Applications of Complex Neutrosophic Sets in Medical Diagnosis Based on Similarity Measures

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Abstract

This paper presents some similarity measures between complex neutrosophic sets. A complex neutrosophic set is a generalization of neutrosophic set whose complex-valued truth membership function, complex-valued indeterminacy membership function, and complex valued falsity membership functions are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. In the present study, we have proposed complex cosine, Dice and Jaccard similarity measures and investigated some of their properties. Finally, complex neutrosophic cosine, Dice and Jaccard similarity measures have been applied to a medical diagnosis problem with complex neutrosophic information.

Introduction

In 1965, Zadeh [1] coined the term degree of membership and first defined the fuzzy set in order to deal with uncertainty. In 1986, Atanassov [2] introduced the degree of non-membership as independent component and defined the intuitionistic fuzzy set. Smarandache [3] introduced the degree of indeterminacy as independent component and defined the neutrosophic set to deal with uncertainty, indeterminacy and inconsistency. To use the concept of neutrosophic set in practical fields such as real scientific and engineering applications, Wang et al.[4] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value. Similarity measures play an important role in the analysis and research of medical diagnosis [5] , pattern recognition [6], decision making [7], and clustering analysis [8] in uncertain, indeterminate and inconsistent environment.

Various similarity measures of SVNSs have been proposed and mainly applied them to decision making problem.

Majumdar and Samanta [9] introduced the similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [10] proposed three vector similarity measures for simplified neutrosophic sets. Ye [11] also proposed improved cosine similarity measure for single valued neutrosophic sets based on cosine function. The same author [12] proposed the similarity measures of SVNSs for multiple attribute group decision making method with completely unknown weights. Ye and Zhang [13] further proposed the similarity measures of SVNSs for decision making problems. Biswas et al. [14] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Pramanik and Mondal [15] proposed rough cosine similarity measure in rough neutrosophic environment. Mondal and Pramanik [16] proposed neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. Mondal and Pramanik [17] proposed refined cotangent similarity measure in single valued neutrosophic environment. The same authors [18] further proposed cotangent similarity measure under rough neutrosophic environments. The same authors [19] further proposed some rough neutrosophic similarity measures and their application to multi attribute decision making.

Recently Ali and Smarandache [20] proposed the concept of complex neutrosophic set. It seems to be very powerful.

In this paper an attempt has been made to establish some similarity measures namely, cosine, Dice and Jaccard similarity measures in complex neutrosophic environment and their applications in medical diagnosis.

Rest of the paper is structured as follows: Section 2 presents neutrosophic and complex neutrosophic preliminaries. Section 3 is devoted to introduce complex Cosine, Dice and Jaccard similarity measure for complex neutrosophic sets and studied some of its properties. Section 4 presents decision making based on complex Dice and Jaccard similarity measure. Section 5 presents the application of complex Cosine, Dice and Jaccard similarity measures in medical diagnosis. Section 6 presents the concluding remarks and future scope of research.

Mathematical Preliminaries

Neutrosophic Set [3]

The concept of neutrosophic set [3] is derived from the new branch of philosophy, namely, neutrosophy [3]. Neutrosophy succeeds in creating different fields of studies because of its capability to deal with the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Definition 1

Let G be a space of points (objects) with generic element in E denoted by y. Then a neutrosophic set N in G is characterized by a truth membership function T_N, an indeterminacy membership function I_N and a falsity membership function F_N. The functions T_N and F_N are real standard or non-standard subsets of $]^-0, 1^+[$ that is T_N: $G \rightarrow]^-0, 1^+[$; I_N: $G \rightarrow]^-0, 1^+[$; F_N: $G \rightarrow]^-0, 1^+[$. The sum of $T_N(y)$, $I_N(y)$, $F_N(y)$ is given by $^-0 \leq \sup T_N(y) + \sup I_N(y) + \sup F_N(y) \leq 3^+$

Definition 2 (complement)[3]

The complement of a neutrosophic set A is denoted by N^c and is defined as follows: $T_{N^c}(y) = \{1^+\} - T_N(y)$; $I_{N^c}(y) = \{1^+\} - I_N(y)$

$F_{N^c}(y) = \{1^+\} - F_N(y)$

Definition 3 (Containment) [3]

A neutrosophic set N is contained in the other neutrosophic set M, $N \subseteq M$ if and only if the following result holds.

$\inf T_N(y) \leq \inf T_M(y)$, $\sup T_N(y) \leq \sup T_M(y)$

$\inf I_N(y) \geq \inf I_M(y)$, $\sup I_N(y) \geq \sup I_M(y)$

$\inf F_N(y) \geq \inf F_M(y)$, $\sup F_N(y) \geq \sup F_M(y)$

for all y in G.

Definition 4 [3]:

Single-valued neutrosophic set. Let G be a universal space of points (objects) with a generic element of G denoted by y.

A single valued neutrosophic set [3] S is characterized by a truth membership function $T_N(y)$, a falsity membership function $F_N(y)$ and indeterminacy function $I_N(y)$ with $T_N(y)$, $F_N(y)$, $I_N(y) \in [0,1]$ for all y in G.

When G is continuous, a SNVS S can be written as follows:

$$S = \int_y \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in G$$

and when G is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in G$$

It should be observed that for a SVNS S,

$$0 \le \sup T_S(y) + \sup F_S(y) + \sup I_S(y) \le 3, \forall y \in G$$

Definition 5: [3]

The complement of a single valued neutrosophic set S is denoted by S^c and is defined as follows:

$${T_S}^c(y) = F_S(y);\ {I_S}^c(y) = 1 - I_S(y);\ {F_S}^c(y) = T_S(y)$$

Definition 6: [3]

A SVNS S_N is contained in the other SVNS S_M, denoted as $S_N \subseteq S_M$ iff $T_{S_N}(y) \le T_{S_M}(y)$; $I_{S_N}(y) \ge I_{S_M}(y)$; $F_{S_N}(y) \ge F_{S_M}(y)$, $\forall y \in G$.

Definition 7: [3]

Two single valued neutrosophic sets S_N and S_M are equal, i.e. $S_N = S_M$, iff, $S_N \subseteq S_M$ and $S_N \supseteq S_M$

Definition 8: (Union) [3]

The union of two SVNSs S_N and S_M is a SVNS S_L, written as $S_L = S_N \cup S_M$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_N and S_M by the following equations

$$T_{S_L}(y) = \max\left(T_{S_N}(y), T_{S_M}(y)\right);$$

$$I_{S_L}(y) = \max\left(I_{S_N}(y), I_{S_M}(y)\right);$$

$$F_{S_L}(y) = \min\left(F_{S_N}(y), F_{S_M}(y)\right) \text{ for all y in G}$$

Definition 9: (Intersection) [3]

The intersection of two SVNSs N and M is a SVNS L, written as $L = N \cap M$. Its truth membership, indeterminacy membership and falsity membership functions are related to N an M by the following equations:

$$T_{S_L}(y) = \min\left(T_{S_N}(y), T_M(y)\right);$$

$$I_{SL}(y) = \max\left(I_{S_N}(y), I_{S_M}(y)\right);$$

$$F_{SL}(y) = \max\left(F_{S_N}(y), F_{SM}(y)\right), \forall y \in G$$

Distance between two neutrosophic sets.

The general SVNS can be presented in the follow form as follows:

$$S = \{(y/(T_S(y), I_S(y), F_S(y))) : y \in G\}$$

Finite SVNSs can be represented as follows:

$$S = \{(y_1/(T_S(y_1), I_S(y_1), F_S(y_1))), \cdots, (y_m/(T_S(y_m), I_S(y_m), F_S(y_m)))\}, \forall y \in G \quad (1)$$

Definition 10: Let

$$S_N = \{(y_1/(T_{S_N}(y_1), I_{S_N}(y_1), F_{S_N}(y_1))), \cdots, (y_n/(T_{S_N}(y_n), I_{S_N}(y_n), F_{S_N}(y_n)))\} \quad (2)$$

$$S_M = \{(x_1/(T_{S_M}(y_1), I_{S_M}(y_1), F_{S_M}(y_1))), \cdots, (x_n/(T_{S_M}(y_n), I_{S_M}(y_n), F_{S_M}(y_n)))\} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance between two SNVS N and M is defined as follows:

$$d_S(S_N, S_M) = \sum_{i=1}^{n} \left\langle \left|T_{S_N}(y) - T_{S_M}(y)\right| + \left|I_{S_N}(y) - I_{S_M}(y)\right| + \left|F_{S_N}(y) - F_{S_M}(y)\right| \right\rangle \quad (4)$$

and normalized Hamming distance between two SNVS N and M is defined as follows:

$$^N d_S(S_N, S_M) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \left|T_{S_N}(y) - T_{S_M}(y)\right| + \left|I_{S_N}(y) - I_{S_M}(y)\right| + \left|F_{S_N}(y) - F_{S_M}(y)\right| \right\rangle \quad (5)$$

with the following properties

1. $0 \le d_S(S_N, S_M) \le 3n$
2. $0 \le {}^N d_S(S_N, S_M) \le 1$

Complex Neutrosophic Set [20]

A complex neutrosophic set S, defined on a universe of discourse X, which is characterized by a truth membership function $T_S(x)$, an indeterminacy membership function $I_S(x)$, and a falsity membership function $F_S(x)$ that assigns a complex-valued grade of $T_S(x)$, $I_S(x)$, $F_S(x)$ in S for all x belongs to X. The values $T_S(x)$, $I_S(x)$, $F_S(x)$ and their sum may all within the unit circle in the complex plane. So it is of the following form,

$T_S(x) = p_S(x)e^{i\mu_S(x)}$, $I_S(x) = q_S(x)e^{i\vartheta_S(x)}$, $F_S(x) = r_S(x)e^{i\omega_S(x)}$

Where, $p_S(x)$, $q_S(x)$, $r_S(x)$ and $\mu_S(x)$, $\vartheta_S(x)$, $\omega_S(x)$ are respectively real valued and $p_S(x)$, $q_S(x)$, $r_S(x) \in [0,1]$ such that $0 \le p_S(x) + q_S(x) + r_S(x) \le 3$

Definition 11: A complex neutrosophic set CN_1 is contained in the other complex neutrosophic set CN_2 denoted as $CN_1 \subseteq CN_2$ iff $p_{CN_1}(x) \le p_{CN_2}(x)$, $q_{CN_1}(x) \le q_{CN_2}(x)$, $r_{CN_1}(x) \le r_{CN_2}(x)$, and $\mu_{CN_1}(x) \le \mu_{CN_2}(x)$, $\vartheta_{CN_1}(x) \le \vartheta_{CN_2}(x)$, $\omega_{CN_1}(x) \le \omega_{CN_2}(x)$.

Definition12: Two complex neutrosophic set CN_1 and CN_2 are equal i.e. $CN_1 = CN_2$ iff $p_{CN_1}(x) = p_{CN_2}(x)$, $q_{CN_1}(x) = q_{CN_2}(x)$, $r_{CN_1}(x) = r_{CN_2}(x)$, $\mu_{CN_1}(x) = \mu_{CN_2}(x)$, $\vartheta_{CN_1}(x) = \vartheta_{CN_2}(x)$, and $\omega_{CN_1}(x) = \omega_{CN_2}(x)$.

Section III

Complex neutrosophic cosine similarity measure

The complex cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two complex neutrosophic sets. Existing cosine similarity measures does not deal with complex neutrosophic sets till now. Therefore, a new cosine similarity measure between complex neutrosophic sets is proposed in 3-D vector space.

Definition3.1: Assume that there are two complex neutrosophic sets namely,

$CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\rangle$ and

$CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\vartheta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\rangle$ in S for all x belongs to X. A complex cosine similarity measure between complex neutrosophic sets CN_1 and CN_2 is proposed as follows:

$$C_{CNS} = \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}}{\sqrt{.a_1b_1+c_1d_1+e_1f_1}\sqrt{.a_2b_2+c_2d_2+e_2f_2}}$$

$a_1 = \text{Re}\,[p_{S_1}(x)e^{i\mu_{S_1}(x)}]$, $b_1 = \text{Im}\,[p_{S_1}(x)e^{i\mu_{S_1}(x)}]$, $a_2 = \text{Re}\,[p_{S_2}(x)e^{i\mu_{S_2}(x)}]$, $b_2 = \text{Im}\,[p_{S_2}(x)e^{i\mu_{S_2}(x)}]$,

$c_1 = \text{Re}\,[q_{S_1}(x)e^{i\vartheta_{S_1}(x)}]$, $d_1 = \text{Im}\,[q_{S_1}(x)e^{i\vartheta_{S_1}(x)}]$, $c_2 = \text{Re}\,[q_{S2}(x)e^{i\vartheta_{S_2}(x)}]$, $d_2 = \text{Im}\,[q_{S_2}(x)e^{i\vartheta_{S2}(x)}]$,

e_1 = Re [$r_{S_1}(x)e^{i\omega_{s1}(x)}$], f_1 = Im [$r_{S_1}(x)e^{i\omega_{s1}(x)}$], e_2 = Re [$r_{S2}(x)e^{i\omega_{s2}(x)}$], f_2 = Im [$r_{S_2}(x)e^{i\omega_{s2}(x)}$]

Let CN_1 and CN_2 be complex neutrosophic sets then,

1. $0 \le C_{CNS}(CN_1, CN_2) \le 1$
2. $C_{CNS}(CN_1, CN_2) = C_{CNS}(CN_2, CN_1)$
3. $C_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$
4. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_1, CN_2)$, and $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_2, CN)$

Proofs:

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $CN_1 = CN_2$, then obviously $C_{CNS}(CN_1, CN_2) = 1$. On the other hand if $C_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

4. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that l_1 = Re [$p_S(x)e^{i\mu_S(x)}$], l_2 = Im [$p_S(x)e^{i\mu_S(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_S(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_S(x)}$], n_1 = Re [$r_S(x)e^{i\omega_S(x)}$], n_2 = Im [$r_S(x)e^{i\omega_S(x)}$]

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.
The cosine function is decreasing function within the interval $[0, \pi/2]$. Hence we can write $C_{CNS}(CN_1, CN) \le C_{CNS}(CN, CN_2)$, and $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_2, CN)$.

Weighted Complex neutrosophic Cosine similarity measure

Definition3.2:

$$C_{WCNS} = \sum_{i=1}^{n} w_i \frac{\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}}{\sqrt{.a_1b_1+c_1d_1+e_1f_1}\sqrt{.a_2b_2+c_2d_2+e_2f_2}}$$

Where, $\sum_{i=1}^{n} w_i = 1$

Complex neutrosophic Dice similarity measure

Definition3.3: Assume that there are two complex neutrosophic sets namely,

$CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{s1}(x)}, q_{S_1}(x)e^{i\vartheta_{s1}(x)}, r_{S_1}(x)e^{i\omega_{s1}(x)} \right\rangle$ and

$CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{s2}(x)}, q_{S_2}(x)e^{i\vartheta_{s2}(x)}, r_{S_2}(x)e^{i\omega_{s2}(x)} \right\rangle$ in S for all x belongs to X. A complex Dice similarity measure between complex neutrosophic sets CN_1 and CN_2 is proposed as follows:

$$D_{CNS} = \sum_{i=1}^{n} \frac{2\left(\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}\right)}{\left(a_1b_1+c_1d_1+e_1f_1\right)+\left(a_2b_2+c_2d_2+e_2f_2\right)}$$

a_1 = Re [$p_{S_1}(x)e^{i\mu_{s1}(x)}$], b_1 = Im [$p_{S_1}(x)e^{i\mu_{s1}(x)}$], a_2 = Re [$p_{S_2}(x)e^{i\mu_{s2}(x)}$], b_2 = Im [$p_{S_2}(x)e^{i\mu_{s2}(x)}$],

c_1 = Re [$q_{S_1}(x)e^{i\vartheta_{s1}(x)}$], d_1 = Im [$q_{S_1}(x)e^{i\vartheta_{s1}(x)}$], c_2 = Re [$q_{S2}(x)e^{i\vartheta_{s2}(x)}$], d_2 = Im [$q_{S_2}(x)e^{i\vartheta_{s2}(x)}$],

e_1 = Re [$r_{S_1}(x)e^{i\omega_{S_1}(x)}$], f_1 = Im [$r_{S_1}(x)e^{i\omega_{S_1}(x)}$], e_2 = Re [$r_{S2}(x)e^{i\omega_{S2}(x)}$], f_2 = Im [$r_{S_2}(x)e^{i\omega_{S2}(x)}$]

Let CN_1 and CN_2 be complex neutrosophic sets then,

1. $0 \le D_{CNS}(CN_1, CN_2) \le 1$
2. $D_{CNS}(CN_1, CN_2) = D_{CNS}(CN_2, CN_1)$
3. $D_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$
4. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $D_{CNS}(CN_1, CN) \le D_{CNS}(CN_1, CN_2)$, and $D_{CNS}(CN_1, CN) \le D_{CNS}(CN_2, CN)$.

Proofs:

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $CN_1 = CN_2$, then obviously $D_{CNS}(CN_1, CN_2) = 1$. On the other hand if $D_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.
This implies that $CN_1 = CN_2$.
4. Let, $CN=\left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)}\right\rangle$ and also assume that l_1 = Re [$p_S(x)e^{i\mu_S(x)}$], l_2 = Im [$p_S(x)e^{i\mu_S(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_S(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_S(x)}$], n_1 = Re [$r_S(x)e^{i\omega_S(x)}$], f_1 = Im [$r_S(x)e^{i\omega_S(x)}$].

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.
Hence we can write $C_{CNS}(CN_1, CN) \le D_{CNS}(CN, CN_2)$, and $D_{CNS}(CN_1, CN) \le D_{CNS}(CN_2, CN)$.

Weighted Complex neutrosophic Dice similarity measure

Definition3.4:

$$D_{WCNS} = \sum_{i=1}^{n} w_i \frac{2\left(\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}\right)}{(a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)}$$

Where, $\sum_{i=1}^{n} w_i = 1$

Complex neutrosophic Jaccard similarity measure

Definition3.5: Assume that there are two complex neutrosophic sets namely,

$CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)}\right\rangle$ and

$CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\vartheta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)}\right\rangle$ in S for all x belongs to X. A complex cosine similarity measure between complex neutrosophic sets CN_1 and CN_2 is proposed as follows:

$$J_{CNS} = \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}}{\left\langle (a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)-\left(\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}\right)\right\rangle}$$

a_1 = Re [$p_{S_1}(x)e^{i\mu_{S_1}(x)}$], b_1 = Im [$p_{S_1}(x)e^{i\mu_{S_1}(x)}$], a_2 = Re [$p_{S_2}(x)e^{i\mu_{S_2}(x)}$], b_2 = Im [$p_{S_2}(x)e^{i\mu_{S_2}(x)}$],

c_1 = Re [$q_{S_1}(x)e^{i\vartheta_{s1}(x)}$], d_1 = Im [$q_{S_1}(x)e^{i\vartheta_{s1}(x)}$], c_2 = Re [$q_{S2}(x)e^{i\vartheta_{s2}(x)}$], d_2 = Im [$q_{S_2}(x)e^{i\vartheta_{s2}(x)}$],

e_1 = Re [$r_{S_1}(x)e^{i\omega_{s1}(x)}$], f_1 = Im [$r_{S_1}(x)e^{i\omega_{s1}(x)}$], e_2 = Re [$r_{S2}(x)e^{i\omega_{s2}(x)}$], f_2 = Im [$r_{S_2}(x)e^{i\omega_{s2}(x)}$].

Let CN_1 and CN_2 be complex neutrosophic sets then,

1. $0 \le J_{CNS}(CN_1, CN_2) \le 1$
2. $J_{CNS}(CN_1, CN_2) = J_{CNS}(CN_2, CN_1)$
3. $C_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$
4. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_1, CN_2)$, and $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_2, CN)$.

Proofs:

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $CN_1 = CN_2$, then obviously $J_{CNS}(CN_1, CN_2) = 1$. On the other hand if $J_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

4. Let, $CN = \left\langle p_S(x)e^{i\mu_s(x)}, q_S(x)e^{i\vartheta_s(x)}, r_S(x)e^{i\omega_s(x)} \right\rangle$ and also assume that l_1 = Re [$p_s(x)e^{i\mu_s(x)}$], l_2 = Im [$p_s(x)e^{i\mu_s(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_s(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_s(x)}$], n_1 = Re [$r_S(x)e^{i\omega_s(x)}$], f_1 = Im [$r_S(x)e^{i\omega_s(x)}$].

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

The cosine function is decreasing function within the interval $[0, \pi/2]$. Hence we can write $J_{CNS}(CN_1, CN) \le J_{CNS}(CN, CN_2)$, and $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_2, CN)$.

Weighted Complex neutrosophic Jaccard similarity measure

Definition3.5:

$$J_{WCNS} = \sum_{i=1}^{n} w_i \frac{\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}}{\left\langle (a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)-\left(\sqrt{a_1b_1a_2b_2} + \sqrt{c_1d_1c_2d_2} + \sqrt{e_1f_1e_2f_2}\right)\right\rangle}$$

Where, $\sum_{i=1}^{n} w_i = 1$

4 Example on medical diagnosis

We consider a medical diagnosis problem for illustration of the proposed approach. Medical diagnosis comprises of uncertainties and increased volume of information available to physicians from new medical technologies. So, all collected information may be in complex neutrosophic form. The three components of a complex neutrosophic set are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. So, to deal more indeterminacy situations in medical diagnosis complex neutrosophic environment is more acceptable.

The process of classifying different set of symptoms under a single name of a disease is very difficult. In some practical situations, there exists possibility of each element within a periodic form of neutrosophic sets. So, medical diagnosis involves more indeterminacy. Complex neutrosophic sets handle this situation. Actually this approach is more flexible, dealing with more indeterminacy areas and easy to use. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis in complex neutrosophic environment.

The main feature of this proposed approach is that it considers complex truth membership, complex indeterminate and complex false membership of each element taking periodic form of neutrosophic sets.

Now, an example of a medical diagnosis is presented. Let P = {P_1, P_2, P_3} be a set of patients, D ={Viral Fever, Malaria, Stomach problem, Chest problem} be a set of diseases and S = {Temperature, Headache, Stomach pain, cough, Chest pain.} be a set of symptoms. Our investigation is to examine the patient and to determine the disease of the patient in complex neutrosophic environment.

Table 1: (Relation-1) the relation between Patients and Symptoms in complex neutrosophic form

Relation-1	Temperature	Headache	Stomach pain	cough	Chest pain
P_1	$\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.2e^{0.8i} \rangle$	$\langle 0.4e^{1.2i}, 0.4e^{1.1i}, 0.3e^{0.7i} \rangle$	$\langle 0.3e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \rangle$	$\langle 0.6e^{1.0i}, 0.5e^{1.2i}, 0.3e^{0.8i} \rangle$	$\langle 0.4e^{1.0i}, 0.3e^{1.0i}, 0.2e^{0.5i} \rangle$
P_2	$\langle 0.7e^{1.3i}, 0.4e^{1.2i}, 0.5e^{0.9i} \rangle$	$\langle 0.4e^{1.5i}, 0.6e^{1.5i}, 0.3e^{0.5i} \rangle$	$\langle 0.5e^{1.4i}, 0.4e^{1.2i}, 0.4e^{1.0i} \rangle$	$\langle 0.6e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \rangle$	$\langle 0.3e^{1.5i}, 0.4e^{1.0i}, 0.5e^{1.0i} \rangle$
P_3	$\langle 0.5e^{0.6i}, 0.5e^{1.2i}, 0.5e^{0.9i} \rangle$	$\langle 0.5e^{1.3i}, 0.4e^{1.2i}, 0.4e^{0.4i} \rangle$	$\langle 0.4e^{1.0i}, 0.4e^{1.0i}, 0.2e^{0.6i} \rangle$	$\langle 0.4e^{1.0i}, 0.5e^{1.1i}, 0.2e^{1.2i} \rangle$	$\langle 0.5e^{1.2i}, 0.2e^{1.2i}, 0.2e^{1.4i} \rangle$

Table 2: Numeric values of a_1, b_1, c_1, d_1, e_1, and f_1

Numeric values → **Alternatives** ↓	$(a_1,b_1), (c_1,d_1), (e_1, f_1)$	$(a_1,b_1), (c_1,d_1), (e_1, f_1)$	$(a_1,b_1), (c_1,d_1), (e_1, f_1)$	$(a_1,b_1), (c_1,d_1), (e_1, f_1)$	$(a_1,b_1), (c_1,d_1), (e_1, f_1)$
	$[a_1b_1, c_1d_1, e_1f_1]$	$[a_1b_1, c_1d_1, e_1f_1]$	$[a_1b_1, c_1d_1, e_1f_1]$	$[a_1b_1, c_1d_1, e_1f_1]$	$[a_1b_1, c_1d_1, e_1f_1]$
	$[(a_1b_1)^{0.5}, (c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5}, (c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5}, (c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5}, (c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5}, (c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$
P_1	(0.324, 0.505), (0.145, 0.373), (0.139, 0.143)	(0.362, 0.932), (0.454, 0.891), (0.765, 0.644)	(0.162, 0.252), (0.216, 0.336), (0.330, 0.226)	(0.324, 0.504), (0.181, 0.280), (0.209, 0.215)	(0.182, 0.356), (0.162, 0.252), (0.175, 0.096)
	[0.164, 0.054, 0.020]	[0.337, 0.406, 0.493]	[0.041, 0.073, 0.075]	[0.163, 0.051, 0.045]	[0.065, 0.041, 0.016]
	[0.405, 0.232, 0.141]	[0.581, 0.637, 0.702]	[0.202, 0.270, 0.274]	[0.404, 0.226, 0.212]	[0.255, 0.202, 0.126]
P_2	(0.187, 0.675), (0.145, 0.373), (0.311, 0.392)	(0.028, 0.399), (0.043, 0.598), (0.263, 0.144)	(0.085, 0.493), (0.145, 0.373), (0.216, 0.336)	(0.324, 0.505), (0.216, 0.336), (0.330, 0.226)	(0.021, 0.299), (0.216, 0.336), (0.270, 0.421)
	[0.126, 0.054, 0.121]	[0.011, 0.026, 0.157]	[0.042, 0.054, 0.073]	[0.164, 0.073, 0.075]	[0.006, 0.073, 0.113]
	[0.355, 0.232, 0.348]	[0.105, 0.161, 0.396]	[0.205, 0.232, 0.271]	[0.405, 0.270, 0.274]	[0.077, 0.270, 0.336]
P_3	(0.413, 0.283), (0.181, 0.466), (0.311, 0.392)	(0.134, 0.482), (0.145, 0.373), (0.368, 0.156)	(0.216, 0.336), (0.216, 0.336), (0.165, 0.113)	(0.216, 0.336), (0.216, 0.336), (0.216, 0.336)	(0.181, 0.466), (0.072, 0.086), (0.034, 0.197)
	[0.117, 0.084, 0.122]	[0.065, 0.054, 0.057]	[0.073, 0.073, 0.019]	[0.073, 0.073, 0.073]	[0.084, 0.006, 0.007]
	[0.342, 0.290, 0.349]	[0.255, 0.232, 0.239]	[0.270, 0.270, 0.138]	[0.270, 0.270, 0.270]	[0.290, 0.077, 0.084]

Table 3: (Relation-2) The relation among Symptoms and Diseases

Relation-2	Viral Fever	Malaria	Stomach problem	Chest problem
Temperature	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.3e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.3i}, 0.4e^{1.4i}, 0.2e^{1.5i} \right\rangle$	$\left\langle 0.5e^{1.4i}, 0.5e^{1.5i}, 0.2e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.5i}, 0.4e^{0.6i}, 0.5e^{0.7i} \right\rangle$
Headache	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.2e^{0.8i} \right\rangle$	$\left\langle 0.4e^{0.7i}, 0.4e^{0.8i}, 0.3e^{0.9i} \right\rangle$	$\left\langle 0.5e^{0.8i}, 0.4e^{0.9i}, 0.2e^{1.0i} \right\rangle$	$\left\langle 0.5e^{0.9i}, 0.4e^{1.0i}, 0.5e^{0.8i} \right\rangle$
Stomach pain	$\left\langle 0.4e^{1.0i}, 0.4e^{1.1i}, 0.4e^{1.2i} \right\rangle$	$\left\langle 0.5e^{1.1i}, 0.2e^{1.2i}, 0.2e^{1.3i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.3i}, 0.5e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.3i}, 0.4e^{1.4i}, 0.3e^{1.5i} \right\rangle$
Cough	$\left\langle 0.3e^{1.4i}, 0.4e^{1.5i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.5i}, 0.5e^{0.6i}, 0.3e^{0.7i} \right\rangle$	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.3e^{0.8i} \right\rangle$	$\left\langle 0.3e^{0.7i}, 0.4e^{0.8i}, 0.4e^{0.9i} \right\rangle$
Chest pain	$\left\langle 0.4e^{0.8i}, 0.4e^{0.9i}, 0.5e^{1.0i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.3e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.4i}, 0.3e^{0.6i}, 0.2e^{0.8i} \right\rangle$

Table 4: (Relation-2) Numeric values of a_2, b_2, c_2, d_2, e_2, and f_2

Numeric values → **Symtoms** ↓				
	(a_2,b_2), (c_2, d_2), (e_2, f_2)	(a_2,b_2), (c_2, d_2), (e_2, f_2)	(a_2,b_2), (c_2, d_2), (e_2, f_2)	(a_2,b_2), (c_2, d_2), (e_2, f_2)
	$[a_2b_2, c_2d_2, e_2f_2]$	$[a_2b_2, c_2d_2, e_2f_2]$	$[a_2b_2, c_2d_2, e_2f_2]$	$[a_2b_2, c_2d_2, e_2f_2]$
	$[(a_2b_2)^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$
Temperature	(0.145, 0.373), (0.068, 0.394), (0.248, 0.170)	(0.160, 0.578), (0.068, 0.394), (0.014, 0.199)	(0.085, 0.493), (0.036, 0.498), (0.165, 0.113)	(0.042, 0.598), (0.330, 0.226), (0.383, 0.322)
	[0.054, 0.026, 0.042]	[0.092, 0.026, 0.003]	[0.042, 0.018, 0.019]	[0.025, 0.075, 0.123]
	[0.232, 0.161, 0.205]	[0.303, 0.161, 0.055]	[0.205, 0.134, 0.138]	[0.158, 0.274, 0.351]
Headache	(0.413, 0.283), (0.306, 0.258), (0.139, 0.143)	(0.306, 0.258), (0.280, 0.283), (0.188, 0.235)	(0.349, 0.358), (0.249, 0.313), (0.108, 0.168)	(0.311, 0.392), (0.216, 0.336), (0.349, 0.359)
	[0.119, 0.079, 0.020]	[0.079, 0.079, 0.044]	[0.125, 0.078, 0.018]	[0.122, 0.073, 0.125]
	[0.345, 0.281, 0.141]	[0.281, 0.281, 0.210]	[0.354, 0.279, 0.134]	[0.349, 0.270, 0.354]
Stomach pain	(0.540, 0.336), (0.182, 0.356), (0.145, 0.373)	(0.227, 0.451), (0.072, 0.186), (0.053, 0.193)	(0.145, 0.373), (0.106, 0.385), (0.085, 0.493)	(0.106, 0.385), (0.068, 0.394), (0.021, 0.299)
	[0.181, 0.061, 0.054]	[0.102, 0.013, 0.010]	[0.054, 0.041, 0.042]	[0.041, 0.027, 0.006]
	[0.425, 0.247, 0.232]	[0.319, 0.114, 0.100]	[0.232, 0.202, 0.205]	[0.202, 0.164, 0.077]

Cough	(0.051, 0.296), (0.028, 0.399), (0.413, 0.283)	(0.028, 0.398), (0.413, 0.283), (0.230, 0.193)	(0.413, 0.283), (0.306, 0.258), (0.209, 0.215)	(0.230, 0.193), (0.279, 0.287), (0.249, 0.313)
	[0.015, 0.011, 0.117]	[0.011, 0.117, 0.044]	[0.117, 0.079, 0.045]	[0.044, 0.080, 0.078]
	[0.122, 0.105, 0.342]	[0.105, 0.342, 0.210]	[0.342, 0.281, 0.212]	[0.210, 0.283, 0.279]
Chest pain	(0.279, 0.287), (0.249, 0.313), (0.270, 0.421)	(0.194, 0.505), (0.149, 0.373), (0.051, 0.296)	(0.149, 0.373), (0.068, 0.394), (0.413, 0.283)	(0.068, 0.394), (0.248, 0.170), (0.139, 0.143)
	[0.080, 0.078, 0.114]	[0.098, 0.056, 0.015]	[0.056, 0.027, 0.117]	[0.027, 0.042, 0.020]
	[0.283, 0.279, 0.338]	[0.313, 0.236, 0.122]	[0.236, 0.164, 0.342]	[0.164, 0.205, 0.141]

Table 5: The Complex cosine neutrosophic Measure between Relation-1 and Relation-2

Complex neutrosophic cosine similarity measure	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
P_1	**0.9303**	0.9272	0.8662	0.8442
P_2	0.8581	0.7512	0.8148	**0.8681**
P_3	**0.9267**	0.8602	0.8409	0.7864

Table 6: The Complex Dice neutrosophic Measure between Relation-1 and Relation-2

Complex neutrosophic Dice similarity measure	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
P_1	**0.8623**	0.8281	0.8596	0.8451
P_2	0.8024	0.7320	0.7935	**0.8307**
P_3	**0.9005**	0.8473	0.8187	0.7672

Table 7: The Complex Jaccard neutrosophic Measure between Relation-1 and Relation-2

Complex neutrosophic Jaccard similarity measure	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
P_1	**0.8595**	0.8114	0.8498	0.8443
P_2	0.8201	0.8019	0.7911	**0.8502**
P_3	**0.8708**	0.8147	0.8469	0.7425

The highest correlation measure (see the Table 5, 6, 7) reflects the proper medical diagnosis. Therefore, all three patient P_1 and P_3 suffer from viral fever and patient P_2 suffers from chest problem.

Conclusion

In this paper, we have proposed three similarity measures namely, cosine, Dice and Jaccard based on complex neutrosophic set. We have also proved some of their basic properties. We have presented their applications in a medical diagnosis problem. The concept presented in this paper can be applied various multiple attribute decision making problems in complex neutrosophic environment.

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