Smarandache 对偶函数的一个计算公式

刘妙华

(空军工程大学理学院,陕西西安,710051)

摘要　对于任意正整数 n,著名的 Smarandache 对偶函数 $s^*(n)$ 定义为使得 $m!/n$ 最大的正整数 m,利用初等方法研究了关于对偶函数 $\sum_{d/n} s^*(d)$,并给出了一个计算公式。

关键词　Smarandache 函数;对偶函数;计算公式

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A Study of Smarandache Dual Function Calculation Formula

LIU Miao-hua

(Science College, Air Force Engineering University, Xi′an 710051, China)

Abstract: For any postive integer n, the famous Smarandache function is defined as making $m!/n$ the largest positive integer m. In the paper, the dual function $\sum_{d/n} s^*(d)$ is studied and a calculation formula is given by using an elementary method.

Key words: smarandache function; dual function; formula

Smarandache 函数是由罗马尼亚著名数论专家 J. Sandor 在文献[1]中首次提出的,并研究了它的各种初等性质,获得了一系列重要结论。关于这个问题,不少学者也做过研究,并且得到了一些有意义的结论。文献[2]中,李洁研究了一个包含 $s^*(n)$ 的无穷级数的敛散性,并获得了一个恒等式。即就是对任意的实数 $\alpha \leqslant 1$,无穷级数 $\sum_{n-1}^{\infty} \frac{s^*(n)}{n^\alpha}$ 是发散的,当 $\alpha > 1$ 时,是收敛的,而且:$\sum_{n-1}^{\infty} \frac{s^*(n)}{n^\alpha} = \zeta(\alpha) \cdot \sum_{n=1}^{\infty} \frac{1}{(n!)^\alpha}$,式中 $\zeta(\alpha)$ 是 Riemann-zeta 函数。注意到 $\zeta(2) = \frac{\pi}{6}$,$\lim_{s\to 1}(s-1)\zeta(s) = 1$,及 $\sum_{n=1}^{\infty} \frac{1}{n!} = e-1$,由上式可以推出:$\sum_{n-1}^{\infty} \frac{s^*(n)}{n^2} = \frac{\pi^2}{6} \sum_{n=1}^{\infty} \frac{1}{(n!)^2}$,此外,文献[3]中,还利用初等方法获得了较强的渐近公式:$\sum_{n\leqslant -1} s^*(n) = (e-1)x + o(\frac{\mathrm{In}^2(x)}{(\mathrm{InIn}x)^2})$。;文献[4]给出了一个包含 Smarandache 对偶函数的方程所有正整数解,文献[5]给出了一个包含 Smarandache 函数的对偶方程的正整数解。关于这一函数以及有关内容也可以参阅文献[6~8]。

本文利用初等方法研究 Smarandache 对偶函数 $\sum_{d/n} s^*(d)$,并给出了一个计算公式。

1　定理及结论

定理　对于正整数 n,关于 Smarandache 对偶函数 $\sum_{d/n} s^*(d)$ 的计算公式为:

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作者简介:刘妙华(1978—),女,陕西咸阳人,讲师,硕士,主要从事数论研究. E-mail:llmmhh_419@163.com

$$\sum_{d/n} s^*(d) = \begin{cases} (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1), & n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k},\ p_i \text{ 是奇素数} \\ (2\alpha+1)(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1), & n = 2^{\alpha} p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k},\ p_1 \neq 3 \\ (2\alpha+1+\alpha_1+3\alpha\alpha_1)(\alpha_2+1)\cdots(\alpha_k+1), & n = 2^{\alpha} p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k},\ p_1 = 3, \alpha = 1,2 \end{cases}$$

式中：p_i 为互不相同的奇素数；α_i，α 是正整数。

2　定理的证明

对于任意正整数 n，当 n 为奇数时，此时对任意 d/n，显然 2 !不整除 n，所以 $s^*(d) = 1$，则 $\sum_{d/n} s^*(d) = \sum_{d/n} 1 = d(n)$，$d(n)$ 表示 Dirichlet 除数函数。因此，分以下几种情况来讨论，为了方便，令 $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ 为 n 的标准素因子分解式，式中 p_i 为奇素数。

2.1　$n = p_1^{\alpha_1} p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}$

$$\sum_{d/n} s^*(d) = \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} 1 = d(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.2　$n = 2p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$

1）当 $p_1 \neq 3$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) = \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} 1 + \sum_{d/p_1^{\alpha_1} p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} 2 = 3d(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = 3(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p = 3$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + \sum_{d/p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2\times 3d) + \cdots + \sum_{d/p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2\times 3^{\alpha_1} d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + 2(\alpha_2+1)\cdots(\alpha_k+1) + 3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1) = (4\alpha_1+3)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.3　$n = 2^2 p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$

1）当 $p_1 \neq 3$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(4d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + 2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + 2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) = 5(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p_1 = 3$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(4d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + \sum_{d/p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2\times 3d) + \cdots + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2\times 3^{\alpha_1} d) + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(4d) + \cdots + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(4\times 3^{\alpha_1} d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + 2(\alpha_2+1)\cdots(\alpha_k+1) + 3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1) + 2(\alpha_2+1)\cdots(\alpha_k+1) + 3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1) = (7\alpha_1+5)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.4　$n = 2^{\alpha} p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$

1）当 $p_1 \neq 3$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \cdots + \sum_{d/p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2^{\alpha} d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + 2\alpha(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) = (2\alpha+1)(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p_1 = 3$，$\alpha = 1,2$ 时，则有：

$$\sum_{d/n} s^*(d) = \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(d) + \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \cdots + \sum_{d/3^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2^{\alpha} d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1) + \sum_{d/p_2^{\alpha_2} \cdots p_k^{\alpha_k}} s^*(2d) + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2\times 3d) + \cdots + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2\times 3^{\alpha_1} d) + \cdots + \sum_{d/p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}} s^*(2^{\alpha} d) + \cdots +$$

$$\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}} s^*(2^{\alpha}\times 3^{\alpha_1}d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2\alpha(\alpha_2+1)\cdots(\alpha_k+1)+3\alpha\alpha_1(\alpha_2+1)\cdots(\alpha_k+1)=(2\alpha+1+\alpha_1+3\alpha\alpha_1)(\alpha_2+1)\cdots(\alpha_k+1)$$。

这样就完成了定理的证明。

3 结语

在 1991 年美国研究出版社出版的《只有问题,没有解答》一书中,F. Smarandache 教授提出了 105 个关于特殊数列、算术函数等未解决的数学问题及猜想,而 Smarandache 对偶函数就是其中一类函数,关于此函数,不少学者专家对此进行了深入研究,并得到了具有理论价值的研究成果。本文通过对对偶函数的研究,得到了此计算公式,更方便进一步研究包含对偶函数的其它问题,并探讨其他 Smarandache 函数的对偶函数的一些性质与结论。

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