{d}_{11}\left(\tilde{b}_{11}, \tilde{b}_1^+\right).$$

so

$$\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)=\mu(c_1,c_2,c_3,c_4)-\mu(c_2,c_3,c_1)=1-0.85=0.15\,;$$

$$\mu\left(B_{\sigma(2)}\right)-\mu\left(B_{\sigma(3)}\right)=\mu(c_1,c_2,c_3)-\mu(c_3,c_1)=0.85-0.65=0.20\,;$$

$$\mu\left(B_{\sigma(3)}\right)-\mu\left(B_{\sigma(4)}\right)=\mu\left(c_1,c_3\right)-\mu\left(c_1\right)=0.65-0.30=0.35\,;$$

$$\mu\left(B_{\sigma(4)}\right)-\mu\left(B_{\sigma(5)}\right)=\mu\left(c_1\right)=0.30\,.$$

Thus,

$$d_{gsnn}^{1}\left(a_1,\tilde{b}^{+}\right)=\frac{1}{3}\Big(\tilde{d}_{14}\left(\tilde{b}_{14},\tilde{b}_4^{+}\right)\left(\mu\left(B_{\sigma(1)}\right)-\mu\left(B_{\sigma(2)}\right)\right)+\tilde{d}_{12}\left(\tilde{b}_{12},\tilde{b}_2^{+}\right)\left(\mu\left(B_{\sigma(2)}\right)-\mu\left(B_{\sigma(3)}\right)\right)+$$
$$\tilde{d}_{13}\left(\tilde{b}_{13},\tilde{b}_3^{+}\right)\left(\mu\left(B_{\sigma(3)}\right)-\mu\left(B_{\sigma(4)}\right)\right)+\tilde{d}_{11}\left(\tilde{b}_{11},\tilde{b}_1^{+}\right)\left(\mu\left(B_{\sigma(4)}\right)-\mu\left(B_{\sigma(5)}\right)\right)\Big)$$
$$=\frac{1}{3}\times\left(0.1001\times0.15+0.1967\times0.2+0.2564\times0.35+0.371\times0.3\right)=0.0851.$$

Similarly, the following results can be obtained:

$$d_{gsnn}^{1}\left(a_1,\tilde{b}^{-}\right)=0.0270\,;\;\; d_{gsnn}^{2}\left(a_2,\tilde{b}^{+}\right)=0.0559;\; d_{gsnn}^{2}\left(a_2,\tilde{b}^{-}\right)=0.0553;$$

$$d_{gsnn}^{3}\left(a_3,\tilde{b}^{+}\right)=0.0842;\; d_{gsnn}^{3}\left(a_3,\tilde{b}^{-}\right)=0.0450;$$

$$d_{gsnn}^{4}\left(a_4,\tilde{b}^{+}\right)=0.0126;\; d_{gsnn}^{4}\left(a_4,\tilde{b}^{-}\right)=0.0889;$$

Step 7. Calculate the closeness coefficient of each alternative.

$$G\left(a_1\right)=\frac{d_{gsnn}^{1}\left(a_1,b^{+}\right)}{d_{gsnn}^{1}\left(a_1,b^{+}\right)+d_{gsnn}^{1}\left(a_1,b^{-}\right)}=\frac{0.0851}{0.0851+0.0270}=0.7591\,;$$

$$G\left(a_2\right)=0.5027;\; G\left(a_3\right)=0.6517;\; G\left(a_4\right)=0.1241.$$

Step 8. Rank the alternatives.

Since $G\left(a_4\right)<G\left(a_2\right)<G\left(a_3\right)<G\left(a_1\right)$. So the final ranking is $a_4\succ a_2\succ a_3\succ a_1$ and the best alternative is a_4 while the worst alternative is a_1.

If the SNCIWG operator is utilized in Step 3, then the collective simplified neutrosophic decision matrix can be obtained as follows:

$$\tilde{R}=\begin{pmatrix} \langle 0.4543,0.1101,0.2102\rangle & \langle 0.4893,0.2000,0.1608\rangle & \langle 0.4045,0.1811,0.3219\rangle & \langle 0.6291,0.1703,0.1703\rangle \\ \langle 0.6272,0.1202,0.2000\rangle & \langle 0.5688,0.1703,0.2115\rangle & \langle 0.4581,0.2406,0.2212\rangle & \langle 0.6794,0.2000,0.1803\rangle \\ \langle 0.4167,0.1404,0.2419\rangle & \langle 0.5000,0.1604,0.2021\rangle & \langle 0.4381,0.1807,0.2102\rangle & \langle 0.6154,0.1820,0.2606\rangle \\ \langle 0.5791,0.1535,0.1202\rangle & \langle 0.6135,0.2115,0.2000\rangle & \langle 0.5191,0.1404,0.1803\rangle & \langle 0.6760,0.1404,0.1604\rangle \end{pmatrix}.$$

Based on Eqs. (21)-(23), the following results can be obtained:

$G(a_1)=7402;\ G(a_2)=0.5138;\ G(a_3)=0.6229;\ G(a_4)=0.1201.$

Since $G(a_4)<G(a_2)<G(a_3)<G(a_1)$. So the final ranking is $a_4 \succ a_2 \succ a_3 \succ a_1$ and the best alternative is a_4 while the worst alternative is a_1.

From the analysis presented above, for the *SNCIWA* and *SNCIWG* operators, the final ranking is always $a_4 \succ a_2 \succ a_3 \succ a_1$. The best alternative is a_4; while the worst alternative is a_1. In order to calculate the actual aggregation values of the alternatives, different aggregation operators can be used and considered as a reflection of the decision makers' preferences. It is also found that *SNCIWA* and *SNCIWG* operators are used to deal with different relationships of the aggregated arguments, which can provide more choices for decision makers. Generally speaking, decision-makers can choose the *SNCIWA* operator to calculate the actual aggregation values of the alternatives, since the calculation is simpler than *SNCIWG* aggregation operator.

6.2 Comparison analysis

In order to validate the feasibility of the proposed decision-making method, a comparative study was conducted with other methods as show in Ye [36, 37, 42] and Peng et al. [44, 45] where the criteria are independent of each other.

For four methods in Ye [36, 37, 42] and Peng et al. [44], some aggregation operators were developed to aggregate the simplified neutrosophic formation first, then the cosine similarity measure, the correlation coefficient and the weighted cross-entropy between each alternative and the ideal alternative were calculated respectively and determine the final ranking order of all alternatives. For the method in Peng et al. [45], some outranking relations were defined. Then an outranking method was proposed to ranking alternatives based on those relations.

However, in those compared methods, they all do not clarify that how to solve a situation where the weight information is unknown. So the comparison analysis was based on the same illustrative example, but the weight of experts and the weight of criteria is determined as follows: $\lambda=(0.1,0.6,0.3)$ and $w=(0.15,0.20,0.35,0.30)$. Then the results by using the different methods can be obtained, which showed in Table 1.

Table 1. The results utilizing the different methods

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Ye [36]	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1
Ye [37]	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
Ye [42]	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
Peng et al. [44]	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1
Peng et al. [45]	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
The proposed method	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1

According to the results presented in Table 1, if the methods [36] are used, then the best alternative is a_2 while the worst alternative is a_1. If the proposed approach and the method of Ye [37, 42] and Peng et al. [45] are used, then the best alternative is a_4 while the worst alternative is a_1. We can see that the result of the proposed approach is different from those using the method of Ye [36] and Peng et al. [44], but is the same as that using the methods of Ye [37, 42] and Peng et al. [45]. For all compared methods and the proposed approach, there are some reasons can interpret the the final rankings to some extent. Firstly, those measure and aggregation operator being involved in those methods are related to some unreasonable operations as we discussed in Examples 1-3. Secondly, different measures and aggregation operator also lead to different rankings and it is intractable for decision makers to confirm their judgments among these operators and measures that have similar characteristics, which always need to a large amount of computation. Finally, the result using the method of Peng et al. [45] is consistent with that of the proposed approach. Although the method was constructed based on reliable theories and may be robust to some degree, they cannot consider the correlation of criteria.

From the analysis presented above, it can be concluded that the main advantages of the approach developed in this paper over the other methods are not only due to its ability to effectively overcome the shortcomings of the compared methods, but also due to its ability to consider the criteria are interactive. This can avoid losing and distorting the preference information provided, which makes the final results better correspond with real life decision-making problems.

7. Conclusion

SNSs can be applied in solving problems with uncertain, imprecise, incomplete and inconsistent information that exist in scientific and engineering situations. Based on related research achievements in IFSs, the novel operations of SNSs were defined in this paper, and two aggragation operators are developed based on Choquet integral, their desirable properties were discussed in detail. Thus, an MCDM method was established based on Choquet integral aggregation operators and TOPSIS method. By using the proposed method, the ranking of all alternatives can be determined and the best one can easily be identified. One illustrative example demonstrated the applicability of the proposed decision-making method. The advantage of this study is that an outranking approach for MCDM problems with SNSs could overcome the shortcomings of existing methods as we discussed earlier. Moreover, the proposed approach could consider the interactive of criteria with simplified neutrosophic formation. The comparison analysis also showed that the final result produced by the proposed method is more precise and reliable than the results produced by existing methods. In the further research, we will continue to study the distance measures of SNNs.

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