On Strong Interval Valued Neutrosophic Graphs

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Abstract

In this paper, we discuss a subclass of interval valued neutrosophic graphs called strong interval valued neutrosophic graphs, which were introduced by Broumi et al. [41]. The operations of Cartesian product, composition, union and join of two strong interval valued neutrosophic graphs are defined. Some propositions involving strong interval valued neutrosophic graphs are stated and proved.

Keyword

Single valued neutrosophic graph, Interval valued neutrosophic graph, Strong interval valued neutrosophic graph, Cartesian product, Composition, Union, Join.

1 Introduction

Neutrosophic set proposed by Smarandache [13, 14] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [30], intuitionistic fuzzy sets [27, 29], interval-valued fuzzy sets [22] and interval-valued intuitionistic fuzzy sets [28]. The neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], the neutrosophic set is easily applied to engineering problems. For this purpose, Smarandache [48] and Wang et al. [17] introduced the concept of a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. The same authors introduced the notion of interval valued neutrosophic sets [18] as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Recently, the concept of single valued neutrosophic set and interval valued neutrosophic sets have been applied in a wide variety of fields including computer science, enginnering, mathematics, medicine and economics [3, 4, 5, 6, 16, 19, 20, 21, 23, 24, 25, 26, 32, 34, 35, 36, 37, 38, 43].

Lots of works on fuzzy graphs and intuitionistic fuzzy graphs [7, 8, 9, 31, 33] have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs fail.

For this purpose, Smarandache [10, 11] defined four main categories of neutrosophic graphs. Two are based on literal indeterminacy (I), called I-edge neutrosophic graph and I-vertex neutrosophic graph; these concepts are studied deeply and has gained popularity among the researchers due to their applications via real world problems [1, 12, 15, 44, 45, 46]. The two others graphs are based on (t, i, f) components and are called (t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph; these concepts are not developed at all. Later on, Broumi et al. [40] introduced a third neutrosophic graph model combining the (t, i, f)-edge and and the (t, i, f)-vertex neutrosophic graph, and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG for short).

The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. The same authors [39] introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. Broumi et al. [41] introduced the concept of interval valued neutrosophic graph, which is a generalization of fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph, interval valued intuitionistic fuzzy graph and single valued neutrosophic graph. Also, Broumi et al. [42] studied some operations on interval valued neutrosophic graphs.

In this paper, motivated by the operations on (crisp) graphs, such as Cartesian product, composition, union and join, we define the operations of Cartesian product, composition, union and join on strong interval valued neutrosophic graphs and investigate some of their properties.

2 Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, fuzzy graph and intuitionistic fuzzy graph, interval valued intuitionstic fuzzy graph and interval valued neutrosophic graph, relevant to the present work.

See especially [2, 7, 8, 13, 17, 40, 41] for further details and background.

Definition 2.1 [13]

Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]−0,1+[define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^{-}0 \leq T_A(x)+ I_A(x)+ F_A(x)\leq 3^{+}. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]−0,1+[.

Since it is difficult to apply NSs to practical problems, Smarandache [48] and Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [17]

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$.

For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1].

A SVNS A can be written as —

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}. \quad (2)$$

Definition 2.3 [7]

A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. i.e σ : V → [0,1] and μ: VxV→[0,1] such that μ(uv) ≤ σ(u) ⋀ σ(v) for all u, v ∈ V where *uv* denotes the edge between *u* and *v* and σ(u) ⋀ σ(v) denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

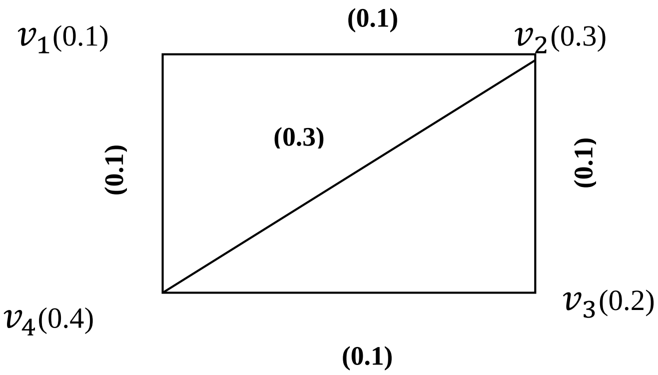

Figure 1. Fuzzy Graph

Definition 2.4 [7]

The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ), if $\tau(u) \leq \sigma(u)$ for all u ∈ V and $\rho(u, v) \leq \mu(u, v)$ for all u, v ∈ V.

Definition 2.5 [8]

An intuitionistic fuzzy graph is of the form G = (V, E), where

i. $V=\{v_1, v_2, \ldots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i)) \leq 1$ for every $v_i \in V$, (i = 1, 2, ..., n),

ii. $E \subseteq V \times V$ where $\mu_2: VxV \rightarrow [0,1]$ and $\gamma_2: VxV \rightarrow [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, (i, j = 1,2, ..., n)

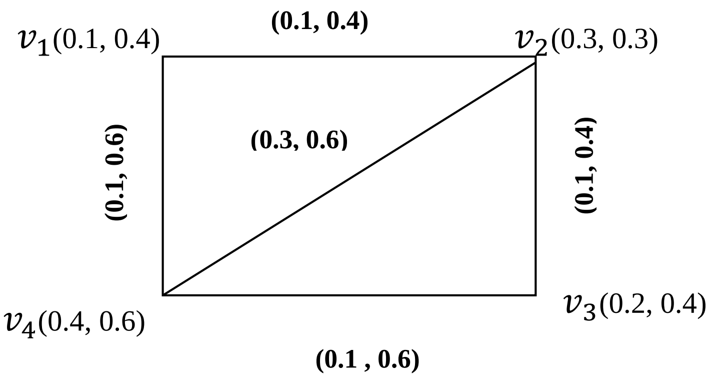

Figure 2. Intuitionistic Fuzzy Graph

Definition 2.6 [40]

Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then A =(T_A, I_A, F_A) is called a single valued neutrosophic relation on B = (T_B, I_B, F_B) if

$T_B(x, y) \leq \min(T_A(x), T_A(y))$

$I_B(x, y) \geq \max(I_A(x), I_A(y))$ and

$F_B(x, y) \geq \max(F_A x), F_A(y))$

for all x, y ∈ X.
A single valued neutrosophic relation A on X is called symmetric if $T_A(x, y) = T_A(y, x)$, $I_A(x, y) = I_A(y, x)$, $F_A(x, y) = F_A(y, x)$ and $T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ and $F_B(x, y) = F_B(y, x)$, for all x, y ∈ X.

Definition 2.7 [2]

An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G = (A, B), where

1) The functions M_A : V→ D [0, 1] and N_A : V→ D [0, 1] denote the degree of membership and non membership of the element x ∈ V, respectively, such that 0 such that $0 \leq M_A(x) + N_A(x) \leq 1$ for all x ∈ V.

2) The functions $M_B : E \subseteq V \times V \to D\,[0, 1]$ and $N_B : E \subseteq V \times V \to D\,[0, 1]$ are defined by

$M_{BL}(x, y)) \leq \min\ (M_{AL}(x), M_{AL}(y))$ and $N_{BL}(x, y)) \geq \max\ (N_{AL}(x), N_{AL}(y))$,

$M_{BU}(x, y)) \leq \min\ (M_{AU}(x), M_{AU}(y))$ and $N_{BU}(x, y)) \geq \max\ (N_{AU}(x), N_{AU}(y))$,

such that

$0 \leq M_{BU}(x, y)) + N_{BU}(x, y)) \leq 1$,

for all (x, y) ∈ E.

Definition 2.8 [41]

By an interval-valued neutrosophic graph of a graph G^* = (V, E) we mean a pair G = (A, B), where A = <[T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]> is an interval-valued neutrosophic set on V and B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]> is an interval-valued neutrosophic relation on E satisfies the following conditions:

1. V= $\{v_1, v_2, \ldots, v_n\}$ such that T_{AL}:V → [0, 1], T_{AU}:V → [0, 1], I_{AL}:V → [0, 1], I_{AU}:V→[0, 1] and F_{AL}:V→[0, 1], F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in$ V, respectively, and

$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in$ V (I = 1, 2, …, n).

2. The functions T_{BL}:V x V →[0, 1], T_{BU}:V x V →[0, 1], I_{BL}:V x V →[0, 1], I_{BU}:V x V →[0, 1] and F_{BL}:V x V →[0,1], F_{BU}:V x V →[0, 1] are such that

$$T_{BL}(\{v_i, v_j\}) \leq \min\,[T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(\{v_i, v_j\}) \leq \min\,[T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(\{v_i, v_j\}) \geq \max[I_{BL}(v_i), I_{BL}(v_j)],$$

$$I_{BU}(\{v_i, v_j\}) \geq \max[I_{BU}(v_i), I_{BU}(v_j)],$$

$$\mathrm{F_{BL}}(\{\mathrm{v_i}, \mathrm{v_j}\}) \geq \max[\mathrm{F_{BL}}(\mathrm{v_i}), \mathrm{F_{BL}}(\mathrm{v_j})],$$

$$\mathrm{F_{BU}}(\{\mathrm{v_i}, \mathrm{v_j}\}) \geq \max[\mathrm{F_{BU}}(\mathrm{v_i}), \mathrm{F_{BU}}(\mathrm{v_j})],$$

denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$$0\leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\})+ F_B(\{v_i, v_j\}) \leq 3,$$

for all $\{v_i, v_j\} \in$ E (i, j = 1, 2,..., n).

We call A the interval valued neutrosophic vertex set of V, B the interval valued neutrosophic edge set of E, respectively. Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, G = (A, B) is a interval valued neutrosophic graph of G*= (V, E) if —

$$T_{BL}(v_i, v_j) \leq \min\,[T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(v_i, v_j) \leq \min\,[T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(v_i, v_j) \geq \max[I_{BL}(v_i), I_{BL}(v_j)],$$

$$I_{BU}(v_i, v_j) \geq \max[I_{BU}(v_i), I_{BU}(v_j)],$$

$$F_{BL}(v_i, v_j) \geq \max[F_{BL}(v_i), F_{BL}(v_j)],$$

$$F_{BU}(v_i, v_j) \geq \max[F_{BU}(v_i), F_{BU}(v_j)], \quad \text{for all } (v_i, v_j) \in \text{E}.$$

Hereafter, we use the notation *xy* for (x, y) an element of E.

3 Strong Interval Valued Neutrosophic Graph

Throught this paper, we denote G^* = (V, E) a crisp graph, and G = (A, B) an interval valued neutrosophic graph.

Definition 3.1

An interval valued neutrosophic graph G = (A, B) is called strong interval valued neutrosophic graph if

$$T_{BL}(xy)= \min\,(T_{AL}(x), T_{AL}(y)),\ I_{BL}(xy) = \max\,(I_{AL}(x), I_{AL}(y)) \text{ and } F_{BL}(xy) = \max\,(F_{AL}(x), F_{AL}(y))$$

$T_{BU}(xy)$= min $(T_{AU}(x), T_{AU}(y))$, $I_{BU}(xy)$ = max $(I_{AU}(x), I_{AU}(y))$ and $F_{BU}(xy)$ =max $(F_{AU}(x), F_{AU}(y))$ such that

0≤$T_{BU}(x,y)$)+ $I_{BU}(x,y)$)+ $F_{BU}(x,y)$) ≤ 3, for all x, y ∈ E.

Example 3.2

Figure 1 is an example for IVNG, G=(A, B) defined on a graph G^* = (V, E) such that V = {x, y, z}, E = {xy, yz, zx}, A is an interval valued neutrosophic set of V.

A={<x, [0.5, 0.7], [0.2, 0.3], [0.1, 0.3]>,<y, [0.6, 0.7], [0.2, 0.4], [0.1, 0.3]>, <z, [0.4, 0.6], [0.1, 0.3], [0.2, 0.4],>},

B={< xy, [0.3, 0.6], [0.2, 0.4], [0.2, 0.4]>, <yz, [0.3, 0.5], [0.2, 0.3], [0.2, 0.4]>, <xz, [0.3, 0.5], [0.1, 0.5], [0.2, 0.4]>}.

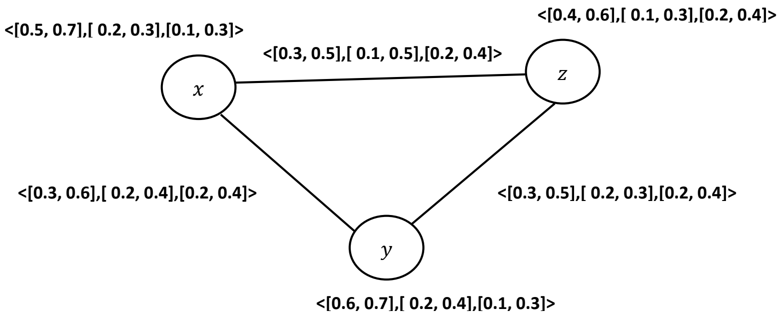

Figure 3. Interval valued neutrosophic graph

Example 3.2

Figure 2 is a SIVNG G = (A, B), where

A={<x, [0.5, 0.7], [0.1, 0.4], [0.1, 0.3]>, <y, [0.6, 0.7],[0.2, 0.3], [0.1, 0.3]>, <z, [0.4, 0.6],[0.2, 0.3], [0.2, 0.4],>},

B={< xy, [0,5, 0.7], [0.2 0.4], [0.1, 0.3]>, <yz, [0.4, 0.6],[0.2, 0.3], [0.2, 0.4]>, <xz, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}

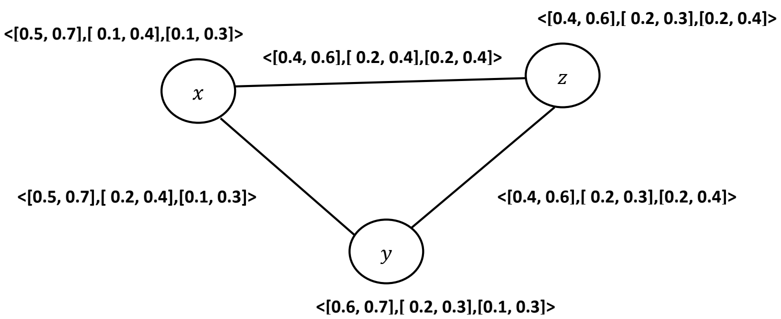

Figure 4. Strong Interval valued neutrosophic graph.

Proposition 3.3

A strong interval valued neutrosophic graph is the generalization of strong interval valued fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then, by setting the indeterminacy-membership and falsity-membership values of vertex set and edge set equals to zero, the strong interval valued neutrosophic graph is reduced to strong interval valued fuzzy graph.

Proposition 3.4

A strong interval valued neutrosophic graph is the generalization of strong interval valued intuitionistic fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then by setting the indeterminacy-membership values of vertex set and edge set equals to zero reduces the strong interval valued neutrosophic graph to strong interval valued intuitionistic fuzzy graph.

Proposition 3.5

A strong interval valued neutrosophic graph is the generalization of strong intuitionistic fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then by setting the indeterminacy-membership, upper truth-membership and upper falsity-membership values of vertex set and edge set equals to zero reduces the strong interval valued neutrosophic graph to strong intuitionistic fuzzy graph.

Proposition 3.6

A strong interval valued neutrosophic graph is the generalization of strong single neutrosophic graph.

Proof

Suppose G = (V, E) is a strong interval valued neutrosophic graph. Then by setting the upper truth-membership equals lower truth-membership, upper indeterminacy-membership equals lower indeterminacy-membership and upper falsity-membership equals lower falsity-membership values of vertex set and edge set reduces the strong interval valued neutrosophic graph to strong single valued neutrosophic graph.

Definition 3.7

Let A_1 and A_2 be interval-valued neutrosophic subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued neutrosophic subsets of E_1 and E_2 respectively. The Cartesian product of two SIVNGs G_1 and G_2 is denoted by $G_1 \times G_2 = (A_1 \times A_2$, $B_1 \times B_2$) and is defined as follows:

1) $(T_{A_1L} \times T_{A_2L})\ (x_1, x_2) = \min\ (T_{A_1L}(x_1), T_{A_2L}(x_2))$
$(T_{A_1U} \times T_{A_2U})\ (x_1, x_2) = \min\ (T_{A_1U}(x_1), T_{A_2U}(x_2))$
$(I_{A_1L} \times I_{A_2L})\ (x_1, x_2) = \max\ (I_{A_1L}(x_1), I_{A_2L}(x_2))$
$(I_{A_1U} \times I_{A_2U})\ (x_1, x_2) = \max\ (I_{A_1U}(x_1), I_{A_2U}(x_2))$
$(F_{A_1L} \times F_{A_2L})\ (x_1, x_2) = \max\ (F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \times F_{A_2U})\ (x_1, x_2) = \max\ (F_{A_1U}(x_1), F_{A_2U}(x_2))$ for all $(x_1, x_2) \in V$

2) $(T_{B_1L} \times T_{B_2L})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1L}(x), T_{B_2L}(x_2y_2))$
$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \times I_{B_2L})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \times I_{B_2U})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \times F_{B_2L})\ ((x, x_2)\ (x, y_2)) = \max\ (F_{A_1L}(x), F_{B_2L}(x_2y_2))$
$(F_{B_1U} \times F_{B_2U})\ ((x, x_2)(x, y_2)) = \max(F_{A_1U}(x), F_{B_2U}(x_2y_2))\ \forall\ x \in V_1$ and $\forall\ x_2y_2 \in E_2$

3) $(T_{B_1L} \times T_{B_2L})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1L}(x_1y_1), T_{A_2L}(z))$
$(T_{B_1U} \times T_{B_2U})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1U}(x_1y_1), T_{A_2U}(z))$
$(I_{B_1L} \times I_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (I_{B_1L}(x_1y_1), I_{A_2L}(z))$
$(I_{B_1U} \times I_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (I_{B_1U}(x_1y_1), I_{A_2U}(z))$
$(F_{B_1L} \times F_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (F_{B_1L}(x_1y_1), F_{A_2L}(z))$
$(F_{B_1U} \times F_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (F_{B_1U}(x_1y_1), F_{A_2U}(z))\ \forall\ z \in V_2$ and $\forall\ x_1y_1 \in E_1$

Proposition 3.7

If G_1 and G_2 are the strong interval valued neutrosophic graphs, then the cartesian product G_1x G_2 is a strong interval valued neutrosophic graph.

Proof

Let G_1 and G_2 are SIVNGs, there exist $x_i, y_i \in E_i$, i= 1, 2 such that

$$T_{B_iL}(x_i, y_i) = \min\ (T_{A_iL}(x_i), T_{A_iL}(y_i)),\ i = 1, 2.$$

$T_{B_iU}(x_i, y_i)$ = min $(T_{A_iU}(x_i), T_{A_iU}(y_i))$, i =1, 2.
$I_{B_iL}(x_i, y_i)$ = max $(I_{A_iL}(x_i), I_{A_iL}(y_i))$, i =1, 2.
$I_{B_iU}(x_i, y_i)$ = max $(I_{A_iU}(x_i), I_{A_iU}(y_i))$, i =1, 2.
$F_{B_iL}(x_i, y_i)$ = max $(F_{A_iL}(x_i), F_{A_iL}(y_i))$, i =1, 2.
$F_{B_iU}(x_i, y_i)$ = max $(F_{A_iU}(x_i), F_{A_iU}(y_i))$, i =1, 2.

Let E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)\ (y_1, z)\ /z \in V_2, x_1y_1 \in E_1\}$.

Consider, $(x, x_2)\ (x, y_2) \in E$, we have

$(T_{B_1L} \times T_{B_2L})\ ((x, x_2)\ (x, y_2))$ = min $(T_{A_1L}(x), T_{B_2L}(x_2y_2))$

= min $(T_{A_1L}(x), T_{A_2L}(x_2), T_{A_2L}(y_2))$

Similarly,

$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)\ (x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_2U}(y_2))$

$(T_{A_1L} \times T_{A_2L})\ (x_1, x_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \times T_{A_2U})\ (x_1, x_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(T_{A_1L} \times T_{A_2L})\ (x_1, y_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(y_2))$

$(T_{A_1U} \times T_{A_2U})\ (x_1, y_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(y_2))$

Min $((T_{A_1U} \times T_{A_2U})\ (x, x_2), (T_{A_1U} \times T_{A_2U})\ (x, y_2))$

= min (min $(T_{A_1U}(x), T_{A_2U}(x_2))$, min $(T_{A_1U}(x), T_{A_2U}(y_2))$)

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_1U}(y_2))$

Hence

$(T_{B_1L} \times T_{B_2L})\ ((x, x_2)\ (x, y_2))$ = min $((T_{A_1L} \times T_{A_2L})\ (x, x_2), (T_{A_1L} \times T_{A_2L})\ (x, y_2))$

$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)\ (x, y_2))$ = min $((T_{A_1U} \times T_{A_2U})\ (x, x_2), (T_{A_1U} \times T_{A_2U})\ (x ,y_2))$.

Similarly, we can show that —

$(I_{B_1L} \times I_{B_2L})\ ((x, x_2)\ (x, y_2))$ = max $((I_{A_1L} \times I_{A_2L})\ (x, x_2), (I_{A_1L} \times I_{A_2L})\ (x, y_2))$

$(I_{B_1U} \times I_{B_2U})\ ((x, x_2)\ (x, y_2))$ = max $((I_{A_1U} \times I_{A_2U})\ (x, x_2), (I_{A_1U} \times I_{A_2U})\ (x, y_2))$.

And also

$(F_{B_1L} \times F_{B_2L})$ $((x, x_2)$ $(x, y_2))$ = max $((F_{A_1L} \times F_{A_2L})$ (x, x_2), $(F_{A_1L} \times F_{A_2L})$ $(x, y_2))$

$(F_{B_1U} \times F_{B_2U})$ $((x, x_2)$ $(x, y_2))$ = max $((F_{A_1U} \times F_{A_2U})$ (x, x_2), $(F_{A_1U} \times F_{A_2U})$ $(x, y_2))$.

Hence, G_1x G_2 strong interval valued neutrosophic graph. This completes the proof.

Proposition 3.8

If G_1 x G_2 is strong interval valued neutrosophic graph, then at least G_1 or G_2 must be strong.

Proof

Let G_1 and G_2 be no strong interval valued neutrosophic graphs; there exists $x_i, y_i \in E_i$, I = 1, 2, such that

$T_{B_iL}(x_i, y_i) <$ min $(T_{A_iL}(x_i), T_{A_iL}(y_i))$, i =1, 2.
$T_{B_iU}(x_i, y_i) <$ min $(T_{A_iU}(x_i), T_{A_iU}(y_i))$, i =1, 2.
$I_{B_iL}(x_i, y_i) >$ max $(I_{A_iL}(x_i), I_{A_iL}(y_i))$, i =1, 2.
$I_{B_iU}(x_i, y_i) >$ max $(I_{A_iU}(x_i), I_{A_iU}(y_i))$, i =1, 2.
$F_{B_iL}(x_i, y_i) >$ max $(F_{A_iL}(x_i), F_{A_iL}(y_i))$, i =1, 2.
F_{B_iU} $(x_i, y_i) >$ max $(F_{A_iU}(x_i), F_{A_iU}(y_i))$, i =1, 2.

Let E= $\{(x, x_2)$ (x, y_2) $/x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)$ (y_1, z) $/z \in V_2, x_1y_1 \in E_1\}$

Consider, (x, x_2) $(x, y_2) \in E$, we have

$(T_{B_1L} \times T_{B_2L})$ $((x, x_2)$ $(x, y_2))$ = min $(T_{A_1L}(x),$ $T_{B_2L}(x_2y_2))$

$<$ min $(T_{A_1L}(x), T_{A_2L}(x_2), T_{A_2L}(y_2))$.

Similarly —

$(T_{B_1U} \times T_{B_2U})$ $((x, x_2)$ $(x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$

$<$ min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_2U}(y_2))$

$(T_{A_1L} \times T_{A_2L})$ (x_1, x_2) = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \times T_{A_2U})$ (x_1, x_2) = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(T_{A_1L} \times T_{A_2L})$ (x_1, y_2) = min $(T_{A_1L}(x_1), T_{A_2L}(y_2))$

$(T_{A_1U} \times T_{A_2U})$ (x_1, y_2) = min $(T_{A_1U}(x_1), T_{A_2U}(y_2))$

min $((T_{A_1U} \times T_{A_2U})$ (x, x_2), $(T_{A_1U} \times T_{A_2U})$ $(x, y_2))$

= min (min $(T_{A_1U}(x), T_{A_2U}(x_2))$, min $(T_{A_1U}(x), T_{A_2U}(y_2)))$

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_1U}(y_2))$.

Hence

$(T_{B_1L} \times T_{B_2L})$ $((x, x_2)$ $(x, y_2)) <$ min $((T_{A_1L} \times T_{A_2L})$ (x, x_2), $(T_{A_1L} \times T_{A_2L})$ $(x, y_2))$,

$(T_{B_1U} \times T_{B_2U})$ $((x, x_2)$ $(x, y_2)) <$ min $((T_{A_1U} \times T_{A_2U})$ (x, x_2), $(T_{A_1U} \times T_{A_2U})$ $(x ,y_2))$.

Similarly, we can show that

$(I_{B_1L} \times I_{B_2L})$ $((x, x_2)$ $(x, y_2)) >$ max $((I_{A_1L} \times I_{A_2L})$ (x, x_2), $(I_{A_1L} \times I_{A_2L})$ $(x, y_2))$,

$(I_{B_1U} \times I_{B_2U})$ $((x, x_2)$ $(x, y_2)) >$ max $((I_{A_1U} \times I_{A_2U})$ (x, x_2), $(I_{A_1U} \times I_{A_2U})$ $(x, y_2))$.

And also

$(F_{B_1L} \times F_{B_2L})$ $((x, x_2)$ $(x, y_2)) >$ max $((F_{A_1L} \times F_{A_2L})$ (x, x_2), $(F_{A_1L} \times F_{A_2L})$ $(x, y_2))$,

$(F_{B_1U} \times F_{B_2U})$ $((x, x_2)$ $(x, y_2)) >$ max $((F_{A_1U} \times F_{A_2U})$ (x, x_2), $(F_{A_1U} \times F_{A_2U})$ $(x, y_2))$.

Hence, G_1 x G_2 is not strong interval valued neutrosophic graph, which is a contradiction. This completes the proof.

Remark 3.9

If G_1 is a SIVNG and G_2 is not a SIVNG, then G_1 x G_2 is need not be an SIVNG.

Example 3.10

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]> ,< b, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]>} and B_1= {< ab, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]>}

G_1

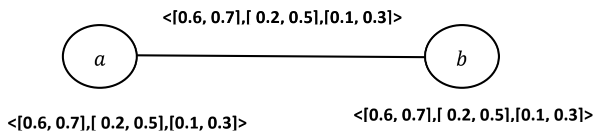

Figure 5. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>, < d, [0.4, 0.6], [0.1,0.3], [0.2, 0.4] >} and B_2= < cd, [0.3,0.5], [0.1, 0.2], [0.3, 0.5]>}.

G_2

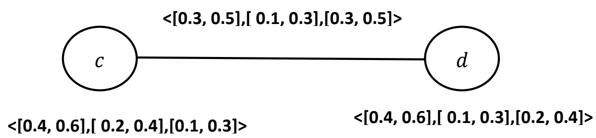

Figure 6. Interval valued neutrosophic G_2.

G_1x G_2= (A_1x A_2, B_1x B_2) is not a SIVNG, where

A_1x A_2= {< (a, c), [0.4, 0.6],[0.2, 0.3], [0.2, 0.4]>, < (a, d), [0.4, 0.6], [0.2, 0.3], [0.2, 0.4]>, < (b, c), [0.4, 0.6], [0.2, 0.6], [0.2, 0.4]>, < (b, d), [0.4, 0.6], [0.3, 0.4], [0.2, 0.4]>},

B_1x B_2= {< ((a, c), (a, d)), [0.3, 0.5], [0.3, 0.5], [0.3, 0.5] >, < ((a, c), (b, c)), [0.4, 0.6], [0.1, 0.4], [0.3, 0.4] >, < ((b, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, d), (b, d)),[0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}. In this example, G_1 is a SIVNG and $G_2$is not a SIVNG, then G_1x $G_2$is not a SIVNG.

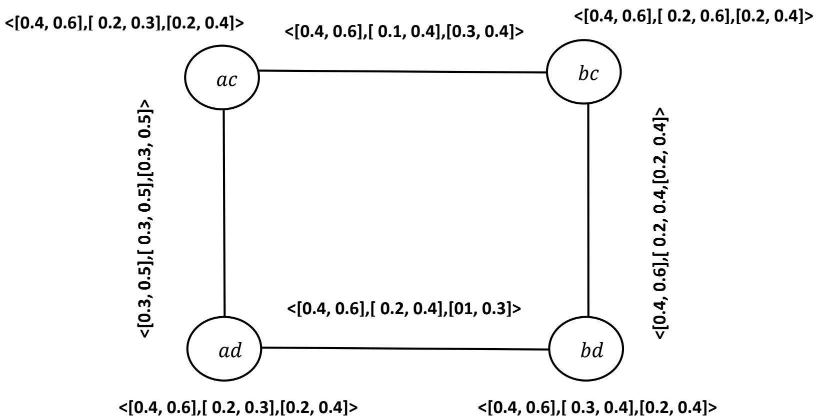

Figure 7. Cartesian product G_1x G_2

Example 3.11

Let G_1= (A_1, B_1) be a SIVNG, where A_1= {< a, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < b, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>} and B_1={< ab, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >,

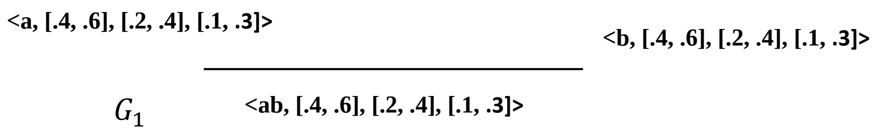

Figure 8. Interval valued neutrosophic G_1.

G_2= (A_2, B_2) is not a SIVIFG, where A_2= {< c, [0.6, 0.7], [0.1, 0.3], [0.1, 0.3] >, < d, [0.6, 0.7], [0.1, 0.3], [0.2, 0.4] >} and B_2= {< cd, [0.5, 0.6], [0.2, 0.4], [0.2, 0.4]>},

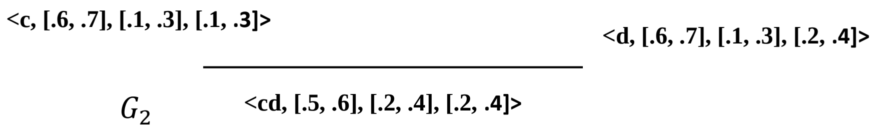

Figure 9. Interval valued neutrosophic G_2.

G_1x G_2= (A_1x A_2, B_1x B_2) is a SIVNG, where

A_1x A_2= {< (a, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (a, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >} and

B_1x B_2= {< ((a, c), (a, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((b, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}. In this example, $G_1$is a SIVNG and $G_2$is not a SIVNG, then G_1x $G_2$is a SIVNG.

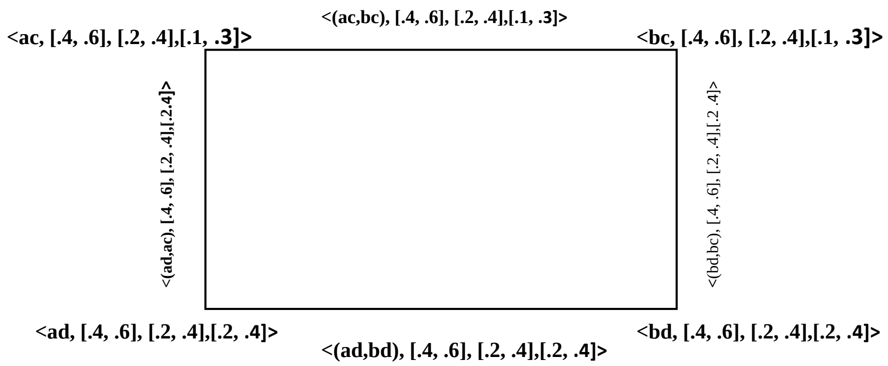

Figure 10. Cartesian product

Proposition 3.12

Let G_1be a strong interval valued neutrosophic graph. Then for any interval valued neutrosophic graph G_2,G_1x G_2 is strong interval valued neutrosophic graph iff

$T_{A_1L}(x_1) \leq T_{B_1L}(x_2y_2)$, $I_{A_1L}(x_1) \geq I_{B_1L}(x_2y_2)$ and $F_{A_1L}(x_1) \geq F_{B_1L}(x_2y_2)$,

$T_{A_1U}(x_1) \leq T_{B_1U}(x_2y_2)$, $I_{A_1U}(x_1) \geq I_{B_1U}(x_2y_2)$ and $F_{A_1U}(x_1) \geq F_{B_1U}(x_2y_2)$, $\forall\ x_1 \in V_1$,$x_2y_2 \in E_2$.

Definition 3.13

Let $A_1$and A_2be interval valued neutrosophic subsets of $V_1$and V_2respectively. Let $B_1$and B_2 interval-valued neutrosophic subsets of E_1 and E_2 respectively. The *composition* of two strong interval valued neutrosophic graphs G_1 and $G_2$is denoted by G_1[G_2] = ($A_1 \circ A_2$, $B_1 \circ B_2$) and is defined as follows

1) $(T_{A_1L} \circ T_{A_2L})\ (x_1, x_2) = \min\ (T_{A_1L}(x_1), T_{A_2L}(x_2))$
   $(T_{A_1U} \circ T_{A_2U})\ (x_1, x_2) = \min\ (T_{A_1U}(x_1), T_{A_2U}(x_2))$
   $(I_{A_1L} \circ I_{A_2L})\ (x_1, x_2) = \max\ (I_{A_1L}(x_1), I_{A_2L}(x_2))$
   $(I_{A_1U} \circ I_{A_2U})\ (x_1, x_2) = \max\ (I_{A_1U}(x_1), I_{A_2U}(x_2))$

$(F_{A_1L} \circ F_{A_2L})\ (x_1, x_2) = \max\ (F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \circ F_{A_2U})\ (x_1, x_2) = \max\ (F_{A_1U}(x_1), F_{A_2U}(x_2))\ \ \forall\ x_1 \in V_1, x_2 \in V_2$

2) $(T_{B_1L} \circ T_{B_2L})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1L}(x), T_{B_2L}(x_2y_2))$
$(T_{B_1U} \circ T_{B_2U})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \circ I_{B_2L})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \circ I_{B_2U})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \circ F_{B_2L})\ ((x, x_2)\ (x, y_2)) = \max\ (F_{A_1L}(x), F_{B_2L}(x_2y_2))$
$(F_{B_1U} \circ F_{B_2U})\ ((x, x_2)(x, y_2)) = \max\ (F_{A_1U}(x), F_{B_2U}(x_2y_2))\ \forall\ x \in V_1, \forall\ x_2y_2 \in E_2$

3) $(T_{B_1L} \circ T_{B_2L})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1L}(x_1y_1), T_{A_2L}(z))$
$(T_{B_1U} \circ T_{B_2U})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1U}(x_1y_1), T_{A_2U}(z))$
$(\mathrm{I}_{B_1L} \circ \mathrm{I}_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (\mathrm{I}_{B_1L}(x_1y_1), \mathrm{I}_{A_2L}(z))$
$(\mathrm{I}_{B_1U} \circ \mathrm{I}_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (\mathrm{I}_{B_1U}(x_1y_1), \mathrm{I}_{A_2U}(z))$
$(\mathrm{F}_{B_1L} \circ \mathrm{F}_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (\mathrm{F}_{B_1L}(x_1y_1), \mathrm{F}_{A_2L}(z))$
$(\mathrm{F}_{B_1U} \circ \mathrm{F}_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (\mathrm{F}_{B_1U}(x_1y_1), \mathrm{F}_{A_2U}(z))\ \forall\ z \in V_2, \forall\ x_1y_1 \in E_1$

4) $(\mathrm{T}_{B_1L} \circ \mathrm{T}_{B_2L})\ ((x_1, x_2)\ (y_1, y_2)) = \min\ (\mathrm{T}_{A_2L}(x_2), \mathrm{T}_{A_2L}(y_2), \mathrm{T}_{B_1L}(x_1y_1))$
$(T_{B_1U} \circ T_{B_2U})\ ((x_1, x_2)\ (y_1, y_2)) = \min\ (T_{A_2U}(x_2), T_{A_2U}(y_2), T_{B_1U}(x_1y_1))$
$(I_{B_1L} \circ I_{B_2L})\ ((x_1, x_2)\ (y_1, y_2)) = \max(I_{A_2L}(x_2), I_{A_2L}(y_2), I_{B_1L}(x_1y_1))$
$(I_{B_1U} \circ I_{B_2U})\ ((x_1, x_2)\ (y_1, y_2))= \max\ (I_{A_2U}(x_2), I_{A_2U}(y_2), I_{B_1U}(x_1y_1))$
$(F_{B_1L} \circ F_{B_2L})\ ((x_1, x_2)\ (y_1, y_2)) = \max(F_{A_2L}(x_2), F_{A_2L}(y_2), F_{B_1L}(x_1y_1))$
$(F_{B_1U} \circ F_{B_2U})((x_1, x_2)\ (y_1, y_2)) = \max\ (F_{A_2U}(x_2), F_{A_2U}(y_2), F_{B_1U}(x_1y_1))$
$\forall (x_1, x_2)(y_1, y_2) \in E^0$ -E , where $E^0 =$ E $\cup \{(x_1, x_2)\ (y_1, y_2)\ | x_1y_1 \in E_1, x_2 \neq y_2\}$.

The following propositions are stated without their proof.

Proposition 3.14

If G_1 and G_2 are the strong interval valued neutrosophic graphs, then the composition $G_1[\ G_2]$ is a strong interval valued neutrosophic graph.

Proposition 3.15

If $G_1[\ G_2]$ is strong interval valued neutrosophic graphs, then at least composition G_1 or G_2 must be strong.

Example 3.16

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3], > ,< b, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3] >} and B_1= {< ab, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3]>}.

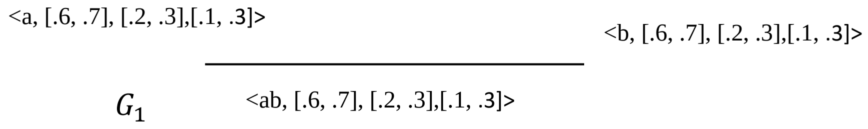

Figure 11. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < d, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >} and B_2=<cd, [0.3, 0.5], [0.2, 0.5], [0.3, 0.5]>}.

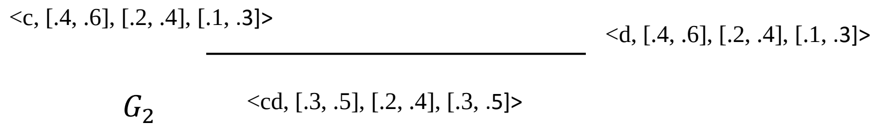

Figure 12. Interval valued neutrosophic G_2.

$G_1[G_2]$= (A_1oA_2, B_1oB_2) is not a SIVNG, where

A_1oA_2= {< (a,c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (a,d), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, d), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >},

B_1oB_2= {< ((a, c), (a, d)), [0.3, 0.5], [0.2, 0.4], [0.3, 0.5] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>, < ((b, c), (b, d)),[0.3, 0.5], [0.2, 0.4], [0.3, 0.5] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((a, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((a, d), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >}. In this example, G_1 is a SIVNG and $G_2$is not a SIVNG, then $G_1[G_2]$ is not a SIVNG.

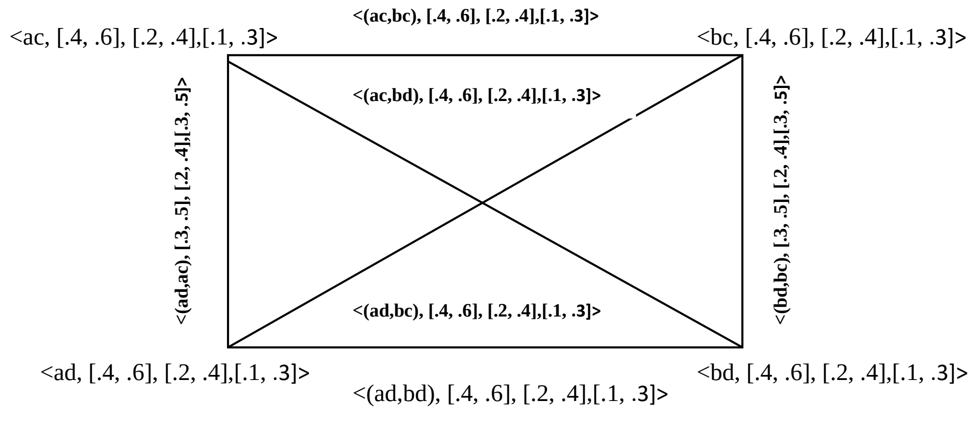

Figure 13. Composition

Example 3.17

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < b, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}and B_1={<ab, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}.

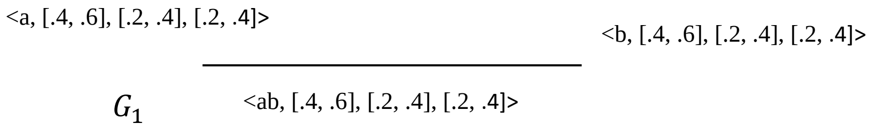

Figure 14. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.6, 0.7], [0.1, 0.3], [0.1, 0.3] >, <d, [0.6, 0.7], [0.2, 0.4], [0.1, 0.3] >} and B_2={< c d, [0.5, 0.6], [0.2, 0.4], [0.2, 0.4] >}.

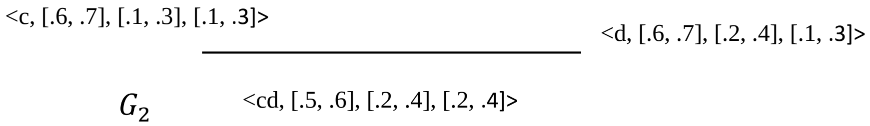

Figure 15. Interval valued neutrosophic G_2.

$G_1[G_2]$ = (A_1oA_2, B_1 oB_2) is a SIVNG, where

A_1oA_2= {< (a, c), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (a, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, d), [0.4, 0.6], [0.2, 0.4],[0.2, 0.4] >} and

B_1 oB_2= {< ((a, c), (a, d)), [0.4, 0.6], [0.2, 0.4] [0.2, 0.4] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((b, c), (b, d)), [0.4, 0.6], [0,2, 0.4] [0.2, 0.4] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, c), (b, d)), [0.4, 0.6], [0.2, 0.4] [0.2, 0.4]>, < ((a, d), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}. In this example, $G_1$is an SIVIFG and $G_2$is not a SIVNG, then $G_1[G_2]$ is a SIVNG.

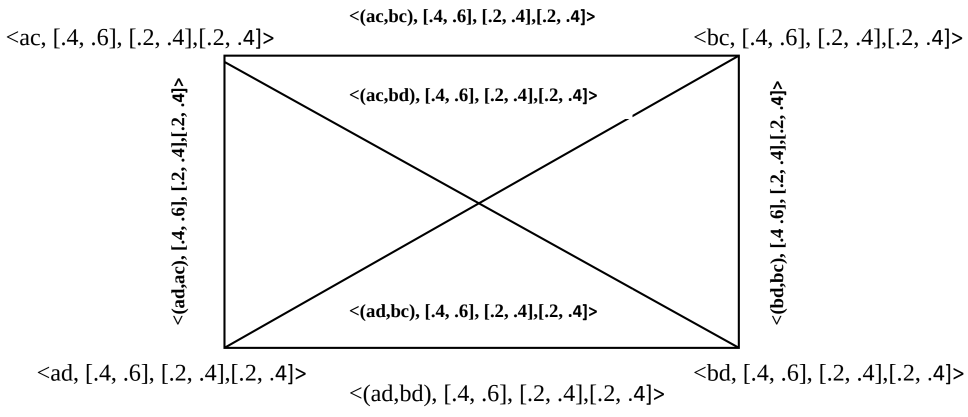

Figure 16. Composition of G_1 and G_2.

Proposition 3.18

Let G_1 be a strong interval valued neutrosophic graph. Then for any interval valued neutrosophic graph G_2,$G_1[G_2]$ is strong interval valued neutrosophic graph iff —

$T_{A_1L}(x_1) \leq T_{B_1L}(x_2y_2)$, $I_{A_1L}(x_1) \geq I_{B_1L}(x_2y_2)$ and $F_{A_1L}(x_1) \geq F_{B_1L}(x_2y_2)$,

$T_{A_1U}(x_1) \leq T_{B_1U}(x_2y_2)$, $I_{A_1U}(x_1) \geq I_{B_1U}(x_2y_2)$ and $F_{A_1U}(x_1) \geq F_{B_1U}(x_2y_2)$, $\forall\, x_1 \in V_1$,$x_2y_2 \in E_2$.

Definition 3.19

Let A_1 and A_2 be interval valued neutrosophic subsets of V_1 and V_2 respectively. Let B_1 and B_1 interval valued neutrosophic subsets of E_1 and E_2 respectively. The join of two strong interval valued neutrosophic graphs G_1 and G_2 is denoted by $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ and is defined as follows

1) $$(T_{A_1L} + T_{A_2L})(x) = \begin{cases} (T_{A_1L} \cup T_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1L}(x) & \text{if } x \in V_1 \\ T_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(T_{A_1U} + T_{A_2U})(x) = \begin{cases} (T_{A_1U} \cup T_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1U}(x) & \text{if } x \in V_1 \\ T_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1L} + I_{A_2L})\,(x) = \begin{cases} (I_{A_1L} \cap I_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1L}(x) & \text{if } x \in V_1 \\ I_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1U} + I_{A_2U})\,(x) = \begin{cases} (I_{A_1U} \cap I_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1U}(x) & \text{if } x \in V_1 \\ I_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1L} + F_{A_2L})\,(x) = \begin{cases} (F_{A_1L} \cap F_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1L}(x) & \text{if } x \in V_1 \\ F_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1U} + F_{A_2U})\,(x) = \begin{cases} (F_{A_1U} \cap F_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1U}(x) & \text{if } x \in V_1 \\ F_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

2) $$(T_{B_1L} + T_{B_2L})\,(xy) = \begin{cases} (T_{B_1L} \cup T_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1L}(xy) & \text{if } xy \in E_1 \\ T_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(T_{B_1U} + T_{B_2U})\,(xy) = \begin{cases} (T_{B_1U} \cup T_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1U}(xy) & \text{if } xy \in E_1 \\ T_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1L} + I_{B_2L})\ (xy) = \begin{cases} (I_{B_1L} \cap I_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1L}(xy) & \text{if } xy \in E_1 \\ I_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1U} + I_{B_2U})\ (xy) = \begin{cases} (I_{B_1U} \cap I_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1U}(xy) & \text{if } xy \in E_1 \\ I_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1L} + F_{B_2L})\ (xy) = \begin{cases} (F_{B_1L} \cap F_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1L}(xy) & \text{if } xy \in E_1 \\ F_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1U} + F_{B_2U})\ (x\ \mathrm{y}) = \begin{cases} (F_{B_1U} \cap F_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1U}(xy) & \text{if } xy \in E_1 \\ F_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

3) $(T_{B_1L} + T_{B_2L})\ (x\ \mathrm{y}) = \min\ (T_{B_1L}(x), T_{B_2L}(x))$
$(T_{B_1U} + T_{B_2U})\ (x\mathrm{y}) = \min\ (T_{B_1U}(x), T_{B_2U}(x))$
$(I_{B_1L} + I_{B_2L})\ (x\mathrm{y}) = \max\ (I_{B_1L}(x), I_{B_2L}(x))$
$(I_{B_1U} + I_{B_2U})\ (x\ \mathrm{y}) = \max\ (I_{B_1U}(x), I_{B_2U}(x)$
$(F_{B_1L} + F_{B_2L})\ (x\mathrm{y}) = \max\ (F_{B_1L}(x), F_{B_2L}(x))$

$(\mathrm{F}_{\mathrm{B_1U}} + \mathrm{F}_{\mathrm{B_2U}})\ (\mathrm{xy}) = \max\ (\mathrm{F}_{\mathrm{B_1U}}(\mathrm{x}), \mathrm{F}_{\mathrm{B_2U}}(\mathrm{x}))$ if $\mathrm{xy} \in \mathrm{E}'$, where E' is the set of all edges joining the nodes of V_1 and V_2 and where we assume $\mathrm{V}_1 \cap \mathrm{V}_2 = \emptyset$.

4 Conclusion

Interval valued neutrosophic set is a generalization of fuzzy set and intuitionistic fuzzy set, interval valued fuzzy set, interval valued intuitionstic fuzzy set and single valued neutrosophic set. Interval valued neutrosophic model gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and single valued neutrosophic models. In this paper, we have discussed a subclass of interval valued neutrosophic graph called strong interval valued neutrosophic graph, and we have introduced some operations, such as Cartesian product, composition and join of two strong interval valued neutrosophic graph, with proofs. In future studies, we plan to extend our research to regular interval valued neutrosophic graphs and irregular interval valued neutrosophic graphs.

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