Generalized Neutrosophic Soft Multi-Attribute Group Decision Making Based on TOPSIS

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Abstract

In this study, we present Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for solving generalized neutrosophic soft multi-attribute group decision making problem. The concept of generalized neutrosophic soft set is the hybridization of the two concepts namely generalized neutrosophic sets and soft sets. In the decision making process, the ratings of alternatives with respect to the parameters are expressed in terms of generalized neutrosophic sets. The evaluator selects the choice parameters and AND operator of generalized neutrosophic soft sets. Generalized neutrosophic soft set is used to aggregate the individual decision maker's opinion into a single opinion based on the performance values of the choice parameters. The weights of the choice parameters are derived from information entropy method. Then, the preference of alternatives is ranked by using TOPSIS method. Finally, a numerical example is solved to show the potential applicability and effectiveness of the proposed method.

Keyword

neutrosophic set, soft set, generalized neutrosophic soft set, TOPSIS, information entropy method, multi-attribute group decision making.

1 Introduction

Multi-attribute group decision making (MAGDM) is the process of determining the best option from a list of feasible alternatives with respect to several predefined attributes offered by the multiple decision makers (DMs).

However, the rating and the weights of the attributes cannot always be preciously assessed in terms of crisp numbers due to the ambiguity of human decision and the complexity of the attributes. In order to overcome the abovementioned difficulties, Zadeh [37] proposed fuzzy set theory by introducing membership function $T_A(x)$ to deal with uncertainty and partial information. Atanassov [3] incorporated the degree of non-membership as independent component and defined intuitionistic fuzzy. Smarandache [28, 29, 30, 31] proposed neutrosophic sets (NSs) by introducing degree of indeterminacy $I_A(x)$ as independent element in intuitionistc fuzzy set for handling incomplete, imprecise, inconsistent information. Later, Salama and Alblowi [27] defined generalized neutrosophic sets (GNSs), where the triplet functions satisfy the condition $T_A(x) \wedge F_A(x) \wedge I_A(x) \leq 0.5$.

In 1999, Molodtsov [23] introduced the notion of soft set theory for dealing with uncertainty and vagueness and the concept has been applied diverse practical fields such as decision making [16, 17, 18, 24], data analysis [38], forecasting [33], optimization [14], etc. Several researchers have incorporated different mathematical hybrid structures such as fuzzy soft sets [10, 11, 19], intuitionistic fuzzy soft set theory [8, 9, 20], possibility fuzzy soft set [2], generalized fuzzy soft sets [22, 35], generalized intuitionistic fuzzy soft [4], possibility intuitionistic fuzzy soft set [5], vague soft set [34], possibility vague soft set [1], neutrosophic soft sets [17], weighted neutrosophic soft sets [16], etc by generalizing and extending classical soft set theory of Molodtsov [23]. Recently, Broumi [7] studied generalized neutrosophic soft sets (GNSSs) and provided some definitions and operations of the concept. He also provided an application of GNSSs in decision making problem. Şahin, and Küçük [25] discussed a method to find out similarity measures of two GNSSs and provided an application of GNSS in decision making problem.

Hwang and Yoon [13] developed Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for solving classical multi-attribute decision making (MADM) problems. Liu et al. [15] proposed a new method based on generalized neutrosophic number Hamacher aggregation operators for MAGDM with single valued neutrosophic numbers. Ye [36] investigated an extended TOPSIS method for solving a MADM problems based on the single valued neutrosophic linguistic numbers under single valued neutrosophic linguistic assessment. Biswas et al. [6] extended the notion of TOPSIS method for MAGDM problems under single valued neutrosophic environment. In the paper, we have demonstrated a new mathematical model for solving generalized neutrosophic soft MAGDM problem based on TOPSIS method.

The content of the paper is structured as follows. Section 2 presents some basic definitions regarding NSs, soft sets, GNSs and GNSSs which will be useful for the construction of the paper. Section 3 is devoted to describe TOPSIS method for solving MAGDM problems under generalized neutrosophic soft environment. Section 4 is devoted to present the algorithm of the proposed TOPSIS method. A numerical problem regarding flat selection is presented to show the applicability of the proposed method in Section 5. Section 6 presents the concluding remarks and future scope of research.

2 Preliminaries

In this section, we present basic definitions regarding NSs, soft sets, GNSs and GNSSs.

2.1 Neutrosophic Set [28, 29, 30, 31]

Consider U be a space of objects with a generic element of U represented by x. Then, a neutrosophic set N on U is represented as follows:

$$N = \{x, \langle \mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) \rangle \mid x \in U\}$$

where, $\mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) : U \to]^{-}0, 1^{+}[$ present respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership of a point $x \in U$ to the set N with the condition $^{-}0 \le \mathrm{T}_N(x) + \mathrm{I}_N(x) + \mathrm{F}_N(x) \le 3^{+}$.

2.2 Generalized Neutrosophic Set [27]

Let U be a universe of discourse, with a generic element in U denoted by x. Then, a generalized neutrosophic set $G \subset U$ is represented as follows:

$$G = \{x, \langle \mathrm{T}_G(x), \mathrm{I}_G(x), \mathrm{F}_G(x) \rangle \mid x \in U\}$$

where, $\mathrm{T}_G(x)$, $\mathrm{I}_G(x)$, $\mathrm{F}_G(x)$ denote respectively the truth-membership function, indeterminacy-membership function, and falsity-membership function of a point $x \in U$ to the set G where the functions satisfy the condition $\mathrm{T}_G(x) \wedge \mathrm{I}_G(x) \wedge \mathrm{F}_G(x) \le 0.5$.

Definition 2.2.1 [21]

The Euclidean distance between two GNSs $S_1 = \{x_i, \langle \mathrm{T}_{S_1}(x_i), \mathrm{I}_{S_1}(x_i), \mathrm{F}_{S_1}(x_i) \rangle \mid x_i \in U\}$ and $S_2 = \{x_i, \langle \mathrm{T}_{S_2}(x_i), \mathrm{I}_{S_2}(x_i), \mathrm{F}_{S_2}(x_i) \rangle \mid x_i \in U\}$ is defined as follows:

$$\mathrm{D}_{\mathrm{Euc}}(S_1, S_2) = \sqrt{\sum_{j=1}^{n} \left\{ (\mathrm{T}_{S_1}(x_j) - \mathrm{T}_{S_2}(x_j))^2 + (\mathrm{I}_{S_1}(x_j) - \mathrm{I}_{S_2}(x_j))^2 + (\mathrm{F}_{S_1}(x_j) - \mathrm{F}_{S_2}(x_j))^2 \right\}} \quad (1)$$

and the normalized Euclidean distance between two GNSs S_1 and S_2 can be defined as follows:

$$D_{Euc}^{N}(S_1,S_2) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{(T_{S_1}(x_j)-T_{S_2}(x_j))^2+(I_{S_1}(x_j)-I_{S_2}(x_j))^2+(F_{S_1}(x_j)-F_{S_2}(x_j))^2\right\}} \quad (2)$$

2.3 Soft set [23]

Let X be a universal set and E be a set of parameters. Consider P (X) represents a power set of X. Also, let F be a non-empty set, where F $\subset$ E. Then, a pair (Θ, F) is called a soft set over U, where Θ is a mapping given by Θ : F $\rightarrow$ P (X).

2.4 Generalized neutrosophic soft sets [7]

Suppose X is a universal set and E is a set of parameters. Let A be a non-empty subset of E and GNS (X) denotes the set of all generalized neutrosophic sets of X. Then, the pair (Θ, A) is termed to be a GNSS over X, where Θ is a mapping given by Θ : A$\rightarrow$ GNS (X).

Example:

Let X be the set of citizens under consideration and E = {very rich, rich, upper-middle-income, middle-income, lower-middle-income, poor, below-poverty-line} be the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence regarding generalized neutrosophic word. Here, to describe GNSS means to indicate very rich citizens, rich citizens, citizens of lower-middle-income, poor citizens, etc. Consider four citizens in the universe X given by X = (x_1, x_2, x_3, x_4) and A = {a_1, a_2, a_3, a_4} be a set of parameters, where a_1, a_2, a_3, a_4 stand for the parameters 'rich', 'middle-income', 'poor', 'below-poverty-line' respectively. Suppose that

Θ (rich) = {< a_1, 0.8, 0.3, 0.2>, < a_2, 0.6, 0.3, 0.3>, < a_3, 0.7, 0.4, 0.2>, < a_4, 0.6, 0.1, 0.2>},
Θ (middle-income) = {< a_1, 0.6, 0.1, 0.1>, < a_2, 0.5, 0.3, 0.4>, < a_3, 0.8, 0.4, 0.3>, < a_4, 0.5, 0.2, 0.2>},
Θ (poor) = {< a_1, 0.8, 0.4, 0.3>, < a_2, 0.6, 0.4, 0.1>, < a_3, 0.7, 0.3, 0.5>, < a_4, 0.7, 0.2, 0.2>},
Θ (below-poverty-line) = {< a_1, 0.8, 0.4, 0.4>, < a_2, 0.6, 0.2, 0.5>, < a_3, 0.5, 0.2, 0.2>, < a_4, 0.7, 0.4, 0.5>}.

Consequently, Θ (rich) represents rich citizens, Θ (middle-income) represents citizens of middle-income, Θ (poor) represents poor citizens and Θ (below-poverty-line) represents citizens of below-poverty-line. Therefore, the tabular representation of GNSS (Θ, A) is given below (see *Table1*).

X	a_1 = rich	a_2 = middle-income	a_3 = poor	a_4 = below-poverty-line
x_1	(0.8, 0.3, 0.2)	(0.6, 0.1, 0.1)	(0.8, 0.4, 0.3)	(0.8, 0.4, 0.4)
x_2	(0.6, 0.3, 0.3)	(0.5, 0.3, 0.4)	(0.6, 0.4, 0.1)	(0.6, 0.2, 0.5)
x_3	(0.7, 0.4, 0.2)	(0.8, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.5, 0.2, 0.2)
x_4	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.2)	(0.7, 0.2, 0.2)	(0.7, 0.4, 0.5)

Table 1. Tabular representation of GNSS (Θ, A)

Definition 2.4.1 [7]

Consider (Θ_1, A) and (Θ_2, B) be two GNSSs over a common universe U. The union (Θ_1, A) and (Θ_2, B) is defined by (Θ_1, A) $\cup$ (Θ_2, B) = (Θ_3, C), where C = A $\cup$ B. The truth-membership, indeterminacy-membership and falsity-membership functions of (Θ_3, C) are presented as follows:

$$\mathrm{T}_{\Theta_3(e)}(m) = \mathrm{T}_{\Theta_1(e)}(m), \text{ if } e \in \Theta_1 - \Theta_2,$$
$$= \mathrm{T}_{\Theta_2(e)}(m), \text{ if } e \in \Theta_2 - \Theta_1,$$
$$= \text{Max}\,(\mathrm{T}_{\Theta_1(e)}(m), \mathrm{T}_{\Theta_2(e)}(m)), \text{ if } e \in \Theta_1 \cap \Theta_2.$$

$$\mathrm{I}_{\Theta_3(e)}(m) = \mathrm{I}_{\Theta_1(e)}(m), \text{ if } e \in \Theta_1 - \Theta_2,$$
$$= \mathrm{I}_{\Theta_2(e)}(m), \text{ if } e \in \Theta_2 - \Theta_1,$$
$$= \text{Min}\,(\mathrm{I}_{\Theta_1(e)}(m), \mathrm{I}_{\Theta_2(e)}(m)), \text{ if } e \in \Theta_1 \cap \Theta_2.$$

$$\mathrm{F}_{\Theta_3(e)}(x) = \mathrm{F}_{\Theta_1(e)}(m), \text{ if } e \in \Theta_1 - \Theta_2,$$
$$= \mathrm{F}_{\Theta_2(e)}(m), \text{ if } e \in \Theta_2 - \Theta_1,$$
$$= \text{Min}\,(\mathrm{F}_{\Theta_1(e)}(m), \mathrm{F}_{\Theta_2(e)}(m)), \text{ if } e \in \Theta_1 \cap \Theta_2.$$

Definition 2.4.2 [7]

Suppose (Θ_1, A) and (Θ_2, B) are two GNSSs over the same universe X The intersection (Θ_1, A) and (Θ_2, B) is defined by (Θ_1, A) $\cap$ (Θ_2, B) = (Θ_4, D), where D = A $\cap$ B ($\neq \varphi$) and the truth-membership, indeterminacy-membership and falsity-membership functions of (Θ_4, D) are defined as follows:

$$\mathrm{T}_{\Theta_4(e)}(x) = \text{Min}\,(\mathrm{T}_{\Theta_1(e)}(m), \mathrm{T}_{\Theta_2(e)}(m)),\ \mathrm{I}_{\Phi_4(e)}(m) = \text{Min}\,(\mathrm{I}_{\Theta_1(e)}(m), \mathrm{I}_{\Theta_2(e)}(m)), \mathrm{F}_{\Phi_4(e)}(m) = \text{Max}\,(\mathrm{F}_{\Theta_1(e)}(m), \mathrm{F}_{\Theta_2(e)}(m)),\ \forall\, e \in \mathrm{D}.$$

Definition 2.4.3 [7]

Let (Θ_1, A) and (Θ_2, B) be two GNSSs over the identical universe U. Then 'AND' operation on (Θ_1, A) and (Θ_2, B) is defined by (Θ_1, A) $\wedge$ (Θ_2, B) = (Θ_5, K), where K = A $\times$ B and the truth-membership, indeterminacy-membership and falsity-membership functions of (Φ_5, A $\times$ B) are defined as follows:

$T_{\Theta_5(\gamma,\delta)}(m)$ = Min ($T_{\Theta_1(\gamma)}(m)$, $T_{\Theta_2(\delta)}(m)$), $I_{\Theta_5(\gamma,\delta)}(m)$ = Min ($I_{\Theta_1(\gamma)}(m)$, $I_{\Theta_2(\delta)}(m)$), $F_{\Theta_5(\gamma,\delta)}(m)$ = Max ($F_{\Theta_1(\gamma)}(m)$, $F_{\Theta_2(\delta)}(m)$), $\forall\ \gamma \in A$, $\forall\ \delta \in B$, $m \in X$.

3 A generalized neutrosophic soft MAGDM based on TOPSIS method

Let C = { C_1, C_2, …, C_n}, (n $\geq$ 2) be a discrete set of alternatives in a MAGDM problem with p DMs. Let q be the total number of parameters involved in the problem, where q_i be number of parameters under the assessment of DM_i (i = 1, 2, …, p) such that q = $\sum_{i=1}^{p} q_i$. The rating of performance value of alternative C_i, (i = 1, 2, …, n) with respect to the choice parameters is provided by the DMs and they can be expressed in terms of GNSs. The procedure for solving neutrosophic soft MAGDM problem based on TOPSIS method is described as follows:

Step 1. Formulation of criterion matrix with SVNSs

Suppose that the rating of alternative C_i (i = 1, 2, …, n) with respect to the choice parameter provided by the s-th (s = 1, 2, …, p) DM is represented by GNSS (Θ_s, H_s), (s = 1, 2, …, p) and they can be presented in matrix form $d^s_{G_{ij}}$ (i = 1, 2, …, n, j = 1, 2, …, q_s; s = 1, 2, …, p). Therefore, criterion matrix for s-th DM can be explicitly formulated as follows:

$$D_G^s = \left\langle d^s_{ij} \right\rangle_{n\times q_s} = \begin{bmatrix} d^s_{11} & d^s_{12} & \dots & d^s_{1q_s} \\ d^s_{21} & d^s_{22} & \dots & d^s_{2q_s} \\ . & . & \dots & . \\ . & . & \dots & . \\ d^s_{n1} & d^s_{n2} & \dots & d^s_{nq_s} \end{bmatrix}$$

Here, d^s_{ij} = ($T^s_{ij}, I^s_{ij}, F^s_{ij}$) where $T^s_{ij}, I^s_{ij}, F^s_{ij} \in [0, 1]$ and $0 \leq T^s_{ij} + I^s_{ij} + F^s_{ij} \leq 3$, i = 1, 2, …, n; j = 1, 2, …, q_s; s =1, 2, …, p.

Step 2. Formulation of combined criterion matrix with GNSs

In the group decision making problem, DMs assessments need to be fused into a group opinion based on the choice parameters of the evaluator. Suppose the evaluator considers *r* number of choice parameters in the decision making situation. Using 'AND' operator of GNSSs proposed by Broumi [7], the resultant GNSSs is placed in the decision matrix D_G as follows:

$$D_G = \langle d_{ij} \rangle_{p\times r} = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1r} \\ d_{21} & d_{22} & \ldots & d_{2r} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ d_{n1} & d_{n2} & \ldots & d_{nr} \end{bmatrix}$$

Here, $d_{ij} = \langle T'_{ij}, I'_{ij}, F'_{ij} \rangle$ where $T'_{ij}, I'_{ij}, F'_{ij} \in [0, 1]$ and $0 \le T'_{ij} + I'_{ij} + F'_{ij} \le 3$, i = 1, 2, ..., n; j = 1, 2, ..., r.

Step 3. Determination of weights of the choice parameters

The evaluator selects the choice parameters in the decision making situation. In general, the weights of the choice parameters are dissimilar and completely unknown to the evaluator. In this paper, we use information entropy method in order to achieve the weights of the choice parameters. The entropy value H_j of the j-th attribute can be defined as follows:

$$H_j = 1 - \frac{1}{r}\sum_{i=1}^{p}(T_{ij}(x_i) + F_{ij}(x_i))\left|I_{ij}(x_i) - I^C_{ij}(x_i)\right|, \ j = 1, 2, \ldots, r \quad (3)$$

Here, $0 \le H_j \le 1$ and the entropy weight [12, 32] of the j-th attribute is obtained from the Eq. as given below.

$$w_j = \frac{1 - H_j}{\sum_{j=1}^{r}(1 - H_j)}, \text{ with } 0 \le w_j \le 1 \text{ and } \sum_{j-1}^{r} w_j = 1. \quad (4)$$

Step 4. Construction of weighted decision matrix

We obtain aggregated weighted decision matrix by multiplying weights (w_j) [26] of the choice parameters and aggregated decision matrix $\langle d_{ij} \rangle_{p\times r}$ as follows:

$$D^w_G = D_G \otimes w = \langle d_{ij} \rangle_{n\times r} \otimes w_j = \langle d^{w_j}_{ij} \rangle_{n\times r} = \begin{bmatrix} d^{w_1}_{11} & d^{w_2}_{12} & \ldots & d^{w_r}_{1r} \\ d^{w_1}_{21} & d^{w_2}_{22} & \ldots & d^{w_r}_{2r} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ d^{w_1}_{n1} & d^{w_2}_{n1} & \ldots & d^{w_r}_{nr} \end{bmatrix}$$

Here, $d_{ij}^{w_j} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ where $T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \in [0, 1]$ and $0 \le T_{ij}^{w_j} + I_{ij}^{w_j} + F_{ij}^{w_j} \le 3$, i = 1, 2, ..., n; j = 1, 2, ..., r.

Step 5. Determination of relative positive ideal solution (RPIS) and relative negative ideal solution (RNIS)

In practical decision making, the attributes are classified into two categories namely benefit type attributes (J_1) and cost type attributes (J_2). Let, R_G^{w+} and R_G^{w-} be the relative positive ideal solution (RPIS) and relative negative ideal solution (RNIS). Then, R_G^{w+} and R_G^{w-} can be defined as follows:

$$R_G^{w+} = (\left\langle T_1^{w_1+}, I_1^{w_1+}, F_1^{w_1+} \right\rangle, \left\langle T_2^{w_2+}, I_2^{w_2+}, F_2^{w_2+} \right\rangle, ..., \left\langle T_r^{w_r+}, I_r^{w_r+}, F_r^{w_r+} \right\rangle)$$

$$R_G^{w-} = (\left\langle T_1^{w_1-}, I_1^{w_1-}, F_1^{w_1-} \right\rangle, \left\langle T_2^{w_2-}, I_2^{w_2-}, F_2^{w_2-} \right\rangle, ..., \left\langle T_r^{w_r-}, I_r^{w_r-}, F_r^{w_r-} \right\rangle)$$

where

$\left\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \right\rangle = <$ [{$\underset{i}{Max}(T_{ij}^{w_j})$ | j∈ J_1}; {$\underset{i}{Min}(T_{ij}^{w_j})$ | j∈J_2}],

[{$\underset{i}{Min}(I_{ij}^{w_j})$ | j∈ J_1}; {$\underset{i}{Max}(I_{ij}^{w_j})$ | j∈ J_2}], [{$\underset{i}{Min}(F_{ij}^{w_j})$ | j∈ J_1};

{$\underset{i}{Max}(F_{ij}^{w_j})$ | j∈ J_2}] >, j = 1, 2, ..., r,

$\left\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \right\rangle = <$ [{$\underset{i}{Min}(T_{ij}^{w_j})$ | j∈ J_1}; {$\underset{i}{Max}(T_{ij}^{w_j})$ | j∈J_2}],

[{$\underset{i}{Max}(I_{ij}^{w_j})$ | j∈ J_1}; {$\underset{i}{Min}(I_{ij}^{w_j})$ | j∈ J_2}], [{$\underset{i}{Max}(F_{ij}^{w_j})$ | j∈ J_1};

{$\underset{i}{Min}(F_{ij}^{w_j})$ | j∈ J_2}] >, j = 1, 2, ..., r.

Step 6. Calculation of distance measure of each alternative from RPIS and RNIS

The normalized Euclidean distance of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the RPIS $\left\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \right\rangle$ for i = 1, 2, ..., n; j = 1, 2, ..., r can be defined as follows:

$$D_{Euc}^{i+}(d_{ij}^{w_j}, d_j^{w_j+})$$

$$\sqrt{\frac{1}{3r}\sum_{j=1}^{r}\left\{(T_{ij}^{w_j}(x_j) - T_j^{w_j+}(x_j))^2 + (I_{ij}^{w_j}(x_j) - I_j^{w_j+}(x_j))^2 + (F_{ij}^{w_j}(x_j) - F_j^{w_j+}(x_j))^2\right\}} \quad (5)$$

Similarly, normalized Euclidean distance of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the RNIS $\left\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \right\rangle$ for i = 1, 2, ..., n; j = 1, 2, ..., r can be written as follows:

$$D_{Euc}^{i-}(d_{ij}^{w_j}, d_j^{w_j-})$$

$$\sqrt{\frac{1}{3r}\sum_{j=1}^{r}\left\{(T_{ij}^{w_j}(x_j)-T_j^{w_j-}(x_j))^2+(I_{ij}^{w_j}(x_j)-I_j^{w_j-}(x_j))^2+(F_{ij}^{w_j}(x_j)-F_j^{w_j-}(x_j))^2\right\}} \quad (6)$$

Step 7. Computation of the relative closeness co-efficient to the neutrosophic ideal solution

The relative closeness co-efficient of each alternative C_i, (i = 1, 2, ..., n) with respect to the RPIS is defined as follows:

$$\rho_i^* = \frac{D_{Euc}^{i-}(d_{ij}^{w_j}, d_j^{w_j-})}{D_{Euc}^{i+}(d_{ij}^{w_j}, d_j^{w_j+}) + D_{Euc}^{i-}(d_{ij}^{w_j}, d_j^{w_j-})} \quad (7)$$

where, $0 \le \rho_i^* \le 1$.

Step 8. Rank the alternatives

We rank the alternatives according to the values of ρ_i^*, i = 1, 2, ..., n and bigger value of ρ_i^*, i = 1, 2, ..., p reflects the better alternative.

4 Proposed TOPSIS algorithm for MAGDM problems

In sum, TOPSIS algorithm for generalized neutrosophic soft MAGDM problems is designed using the following steps:

Step 1. Formulate the criterion matrix D_G^s of the s-th decision maker, s = 1, 2, …, p.

Step 2. Establish the aggregated decision matrix D_G using AND operator GNSSs the based on the choice parameters of the evaluator.

Step 3. Determine the weight (w_j) of the choice parameters using Eq. (4).

Step 4. Construct the weighted aggregated decision matrix $D_G^w = \left\langle d_{ij}^{w_j} \right\rangle_{n\times r}$.

Step 5. Identify the relative positive ideal solution (R_G^{w+}) and relative negative ideal solution (R_G^{w-}).

Step 6. Compute the normalized Euclidean distance of each alternative from relative positive ideal solution (R_G^{w+}) and relative negative ideal solution (R_G^{w-}) by Eqs. (5) and (6) respectively.

Step 7. Calculate the relative closeness co-efficient ρ_i^* using Eq. (7) of each alternative C_i.

Step 8. Rank the preference order of alternatives according to the order of their relative closeness.

5 A numerical example

Let $F = \{f_1, f_2, f_3, f_4\}$ be the set of flats characterized by different locations, prices and constructions and E = {very good, good, average good, below average, bad, very costly, costly, moderate, cheap, new-construction, not so new-constructions, old-constructions, very old-constructions} be the set of parameters. Assume that E_1 = {very good, good}, E_2 = {very costly, costly, moderate}, E_3 = {new-construction, not so new-construction} are three subsets of E. Let the GNSSs (Θ_1, E_1), (Θ_2, E_2), (Θ_3, E_3) stand for the flats 'having diverse locations', 'having diverse prices', 'having diverse constructions' respectively and they are computed by the three DMs namely DM_1, DM_2 and DM_3 respectively. The criterion decision matrices for DM_1, DM_2 and DM_3 are presented (see *Table 2, Table 3, Table 4*) respectively as follows:

U	α_1 = very good	α_2 = good
f_1	(0.9, 0.3, 0.5)	(0.5, 0.3, 0.4)
f_2	(0.6, 0.4, 0.3)	(0.5, 0.2, 0.4)
f_3	(0.8, 0.2, 0.3)	(0.7, 0.5, 0.4)
f_4	(0.7, 0.2, 0.1)	(0.7, 0.5, 0.4)

Table 2: Tabular form of GNSS (Θ_1, E_1)

U	β_1 = very costly	β_2 = costly	β_3 = moderate
f_1	(0.9, 0.3, 0.1)	(0.7, 0.3, 0.4)	(0.6, 0.2, 0.4)
f_2	(0.8, 0.3, 0.2)	(0.6, 0.5, 0.4)	(0.5, 0.4, 0.3)
f_3	(0.8, 0.5, 0.4)	(0.7, 0.2, 0.3)	(0.8, 0.3, 0.2)
f_4	(0.7, 0.2, 0.4)	(0.8, 0.4, 0.5)	(0.6, 0.5, 0.3)

Table 3: Tabular form of GNSS (Θ_2, E_2)

U	λ_1 = new-construction	λ_2 = not so new-construction
f_1	(0.8, 0.4, 0.2)	(0.7, 0.4, 0.3)
f_2	(0.9, 0.1, 0.1)	(0.6, 0.3, 0.1)
f_3	(0.5, 0.4, 0.4)	(0.8, 0.3, 0.4)
f_4	(0.4, 0.3, 0.4)	(0.6, 0.3, 0.4)

Table 4: Tabular form of GNSS (Θ_3, E_3)

The proposed TOPSIS method for solving generalized soft MAGDM problem is presented in the following steps.

Step 1: If the evaluator wishes to perform the operation '(Θ_1, E1) AND (Θ_2, E2)' then we will get 2×3 parameters of the form μ_{ij}, where $\mu_{ij} = \alpha_i \wedge \beta_j$, for i = 1, 2; j = 1, 2, 3. Let S = {$\mu_{12}, \mu_{13}, \mu_{21}, \mu_{22}, \mu_{23}$} be the set of choice parameters of the evaluator, where μ_{12} = (very good, costly), μ_{13} = (very good, moderate), μ_{21} = (good, very costly), etc. (see *Table 5*).

U	μ_{12}	μ_{13}	μ_{21}	μ_{22}	μ_{23}
f_1	(0.7, 0.3, 0.5)	(0.6, 0.2, 0.5)	(0.5, 0.3, 0.4)	(0.5, 0.3, 0.4)	(0.5, 0.2, 0.4)
f_2	(0.6, 0.4, 0.4)	(0.5, 0.4, 0.3)	(0.5, 0.2, 0.4)	(0.5, 0.2, 0.4)	(0.5, 0.2, 0.4)
f_3	(0.7, 0.2, 0.3)	(0.8, 0.2, 0.3)	(0.7, 0.5, 0.4)	(0.7, 0.2, 0.4)	(0.7, 0.3, 0.4)
f_4	(0.7, 0.2, 0.5)	(0.6, 0.2, 0.3)	(0.6, 0.2, 0.4)	(0.6, 0.3, 0.5)	(0.6, 0.3, 0.4)

Table 5: Tabular form of '(Θ_1, E1) AND (Θ_2, E2)'

Now the evaluator desires to compute (Θ_5, T) from (Θ_4, S) AND (Θ_3, E3) for the specified parameters T = {$\mu_{13} \wedge \lambda_1, \mu_{22} \wedge \lambda_1, \mu_{12} \wedge \lambda_2, \mu_{21} \wedge \lambda_2$}, where $\mu_{13} \wedge \lambda_1$ denotes (very good, moderate, new-construction), $\mu_{12} \wedge \lambda_2$ represents (very good, costly, not so new construction), etc, (see *Table 6*).

U	$\mu_{13} \wedge \lambda_1$	$\mu_{22} \wedge \lambda_1$	$\mu_{12} \wedge \lambda_2$	$\mu_{21} \wedge \lambda_2$
f_1	(0.6, 0.2, 0.5)	(0.5, 0.3, 0.4)	(0.7, 0.3, 0.5)	(0.5, 0.3, 0.4)
f_2	(0.5, 0.1, 0.3)	(0.5, 0.1, 0.4)	(0.6, 0.3, 0.4)	(0.5, 0.2, 0.4)
f_3	(0.5, 0.2, 0.4)	(0.5, 0.2, 0.4)	(0.7, 0.2, 0.4)	(0.7, 0.3, 0.4)
f_4	(0.4, 0.2, 0.4)	(0.4, 0.3, 0.5)	(0.6, 0.2, 0.5)	(0.6, 0.2, 0.4)

Table 6: Tabular form of '(Θ_4, S) AND (Θ_3, E3)'

Step 2. Computation of the weights of the parameters

Entropy value H_j (j = 1, 2, 3, 4) of the j-th choice parameter can be determined from Eq. (3) as follows:
H_1 = 0.42, H_2 = 0.505, H_3 = 0.45, H_4 = 0.515.
Then, normalized entropy weights are obtained as follows:
w_1 = 0.2712, w_2 = 0.2318, w_3 = 0.2564, w_4 = 0.2406, where $\sum_{j-1}^{4} w_j = 1$.

Step 3. Formulation of weighted decision matrix of the choice parameters

The tabular form of the weighted decision matrix is presented in the *Table 7*).

U	$w_1 \otimes (\mu_{13} \wedge \lambda_1)$	$w_2 \otimes (\mu_{22} \wedge \lambda_1)$	$w_3 \otimes (\mu_{12} \wedge \lambda_2)$	$w_4 \otimes (\mu_{21} \wedge \lambda_2)$
f_1	(0.22, 0.6463, 0.8286)	(0.1484, 0.7565, 0.8086)	(0.2656, 0.7344, 0.8372)	(0.1536, 0.7485, 0.8022)
f_2	(0.1714, 0.5356, 0.7214)	(0.1484, 0.5864, 0.8086)	(0.2094, 0.7344, 0.7906)	(0.1536, 0.6789, 0.8022)
f_3	(0.1714, 0.6463, 0.78)	(0.1484, 0.6886, 0.8086)	(0.2656, 0.6619, 0.7906)	(0.2515, 0.7485, 0.8022)
f_4	(0.1294, 0.6463, 0.78)	(0.1167, 0.7565, 0.8516)	(0.2094, 0.6619, 0.8372)	(0.1978, 0.6789, 0.8022)

Table 7: Tabular form of weighted decision matrix

Step 4. Determination of RPIS and RNIS

The RPIS (R_G^+) and RNIS (R_G^-) can be obtained from the weighted decision matrix as follows:

R_G^+ = < (0.22, 0.5356, 0.7214); (0.1484, 0.5864, 0.8086); (0.2656, 0.6619, 0.7906); (0.2515, 0.6789, 0.8022) >

R_G^- = < (0.1294, 0.6453, 0.8286); (0.1167, 0.7565, 0.8516); (0.2094, 0.7344, 0.8372); (0.1536, 0.7485, 0.8022) >

Step 5. Determine the distance measure of each alternative from the RPIS and RNIS

Using Eq. (5), the distance measures of each alternative from the RPIS are obtained as follows:

$D_{Euc}^{1+} = 0.0788$, $D_{Euc}^{2+} = 0.0412$, $D_{Euc}^{3+} = 0.0527$, $D_{Euc}^{4+} = 0.0730$.

Similarly, the distance measures of each alternative from the RNIS are obtained using Eq. (6) as follows:

$D_{Euc}^{1-} = 0.0344$, $D_{Euc}^{2-} = 0.0731$, $D_{Euc}^{3-} = 0.0514$, $D_{Euc}^{4-} = 0.0346$.

Step 6. Calculate the relative closeness coefficient

We now compute the relative closeness co-efficient ρ_i^*, i = 1, 2, 3, 4 using Eq. (7) as follows:

$\rho_1^* = 0.3039$, $\rho_2^* = 0.6395$, $\rho_3^* = 0.4938$, $\rho_4^* = 0.3216$.

Step 7. Rank the alternatives

The ranking order of alternatives based on the relative closeness coefficient is presented as follows:

$$C_2 \succ C_3 \succ C_4 \succ C_1.$$

Therefore, C_2 is the best alternative.

6 Conclusion

In this paper, we have proposed a TOPSIS method for solving MAGDM problem with generalized neutrosophic soft information. In the decision making context, the rating of performance values of the alternatives with respect to the parameters are presented in terms of GNSSs. We employ AND operator of GNSSs to combine opinions of the DMs based on the choice parameters of the evaluator. We construct weighted decision matrix after obtaining the weights of the choice parameters by using information entropy method. Then, we define RPIS and RNIS from the weighted decision matrix and Euclidean distance measure is used to compute distances of each alternative from RPISs as well as RNISs. Finally, relative closeness co-efficient of each alternative is calculated in order to select the best alternative. The authors expect that the proposed concept can be useful in dealing with diverse MAGDM problems such as personnel and project selections, manufacturing systems, marketing research problems and various other management decision problems.

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