Isomorphism of Bipolar Single Valued Neutrosophic Hypergraphs

Muhammad Aslam Malik[1], Ali Hassan[2], Said Broumi[3], Assia Bakali[4], Mohamed Talea[5], Florentin Smarandache[6]

[1] Department of Mathematics, University of Punjab, Lahore, Pakistan
aslam@math.pu.edu.pk

[2] Department of Mathematics, University of Punjab, Lahore, Pakistan
alihassan.iiui.math@gmail.com

[3, 5] University Hassan II, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com

[4] Ecole Royale Navale, Casablanca, Morocco
assiabakali@yahoo.fr

[6] University of New Mexico, Gallup, NM, USA
smarand@unm.edu

Abstract

In this paper, we introduce the homomorphism, the weak isomorphism, the co-weak isomorphism, and the isomorphism of the bipolar single valued neutrosophic hypergraphs. The properties of order, size and degree of vertices are discussed. The equivalence relation of the isomorphism of the bipolar single valued neutrosophic hypergraphs and the weak isomorphism of bipolar single valued neutrosophic hypergraphs, together with their partial order relation, is also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism, bipolar single valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic set - proposed by Smarandache [8] as a generalization of the fuzzy set [14], intuitionistic fuzzy set [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy set [13] theories - is a mathematical tool created to deal with incomplete, indeterminate and inconsistent information in the real world. The characteristics of the neutrosophic set are the truth-membership function (t), the indeterminacy-membership function (i), and the falsity membership function (f), which take values within the real standard or non-standard unit interval $]^-0 , 1^+[$.

A subclass of the neutrosophic set, the single-valued neutrosophic set (SVNS), was intoduced by Wang et al. [9]. The same authors [10] also introduced a generalization of the single valued neutrosophic set, namely the interval valued neutrosophic set (IVNS), in which the three membership functions are independent, and their values belong to the unit interval [0, 1]. The IVNS is more precise and flexible than the single valued neutrosophic set.

More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

In this paper, we extend the isomorphism of the bipolar single valued neutrosophic hypergraphs, and introduce some of their relevant properties.

1 Preliminaries

Definition 2.1

A hypergraph is an ordered pair H = (X, E), where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of subsets of X.

(3) E_j are non-void for j = 1, 2, 3, ..., m, and $\cup_j (E_j) = X$.

The set X is called 'set of vertices', and E is denominated as the 'set of edges' (or 'hyper-edges').

Definition 2.2

A fuzzy hypergraph H = (X, E) is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

The collection $E = \{E_1, E_2, E_3, \dots., E_m\}$ is a collection of edge set of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form H = (X, E, R), where $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of fuzzy subsets of X, that is $E_j : X \to [0, 1]$, j = 1, 2, 3, ..., m, and $R : E \to [0, 1]$ is the fuzzy relation of the fuzzy subsets E_j, such that:

$$R(x_1, x_2, \dots, x_r) \le \min(E_j(x_1), \dots, E_j(x_r)), \quad (1)$$

for all $\{ x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by its truth membership function T_A(x), its indeterminacy membership function I_A(x), and its falsity membership function F_A(x). For each point, x ∈ X; T_A(x), I_A(x), F_A(x) ∈ [0, 1].

Definition 2.6

A single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \dots, m$, and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyperedges).

Proposition 2.7

The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

Note that a given SVNHG$H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$, is the collection of the non-empty family of SVN subsets of X, and R is the SVN relation of the SVN subsets E_j, such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.8

Let X be a space of points (objects) with generic elements in X denoted by x.

A bipolar single valued neutrosophic set A (BSVNS A) is characterized by the positive truth membership function $PT_A(x)$, the positive indeterminacy membership function $PI_A(x)$, the positive falsity membership function $PF_A(x)$, the negative truth membership function $NT_A(x)$, the negative indeterminacy membership function $NI_A(x)$, and the negative falsity membership function $NF_A(x)$.

For each point x ∈X; $PT_A(x)$, $PI_A(x)$,$PF_A(x)$ ∈ [0, 1], and $NT_A(x)$, $NI_A(x)$, $NF_A(x)$ ∈ [-1, 0].

Definition 2.9

A bipolar single valued neutrosophic hypergraph is an ordered pair *H = (X, E),* where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of BSVNSs of *X*.

(3) $E_j \neq 0 = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, 3, \ldots, m$, and $\bigcup_j Supp(E_j) = X$.

The set *X* is called the 'set of vertices' and *E* is called the 'set of BSVN-edges' (or 'IVN-hyper-edges'). Note that a given BSVNHG$H = (X, E, R)$, with underlying set *X*, where $E = \{E_1, E_2, \ldots, E_m\}$ is the collection of non-empty family of BSVN subsets of *X*, and *R* is the BSVN relation of BSVN subsets E_j such that:

$$R_{PT}(x_1, x_2, \ldots, x_r) \leq \min([PT_{E_j}(x_1)], \ldots., [PT_{E_j}(x_r)]), \quad (5)$$

$$R_{PI}(x_1, x_2, \ldots, x_r) \geq \max([PI_{E_j}(x_1)], \ldots., [PI_{E_j}(x_r)]), \quad (6)$$

$$R_{PF}(x_1, x_2, \ldots, x_r) \geq \max([PF_{E_j}(x_1)], \ldots., [PF_{E_j}(x_r)]), \quad (7)$$

$$R_{NT}(x_1, x_2, \ldots, x_r) \geq \max([NT_{E_j}(x_1)], \ldots., [NT_{E_j}(x_r)]), \quad (8)$$

$$R_{NI}(x_1, x_2, \ldots, x_r) \leq \min([NI_{E_j}(x_1)], \ldots., [NI_{E_j}(x_r)]), \quad (9)$$

$$R_{NF}(x_1, x_2, \ldots, x_r) \leq \min([NF_{E_j}(x_1)], \ldots., [NF_{E_j}(x_r)]), \quad (10)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X.*

Proposition 2.10

The bipolar single valued neutrosophic hypergraph is the generalization of the fuzzy hypergraph, intuitionistic fuzzy hypergraph, bipolar fuzzy hypergraph and intuitionistic fuzzy hypergraph.

Example 2.11

Consider the BSVNHG *H = (X, E, R),* with underlying set *X = {a, b, c},* where *E = {A, B}*, and *R* defined in *Tables* below:

H	A	B
a	(0.2, 0.3, 0.9, -0.2, -0.2, -0.3)	(0.5, 0.2, 0.7, -0.4, -0.2, -0.3)
b	(0.5, 0.5, 0.5, -0.4, -0.3, -0.3)	(0.1, 0.6, 0.4, -0.9, -0.3,-0.4)
c	(0.8, 0.8, 0.3, -0.9, -0.2, -0.3)	(0.5, 0.9, 0.8, -0.1, -0.2, -0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.5
B	0.1	0.9	0.8	-0.1	-0.5	-0.6

By routine calculations, *H = (X, E, R)* is BSVNHG.

3 Isomorphism of BSVNHGs

Definition 3.1

A homomorphism *f: H →K* between two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a mapping *f: X → Y* which satisfies the conditions:

$$\min[PT_{E_j}(x)] \leq \min[PT_{F_j}(f(x))], \tag{11}$$

$$\max[PI_{E_j}(x)] \geq \max[PI_{F_j}(f(x))], \tag{12}$$

$$\max[PF_{E_j}(x)] \geq \max[PF_{F_j}(f(x))], \tag{13}$$

$$\max[NT_{E_j}(x)] \geq \max[NT_{F_j}(f(x))], \tag{14}$$

$$\min[NI_{E_j}(x)] \leq \min[NI_{F_j}(f(x))], \tag{15}$$

$$\min[NF_{E_j}(x)] \leq \min[NF_{F_j}(f(x))], \tag{16}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \leq S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{17}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{18}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{19}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Example 3.2

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R*and*S*, which are defined in *Tables* given below:

H	A	B
a	(0.2, 0.3, 0.9,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.4,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.9,-0.3,-0.4)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.1,-0.2,-0.3)

K	C	D
x	(0.3, 0.2, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.3,-0.6,-0.1,-0.2)
y	(0.2, 0.4, 0.2,-0.4,-0.2,-0.3)	(0.3, 0.2, 0.1,-0.7,-0.2,-0.1)
z	(0.5, 0.8, 0.2,-0.2,-0.1,-0.3)	(0.9, 0.7, 0.1,-0.2,-0.1,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.5
B	0.1	0.9	0.8	-0.1	-0.5	-0.6

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.3	-0.1	-0.2	-0.3
D	0.1	0.7	0.3	-0.1	-0.2	-0.3

and $f : X \rightarrow Y$ defined by: $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a homomorphism between H and K.

Definition 3.3

A weak isomorphism $f : H \rightarrow K$ between two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \rightarrow Y$ which satisfies f is homomorphism, such that:

$$\min[PT_{E_j}(x)] \leq \min[PT_{F_j}(f(x))], \tag{23}$$

$$\max[PI_{E_j}(x)] \geq \max[PI_{F_j}(f(x))], \tag{24}$$

$$\max[PF_{E_j}(x)] \geq \max[PF_{F_j}(f(x))], \tag{25}$$

$$\max[NT_{E_j}(x)] \geq \max[NT_{F_j}(f(x))], \tag{26}$$

$$\min[NI_{E_j}(x)] \leq \min[NI_{F_j}(f(x))], \tag{27}$$

$$\min[NF_{E_j}(x)] \leq \min[NF_{F_j}(f(x))], \tag{28}$$

for all $x \in X$.

Note

The weak isomorphism between two BSVNHGs preserves the weights of vertices.

Example 3.4

Consider the two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined by *Tables* given below, and $f: X \rightarrow Y$ defined by: $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a weak isomorphism between H and K.

H	A	B
a	(0.2, 0.3, 0.9,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.4,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.9,-0.3,-0.4)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.1,-0.2,-0.3)

K	C	D
x	(0.2, 0.3, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.8,-0.6,-0.1,-0.4)
y	(0.2, 0.4, 0.2,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.5,-0.6,-0.2,-0.3)
z	(0.5, 0.8, 0.9,-0.2,-0.2,-0.3)	(0.9, 0.9, 0.1,-0.1,-0.3,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.3
B	0.1	0.9	0.9	-0.1	-0.3	-0.5

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.3	-0.2
D	0.1	0.9	0.8	-0.1	-0.3	-0.4

Definition 3.5

A co-weak isomorphism *f: H → K* between two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y* which satisfies *f* is homomorphism, such that:

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (29)$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (30)$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (31)$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (32)$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (33)$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (34)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Note

The co-weak isomorphism between two BSVNHGs preserves the weights of edges.

Example 3.6

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in *Tables* given below, and *f : X → Y* defined by: *f(a)=x* , *f(b)=y* and *f(c)*=z. Then, by routine calculations, *f: H → K* is a co-weak isomorphism between *H* and *K*.

H	A	B
a	(0.2, 0.3, 0.9,-0.4,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.1,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.2,-0.3)	(0.1, 0.6, 0.4,-0.4,-0.2,-0.3)
c	(0.8, 0.8, 0.3,-0.1,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.4,-0.2,-0.3)

K	C	D
x	(0.3, 0.2, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.3,-0.4,-0.2,-0.3)
y	(0.2, 0.4, 0.2,-0.4,-0.2,-0.3)	(0.3, 0.2,0.1,-0.9,-0.2,-0.3)
z	(0.5, 0.8, 0.2,-0.1,-0.2,-0.3)	(0.9, 0.7, 0.1,-0.1,-0.2,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.2	-0.3
B	0.1	0.9	0.8	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.2	-0.3
D	0.1	0.9	0.8	-0.1	-0.2	-0.3

Definition 3.7

An isomorphism $f : H \to K$ between two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \to Y$ which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{35}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{36}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{37}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{38}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{39}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{40}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{41}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{42}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{43}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{44}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{45}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{46}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Note

The isomorphism between two BSVNHGs preserves the both weights of vertices and weights of edges.

Example 3.8

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined by *Tables* given below:

H	A	B
a	(0.2, 0.3, 0.7,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.6,-0.6,-0.6)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.1,-0.2,-0.7)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.4)	(0.5, 0.9, 0.8,-0.7,-0.2,-0.3)

K	C	D
x	(0.2, 0.3, 0.2,-0.2,-0.2,-0.4)	(0.2, 0.1, 0.8,-0.3,-0.2,-0.3)
y	(0.2, 0.4, 0.2,-0.6,-0.2,-0.3)	(0.1, 0.6, 0.5,-0.1,-0.2,-0.7)
z	(0.5, 0.8, 0.7,-0.4,-0.3,-0.3)	(0.9, 0.9, 0.1,-0.9,-0.6,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.3	-0.4
B	0.0	0.9	0.8	-0.1	-0.7	-0.8

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.3	-0.4
D	0.0	0.9	0.8	-0.1	-0.7	-0.8

and $f : X \to Y$ defined by: *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, $f: H \to K$ is an isomorphism between *H* and *K*.

Definition 3.9

Let *H = (X, E, R)* be a BSVNHG, then the order of *H* is denoted and defined by as follows:

$$O(H) = (\sum min\left(PT_{E_j}(x)\right), \sum max\left(PI_{E_j}(x)\right), \sum max\left(PF_{E_j}(x)\right), \sum max\left(NT_{E_j}(x)\right), \sum min\left(NI_{E_j}(x)\right), \sum min\left(NF_{E_j}(x)\right)) \tag{47}$$

The size of *H* is denoted and defined by:

$$S(H) = (\sum R_{PT}(E_j), \sum R_{PI}(E_j), \sum R_{PF}(E_j), \sum R_{NT}(E_j), \sum R_{NI}(E_j), \sum R_{NF}(E_j)) \tag{48}$$

Theorem 3.10

Let *H = (X, E, R)* and *K = (Y, F, S)* be two BSVNHGs such that *H* is isomorphic to *K*, then:

(1) O(H) = O(K),

(2) S(H) = S(K).

Proof

Let *f*: $H \rightarrow K$ be an isomorphism between two BSVNHGs H and K with underlying sets *X* and *Y* respectively; then, by definition:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \quad (49)$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \quad (50)$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \quad (51)$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \quad (52)$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \quad (53)$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \quad (54)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (55)$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (56)$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (57)$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (58)$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (59)$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (60)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Consider:

$$O_{PT}(H) = \sum \min PT_{E_j}(x) = \sum \min PT_{F_j}(f(x)) = O_{PT}(K) \quad (61)$$

$$O_{NT}(H) = \sum \max NT_{E_j}(x) = \sum \max NT_{F_j}(f(x)) = O_{NT}(K) \quad (62)$$

Similarly, $O_{PI}(H) = O_{PI}(K)$ and $O_{PF}(H) = O_{PF}(K)$, $O_{NI}(H) = O_{NI}(K)$ and $O_{NF}(H) = O_{NF}(K)$**,** hence *O(H) = O(K).*

Next:

$$S_{PT}(H) = \sum R_{PT}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{PT}(f(x_1), f(x_2), \dots, f(x_r)) = S_{PT}(K). \quad (63)$$

Similarly,

$$S_{NT}(H) = \sum R_{NT}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{NT}(f(x_1), f(x_2), \dots, f(x_r)) = S_{NT}(K). \quad (64)$$

and $S_{PI}(H) = S_{PI}(K)$, $S_{PF}(H) = S_{PF}(K)$, $S_{NI}(H) = S_{NI}(K)$, $S_{NF}(H) = S_{NF}(K)$, hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem need not to be true in general.

Example 3.12

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in *Tables* given below:

H	A	B
a	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.14, 0.5, 0.3, -0.1, -0.2, -0.3)
b	(0.0,0.0,0.0, 0.0, 0.0, 0.0)	(0.2, 0.5, 0.3, -0.4, -0.2, -0.3)
c	(0.33, 0.5, 0.3, -0.4, -0.2, -0.3)	(0.16, 0.5, 0.3, -0.1, -0.2, -0.3)
d	(0.5, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.0,0.0,0.0, 0.0, 0.0, 0.0)

K	C	D
w	(0.14, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.2, 0.5, 0.33, -0.4, -0.2, -0.3)
x	(0.16, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.33,0.5, 0.33, -0.1, -0.2, -0.3)
y	(0.25, 0.5, 0.3, -0.1,-0.2, -0.3)	(0.2, 0.5, 0.33, -0.1, -0.2, -0.3)
z	(0.5, 0.5, 0.3, -0.4, -0.2, -0.3)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.5	0.3	-0.1	-0.2	-0.3
B	0.14	0.5	0.3	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.14	0.5	0.3	-0.1	-0.2	-0.3
D	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=*z.

Here, *O(H) = (1.0,2.0, 1.2,-0.7,-0.8,-1.2) = O(K)* and *S(H)=(0.34, 1.0, 0.9, -0.2, -0.4, -0.9)=S(K)*, but, by routine calculations, *H* is not an isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two BSVNHGs *H* and *K* preserves the orders.

Remark 3.14

The converse of the above corollary need not to be true in general.

Example 3.15

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in *Tables* given below**,** where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=z:*

H	A	B
a	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.14, 0.5, 0.3,-0.4,-0.2,-0.3)
b	(0.0,0.0,0.0,0.0,0.0,0.0)	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)
c	(0.33, 0.5, 0.3,-0.4,-0.2,-0.3)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
d	(0.5, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

K	C	D
w	(0.14, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
x	(0.0, 0.0, 0.0,0.0,0.0,0.0)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
y	(0.25, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
z	(0.5, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.0, 0.0, 0.0,0.0,0.0,0.0)

Here, *O(H)= (1.0, 2.0, 1.2, -0.4, -0.8, -1.2) = O(K),* but, by routine calculations, *H* is not a weak isomorphism to *K.*

Corollary 3.16

The co-weak isomorphism between any two BSVNHGs *H* and *K* preserves sizes.

Remark 3.17

The converse of the above corollary need not to be true in general.

Example 3.18

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z},* where *E = {A, B}, F = {C, D}*,*R* and *S* are defined in *Tables* given below,

H	A	B
a	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.14, 0.5, 0.3, -0.1,-0.2,-0.3)
b	(0.0,0.0,0.0,0.0,0.0,0.0)	(0.16, 0.5, 0.3, -0.1,-0.2,-0.3)
c	(0.3, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
d	(0.5, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

K	C	D
w	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.2, 0.5, 0.3, -0.1,-0.2,-0.3)
x	(0.14,0.5,0.3, -0.1,-0.2,-0.3)	(0.25, 0.5, 0.3, -0.1,-0.2,-0.3)
y	(0.5, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
z	(0.3, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.5	0.3	-0.1	-0.2	-0.3
B	0.14	0.5	0.3	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.14	0.5	0.3	-0.1	-0.2	-0.3
D	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=*z.

Here *S(H)= (0.34, 1.0, 0.6, -0.2, -0.4, -0.6) = S(K),* but, by routine calculations, *H* is not a co-weak isomorphism to *K.*

Definition 3.19

Let *H = (X, E, R)* be a BSVNHG, then the degree of vertex x_i, which is denoted and defined by:

$$\deg(x_i) = (deg_{PT}(x_i), deg_{PI}(x_i), deg_{PF}(x_i), deg_{NT}(x_i), deg_{NI}(x_i), deg_{NF}(x_i)) \quad (65)$$

where:

$$deg_{PT}(x_i) = \sum R_{PT}(x_1, x_2, \dots, x_r), \quad (66)$$

$$deg_{PI}(x_i) = \sum R_{PI}(x_1, x_2, \dots, x_r), \quad (67)$$

$$deg_{PF}(x_i) = \sum R_{PF}(x_1, x_2, \dots, x_r), \quad (68)$$

$$deg_{NT}(x_i) = \sum R_{NT}(x_1, x_2, \dots, x_r), \quad (69)$$

$$deg_{NI}(x_i) = \sum R_{NI}(x_1, x_2, \dots, x_r), \quad (70)$$

$$deg_{NF}(x_i) = \sum R_{NF}(x_1, x_2, \dots, x_r), \quad (71)$$

for $x_i \neq x_r$.

Theorem 3.20

If *H* and *K* be two isomorphic BSVNHGs, then the degree of their vertices are preserved.

Proof

Let *f: H → K* be an isomorphism between two BSVNHGs *H* and *K* with underlying sets *X* and *Y* respectively, then, by definition, we have:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \quad (72)$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \quad (73)$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \quad (74)$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \quad (75)$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \quad (76)$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \quad (77)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{78}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{79}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{80}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{81}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{82}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{83}$$

for all $\{ x_1, x_2, \dots, x_r\}$ subsets of *X*.

Consider:

$$deg_{PT}(x_i) = \sum R_{PT}(x_1, x_2, \dots, x_r)$$
$$= \sum S_{PT}(f(x_1), f(x_2), \dots, f(x_r))$$
$$= deg_{PT}(f(x_i)), \tag{84}$$

and similarly:

$$deg_{NT}(x_i) = deg_{NT}(f(x_i)), \tag{85}$$

$$deg_{PI}(x_i) = deg_{PI}(f(x_i)), deg_{PF}(x_i) = deg_{PF}(f(x_i)) \tag{86}$$

$$deg_{NI}(x_i) = deg_{NI}(f(x_i)), deg_{NF}(x_i) = deg_{NF}(f(x_i)) \tag{87}$$

Hence:

$$deg(x_i) = deg(f(x_i)). \tag{88}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined by *Tables* given below:

H	A	B
a	(0.5, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.3, 0.5, 0.3, -0.1, -0.2, -0.3)
b	(0.25, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)

K	C	D
x	(0.3, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.5,0.5,0.3, -0.1, -0.2, -0.3)
y	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.25, 0.5, 0.3, -0.1, -0.2, -0.3)

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.5	0.3	-0.1	-0.2	-0.3
D	0.25	0.5	0.3	-0.1	-0.2	-0.3

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.25	0.5	0.3	-0.1	-0.2	-0.3
B	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=x, f(b)=y,* here *deg(a) = (0.8, 1.0, 0.6, -0.2, -0.4, -0.6) = deg(x)* and *deg(b) = (0.45, 1.0, 0.6, -0.2, -0.4, -0.6) = deg(y).*

But *H* is not isomorphic to *K*, i.e. *H* is neither weak isomorphic, nor co-weak isomorphic to *K*.

Theorem 3.23

The isomorphism between BSVNHGs is an equivalence relation.

Proof

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be BSVNHGs with underlying sets X, Y and *Z*, respectively:

Reflexive

Consider the map (identity map) *f : X → X* defined as follows: *f(x) = x* for all *x* ∈ *X*, since the identity map is always bijective and satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{E_j}(f(x))], \tag{89}$$

$$\max[PI_{E_j}(x)] = \max[PI_{E_j}(f(x))], \tag{90}$$

$$\max[PF_{E_j}(x)] = \max[PF_{E_j}(f(x))], \tag{91}$$

$$\max[NT_{E_j}(x)] = \max\left[NT_{E_j}(f(x))\right], \tag{92}$$

$$\min[NI_{E_j}(x)] = \min[NI_{E_j}(f(x))], \tag{93}$$

$$\min[NF_{E_j}(x)] = \min[NF_{E_j}(f(x))], \tag{94}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = R_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{95}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = R_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{96}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = R_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{97}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = R_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{98}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = R_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{99}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = R_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{100}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Hence *f* is an isomorphism of BSVNHG *H* to itself.

Symmetric

Let *f*: $X \rightarrow Y$ be an isomorphism of H and K, then f is a bijective mapping defined as $f(x) = y$ for all $x \in X$.

Then, by definition:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \quad (101)$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \quad (102)$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \quad (103)$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \quad (104)$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \quad (105)$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \quad (106)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (107)$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (108)$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (109)$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (101)$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (111)$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (112)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since f is bijective, then we have:

$$f^{-1}(y) = x \text{ for all } y \in Y.$$

Thus, we get:

$$\min[PT_{E_j}(f^{-1}(y))] = \min[PT_{F_j}(y)], \quad (113)$$

$$\max[PI_{E_j}(f^{-1}(y))] = \max[PI_{F_j}(y)], \quad (114)$$

$$\max[PF_{E_j}(f^{-1}(y))] = \max[PF_{F_j}(y)], \quad (115)$$

$$\max[NT_{E_j}(f^{-1}(y))] = \max[NT_{F_j}(y)], \quad (116)$$

$$\min[NI_{E_j}(f^{-1}(y))] = \min[NI_{F_j}(y)], \quad (117)$$

$$\min[NF_{E_j}(f^{-1}(y))] = \min[NF_{F_j}(y)], \quad (118)$$

for all $x \in X$.

$$R_{PT}(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)) = S_{PT}(y_1, y_2, \dots, y_r), \quad (119)$$

$$R_{PI}(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)) = S_{PI}(y_1, y_2, \dots, y_r), \quad (120)$$

$$R_{PF}(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)) = S_{PF}(y_1, y_2, \dots, y_r), \quad (121)$$

$$R_{NT}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NT}(y_1, y_2, \dots, y_r), \quad (122)$$
$$R_{NI}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NI}(y_1, y_2, \dots, y_r), \quad (123)$$
$$R_{NF}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NF}(y_1, y_2, \dots, y_r), \quad (124)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Hence, we have a bijective map $f^{-1} : Y \rightarrow X$ which is an isomorphism from K to H.

Transitive

Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two isomorphism of BSVNHGs of H onto K and K onto M, respectively. Then $g\ o\ f$ is bijective mapping from X to Z, where $g\ o\ f$ is defined as $(g\ o\ f)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is an isomorphism, then by definition $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}\big(f(x)\big)], \quad (125)$$
$$\max[PI_{E_j}(x)] = \max[PI_{F_j}\big(f(x)\big)], \quad (126)$$
$$\max[PF_{E_j}(x)] = \max[PF_{F_j}\big(f(x)\big)], \quad (127)$$
$$\max[NT_{E_j}(x)] = \max[NT_{F_j}\big(f(x)\big)], \quad (128)$$
$$\min[NI_{E_j}(x)] = \min\left[NI_{F_j}\big(f(x)\big)\right], \quad (129)$$
$$\min[NF_{E_j}(x)] = \min[NF_{F_j}\big(f(x)\big)], \quad (130)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (131)$$
$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (132)$$
$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (133)$$
$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (134)$$
$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (135)$$
$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (136)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g : Y \rightarrow Z$ is an isomorphism, then by definition $g(y) = z$ for all $y \in Y$ satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{G_j}\big(g(y)\big)], \quad (137)$$
$$\max[PI_{F_j}(y)] = \max[PI_{G_j}\big(g(y)\big)], \quad (138)$$

$$\max[PF_{F_j}(y)] = \max[PF_{G_j}(g(y))], \tag{139}$$

$$\max[NT_{F_j}(y)] = \max\left[NT_{G_j}(g(y))\right], \tag{140}$$

$$\min[NI_{F_j}(y)] = \min[NI_{G_j}(g(y))], \tag{141}$$

$$\min[NF_{F_j}(y)] = \min\left[NF_{G_j}(g(y))\right], \tag{142}$$

for all $x \in X$.

$$S_{PT}(y_1, y_2, \dots, y_r) = W_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{143}$$

$$S_{PI}(y_1, y_2, \dots, y_r) = W_{PI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{144}$$

$$S_{PF}(y_1, y_2, \dots, y_r) = W_{PF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{145}$$

$$S_{NT}(y_1, y_2, \dots, y_r) = W_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{146}$$

$$S_{NI}(y_1, y_2, \dots, y_r) = W_{NI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{147}$$

$$S_{NF}(y_1, y_2, \dots, y_r) = W_{NF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{148}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of *Y*.

Thus, from above equations we conclude that:

$$\min[PT_{E_j}(x)] = \min[PT_{G_j}(g(f(x)))], \tag{149}$$

$$\max[PI_{E_j}(x)] = \max[PI_{G_j}(g(f(x)))], \tag{150}$$

$$\max[PF_{E_j}(x)] = \max[PF_{G_j}(g(f(x)))], \tag{151}$$

$$\max[NT_{E_j}(x)] = \max\left[NT_{G_j}\left(g(f(x))\right)\right], \tag{152}$$

$$\min[NI_{E_j}(x)] = \min[NI_{G_j}(g(f(x)))], \tag{153}$$

$$\min[NF_{E_j}(x)] = \min[NF_{G_j}(g(f(x)))], \tag{154}$$

for all $x \in X$.

$$R_{PT}(x_1, \dots, x_r) = W_{PT}(g(f(x_1)), \dots, g(f(x_r))), \tag{155}$$

$$R_{PI}(x_1, \dots, x_r) = W_{PI}(g(f(x_1)), \dots, g(f(x_r))), \tag{156}$$

$$R_{PF}(x_1, \dots, x_r) = W_{PF}(g(f(x_1)), \dots, g(f(x_r))), \tag{157}$$

$$R_{NT}(x_1, \dots, x_r) = W_{NT}(g(f(x_1)), \dots, g(f(x_r))), \tag{158}$$

$$R_{NI}(x_1, \dots, x_r) = W_{NI}(g(f(x_1)), \dots, g(f(x_r))), \tag{159}$$

$$R_{NF}(x_1, \dots, x_r) = W_{NF}(g(f(x_1)), \dots, g(f(x_r))), \tag{160}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Therefore $g\ o\ f$ is an isomorphism between *H* and *M*.

Hence, the isomorphism between BSVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between BSVNHGs satisfies the partial order relation.

Proof

Let $H = (X, E, R)$, $K = (Y, F, S)$ and $M = (Z, G, W)$ be BSVNHGs with underlying sets X, Y and Z, respectively:

Reflexive

Consider the map (identity map) $f : X \to X$ defined as follows: $f(x)=x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{E_j}(f(x))], \tag{161}$$

$$\max[PI_{E_j}(x)] = \max[PI_{E_j}(f(x))], \tag{162}$$

$$\max[PF_{E_j}(x)] = \max[PF_{E_j}(f(x))], \tag{163}$$

$$\max[NT_{E_j}(x)] = \max[NT_{E_j}(f(x))], \tag{164}$$

$$\min[NI_{E_j}(x)] = \min[NI_{E_j}(f(x))], \tag{165}$$

$$\min[NF_{E_j}(x)] = \min[NF_{E_j}(f(x))], \tag{166}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \le R_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{167}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \ge R_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{168}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \ge R_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{169}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \ge R_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{170}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \le R_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{171}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \le R_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{172}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.
Hence, f is a weak isomorphism of BSVNHG H to itself.

Anti-symmetric

Let f be a weak isomorphism between H onto K, and g be a weak isomorphic between K and H, that is $f: X \to Y$ is a bijective map defined by: $f(x) = y$ for all $x \in X$ satisfying the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{173}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{174}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{175}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \quad (176)$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \quad (177)$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \quad (178)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (179)$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (180)$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (181)$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (182)$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (183)$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (184)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since g is also bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{E_j}(g(y))], \quad (185)$$

$$\max[PI_{F_j}(y)] = \max[PI_{E_j}(g(y))], \quad (186)$$

$$\max[PF_{F_j}(y)] = \max[PF_{E_j}(g(y))], \quad (187)$$

$$\max[NT_{F_j}(y)] = \max[NT_{E_j}(g(y))], \quad (188)$$

$$\min[NI_{F_j}(y)] = \min[NI_{E_j}(g(y))], \quad (189)$$

$$\min[NF_{F_j}(y)] = \min[NF_{E_j}(g(y))], \quad (190)$$

for all $y \in Y$.

$$R_{PT}(y, y_2, \dots, y_r) \leq S_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \quad (191)$$

$$R_{PI}(y_1, y_2, \dots, y_r) \geq S_{PI}(f(y_1), f(y_2), \dots, f(y_r)), \quad (192)$$

$$R_{PF}(y_1, y_2, \dots, y_r) \geq S_{PF}(f(y_1), f(y_2), \dots, f(y_r)), \quad (193)$$

$$R_{NT}(y, y_2, \dots, y_r) \geq S_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \quad (194)$$

$$R_{NI}(y_1, y_2, \dots, y_r) \leq S_{NI}(f(y_1), f(y_2), \dots, f(y_r)), \quad (195)$$

$$R_{NF}(y_1, y_2, \dots, y_r) \leq S_{NF}(f(y_1), f(y_2), \dots, f(y_r)), \quad (196)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only whenever H and K have same number of edges and corresponding edge have same weights, hence H is identical to K.

Transitive

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of BSVNHGs of H onto K and K onto M, respectively. Then $g \ o \ f$ is bijective mapping from X to Z, where $g \ o \ f$ is defined as $(g \ o \ f)(x) = g(f(x))$ for all $x \in X$.

Since f is a weak isomorphism, then by definition $f(x) = y$ for all $x \in X$ which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{197}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{198}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{199}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{200}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{201}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{202}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \leq S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{203}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{204}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{205}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{206}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{207}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{208}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g : Y \to Z$ is a weak isomorphism, then by definition $g(y) = z$ for all $y \in Y$, satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{G_j}(g(y))], \tag{209}$$

$$\max[PI_{F_j}(y)] = \max[PI_{G_j}(g(y))], \tag{210}$$

$$\max[PF_{F_j}(y)] = \max[PF_{G_j}(g(y))], \tag{211}$$

$$\max[NT_{F_j}(y)] = \max[NT_{G_j}(g(y))], \tag{212}$$

$$\min[NI_{F_j}(y)] = \min[NI_{G_j}(g(y))], \tag{213}$$

$$\min[NF_{F_j}(y)] = \min[NF_{G_j}(g(y))], \tag{214}$$

for all $x \in X$.

$$S_{PT}(y_1, y_2, \dots, y_r) \leq W_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{215}$$

$$S_{PI}(y_1, y_2, \dots, y_r) \geq W_{PI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{216}$$

$$S_{PF}(y_1, y_2, \dots, y_r) \geq W_{PF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{217}$$

$$S_{NT}(y_1, y_2, \dots, y_r) \geq W_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{218}$$

$$S_{NI}(y_1, y_2, \dots, y_r) \leq W_{NI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{219}$$

$$S_{NF}(y_1, y_2, \dots, y_r) \leq W_{NF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{220}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from above equations, we conclude that:

$$\min[PT_{E_j}(x)] = \min[PT_{G_j}(g(f(x)))], \quad (221)$$

$$\max[PI_{E_j}(x)] = \max[PI_{G_j}(g(f(x)))], \quad (222)$$

$$\max[PF_{E_j}(x)] = \max[PF_{G_j}(g(f(x)))], \quad (223)$$

$$\max[NT_{E_j}(x)] = \max[NT_{G_j}(g(f(x)))], \quad (224)$$

$$\min[NI_{E_j}(x)] = \min[NI_{G_j}(g(f(x)))], \quad (225)$$

$$\min[NF_{E_j}(x)] = \min[NF_{G_j}(g(f(x)))], \quad (226)$$

for all $x \in X$.

$$R_{PT}(x_1, \dots, x_r) \leq W_{PT}(g(f(x_1)), \dots, g(f(x_r))), \quad (227)$$

$$R_{PI}(x_1, \dots, x_r) \geq W_{PI}(g(f(x_1)), \dots, g(f(x_r))), \quad (228)$$

$$R_{PF}(x_1, \dots, x_r) \geq W_{PF}(g(f(x_1)), \dots, g(f(x_r))), \quad (229)$$

$$R_{NT}(x_1, \dots, x_r) \geq W_{NT}(g(f(x_1)), \dots, g(f(x_r))), \quad (230)$$

$$R_{NI}(x_1, \dots, x_r) \leq W_{NI}(g(f(x_1)), \dots, g(f(x_r))), \quad (231)$$

$$R_{NF}(x_1, \dots, x_r) \leq W_{NF}(g(f(x_1)), \dots, g(f(x_r))), \quad (232)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore $g\ o\ f$ is a weak isomorphism between H and M.

Hence, the weak isomorphism between BSVNHGs is a partial order relation.

4 Conclusion

The bipolar single valued neutrosophic hypergraph can be applied in various areas of engineering and computer science. In this paper, the isomorphism between BSVNHGs is proved to be an equivalence relation and the weak isomorphism is proved to be a partial order relation. Similarly, it can be proved that co-weak isomorphism in BSVNHGs is a partial order relation.

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