Neutrosophic Index Numbers: Neutrosophic Logic Applied In The Statistical Indicators Theory

Florentin Smarandache[1], Gheorghe Savoiu[2]

[1] University of New Mexico
705 Gurley Ave., Gallup, NM 87301, USA
smarand@unm.edu

[2] University of Pitesti
8 Calea Bucuresti, 110133 Pitesti, Romania
gheorghe.savoiu@upit.ro

Abstract

Neutrosophic numbers easily allow modeling uncertainties of prices universe, thus justifying the growing interest for theoretical and practical aspects of arithmetic generated by some special numbers in our work. At the beginning of this paper, we reconsider the importance in applied research of instrumental discernment, viewed as the main support of the final measurement validity. Theoretically, the need for discernment is revealed by decision logic, and more recently by the new neutrosophic logic and by constructing neutrosophic-type index numbers, exemplified in the context and applied to the world of prices, and, from a practical standpoint, by the possibility to use index numbers in characterization of some cyclical phenomena and economic processes, e.g. inflation rate. The neutrosophic index numbers or neutrosophic indexes are the key topic of this article. The next step is an interrogative and applicative one, drawing the coordinates of an optimized discernment centered on neutrosophic-type index numbers. The inevitable conclusions are optimistic in relation to the common future of the index method and neutrosophic logic, with statistical and economic meaning and utility.

Keyword

neutrosophic-tendential fuzzy logic, neutrosophic logic, neutrosophic index, index statistical method, price index, interpreter index, neutrosophic interpreter index.

1 Introduction

Any decision, including the statistical evaluation in the economy, requires three major aspects, distinct but interdependent to a large extent, starting with *providing the needed knowledge to a certain level of credibility* (reducing uncertainty, available knowledge being incomplete and unreliable in different proportions, and the condition of certainty rarely being encountered in practice, the determinism essentially characterizing only the theory), then by the *discernment of chosing the decision option*, and, finally, by *obtaining the instrumental and quantified consensus*. In the hierarchy of measurement results qualities, the discernment of instrumental choice – by selection of the tool, of the technics, or of the method from the alternative options that characterizes all available solutions – should be declared the fundamental property of applied research. Moreover, the discernment can be placed on a scale intensity, from experimental discernment or decison discernment, selected according to the experience acquired in time, then ascending a “ladder” revealed by perpetual change of the continuous informational discernment or by the discernment obtained through knowledge from new results of research in specific activity, until the final stage of intuitional discernment (apparently rational, but mostly based on intuition), in fact the expression of a researcher's personal reasoning.

In summary, the process of making a measurement decision, based on a spontaneous and intuitive personal judgment, contains a referential system that experiences, more or less by chance, different quantifying actions satisfying to varying degrees the needs of which the system is aware in a fairly nuanced manner. The actions, the tools, the techniques and the measurement methods that are experienced as satisfactory will be accepted, resumed, fixed and amplified as accurate, and those that are experienced as unsatisfactory will be remove from the beginning. A modern discernment involves completing all the steps of the described “ladder”, continuously exploiting the solutions or the alternatives enabling the best interpretation, ensuring the highest degree of differentiation, offering the best diagnostic, leading to the best treatment, with the most effective impact in real time. However, some modern measurement theories argue that human social systems, in conditions of uncertainity, resort to a simplified decision-making strategy, respectively the adoption of the first satisfactory solution, coherentely formulated, accepted by relative consensus (the Dow Jones index example is a perennial proof in this respect).

Neutrosophic logic facilitates the discernment in relation to natural language, and especially with some of its terms, often having arbitrary values. An example in this regard is the formulation of common market economies: “inflation is low and a slight increase in prices is reported,” a mathematical imprecise formulation since it is not exactly known which is the percentage of price increase; still, if it comes about a short period of time and a well defined market of a product, one can make the assumption that a change to the current price is between 0% and 100% compared to the last (basic) price.

A statement like – "if one identifies a general increase in prices close to zero" (or an overall increase situated between three and five percent, or a general increase around up to five percent), "then the relevant market enjoys a low inflation" – has a corresponding degree of truth according to its interpretation in the context it was issued. However, the information must be interpreted accordingly to a certain linguistic value, because it can have different contextual meanings (for Romania, an amount of 5% may be a low value, but for the EU even an amount of five percent is certainly a very high one).

Neutrosophic logic, by employing neutrosophic-type sets and corresponding membership functions, could allow detailing the arrangement of values covering the area of representation of a neutrosophic set, as well as the correspondence between these values and their degree of belonging to the related neutrosophic set, or by employing neutrosophic numbers, especially the neutrosophic indexes explained in this paper; and could open new applied horizons, e.g. price indexes that are, in fact, nothing else than interpreters, but more special – on the strength of their special relationship with the reality of price universe.

2 Neutrosophic Logic

First of all, we should define what the Logic is in general, and then the Neutrosophic Logic in particular.

Although considered elliptical by Nae Ionescu, the most succinct and expressive metaphorical definition of the Logic remains that – Logic is "the thinking that thinks itself."

The Logic has indisputable historical primacy as science. The science of Logic seeks a finite number of consequences, operating with sets of sentences and the relationship between them. The consequence's or relationship's substance is exclusively predicative, modal, or propositional, such generating Predicative Logic, Modal Logic, or Propositional Logic.

A logic calculation can be syntactic (based on evidence) and semantic (based on facts). The Classical Logic, or the Aristotelian excluded middle logic, operates only with the notions of truth and false, which makes it inappropriate to the vast majority of real situations, which are unclear or imprecise. According to the same traditional approach, an object could either belong or not belong to a set.

The essence of the new Neutrosophic Logic is based on the notion of vagueness: a neutrosophic sentence may be only true to a certain extent; the notion of belonging benefits from a more flexible interpretation, as more items may belong to a set in varying degrees. The first imprecision based logic (early neutrosophic) has existed since 1920, as proposed by the Polish mathematician and logician Jan Łukasiewicz, which expanded the truth of a proposition to all real numbers in the range [0; 1], thus generating the possibility theory, as reasoning method in conditions of inaccuracy and incompleteness [1].

In early fuzzy logic, a neutrosophic-tendential logic of Łukasiewicz type, this paradox disappears, since if φ has the value of 0.5, its own negation will have the same value, equivalent to φ. This is already a first step of a potentially approachable gradation, by denying the true statement (1) by the false statement (φ) and by the new arithmetic result of this logic, namely the new value 1- φ.

In 1965, Lotfi A. Zadeh extended the possibility theory in a formal system of fuzzy mathematical logic, focused on methods of working using nuanced terms of natural language. Zadeh introduced the degree of membership/truth (t) in 1965 and defined the *fuzzy set*.

Atanassov introduced the degree of nonmembership/falsehood (f) in 1986 and defined the *intuitionistic fuzzy set*.

Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998), and defined the neutrosophic set. In 2013, he refined the neutrosophic set to *n* components:

$$t_1, t_2, \dots t_j;\ i_1, i_2, \dots, i_k;\ f_1, f_2, \dots, f_2,$$

where $j + k + l = n > 3$.

The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book. Etymologically, "neutro-sophy" (noun) [French *neutre* <Latin *neuter*, *neutral*, and Greek *sophia*, skill/wisdom] means "knowledge of neutral thought", while "neutrosophic" (adjective), means "having the nature of, or having the characteristic of Neutrosophy".

Going over, in fuzzy set, there is only a degree (percentage) of belonging of an element to a set (Zadeh, 1965). Atanassov introduced in 1986 the degree (percentage) of non-belonging of an element to a set, and developed the intuitionistic fuzzy set. Smarandache introduced in 1995 the degree (percentage) of indeterminacy of belonging, that is: we do not know if an element belongs, or does not belong to a set), defining the neutrosophic set.

The neutrosophy, in general, is based on the neutral part, neither membership nor non-membership, and in neutrosophic logic, in particular: neither true, nor false, but in between them. Therefore, an element $x(t, i, f)$ belongs to a neutrosophic set M in the following way: x is $t\%$ in M, $i\%$ indeterminate belonging, and $f\%$ does not belong. Or we can look at this issue in probabilistic terms as such: the chance for the element x to belong to the set M is $t\%$, the indeterminate chance to belong is $i\%$, and the chance not to belong is $f\%$.

In normalized cases, $t + i + f = 1$ (100%), but in general, if the information about the possibility of membership of the element x in the set M is independently sourced (not communicating with one another, so not influencing each other), then it may be that $0 \leq t + i + f \leq 3$.

In more general or approximated cases, t, i, f can be included intervals in [0, 1], or even certain subsets included in [0, 1], i.e. when working with inaccurate, wrong, contradictory, vague data.

In 1972, S.S.L. Chang and L. A. Zadeh sketched the use of fuzzy logic (also of tendential-neutrosophic logic) in conducting technological processes by introducing the concept of linguistic variables defined not by numbers, but as a variable in linguistic terms, clearly structured by letters of words. The linguistic variables can be decomposed into a multitude of terms, covering the full range of the considered parameter.

On the other hand, unlike the classical logic (Aristotelian, mathematic and boolean), which work exclusively with two exact numerical values (0 for false and 1 for true), the fuzzy early-neutrosophic logic was able to use a wide continuous spectrum of logical values in the range [0, 1], where 0 indicates complete falsity, and 1 indicates complete truth. However, if an object, in classical logic, could belong to a set (1) or not belong to a set (0), the neutrosophic logic redefines the object's degree of membership to the set, taking any value between 0 and 1. The linguistic refinement could be fuzzy tendential-neutrosophically redefined, both logically and mathematically, by inaccuracy, by indistinctness, by vagueness. The mathematical clarification of imprecision and vagueness, the more elastic formal interpretation of membership, the representation and the manipulation of nuanced terms of natural language, all these characterize today, after almost half a century, the neutrosophic logic.

The first major application of the neutrosophic logical system has been carried out by L.P. Holmblad and J.J. Ostergaard on a cement kiln automation [2], in 1982, followed by more practical various uses, as in high traffic intersections or water treatment plants. The first chip capable of performing the inference in a decision based on neutrosophic logic was conducted in 1986 by Masaki Togai and Hiroyuki Watanabe at AT&T Bell Laboratories, using the digital implementation of *min-max* type logics, expressing elementary union and intersection operations [3].

A neutrosophic-tendential fuzzy set, e.g. denoted by *F*, defined in a field of existence *U*, is characterized by a membership function $\mu F(x)$ which has values in the range [0, 1] and is a generalization of the concise set [4], where the belonging function takes only one of two values, zero and one. The membership function provides a measure of the degree of similarity of an element *U* of neutrosophic-tendential fuzzy subset *F.* Unlike the concise sets and subsets, characterized by net frontiers, the frontiers of the neutrosophic-tendential fuzzy sets and subsets are made from regions where membership function values gradually fade out until they disappear, and the areas of frontiers of these nuanced subsets may overlap, meaning that the elements from these areas may belong to two neighboring subsets at the same time.

As a result of the neutrosophic-tendential fuzzy subset being characterized by frontiers, which are not net, the classic inference reasoning, expressed by a *Modus Ponens* in the traditional logic, of form:

$(p\rightarrow (p\rightarrow q))\rightarrow q$, i.e.: *premise:* if *p,* then *q*
fact: p
consequence: q,

becomes a *generalized Modus Ponens,* according to the neutrosophic-tendential fuzzy logic and under the new rules of inference suggested from the very beginning by Lotfi A. Zadeh [5], respectively in the following expression:

premise: if *x* is *A*, then *y* is *B*
fact: x is *A'*
consequence: y is *B'*, where $B' = A' \text{o} (A\rightarrow B)$.

(*Modus ponens* from classical logic could have the rule *max-min* as correspondent in neutrosophic-tendential fuzzy logic).

This inference reasoning, which is essentially the basis of the neutrosophic-tendential fuzzy logic, generated the use of expression "approximate reasoning", with a nuanced meaning. Neutrosophic-tendential fuzzy logic can be considered a first extension of meanings of the incompleteness theory to date, offering the possibility of representing and reasoning with common knowledge, ordinary formulated, therefore having found applicability in many areas.

The advantage of the neutrosophic-tendential fuzzy logic was the existence of a huge number of possibilities that must be validated at first. It could use linguistic modifiers of the language to appropriate the degree of imprecision represented by a neutrosophic-tendential fuzzy set, just having the natural language as example, where people alter the degree of ambiguity of a sentence using adverbs as *incredibly, extremely, very,* etc. An adverb can modify a verb, an adjective, another adverb, or the entire sentence.

After designing and analyzing a logic system with neutrosophic-tendential fuzzy sets [6], one develops its algorithm and, finally, its program incorporating specific applications, denoted as neutrosophic-tendential fuzzy controller. Any neutrosophic-tendential fuzzy logic consists of four blocks: the fuzzyfication (transcribing by the membership functions in neutrosophic input sets), the basic rules block (which contains rules, mostly described in a conditional manner, drawn from concise numerical data in a single collection of specific judgments, expressed in linguistic terms, having neutrosophic sets associated in the process of inference or decision), the inference block (transposing by neutrosophic inferential procedures nuanced input sets into nuanced output sets), and the defuzzyfication (transposing nuanced output sets in the form of concise numbers).

The last few decades are increasingly dominated by artificial intelligence, especially by the computerized intelligence of experts and the expert-systems; alongside, the tendential-neutrosophic fuzzy logic has gradually imposed itself, being more and more commonly used in tendential-neutrosophic fuzzy control of subways and elevators systems, in tendential-neutrosophic fuzzy-controlled household appliances (washing machines, microwave ovens, air conditioning, so on), in voice commands of tendential-neutrosophic fuzzy types, like *up*, *land*, *hover*, used to drive helicopters without men onboard, in tendential-neutrosophic fuzzy cameras that maps imaging data in medical lens settings etc.

In that respect, a bibliography of theoretical and applied works related to the tendential-neutrosophic fuzzy logic, certainly counting thousands of articles and books, and increasing at a fast pace, proves the importance of the discipline.

3 Construction of Sets and Numbers of Neutrosophic Type in the Universe of Prices heading

As it can be seen from almost all fields of science and human communication, natural language is structured and prioritized through logical nuances of terms. Valorisation of linguistic nuances through neutrosophic-tendential fuzzy logic, contrary to traditional logic, after which an object may belong to a set or may not belong to a set, allow the use with a wide flexibility of the concept of belonging [7].

Neutrosophic-tendential fuzzy numbers are used in practice to represent more precisely defined approximate values. For example, creating a budget of a business focused on selling a new technology, characterized by uncertainty in relation to the number of firms that have the opportunity to purchase it for a prices ensuring a certain profit of the producer, a price situated between 50 and 100 million lei, with the highest possible range in the interval situated somewhere between 70 and 75, provides, among other things, a variant to define concretely a neutrosophic-tendential fuzzy number *Z*, using the set of pairs (offered contractual price, possibility, or real degree of membership), that may lead to a steady price: Z = [(50, 0), (60, 0.5), (70, 1), (75, 1), (85; 0.5) (100, 0)].

Given that *X* represents a universe of discourse, with a linguistic variable referring to the typical inflation or to a slight *normal-upward* shift of the price of a product, in a short period of time and in a well-defined market, specified by the elements *x*, it can be noted $\triangle p$, where $\triangle p=(p_1-p_0)/p_0$. In the following exemplification, the values of $\triangle p$ are simultaneously considered positive for the beginning and also below 1 (it is not hypothetically allowed, in a short period of time, a price increase more than double the original price, respectively the values of $\triangle p$ are situated in the interval between 0 and 1). A neutrosophic-tendential fuzzy set *A* of a universe of discourse *X* is defined or it is characterized by a function of belonging $\mu A(x)$ or $\mu A(\triangle p)$, associating to each item *x* or $\triangle p$ a degree of membership in the set *A,* as described by the equation:

$$\mu A(x): X \rightarrow [0,1] \text{ or } \mu A(\triangle p): X \rightarrow [0,1]. \quad (1)$$

To graphically represent a neutrosophic-tendential fuzzy set, we must first define the function of belonging, and thus the solution of spacial unambiguous definition is conferred by the coordinates *x* and $\mu A(x)$ or $\triangle p$ and $\mu A(\triangle p)$:

$$A = \{[x, \mu A(x)] \mid x \in [0,1]\}$$

or

$$A = \{[\triangle p, \mu A(\triangle p)] \mid \triangle p \in [0,1]\} \quad (2)$$

A finite universe of discourse $X=\{x_1,x_2,...,x_n\}$ or $X=\{\triangle p_1,\triangle p_2,...,\triangle p_n\}$ can redeem, for simplicity, a notation of type:

$$A = \{\mu_1/x_1 + \mu_2/x_2 + ... + \mu_n/x_n\},$$

respectively $A = \{\mu_1/\triangle p_1 + \mu_1/\triangle p_2 + ... + \mu_n/\triangle p_n\}$.

For example, in the situation of linguistic variable "a slight increase in price," one can detail multiple universes of discourse, be it a summary one *X* = {0, 10, 20, 100}, be it an excessive one *X* = {0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100}, the breakdowns being completed by membership functions for percentage values of variable $\triangle p$, resorting either to a reduced notation:

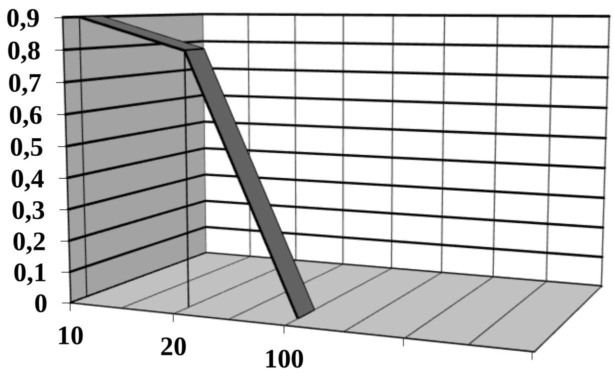
A = [0/1 + 10 / 0,9 + 20 / 0,8 + 100 / 0],

or to an extended one:

A = [0/1 + 10/0,9 + 20/0,8 + 30/0,7 + 40/0,6 + 50/0,5 + 60/0,4 + 70/0,3 + 80/0,2 + 90/0,1 + 100/0].

The meaning of this notations starts with inclusion in the slight increase of a both unchanged price, where the difference between the old price of 20 lei and the new price of a certain product is nil, thus the unchanged price belonging 100% to the set of "slight increase of price", and of a changed price of 22 lei, where $\triangle p = (p_1 - p_0) / p_0 = 0{,}1$ or 10%, therefore belonging 90% to the set of "slight increase of price", … , and, finally, even the price of 40 lei, in proportion of 0% (its degree of belonging to the analyzed set being 0).

Let us represent graphically, in a situation of a summary inflationary discourse:



Graphic 2. Neutrosophic-tendential excesively described fuzzy set.

To define a neutrosophic number, some other important concepts are required from the theory of neutrosophic set:

- the *support* of *A* or the strict subset of *X,* whose elements have nonzero degrees of belonging in *A*:
  supp(*A*) = $\{x \in X \mid \mu A(x) > 0\}$
  or supp (*A)* = $\{\triangle p \in X \mid \mu A(\triangle p) > 0\}$, (3)
- the *height* of *A* or the highest value of membership function [8]:
  h(*A*) = $\sup \mu A(x)$, where $x \in X$
  or h(*A*) = $\sup \mu A(\triangle p)$, where $\triangle p \in X$, (4)
- the *nucleus* of *A* or the *strict subset* of *X*, whose elements have unitary degrees of belonging in *A*:
  n(*A*) = $\{x \in X \mid \mu A(x) = 1\}$

or $n(A) = \{\triangle p \in X \mid \mu A (\triangle p) = 1\}$, (5)

- the *subset A of subset B* of neutrosophic-tendential fuzzy type: for *A* and *B* neutrosophic subsets of *X*, A becomes a *subset of B* if $\mu A (X) \leq \mu B (X)$, in the general case of any $x \in X$, (6)
- *neutrosophic-tendential fuzzy subsets equal* to *X* or $A = B \Leftrightarrow \mu A (X) = \mu B (X)$, if $A \subset B$ şi $B \subset A$. (7)

The first three operations with neutrosophic-tendential fuzzy set according to their importance are broadly the same as those of classical logic (reunion, intersection, complementarity etc.), being defined in the neutrosophic-tendential fuzzy logic by characteristic membership functions. If A and B are two fuzzy or nuanced neutrosophic-tendential subsets, described by their membership functions μA(x) or μB(x), one gets the following results:

a. The neutrosophic-tendential fuzzy *reunion* is defined by the membership function: $\mu A \cup B(x) = \max[\mu A(x), \mu B(x)]$;

b. The neutrosophic-tendential fuzzy *intersection* is rendered by the expression: $\mu A \cap B(x) = \min[\mu A(x), \mu B(x)]$;

c. The neutrosophic-tendential fuzzy *complementarity* is theor-etically identic with the belonging function: $\mu B(x) = 1- \mu B(x)$.

The neutrosophic-tendential fuzzy logic does not respect the classical principles of excluded middle and noncontradiction. For the topic of this article, a greater importance presents the arithmetic of neutrosophic-tendential fuzzy numbers useful in building the neutrosophic indexes and mostly the interpret indexes.

The neutrosophic-tendential fuzzy numbers, by their nuanced logic, allow a more rigorous approach of indexes in general and, especially, of interpreter indexes and price indexes, mathematically solving a relatively arbitrary linguistic approach of inflation level.

The arguments leading to the neutrosophic-type indexes solution are:

1. The inflation can be corectly defined as the rate of price growth ($\triangle p$), in relation to either the past price, when $\triangle p = (p_1-p_0) / p_0$, or an average price, and then $\triangle p = (p_1-p_m) / p_m$ (the index from which this rate will be extracted, just as inflation is extracted from IPCG as soon as it was quantified, will be a neutrosophic-type index number purely expressing a mathematical coefficient).

2. The denominator or the *reference base* of statistical index, from which the rate defining the inflation is extracted, is the most important value; the optimal choice acquires a special significance, while the numerator *reported level* of statistical index is the signal of variation or stationarity of the studied phenomenon. Similarly, in the nuanced logic of neutrosophic-tendential fuzzy numbers, the denominator value of $\triangle p$ (either p_0, or p_m) still remains essential, keeping the validity of the index paradox, as a sign of evolution or variation, to be fundamentally dependent on denominator, although apparently it seems to be signified by the nominator.
3. The prices of any economy can be represented as a universe of discourse *X*, with a linguistic variable related to typical inflation or to a slight *normal-upward* shift of a product price, in a short period of time and in a well-defined market, specified by the elements *x*, the variable being denoted by $\triangle p$, where $\triangle p = (p_1\text{-}p_0) / p_0$ or $\triangle p = (p_1\text{-}p_m) / p_m$.
4. The values of $\triangle p$ can be initially considered both positive and negative, but still smaller than 1. This is normal and in fact a price increase more than double the original price can not even be admitted in a short interval of time (usually a decade or a month), respectively the values of $\triangle p$ are initially placed in the interval between -1 and 1, so that in the end $\Sigma\triangle p/n$, where *n* represents the number of registered prices, the overwhelming majority of real cases to belong to the interval [0;1].
5. All operations generated by the specific arithmetic of constructing a neutrosophic number or a neutrosophic-type index are possible in the nuanced logic of neutrosophic numbers, finally being accepted even negative values or deflation processes (examples 1 and 2).
6. The *equations* with neutrosophic-tendential fuzzy numbers and the *functions* specified by neutrosophic-tendential fuzzy numbers offer a much better use in constructing the hedonic functions – that were the relative computing solution of price dynamics of new products replacing in the market the technologically obsolete products, a solution often challenged in contemporary statistics of inflation. Example 3 resolves more clearly the problem of products substitution due to new technologies, but placing the divergences in the plane of correctness of the functions specified by neutrosophic numbers,

regarding the measurement of price increases or of inflationary developments.

7. Some current calculation procedures capitalize the simplified notation

   $A= \{\mu_1/x_1+\mu_2/x_2+...+\mu_n/x_n\}$,

   respectively $A=\{\mu_1/\triangle p_1+\mu_1/\triangle p_2+...+\mu_n/\triangle p_n\}$.

   Even the calculation formula of IPCG of Laspeyres type constitutes a way to build an anticipation method of constructing neutrosophic-tendential fuzzy numbers. Thus IPCG = $\frac{\Sigma I^p (p_0 q_0)}{\Sigma (p_0 q_0)}$, where

   $\frac{(p_0 q_0)}{\Sigma (p_0 q_0)} = C_p$ where I^p = the index of month *t* compared to the average price and C_p = weighting coefficient, finally becomes IPCG = $\Sigma\, I^p \times C_p$, for each item or group of expenditures being required the values $\triangle p$ and C_p.

4 Index Numbers or Statistical Indexes

In Greek, *deixis* means "to indicate", which makes the indicator to be that *which indicates* (etymologically). An indicator linguistically defines the situation, the time and the subject of an assertion. The concept of linguistic indicator becomes indicial exclusively in practical terms, respectively the pragmatism turns an indicator into an index as soon as the addressee and the recipient are clarified. The indicial character is conferred by specifying the addressee, but especially the recipient, and by determining the goals that created the indicator. The indicial is somehow similar to the symptom or to the syndrome in an illness metaphor of a process, phenomenon or system, be it political, economic or social.

The symptom or the factor analysis of illness coincides with its explanatory fundamental factor, and a preventive approach of the health of a process, a phenomenon or a system obliges to the preliminary construction of indexes. The index is also a specific and graphic sign which reveals its character as iconic or reflected sign. The iconicity degree or the coverage depth in specific signs increases in figures, tables, or charts, and reaches a statistical peak with indexes. The statistical index reflects more promptly the information needed for a correct diagnosis, in relation to the flow chart and the table. The systemic approach becomes salutary. The indexes, gathered in systems, generates the systematic indicial significance, characterized by:

- in-depth approach of complex phenomena,
- temporal and spatial ongoing investigation,
- diversification of recipients,
- extending intension (of sense) and increasing the extension coverage (of described reality),
- gradual appreciation of development,
- motivating the liasons with described reality,
- ensuring practical conditions that are necessary for clustering of temporal primary indexes or globalization of regional indexes,
- diversification of addressees (sources) and recipients (beneficiaries),
- limiting restrictions of processing,
- continued expansion of the range of phenomena and processes etc.

The complexity and the promptness of the indicial overpass any other type of complexity and even promptness.

After three centuries of existing, the index method is still the method providing the best statistical information, and the advanced importance of indexes is becoming more evident in the expediency of statistical information. The assessments made by means of indexes offer qualitatively the pattern elements defining national economies, regional or community and, ultimately, international aggregates. Thinking and practice of the statistical work emphasize the relevance of factorial analysis by the method of index, embodied in the interpreter (price) indexes of inflation, in the efficient use of labor indexes etc. Because the favorite field of indexes is the economic field, they gradually became key economic indicators. The indexes are used in most comparisons, confrontations, territorial and temporal analysis – as measuring instruments. [9]

Originating etymologically in Greek *deixis,* which became in latin *index,* the index concept has multiple meanings, e.g. *index, indicator, title, list, inscription*. These meanings have maintained and even have enriched with new one, like *hint, indication, sign*. The statistical index is accepted as method, system, report or reference, size or relative indicator, average value of relative sizes or relative average change, instrument or measurement of relative change, pure number or adimensional numerical expression, simplified representation by substituting raw data, mathematical function or distinctive value of the axiomatic index theory etc.

Defined as *pure number* or adimensional numerical expression, the index is a particular form of "numerical purity", namely of independence in relation to the measurement unit of comparable size. The term "index" was first applied to dynamic data series and is expressed as a relative number. Even today, it is considered statistically an adimensional number, achieved in relation either to two values of the same simple variable corresponding to two different periods of time or space, or to two sizes of a complex indicator, whose simple sizes are heterogeneous and can not be directly added together. The first category is that of individual (particular or elementary) indexes, and the second, known as synthetic or group indexes category, which is indeed the most important. Considered as a variation scheme of a single or of multiple sizes or phenomena, the index is a simplified representation by substituting raw data by their report, aimed at rebuilding the evolution of temporal and spatial observed quantities. Whenever a variable changes its level in time or space, a statistical index is born (Henri Guitton). Approached as statistical and mathematical function, the index generated a whole axiomatic theory which defines it as an economic measure, a function F: $D \rightarrow \mathbb{R}$, which projects a set or a set D of economic interest goals (information and data) into a set or a set of real numbers $\mathbb{R}$, which satisfies a system of relevant economic conditions – for example, the properties of monotony, homogeneity or homothety or relative identity (Wolfgang Eichhorn).

Thus, the concept of "index" is shown by a general method of decomposition and factorial analysis; it is used in practice mainly as system. The index is defined either as a report or a reference which provides a characteristic number, or as synthetic relative size, either as relative indicator (numerical adimensional indicator), or as pure number, either in the condensed version as the weighted average of relative sizes or the measure of the average relative change of variables at their different time moments, different spaces or different categories, and, last but not least, as a simplified mathematical representation, by substituting the raw data by their report through a function with the same name – index function - respectively F: $D \rightarrow \mathbb{R}$, where $F(z_1, z_2, \ldots, z_k) = z_1 / z_2$, with z representing a specific variable and D the set of goals, information and data of (economic) interest, and $\mathbb{R}$ is the set of real numbers. [10; 11; 12; 13; 14]

The above mentioned properties means the following:

→ MONOTONY (A)

> An index is greater than the index of whose variables resultative vector is less than the initial index vector, all other conditions being constant:

$z_1 / z_2 > x_1 / x_2 \Rightarrow F(\underline{z}) > F(\underline{x})$

or:

$z_1 \nearrow$ → $F(\underline{z})$ is strictly increasing

$z_2 \searrow$ → $F(\underline{z})$ is strictly decreasing

$z_i = ct$ → $F(\underline{z})$ is constant, where $i = \overline{3k}$

(where z is the vector of objective economic phenomenon, and $\underline{z}$ a correspondent real number).

→ HOMOGENEITY (A)

If all variables "z" have a common factor λ, the resulting index $F(\lambda \underline{z})$ is equal to the product of the common factor λ and the calculated index, if a multiplication factor λ is absent.

- *of 1st degree* (cu referire la z_1)

$F(\lambda z_1, z_2, \ldots, z_k) = \lambda F(\underline{z})$ for any $z > 0$ and $\lambda > 0$

- *of "zero" degree*

$F(\lambda \underline{z}) = F(\underline{z})$ for any $\lambda > 0$.

→ IDENTITY (A) ("STATIONARY")

If there is no change of variables ($z_1 = z_2$), the index is uniform or stationary regardless of other conditions.

$F(1, 1, z_3, \ldots, z_k) = 1$ for any $z_3, \ldots, z_k$ (for description simplification of F, we considered $z_1 = z_2 = 1$).

→ ADDITIVITY (T)

If the variable z is expressed in terms of its original value through an algebraic sum ($z_1 = z_2 + \bar{z}$), the new index $F(z_2 + \bar{z})$ is equal to the algebraic sum of generated indexes $F(z_2) + F(\bar{z})$

$F(z_2 + \bar{z}) = F(z_2) + F(\bar{z})$.

→ MULTIPLICATION (T)

If the variable z is multiplied by the values $(\lambda_1, \ldots, \lambda_k) \in \mathbb{R}_+$, than the resulting index $F(\lambda_1 z_1, \lambda_2 z_2, \ldots, \lambda_k z_k)$ is equal to the product between the differentially multiplied variable z $(\lambda_1, \ldots, \lambda_k)$ and the initial index $F(\underline{z})$

$F(\lambda_1 z_1, \dots, \lambda_k z_k) = z(\lambda_1, \dots, \lambda_k) F(\underline{z})$,

where $\lambda_i \in \mathbb{R}_+$ and $i = \overline{1\,k}$.

→ QUASILINEARITY (T)

If $a_1, a_2, \dots, a_k$ and b are real constant, and $a_1, a_2, \dots, a_k \neq 0$, and given the continuous and strictly monotone function f: $\mathbb{R}_+ \rightarrow \mathbb{R}$, having the inverse function f^{-1}, it verifies the relation:

$F(\underline{z}) = f^{-1}[a_1 f(z_1) + a_2 f(z_2) + \dots + a_k f(z_k) + b]$.

→ DIMENSIONALITY (A)

If all variables z_1 and z_2 are multiplied by a certain factor λ, the resulting index is equal to the initial index, as the case of the multiplying by λ would not have been existed.

$F(\lambda z_1, \lambda z_2, \dots, \lambda z_k) = \frac{\lambda}{\lambda} F(\underline{z}) = F(\underline{z})$, for any $z > 0$ and $\lambda > 0$.

→ INTERIORITY (T) ("AVERAGE VALUE")

The index $F(\underline{z})$ should behave as an average value of individual indicices, being inside the interval of minimum and maximum value

$$\min\left\{\frac{z_{1i}}{z_{2i}}\right\} \leq F(\underline{z}) \leq \max\left\{\frac{z_{1i}}{z_{2i}}\right\}.$$

→ MEASURABILITY (A)

The index $F(\underline{z})$ is independent, respectively it is unaffected by the measurement units in which the variables are denominated

$$F\left(\frac{z_1}{\lambda_1}, \dots, \frac{z_K}{\lambda_K}; \lambda_1 z_1, \dots, \lambda_k z_k\right) = F(z_1, z_2, \dots, z_k) = F(\underline{z}).$$

→ PROPORTIONALITY (T)

(Homogeneity of 1st degree of a stationary initial index)

If an index is in the state of identity, respectively $F(1, 1, z_3, \dots, z_k) = 1$ for any $z_3, \dots, z_k$, the proportional increase of variable z_1 by turning it from to λ lead to a similar valure of the obtained index $F(\lambda, 1, z_3, \dots, z_k) = \lambda$ (where $\lambda \in \mathbb{R}_+$)

→ REVERSIBILITY (T) (ANTISYMMETRY AND SYMMETRY)

Considered as an axiom, the reversibility implies a double interpretation:

- the reversibility temporal or territorial approach generates an antisymmetry of Fisher type, respectively the index calculated as a report between the current period level or the compared space and the period level or reference space must be an inverse amount of the calculated index as report between the period level or reference space and the current period level or the compared space:

$$F(z_1, z_2, \ldots, z_k) \cdot \frac{1}{F(z_2, z_1, \ldots, z_k)} = 1$$

- the factorial approach generates a symmetry of Fisher type, respectively, if the phenomenon was split into qualitative and quantitative factors ($z_1 = \Sigma n_1\theta_1$ and $z_2 = \Sigma n_0\theta_0$), changing index factors does not modify the product of new indexes (symmetry of "crossed" indexes)

$$\left(\frac{\sum n_1\theta_1}{\sum n_1\theta_0} \cdot \frac{\sum n_1\theta_0}{\sum n_0\theta_0} = \frac{\sum n_1\theta_1}{\sum n_0\theta_0}\right).$$

→ CIRCULARITY (T) (TRANZITIVITY OR CONCATENATION)

The product of successive indexes represents a closed circle, respectively an index of the first level reported to the top level of the variable.

$$F(z_1, z_2, \ldots, z_k) \cdot F(z_2, z_3, \ldots, z_k) \cdot F(z_{i-1}, z_i, \ldots, z_k) = F(z_1, z_i, \ldots, z_k).$$

→ DETERMINATION (T) (CONTINUITY)

If any scalar argument in $F(z_1, z_2, \ldots, z_k)$ tends to zero, then $F(\underline{z})$ tends as well to a unique positive value of a real number (all other variable-dependent values).

→ AGGREGATION (A) (INDEX OF INDEXES)

The index of a set of variables is equal to an aggregated index when it is derived from indexes of each group sizes. Let all sizes: $z_n = F(z_1,z_2,\ldots,z_n)$ be partial indexes; the index F is aggregative if $F_n(\underline{z}) = F[F(z_1,z_2,\ldots,z_n)]$.

→ EXPANSIBILITY (A) (- specific to aggregate indexes)

$F_n(\underline{z}) < F_{n+1}(\underline{z},0)$.

→ PRESERVING THE VALUE INDEX *(Theorem)*

The aggregated index, written in the form of average index, which corresponds to a value index equal to the real value index, preserves the value index.

→ UNICITY *(Theorem)*

An index *F* is not accepted as unique index if there exists two indices $F_1 \neq F_2$ such that:

$$F(z_1, z_2, \ldots, z_k) = \begin{cases} F_1 \quad for \;\; k \in K_1, where \;\; K_1, K_2 \in N \;\; \grave{\imath} \;\; k = \overline{1K} \\ F_2 \quad for \; k \in K_2 \end{cases}$$

where *z* is the variable, *k* the variables set, K_1 and K_2 are two subsets of the set *N*, such that $K_1 \cup K_2 = N$ and $K_1 \cap K_2 = \varnothing$. This property requires the index calculation algorithm to be the same for all analyzed variables.

→ The USE OF INDEXES

This is a property resulting from data promptitude and data availability, easiness and rapidity of calculation, from simplicity of formula and of weighting system, from truthfulness of base and practical construction of indexes.

As shown, the axiomatic theory of economy is in fact a sum of properties-conditions mostly expressed by *axioms* (**A**) defining indexes, and by *theorems* and corollaries thereof derived from axioms and from tests (**T**) whose role is also important in the construction of indexes. Depending on the system of indexes they belong to, and on the specific use, the required properties are layered by Helmut Diehl in:

→ *basic requirements* – imposed by specific circumstances of the project;
→ *required properties* – ensuring fundamental qualities and operational consistency;
→ *desirable properties* – providing some technical facilities and even some theoretical elegance;
→ *special properties* – generated by construction and method.

Gathering specific characteristics in a definition as general as possible, the index is considered an indicator, *a statistical category*, expressed through a synthetic size that renders the relative variation between two states – one “actual” (or territoriality of interest), another “baseline” – of a phenomenon, or *a relative number* resulted by the comparison of a statistical indicator values, *a measure* of the relative change of variables at different time points and in different spaces, or in different categories, set in relation to a certain characteristic feature.

The evolution in time of indexes required for over three centuries solving all sort of theoretical and methodological problems regarding the method calculation, including formula, the base choice, the weighting system and, especially, the practical construction. [10; 11; 12; 13; 14]

Process optimization of this issue is not definitively over even though its history is quite eventful, as summarized in Box No. 1, below. Moreover, even this paper is only trying to propose a new type of neutrosophic index or a neutrosophic-type index number.

Box No. 1

The index – appeared, as the modern statistics, in the school of political aritmetics – has as father an Anglican Bishop, named William Fleetwood. The birth year of the first interpreter index is 1707; it was recorded by studying the evolution of prices in England between 1440 and 1707, a work known under the title "Chicon Preciosum". The value of this first index was 30/5, respectively 600,0%, and it was built on the simple arithmetical mean of eight products: wheat, oats, beans, clothing, beer, beef, sheepmeat and ham. Moreover, the world prices – a world hardly approachable because of specific amplitude, sui generis heterogeneity and apparently infinite trend – was transformed into a homogeneous population through interpreter indexes. In 1738, Dutot C. examines the declining purchasing power of the French currency between 1515 and 1735, through a broader interpreter index, using the following formula:

(1.1) *Dutot* Index: $\frac{p_1 + p_2 + \ldots + p_n}{\mathrm{P}_1 + \mathrm{P}_2 + \ldots + \mathrm{P}_n} = \frac{\sum_{i=1}^{n} p_i}{\sum_{i=1}^{n} \mathrm{P}_i}$, where: p_i and P_i = prices of current period vs. basic period.

If you multiply the numerator and denominator index by (1/n), the calculation formula of *Dutot* index becomes a mean report, respectively:

$(\sum_{i=1}^{n} p_i /\mathrm{n}):(\sum_{i=1}^{n} \mathrm{P}_i /\mathrm{n})$.

To quantify the effect of the flow of precious metals in Europe after the discovery of the Americas, the Italian historian, astronomer and economist Gian Rinaldo Carli, in 1764, used the simple arithmetic mean for three products, i.e. wheat, wine and oil, in constructing the interpreter index determined for 1500 and 1750:

(1.2) *Carli* Index: $\frac{1}{n}\left(\frac{p_1}{\mathrm{P}_1} + \frac{p_2}{\mathrm{P}_2} + \ldots + \frac{p_n}{\mathrm{P}_n}\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{p_i}{\mathrm{P}_i}$

As William Fleetwood has the merit of being the first to homogenize the heterogeneous variables through their ratio, using the results to ensure the necessary comparisons, the same way Dutot and Carli are praiseworthy for generating the "adimensionality" issue, namely the transformation of absolute values into relative values, generally incomparable or not reducible to a central (essential or typical) value (a value possessing an admissible coefficient of variation in statistical terms). But the most important improvement in index construction, streamlining its processing, belongs to Englishman Arthur Young, by introducing the *weight* (ponderation), i.e. coefficients meant to point the *relative importance* of the various items that are part of the index.

Young employed two weighting formulas, having as a starting point either *Dutot*:

(1.3) *Young* Index (1):

$$\frac{p_1k_1 + p_2k_2 + \ldots + p_nk_n}{P_1K_1 + P_2K_2 + \ldots + P_nK_n} = \frac{\sum_{i=1}^{n} p_ik_i}{\sum_{i=1}^{n} P_iK_i},$$

where k_i= coefficient of importance of product *i*,

or *Carli*:

(1.4) *Young* Index (2):

$$\frac{1}{\sum_{i=1}^{n} C_i}\left(\frac{p_1}{P_1}C_1 + \frac{p_2}{P_2} + C_2 \ldots + \frac{p_n}{Pn}C_n\right) = \frac{1}{\sum_{i=1}^{n} C_i} \times \sum_{i=1}^{n} \frac{p_i}{P_i} \times C_i = \sum_{i=1}^{n} \frac{p_i}{P_i} \times \frac{C_i}{\sum_{i=1}^{n} C_i},$$

where $\frac{C_i}{\sum_{i=1}^{n} C_i}$ = weighting coefficient and $\sum_{i=1}^{n} (c.p.)_i = 1$.

After *Young solution* from 1812, the new problem of designing indexes has become the effect of weight variations. Sir George Shuckburgh Evelyn introduced, in 1798, the concept of "basic year", thus anticipating *the dilemma of base selection and of construction of the weighting system.* In 1863, by the index calculated as geometric mean of individual indexes, Stanley Jevons extended the issue to the formula:

(1.5) *Jevons* Index:

$$\sqrt[n]{\prod_{i=1}^{n} \frac{p_i}{P_i}}$$

Jevons does not distinguish between individual indexes, giving them the same importance.

Two indexes imposed by the German school of statistics remain today, like the two terrestrial poles, structural limits of *weighting systems*. The first is the index of Etienne Laspeyres, produced in 1864, using basic period weighting, and the second is the index of Hermann Paasche, drafted in 1874, using the current period as weighting criterion:

(1.6) *Laspeyres* Index: $\frac{\sum p_{i1}q_{i0}}{\sum p_{i0}q_{i0}}$ or $\frac{\sum p_{i0}q_{i1}}{\sum p_{i0}q_{i0}}$ and

(1.7) *Paasche* Index: $\frac{\sum p_{i1}q_{i1}}{\sum p_{i0}q_{i1}}$ or $\frac{\sum p_{i1}q_{i1}}{\sum p_{i1}q_{i0}}$, where:

p_{i0}, p_{i1} = basic period prices (0) and current period prices (1)

q_{i0}, q_{i1} = basic period quantities (0) and current period quantities (1).

Although the provided indexes only checks the identity condition ($I_{1/1}^{X}$ =X_1/X_1 =1) from Fischer's tests for elementary indexes, however they are the most commonly used in practice due to the economic content of each construction. Several "theoretical" indexes were placed close to the Laspeyres and Paasche indexes, but with the loss of specific business content, and different of Ladislaus von Bortkiewicz relationship. They can be called unreservedly indexes of "*mesonic*"-type, based on authors' wishes to situate the values within the difference (P – L), to provide a solution of equilibrium between the two limit values in terms of choosing of base. Along with the two weighting systems, other issues are born, like weighting constancy and inconsistency, or connecting the bases on the extent of aging or disuse. Of the most popular "*mesonic*"-type index formulas [5], there are the constructions using common, ordinary statistics. The simple arithmetic mean of Laspeyres and Paasche indexes is known as *Sidgwik – Drobisch* index.

(1.8) *Sidgwig -Drobisch* Index: $\frac{L+P}{2}$

The arithmetic mean of the quantities of the two periods (thus becoming weight) generates the *Marshall - Edgeworth* index or *Bowley - Edgeworth* index (1885 - 1887).

(1.9) *Marshall – Edgeworth* Index: $\frac{\sum p_{i1}(q_{i0}+q_{i1})}{\sum p_{i0}(q_{i0}+q_{i1})}$

The geometric mean of quantities in the two periods converted in weights fully describes the *Walsh* index (1901).

(1.10) *Walsh* Index: $\frac{\sum p_{i1}\sqrt{(q_{i1}\times q_{i0})}}{\sum p_{i0}\sqrt{(q_{i1}\times q_{i0})}}$

The simple geometric mean of Laspeyres and Paasche indexes is none other than the well-known *Fisher* index (1922).

(1.11) *Fisher* Index: $\sqrt{(L\times P)}$

The index checks three of the four tests of its author, Irving Fisher: the identity test, the symmetry test, or the reversibility-in-time test and the completeness test, or the factors reversibility test. The only test that is not entirely satisfied is the chaining (circularity) test. The advantage obtained by the reversibility of Fisher index:

(1.12) $F_{0/1} = \sqrt{(L_{1/0}\times P_{1/0})} = \frac{1}{\sqrt{(L_{1/0}\times P_{1/0})}} = \frac{1}{F_{1/0}}$,

is unfortunately offset by the disadvantage caused by the lack of real economic content. A construction with real practical valences is that of *R.H.I. Palgrave* (1886), which proposed a calculation formula of an arithmetic average index weighted by the total value of goods for the current period ($v_{1i} = p_{1i} \cdot q_{1i}$) :

(1.13) *Palgrave* Index: $\frac{\sum i_{1/0}\times(p_{1i}q_{i1})}{\sum p_{i1}q_{i1}} = \frac{\sum i_{1/0}\times(v_{1i})}{\sum v_{1i}}$.

The series of purely theoretical or generalized indexes is unpredictable and full of originality.

Cobb - Douglas solution (1928) is a generalization of Jevons index, using unequal weights and fulfilling three of Fisher's tests (less the completeness or the reversibility of factors):

(1.14) *Cobb – Douglas* Index: $\prod_{i=1}^{n}\left(\frac{p_i}{P_i}\right)^{\alpha_i}$, where $\alpha_i > 0$ and $\sum_{i=1}^{n}\alpha_i = 1$.

Stuvel version, an index combining the Laspeyres index „of price factor" (L^p) and the Laspeyres index „of quantity factor" (L^q), proposed in 1957, exclusively satisfies the condition of identity as its source:

(1.15) *Stuvel* Index: $\frac{L^p\text{-}P^q}{2}+\sqrt{\frac{\left(L^p\text{-}P^q\right)^2}{4}+I^{(p\times q)}}$

(where $I^{(pxq)}$ = total variation index)

Another construction, inspired this time from the „experimental" design method, based on the factorial conception, but economically ineffective, lacking such a meaning, is R.S. Banerjee index (1961), a combination of indexes as well, but of Laspeyres type and Paasche type:

(1.16) *Banerjee* Index: $\frac{L+1}{\frac{1}{P}+1}=\frac{P(L+1)}{(P+1)}$

A true turning point of classical theorizing in index theory is the *autoregressive* index.

(1.17) Autoregressive Index: $\frac{\sum\left(p_i P_i a_i^2\right)}{\sum\left(P_i\right)^2\times a_i^2}$,

Therefore, a_i means the quantities of products or weights (importance) coefficients. This only verifies the provided identity, although conditionally constructed, respectively:

$$\sum\left[p_i - P_i \times I_{\text{AUTOREGRESSIVE}}\right]^2 = \text{minimum.}$$

Torngvist (1936) and *Divisia* (1925) indexes are results of generalizations of mathematical type, defining the following relationships:

(1.18) ln (Torngvist Index) = $\sum\frac{1}{2}\left[\frac{p_i q_i}{\sum p_i q_i}+\frac{P_i Q_i}{\sum P_i Q_i}\right]\times\ln\frac{p_i}{P_i}$,

where: $\frac{p_i q_i}{\sum p_i q_i}$ and $\frac{P_i Q_i}{\sum P_i Q_i}$ are weights of specific transactions values $p_i q_i$ and $P_i Q_i$.

The usual shape under which one meets the *Divisia* index is:

(1.19) $P_{0t}\ Q_{0t} = \frac{\sum p_{it} q_{it}}{\sum p_{i0} q_{i0}}$ as averaged value in a relationship determined by individual prices indexes, respectively:

$$P\left(i_{p1}+i_{p2}+\ldots+i_{pn}\right)=i_{pi}.$$

Contemporary multiplication processes of indexes calculation formulas have two trends, *one already visible of extrem axiomatization and mathematization,* based on *Torngvist* and *Divisia* indexes models, which culminated with the school of axiomatic indexes, and another, of resumption of the logic stream of economic significance of index construction, specific for the latest international constructions at the end of the twentieth century, respectively the integration variants of additive construction patterns or additive-multiplicative mixed models, close to the significance of real phenomena. In this regard, one could summary present the comparative advantage index or David Neven index (1895).

(1.20) *David Neven* Index:

$\left(\frac{x_k}{\sum x_k} - \frac{m_k}{\sum m_k}\right) \times 100$, where *x* and *m* are values of exports and imports in the industry *k*. The index belongs to the range of values (-100%; 100%), but rarely achieves in practice higher values than 10% or lower than – 10%. etc.

In the theory and practice of index numbers construction, to quantify and interpret the degree and the direction of the weights influence, use is made of Bortkiewicz relationship [15]. This specific relationship is based on factorial indexes and yields to the following equality:

$$I_{1/0}^{x(f_1)} : I_{1/0}^{x(f_0)} = 1 + r_{x_i f_i} \bullet Cv_{x_i} \bullet Cv_{f_i} \quad (1.21)$$

where:

$r_{x_i f_i}$ is a simple linear correlation coefficient between individual indexes of the qualitative factor x_i and individual indexes of the weights (respectively, individual indexes of the qualitative factor f_i),

Cv_{x_i} is the coefficient of variation of individual indexes of variable x to their environmental index,

Cv_{f_i} is the coefficient of variation of individual indexes of weights towards their environment index,

while $I_{1/0}^{x(f_1)} = \frac{\sum x_1 f_1}{\sum x_0 f_1}$ and $I_{1/0}^{x(f_0)} = \frac{\sum x_1 f_0}{\sum x_0 f_0}$.

The interpretation of that relationship shows that *the weighting system does not influence the index of a numerically expressed group variable, if the product of the three factors is null*, respectively $r_{x_i f_i} \times Cv_{x_i} \times Cv_{f_i} = 0$.

This is possible in three distinct situations:

a) $r_{x_i f_i} = 0 \Rightarrow x_i$ and f_i are independent to each other (there is no connection between individual indexes i^x and i^f),

b) $Cv_{x_i} = 0$ ⇒ the absence of any variation on the part of *x* or *f*

c) $Cv_{f_i} = 0$ (individual indexes are equal to the average index).

Product sign of factors $r_{x_i f_i} \times Cv_{x_i} \times Cv_{f_i}$ *is positive or negative depending on* $r_{x_i f_i}$*, the sign of the latter being decisive.*

The interpretation of the influence of the weighting systems on the value of a synthetic index is based on the following three cases:

- The synthetic index calculated using current period weights is equal with the same index calculated with the weights of the basic period when at least one of the factors is equal to „0”.
- The synthetic index calculated using current period weights is bigger in value than the index calculated with the weights of the basic period when the three factors are different from „0” and the simple linear correlation coefficient is positive.
- The synthetic index calculated using current period weights is lower in value than the index calculated with the weights of the basic period when the three factors are different from „0” and the simple linear correlation coefficient is negative.

Applying Bortkiewicz's relationship to the interpretation of statistical indexes offers the opportunity to check the extent and direction to which the weighting system that is employed influence the value of the indexes.

The conclusive instauration of a sign in language, be it gradually, is a lengthy process, where the sign (the representative or the signifier) replaces at a certain moment the representative (the signifier). The sign substitutes an object and can express either a quality (*qualisign*), or a current existence (*synsign*), or a general law (*legisign*). Thus, the index appears as sign together with an icon (e.g.: a chart, a graphic), a symbol (e.g.: currency), a rhema (e.g.: the mere posibility), a dicent (e.g.: a fact), an argument (e.g.: a syllogism) etc. The semiotic *index* can be defined as a sign that loses its sign once the object disappears or it is destroyed, but it does not lose this status if there is no interpreter. The index can therefore easily become its own interpreter sign. Currency as sign takes nearly all detailed semiotic forms, e.g. qualisign or hard currency, symbol of a broad range of sciences, or legisign specific to monetary and banking world. As the world's history is marked by inflation, and currency implicitly, as briefly described in Box nr. 2 below, likewise the favorite index of the inflationary phenomenon remains the interpreter index.

Box No. 2

The inflation – an evolution perceived as diminishing the value or purchasing power of the domestic currency, defined either as an imbalance between a stronger domestic price growth and an international price growth, or as a major macroeconomic imbalance of material-monetary kind and practically grasped as a general and steady increase in prices – appeared long before economics. Inflationary peak periods or “critical moments” occurred in the third century, at the beginning of sixteenth century, during the entire eighteenth and the twentieth centuries. The end of the third century is marked by inflation through currency, namely excessive uncovered currency issuance in the Roman Empire, unduely and in vain approached by the Emperor Diocletian in 301 by a “famous” edict of maximum prices which sanctioned the “crime” of price increase by death penalty. The Western Roman Empire collapsed and the reformer of the Eastern Roman Empire, Constantine the Great, imposed an imperial currency, called “solidus” or "nomisma", after 306, for almost 1000 years. The beginning of the sixteenth century, due to the great geographical discoveries, brings, together with gold and silver from the "new world", over four times price increases, creating problems throughout Europe by precious metal excess of Spain and Portugal, reducing the purchasing power of their currencies and, finally, of all European money. If the seventeenth century is a century of inflationary "princes", which were maintaining wars by issuing calp fluctuating currency, the twentieth century distinguishes itself by waves of inflation, e.g. the inflation named "Great Depression began in Black Thursday", or the economic crisis in 1930, the inflation hidden in controlled and artificial imposed prices of "The Great Planning", the inflation caused by price evolutions of oil barrel, or sometimes galloping inflation of Eastern European countries' transition to market economy. Neither the "edicts" or the "assignats" of Catherine II, as financial guarantees of currency, nor the imposed or controlled prices were perennial solutions against inflation.

Inflation is driven, par excellence, by the term "excess": excessive monetary emission or inflation through currency, excessive solvable demand or inflation by demand, excessive nominal demand, respectively by loan or loan inflation, excessive cost or cost-push inflation; but rarely by the term "insufficiency", e.g. insufficient production, or supply inflation. Measurement of overall and sustained price growth – operation initiated by Bishop William Fleetwood in 1707 by estimating at about 500% the inflation present in the English economy between 1440 and 1707 – lies on the statistical science and it materializes into multiple specific assessment tools, all bearing the name of price indexes, which originated in interpreter indexes. Modern issues impose new techniques, e.g. econophysics modeling, or modeling based on neutrosophic numbers resulted from nuanced logic.

5 Neutrosofic Index Numbers or Neutrosophic-Type Interpreter Indexes

Created in the full-of-diversity world of prices, the first index was one of *interpreter* type. The term "interpreter" must be understood here by the originary meaning of its Latin component, respectively *inter* = between (middle, implicit mediation) and *pretium* = price. [16]

The distinct national or communautaire definitions, assigned to various types of price indexes, validate, by synthesizing, the statement that the interpreter index has, as constant identical components, the following features:

- measuring tool that provides an *estimate* of price trends (consumer goods in PCI, industrial goods în IPPI or import/export, rent prices, building cost etc.);
- *alienation* of goods and services (respectively, actual charged prices and tariffs);
- *price change between a fixed period* (called basic period or reference period) and *a variable period* (called current period).

The most used interpreter indexes are the following:

- PCI – Prices of Consumer (goods and services) Index measures the overall evolution of prices for bought goods and tariffs of services, considered the main tool for assessing inflation;
- IPPI – Industrial Producers' Price Index of summarizes developments and changes in average prices of products manufactured and supplied by domestic producers, actually charged in the first stage of commercialization, used both for deflating industrial production valued at current prices, and for determining inflation within "producer prices". This index is one of the few indexes endowed with power of "premonition", a true Cassandra of instruments in the so-populated world of instruments measuring inflation. Thus, IIPP anticipates the developments of IPCG. The analysis of the last 17 years shows a parallel dynamics of evolution of the two statistical tools for assessing inflation, revealing the predictive ability of IPCG dynamics, starting from the development of IIPP;
- UVI – Unit Value Index of export / import contracts characterizes the price dynamics of export / import, expanding representative goods price changes ultimately providing for products a coverage rate of maximum 92%, allowing deflation through indicators characterizing the foreign trade, and even calculating the exchange ratio;
- CLI – Cost of Living Index shows which is the cost at market prices in the current period, in order to maintain the standard of living achieved in the basic period, being calculated as a ratio between this hypothetical cost and the actual cost (consumption) of the basic period; the need for this type of interpreter index is obvious above all in the determination of real wages and real income;
- IRP – Index of Retail Price sets the price change for all goods sold through the retail network, its importance as a tool to measure inflation within "retail prices" being easily noticed;
- BCI – Building Cost Index assesses price changes in housing construction, serving for numerous rental indexation, being used independently or within IPCG, regardless of the chosen calculation method;
- FPPI-F – food products price index measures changes in prices of food products on the *farm market* (individual or associated

farmers market), providing important information about inflation on this special market;

- GDP deflator index or *the implicit deflator of GDP* – GDP price index that is not calculated directly by measuring price changes, but as a result of the ratio between nominal GDP or in current prices and GDP expressed in comparable prices (after separately deflating the individual components of this macroeconomic indicator); GDP deflator has a larger coverage as all other price indexes.

The main elements of the construction of an interpreter index refer to official name, construction aims, official computing base, weighting coefficients, sources, structure, their coverage and limits, choosing of the weighting system, of the calculation formula, method of collection, price type and description of varieties, product quality, seasonality and specific adjustments, processing and analysis of comparable sources, presentation, representation and publication. The instrumental and applied description of consumer goods price index has as guidelines: definition, the use advantages and the use disadvantages, the scope, data sources, samples used in construction, the weighting system, the actual calculation, the inflation calculated as the rate of IPCG, specific indicators of inflation, uses of IPCG and index of purchasing power of the national currency.

As there seems natural, there is a statistical correlation and a gap between two typical constructions of price index and interpreter index, IPPI and PCI. Any chart, a chronogram or a historiogram, shows the evolution of both the prices of goods purchased, and of paid services that benefited common people (according to the consumer goods price index), and of the industrial goods prices that went out of the enterprises' gate (according to the producers' price index) and are temporarily at intermediaries, following to reach the consumers in a time period from two weeks to six months, depending on the length of "commercial channel".

The interpreter indexes are statistical tools – absolutely necessary in market economies – allowing substitution of adjectival-type characterizations of inflation within an ordinal scale. As the variable measured on an ordinal scale is equipped with a relationship of order, the following ordering becomes possible:

- the level of subnormal inflation (between 0 and 3%);
- the level of (infra)normal inflation (Friedman model with yearly inflation between 3 and 5%);
- the level of moderate inflation (between 5 and 10% yearly);
- the level of maintained inflation (between 10 and 20% yearly);

- the level of persistent inflation (between 20 and 100% yearly);
- the level of enforced inflation (between 100 and 200% yearly);
- the level of accelerated inflation (between 200 and 300% yearly);
- the level of excessive inflation (over 300% yearly).

Knowing the correct level of inflation, the dynamics and the estimates of short-term price increase allow development appreciation of value indicators in real terms. The consumer goods price index, an interpreter index that can inflate or deflate all nominal value indicators, remains a prompt measurement tool of inflation at the micro and macroeconomic level.

Any of the formulas or of the classical and modern weighting systems used in price indexes' construction can be achieved by neutrosophic-tendential fuzzy numbers following operations that can be performed in neutrosophic arithmetics. A random example [17, 18, 19] relative to historical formulas and classical computing systems (maintaining the traditional name of "Index Number") is detailed for the main indexes used to measure inflation, according to the data summarized in Table 1.

The statistical data about the price trends and the quantities of milk and cheese group are presented below for two separate periods:

Table 1.

Product	Basic price p_o	Current price p_t	Total expenses in the basic period ($p_o q_o$)	Total expenses in the basic period with current prices ($p_t q_o$)	Total expenses in current period with basic prices ($p_0 q_t$)	Total expenses in the current period ($p_t q_t$)	Weighting coefficients (Cp_0) $p_o q_o / \Sigma p_o q_o$	Quantities of products bought in the basic period (q_o)	Quantities of products bought in the current period (q_t)
Milk	1,20	1,70	12,0	17,0	10,8	15,3	15,6	10	9
Butter	1,90	1,70	15,2	13,6	13,3	11,9	19,8	8	7
Yogurt	0,85	0,90	3,4	3,6	5,1	5,4	4,4	4	6
Sour cream	1,25	3,00	1,25	3,0	1,25	3,0	1,6	1	1
Cheese	7,50	8,00	45,0	48,0	52,5	56,0	58,6	6	7
Total group	12,70	15,30	76,85	85,2	82,95	91,6	100,0	-	-

A. Historical solutions (unorthodox) focused on calculating formula for the simple aggregate and unweighted index (quantities are not taken into account, although there have been changes as a result of price developments)

$$\text{I. Index Number} = \frac{\sum p_t}{\sum po} = \frac{15,30}{12,70} = 1,205.$$

The inflation rate extracted from index = 0,205 or 20,5%.

B. Contemporary solutions focused on formula for calculating the aggregate weighted index in classical system

I. Index Number by classical Laspeyres formula =

$$\frac{\sum(ptqo)}{\sum(poqo)} = \frac{85,2}{76,85} = 1,109.$$

The inflation rate extracted from index = 0,109 or 10,9%.

II. Index Number expressed by relative prices or individual prices indexes by Laspeyres formula =

$$\frac{\sum \frac{pt}{po}(poqo)}{\sum(poqo)} = \frac{85,2}{76,85} = 1,109 \text{ or } \sum \frac{pt}{po} \times Cp_0 = 1,109.$$

The inflation rate extracted from index = 0,109 or 10,9%.

III. Index Number by Paasche formula =

$$\frac{\sum(ptqt)}{\sum(poqt)} = \frac{91,6}{82,95} = 1,104.$$

The inflation rate extracted from index = 0,104 or 10,4%.

IV. Index Number by Fisher formula =

$$\sqrt{\text{Laspeyres Index Number} \times \text{ Paasche Index Number}} =$$

$$= \sqrt{1,109 \times 1,104} = 1,106.$$

The inflation rate extracted from index = 0,106 or 10,6%.

V. Index Number by Marshall-Edgeworth formula =

$$\frac{\sum\left[pt(qo+qt)\right]}{\sum\left[po(qo+qt)\right]} = \frac{176,8}{159,8} = 1,106.$$

The inflation rate extracted from index = 0,106 or 10,6%.

VI. Index Number by Tornqvist formula =

$$\Pi\left(\frac{pt}{po}\right)^{w} \text{ where } \quad w = \frac{p_o q_o}{2\sum p_o q_o} + \frac{p_t q_t}{2\sum p_t q_t} =$$

$$= \left(\frac{1,7}{1,2}\right)^{0,1616} \times \left(\frac{1,7}{1,9}\right)^{0,1639} \times \left(\frac{0,9}{0,85}\right)^{0,0516} \times \left(\frac{3,0}{1,25}\right)^{0,0245} \times \left(\frac{8,0}{7,5}\right)^{0,5985} = 1,106.$$

The inflation rate extracted from index = 0,106 or 10,6%.

As one can see, three Index Numbers or price indexes in Fisher, Marshall-Edgeworth and Tornqvist formulas lead to the same result of inflation of 10.6%, which is placed in median position in relation to the Laspeyres and Paasche indexes.

However, the practice imposed Laspeyres index because of obtaining a high costs and a relatively greater difficulty of weighting coefficients in the current period (t). [17]

C. Neutrosophic index-based computing solutions

Starting from the definition of "a slight increase in price" variable, denoted by $\triangle p$, where $\triangle p=(p_1-p_0)/p_0$, data from Table 1 are recalculated in Table 2, below and defines the same *unorthodox* but classic solutions (especially in the last two columns).

Table 2.

Product	Basic price p_o	Current price p_t	Quantities of products bought in the basic period (q_o)	Quantities of products bought in the current period (q_t)	Total expenses in the basic period (p_oq_o)	Total expenses in the current period ($p_t q_t$)	Classic Index Number (p_t / p_0)	$\triangle p=(p_1-p_0)/p_0$	$\triangle pq=(p_1q_1-p_0q_0)/p_0q_0$
Milk	1.20	1.70	10	9	12.0	15.3	1.4167	0.4167	0.2750
Butter	1.90	1.70	8	7	15.2	11.9	0.8947	-0.1053	-0.2171
Yogurt	0.85	0.90	4	6	3.4	5.4	1.0588	0.0588	0.5882
Sour cream	1.25	3.00	1	1	1.25	3.0	2.4000	1.4000	1.4000
Cheese	7.50	8.00	6	7	45.0	56.0	1.0667	0.0667	0.2444
Total group	12.70	15.30	-	-	76.85	91.6	1.2047	0.2047	0.1919

The identical values of Fisher, Marshall-Edgeworth and Tornqvist indices offer a hypothesis similar with the neutrosophic statistics and especially with neutrosophic frequencies. The highest similarity with the idea of neutrosophic statistics consists of the Tornqvist formula's solution. The calculus of the absolute and relative values for necessary neutrosophic frequencies is described in the Table 3.

Table 3.

Product	Total expenses in the basic period (p_0q_0)	Weighting coefficients (Cp_0) = $p_0q_0/\Sigma p_0q_0$	Total expenses in the basic period with current prices (p_t q_o)	Weighting coefficients (Cp_{t0}) $p_tq_0/\Sigma p_tq_0$	Total expenses in current period with basic prices (p_0q_t)	Weighting coefficients (Cp_{0t}) = $p_0q_t/\Sigma p_0q_t$	Total expenses in the current period (p_t q_t)	Weighting coefficients (Cp_t) = $p_tq_t/\Sigma p_tq_t$
Milk	12.0	**15.62**	17.0	*20.0*	10.8	*13.0*	15.3	**16.70**
Butter	15.2	**19.78**	13.6	*16.0*	13.3	*16.0*	11.9	**12.99**
Yogurt	3.4	**4.42**	3.6	*4.2*	5.1	*6.2*	5.4	**5.90**
Sour cream	1.25	**1.63**	3.0	*3.5*	1.25	*1.5*	3.0	**3.28**
Cheese	45.0	**58.55**	48.0	*56.3*	52.5	*63.3*	56.0	**61.13**
Total group	76.85	**100.00**	85.2	*100.0*	82.95	*100.0*	91.6	**100.00**

In this situation, the construction of major modern indexes is the same as the practical application of statistical frequencies of neutrosophic type generating neutrosophic indexes in the seemingly infinite universe of prices specific to inflation phenomena, as a necessary combination between classical indexes and thinking and logic of frequencial neutrosophic statistics [20; 21; 22; 2. 3].

Table 4.

Product	Classic Index Number (p_t / p_0) (unorthodox)	Relative Neutrosophic Frequency **RNF(0)** Weighting coefficients (Cp_0) = $p_0q_0/\Sigma p_0q_0$	Relative Neutrosophic Frequency RNF(t.0) Weighting coefficients (Cp_{t0}) $p_tq_0/\Sigma p_tq_0$	Relative Neutrosophic Frequency RNF(0.t) Weighting coefficients (Cp_{0t}) = $p_0q_t/\Sigma p_0q_t$	Relative Neutrosophic Frequency **RNF(t)** Weighting coefficients (Cp_t) = $p_tq_t/\Sigma p_tq_t$	w = [$RNF_{(0)}$ + $RNF_{(t)}$] : 2
Milk	1.4167	**15.62**	*20.0*	*13.0*	**16.70**	16.16 % or 0.1616
Butter	0.8947	**19.78**	*16.0*	*16.0*	**12.99**	16.39 % or 0.1639
Yogurt	1.0588	**4.42**	*4.2*	*6.2*	**5.90**	5.16 % or 0.0516
Sour cream	2.4000	**1.63**	*3.5*	*1.5*	**3.28**	2.45 % or 0.0245
Cheese	1.0667	**58.55**	*56.3*	*63.3*	**61.13**	59.84 % or 0.5984
Total group	1.2047	**100.0**	*100.0*	*100.0*	**100.00**	100.00 or 1.0000

In this case, index of Tornqvist type is determined exploiting the relative statistical frequencies of neutrosophic type consisting of column values (Cp_0) *and* (Cp_t) *according to the new relations:*

$$\prod\left(\frac{pt}{po}\right)^{w} \text{ where } w = \frac{p_o q_o}{2\sum p_o q_o} + \frac{p_t q_t}{2\sum p_t q_t} = [RNF_{(0)} + RNF_{(t)}] : 2$$

Finally, applying the values in Table 4 shows that the result $\prod\left(\frac{pt}{po}\right)^{w}$ is identical.

$$\prod\left(\frac{pt}{po}\right)^{w} = 1.4167^{0.1616} \times 0.8947^{0.1639} \times 1.0588^{0.0516} \times 2.4^{0.0245} \times 1.0667^{0.5985} = 1.106$$

5 Conclusion

Over time, the index became potentially-neutrosophic, through the weighting systems of the classical indexes, especially after Laspeyres and Paasche. This journey into the world of indexes method merely proves that, with Tornqvist, we are witnessing the birth of neutrosophic index, resulting from applying predictive statistical neutrosophic frequencies, still theoretically not exposed by the author of this kind of thinking, actually the first author of the present article.

Future intention of the authors is to exceed, by neutrosophic indexes, the level of convergence or even emergence of unorthodox classical indexes, delineating excessive prices (high or low) by transforming into probabilities the classical interval [0 ; 1], either by the limiting values of Paasche and Laspeyres indexes, redefined as reporting base, or by detailed application of the neutrosophic thinking into statistical space of effective prices, covered by the standard interpreter index calculation (the example of PCI index is eloquent through its dual reference to time and space as determination of ten-years average index type, by arithmetic mean, and as determination of local average index type, by geometric mean of a large number of territories according to EU methodology, EUROSTAT).

6 Notes and Bibliography

[1] An information can be considered incomplete in relation to two scaled qualitative variables. The first variable is trust given to the information by the source. by the measuring instrument or by the degree of professionalism of the expert who analyze it. the final scale of uncertainty having as lower bound the completely uncertain information and as upper bound the completely definite information. The second variable is accuracy of the information content. the information benefiting of a sure content on the scale of imprecision only when the set of specified values is single-tone. i.e. holding a unique value.

[2] Holmblad L.P.. Ostergaard J.J., *Control of a cement kiln by neutrosophic logic techniques*. Proc. Conf. on Eighth IFAC. Kyoto. Japan (1981). pp. 809-814.

[3] Togai M., Watanabe H., *A VLSI implementation of a neutrosophic-inference engine: toward an expert system on a chip*. In „Information Sciences: An International Journal archive". Volume 38. Issue 2 (1986). pp. 147-163.

[4] A concise set *A* in existence domain *U* (providing the set of values allowed for a variable) can be objectively defined by:

a. listing all the items contained;

b. finding a definition conditions such specified: A= {x|x knowing certain conditions};

c. introduction of a zero-one membership function for A. denoted by μA(x) or characteristic (community. discriminatory or indicative) function {where: A≥μA(x)=1. if... and μA(x)=0 if...}, the subet A being thus equivalent to the function of belonging.

meaning that, mathematically, knowing µA(x) becomes equivalent to know A itself.

[5] When A' = A and B' = B. the rule identifies or becomes the particular case of a classical Modus Ponens. (Zadeh. L. A.. 1994. *Foreword*. in II. Marks J.F. ed., *The Neutrosophic Logic Technology and its Applications*. IEEE Publications).

[6] The logic system with tendential-neutrosophic fuzzy sets essentially transposes highly nuanced words in concise numbers; it was introduced in the vocabulary of contemporary logic by E. H. Mamdani in 1983. at San Francisco IEEE conference about tendential-neutrosophic fuzzy systems.

[7] Gâlea Dan. Leon Florin. *Teoria mulţimilor neutrosofice* [Neutrosophic Set Theory]. Biblioteca virtuală de Inteligenţă artificială. Universitatea Tehnică "Gh. Asachi" Iaşi. Facultatea de Automatică și Calculatoare. http://eureka.cs.tuiasi/. pag. 1-9

[8] The height value mathematically described by the relationship (4) classifies the neutrosophic-tendential fuzzy subsets in *normal* or *normalized*. when h (A) = 1. i.e. $\exists x \in X$ such that µ A (x) = 1 or $\exists(\Delta p) \in X$. such that µ A (△p) =1; and in *subnormal* or *subnormalized* when it appears impossible the existence of a height equal to 1. (Gâlea Dan. Leon Florin. *Teoria mulţimilor fuzzy / Fuzzy Set Theory*).

[9] Gheorghe Săvoiu. (2007). *Statistica. Un mod ştiinţific de gândire* [Statistics. A scientific way of thinking]. Bucharest (Romania): Universitară. pp. 244-258.

[10] Gheorghe Săvoiu. (2001). *Universul preţurilor şi indicii interpret* [The Universe of Prices and the Interpreter Indexes]. Piteşti (Romania): Independenţa Economică. p. 71.

[11] Ioan Moldovan. (1999). *Metoda de calcul și estimare a indicilor costului vieții* [Method of calculation and estimation of cost of living indices]. Cluj (Romania): Focus.

[12] Gheorghe Săvoiu. (2009). *Two important traditional prices' indices for the measurement of inflation and cost-of-living in Romania, during the last century*. Scientific Bulletin – Economic Sciences. Vol. 8 (14). pp. 1-8.

[13] Gheorghe Săvoiu. Ion Iorga-Siman. (2011). *The concept of time in the physical way of thinking, and its impact on knowledge and the evaluation of inflation as an economic phenomenon*. In "Econophysics. Sociophysics & Other Multidisciplinary Sciences Journal" (ESMSJ). Vol. 1 (2). pp. 25-35.

[14] Gheorghe Săvoiu. (2013). *A Statistical Applied Method. Drawing on the Consumer Price Index and its Investigative Qualities*. Romanian

Statistical Review Supplement. Romanian Statistical Review. vol. 61(2)/ 2013. pp. 116-122.

[15] Ladislauss von Bortkiewicz (1868-1931), statistician, mathematician, economist and demographer of German origin, professor at Petersburg and Berlin, with an essential contribution to the index theory by the rapport built to assess the influence share in an aggregate index.

[16] Săvoiu G.. (2003). Statistică generală. Argumente în favoarea formării gândirii statistice [General Statistics. Arguments in favor of the Statistical Thinking Training]. Piteşti (Romania): Independenţa Economică.

[17] Statistics New Zealand (1995). Consumers Price Index Revision 1993. New Zealand.

[18] OECD (1994). Sources and Methods Consumer Price Indexes. France.

[19] International Labour Office (1987). Sources and Methods Labour Statistics. Vol. 1: Consumer Price Indexes. Geneva.

[20] L. A. Zadeh. Fuzzy Sets. In „Inform. and Control”. 8 (1965). pp. 338-353.

[21] K. T. Atanassov. Intuitionistic Fuzzy Set. In „Fuzzy Sets and Systems”. 20(1) (1986). pp. 87-96.

[22] Florentin Smarandache (1998). *Neutrosophy. Neutrosophic Probability, Set, and Logic*. Amer. Res. Press. Rehoboth. USA. 105 pages.

[23] Florentin Smarandache. *n-Valued Refined Neutrosophic Logic and Its Applications in Physics*. In „Progress in Physics”. Vol. 4 (2013). pp. 143-146.