Complex Neutrosophic Similarity Measures in Medical Diagnosis

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Abstract

This paper presents some similarity measures between complex neutrosophic sets. A complex neutrosophic set is a generalization of neutrosophic set whose complex-valued truth membership function, complex-valued indeterminacy membership function, and complex valued falsity membership functions are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. In the present study, we have proposed neutrosophic complex cosine, Dice and Jaccard similarity measures and investigated some of their properties. Finally, complex neutrosophic cosine, Dice and Jaccard similarity measures have been applied to a medical diagnosis problem with complex neutrosophic information.

Key Words: neutrosophic set; single valued neutrosophic set; complex neutrosophic sets; medical diagnosis; similarity measures

1. Introduction

It is avowed that uncertainty plays an important role in modeling real world problems. So it is necessary to bridge the gap between mathematical models and uncertainty and their explorative explanations. This gap can be found in problems of mathematics, operations research, biological and social sciences, modern technology and other applied sciences. In 1965, Zadeh [1] proposed the new concept of mathematics namely fuzzy sets (FS). In fuzzy set theory, the sum of membership and non-membership degrees of an element of a fuzzy set is equal to one. However, there exist some situations where the sum of membership and non membership degrees are not equal to one. In order to handle such situations Atanassov [2] introduced intuitionistic fuzzy set (IFS). Each element of an intuitionistic fuzzy set is assigned by membership and non-membership degrees, where the sum of the two degrees is less than one.

The concept of intuitionistic fuzzy set has been widely studied and applied in many areas such as decision-making problems [3, 4, 5], selection problem [6, 7], educational problem [8], medical diagnosis [9, 10, 11] etc. Smarandache [12] introduced the degree of indeterminacy as independent component and defined the neutrosophic set to deal with uncertainty, indeterminacy and inconsistency. To use the concept of neutrosophic set in practical fields such as real scientific and engineering applications, Wang et al.[13] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value. Similarity measures play an important role in the analysis and research of medical diagnosis [14] , pattern recognition [15], decision making [16, 17], and clustering analysis [18] in uncertain, indeterminate and inconsistent environment. Various similarity measures of SVNSs have been proposed in the literature.. Majumdar and Samanta [19] introduced the similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [20] proposed three vector similarity measures for simplified neutrosophic sets. Ye [21] also proposed improved cosine similarity measure for single valued neutrosophic sets based on cosine function. The same author [22] proposed the similarity measures of SVNSs for multiple attribute group decision making method with completely unknown weights. Ye and Zhang [23] further proposed the similarity measures of SVNSs for decision making problems. Biswas et al. [24] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Pramanik and Mondal [25] proposed rough cosine similarity measure in rough neutrosophic environment. Mondal and Pramanik [26] proposed neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. Mondal and Pramanik [27] proposed refined cotangent similarity measure in single valued neutrosophic environment. The same authors [28] further proposed cotangent similarity measure under rough neutrosophic environments. The same authors [29] further proposed some rough neutrosophic similarity measures and their application to multi attribute decision making.

Ramot et al. [30] introduced a concept of complex fuzzy sets (CFS). It is an extension of fuzzy sets. Here, membership function $z = r_s e^{iws(x)}$ where, $i = (-1)^{0.5}$ which ranges in a unit circle. The membership function is defined for the complex fuzzy set as $r_s e^{iws(x)}$. Here, $r_s(x)$ is the amplitude term and $w_s(x)$ is the phase term. $r_s(x)$ ranges in the interval [0, 1] and $w_s(x)$ is a periodic function. Ramot et al. [31] also proposed different complex fuzzy operations like union, intersection, complement etc. The amplitude term explains the idea of "fuzziness" and phase term implies declaration of complex fuzzy set. Chen et al. [32] proposed a neuro-fuzzy system architecture rule as a practical application of complex fuzzy logic.

Alkouri and Salleh [33] introduced complex intuitionistic fuzzy set (CIFS). CIFS is a generalization of complex fuzzy set. Complex fuzzy set is transformed into complex intuitionistic fuzzy set by adding complex-valued non-membership grade.

The complex intuitionistic fuzzy sets can deal the problems involving uncertainty and periodicity simultaneously. The concept of phase term is extended in complex intuitionistic fuzzy set which appears in several prominent concepts such as distance measure, Cartesian products, projections, relations, and so on. The complex fuzzy set has one additional phase. Complex intuitionistic fuzzy set has two additional phase terms. Recently Ali and Smarandache [34] proposed the concept of complex neutrosophic set. It seems to be very powerful. In this paper an attempt has been made to establish some similarity measures namely, cosine, Dice and Jaccard similarity measures in complex neutrosophic environment and their applications in medical diagnosis.

Rest of the paper is structured as follows: Section 2 presents neutrosophic and complex neutrosophic preliminaries. In Section we introduce complex Cosine, Dice and Jaccard similarity measure for complex neutrosophic sets and establish some of thier properties. Section 4 is devoted to present new method of medical diagnosis based on complex Dice and Jaccard similarity measures. Section 5 presents an application of complex Cosine, Dice and Jaccard similarity measures in medical diagnosis. Section 6 presents the concluding remarks and future scope of research.

2. Mathematical Preliminaries

Definition 2.1

Let G be a space of points with generic element in E denoted by y. Then a neutrosophic set P in G is characterized by a truth membership function T_P, an indeterminacy membership function I_P and a falsity membership function F_P. The functions T_P and F_P are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ that is T_P: $G \rightarrow\ ]^{-}0, 1^{+}[$; I_P: $G \rightarrow\ ]^{-}0, 1^{+}[$; F_P: $G \rightarrow\ ]^{-}0, 1^{+}[$. The sum of $T_P(y), I_P(y), F_P(y)$ is given by $^{-}0 \leq \sup T_P(y) + \sup I_P(y) + \sup F_P(y) \leq 3^{+}$

Definition 2.2

The complement of a neutrosophic set P is denoted by P^c and is defined as follows: $T_{P^c}(y) = \{1^+\} - T_P(y)$; $I_{P^c}(y) = \{1^+\} - I_P(y)$; $F_{P^c}(y) = \{1^+\} - F_P(y)$.

Definition 2.3

A neutrosophic set P is contained in the other neutrosophic set Q, $P \subseteq Q$ if and only if the following result holds.

$\inf T_P(y) \le \inf T_Q(y), \ \sup T_P(y) \le \sup T_Q(y)$;

$\inf I_P(y) \ge \inf I_Q(y), \ \sup I_P(y) \ge \sup I_Q(y)$;

$\inf F_P(y) \ge \inf F_Q(y), \ \sup F_P(y) \ge \sup F_Q(y)$, for all y in G.

Definition 2.4 Single-valued neutrosophic set

Let G be a universal space of points with generic element of G denoted by y. A single valued neutrosophic set S is characterized by a truth membership function $T_S(y)$, a falsity membership function $F_S(y)$ and indeterminacy function $I_S(y)$ such that $T_S(y)$, $F_S(y)$, $I_S(y) \in [0, 1]$ for all y in G.

When G is continuous, a SNVS S can be written as follows:

$$S = \int_y \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in G$$

and when G is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in G$$

It should be noted that for a SVNS S, $0 \le \sup T_S(y) + \sup F_S(y) + \sup I_S(y) \le 3, \ \forall y \in G$.

Definition 2.5

The complement of a single valued neutrosophic set S is denoted by S^c and is defined as follows:

$T_S{}^c(y) = F_S(y)$; $I_S{}^c(y) = 1 - I_S(y)$; $F_S{}^c(y) = T_S(y)$

Definition 2.6

A SVNS S_1 is contained in the other SVNS S_2, denoted as $S_1 \subseteq S_2$ if and only if $T_{S_1}(y) \le T_{S_2}(y)$; $I_{S_1}(y) \ge I_{S_2}(y)$; $F_{S_1}(y) \ge F_{S_2}(y)$, $\forall y \in G$.

Definition 2.7

Two single valued neutrosophic sets S_1 and S_2 are equal, i.e. $S_1 = S_{M2}$, if and only if, $S_1 \subseteq S_2$ and $S_1 \supseteq S_2$.

Definition 2.8

The union of two SVNSs S_1 and S_2 is a SVNS S_3, written as $S_3 = S_1 \cup S_2$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_1 and S_2 by the following equations

$T_{S_3}(y) = \max(T_{S_1}(y), T_{S_2}(y))$;

$I_{S_3}(y) = \max(I_{S_1}(y), I_{S_2}(y))$;

$F_{S3}(y) = \min(F_{S_1}(y), F_{S_2}(y))$ for all y in G.

Definition 2.9

The intersection of two SVNSs S_1 and M is a SVNS S_2, written as $S_3 = S_1 \cap S_2$. The truth membership, indeterminacy membership and falsity membership functions c an be defined as follows:

$T_{S_3}(y) = \min(T_{S_1}(y), T_2(y))$;

$I_{S_3}(y) = \max(I_{S_1}(y), I_{S_2}(y))$;

$F_{S_3}(y) = \max(F_{S_N}(y), F_{SM}(y)), \forall y \in G$.

Definition 2.10 Distance between two neutrosophic sets

The general SVNS can be presented in the follow form as follows:

$S = \{(y/(T_S(y), I_S(y), F_S(y))) : y \in G\}$

Finite SVNSs can be represented as follows:

$$S = \{(y_1/(T_S(y_1), I_S(y_1), F_S(y_1))), \cdots, (y_m/(T_S(y_m), I_S(y_m), F_S(y_m)))\}, \forall y \in G \quad (1)$$

Definition 2.11

$$S_1 = \{(y_1/(T_{S_1}(y_1), I_{S_1}(y_1), F_{S_1}(y_1))), \cdots, (y_n/(T_{S_1}(y_n), I_{S_1}(y_n), F_{S_1}(y_n)))\} \quad (2)$$

$$S_2 = \{(y_1/(T_{S_2}(y_1), I_{S_2}(y_1), F_{S_2}(y_1))), \cdots, (y_n/(T_{S_2}(y_n), I_{S_2}(y_n), F_{S_2}(y_n)))\} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance between two SNVS S_1 and S_2 can be defined as follows:

$$d(S_1, S_2) = \sum_{i=1}^{n} \left\langle \left|T_{S_1}(y) - T_{S_2}(y)\right| + \left|I_{S_1}(y) - I_{S_2}(y)\right| + \left|F_{S_1}(y) - F_{S_2}(y)\right| \right\rangle \quad (4)$$

and normalized Hamming distance between two SNVS $S_1$and S_2 can be defined as follows:

$$^{N}d(S_1, S_2) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \left|T_{S^{\cdot}}(y) - T_{S_1}(y)\right| + \left|I_{S^{\cdot}}(y) - I_{S_1}(y)\right| + \left|F_{S^{\cdot}}(y) - F_{S_1}(y)\right| \right\rangle \quad (5)$$

with the following properties

1. $0 \leq d(S_1, S_2) \leq 3n$
2. $0 \leq {}^{N}d(S_1, S_2) \leq 1$

2.1 Complex fuzzy set [30]

A complex fuzzy set S, defined on a universe of discourse X, is characterized by a membership function $\eta_S(x)$ that assigns any element $x \in X$ a complex-valued grade of membership in S. The values $\eta_S(x)$ all lie within the unit circle in the complex plane, and thus all of the form $p_S(x).e^{i.\mu_S(x)}$ where, $p_S(x)$, $\mu_S(x)$ are both real valued and $p_S(x)\in[0, 1]$. Here, $p_S(x)$ is termed as amplitude term and $e^{i.\mu_S(x)}$ is termed as phase term. The complex fuzzy set may be represented in the set form as $S=\{(x,\eta_S(x))\}: x\in X$

The complex fuzzy set is denoted as CFS. We now present some set operations of complex fuzzy sets.

Definition 2.1.1

Let S be a complex fuzzy set on X and $\eta_S(x)=p_S(x).e^{i.\mu_S(x)}$ its complex-valued membership function. The complement of S, denoted as c(S) and is specified by a function

$$\eta_{c(S)}(x)=p_{c(S)}(x).e^{i.\mu_{c(s)}(x)}=(1-p_{c(S)})(x).e^{i.(2\pi-\mu_{c(s)}(x))}$$

Definition 2.1.2

Let A and B be two complex fuzzy sets on X, and $\eta_A(x)=p_A(x).e^{i.\mu_A(x)}$ and $\eta_B(x)=p_B(x).e^{i.\mu_B(x)}$ be their membership functions respectively. The union of A and B is denoted as $A\bigcup B$ which is characterized by a function $\eta_{A\cup B}(x)=p_{A\cup B}(x).e^{i.\mu_{A\cup B}(x)}=\max(p_A(x), p_B(x)).e^{i.[\max(\mu_A(x), \mu_B(x))}$

Definition 2.1.3

Let A and B be two complex fuzzy sets on X, and $\eta_A(x)=p_A(x).e^{i.\mu_A(x)}$ and $\eta_B(x)=p_B(x).e^{i.\mu_B(x)}$ be their membership functions respectively. The intersection of A and B is denoted as $A\bigcap B$ which is characterized by a function $\eta_{A\cap B}(x)=p_{A\cap B}(x).e^{i.\mu_{A\cap B}(x)}=\min(p_A(x), p_B(x)).e^{i.[\min(\mu_A(x), \mu_B(x))}$

Definition 2.1.4

Let A and B be two complex fuzzy sets on X, and $\eta_A(x)=p_A(x).e^{i.\mu_A(x)}$ and $\eta_B(x)=p_B(x).e^{i.\mu_B(x)}$ be their membership functions respectively. The complex fuzzy product of A and B is denoted as $A\circ B$ which is characterized by a function

$$\eta_{A\circ B}(x)=p_{A\circ B}(x).e^{i.\mu_{A\circ B}(x)}=(p_A(x).\ p_B(x)).e^{i..\frac{\mu_A(x).\ \mu_B(x)}{2\pi}}$$

Definition 2.1.5 δ **equality of Complex Fuzzy sets** [30]

Let A and B be two complex fuzzy sets on X, and $\eta_A(x)=p_A(x).e^{i.\mu_A(x)}$ and $\eta_B(x)=p_B(x).e^{i.\mu_B(x)}$ be their membership functions respectively. Now, A and B are δ equal if and only if $d(A,B)\le 1-\delta$ if where $0\le\delta\le 1$.

2.2 Complex intuitionistic fuzzy set [33]

A complex intuitionistic fuzzy set S, defined on a universe of discourse X, is characterized by a membership function $\eta_S(x)$ and a non membership function $\psi_S(x)$ that assigns any element $x\in X$ a complex-valued grade of membership in S. The values $\eta_S(x)$ and $\psi_S(x)$ lie within the unit circle in the complex plane, and thus all of the form $p_S(x).e^{i.\mu_S(x)}$ and $q_S(x).e^{i.\vartheta_S(x)}$ where, $p_S(x)$, $\mu_S(x)$, $q_S(x)$ and $\vartheta_S(x)$ are both real valued and $q_S(x)$, $p_S(x)\in[0, 1]$. Here, $p_S(x)$ and $q_S(x)$ are expressed as amplitude terms and $e^{i.\mu_S(x)}$ and $e^{i.\vartheta_S(x)}$ expressed as phase terms. The complex intuitionistic fuzzy set is represented in the set form as $S=\{(x,\eta_S(x),\psi_S(x))\}: x\in X$

The complex intuitionistic fuzzy set is denoted as CIFS. Some set operations of complex intuitionistic fuzzy sets are given below.

Definition 2.2.1 Complement of Complex Intuitionistic Fuzzy set

Let S be a complex intuitionistic fuzzy set on X and $\eta_S(x)=p_S(x).e^{i.\mu_S(x)}$ and $\eta_S(x)=q_S(x).e^{i.\mu_S(x)}$ be it complex-valued membership function and non membership function respectively. The complement of S denoted as c(S) and is expressed by $\eta_{c(S)}(x)$ as follows.

$$\eta_{c(S)}(x)=p_{c(S)}(x).e^{i.\mu_{c(s)}(x)}=(p_{c(S)})(x).e^{i.(2\pi-\mu_{c(s)}(x))} \text{ and } \psi_{c(S)}(x)=q_{c(S)}(x).e^{i.\vartheta_{c(s)}(x)}=(q_{c(S)})(x).e^{i.(2\pi-\vartheta_{c(s)}(x))}$$

Definition 2.2.2 Union of Complex intuitionistic Fuzzy sets

Let A and B be two complex intuitionistic fuzzy sets on X, and $\eta_S(x) = p_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = q_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = p_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = q_S(x).e^{i.\mu_s(x)}$ be their membership functions and non membership function respectively. The union of A and B is denoted as $A \cup B$ which is expressed by $\eta_{A\cup B}(x)$ as follows.

$$\eta_{A\cup B}(x) = p_{A\cup B}(x).e^{i.\mu_{A\cup B}(x)} = \max\left(p_A(x),\, p_B(x)\right).e^{i.[\max(\mu_A(x),\, \mu_B(x)]}$$

$$\eta_{A\cup B}(x) = p_{A\cup B}(x).e^{i.\mu_{A\cup B}(x)} = \min\left(p_A(x),\, p_B(x)\right).e^{i.[\min(\mu_A(x),\, \mu_B(x)]}.$$

Definition 2.2.3 Intersection of Complex intuitionistic Fuzzy sets

Let A and B be two complex intuitionistic fuzzy sets on X, and $\eta_S(x) = p_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = q_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = p_S(x).e^{i.\mu_s(x)}$ and $\eta_S(x) = q_S(x).e^{i.\mu_s(x)}$ be their membership functions and non membership function respectively. The intersection of A and B is denoted as $A \cap B$ and can be defined as follows:

$$\eta_{A\cup B}(x) = p_{A\cup B}(x).e^{i.\mu_{A\cup B}(x)} = \min\left(p_A(x),\, p_B(x)\right).e^{i.[\min(\mu_A(x),\, \mu_B(x)]}$$

$$\eta_{A\cup B}(x) = p_{A\cup B}(x).e^{i.\mu_{A\cup B}(x)} = \max\left(p_A(x),\, p_B(x)\right).e^{i.[\max(\mu_A(x),\, \mu_B(x)]}.$$

2.3 Complex Neutrosophic Set [34]

A complex neutrosophic set S on a universe of discourse X, which is characterized by a truth membership function $T_S(x)$, an indeterminacy membership function $I_S(x)$, and a falsity membership function $F_S(x)$ that identifies a complex-valued grade of $T_S(x)$, $I_S(x)$, $F_S(x)$ in S for all x belongs to X. The values $T_S(x)$, $I_S(x)$, $F_S(x)$. Their sum is within the unit circle in the complex plane. So it can be expressed as follows.

$$T_S(x) = p_S(x)e^{i\mu_S(x)},\ I_S(x) = q_S(x)e^{i\vartheta_S(x)},\ F_S(x) = r_S(x)e^{i\omega_S(x)}$$

Where, $p_S(x)$, $q_S(x)$, $r_S(x)$ and $\mu_S(x)$, $\vartheta_S(x)$, $\omega_S(x)$ are respectively real valued and $p_S(x)$, $q_S(x)$, $r_S(x) \in [0,1]$ such that $0 \le p_S(x) + q_S(x) + r_S(x) \le 3$

Definition 2.3.1

A complex neutrosophic set CN_1 is contained in the other complex neutrosophic set CN_2 denoted as $CN_1 \subseteq CN_2$ if and only if $p_{CN_1}(x) \le p_{CN_2}(x)$, $q_{CN_1}(x) \le q_{CN_2}(x)$, $r_{CN_1}(x) \le r_{CN_2}(x)$, and $\mu_{CN_1}(x) \le \mu_{CN_2}(x)$, $\vartheta_{CN_1}(x) \le \vartheta_{CN_2}(x)$, $\omega_{CN_1}(x) \le \omega_{CN_2}(x)$.

Definition 2.3.2

Two complex neutrosophic sets CN_1 and CN_2 are equal i.e. $CN_1 = CN_2$ if and only if $p_{CN_1}(x) = p_{CN_2}(x)$, $q_{CN_1}(x) = q_{CN_2}(x)$, $r_{CN_1}(x) = r_{CN_2}(x)$, $\mu_{CN_1}(x) = \mu_{CN_2}(x)$, $\vartheta_{CN_1}(x) = \vartheta_{CN_2}(x)$, and $\omega_{CN_1}(x) = \omega_{CN_2}(x)$.

Definition 2.3.3 Complex Neutrosophic number (CNN)

A complex neutrosophic number (CNN) in a complex neutrosophic set S, can be defined as three complex components. It can be expressed as ($T_S(x)$, $I_S(x)$, $F_S(x)$). Here, $T_S(x) = (p_S(x)e^{i\mu_S(x)}$, $I_S(x) = q_S(x)e^{i\vartheta_S(x)}$, $F_S(x) = r_S(x)e^{i\omega_S(x)}$ and $p_S(x)$, $q_S(x)$, $r_S(x)$, $\mu_S(x)$, $\vartheta_S(x)$, $\omega_S(x)$ are respectively real valued and $p_S(x)$, $q_S(x)$, $r_S(x) \in [0,1]$ such that $0 \le p_S(x) + q_S(x) + r_S(x) \le 3$.

3. Complex neutrosophic similarity measures

3.1 Complex neutrosophic cosine similarity measure (CNCSM)

The complex cosine similarity measure is defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two complex neutrosophic sets. Literature review suggests that cosine similarity measure with complex neutrosophic sets has not been defined. Therefore, a new cosine similarity measure between complex neutrosophic sets is proposed in 3-D vector space.

Definition 3.1.1 Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{s_1}(x)}, q_{S_1}(x)e^{i\vartheta_{s_1}(x)}, r_{S_1}(x)e^{i\omega_{s_1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{s_2}(x)}, q_{S_2}(x)e^{i\vartheta_{s_2}(x)}, r_{S_2}(x)e^{i\omega_{s_2}(x)} \right\rangle$ in S for all x belongs to X. A complex cosine similarity measure between complex neutrosophic sets CN_1 and CN_2 is defined as follows:

$$C_{CNS} = \frac{1}{n}\Sigma_{i=1}^{n}\frac{\left((a_1b_1a_2b_2)^{0.5}+(c_1d_1c_2d_2)^{0.5}+(e_1f_1e_2f_2)^{0.5}\right)}{\left((a_1b_1+c_1d_1+e_1f_1)^{0.5}.(a_2b_2+c_2d_2+e_2f_2)^{0.5}\right)} \tag{6}$$

$a_1 = \mathrm{Re}\,[\, p_{S_1}(x)e^{i\mu_{s_1}(x)}\,]$, $b_1 = \mathrm{Im}\,[\, p_{S_1}(x)e^{i\mu_{s_1}(x)}\,]$, $a_2 = \mathrm{Re}\,[\, p_{S_2}(x)e^{i\mu_{s_2}(x)}\,]$, $b_2 = \mathrm{Im}\,[\, p_{S_2}(x)e^{i\mu_{s_2}(x)}\,]$,

$c_1 = \mathrm{Re}\,[\, q_{S_1}(x)e^{i\vartheta_{s_1}(x)}\,]$, $d_1 = \mathrm{Im}\,[\, q_{S_1}(x)e^{i\vartheta_{s_1}(x)}\,]$, $c_2 = \mathrm{Re}\,[\, q_{S2}(x)e^{i\vartheta_{s_2}(x)}\,]$, $d_2 = \mathrm{Im}\,[\, q_{S_2}(x)e^{i\vartheta_{s2}(x)}\,]$,

$e_1 = \mathrm{Re}\,[\, r_{S_1}(x)e^{i\omega_{s_1}(x)}\,]$, $f_1 = \mathrm{Im}\,[\, r_{S_1}(x)e^{i\omega_{s_1}(x)}\,]$, $e_2 = \mathrm{Re}\,[\, r_{S2}(x)e^{i\omega_{s_2}(x)}\,]$, $f_2 = \mathrm{Im}\,[\, r_{S_2}(x)e^{i\omega_{s_2}(x)}\,]$

Where, “Re” indicates real part and “Im” indicates imaginary part of corresponding complex number.

Let CN_1 and CN_2 be complex neutrosophic sets then,

I. $0 \le C_{CNS}(CN_1, CN_2) \le 1$

II. $C_{CNS}(CN_1, CN_2) = C_{CNS}(CN_2, CN_1)$

III. $C_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_1, CN_2)$, and $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_2, CN)$.

Proofs:

I. It is obvious because all positive values of cosine function are within 0 and 1.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then obviously $C_{CNS}(CN_1, CN_2) = 1$. On the other hand if $C_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_s(x)}, q_S(x)e^{i\vartheta_s(x)}, r_S(x)e^{i\omega_s(x)} \right\rangle$ and also assume that $l_1 = \mathrm{Re}\,[\, p_s(x)e^{i\mu_s(x)}\,]$, $l_2 = \mathrm{Im}\,[\, p_s(x)e^{i\mu_s(x)}\,]$, $m_1 = \mathrm{Re}\,[\, q_S(x)e^{i\vartheta_s(x)}\,]$, $m_2 = \mathrm{Im}\,[\, q_S(x)e^{i\vartheta_s(x)}\,]$, $n_1 = \mathrm{Re}\,[\, r_S(x)e^{i\omega_s(x)}\,]$, $n_2 = \mathrm{Im}\,[\, r_S(x)e^{i\omega_s(x)}\,]$

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

The cosine function is decreasing function within the interval $[0, \pi/2]$. Hence we can write $C_{CNS}(CN_1, CN) \le C_{CNS}(CN, CN_2)$, and $C_{CNS}(CN_1, CN) \le C_{CNS}(CN_2, CN)$.

3.2 Weighted Complex neutrosophic Cosine similarity measure (WCNCSM)

Definition 3.2.1

Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{s_1}(x)}, q_{S_1}(x)e^{i\vartheta_{s_1}(x)}, r_{S_1}(x)e^{i\omega_{s_1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{s_2}(x)}, q_{S_2}(x)e^{i\vartheta_{s_2}(x)}, r_{S_2}(x)e^{i\omega_{s_2}(x)} \right\rangle$ in S for all x belongs to X. A weighted complex cosine similarity measure between complex neutrosophic sets CN_1 and CN_2 can be defined as follows:

$$C_{WCNS} = \Sigma_{i=1}^{n} w_i \frac{\left((a_1b_1a_2b_2)^{0.5}+(c_1d_1c_2d_2)^{0.5}+(e_1f_1e_2f_2)^{0.5}\right)}{\left((a_1b_1+c_1d_1+e_1f_1)^{0.5}.(a_2b_2+c_2d_2+e_2f_2)^{0.5}\right)} \tag{7}$$

Where, $\sum_{i=1}^{n} w_i = 1$

Let CN_1 and CN_2 be complex neutrosophic sets then,

I. $0 \le C_{WCNS}(CN_1, CN_2) \le 1$

II. $C_{WCNS}(CN_1, CN_2) = C_{WCNS}(CN_2, CN_1)$

III. $C_{WCNS}(CN_1, CN_2) = 1$, if and only if $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $C_{WCNS}(CN_1, CN) \le C_{WCNS}(CN_1, CN_2)$, and $C_{WCNS}(CN_1, CN) \le C_{WCNS}(CN_2, CN)$

Proofs:

I. Since $\sum_{i=1}^{n} W_i = 1$ and all positive values of cosine function are within 0 and 1, it can be written as $0 \le C_{WCNS}(CN_1, CN_2) \le 1$.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then $C_{WCNS}(CN_1, CN_2) = 1$. On the other hand if $C_{WCNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that $l_1 = \text{Re}\,[\, p_s(x)e^{i\mu_s(x)}]$, $l_2 = \text{Im}\,[\, p_s(x)e^{i\mu_s(x)}]$, $m_1 = \text{Re}\,[\, q_S(x)e^{i\vartheta_s(x)}]$, $m_2 = \text{Im}\,[\, q_S(x)e^{i\vartheta_s(x)}]$, $n_1 = \text{Re}\,[\, r_S(x)e^{i\omega_s(x)}]$, $n_2 = \text{Im}\,[\, r_S(x)e^{i\omega_s(x)}]$

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

The cosine function is decreasing function within the interval $[0, \pi/2]$. Hence we can write

$C_{WCNS}(CN_1, CN) \le C_{WCNS}(CN, CN_2)$, and $C_{WCNS}(CN_1, CN) \le C_{WCNS}(CN_2, CN)$.

3.3 Complex neutrosophic Dice similarity measure (CNDSM)

Definition 3.3.1

Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\vartheta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\rangle$ in S for all x belongs to X. A complex Dice similarity measure between complex neutrosophic sets CN_1 and CN_2 can be defined as follows:

$$D_{CNS} = \sum_{i=1}^{n} \frac{2\left((a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}\right)}{(a_1b_1 + c_1d_1 + e_1f_1) + (a_2b_2 + c_2d_2 + e_2f_2)} \tag{8}$$

$a_1 = \text{Re}\,[\, p_{S_1}(x)e^{i\mu_{S_1}(x)}]$, $b_1 = \text{Im}\,[\, p_{S_1}(x)e^{i\mu_{S_1}(x)}]$, $a_2 = \text{Re}\,[\, p_{S_2}(x)e^{i\mu_{S_2}(x)}]$, $b_2 = \text{Im}\,[\, p_{S_2}(x)e^{i\mu_{S_2}(x)}]$,

$c_1 = \text{Re}\,[\, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}]$, $d_1 = \text{Im}\,[\, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}]$, $c_2 = \text{Re}\,[\, q_{S2}(x)e^{i\vartheta_{S_2}(x)}]$, $d_2 = \text{Im}\,[\, q_{S_2}(x)e^{i\vartheta_{S2}(x)}]$,

$e_1 = \text{Re}\,[\, r_{S_1}(x)e^{i\omega_{S_1}(x)}]$, $f_1 = \text{Im}\,[\, r_{S_1}(x)e^{i\omega_{S_1}(x)}]$, $e_2 = \text{Re}\,[\, r_{S2}(x)e^{i\omega_{S_2}(x)}]$, $f_2 = \text{Im}\,[\, r_{S_2}(x)e^{i\omega_{S_2}(x)}]$

Where, "Re" indicates real part and "Im" indicates imaginary part of corresponding complex number.

Let CN_1 and CN_2 be complex neutrosophic sets then,

III. $0 \le D_{CNS}(CN_1, CN_2) \le 1$

III. $D_{CNS}(CN_1, CN_2) = D_{CNS}(CN_2, CN_1)$

III. $D_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $D_{CNS}(CN_1, CN) \leq D_{CNS}(CN_1, CN_2)$, and $D_{CNS}(CN_1, CN) \leq D_{CNS}(CN_2, CN)$.

Proofs:

I. Since, $2((a_1b_1a_2b_2)^{0.5}+(c_1d_1c_2d_2)^{0.5}+(e_1f_1e_2f_2)^{0.5}) \leq (a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)$ it can be written as $0 \leq D_{CNS}(CN_1, CN_2) \leq 1$.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then obviously $D_{CNS}(CN_1, CN_2) = 1$. On the other hand if $D_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that l_1 = Re [$p_s(x)e^{i\mu_s(x)}$], l_2 = Im [$p_s(x)e^{i\mu_s(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_S(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_S(x)}$], n_1 = Re [$r_S(x)e^{i\omega_S(x)}$], f_1 = Im [$r_S(x)e^{i\omega_S(x)}$].

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \leq a_2b_2 \leq l_1l_2$, $c_1d_1 \geq c_2d_2 \geq m_1m_2$, $e_1f_1 \geq e_2f_2 \geq n_1n_2$.

Hence we can write $D_{CNS}(CN_1, CN) \leq D_{CNS}(CN, CN_2)$, and $D_{CNS}(CN_1, CN) \leq D_{CNS}(CN_2, CN)$.

3.4 Weighted Complex neutrosophic Dice similarity measure (WCNDSM)

Definition 3.4.1

Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S1}(x)}, q_{S_1}(x)e^{i\vartheta_{S1}(x)}, r_{S_1}(x)e^{i\omega_{S1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S2}(x)}, q_{S_2}(x)e^{i\vartheta_{S2}(x)}, r_{S_2}(x)e^{i\omega_{S2}(x)} \right\rangle$ in S for all x belongs to X. A weighted complex Dice similarity measure between complex neutrosophic sets CN_1 and CN_2 can be defined as follows:

$$D_{WCNS} = \sum_{i=1}^{n} w_i \frac{2\left((a_1b_1a_2b_2)^{0.5}+(c_1d_1c_2d_2)^{0.5}+(e_1f_1e_2f_2)^{0.5}\right)}{(a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)} \tag{9}$$

Where, $\sum_{i=1}^{n} w_i = 1$

Let CN_1 and CN_2 be complex neutrosophic sets then,

I. $0 \leq D_{WCNS}(CN_1, CN_2) \leq 1$

II. $D_{WCNS}(CN_1, CN_2) = D_{WCNS}(CN_2, CN_1)$

III. $D_{WCNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $D_{WCNS}(CN_1, CN) \leq D_{WCNS}(CN_1, CN_2)$, and $D_{WCNS}(CN_1, CN) \leq D_{WCNS}(CN_2, CN)$

Proofs:

I. Since, $\sum_{i=1}^{n} w_i = 1$ and $2((a_1b_1a_2b_2)^{0.5}+(c_1d_1c_2d_2)^{0.5}+(e_1f_1e_2f_2)^{0.5}) \leq (a_1b_1+c_1d_1+e_1f_1)+(a_2b_2+c_2d_2+e_2f_2)$ it can be written as $0 \leq D_{WCNS}(CN_1, CN_2) \leq 1$.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then obviously $D_{WCNS}(CN_1, CN_2) = 1$. On the other hand if $D_{WCNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that l_1 = Re [$p_s(x)e^{i\mu_s(x)}$], l_2 = Im [$p_s(x)e^{i\mu_s(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_S(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_S(x)}$], n_1 = Re [$r_S(x)e^{i\omega_S(x)}$], n_2 = Im [$r_S(x)e^{i\omega_S(x)}$]

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

Hence we can write $D_{WCNS}(CN_1, CN) \le D_{WCNS}(CN, CN_2)$, and $D_{WCNS}(CN_1, CN) \le D_{WCNS}(CN_2, CN)$.

3.5 Complex neutrosophic Jaccard similarity measure (CNJSM)

Definition 3.5.1

Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\vartheta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\rangle$ in S for all x belongs to X. A complex Jaccard similarity measure between complex neutrosophic sets CN_1 and CN_2 can be defined as follows:

$$J_{CNS} = \frac{1}{n}\Sigma_{i=1}^{n} \frac{(a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}}{\left\langle (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) - (a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5} \right\rangle} \quad (10)$$

a_1 = Re [$p_{S_1}(x)e^{i\mu_{s_1}(x)}$], b_1 = Im [$p_{S_1}(x)e^{i\mu_{s_1}(x)}$], a_2 = Re [$p_{S_2}(x)e^{i\mu_{s_2}(x)}$], b_2 = Im [$p_{S_2}(x)e^{i\mu_{s_2}(x)}$],

c_1 = Re [$q_{S_1}(x)e^{i\vartheta_{s_1}(x)}$], d_1 = Im [$q_{S_1}(x)e^{i\vartheta_{s_1}(x)}$], c_2 = Re [$q_{S2}(x)e^{i\vartheta_{s_2}(x)}$], d_2 = Im [$q_{S_2}(x)e^{i\vartheta_{s2}(x)}$],

e_1 = Re [$r_{S_1}(x)e^{i\omega_{s_1}(x)}$], f_1 = Im [$r_{S_1}(x)e^{i\omega_{s_1}(x)}$], e_2 = Re [$r_{S2}(x)e^{i\omega_{s_2}(x)}$], f_2 = Im [$r_{S_2}(x)e^{i\omega_{s_2}(x)}$].

Where, “Re” indicates real part and “Im” indicates imaginary part of corresponding complex number.

Let CN_1 and CN_2 be complex neutrosophic sets then,

I. $0 \le J_{CNS}(CN_1, CN_2) \le 1$

II. $J_{CNS}(CN_1, CN_2) = J_{CNS}(CN_2, CN_1)$

III. $J_{CNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_1, CN_2)$, and $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_2, CN)$.

Proofs:

I. Since, $((a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}) \le (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) - ((a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5})$ it can be written as $0 \le J_{CNS}(CN_1, CN_2) \le 1$.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then obviously $J_{CNS}(CN_1, CN_2) = 1$. On the other hand if $J_{CNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that l_1 = Re [$p_s(x)e^{i\mu_s(x)}$],

l_2 = Im [$p_s(x)e^{i\mu_s(x)}$], m_1 = Re [$q_S(x)e^{i\vartheta_S(x)}$], m_2 = Im [$q_S(x)e^{i\vartheta_S(x)}$], n_1 = Re [$r_S(x)e^{i\omega_S(x)}$], f_1 = Im [$r_S(x)e^{i\omega_S(x)}$].

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

Hence we can write $J_{CNS}(CN_1, CN) \le J_{CNS}(CN, CN_2)$, and $J_{CNS}(CN_1, CN) \le J_{CNS}(CN_2, CN)$.

3.6 Weighted Complex neutrosophic Jaccard similarity measure (WCNJSM)

Definition 3.6.1

Assume that there are two complex neutrosophic sets namely, $CN_1 = \left\langle p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\vartheta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\rangle$ and $CN_2 = \left\langle p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\vartheta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\rangle$ in S for all x belongs to X. A weighted complex Jaccard similarity measure between complex neutrosophic sets CN_1 and CN_2 can defined as follows:

$$J_{WCNS} = \sum_{i=1}^{n} w_i \frac{(a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}}{\left\langle (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) - (a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5} \right\rangle} \tag{11}$$

Where, $\sum_{i=1}^{n} w_i = 1$

Let CN_1 and CN_2 be complex neutrosophic sets then,

I. $0 \le J_{WCNS}(CN_1, CN_2) \le 1$

II. $J_{WCNS}(CN_1, CN_2) = J_{WCNS}(CN_2, CN_1)$

III. $J_{WCNS}(CN_1, CN_2) = 1$, iff $CN_1 = CN_2$

IV. If CN is a CNS in S and $CN_1 \subset CN_2 \subset CN$ then, $J_{WCNS}(CN_1, CN) \le J_{WCNS}(CN_1, CN_2)$, and $J_{WCNS}(CN_1, CN) \le J_{WCNS}(CN_2, CN)$

Proofs:

I. Since $\sum_{i=1}^{n} w_i = 1$ and $((a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}) \le (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) - ((a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5})$ it can be written as $0 \le J_{WCNS}(CN_1, CN_2) \le 1$.

II. It is obvious that the proposition is true.

III. When $CN_1 = CN_2$, then obviously $J_{WCNS}(CN_1, CN_2) = 1$. On the other hand if $J_{WCNS}(CN_1, CN_2) = 1$ then, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $e_1 = e_2$, $f_1 = f_2$.

This implies that $CN_1 = CN_2$.

IV. Let, $CN = \left\langle p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\vartheta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\rangle$ and also assume that $l_1 = \mathrm{Re}[p_s(x)e^{i\mu_s(x)}]$, $l_2 = \mathrm{Im}[p_s(x)e^{i\mu_s(x)}]$, $m_1 = \mathrm{Re}[q_S(x)e^{i\vartheta_s(x)}]$, $m_2 = \mathrm{Im}[q_S(x)e^{i\vartheta_s(x)}]$, $n_1 = \mathrm{Re}[r_S(x)e^{i\omega_s(x)}]$, $n_2 = \mathrm{Im}[r_S(x)e^{i\omega_s(x)}]$

If $CN_1 \subset CN_2 \subset CN$ then we can write $a_1b_1 \le a_2b_2 \le l_1l_2$, $c_1d_1 \ge c_2d_2 \ge m_1m_2$, $e_1f_1 \ge e_2f_2 \ge n_1n_2$.

Hence we can write $J_{WCNS}(CN_1, CN) \le J_{WCNS}(CN, CN_2)$, and $J_{WCNS}(CN_1, CN) \le J_{WCNS}(CN_2, CN)$.

4. Methodology of medical diagnosis

Assume that, $H_1, H_2, ..., H_m$ be a discrete set of patients, $D_1, D_2, ..., D_n$ be the set of diseases, and $A_1, A_2, ..., A_k$ be a set of symptoms. The decision-maker provides the ranking of diseases with respect to each symptom. Medical diagnosis procedure under complex neutrosophic environment based on Cosine, Dice and Jaccard similarity measure camn be presented using the as following steps.

Step 1: Determination the relation between patients and symptoms

The ranking presents the performances of patients H_i (i = 1, 2,..., m) against the symptoms A_j (j = 1, 2, ..., k). The complex neutrosophic values associated with the patients and their symptoms for diagnosis problem can be presented in the decision matrix (see the table 1).

Table 1: The relation between Patients and Symptoms (R-1)

$R-1$	A_1	A_2	$\cdots$	A_k
H_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	$\cdots$	$\langle T_{1k}, I_{1k}, F_{1k} \rangle$
H_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	$\cdots$	$\langle T_{2k}, I_{2k}, F_{2k} \rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
H_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	$\cdots$	$\langle T_{mk}, I_{mk}, F_{mk} \rangle$

Here $\langle T_{ij}, I_{ij}, F_{ij} \rangle$ (i = 1, 2,..., m; j = 1, 2, ..., k) is the complex neutrosophic number associated to the i-th patient and the j-th symptom.

Step 2: Determination of the relation between symptoms and diseases

The relation between symptoms A_j (j = 1, 2, ..., k) and diseases D_t (t = 1, 2, ..., n) in terms of complex neutrosophic numbers can be presented in the decision matrix (see the table 2).

Table 2: The relation between symptoms and diseases (R-2)

$R-2$	D_1	D_2	$\cdots$	D_n
A_1	$\langle \xi_{11}, \eta_{11}, \zeta_{11} \rangle$	$\langle \xi_{12}, \eta_{12}, \zeta_{12} \rangle$	$\cdots$	$\langle \xi_{1n}, \eta_{1n}, \zeta_{1n} \rangle$
A_2	$\langle \xi_{21}, \eta_{21}, \zeta_{21} \rangle$	$\langle \xi_{22}, \eta_{22}, \zeta_{22} \rangle$	$\cdots$	$\langle \xi_{2n}, \eta_{2n}, \zeta_{2n} \rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_k	$\langle \xi_{k1}, \eta_{k1}, \zeta_{k1} \rangle$	$\langle \xi_{k2}, \eta_{k2}, \zeta_{k2} \rangle$	$\cdots$	$\langle \xi_{kn}, \eta_{kn}, \zeta_{kn} \rangle$

Here $\langle \xi_{ij}, \eta_{ij}, \zeta_{ij} \rangle$ (i = 1, 2,..., k; j = 1, 2, ..., n) is the complex neutrosophic number associated to the i-th symptom and the j-th disease.

Step 3: Determination of the similarity measures

Determine the complex cosine, Dice and Jaccard similarity measures C_{CNS}, D_{CNS} and J_{CNS} between the table 1 and the table 2 using equation (6), equation (8) and equation (10).

Step 4: Ranking the alternatives

Ranking of diseases can be prepared based on the descending order of complex cosine, Dice and Jaccard similarity measures. The disease corresponding to highest similarity value reflects that patient H_i (i = 1, 2,..., m) suffering from that disease.

Step 5: End

5. Example on medical diagnosis

We consider a medical diagnosis problem for illustration of the proposed approach. Medical diagnosis comprises of uncertainties and increased volume of information available to physicians from new medical technologies. So, all collected information may be in complex neutrosophic form. The three components of a complex neutrosophic set are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. So, to deal more indeterminacy situations in medical diagnosis complex neutrosophic environment is more acceptable.

The process of classifying different set of symptoms under a single name of a disease is very difficult. In some practical situations, there exists possibility of each element within a periodic form of neutrosophic sets. So, medical diagnosis involves more indeterminacy. Complex neutrosophic sets handle this situation. Actually this approach is more flexible, dealing with more indeterminacy areas and easy to use. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis in complex neutrosophic environment.

The main feature of this proposed approach is that it considers complex truth membership, complex indeterminate and complex false membership of each element taking periodic form of neutrosophic sets.

Now, consider an example of a medical diagnosis. Assume that $H = \{H_1, H_2, H_3\}$ be a set of patients, D = {Viral Fever, Malaria, Stomach problem, Chest problem} be a set of diseases and A = {Temperature, Headache, Stomach pain, cough, Chest pain.} be a set of symptoms. Our investigation is to examine the patient and to determine the disease of the patient in complex neutrosophic environment.

Step 1: Determination the relation between patients and symptoms

In the diagnosis process the relation between Patients and Symptoms in complex neutrosophic form has been presented in the decision matrix as follows (see table 3).

Table 3: Relation between Patients and Symptoms in complex neutrosophic form (R-1)

R-1	Temperature	Headache	Stomach pain	cough	Chest pain
H_1	$\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.2e^{0.8i} \rangle$	$\langle 0.4e^{1.2i}, 0.4e^{1.1i}, 0.3e^{0.7i} \rangle$	$\langle 0.3e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \rangle$	$\langle 0.6e^{1.0i}, 0.5e^{1.2i}, 0.3e^{0.8i} \rangle$	$\langle 0.4e^{1.0i}, 0.3e^{1.0i}, 0.2e^{0.5i} \rangle$
H_2	$\langle 0.7e^{1.3i}, 0.4e^{1.2i}, 0.5e^{0.9i} \rangle$	$\langle 0.4e^{1.5i}, 0.6e^{1.5i}, 0.3e^{0.5i} \rangle$	$\langle 0.5e^{1.4i}, 0.4e^{1.2i}, 0.4e^{1.0i} \rangle$	$\langle 0.6e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \rangle$	$\langle 0.3e^{1.5i}, 0.4e^{1.0i}, 0.5e^{1.0i} \rangle$
H_3	$\langle 0.5e^{0.6i}, 0.5e^{1.2i}, 0.5e^{0.9i} \rangle$	$\langle 0.5e^{1.3i}, 0.4e^{1.2i}, 0.4e^{0.4i} \rangle$	$\langle 0.4e^{1.0i}, 0.4e^{1.0i}, 0.2e^{0.6i} \rangle$	$\langle 0.4e^{1.0i}, 0.5e^{1.1i}, 0.2e^{1.2i} \rangle$	$\langle 0.5e^{1.2i}, 0.2e^{1.2i}, 0.2e^{1.4i} \rangle$

Numerical values of $(a_1b_1)^{0.5}$, $(c_1d_1)^{0.5}$ and $(e_1f_1)^{0.5}$ corresponding to each CNN (from table 3) has been presented in the following matrix (see table 4).

Table 4: Numerical values of $(a_1b_1)^{0.5}$, $(c_1d_1)^{0.5}$ and $(e_1f_1)^{0.5}$ corresponding to each CNN (from table 3)

Patients	Temperature	Headache	Stomach pain	cough	Chest pain
	$[(a_1b_1)^{0.5},(c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5},(c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5},(c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5},(c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$	$[(a_1b_1)^{0.5},(c_1d_1)^{0.5}, (e_1f_1)^{0.5}]$

H_1	[0.405, 0.232, 0.141]	[0.581, 0.637, 0.702]	[0.202, 0.270, 0.274]	[0.404, 0.226, 0.212]	[0.255, 0.202, 0.126]
H_2	[0.355, 0.232, 0.348]	[0.105, 0.161, 0.396]	[0.205, 0.232, 0.271]	[0.405, 0.270, 0.274]	[0.077, 0.270, 0.336]
H_3	[0.342, 0.290, 0.349]	[0.255, 0.232, 0.239]	[0.270, 0.270, 0.138]	[0.270, 0.270, 0.270]	[0.290, 0.077, 0.084]

Step 2: Determination of the relation between symptoms and diseases

The relation between symptoms namely, temperature, headache, stomach pain, cough and diseases namely, viral fever, malaria, stomach pain, chest pain in terms of complex neutrosophic numbers has been presented in the following decision matrix (see the table 5).

Table 5: Relation among Symptoms and Diseases in complex neutrosophic form (R-2)

R-2	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
Temperature	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.3e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.3i}, 0.4e^{1.4i}, 0.2e^{1.5i} \right\rangle$	$\left\langle 0.5e^{1.4i}, 0.5e^{1.5i}, 0.2e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.5i}, 0.4e^{0.6i}, 0.5e^{0.7i} \right\rangle$
Headache	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.2e^{0.8i} \right\rangle$	$\left\langle 0.4e^{0.7i}, 0.4e^{0.8i}, 0.3e^{0.9i} \right\rangle$	$\left\langle 0.5e^{0.8i}, 0.4e^{0.9i}, 0.2e^{1.0i} \right\rangle$	$\left\langle 0.5e^{0.9i}, 0.4e^{1.0i}, 0.5e^{0.8i} \right\rangle$
Stomach pain	$\left\langle 0.4e^{1.0i}, 0.4e^{1.1i}, 0.4e^{1.2i} \right\rangle$	$\left\langle 0.5e^{1.1i}, 0.2e^{1.2i}, 0.2e^{1.3i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.3i}, 0.5e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.3i}, 0.4e^{1.4i}, 0.3e^{1.5i} \right\rangle$
Cough	$\left\langle 0.3e^{1.4i}, 0.4e^{1.5i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.5i}, 0.5e^{0.6i}, 0.3e^{0.7i} \right\rangle$	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.3e^{0.8i} \right\rangle$	$\left\langle 0.3e^{0.7i}, 0.4e^{0.8i}, 0.4e^{0.9i} \right\rangle$
Chest pain	$\left\langle 0.4e^{0.8i}, 0.4e^{0.9i}, 0.5e^{1.0i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.3e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.4i}, 0.3e^{0.6i}, 0.2e^{0.8i} \right\rangle$

Numerical values of $(a_2b_2)^{0.5}$, $(c_2d_2)^{0.5}$and $(e_2f_2)^{0.5}$ corresponding to each CNN (from table 5) is presented in the following matrix (see the table 6).

Table 6: Numerical values of $(a_2b_2)^{0.5}$, $(c_2d_2)^{0.5}$and $(e_2f_2)^{0.5}$ corresponding to each CNN (from table 5)

Symtoms	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5}, (e_2f_2)^{0.5}]$
Temperature	[0.232, 0.161, 0.205]	[0.581, 0.637, 0.702]	[0.205, 0.134, 0.138]	[0.158, 0.274, 0.351]
Headache	[0.345, 0.281, 0.141]	[0.281, 0.281, 0.210]	[0.354, 0.279, 0.134]	[0.349, 0.270, 0.354]

Stomach pain	[0.425, 0.247, 0.232]	[0.319, 0.114, 0.100]	[0.232, 0.202, 0.205]	[0.202, 0.164, 0.077]
Cough	[0.122, 0.105, 0.342]	[0.105, 0.342, 0.210]	[0.342, 0.281, 0.212]	[0.210, 0.283, 0.279]
Chest pain	[0.283, 0.279, 0.338]	[0.313, 0.236, 0.122]	[0.236, 0.164, 0.342]	[0.164, 0.205, 0.141]

Step 3: Determination of the similarity measures

The complex cosine, Dice and Jaccard similarity measures C_{CNS}, D_{CNS} and J_{CNS} between the table 3 and the table 5 using equation (6), equation (8) and equation (10) have been presented in the table 7, the table 8 and the table 9.

Table 7: Complex neutrosophic cosine similarity measure between R-1 and R-2

CNCSM	Viral Fever	Malaria	Stomach problem	Chest problem
H_1	**0.9303**	0.9272	0.8662	0.8442
H_2	0.8581	0.7512	0.8148	**0.8681**
H_3	**0.9267**	0.8602	0.8409	0.7864

Table 8: Complex neutrosophic Dice similarity measure between R-1 and R-2

CNDSM	Viral Fever	Malaria	Stomach problem	Chest problem
H_1	**0.8623**	0.8281	0.8596	0.8451
H_2	0.8024	0.7320	0.7935	**0.8307**
H_3	**0.9005**	0.8473	0.8187	0.7672

Table 9: Complex neutrosophic Jaccard similarity measure between R-1 and R-2

CNJSM	Viral Fever	Malaria	Stomach problem	Chest problem
H_1	**0.8595**	0.8114	0.8498	0.8443
H_2	0.8201	0.8019	0.7911	**0.8502**
H_3	**0.8708**	0.8147	0.8469	0.7425

Step 4: Ranking the alternatives

The highest correlation measure from the table 7, table 8 and table 9 reflects the proper medical diagnosis. Therefore, patients H_1 and H_3 suffer from viral fever and patient H_2 suffers from chest problem.

Step 4: End.

Conclusion

In this paper, we have proposed three similarity measures namely, Cosine, Dice and Jaccard similarity measures based on complex neutrosophic sets. We have also proved some of their basic properties. We have presented their applications in a medical diagnosis problem. The concept presented in this paper can be applied to multiple attribute decision making problems, pattern recognition, personnel selection, artificial intelligence in complex neutrosophic environment.

References

[1] Zadeh LA. Fuzzy sets. Inf Cont 1965; 8: 338-353.
[2] Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets Systs 1986: 20: 87-96.
[3] Pankowska A, Wygralak M. General IF-sets with triangular norms and their applications to group decision making. Inf Sci 2006; 176: 2713–2754.
[4] Xu Z. Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. Fuzzy Optimum Decision Making 2007; 6: 109-121.
[5] Xu Z. Intuitionistic preference relations and their application in group decision making. Inf Sci 2007; 177: 2363–2379.
[6] Mondal K, Pramanik S. Intuitionistic fuzzy multicriteria group decision making approach to quality-brick selection problem. J Appl Quant Methods 2014; 9(2): 35-50.
[7] Karsak EE. Personnel Selection Using a Fuzzy MCDM Approach Based on Ideal and Anti-Ideal Solutions. Lecture Notes Econ Math Systs 2001; 507: 393–402.
[8] Pramanik S, Mukhopadhyaya D. Grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Int J Comp Appl 2011; 34(10): 21-29.
[9] Szmidt E, Kacprzyk J. Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets. Notes intuitionistic fuzzy sets 2004; 10(4): 61-69.
[10] De SK, Biswas R, Roy AR. An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets Systs 2011; 117: 209–213.
[11] Khatibi V, Montazer GA. Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition. Artificial Intelligence Med 2009; 47: 43-52.
[12] Smarandache F. A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability, and neutrosophic statistics. Rehoboth: American Research Press; 1998.
[13] Wang H, Smarandache F, Zhang YQ, Sunderraman R. Single valued neutrosophic, sets. Multispace Multistructure 2010; 4: 410–413.
[14] Broumi S, Smarandache F. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets Systs 2014; 6: 43-49.
[15] Guo Y, Cheng HD. New neutrosophic approach to image segmentation. Pattern Recognition 2009; 42: 587–595.
[16] Ye J, Zhang Q. Single valued neutrosophic similarity measures for multiple attribute decision-making. Neutrosophic Sets Systs 2014; 2: 48-54.
[17] Ye J. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. Int J Fuzzy Systs 2014; 16(2): 204-215.
[18] Ye J. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. J Intel Systs 2014; 23(4): 379–389.
[19] Majumdar P, Samanta SK. On similarity and entropy of neutrosophic sets. J Intell Fuzzy Systs 2014; 26: 1245–1252.
[20] Ye J. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. Int J Fuzzy Systs 2014; 16(2): 204-215.
[21] Ye J, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial Intelligence in Medicine 2015; 63: 171-179.
[22] Ye J. Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. J Intell Fuzzy Systs 2014; 27(6): 2927-2935.
[23] Ye J, Zhang QS. Single valued neutrosophic similarity measures for multiple attribute decision making. Neutrosophic Sets Systs 2014; 2: 48-54.

[24] Biswas P, Pramanik S, Giri BC. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic sets Systs 2015; 8: 48-58.
[25] Pramanik S, Mondal K. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global J Adv Research 2015; 2(1): 212-220.
[26] Mondal K, Pramanik S. Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. J New Theory 2015; 8: 41-50.
[27] Mondal K, Pramanik S. Neutrosophic refined similarity measure based on cotangent function and its application to multi attribute decision making. Global J Adv Research 2015; 2(2): 486-496.
[28] Pramanik S, Mondal K, Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. J New Theory 2015; 4: 90-102.
[29] Pramanik S, Mondal K. Some rough neutrosophic similarity measure and their application to multi attribute decision making. Global J Engine Sci Research Manage 2015; 2(7): 61-74.
[30] Ramot D, Milo R, Friedman M, Kandel A. Complex fuzzy sets. IEEE Transaction Fuzzy Systs 2002; 10(2): 171–186.
[31] Ramot D, Friedman M, Langholz G, Kandel A. Complex fuzzy logic. IEEE Transaction Fuzzy Systs 2003; 11(4): 450–461.
[32] Chen Z, Aghakhani S, Man J, Dick S. ANCFIS: a neurofuzzy architecture employing complex fuzzy sets. IEEE Transactions Fuzzy Systs 2011; 19(2): 305–322.
[33] Alkouri A, Salleh A. Complex intuitionistic fuzzy sets. International Conference on Fundamental and Applied Sciences, AIP Conference Proceedings 2012; 1482: 464–470.
[34] Ali M, Smarandache S. Complex neutrosophic set. Neural Comput Appl 2016; 25: 1-18.

Highlights

- We propose complex neutrosophic cosine, Dice and Jaccard similarity measures.
- We establish some of the properties of complex neutrosophic cosine, Dice and Jaccard similarity measures.
- We present an application of neutrosophic complex cosine, Dice and Jaccard similarity measures have to medical diagnosis problem with complex neutrosophic information.
- We conclude that the proposed similarity measures can be applied in multi attribute decision making, pattern recognition, personnel selection, etc problems.

Table 1: The relation between Patients and Symptoms (R-1)

$R-1$	A_1	A_2	$\cdots$	A_k
H_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	$\cdots$	$\langle T_{1k}, I_{1k}, F_{1k} \rangle$
H_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	$\cdots$	$\langle T_{2k}, I_{2k}, F_{2k} \rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
H_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	$\cdots$	$\langle T_{mk}, I_{mk}, F_{mk} \rangle$

Table 2: The relation between symptoms and diseases (R-2)

$R-2$	D_1	D_2	$\cdots$	D_n
A_1	$\langle \xi_{11}, \eta_{11}, \zeta_{11} \rangle$	$\langle \xi_{12}, \eta_{12}, \zeta_{12} \rangle$	$\cdots$	$\langle \xi_{1n}, \eta_{1n}, \zeta_{1n} \rangle$
A_2	$\langle \xi_{21}, \eta_{21}, \zeta_{21} \rangle$	$\langle \xi_{22}, \eta_{22}, \zeta_{22} \rangle$	$\cdots$	$\langle \xi_{2n}, \eta_{2n}, \zeta_{2n} \rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_k	$\langle \xi_{k1}, \eta_{k1}, \zeta_{k1} \rangle$	$\langle \xi_{k2}, \eta_{k2}, \zeta_{k2} \rangle$	$\cdots$	$\langle \xi_{kn}, \eta_{kn}, \zeta_{kn} \rangle$

Table 3: Relation between Patients and Symptoms in complex neutrosophic form (R-1)

R-1	Temperature	Headache	Stomach pain	cough	Chest pain
H_1	$\left\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.2e^{0.8i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.1i}, 0.3e^{0.7i} \right\rangle$	$\left\langle 0.3e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.5e^{1.2i}, 0.3e^{0.8i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.3e^{1.0i}, 0.2e^{0.5i} \right\rangle$
H_2	$\left\langle 0.7e^{1.3i}, 0.4e^{1.2i}, 0.5e^{0.9i} \right\rangle$	$\left\langle 0.4e^{1.5i}, 0.6e^{1.5i}, 0.3e^{0.5i} \right\rangle$	$\left\langle 0.5e^{1.4i}, 0.4e^{1.2i}, 0.4e^{1.0i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \right\rangle$	$\left\langle 0.3e^{1.5i}, 0.4e^{1.0i}, 0.5e^{1.0i} \right\rangle$
H_3	$\left\langle 0.5e^{0.6i}, 0.5e^{1.2i}, 0.5e^{0.9i} \right\rangle$	$\left\langle 0.5e^{1.3i}, 0.4e^{1.2i}, 0.4e^{0.4i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.4e^{1.0i}, 0.2e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.5e^{1.1i}, 0.2e^{1.2i} \right\rangle$	$\left\langle 0.5e^{1.2i}, 0.2e^{1.2i}, 0.2e^{1.4i} \right\rangle$

Table 3: Relation between Patients and Symptoms in complex neutrosophic form (R-1)

R-1	**Temperature**	**Headache**	**Stomach pain**	**cough**	**Chest pain**
H_1	$\left\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.2e^{0.8i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.1i}, 0.3e^{0.7i} \right\rangle$	$\left\langle 0.3e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.5e^{1.2i}, 0.3e^{0.8i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.3e^{1.0i}, 0.2e^{0.5i} \right\rangle$
H_2	$\left\langle 0.7e^{1.3i}, 0.4e^{1.2i}, 0.5e^{0.9i} \right\rangle$	$\left\langle 0.4e^{1.5i}, 0.6e^{1.5i}, 0.3e^{0.5i} \right\rangle$	$\left\langle 0.5e^{1.4i}, 0.4e^{1.2i}, 0.4e^{1.0i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.4e^{1.0i}, 0.4e^{0.6i} \right\rangle$	$\left\langle 0.3e^{1.5i}, 0.4e^{1.0i}, 0.5e^{1.0i} \right\rangle$
H_3	$\left\langle 0.5e^{0.6i}, 0.5e^{1.2i}, 0.5e^{0.9i} \right\rangle$	$\left\langle 0.5e^{1.3i}, 0.4e^{1.2i}, 0.4e^{0.4i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.4e^{1.0i}, 0.2e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.0i}, 0.5e^{1.1i}, 0.2e^{1.2i} \right\rangle$	$\left\langle 0.5e^{1.2i}, 0.2e^{1.2i}, 0.2e^{1.4i} \right\rangle$

Table 5: Relation among Symptoms and Diseases in complex neutrosophic form (R-2)

R-2	Viral Fever	Malaria	Stomach problem	Chest problem
Temperature	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.3e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.3i}, 0.4e^{1.4i}, 0.2e^{1.5i} \right\rangle$	$\left\langle 0.5e^{1.4i}, 0.5e^{1.5i}, 0.2e^{0.6i} \right\rangle$	$\left\langle 0.6e^{1.5i}, 0.4e^{0.6i}, 0.5e^{0.7i} \right\rangle$
Headache	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.2e^{0.8i} \right\rangle$	$\left\langle 0.4e^{0.7i}, 0.4e^{0.8i}, 0.3e^{0.9i} \right\rangle$	$\left\langle 0.5e^{0.8i}, 0.4e^{0.9i}, 0.2e^{1.0i} \right\rangle$	$\left\langle 0.5e^{0.9i}, 0.4e^{1.0i}, 0.5e^{0.8i} \right\rangle$
Stomach pain	$\left\langle 0.4e^{1.0i}, 0.4e^{1.1i}, 0.4e^{1.2i} \right\rangle$	$\left\langle 0.5e^{1.1i}, 0.2e^{1.2i}, 0.2e^{1.3i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.3i}, 0.5e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.3i}, 0.4e^{1.4i}, 0.3e^{1.5i} \right\rangle$
Cough	$\left\langle 0.3e^{1.4i}, 0.4e^{1.5i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.5i}, 0.5e^{0.6i}, 0.3e^{0.7i} \right\rangle$	$\left\langle 0.5e^{0.6i}, 0.4e^{0.7i}, 0.3e^{0.8i} \right\rangle$	$\left\langle 0.3e^{0.7i}, 0.4e^{0.8i}, 0.4e^{0.9i} \right\rangle$
Chest pain	$\left\langle 0.4e^{0.8i}, 0.4e^{0.9i}, 0.5e^{1.0i} \right\rangle$	$\left\langle 0.6e^{1.0i}, 0.4e^{1.2i}, 0.3e^{1.4i} \right\rangle$	$\left\langle 0.4e^{1.2i}, 0.4e^{1.4i}, 0.5e^{0.6i} \right\rangle$	$\left\langle 0.4e^{1.4i}, 0.3e^{0.6i}, 0.2e^{0.8i} \right\rangle$

Table 6: Numerical values of $(a_2b_2)^{0.5}$, $(c_2d_2)^{0.5}$and $(e_2f_2)^{0.5}$ corresponding to each CNN (from table 5)

Symtoms	Viral Fever	Malaria	Stomach problem	Chest problem
	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5},(e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5},(e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5},(e_2f_2)^{0.5}]$	$[(a_2b_2)^{0.5},(c_2d_2)^{0.5},(e_2f_2)^{0.5}]$
Temperature	[0.232, 0.161, 0.205]	[0.581, 0.637, 0.702]	[0.205, 0.134, 0.138]	[0.158, 0.274, 0.351]
Headache	[0.345, 0.281, 0.141]	[0.281, 0.281, 0.210]	[0.354, 0.279, 0.134]	[0.349, 0.270, 0.354]
Stomach pain	[0.425, 0.247, 0.232]	[0.319, 0.114, 0.100]	[0.232, 0.202, 0.205]	[0.202, 0.164, 0.077]
Cough	[0.122, 0.105, 0.342]	[0.105, 0.342, 0.210]	[0.342, 0.281, 0.212]	[0.210, 0.283, 0.279]
Chest pain	[0.283, 0.279, 0.338]	[0.313, 0.236, 0.122]	[0.236, 0.164, 0.342]	[0.164, 0.205, 0.141]

Table 7: Complex neutrosophic cosine similarity measure between R-1 and R-2

CNCSM	Viral Fever	Malaria	Stomach problem	Chest problem
H_1	**0.9303**	0.9272	0.8662	0.8442
H_2	0.8581	0.7512	0.8148	**0.8681**
H_3	**0.9267**	0.8602	0.8409	0.7864

Table 8: Complex neutrosophic Dice similarity measure between R-1 and R-2

CNDSM	Viral Fever	Malaria	Stomach problem	Chest problem
H_1	**0.8623**	0.8281	0.8596	0.8451
H_2	0.8024	0.7320	0.7935	**0.8307**
H_3	**0.9005**	0.8473	0.8187	0.7672

Table 9: Complex neutrosophic Jaccard similarity measure between R-1 and R-2

CNJSM	**Viral Fever**	**Malaria**	**Stomach problem**	**Chest problem**
H_1	**0.8595**	0.8114	0.8498	0.8443
H_2	0.8201	0.8019	0.7911	**0.8502**
H_3	**0.8708**	0.8147	0.8469	0.7425