

Interval Valued Neutrosophic Graphs

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Abstract

The notion of interval valued neutrosophic sets is a generalization of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionistic fuzzy sets and single valued neutrosophic sets. We apply for the first time to graph theory the concept of interval valued neutrosophic sets, an instance of neutrosophic sets. We introduce certain types of interval valued neutrosophic graphs (IVNG) and investigate some of their properties with proofs and examples.

Keyword

Interval valued neutrosophic set, Interval valued neutrosophic graph, Strong interval valued neutrosophic graph, Constant interval valued neutrosophic graph, Complete interval valued neutrosophic graph, Degree of interval valued neutrosophic graph.

1 Introduction

Neutrosophic sets (NSs) proposed by Smarandache [13, 14] are powerful mathematical tools for dealing with incomplete, indeterminate and inconsistent information in real world. They are a generalization of fuzzy sets [31], intuitionistic fuzzy sets [28, 30], interval valued fuzzy set [23] and interval-valued intuitionistic fuzzy sets theories [29].

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $] -0, 1+[$. In order to conveniently practice NS in real life applications, Smarandache [53] and Wang et al. [17] introduced the concept of single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets.

The same authors [16, 18] introduced as well the concept of interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which three membership functions are independent, and their values included into the unit interval $[0, 1]$.

More on single valued neutrosophic sets, interval valued neutrosophic sets and their applications may be found in [3, 4, 5, 6, 19, 20, 21, 22, 24, 25, 26, 27, 39, 41, 42, 43, 44, 45, 49].

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving a combinatorial problem in different areas, such as geometry, algebra, number theory, topology, optimization or computer science. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph.

The extension of fuzzy graph [7, 9, 38] theory have been developed by several researchers, including intuitionistic fuzzy graphs [8, 32, 40], considering the vertex sets and edge sets as intuitionistic fuzzy sets. In interval valued fuzzy graphs [33, 34], the vertex sets and edge sets are considered as interval valued fuzzy sets. In interval valued intuitionistic fuzzy graphs [2, 48], the vertex sets and edge sets are regarded as interval valued intuitionistic fuzzy sets. In bipolar fuzzy graphs [35, 36], the vertex sets and edge sets are considered as bipolar fuzzy sets. In m-polar fuzzy graphs [37], the vertex sets and edge sets are regarded as m-polar fuzzy sets.

But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions fail. In order to overcome the failure, Smarandache [10, 11, 12, 51] defined four main categories of neutrosophic graphs: I-edge neutrosophic graph, I-vertex neutrosophic graph [1, 15, 50, 52], (t, i, f) -edge neutrosophic graph and (t, i, f) -vertex neutrosophic graph. Later on, Broumi et al. [47] introduced another neutrosophic graph model. This model allows the attachment of truth-membership (t), indeterminacy –membership (i) and falsity-membership (f) degrees both to vertices and edges. A neutrosophic graph model that generalizes the fuzzy graph and intuitionistic fuzzy graph is called single valued neutrosophic graph (SVNG). Broumi [46] introduced as well the neighborhood degree of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph, as generalizations of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph.

In this paper, we focus on the study of interval valued neutrosophic graphs.

2 Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, fuzzy graph, intuitionistic fuzzy graph, single valued neutrosophic graphs, relevant to the present work. See especially [2, 7, 8, 13, 18, 47] for further details and background.

Definition 2.1 [13]

Let X be a space of points (objects) with generic elements in X denoted by x ; then the neutrosophic set A (NS A) is an object having the form $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$, where the functions $T, I, F: X \rightarrow]-0, 1+[$ define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition:

$$-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]-0, 1+[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [17]

Let X be a space of points (objects) with generic elements in X denoted by x . A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$$

Definition 2.3 [7]

A fuzzy graph is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ . i.e $\sigma : V \rightarrow [0, 1]$ and $\mu: V \times V \rightarrow [0, 1]$, such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E .

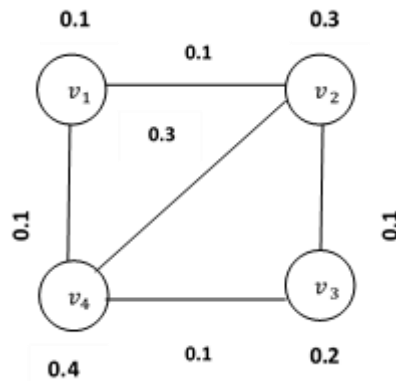


Figure 1: Fuzzy Graph

Definition 2.4 [7]

The fuzzy subgraph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma, \mu)$, if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u, v) \leq \mu(u, v)$ for all $u, v \in V$.

Definition 2.5 [8]

An Intuitionistic fuzzy graph is of the form $G = (V, E)$, where

- i. $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$, for every $v_i \in V$, ($i = 1, 2, \dots, n$);
- ii. $E \subseteq V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $\gamma_2: V \times V \rightarrow [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, ($i, j = 1, 2, \dots, n$)

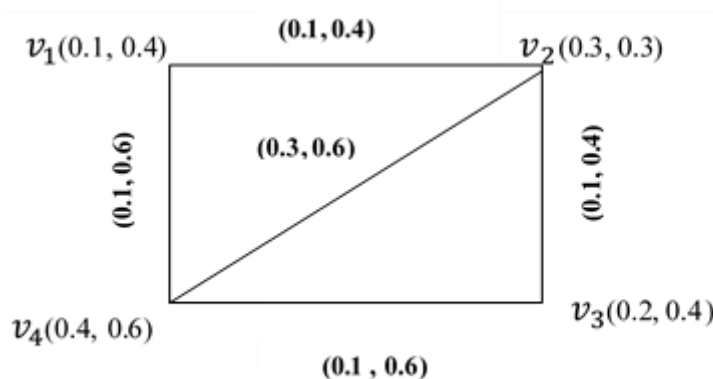


Figure 2: Intuitionistic Fuzzy Graph

Definition 2.6 [2]

An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (A, B)$, where

1) The functions $M_A : V \rightarrow D [0, 1]$ and $N_A : V \rightarrow D [0, 1]$ denote the degree of membership and non-membership of the element $x \in V$, respectively, such that $0 \leq M_A(x) + N_A(x) \leq 1$ for all $x \in V$.

2) The functions $M_B : E \subseteq V \times V \rightarrow D [0, 1]$ and $N_B : E \subseteq V \times V \rightarrow D [0, 1]$ are defined by:

$$M_{BL}(x, y) \leq \min (M_{AL}(x), M_{AL}(y)),$$

$$N_{BL}(x, y) \geq \max (N_{AL}(x), N_{AL}(y)),$$

$$M_{BU}(x, y) \leq \min (M_{AU}(x), M_{AU}(y)),$$

$$N_{BU}(x, y) \geq \max (N_{AU}(x), N_{AU}(y)),$$

such that

$$0 \leq M_{BU}(x, y) + N_{BU}(x, y) \leq 1, \text{ for all } (x, y) \in E.$$

Definition 2.7 [47]

Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X . If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X , then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$, if

$$T_B(x, y) \leq \min(T_A(x), T_A(y)),$$

$$I_B(x, y) \geq \max(I_A(x), I_A(y)),$$

$$F_B(x, y) \geq \max(F_A(x), F_A(y)),$$

for all $x, y \in X$.

A single valued neutrosophic relation A on X is called symmetric if

$$T_A(x, y) = T_A(y, x), I_A(x, y) = I_A(y, x), F_A(x, y) = F_A(y, x)$$

$$T_B(x, y) = T_B(y, x), I_B(x, y) = I_B(y, x)$$

$$F_B(x, y) = F_B(y, x),$$

for all $x, y \in X$.

Definition 2.8 [47]

A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$, where

1) The functions $T_A : V \rightarrow [0, 1]$, $I_A : V \rightarrow [0, 1]$ and $F_A : V \rightarrow [0, 1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3,$$

for all $v_i \in V$ ($i=1, 2, \dots, n$).

2) The functions $T_B: E \subseteq V \times V \rightarrow [0, 1]$, $I_B: E \subseteq V \times V \rightarrow [0, 1]$ and $F_B: E \subseteq V \times V \rightarrow [0, 1]$ are defined by

$$T_B(\{v_i, v_j\}) \leq \min [T_A(v_i), T_A(v_j)],$$

$$I_B(\{v_i, v_j\}) \geq \max [I_A(v_i), I_A(v_j)],$$

$$F_B(\{v_i, v_j\}) \geq \max [F_A(v_i), F_A(v_j)],$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3,$$

for all $\{v_i, v_j\} \in E$ ($i, j = 1, 2, \dots, n$).

We call A the single valued neutrosophic vertex set of V , and B the single valued neutrosophic edge set of E , respectively. Note that B is a symmetric single valued neutrosophic relation on A . We use the notation (v_i, v_j) for an element of E . Thus, $G = (A, B)$ is a single valued neutrosophic graph of $G^* = (V, E)$ if

$$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)],$$

$$I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)],$$

$$F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)],$$

for all $(v_i, v_j) \in E$.

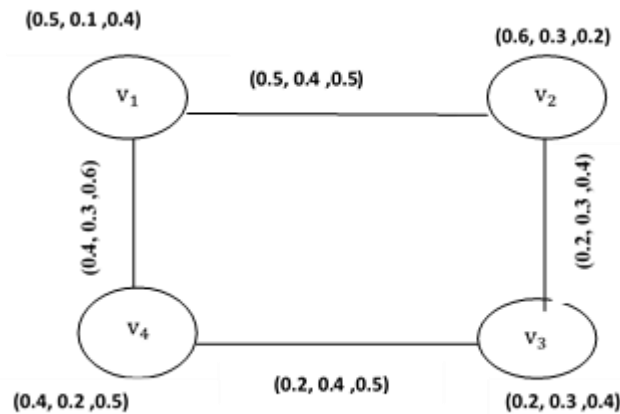


Figure 3: Single valued neutrosophic graph

Definition 2.9 [47]

A partial SVN-subgraph of SVN-graph $G = (A, B)$ is a SVN-graph $H = (V', E')$ such that

- (i) $V' \subseteq V$, where $T'_A(v_i) \leq T_A(v_i)$, $I'_A(v_i) \geq I_A(v_i)$, $F'_A(v_i) \geq F_A(v_i)$, for all $v_i \in V$.
- (ii) $E' \subseteq E$, where $T'_B(v_i, v_j) \leq T_B(v_i, v_j)$, $I'_{Bij} \geq I_B(v_i, v_j)$, $F'_B(v_i, v_j) \geq F_B(v_i, v_j)$, for all $(v_i, v_j) \in E$.

Definition 2.10 [47]

A SVN-subgraph of SVN-graph $G = (V, E)$ is a SVN-graph $H = (V', E')$ such that

- (i) $V' = V$, where $T'_A(v_i) = T_A(v_i)$, $I'_A(v_i) = I_A(v_i)$, $F'_A(v_i) = F_A(v_i)$ for all v_i in the vertex set of V' .
- (ii) $E' = E$, where $T'_B(v_i, v_j) = T_B(v_i, v_j)$, $I'_B(v_i, v_j) = I_B(v_i, v_j)$, $F'_B(v_i, v_j) = F_B(v_i, v_j)$ for every $(v_i, v_j) \in E$ in the edge set of E' .

Definition 2.11 [47]

Let $G = (A, B)$ be a single valued neutrosophic graph. Then the degree of any vertex v is the sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex v denoted by $d(v) = (d_T(v), d_I(v), d_F(v))$, where

$$d_T(v) = \sum_{u \neq v} T_B(u, v) \text{ denotes degree of truth-membership vertex,}$$

$$d_I(v) = \sum_{u \neq v} I_B(u, v) \text{ denotes degree of indeterminacy-membership vertex,}$$

$$d_F(v) = \sum_{u \neq v} F_B(u, v) \text{ denotes degree of falsity-membership vertex.}$$

Definition 2.12 [47]

A single valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ is called strong single valued neutrosophic graph, if

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)],$$

$$I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)],$$

$$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)],$$

for all $(v_i, v_j) \in E$.

Definition 2.13 [47]

A single valued neutrosophic graph $G = (A, B)$ is called complete if

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)],$$

$$I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)],$$

$$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)],$$

for all $v_i, v_j \in V$.

Definition 2.14 [47]

The complement of a single valued neutrosophic graph $G(A, B)$ on G^* is a single valued neutrosophic graph \bar{G} on G^* , where:

1. $\bar{A} = A$
2. $\bar{T}_A(v_i) = T_A(v_i)$, $\bar{I}_A(v_i) = I_A(v_i)$, $\bar{F}_A(v_i) = F_A(v_i)$, for all $v_j \in V$.
3. $\bar{T}_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)] - T_B(v_i, v_j)$
 $\bar{I}_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)] - I_B(v_i, v_j)$, and
 $\bar{F}_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)] - F_B(v_i, v_j)$,

for all $(v_i, v_j) \in E$.

Definition 2.15 [18]

Let X be a space of points (objects) with generic elements in X denoted by x . An interval valued neutrosophic set (for short IVNS A) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X , we have that $T_A(x) = [T_{AL}(x), T_{AU}(x)]$, $I_A(x) = [I_{AL}(x), I_{AU}(x)]$, $F_A(x) = [F_{AL}(x), F_{AU}(x)] \subseteq [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.16 [18]

An IVNS A is contained in the IVNS B , $A \subseteq B$, if and only if $T_{AL}(x) \leq T_{BL}(x)$, $T_{AU}(x) \leq T_{BU}(x)$, $I_{AL}(x) \geq I_{BL}(x)$, $I_{AU}(x) \geq I_{BU}(x)$, $F_{AL}(x) \geq F_{BL}(x)$ and $F_{AU}(x) \geq F_{BU}(x)$ for any x in X .

Definition 2.17 [18]

The union of two interval valued neutrosophic sets A and B is an interval neutrosophic set C , written as $C = A \cup B$, whose truth-membership, indeterminacy-membership, and false membership are related to A and B by

$$\begin{aligned} T_{CL}(x) &= \max (T_{AL}(x), T_{BL}(x)) \\ T_{CU}(x) &= \max (T_{AU}(x), T_{BU}(x)) \\ I_{CL}(x) &= \min (I_{AL}(x), I_{BL}(x)) \\ I_{CU}(x) &= \min (I_{AU}(x), I_{BU}(x)) \\ F_{CL}(x) &= \min (F_{AL}(x), F_{BL}(x)) \\ F_{CU}(x) &= \min (F_{AU}(x), F_{BU}(x)) \end{aligned}$$

for all x in X .

Definition 2.18 [18]

Let X and Y be two non-empty crisp sets. An interval valued neutrosophic relation $R(X, Y)$ is a subset of product space $X \times Y$, and is characterized by the truth membership function $T_R(x, y)$, the indeterminacy membership function $I_R(x, y)$, and the falsity membership function $F_R(x, y)$, where $x \in X$ and $y \in Y$ and $T_R(x, y), I_R(x, y), F_R(x, y) \subseteq [0, 1]$.

3 Interval Valued Neutrosophic Graphs

Throughout this paper, we denote $G^* = (V, E)$ a crisp graph, and $G = (A, B)$ an interval valued neutrosophic graph.

Definition 3.1

By an interval-valued neutrosophic graph of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A = \langle [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}] \rangle$ is an interval-valued neutrosophic set on V ; and $B = \langle [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}] \rangle$ is an interval-valued neutrosophic relation on E satisfying the following condition:

- 1) $V = \{v_1, v_2, \dots, v_n\}$, such that $T_{AL}: V \rightarrow [0, 1]$, $T_{AU}: V \rightarrow [0, 1]$, $I_{AL}: V \rightarrow [0, 1]$, $I_{AU}: V \rightarrow [0, 1]$ and $F_{AL}: V \rightarrow [0, 1]$, $F_{AU}: V \rightarrow [0, 1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3,$$

for all $v_i \in V$ ($i=1, 2, \dots, n$)

- 2) The functions $T_{BL}: V \times V \rightarrow [0, 1]$, $T_{BU}: V \times V \rightarrow [0, 1]$, $I_{BL}: V \times V \rightarrow [0, 1]$, $I_{BU}: V \times V \rightarrow [0, 1]$ and $F_{BL}: V \times V \rightarrow [0, 1]$, $F_{BU}: V \times V \rightarrow [0, 1]$ are such that

$$T_{BL}(\{v_i, v_j\}) \leq \min [T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(\{v_i, v_j\}) \leq \min [T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(\{v_i, v_j\}) \geq \max [I_{BL}(v_i), I_{BL}(v_j)],$$

$$I_{BU}(\{v_i, v_j\}) \geq \max [I_{BU}(v_i), I_{BU}(v_j)],$$

$$F_{BL}(\{v_i, v_j\}) \geq \max [F_{BL}(v_i), F_{BL}(v_j)],$$

$$F_{BU}(\{v_i, v_j\}) \geq \max [F_{BU}(v_i), F_{BU}(v_j)],$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$$

for all $\{v_i, v_j\} \in E$ ($i, j = 1, 2, \dots, n$).

We call A the interval valued neutrosophic vertex set of V, and B the interval valued neutrosophic edge set of E, respectively. Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, $G = (A, B)$ is an interval valued neutrosophic graph of $G^* = (V, E)$ if

$$\begin{aligned}
 T_{BL}(v_i, v_j) &\leq \min[T_{AL}(v_i), T_{AL}(v_j)], \\
 T_{BU}(v_i, v_j) &\leq \min[T_{AU}(v_i), T_{AU}(v_j)], \\
 I_{BL}(v_i, v_j) &\geq \max[I_{BL}(v_i), I_{BL}(v_j)], \\
 I_{BU}(v_i, v_j) &\geq \max[I_{BU}(v_i), I_{BU}(v_j)], \\
 F_{BL}(v_i, v_j) &\geq \max[F_{BL}(v_i), F_{BL}(v_j)], \\
 F_{BU}(v_i, v_j) &\geq \max[F_{BU}(v_i), F_{BU}(v_j)] \text{ — for all } (v_i, v_j) \in E.
 \end{aligned}$$

Example 3.2

Consider a graph G^* , such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be a interval valued neutrosophic subset of V and B a interval valued neutrosophic subset of E, denoted by

	v_1	v_2	v_3
T_{AL}	0.3	0.2	0.1
T_{AU}	0.5	0.3	0.3
I_{AL}	0.2	0.2	0.2
I_{AU}	0.3	0.3	0.4
F_{AL}	0.3	0.1	0.3
F_{AU}	0.4	0.4	0.5

	v_1v_2	v_2v_3	v_3v_1
T_{BL}	0.1	0.1	0.1
T_{BU}	0.2	0.3	0.2
I_{BL}	0.3	0.4	0.3
I_{BU}	0.4	0.5	0.5
F_{BL}	0.4	0.4	0.4
F_{BU}	0.5	0.5	0.6

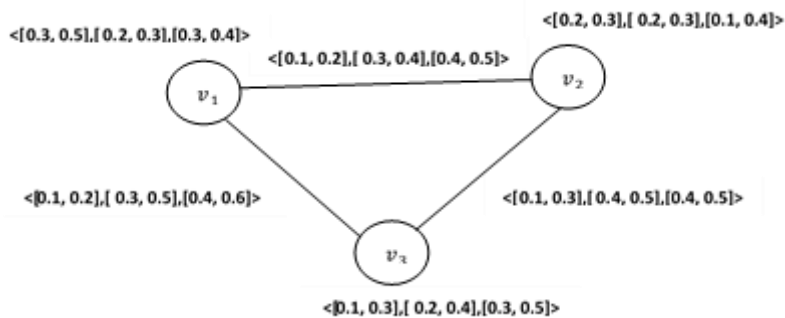


Figure 4: G: Interval valued neutrosophic graph

In *Figure 4*,

(i) $(v_1, <[0.3, 0.5],[0.2, 0.3],[0.3, 0.4]>)$ is an interval valued neutrosophic vertex or IVN-vertex.

(ii) $(v_1v_2, <[0.1, 0.2], [0.3, 0.4], [0.4, 0.5]>)$ is an interval valued neutrosophic edge or IVN-edge.

(iii) $(v_1, <[0.3, 0.5], [0.2, 0.3], [0.3, 0.4]>)$ and $(v_2, <[0.2, 0.3],[0.2, 0.3],[0.1, 0.4]>)$ are interval valued neutrosophic adjacent vertices.

(iv) $(v_1v_2, <[0.1, 0.2], [0.3, 0.4], [0.4, 0.5]>)$ and $(v_1v_3, <[0.1, 0.2],[0.3, 0.5], [0.4, 0.6]>)$ are an interval valued neutrosophic adjacent edge.

Remarks

(i) When $T_{BL}(v_i, v_j) = T_{BU}(v_i, v_j) = I_{BL}(v_i, v_j) = I_{BU}(v_i, v_j) = F_{BL}(v_i, v_j) = F_{BU}(v_i, v_j)$ for some i and j , then there is no edge between v_i and v_j . Otherwise there exists an edge between v_i and v_j .

(ii) If one of the inequalities is not satisfied in (1) and (2), then G is not an IVNG. The interval valued neutrosophic graph G depicted in *Figure 3* is represented by the following adjacency matrix M_G —

$$M_G = \begin{bmatrix} < [0.3, 0.5], [0.2, 0.3], [0.3, 0.4] > & < [0.1, 0.2], [0.3, 0.4], [0.4, 0.5] > & < [0.1, 0.2], [0.3, 0.5], [0.4, 0.6] > \\ < [0.1, 0.2], [0.3, 0.4], [0.4, 0.5] > & < [0.2, 0.3], [0.2, 0.3], [0.1, 0.4] > & < [0.1, 0.3], [0.4, 0.5], [0.4, 0.5] > \\ < [0.1, 0.2], [0.3, 0.5], [0.4, 0.6] > & < [0.1, 0.3], [0.4, 0.5], [0.4, 0.5] > & < [0.1, 0.3], [0.2, 0.4], [0.3, 0.5] > \end{bmatrix}$$

Definition 3.3

A partial IVN-subgraph of IVN-graph $G = (A, B)$ is an IVN-graph $H = (V', E')$ such that —

(i) $V' \subseteq V$, where $T'_{AL}(v_i) \leq T_{AL}(v_i), T'_{AU}(v_i) \leq T_{AU}(v_i), I'_{AL}(v_i) \geq I_{AL}(v_i), I'_{AU}(v_i) \geq I_{AU}(v_i), F'_{AL}(v_i) \geq F_{AL}(v_i), F'_{AU}(v_i) \geq F_{AU}(v_i)$, for all $v_i \in V$.

(ii) $E' \subseteq E$, where $T'_{BL}(v_i, v_j) \leq T_{BL}(v_i, v_j), T'_{BU}(v_i, v_j) \leq T_{BU}(v_i, v_j), I'_{BL}(v_i, v_j) \geq I_{BL}(v_i, v_j), I'_{BU}(v_i, v_j) \geq I_{BU}(v_i, v_j), F'_{BL}(v_i, v_j) \geq F_{BL}(v_i, v_j), F'_{BU}(v_i, v_j) \geq F_{BU}(v_i, v_j)$, for all $(v_i, v_j) \in E$.

Definition 3.4

An IVN-subgraph of IVN-graph $G = (V, E)$ is an IVN-graph $H = (V', E')$ such that

(i) $T'_{AL}(v_i) = T_{AL}(v_i), T'_{AU}(v_i) = T_{AU}(v_i), I'_{AL}(v_i) = I_{AL}(v_i), I'_{AU}(v_i) = I_{AU}(v_i), F'_{AL}(v_i) = F_{AL}(v_i), F'_{AU}(v_i) = F_{AU}(v_i)$, for all v_i in the vertex set of V' .

(ii) $E' = E$, where $T'_{BL}(v_i, v_j) = T_{BL}(v_i, v_j), T'_{BU}(v_i, v_j) = T_{BU}(v_i, v_j)$,

$I'_{BL}(v_i, v_j) = I_{BL}(v_i, v_j)$, $I'_{BU}(v_i, v_j) = I_{BU}(v_i, v_j)$, $F'_{BL}(v_i, v_j) = F_{BL}(v_i, v_j)$, $F'_{BU}(v_i, v_j) = F_{BU}(v_i, v_j)$, for every $(v_i, v_j) \in E$ in the edge set of E' .

Example 3.5

G_1 in Figure 5 is an IVN-graph, H_1 in Figure 6 is a partial IVN-subgraph and H_2 in Figure 7 is a IVN-subgraph of G_1 .

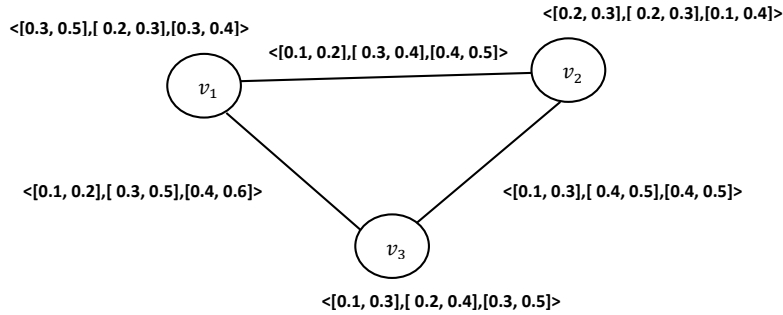


Figure 5: G_1 , an interval valued neutrosophic graph

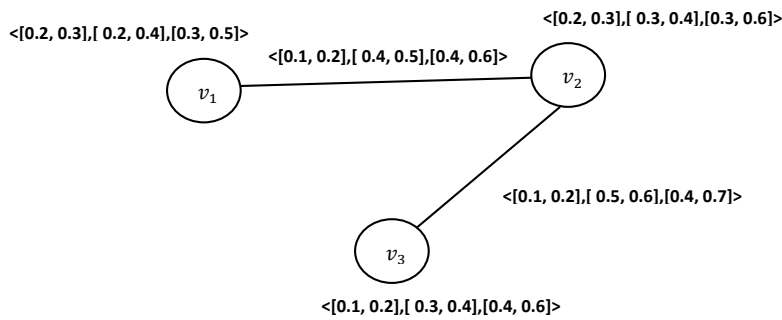


Figure 6: H_1 , a partial IVN-subgraph of G_1

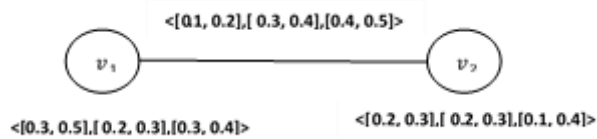


Figure 7: H_2 , an IVN-subgraph of G_1

Definition 3.6

The two vertices are said to be adjacent in an interval valued neutrosophic graph $G = (A, B)$ if —

$$T_{BL}(v_i, v_j) = \min[T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(v_i, v_j) = \min[T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(v_i, v_j) = \max[I_{AL}(v_i), I_{AL}(v_j)]$$

$$I_{BU}(v_i, v_j) = \max[I_{AU}(v_i), I_{AU}(v_j)]$$

$$F_{BL}(v_i, v_j) = \max[F_{AL}(v_i), F_{AL}(v_j)]$$

$$F_{BU}(v_i, v_j) = \max[F_{AU}(v_i), F_{AU}(v_j)]$$

In this case, v_i and v_j are said to be neighbours and (v_i, v_j) is incident at v_i and v_j also.

Definition 3.7

A path P in an interval valued neutrosophic graph $G = (A, B)$ is a sequence of distinct vertices $v_0, v_1, v_2, \dots, v_n$ such that $T_{BL}(v_{i-1}, v_i) > 0, T_{BU}(v_{i-1}, v_i) > 0, I_{BL}(v_{i-1}, v_i) > 0, I_{BU}(v_{i-1}, v_i) > 0$ and $F_{BL}(v_{i-1}, v_i) > 0, F_{BU}(v_{i-1}, v_i) > 0$ for $0 \leq i \leq n-1$. Here $n \geq 1$ is called the length of the path P . A single node or vertex v_i may also be considered as a path. In this case, the path is of the length $([0, 0], [0, 0], [0, 0])$. The consecutive pairs (v_{i-1}, v_i) are called edges of the path. We call P a cycle if $v_0 = v_n$ and $n \geq 3$.

Definition 3.8

An interval valued neutrosophic graph $G = (A, B)$ is said to be connected if every pair of vertices has at least one interval valued neutrosophic path between them, otherwise it is disconnected.

Definition 3.9

A vertex $v_j \in V$ of interval valued neutrosophic graph $G = (A, B)$ is said to be an isolated vertex if there is no effective edge incident at v_j .

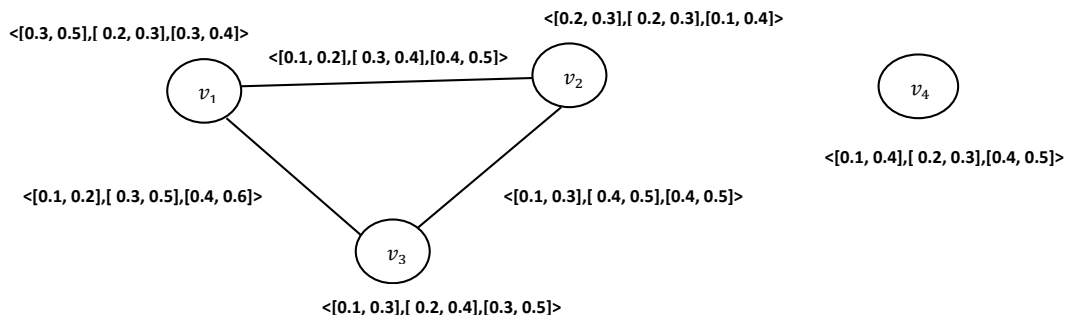


Figure 8. Example of interval valued neutrosophic graph

In Figure 8, the interval valued neutrosophic vertex v_4 is an isolated vertex.

Definition 3.10

A vertex in an interval valued neutrosophic $G = (A, B)$ having exactly one neighbor is called a *pendent vertex*. Otherwise, it is called *non-pendent vertex*. An edge in an interval valued neutrosophic graph incident with a pendent vertex is called a *pendent edge*. Otherwise it is called *non-pendent edge*. A

vertex in an interval valued neutrosophic graph adjacent to the pendent vertex is called a support of the pendent edge.

Definition 3.11

An interval valued neutrosophic graph $G = (A, B)$ that has neither self-loops nor parallel edge is called simple interval valued neutrosophic graph.

Definition 3.12

When a vertex v_i is end vertex of some edges (v_i, v_j) of any IVN-graph $G = (A, B)$. Then v_i and (v_i, v_j) are said to be incident to each other.

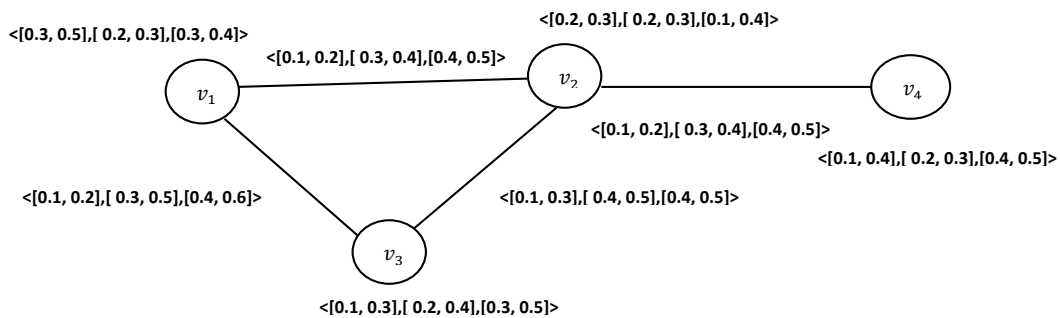


Figure 9. Incident IVN-graph.

In this graph v_2v_1 , v_2v_3 and v_2v_4 are incident on v_2 .

Definition 3.13

Let $G = (A, B)$ be an interval valued neutrosophic graph. Then the degree of any vertex v is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex v denoted by –

$$d(v) = ([d_{TL}(v), d_{TU}(v)], [d_{IL}(v), d_{IU}(v)], [d_{FL}(v), d_{FU}(v)]),$$

where:

$d_{TL}(v) = \sum_{u \neq v} T_{BL}(u, v)$ denotes the degree of lower truth-membership vertex;
 $d_{TU}(v) = \sum_{u \neq v} T_{BU}(u, v)$ denotes the degree of upper truth-membership vertex;

$d_{IL}(v) = \sum_{u \neq v} I_{BL}(u, v)$ denotes the degree of lower indeterminacy-membership vertex;

$d_{IU}(v) = \sum_{u \neq v} I_{BU}(u, v)$ denotes the degree of upper indeterminacy-membership vertex;

$d_{FL}(v) = \sum_{u \neq v} F_{BL}(u, v)$ denotes the degree of lower falsity-membership vertex;

$d_{FU}(v) = \sum_{u \neq v} F_{BU}(u, v)$ denotes the degree of upper falsity-membership vertex.

Example 3.14

Let us consider an interval valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.

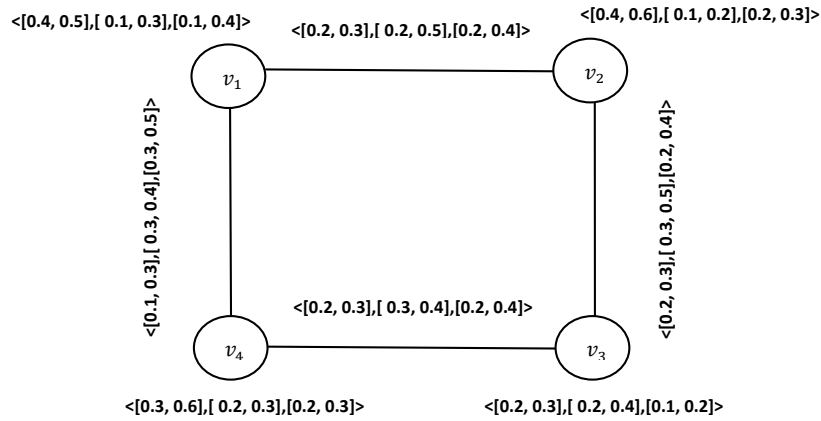


Figure 10: Degree of vertex of interval valued neutrosophic graph

We have the degree of each vertex as follows:

$$d(v_1) = ([0.3, 0.6], [0.5, 0.9], [0.5, 0.9]), d(v_2) = ([0.4, 0.6], [0.5, 1.0], [0.4, 0.8]),$$

$$d(v_3) = ([0.4, 0.6], [0.6, 0.9], [0.4, 0.8]), d(v_4) = ([0.3, 0.6], [0.6, 0.8], [0.5, 0.9]).$$

Definition 3.15

An interval valued neutrosophic graph $G = (A, B)$ is called constant if degree of each vertex is $k = ([k_{1L}, k_{1U}], [k_{2L}, k_{2U}], [k_{3L}, k_{3U}])$. That is $d(v) = ([k_{1L}, k_{1U}], [k_{2L}, k_{2U}], [k_{3L}, k_{3U}])$, for all $v \in V$.

Example 3.16

Consider an interval valued neutrosophic graph G such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.

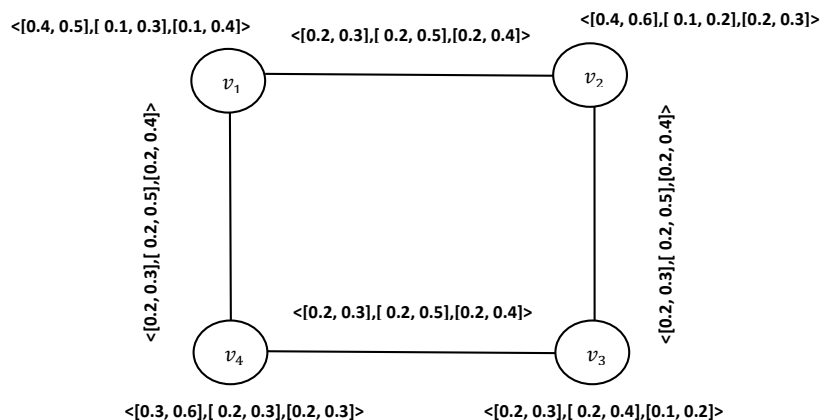


Figure 11. Constant IVN-graph.

Clearly, G is constant IVN-graph since the degree of v_1, v_2, v_3 and v_4 is $([0.4, 0.6], [0.4, 1], [0.4, 0.8])$

Definition 3.17

An interval valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ is called strong interval valued neutrosophic graph if

$$T_{BL}(v_i, v_j) = \min[T_{AL}(v_i), T_{AL}(v_j)], T_{BU}(v_i, v_j) = \min[T_{AU}(v_i), T_{AU}(v_j)]$$

$$I_{BL}(v_i, v_j) = \max[I_{AL}(v_i), I_{AL}(v_j)], I_{BU}(v_i, v_j) = \max[I_{AU}(v_i), I_{AU}(v_j)]$$

$$F_{BL}(v_i, v_j) = \max[F_{AL}(v_i), F_{AL}(v_j)], F_{BU}(v_i, v_j) = \max[F_{AU}(v_i), F_{AU}(v_j)], \quad \text{for all } (v_i, v_j) \in E.$$

Example 3.18

Consider a graph G^* such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be an interval valued neutrosophic subset of V and let B an interval valued neutrosophic subset of E denoted by:

	v_1	v_2	v_3
T_{AL}	0.3	0.2	0.1
T_{AU}	0.5	0.3	0.3
I_{AL}	0.2	0.2	0.2
I_{AU}	0.3	0.3	0.4
F_{AL}	0.3	0.1	0.3
F_{AU}	0.4	0.4	0.5

	v_1v_2	v_2v_3	v_3v_1
T_{BL}	0.2	0.1	0.1
T_{BU}	0.3	0.3	0.3
I_{BL}	0.2	0.2	0.2
I_{BU}	0.3	0.4	0.4
F_{BL}	0.3	0.3	0.3
F_{BU}	0.4	0.4	0.5

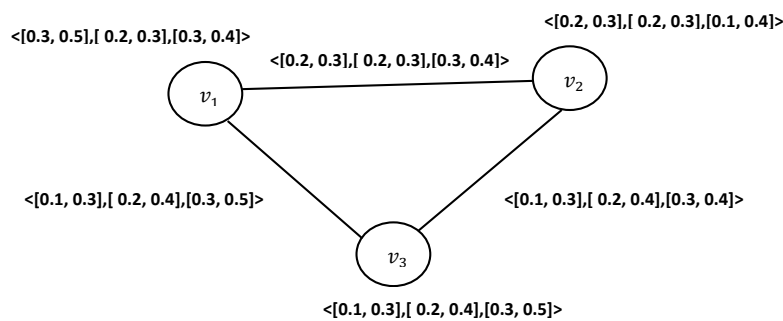


Figure 12. Strong IVN-graph.

By routing computations, it is easy to see that G is a strong interval valued neutrosophic of G^* .

Proposition 3.19

An interval valued neutrosophic graph is the generalization of interval valued fuzzy graph

Proof

Suppose $G = (V, E)$ be an interval valued neutrosophic graph. Then by setting the indeterminacy-membership and falsity-membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to interval valued fuzzy graph.

Proposition 3.20

An interval valued neutrosophic graph is the generalization of interval valued intuitionistic fuzzy graph

Proof

Suppose $G = (V, E)$ is an interval valued neutrosophic graph. Then by setting the indeterminacy-membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to interval valued intuitionistic fuzzy graph.

Proposition 3.21

An interval valued neutrosophic graph is the generalization of intuitionistic fuzzy graph.

Proof

Suppose $G = (V, E)$ is an interval valued neutrosophic graph. Then by setting the indeterminacy-membership, upper truth-membership and upper falsity-membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to intuitionistic fuzzy graph.

Proposition 3.22

An interval valued neutrosophic graph is the generalization of single valued neutrosophic graph.

Proof

Suppose $G = (V, E)$ is an interval valued neutrosophic graph. Then by setting the upper truth-membership equals lower truth-membership, upper

indeterminacy-membership equals lower indeterminacy-membership and upper falsity-membership equals lower falsity-membership values of vertex set and edge set reduces the interval valued neutrosophic graph to single valued neutrosophic graph.

Definition 3.23

The complement of an interval valued neutrosophic graph $G(A, B)$ on G^* is an interval valued neutrosophic graph \bar{G} on G^* where:

1. $\bar{A} = A$
2. $\bar{T}_{AL}(v_i) = T_{AL}(v_i)$, $\bar{T}_{AU}(v_i) = T_{AU}(v_i)$, $\bar{I}_{AL}(v_i) = I_{AL}(v_i)$, $\bar{I}_{AU}(v_i) = I_{AU}(v_i)$, $\bar{F}_{AL}(v_i) = F_{AL}(v_i)$, $\bar{F}_{AU}(v_i) = F_{AU}(v_i)$,

for all $v_j \in V$.

3. $\bar{T}_{BL}(v_i, v_j) = \min [T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j)$,
 $\bar{T}_{BU}(v_i, v_j) = \min [T_{AU}(v_i), T_{AU}(v_j)] - T_{BU}(v_i, v_j)$,
 $\bar{I}_{BL}(v_i, v_j) = \max [I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j)$,
 $\bar{I}_{BU}(v_i, v_j) = \max [I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j)$,

and

$$\bar{F}_{BL}(v_i, v_j) = \max [F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j),$$

$$\bar{F}_{BU}(v_i, v_j) = \max [F_{AU}(v_i), F_{AU}(v_j)] - F_{BU}(v_i, v_j),$$

for all $(v_i, v_j) \in E$

Remark 3.24

If $G = (V, E)$ is an interval valued neutrosophic graph on G^* . Then from above definition, it follow that \bar{G} is given by the interval valued neutrosophic graph $\bar{\bar{G}} = (\bar{\bar{V}}, \bar{\bar{E}})$ on G^* where $\bar{\bar{V}} = V$ and –

$$\bar{\bar{T}}_{BL}(v_i, v_j) = \min [T_{AL}(v_i), T_A(v_j)] - T_{BL}(v_i, v_j),$$

$$\bar{\bar{T}}_{BU}(v_i, v_j) = \min [T_{AU}(v_i), T_A(v_j)] - T_{BU}(v_i, v_j),$$

$$\bar{\bar{I}}_{BL}(v_i, v_j) = \max [I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j),$$

$$\bar{\bar{I}}_{BU}(v_i, v_j) = \max [I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j),$$

and

$$\bar{\bar{F}}_{BL}(v_i, v_j) = \max [F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j), \bar{\bar{F}}_{BU}(v_i, v_j)$$

$$= \max [F_{AU}(v_i), F_{AU}(v_j)] - F_{BU}(v_i, v_j), \text{ For all } (v_i, v_j) \in E.$$

Thus $\overline{\overline{T_{BL}}} = T_{BL}, \overline{\overline{T_{BU}}} = T_{BU}, \overline{\overline{I_{BL}}} = I_{BL}, \overline{\overline{I_{BU}}} = I_{BU}$, and $\overline{\overline{F_{BL}}} = F_{BL}, \overline{\overline{F_{BU}}} = F_{BU}$ on V , where $E = ([T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}])$ is the interval valued neutrosophic relation on V . For any interval valued neutrosophic graph G , $\overline{\overline{G}}$ is strong interval valued neutrosophic graph and $G \subseteq \overline{\overline{G}}$.

Proposition 3.25

$G = \overline{\overline{G}}$ if and only if G is a strong interval valued neutrosophic graph.

Proof

It is obvious.

Definition 3.26

A strong interval valued neutrosophic graph G is called self complementary if $G \cong \overline{\overline{G}}$, where $\overline{\overline{G}}$ is the complement of interval valued neutrosophic graph G .

Example 3.27

Consider a graph $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Consider an interval valued neutrosophic graph G .

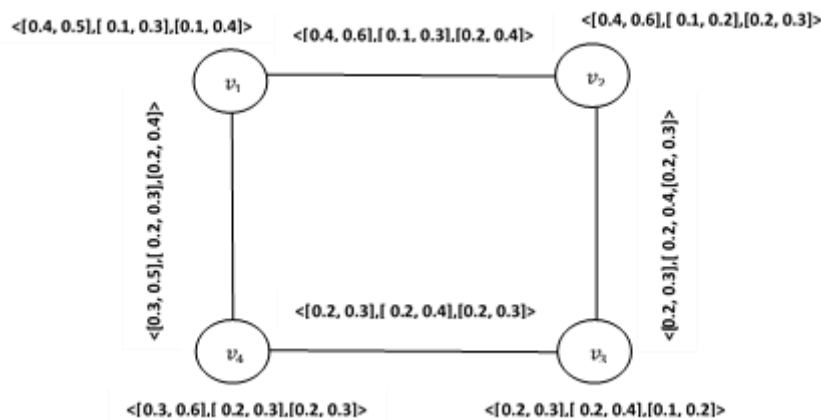


Figure 13. G: Strong IVN- graph

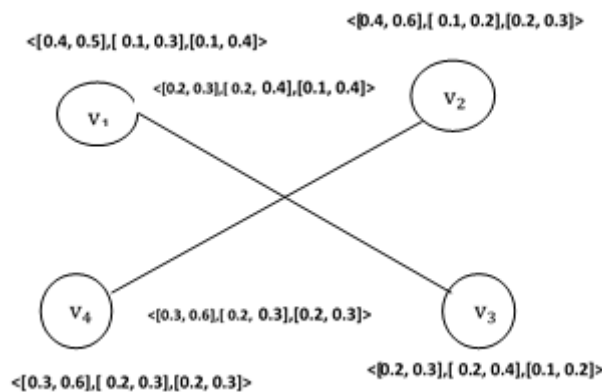


Figure 14. $\overline{\overline{G}}$ Strong IVN- graph

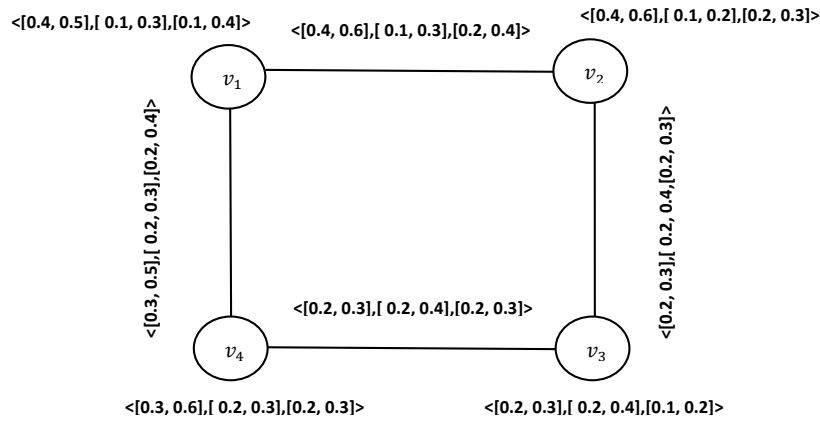


Figure 15. $\bar{\bar{G}}$ Strong IVN- graph

Clearly, $G \cong \bar{\bar{G}}$, hence G is self complementary.

Proposition 3.26

Let $G=(A, B)$ be a *strong* interval valued neutrosophic graph. If –

$$T_{BL}(v_i, v_j) = \min [T_{AL}(v_i), T_{AL}(v_j)]$$

$$T_{BU}(v_i, v_j) = \min [T_{AU}(v_i), T_{AU}(v_j)]$$

$$I_{BL}(v_i, v_j) = \max [I_{AL}(v_i), I_{AL}(v_j)]$$

$$I_{BU}(v_i, v_j) = \max [I_{AU}(v_i), I_{AU}(v_j)]$$

$$F_{BL}(v_i, v_j) = \max [F_{AL}(v_i), F_{AL}(v_j)]$$

$$F_{BU}(v_i, v_j) = \max [F_{AU}(v_i), F_{AU}(v_j)]$$

for all $v_i, v_j \in V$, then G is self complementary.

Proof

Let $G = (A, B)$ be a strong interval valued neutrosophic graph such that –

$$T_{BL}(v_i, v_j) = \min [T_{AL}(v_i), T_{AL}(v_j)];$$

$$T_{BU}(v_i, v_j) = \min [T_{AU}(v_i), T_{AU}(v_j)];$$

$$I_{BL}(v_i, v_j) = \max [I_{AL}(v_i), I_{AL}(v_j)];$$

$$I_{BU}(v_i, v_j) = \max [I_{AU}(v_i), I_{AU}(v_j)];$$

$$F_{BL}(v_i, v_j) = \max [F_{AL}(v_i), F_{AL}(v_j)];$$

$$F_{BU}(v_i, v_j) = \max [F_{AU}(v_i), F_{AU}(v_j)],$$

for all $v_i, v_j \in V$, then $G \approx \bar{\bar{G}}$ under the identity map $I: V \rightarrow V$, hence G is self complementary.

Proposition 3.27

Let G be a self complementary interval valued neutrosophic graph. Then —

$$\begin{aligned}\sum_{v_i \neq v_j} T_{BL}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \min [T_{AL}(v_i), T_{AL}(v_j)] \\ \sum_{v_i \neq v_j} T_{BU}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \min [T_{AU}(v_i), T_{AU}(v_j)] \\ \sum_{v_i \neq v_j} I_{BL}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \max [I_{AL}(v_i), I_{AL}(v_j)] \\ \sum_{v_i \neq v_j} I_{BU}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \max [I_{AU}(v_i), I_{AU}(v_j)] \\ \sum_{v_i \neq v_j} F_{BL}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \max [F_{AL}(v_i), F_{AL}(v_j)] \\ \sum_{v_i \neq v_j} F_{BU}(v_i, v_j) &= \frac{1}{2} \sum_{v_i \neq v_j} \max [F_{AU}(v_i), F_{AU}(v_j)].\end{aligned}$$

Proof

If G be a self complementary interval valued neutrosophic graph. Then there exist an isomorphism $f: V_1 \rightarrow V_1$ satisfying

$$\begin{aligned}\overline{T_{V_1}}(f(v_i)) &= T_{V_1}(f(v_i)) = T_{V_1}(v_i) \\ \overline{I_{V_1}}(f(v_i)) &= I_{V_1}(f(v_i)) = I_{V_1}(v_i) \\ \overline{F_{V_1}}(f(v_i)) &= F_{V_1}(f(v_i)) = F_{V_1}(v_i)\end{aligned}$$

for all $v_i \in V_1$, and —

$$\begin{aligned}\overline{T_{E_1}}(f(v_i), f(v_j)) &= T_{E_1}(f(v_i), f(v_j)) = T_{E_1}(v_i, v_j) \\ \overline{I_{E_1}}(f(v_i), f(v_j)) &= I_{E_1}(f(v_i), f(v_j)) = I_{E_1}(v_i, v_j) \\ \overline{F_{E_1}}(f(v_i), f(v_j)) &= F_{E_1}(f(v_i), f(v_j)) = F_{E_1}(v_i, v_j)\end{aligned}$$

for all $(v_i, v_j) \in E_1$.

We have

$$\begin{aligned}\overline{T_{E_1}}(f(v_i), f(v_j)) &= \min [\overline{T_{V_1}}(f(v_i)), \overline{T_{V_1}}(f(v_j))] - T_{E_1}(f(v_i), f(v_j)) \\ \text{i.e., } T_{E_1}(v_i, v_j) &= \min [T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(f(v_i), f(v_j)) \\ T_{E_1}(v_i, v_j) &= \min [T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(v_i, v_j).\end{aligned}$$

That is —

$$\begin{aligned}\sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \min [T_{V_1}(v_i), T_{V_1}(v_j)] \\ \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \max [I_{V_1}(v_i), I_{V_1}(v_j)] \\ \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \max [F_{V_1}(v_i), F_{V_1}(v_j)]\end{aligned}$$

$$\begin{aligned} 2 \sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \min [T_{V_1}(v_i), T_{V_1}(v_j)] \\ 2 \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \max [I_{V_1}(v_i), I_{V_1}(v_j)] \\ 2 \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) &= \sum_{v_i \neq v_j} \max [F_{V_1}(v_i), F_{V_1}(v_j)]. \end{aligned}$$

From these equations, *Proposition 3.27* holds.

Proposition 3.28

Let G_1 and G_2 be strong interval valued neutrosophic graph, $\overline{G_1} \approx \overline{G_2}$ (isomorphism).

Proof

Assume that G_1 and G_2 are isomorphic, there exists a bijective map $f: V_1 \rightarrow V_2$ satisfying

$$\begin{aligned} T_{V_1}(v_i) &= T_{V_2}(f(v_i)), \\ I_{V_1}(v_i) &= I_{V_2}(f(v_i)), \\ F_{V_1}(v_i) &= F_{V_2}(f(v_i)), \end{aligned}$$

for all $v_i \in V_1$, and

$$\begin{aligned} T_{E_1}(v_i, v_j) &= T_{E_2}(f(v_i), f(v_j)), \\ I_{E_1}(v_i, v_j) &= I_{E_2}(f(v_i), f(v_j)), \\ F_{E_1}(v_i, v_j) &= F_{E_2}(f(v_i), f(v_j)), \end{aligned}$$

for all $(v_i, v_j) \in E_1$.

By *Definition 3.21*, we have

$$\begin{aligned} \overline{T_{E_1}}(v_i, v_j) &= \min [T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(v_i, v_j) \\ &= \min [T_{V_2}(f(v_i)), T_{V_2}(f(v_j))] - T_{E_2}(f(v_i), f(v_j)), \\ &= \overline{T_{E_2}}(f(v_i), f(v_j)), \\ \overline{I_{E_1}}(v_i, v_j) &= \max [I_{V_1}(v_i), I_{V_1}(v_j)] - I_{E_1}(v_i, v_j) \\ &= \max [I_{V_2}(f(v_i)), I_{V_2}(f(v_j))] - I_{E_2}(f(v_i), f(v_j)), \\ &= \overline{I_{E_2}}(f(v_i), f(v_j)), \\ \overline{F_{E_1}}(v_i, v_j) &= \min [F_{V_1}(v_i), F_{V_1}(v_j)] - F_{E_1}(v_i, v_j) \\ &= \min [F_{V_2}(f(v_i)), F_{V_2}(f(v_j))] - F_{E_2}(f(v_i), f(v_j)), \\ &= \overline{F_{E_2}}(f(v_i), f(v_j)), \end{aligned}$$

for all $(v_i, v_j) \in E_1$, hence $\overline{G_1} \approx \overline{G_2}$. The converse is straightforward.

4 Complete Interval Valued Neutrosophic Graphs

Definition 4.1

An interval valued neutrosophic graph $G = (A, B)$ is called complete if

$$T_{BL}(v_i, v_j) = \min(T_{AL}(v_i), T_{AL}(v_j)), T_{BU}(v_i, v_j) = \min(T_{AU}(v_i), T_{AU}(v_j)),$$

$$I_{BL}(v_i, v_j) = \max(I_A(v_i), I_A(v_j)), I_{BU}(v_i, v_j) = \max(I_{AU}(v_i), I_{AU}(v_j)),$$

and

$$F_{BL}(v_i, v_j) = \max(F_A(v_i), F_A(v_j)), F_{BU}(v_i, v_j) = \max(F_{AU}(v_i), F_{AU}(v_j)),$$

for all $v_i, v_j \in V$.

Example 4.2

Consider a graph $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_1v_3, v_2v_3, v_1v_4, v_3v_4, v_2v_4\}$, then $G = (A, B)$ is a complete interval valued neutrosophic graph of G^* .

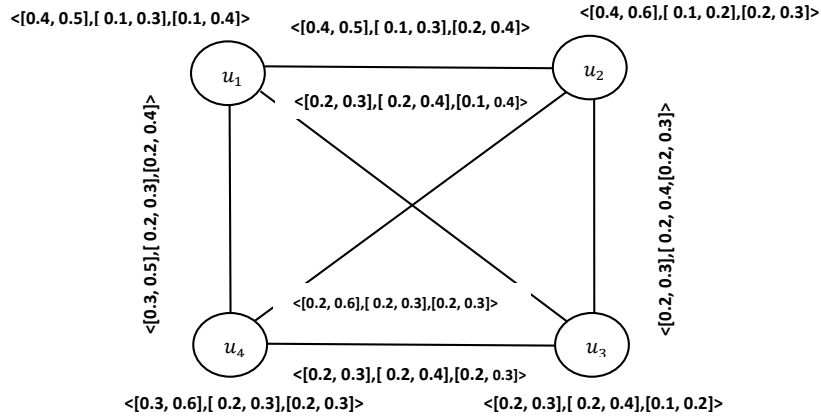


Figure17: Complete interval valued neutrosophic graph

Definition 4.3

The complement of a complete interval valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ is an interval valued neutrosophic complete graph $\bar{G} = (\bar{A}, \bar{B})$ on $G^* = (V, \bar{E})$, where

$$1. \bar{V} = V$$

$$2. \bar{T}_{AL}(v_i) = T_{AL}(v_i), \bar{T}_{AU}(v_i) = T_{AU}(v_i), \bar{I}_{AL}(v_i) = I_{AL}(v_i), \bar{I}_{AU}(v_i) = I_{AU}(v_i), \bar{F}_{AL}(v_i) = F_{AL}(v_i), \bar{F}_{AU}(v_i) = F_{AU}(v_i), \text{ for all } v_j \in V.$$

$$3. \overline{T}_{BL}(v_i, v_j) = \min [T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j),$$

$$\overline{T}_{BU}(v_i, v_j) = \min [T_{AU}(v_i), T_{AU}(v_j)] - T_{BU}(v_i, v_j),$$

$$\overline{I}_{BL}(v_i, v_j) = \max [I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j),$$

$$\overline{I}_{BU}(v_i, v_j) = \max [I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j),$$

and

$$\overline{F}_{BL}(v_i, v_j) = \max [F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j),$$

$$\overline{F}_{BU}(v_i, v_j) = \max [F_{AU}(v_i), F_{AU}(v_j)] - F_{BU}(v_i, v_j),$$

for all $(v_i, v_j) \in E$.

Proposition 4.4

The complement of complete IVN-graph is a IVN-graph with no edge. Or if G is a complete, then in \overline{G} the edge is empty.

Proof

Let $G = (A, B)$ be a complete IVN-graph. So

$$\begin{aligned} T_{BL}(v_i, v_j) &= \min(T_{AL}(v_i), T_{AL}(v_j)), T_{BU}(v_i, v_j) = \min(T_{AU}(v_i), \\ T_{AU}(v_j)), I_{BL}(v_i, v_j) &= \max(I_{AL}(v_i), I_{AL}(v_j)), I_{BU}(v_i, v_j) = \max \\ (I_{AU}(v_i), I_{AU}(v_j)) \text{ and } F_{BL}(v_i, v_j) &= \max(F_{AL}(v_i), F_{AL}(v_j)), \\ F_{BU}(v_i, v_j) &= \max(F_{AU}(v_i), F_{AU}(v_j)), \text{ for all } v_i, v_j \in V \end{aligned}$$

Hence in \overline{G} ,

$$\begin{aligned} \overline{T}_{BL}(v_i, v_j) &= \min(T_{AL}(v_i), T_{AL}(v_j)) - T_{BL}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \min(T_{AL}(v_i), T_{AL}(v_j)) - \min(T_{AL}(v_i), T_{AL}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0 \text{ for all } i, j, \dots, n \end{aligned}$$

$$\begin{aligned} \overline{T}_{BU}(v_i, v_j) &= \min(T_{AU}(v_i), T_{AU}(v_j)) - T_{BU}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \min(T_{AU}(v_i), T_{AU}(v_j)) - \min(T_{AU}(v_i), T_{AU}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0 \text{ for all } i, j, \dots, n. \end{aligned}$$

and

$$\begin{aligned} \overline{I}_{BL}(v_i, v_j) &= \max(I_{AL}(v_i), I_{AL}(v_j)) - I_{BL}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \max(I_{AL}(v_i), I_{AL}(v_j)) - \max(I_{AL}(v_i), I_{AL}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0 \text{ for all } i, j, \dots, n \end{aligned}$$

$$\begin{aligned} \bar{I}_{BU}(v_i, v_j) &= \max(I_{AU}(v_i), I_{AU}(v_j)) - I_{BU}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \max(I_{AU}(v_i), I_{AU}(v_j)) - \max(I_{AU}(v_i), I_{AU}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0 \text{ for all } i, j, \dots, n. \end{aligned}$$

Also

$$\begin{aligned} \bar{F}_{BL}(v_i, v_j) &= \max(F_{AL}(v_i), F_{AL}(v_j)) - F_{BL}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \max(F_{AL}(v_i), I_{AL}(v_j)) - \max(F_{AL}(v_i), F_{AL}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0, \text{ for all } i, j, \dots, n. \end{aligned}$$

$$\begin{aligned} \bar{F}_{BU}(v_i, v_j) &= \max(F_{AU}(v_i), F_{AU}(v_j)) - F_{BU}(v_i, v_j) \text{ for all } i, j, \dots, n \\ &= \max(F_{AU}(v_i), F_{AU}(v_j)) - \max(F_{AU}(v_i), F_{AU}(v_j)) \text{ for all } i, j, \dots, n \\ &= 0, \text{ for all } i, j, \dots, n. \end{aligned}$$

Thus

$$\begin{aligned} &([\bar{T}_{BL}(v_i, v_j), \bar{T}_{BU}(v_i, v_j)], [\bar{I}_{BL}(v_i, v_j), \bar{I}_{BU}(v_i, v_j)], \\ &[\bar{F}_{BL}(v_i, v_j), \bar{F}_{BU}(v_i, v_j)]) = ([0, 0], [0, 0], [0, 0]). \end{aligned}$$

Hence, the edge set of \bar{G} is empty if G is a complete IVN-graph.

5 Conclusion

Interval valued neutrosophic sets is a generalization of the notion of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionistic fuzzy sets and single valued neutrosophic sets.

Interval valued neutrosophic model gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and single valued neutrosophic models.

In this paper, we have defined for the first time certain types of interval valued neutrosophic graphs, such as strong interval valued neutrosophic graph, constant interval valued neutrosophic graph and complete interval valued neutrosophic graphs.

In future study, we plan to extend our research to regular interval valued neutrosophic graphs and irregular interval valued neutrosophic.

References

- [1] V. Devadoss, A. Rajkumar & N. J. P. Praveena. *A Study on Miracles through Holy Bible using Neutrosophic Cognitive Maps (NCMS)*, in "International Journal of Computer Applications", 69(3), 2013.
- [2] Mohamed Ismayil and A. Mohamed Ali. *On Strong Interval-Valued Intuitionistic Fuzzy Graph*, in "International Journal of Fuzzy Mathematics and Systems", Volume 4, Number 2, 2014, pp. 161-168.
- [3] Q. Ansari, R. Biswas & S. Aggarwal. *Neutrosophic classifier: An extension of fuzzy classifier*, in "Applied Soft Computing", 13, 2013, pp. 563-573, <http://dx.doi.org/10.1016/j.asoc.2012.08.002>.
- [4] Aydoğdu, *On Similarity and Entropy of Single Valued Neutrosophic Sets*, in "Gen. Math. Notes", Vol. 29, No. 1, 2015, pp. 67-74.
- [5] Q. Ansari, R. Biswas & S. Aggarwal. *Neutrosophication of Fuzzy Models*, poster presentation at IEEE Workshop On Computational Intelligence: Theories, Applications and Future Directions (hosted by IIT Kanpur), 14th July 2013.
- [6] Q. Ansari, R. Biswas & S. Aggarwal. *Extension to fuzzy logic representation: Moving towards neutrosophic logic - A new laboratory rat*, at the Fuzzy Systems IEEE International Conference, 2013, 1-8, doi:10.1109/FUZZ-IEEE.2013.6622412.
- [7] Nagoor Gani and M. Basheer Ahamed. *Order and Size in Fuzzy Graphs*, in "Bulletin of Pure and Applied Sciences", Vol 22E, No. 1, 2003, pp. 145-148.
- [8] Nagoor Gani, A. and S. Shajitha Begum. *Degree, Order and Size in Intuitionistic Fuzzy Graphs*, in "International Journal of Algorithms, Computing and Mathematics", (3)3, 2010.
- [9] Nagoor Gani and S. R. Latha. *On Irregular Fuzzy Graphs*, in "Applied Mathematical Sciences", Vol. 6, no. 11, 2012, pp. 517-523.
- [10] F. Smarandache. *Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies*, in "Neutrosophic Sets and Systems", Vol. 9, 2015, pp. 58- 63.
- [11] F. Smarandache. *Types of Neutrosophic Graphs and Neutrosophic Algebraic Structures together with their Applications in Technology*, seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania, 6 June 2015.
- [12] F. Smarandache, *Symbolic Neutrosophic Theory*, Europanova asbl, Brussels, 2015, 195p.
- [13] F. Smarandache. *Neutrosophic set - a generalization of the intuitionistic fuzzy set*, Granular Computing, 2006 IEEE International Conference, (2006)38 – 42, DOI: 10.1109/GRC.2006.1635754.

- [14] F. Smarandache. *A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set*, Granular Computing (GrC), 2011 IEEE International Conference, 2011, pp. 602 – 606, DOI 10.1109/GRC.2011.6122665.
- [15] Gaurav Garg, Kanika Bhutani, Megha Kumar and Swati Aggarwal. *Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms*, FUZZ-IEEE 2015 (IEEE International conference on fuzzy systems).
- [16] H. Wang, Y. Zhang, R. Sunderraman. *Truth-value based interval neutrosophic sets*, Granular Computing, 2005 IEEE International Conference, Vol. 1, 2005, pp. 274 - 277
DOI: 10.1109/GRC.2005.1547284.
- [17] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single valued Neutrosophic Sets*, Multisspace and Multistructure 4, 2010, pp. 410-413.
- [18] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderram. *An Interval neutrosophic sets and logic: theory and applications in computing*. Hexis, Arizona, 2005.
- [19] H. Y. Zhang , J. Q. Wang , X. H. Chen. *Interval neutrosophic sets and their application in multicriteria decision making problems*, The Scientific World Journal, (2014), DOI:10.1155/2014/ 645953.
- [20] H, Zhang, J. Wang, X. Chen. *An outranking approach for multi-criteria decision-making problems with interval valued neutrosophic sets*, Neural Computing and Applications, 2015, pp. 1-13.
- [21] H.Y. Zhang, P. Ji, J.Q. Wang & X. H. Chen. *An Improved Weighted Correlation Coefficient Based on Integrated Weight for Interval Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems*, International Journal of Computational Intelligence Systems, V8, Issue 6 (2015), DOI:10.1080/18756891.2015.1099917.
- [22] Deli, M. Ali, F. Smarandache. *Bipolar neutrosophic sets and their application based on multi-criteria decision making problems*, Advanced Mechatronic Systems (ICAMechS), International Conference, 2015, pp. 249 – 254.
- [23] Turksen. *Interval valued fuzzy sets based on normal forms*, Fuzzy Sets and Systems, vol. 20, 1986, pp. 191-210 .
- [24] J. Ye. *vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making*, International Journal of Fuzzy Systems, Vol. 16, No. 2, 2014, pp. 204-211.
- [25] J. Ye. *Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method*, Journal of Intelligent Systems 23(3), 2014, pp. 311–324.

- [26] J. Ye. *Similarity measures between interval neutrosophic sets and their applications in Multi-criteria decision-making*, Journal of Intelligent and Fuzzy Systems, 26, 2014, pp. 165-172.
- [27] J. Ye. *Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making*, Journal of Intelligent & Fuzzy Systems (27), 2014, pp. 2231-2241.
- [28] K. Atanassov. *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, vol. 20, 1986, pp. 87-96.
- [29] K. Atanassov and G. Gargov. *Interval valued intuitionistic fuzzy sets*, Fuzzy Sets and Systems, vol. 31, 1989, pp. 343-349.
- [30] K. Atanassov. *Intuitionistic fuzzy sets: theory and applications*, Physica, New York, 1999.
- [31] L. Zadeh. *Fuzzy sets*, Inform and Control, 8, 1965, pp. 338-353.
- [32] M. Akram and B. Davvaz, *Strong intuitionistic fuzzy graphs*, Filomat, vol. 26, no. 1, 2012, pp. 177-196.
- [33] M. Akram and W. A. Dudek, *Interval-valued fuzzy graphs*, Computers & Mathematics with Applications, vol. 61, no. 2, 2011, pp. 289-299.
- [34] M. Akram, *Interval-valued fuzzy line graphs*, in "Neural Computing and Applications", vol. 21, pp. 145-150, 2012.
- [35] M. Akram, *Bipolar fuzzy graphs*, Information Sciences, vol. 181, no. 24, 2011, pp. 5548-5564.
- [36] M. Akram, *Bipolar fuzzy graphs with applications*, Knowledge Based Systems, vol. 39, 2013, pp. 1-8.
- [37] M. Akram and A. Adeel, *m-polar fuzzy graphs and m-polar fuzzy line graphs*, Journal of Discrete Mathematical Sciences and Cryptography, 2015.
- [38] P. Bhattacharya. *Some remarks on fuzzy graphs*, Pattern Recognition Letters 6, 1987, pp. 297-302.
- [39] P. Liu and L. Shi. *The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making*, Neural Computing and Applications, 26 (2), 2015, pp. 457-471.
- [40] R. Parvathi and M. G. Karunambigai. *Intuitionistic Fuzzy Graphs*, Computational Intelligence, Theory and applications, International Conference in Germany, Sept. 18-20, 2006.
- [41] R. Rıdvan, A. Küçük,. *Subsethood measure for single valued neutrosophic sets*, Journal of Intelligent & Fuzzy Systems, vol. 29, no. 2, 2015, pp. 525-530, DOI: 10.3233/IFS-141304.
- [42] R. Şahin. *Cross-entropy measure on interval neutrosophic sets and its applications in multicriteria decision making*, Neural Computing and Applications, 2015, pp. 1-11.

- [43] S. Aggarwal, R. Biswas, A. Q. Ansari. *Neutrosophic modeling and control*, Computer and Communication Technology (ICCCT), International Conference, 2010, pp. 718 – 723, DOI: 10.1109/ICCCT.2010.5640435
- [44] S. Broumi, F. Smarandache. *New distance and similarity measures of interval neutrosophic sets*, Information Fusion (FUSION), 2014 IEEE 17th International Conference, 2014, pp. 1 – 7.
- [45] S. Broumi, F. Smarandache, *Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making*, Bulletin of Pure & Applied Sciences - Mathematics and Statistics, 2014, pp. 135-155, DOI: 10.5958/2320-3226.2014.00006.X.
- [46] S. Broumi, M. Talea, F. Smarandache, *Single Valued Neutrosophic Graphs: Degree, Order and Size*, 2016, submitted.
- [47] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *Single Valued Neutrosophic Graphs*, Journal of new theory, 2016, under process.
- [48] S. N. Mishra and A. Pal, *Product of Interval Valued Intuitionistic fuzzy graph*, Annals of Pure and Applied Mathematics, Vol. 5, No. 1, 2013, pp. 37-46.
- [49] Y. Hai-Long, G. She, Yanhonge, L. Xiuwu. *On single valued neutrosophic relations*, Journal of Intelligent & Fuzzy Systems, vol. Preprint, no. Preprint, 2015, pp. 1-12.
- [50] W. B. Vasantha Kandasamy and F. Smarandache. *Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps*, 2013.
- [51] W. B. Vasantha Kandasamy, K. Ilanthenral and Florentin Smarandache. *Neutrosophic Graphs: A New Dimension to Graph Theory*, Kindle Edition, 2015.
- [52] W.B. Vasantha Kandasamy and F. Smarandache. *Analysis of social aspects of migrant laborers living with HIV/AIDS using Fuzzy Theory and Neutrosophic Cognitive Maps*, Xiquan, Phoenix, 2004.
- [53] Florentin Smarandache, *Neutrosophy. Neutrosophic Probability, Set, and Logic*, Amer. Res. Press, Rehoboth, USA, 105 p., 1998; <http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf> (last edition); reviewed in Zentralblatt für Mathematik (Berlin, Germany), <https://zbmath.org/?q=an:01273000>
- [54] F. Smarandache, *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics*, 168 p., Pons Editions, Bruxelles, Belgique, 2016, on Cornell University's website: <https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf> and in France at the international scientific database: <https://hal.archives-ouvertes.fr/hal-01340830>