Neutrosophic Actions, Prevalence Order, Refinement of Neutrosophic Entities, and Neutrosophic Literal Logical Operators

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Abstract

In this chapter we define for the first time three neutrosophic actions and their properties. We then introduce the prevalence order on $\{T, I, F\}$ with respect to a given neutrosophic operator "o", which may be subjective - as defined by the neutrosophic experts. And the refinement of neutrosophic entities <A>, <neutA>, and <antiA>.

Then we extend the classical logical operators to neutrosophic literal logical operators and to refined literal logical operators, and we define the refinement neutrosophic literal space.

Keywords

neutrosophy, neutrosophics, neutrosophic actions, prevalence order, neutrosophic operator, refinement of neutrosophic entities, neutrosophic literal logical operators, refined literal logical operators, refinement neutrosophic literal space.

1 Introduction

In Boolean Logic, a proposition $\mathcal{P}$ is either true (T), or false (F). In Neutrosophic Logic, a proposition $\mathcal{P}$ is either true (T), false (F), or indeterminate (I).

For example, in Boolean Logic the proposition $\mathcal{P}_1$:

"1+1=2 (in base 10)"

is true, while the proposition $\mathcal{P}_2$:

"1+1=3 (in base 10)"

is false.

In neutrosophic logic, besides propositions $\mathcal{P}_1$ (which is true) and $\mathcal{P}_2$ (which is false), we may also have proposition $\mathcal{P}_3$:

"1+1= ?(in base 10)",

which is an incomplete/indeterminate proposition (neither true, nor false).

1.1 Remark

All conjectures in science are indeterminate at the beginning (researchers not knowing if they are true or false), and later they are proved as being either true, or false, or indeterminate in the case they were unclearly formulated.

2 Notations

In order to avoid confusions regarding the operators, we note them as:

Boolean (classical) logic:

$$\neg, \quad \wedge, \quad \vee, \quad \underline{\vee}, \quad \rightarrow, \quad \leftrightarrow$$

Fuzzy logic:

$$\underset{F}{\neg}, \quad \underset{F}{\wedge}, \quad \underset{F}{\vee}, \quad \underset{F}{\underline{\vee}}, \quad \underset{F}{\rightarrow}, \quad \underset{F}{\leftrightarrow}$$

Neutrosophic logic:

$$\underset{N}{\neg}, \quad \underset{N}{\wedge}, \quad \underset{N}{\vee}, \quad \underset{N}{\underline{\vee}}, \quad \underset{N}{\rightarrow}, \quad \underset{N}{\leftrightarrow}$$

3 Three Neutrosophic Actions

In the frame of neutrosophy, we have considered [1995] for each entity ⟨A⟩, its opposite ⟨antiA⟩, and their neutrality ⟨neutA⟩ {i.e. neither ⟨A⟩, nor ⟨antiA⟩}. Also, by ⟨nonA⟩ we mean what is not ⟨A⟩, i.e. its opposite ⟨antiA⟩, together with its neutral(ity) ⟨neutA⟩; therefore:

$$\langle \text{non}A\rangle = \langle \text{neut}A\rangle \vee \langle \text{anti}A\rangle.$$

Based on these, we may straightforwardly introduce for the first time the following neutrosophic actions with respect to an entity <A>:

1. To neutralize (or to neuter, or simply to neut-ize) the entity <A>. [As a noun: neutralization, or neuter-ization, or simply neutization.] We denote it by <neutA> or neut(A).
2. To antithetic-ize (or to anti-ize) the entity <A>. [As a noun: antithetic-ization, or anti-ization.] We denote it by <antiA> ot anti(A).

This action is 100% opposition to entity <A> (strong opposition, or strong negation).

3. To non-ize the entity <A>. [As a noun: non-ization]. We denote it by <nonA> or non(A).
It is an opposition in a percentage between (0, 100]% to entity <A> (weak opposition).

Of course, not all entities <A> can be neutralized, or antithetic-ized, or non-ized.

3.1 Example

Let

⟨A⟩="Phoenix Cardinals beats Texas Cowboys".

Then,

⟨neutA⟩="\"Phoenix Cardinals has a tie game with Texas Cowboys\"";

⟨antiA⟩="\"Phoenix Cardinals is beaten by Texas Cowboys\"";

⟨nonA⟩="\"Phoenix Cardinals has a tie game with Texas Cowboys,"
"or Phoenix Cardinals is beaten by Texas Cowboys\"."

3.2 Properties of the Three Neutrosophic Actions

$$\text{neut}(\langle \text{anti}A \rangle) = \text{neut}(\langle \text{neutA} \rangle) = \text{neut}(A);$$

$$\text{anti}(\langle \text{anti}A \rangle) = A;\ \text{anti}(\langle \text{neut}A \rangle) = \langle A \rangle \text{ or } \langle \text{anti}A \rangle;$$

$$\text{non}(\langle \text{anti}A \rangle) = \langle A \rangle \text{ or } \langle \text{neut}A \rangle;\ \text{non}(\langle \text{neut}A \rangle) = \langle A \rangle \text{ or } \langle \text{anti}A \rangle.$$

4 Neutrosophic Actions' Truth-Value Tables

Let's have a logical proposition P, which may be true (T), Indeterminate (I), or false (F) as in previous example. One applies the neutrosophic actions below.

4.1 Neutralization (or Indetermination) of P

neut(P)	T	I	F
	I	I	I

4.2 Antitheticization (Neutrosophic Strong Opposition to P)

anti(P)	T	I	F
	F	$T \vee F$	T

4.3 Non-ization (Neutrosophic Weak Opposition to P):

non(P)	T	I	F
	$I \vee F$	$T \vee F$	$T \vee I$

5 Refinement of Entities in Neutrosophy

In neutrosophy, an entity ⟨A⟩ has an opposite ⟨antiA⟩ and a neutral ⟨neutA⟩. But these three categories can be refined in sub-entities $\langle A\rangle_1, \langle A\rangle_2, \dots, \langle A\rangle_m$, and respectively $\langle \text{neut}A\rangle_1, \langle \text{neut}A\rangle_2, \dots, \langle \text{neut}A\rangle_n$, and also $\langle \text{anti}A\rangle_1$, $\langle \text{anti}A\rangle_2, \dots, \langle \text{anti}A\rangle_p$, where m, n, p are integers ≥1, but $m + n + p \geq 4$ (meaning that at least one of ⟨A⟩, ⟨antiA⟩ or ⟨neutA⟩ is refined in two or more sub-entities).

For example, if ⟨A⟩=white color, then
⟨antiA⟩=black color,
while ⟨neutA⟩=colors different from white and black.

If we refine them, we get various nuances of white color: $\langle A\rangle_1, \langle A\rangle_2$, ..., and various nuances of black color: $\langle \text{anti}A\rangle_1, \langle \text{anti}A\rangle_2$, ..., and the colors in between them (red, green, yellow, blue, etc.): $\langle \text{neut}A\rangle_1, \langle \text{neut}A\rangle_2, \dots$.

Similarly as above, we want to point out that not all entities <A> and/or their corresponding (if any) <neutA> and <antiA> can be refined.

6 The Prevalence Order

Let’s consider the classical literal (symbolic) truth (T) and falsehood (F).

In a similar way, for neutrosophic operators we may consider the literal (symbolic) truth (T), the literal (symbolic) indeterminacy (I), and the literal (symbolic) falsehood (F).

We also introduce the prevalence order on $\{T, I, F\}$ with respect to a given binary and commutative neutrosophic operator “o”.

The neutrosophic operators are: neutrosophic negation, neutrosophic conjunction, neutrosophic disjunction, neutrosophic exclusive disjunction, neutrosophic Sheffer's stroke, neutrosophic implication, neutrosophic equivalence, etc.

The prevalence order is partially objective (following the classical logic for the relationship between T and F), and partially subjective (when the indeterminacy I interferes with itself or with T or F).

For its subjective part, the prevalence order is determined by the neutrosophic logic expert in terms of the application/problem to solve, and also depending on the specific conditions of the application/problem.

For $X \neq Y$, we write X℗Y, or $X \succ_o Y$, and we read X prevails to Y with respect to the neutrosophic binary commutative operator "o", which means that $XoY = X$.

Let's see the below examples. We mean by "o": conjunction, disjunction, exclusive disjunction, Sheffer's stroke, and equivalence.

7 Neutrosophic Literal Operators & Neutrosophic Numerical Operators

7.1 If we mean by neutrosophic literal proposition, a proposition whose truth value is a letter: either T or I or F. The operators that deal with such logical propositions are called neutrosophic literal operators.

7.2 And by neutrosophic numerical proposition, a proposition whose truth value is a triple of numbers (or in general of numerical subsets of the interval [0, 1]), for examples A(0.6, 0.1, 0.4) or B([0, 0.2], {0.3, 0.4, 0.6}, (0.7, 0.8)). The operators that deal with such logical propositions are called neutrosophic numerical operators.

8 Truth-Value Tables of Neutrosophic Literal Operators

In Boolean Logic, one has the following truth-value table for negation:

8.1 Classical Negation

¬	T	F
	F	T

In Neutrosophic Logic, one has the following neutrosophic truth-value table for the neutrosophic negation:

8.2 Neutrosophic Negation

$\neg_N$	T	I	F
	Ⓕ	*I*	Ⓣ

So, we have to consider that the negation of I is I, while the negations of T and F are similar as in classical logic.

In classical logic, one has:

8.3 Classical Conjunction

∧	T	F
T	*T*	*F*
F	*F*	*F*

In neutrosophic logic, one has:

8.4 Neutrosophic Conjunction (AND_N), version 1

$\wedge_N$	T	I	F
T	Ⓣ	*I*	Ⓕ
I	*I*	*I*	*I*
F	Ⓕ	*I*	Ⓕ

The objective part (circled literal components in the above table) remains as in classical logic, but when indeterminacy *I* interferes, the neutrosophic expert may choose the most fit prevalence order.

There are also cases when the expert may choose, for various reasons, to entangle the classical logic in the objective part. In this case, the prevalence order will be totally subjective.

The prevalence order works for classical logic too. As an example, for classical conjunction, one has $F \succ_c T$, which means that $F \wedge T = F$. While the prevalence order for the neutrosophic conjunction in the above tables was:

$$I \succ_c F \succ_c T,$$

which means that $I \wedge_N F = I$, and $I \wedge_N T = I$.

Other prevalence orders can be used herein, such as:

$$F \succ_c I \succ_c T,$$

and its corresponding table would be:

8.5 Neutrosophic Conjunction (AND_N), version 2

$\wedge_N$	T	I	F
T	Ⓣ	I	Ⓕ
I	I	I	F
F	Ⓕ	F	Ⓕ

which means that $F_{\wedge_N} I = F$ and $I_{\wedge_N} I = I$; or another prevalence order:

$$F \succ_c T \succ_c I,$$

and its corresponging table would be:

8.6 Neutrosophic Conjunction (AND_N), version 3

$\wedge_N$	T	I	F
T	Ⓣ	T	Ⓕ
I	T	I	F
F	Ⓕ	F	Ⓕ

which means that $F_{\wedge_N} I = F$ and $T_{\wedge_N} I = T$.

If one compares the three versions of the neutrosophic literal conjunction, one observes that the objective part remains the same, but the subjective part changes.

The subjective of the prevalence order can be established in an optimistic way, or pessimistic way, or according to the weights assigned to the neutrosophic literal components *T, I, F* by the experts.

In a similar way, we do for disjunction. In classical logic, one has:

8.7 Classical Disjunction

$\vee$	T	F
T	T	T
F	T	F

In neutrosophic logic, one has:

8.8 Classical Disjunction (OR_N)

$\vee_N$	T	I	F
T	Ⓣ	T	Ⓣ
I	T	I	F
F	Ⓣ	F	Ⓕ

where we used the following prevalence order:

$$T \succ_d F \succ_d I,$$

but the reader is invited (as an exercise) to use another prevalence order, such as:

$$T \succ_d I \succ_d F,$$

Or

$$I \succ_d T \succ_d F, \text{ etc.,}$$

for all neutrosophic logical operators presented above and below in this paper.

In classical logic, one has:

8.9 Classical Exclusive Disjunction

$\veebar$	T	F
T	F	T
F	T	F

In neutrosophic logic, one has:

8.10 Neutrosophic Exclusive Disjunction

$\veebar_N$	T	I	F
T	Ⓕ	T	Ⓣ
I	T	I	F
F	Ⓣ	F	Ⓕ

using the prevalence order

$$T \succ_d F \succ_d I.$$

In classical logic, one has:

8.11 Classical Sheffer's Stroke

$\vert$	T	F
T	F	T
F	T	T

In neutrosophic logic, one has:

8.12 Neutrosophic Sheffer's Stroke

$\vert_N$	T	I	F
T	Ⓕ	T	Ⓣ
I	T	I	I
F	Ⓣ	I	Ⓣ

using the prevalence order

$$T \succ_d I \succ_d F.$$

In classical logic, one has:

8.13 Classical Implication

→	T	F
T	*T*	*F*
F	*T*	*T*

In neutrosophic logic, one has:

8.14 Neutrosophic Implication

$\rightarrow_N$	T	I	F
T	Ⓣ	*I*	Ⓕ
I	*T*	*T*	*F*
F	Ⓣ	*T*	Ⓣ

using the subjective preference that $I \rightarrow_N T$ is true (because in the classical implication T is implied by anything), and $I \rightarrow_N F$ is false, while $I \rightarrow_N I$ is true because is similar to the classical implications $T \rightarrow T$ and $F \rightarrow F$, which are true.

The reader is free to check different subjective preferences.

In classical logic, one has:

8.15 Classical Equivalence

↔	T	F
T	*T*	*F*
F	*F*	*T*

In neutrosophic logic, one has:

8.15 Neutrosophic Equivalence

$\leftrightarrow_N$	T	I	F
T	Ⓣ	I	Ⓕ
I	I	T	I
F	Ⓕ	I	Ⓣ

using the subjective preference that $I \leftrightarrow_N I$ is true, because it is similar to the classical equivalences that $T \rightarrow T$ and $F \rightarrow F$ are true, and also using the prevalence:

$$I \succ_e F \succ_e T.$$

9 Refined Neutrosophic Literal Logic

Each particular case has to be treated individually.

In this paper, we present a simple example. Let's consider the following neutrosophic logical propositions:

T = Tomorrow it will rain or snow.
T is split into
→ Tomorrow it will rain.
→ Tomorrow it will snow.
F = Tomorrow it will neither rain nor snow.
F is split into
→ Tomorrow it will not rain.
→ Tomorrow it will not snow.
I = Do not know if tomorrow it will be raining, nor if it will be snowing.
I is split into
→ Do not know if tomorrow it will be raining or not.
→ Do not know if tomorrow it will be snowing or not.

Then:

$\neg_N$	T_1	T_2	I_1	I_2	F_1	F_2
	F_1	F_2	$T_1 \vee F_1$	$T_2 \vee F_2$	T_1	T_2

It is clear that the negation of T_1 (Tomorrow it will raining) is F_1 (Tomorrow it will not be raining). Similarly for the negation of T_2, which is F_2.

But, the negation of I_1 (Do not know if tomorrow it will be raining or not) is "Do know if tomorrow it will be raining or not", which is equivalent to "We know that tomorrow it will be raining" (T_1), or "We know that tomorrow it will not be raining" (F_1).

Whence, the negation of I_1 is $T_1 \vee F_1$, and similarly, the negation of I_2 is $T_2 \vee F_2$.

9.1 Refined Neutrosophic Literal Conjunction Operator

$\wedge_N$	T_1	T_2	I_1	I_2	F_1	F_2
T_1	T_1	$T_{1\,2}$	I_1	I_2	F_1	F_2
T_2	$T_{1\,2}$	T_2	I_1	I_2	F_1	F_2
I_1	I_1	I_1	I_1	I	F_1	F_2
I_2	I_2	I_2	I	I_2	F_1	F_2
F_1	F_1	F_1	F_1	F_1	F_1	F
F_2	F_2	F_2	F_2	F_2	F	F_2

where $T_{1\,2} = T_1 \wedge T_2 =$ "Tomorrow it will rain and it will snow".

Of course, other prevalence orders can be studied for this particular example.

With respect to the neutrosophic conjunction, F_l prevail in front of I_k, which prevail in front of T_j, or $F_l \succ I_k \succ T_j$, for all $l, k, j \in \{1, 2\}$.

9.2 Refined Neutrosophic Literal Disjunction Operator

$\vee_N$	T_1	T_2	I_1	I_2	F_1	F_2
T_1	T_1	T	T_1	T_1	T_1	T_1
T_2	T	T_2	T_2	T_2	T_2	T_2
I_1	T_1	T_2	I_1	I	F_1	F_2
I_2	T_1	T_2	I	I_2	F_1	F_2
F_1	T_1	T_2	F_1	F_1	F_1	$F_1 \vee F_2$
F_2	T_1	T_2	F_2	F_2	$F_1 \vee F_2$	F_2

With respect to the neutrosophic disjunction, T_j prevail in front of F_l, which prevail in front of I_k, or $T_j \succ F_l \succ I_k$, for all $j, l, k \in \{1, 2\}$.

For example, $T_1 \vee T_2 = T$, but $F_1 \vee F_2 \notin \{T, I\ F\} \cup \{T_1, \mathrm{T}_2, I_1, \mathrm{I}_2, F_1, \mathrm{F}_2\}$.

9.3 Refined Neutrosophic Literal Space

The Refinement Neutrosophic Literal Space $\{T_1, T_2, I_1, I_2, F_1, F_2\}$ is not closed under neutrosophic negation, neutrosophic conjunction, and neutrosophic disjunction. The reader can check the closeness under other neutrosophic literal operations.

A neutrosophic refined literal space

$$S_N = \{T_1, T_2, \dots, T_p;\ I_1, I_2, \dots, I_r;\ F_1, F_2, \dots, F_s\},$$

where p, r, s are integers ≥ 1, is said to be closed under a given neutrosophic operator "θ_N", if for any elements $X, Y \in S_N$ one has $X_{\theta_N} Y \in S_N$.

Let's denote the extension of S_N with respect to a single θ_N by:

$$S_{N_1}^C = (S_N, \theta_N).$$

If S_N is not closed with respect to the given neutrosophic operator θ_N, then $S_{N_1}^C \neq S_N$, and we extend S_N by adding in the new elements resulted from the operation $X\theta_N Y$, let's denote them by $A_1, A_2, \dots A_m$.

Therefore,

$$S_{N_1}^C \neq S_N \cup \{A_1, A_2, \dots A_m\}.$$

$S_{N_1}^C$ encloses S_N.

Similarly, we can define the closeness of the neutrosophic refined literal space S_N with respect to the two or more neutrosophic operators $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$, for $w \geq 2$.

S_N is closed under $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$ if for any $X, Y \in S_N$ and for any $i \in \{1, 2, \dots, w\}$ one has $X_{\theta_{i_N}} Y \in S_N$.

If S_N is not closed under these neutrosophic operators, one can extend it as previously.

Let's consider: $S_{N_w}^C = (S_N, \theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N})$, which is S_N closed with respect to all neutrosophic operators $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$, then $S_{N_w}^C$ encloses S_N.

10 Conclusion

We have defined for the first time three neutrosophic actions and their properties. We have introduced the prevalence order on $\{T, I, F\}$ with respect to a given neutrosophic operator "o", the refinement of neutrosophic entities <A>, <neutA>, and <antiA>, and the neutrosophic literal logical operators and refined literal logical operators, and the refinement neutrosophic literal space.

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