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研究简报

数 学

关于 Smarandache对偶函数的相关均值

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摘 要 对任意正整数 n, Smarandache LCM对偶函数是满足 $[1,2,\cdots,k]\mid n$的最小正整数,其中 $[1,2,\cdots,k]$代表 $1,2,\cdots,k$的最小公倍数。用初等方法研究 $\frac{SL^*(n)}{n}$,并给出一个有趣的渐近公式。

关键词 Smarandache LCM对偶函数 均值 渐近公式
中图法分类号 O12; 文献标志码 A

1 Introduction and result

For any positive integer n, the famous F. Smarandache LCM function SL(n) defined as the smallest positive integer k such that $n\mid[1,2,\cdots,k]$, where $[1,2,\cdots,k]$ denotes the least common multiple of $1,2,\cdots,k$. That is

$$SL(n)=\min\{k: k\in N, n\mid[1,2,\cdots,k]\}.$$

About the elementary properties of SL(n), many authors had studied it and obtained some interesting results, see references [2] and [3]. For example, Murthy[2] proved that if n be prime, then $SL(n)=S(n)$, where $S(n)=\min\{m: n\mid m!, m\in N\}$ be the F. Smarandache function. Simultaneously, Murthy[2] also proposed the following problem:

$$SL(n)=S(n), S(n)\neq n \qquad (1)$$

Le Maohua[3] solved this problem completely, and proved the following conclusion: Every positive integer n satisfying equa. (1) can be expressed as

$$n=12 \quad \text{or} \quad n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_r^{\alpha_r}p,$$

Where $p_1, p_2, \cdots, p_r, p$ are distinct primes and $\alpha_1, \alpha_2, \cdots, \alpha_r$ are positive integers satisfying $p>p_i^{\alpha_i}$, $i=1,2,\cdots,r$.

Zhongtian Lü[4] proved that for any real number $x>1$ and fixed positive integer k, we have the asymptotic formula

$$\sum_{n\leqslant x}SL(n)=\frac{\pi^2}{12}\frac{x^2}{\ln x}+\sum_{i=2}^{k}\frac{c_i x^2}{\ln^i x}+o\left(\frac{x^2}{\ln^{k+1}x}\right),$$

Where c_i $(i=2,3,\cdots,k)$ are computable constants.

Chengliang Tian[5] defined the F. Smarandache LCM dual function $SL^*(n)$ as follows:

$$SL^*(n)=\max\{k: k\in N, [1,2,\cdots,k]\mid n\}.$$

For example, $SL^*(1)=1$, $SL^*(2)=2$, $SL^*(3)=1$, $SL^*(4)=2$, $SL^*(5)=1$, $SL^*(6)=3$, $SL^*(7)=1$,

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$SL^*(8)=2$, $SL^*(9)=1$, $SL^*(10)=2$, …, Obviously, if n is an odd number, then $SL^*(n)=1$. If n is an even number, the $SL^*(n)\geqslant 2$. About the mean value properties of $SL^*(n)$, Chengliang Tian[5] proved that for any real number $x>1$, we have the asymptotic formula

$$\sum_{n\leqslant x} SL^*(n) = cx + O(\ln^2 x),$$

Where $c=\sum_{\alpha=1}^{\infty}\sum_{p}\frac{(p^{\alpha}-1)(p-1)}{[1,2,\cdots,p^{\alpha}]}$ is a constant.

The main purpose of this paper is to use elementary methods to study the mean value property of $\frac{SL^*(n)}{n}$, and give an interesting asymptotic formula for it. That is, we shall proved the following conclusion.

Theorem. For any real number $x\geqslant 2$, we have the asymptotic formula

$$\sum_{n\leqslant x}\frac{SL^*(n)}{n} = c+\frac{c}{x}+O\left(\frac{\ln^2 x}{x}\right).$$

Where $c=\sum_{\alpha=1}^{\infty}\sum_{p}\frac{(p^{\alpha}-1)(p-1)}{[1,2,\cdots,p^{\alpha}]}$ is a constant.

2 Some useful lemmas

To complete the proof of the theorem, we need the following lemmas.

Lemma 1. For any positive integer n, let n has the prime powers factorization $n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_r^{\alpha_r}p$, if $P(n)>\sqrt{n}$, then we have the identity

$$SL^*(n)=P(n).$$

Where $P(n)$ denotes the greatest prime divisor of n.

Proof. From the prime powers factorization of n, we may immediately get

$$p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_r^{\alpha_r}<\sqrt{n}.$$

Then we have

$$p_i^{\alpha_i}\mid P(n)!,\quad i=1,2,\cdots,r.$$

Thus we can easily obtain $n\mid P(n)!$, but $P(n)\nmid(P(n)-1)$, so we have

$$SL^*(n)=P(n).$$

This completes the proof of Lemma 1.

Lemma 2. For any real number $x>1$, we have the asymptotic formula

$$\sum_{n\leqslant x} SL^*(n) = cx + O(\ln^2 x),$$

Where $c=\sum_{\alpha=1}^{\infty}\sum_{p}\frac{(p^{\alpha}-1)(p-1)}{[1,2,\cdots,p^{\alpha}]}$ is a constant.

Proof. See reference [5].

3 Proof of the theorem

In this section, we shall complete the proof of theorem. First applying the Abel's summation[6], and note that the results of lemma 1 and lemma 2, we may have

$$\sum_{n\leqslant x}\frac{SL^*(n)}{n}=\frac{1}{x}\sum_{n\leqslant x}SL^*(n)+\int_1^x\frac{1}{t^2}\left(\sum_{n\leqslant t}SL^*(t)\right)dt=\frac{1}{x}(cx+O(\ln^2 x))+\int_1^x\frac{1}{t^2}(ct+O(\ln^2 t))dt=c+O\left(\frac{\ln^2 x}{x}\right)+\int_1^x\frac{c}{t}dt+O\left(\int_1^x\frac{\ln^2 t}{t}dt\right)=c+\frac{c}{x}+O\left(\frac{x}{\ln^2 x}\right).$$

Where $c=\sum_{\alpha=1}^{\infty}\sum_{p}\frac{(p^{\alpha}-1)(p-1)}{[1,2,\cdots,p^{\alpha}]}$ is a constant.

This completes the proof of Theorem.

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Mean Value Involving the Smarandache LCM Dual Function

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[Abstract] For any positive integer n, the F. Smarandache LCM dual function $SL^*(n)$ is defined as the grestest positive integer k such that [1, 2, …, k] divides n. The main purpose is to use elementary methods to study the mean value property of $\frac{SL^*(n)}{n}$, and give an interesting asymptotic formula for it.

[Key words] F. Smarandache LCM function; mean value; asymptotic formula

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Survey on Research Dynamic Voltage Restorer

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[Abstract] Dynamic Voltage Restorer (DVR) is a series compensating device. It is the most economical and effective means to solve the power quality problems, especially the voltage sags, because of its good dynamic characteristics and high cost-effective characteristic. The DVR's function, topology and its feature are mainly introduced, analyzed the research actuality and the existing problems of DVRs, and finally discussed the DVR's development direction and trends.

[Key words] power quality; voltage sag; dynamic voltage restorer (DVR)