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A SUCCESSFUL APPLICATION OF DSMT IN SONAR GRID MAP BUILDING AND COMPARISON WITH DST-BASED APPROACH

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Abstract. *Robot perception is becoming more and more popular with the development of artificial intelligence in computer science. Many sensors are usually involved to get a better perception of the surrounding unknown environment, especially multi-sonar sensors because of their low cost, simplicity and convenience. However the information acquired from multi-sonar sensors in building maps presents characteristics of uncertainty, imprecision and even high conflict, which have a great impact on the low quality of robot perception and this important problem must be solved. In this paper, a new method of information fusion based on DSmT (Dezert-Smarandache Theory) is introduced for dealing with uncertain, imprecise and conflicting multi-sonar information. A Pioneer II mobile robot with onboard multi-sonar sensors evolving in an unknown environment has been chosen for our experimental platform. This robot can perceive the environment through sonar sensors coupled with classic DSm model and the construction of a general basic belief assignment function (gbbaf), so that the global evidence grid map can be built using the classic DSm fusion rule. Our experimental results show clearly that the rate of accuracy, the quality of grid map building and the time for building the global map are much better than the performances obtained in the same conditions with the classical DST-based approach and Demspter's fusion rule, especially when we deal with highly conflicting information from sonar grid maps building. Our results proves that DSmT is more efficient and successful than DST for this type of application. In short, this study proposes a new method for building map, and supplies a new interesting shortcut for human-computer interface for mobile robot exploring unknown environment.*
Keywords: DSmT, Information fusion, Grid map building, DST, Uncertainty management

1. **Introduction.** To explore the unknown environment by applying and deploying robots, both in research and in industrial contexts, we must develop more powerful and flexible robotic systems exhibiting higher degrees of autonomy and able to sense, plan, and operate in unstructured environments [1]. In order to realize the aim, we need to make the robot interact coherently with its world, both by being able to recover robust and useful spatial descriptions of its surroundings using sensory information and by efficiently utilizing

these descriptions in an appropriate short-term and long-term planning and decision-making activities. For example, robot rovers such as Spirit and Opportunity developed for planetary and space exploration, or autonomous submersibles devoted to submarine prospecting and surveying, have to deal with unexpected circumstances and require the ability to handle complex and rough environments with little or no prior knowledge of terrain. In addition, mobile robots developed for factory automation purposes or operation in hazardous mining environments or nuclear facilities generally can be expected to operate in more constrained situations and to have access to precompiled maps derived from plant blueprints. However, the accumulative substantial positional errors occur from the inertial or dead-reckoning navigation, especially for a long distance travel. Therefore, a contradiction occurs between self-localization and map building, which can be compared to the famous *chickens and eggs* puzzle [2, 3]. To solve this puzzle, some experts in the robot field have proposed many methods in self-localization [5] and map building respectively. In this study, we mainly apply DSmT (Dezert-Smarandache Theory [6, 7], which is a new theory of information fusion proposed by Jean Dezert and Florentin Smarandache based on the natural extension of DST (Dempster-Shafer Theory [8]) to sonar maps building with grid method [4]. The sonar map building with grid method remains so far one of the most successful and popular methods for building map in robotics. Although sonar sensors own inherent restrictions due to their physical characteristics, DSmT approach presents a powerful ability in dealing with uncertainty, imprecision and even high conflict information induced by the inherent sonar sensors physical limitations. Of course, with regard to the positional errors, we need to adopt a self-localization method. In this study, because the experiment environment is relatively small, and the accumulative error is very small, the self-localization method is ignored. Before using DSmT approach in robotics, a previous attempt for map building of mobile robot based on DST has been proposed in literature by Su et al. in [9]. But it is very difficult to deal with highly conflicting information when using Dempster's rule, because as soon as the level of conflict increases the Dempster's fusion rule becomes inefficient and can generate counter-intuitive results. In the sequel, we provide a detailed comparison in maps building based on the two approaches (i.e. those based on DSmT and DST). The experiment result shows clearly the advantage to build map with DSmT rather than with DST.

This paper is organized as follows: in Section 2, we briefly review the basics of DSmT; in Section 3, we present the sonar model for uncertainty representation used in the DSm framework; in Section 4, we present the map DSmT-based building process through our self-developing software when the mobile robot is evolving in the unknown environment. We also compare the results with those obtained from DST-based map building approach. It is shown the superiority of DSmT approach over the DST one. Concluding remarks are then given in Section 5.

2. Simple Review of DSmT.

2.1. The frame of discernment for DSmT. Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, here Θ is the frame of discernment, which includes n finite elements θ_i $(i = 1, 2, \cdots, n)$. In the free DSm model[1], denoted $\mathcal{M}^f(\Theta)$ [6], the elements $\theta_i \in \Theta$ are not precisely defined and separated,

[1]This is the model adopted in this study.

so that no refinement of Θ in a new larger set Θ_{ref} of disjoint elementary hypotheses is possible.

2.2. **Hyper-power set of DSmT.** The hyper-power set D^Θ is defined as the set of all propositions from Θ with $\cap$ and $\cup$ operators, such that:

a) $\emptyset, \theta_1, \theta_2, \cdots, \theta_n \in D^\Theta$.
b) If $A, B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$.
c) No other elements belong to D^Θ, except those obtained by using rules a) or b).

2.3. **General belief and plausibility functions.** Let's consider a general frame of discernment $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$. For every evidential source S, let us define a mapping $m(.) : D^\Theta \to [0, 1]$ associated to it (so abandoning Shafer's model since here we work on hyper-power set D^Θ rather than classical power set 2^Θ) because wants to take into account the inherent fuzzy/vague/relative nature of elements θ_i $(i = 1, 2, \cdots, n)$ so that the elements can be truly non-exclusive and no refinement of Θ into a new finer exclusive frame of discernment Θ_{ref} is possible. The mapping $m(.)$ is called a generalized basic belief assignment function (gbbaf) when it satisfies

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in D^\Theta} m(A) = 1. \tag{1}$$

The general belief function and plausibility function are defined respectively in almost the same manner as within the DST [8], i.e.

$$\mathrm{Bel}(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B) \tag{2}$$

$$\mathrm{Pl}(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B) \tag{3}$$

2.4. **Classical (free) DSmT rule of combination.** Let $\mathcal{M}^f(\Theta)$ be a free DSm model. The classical (free) DSm rule of combination (denoted (DSmC) for short) for $k \geq 2$ sources is given $\forall A \neq \emptyset$, and $A \in D^\Theta$ as follows:

$$m_{\mathcal{M}^f(\Theta)}(A) = [m_1 \oplus \cdots \oplus m_k](A) = \sum_{\substack{X_1, \cdots, X_k \in D^\Theta \\ X_1 \cap \cdots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i) \tag{4}$$

3. **Modeling for Sonar Grid Information.** Sonar sensors' working principle (shown as Figure 1) is: producing sheaves of cone-shaped wave, and detecting the objects by receiving the reflected wave. Due to the restriction of sonar physical characteristic, metrical data behaves out uncertainty as follows:

(a) Beside its own error of making, the influence of external environment is also very important (for example, the temperature, the humidity, the atmospheric pressure and so on).

(b) Because the sound wave spreads outwards in the form of loudspeaker, and there exists a cone-shaped angle, we cannot know the true position of objects detected among the fan-shaped area, with the enlargement of distance between sonar and the object/obstacle.

(c) The use of a lot of sonar sensors will generate interferences between the sensors. For example, when the i*th* sonar gives out detecting wave towards an object of irregular shape, if the angle of incidence is too large, the sonar wave might be reflected out of the receiving range of the i*th* sonar sensor or also might be received by other sonar sensors.

(d) Because sonar sensors utilize the reflection principle of sound wave, when an object absorbs significantly a sound wave, the sonar sensor measurement might become invalid or just unavailable at all.

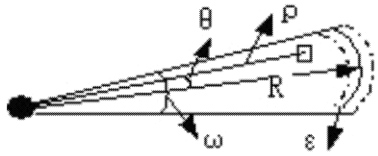

FIGURE 1. Sketch of the principle of sonar

3.1. **Modeling of uncertain sonar sensor based on DSmT.** Pointing to the characteristics of sonar's measurement, we construct a model of uncertain information acquired from grid map using sonar based on DSmT. Here we suppose there are only two hypotheses in the frame committed to each cell of the grid map, that is, $\Theta = \{\theta_1, \theta_2\}$, where θ_1 means *grid cell is empty*, θ_2 means *grid cell is occupied*. The hyper-power set of Θ is then defined as $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cap \theta_2, \theta_1 \cup \theta_2\}$. Every grid cell in environment is scanned $k \geq 2$ times, each of which is viewed as source of evidence. Then we may define a set of map aiming to every source of evidence and construct the general basic belief assignment functions (gbbaf) as follows: $m(\theta_1)$ is defined as the gbbaf for grid-unoccupancy (emptiness); $m(\theta_2)$ is defined as the gbbaf for grid-occupancy; $m(\theta_1 \cap \theta_2)$ is defined as the gbbaf for holding grid-unoccupancy and occupancy simultaneous (conflict). $m(\theta_1 \cup \theta_2)$ is defined as the gbbaf for grid-ignorance due to the restriction of knowledge and experience presently[2], it reflects the degree of ignorance of grid-unoccupancy or occupancy.

The gbbaf $m(.) : D^\Theta \to [0, 1]$ is then constructed according to the formulas (5)-(8) taking into account the physical characteristics of sonar sensors, i.e.

$$m(\theta_1) = E(\rho) \cdot E(\theta) = \begin{cases} (1 - (\rho/(R-2\epsilon))^2) \cdot (1 - \lambda/2) & R_{min} \leq \rho \leq R-2\epsilon, \quad 0 \leq \theta \leq \omega/2 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$m(\theta_2) = O(\rho) \cdot O(\theta) = \begin{cases} \exp(-3\rho_\nu(\rho - R)^2) \cdot \lambda & R_{min} \leq \rho \leq R+2\epsilon, \quad 0 \leq \theta \leq \omega/2 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$m(\theta_1 \cap \theta_2) = \begin{cases} (1 - (2(\rho - (R - 2\epsilon))/R)^2) & R_{min} \leq \rho \leq R+\epsilon, \quad 0 \leq \theta \leq \omega/2 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

[2]Here referring to the gbbaf for these grids still not scanned presently.

$$m(\theta_1 \cup \theta_2) = \begin{cases} \tanh(2(\rho - R))(1-\lambda) & R \le \rho \le R+2\epsilon, \quad 0 \le \theta \le \omega/2 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

with

$$\lambda = \begin{cases} 1 - (2\theta/\omega)^2 & 0 \le |\theta| \le \omega/2 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where $E(\theta) = 1 - \lambda, O(\theta) = \lambda, E(\rho) = (1 - (\rho/(R - 2\epsilon))^2), O(\rho) = \exp(-3\rho_\nu(\rho - R)^2)$. ρ_ν in (6) is defined as the adjusting function of environment (when we don't consider the effect of the uncertainty of angle θ on the gbbaf). Figure 2 shows the effect of ρ on gbbaf $m(.)$.

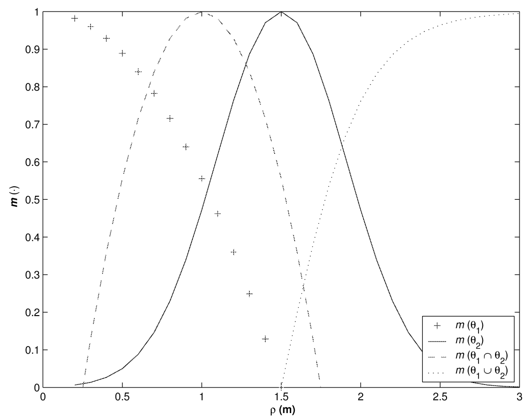

FIGURE 2. $m(\cdot)$ as function of ρ given by (5)-(8).

Seen from Figure 2, gbbaf reflects really out the characteristics of sonar information with the shift of ρ in the course of building grid map. Here we assume the range of sonar sensor from $0.2m \sim 3m$. Highly conflicting information occurs at the point of intersection of two curves between $m(\theta_1)$ and $m(\theta_2)$. The maximum $m(\theta_2)$ is reached at distance R, while the maximum of $m(\theta_1)$ is reached at $R_{min} = 0.2m$. Of course, in order to satisfy the definition of DSmT, and assure the sum of all mass of to be one, we must renormalize them while acquiring sonar grip information.

4. **Modeling Based on DST.** Su et al. have proposed in [9] a solution to robot's map building problem in the classical DST framework. In order to solve the uncertainty of sonar sensors, we can suppose a uniform distribution of probability of existence of the obstacles in the area above the segment AB; that is, points to the part in Figure 3. The area of part 2 in Figure 3 represents the empty area.

Therefore, in the *DST* framework and as in DSmT, we consider also only two hypotheses in the frame of discernment, that is, $\Theta = \{\theta_1, \theta_2\}$, but we consider them as *truly exclusive* and so we work on classical power set $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. θ_1 and θ_2 represent respectively the emptiness and occupancy states of the grid cell under consideration in the map. In DST, these two states are considered as totally disjointed. For example, θ_2

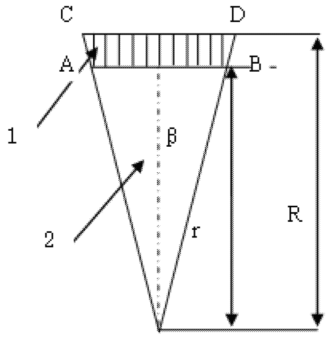

FIGURE 3. Simplified representation of Shafer's model for sonar

represents existence of obstacle, that is, the grid cell is full or occupied; θ_1 represents non-existence of an obstacle, that is, the grid is empty or unoccupied; $\theta_1 \cup \theta_2$ represents the ignorance of grid for the limitation of current knowledge and experience. In [9], only two states θ_1, θ_2 are used, that is, we must define a basic belief assignment function $m(\theta_1)(x, y)$ and $m(\theta_2)(x, y)$. Here cell (x, y) refers to the location of grid cell in the environment. So the basic belief assignment functions for the area of part 1 of Figure 3 are given as follows:

$$m(\theta_1)(x,y) = 0, \qquad \text{and} \qquad m(\theta_2)(x,y) = \frac{1}{2r\tan(\beta)} \tag{10}$$

where all cells (x, y) are defined as in part 1 in Figure 3; that is, there is a greater probability of existence of obstacle near the location, where the sound is returned. Here r refers to the distance, which is the vertical distance from the spot of sonar emission to the segment/chord AB. R refers to the sonar reading/measurement.

While aiming to the area of part 2, generally speaking, the obstacle doesn't exist, so that we define the basic belief assignment functions as follows:

$$m(\theta_1)(x,y) = 1, \qquad \text{and} \qquad m(\theta_2)(x,y) = 0 \tag{11}$$

Dempster's rule of combination for two sources is given by [8]: $m(\emptyset) = 0$ and $\forall A \neq \emptyset \in 2^\Theta$,

$$m(A) = \frac{\displaystyle\sum_{\substack{X,Y \in 2^\Theta \\ X \cap Y = A}} m_1(X)m_2(Y)}{1 - \displaystyle\sum_{\substack{X,Y \in 2^\Theta \\ X \cap Y = \emptyset}} m_1(X)m_2(Y)} \tag{12}$$

5. Experimental Results.

5.1. **Fusion of sonar information.** The fusion arithmetic can be realized through our self-developing software platform shown in Figure 5, which is based on the developing tool of Visual C++6.0 and OpenGL. Here the platform serves as a client-end, which can connect the server-end (also developed under the Linux by ourselves, which connects the

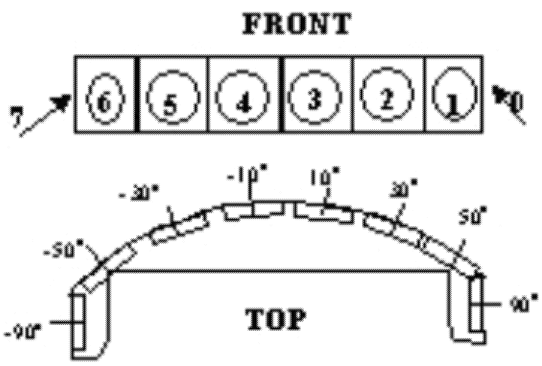

FIGURE 4. Sketch of the layout of sonars

Pioneer II mobile robot and the client). We upload the server procedure to embedded Linux system of Pioneer II mobile robot through the file transfer protocol (FTP).

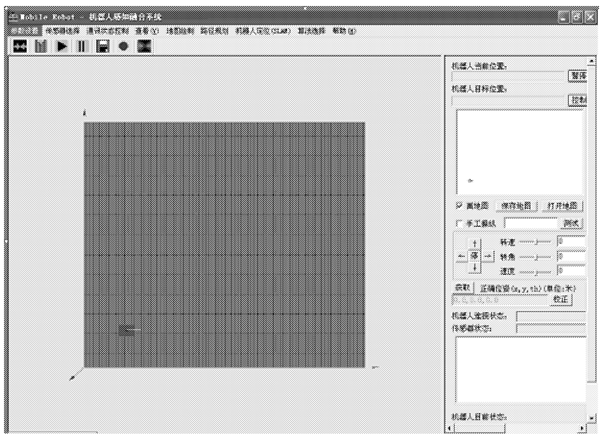

FIGURE 5. Software platform for robot intelligent perception

When the robot moves in the environment (see Figure 6), the sever-end collects much information (i.e. the location of robot, its velocity, sensors measurements, etc.) from sensors onboard. Through the protocol of TCP/IP, the client-end can get any information from the sever-end and fuse them.

To construct the map of the unknown environment, self-localization of the mobile robot is needed, so in our system, a self-calibration and localization arithmetic based on Hough transformation of laser line feature is applied. However, when self-localization doesn't work, manual calibration and localization in our system is very helpful. Of course, in this study, because the environment is relative small and the accumulated error is very small, the self-localization arithmetic is not considered here. Pioneer II mobile robot carries 16 Sonar sensors used as the main perception tool because of their advantage of low cost, simplicity and convenience. Only 8 front sonar sensors are shown on Figure 4 and their distribution is asymmetrical.

FIGURE 6. The original environment where mobile robot evolves

Here we only give the procedure of fusion based on DSmT in Figure 7. In fact, DST fusion process is very similar to DSmT. Fusion steps are listed as follow:

1. Initialize the parameters of robot (i.e. initial location, velocity, etc.).
2. Acquire 16 sonar measurements, and robot's location, when the robot is moving (set also the time stamp; the measurement period used in our experiment is 100 ms).
3. Compute the gbbf of the fan-form area detected by each sonar sensor according to (5)-(8).
4. Whether some cells are scanned renewedly by sonar sensors (same sonar in different locations, or different sonar sensors, of course, here we suppose that each sonar sensor has the same characteristics)? if yes, go to next step, otherwise, go to step 6.
5. Compute the new basic belief masses according to (4) when DSmT is used (or using (12) when DST is used), and save and replace the old ones.
6. Compute the credibility of occupancy of the grid-cells, which have been fused according to (2).
7. Update the map of the environment. (start the second timer, of which interval is 100 ms). Note that the more the fusion times one has, and the more precise the perception of the environment is. Whether the robot stops receiving sonar data? If yes, stop fusion and exit; otherwise, go back to step 2.

5.2. **Comparison of fusion results between DSmT and DST.** The environment's size is $4550mm \times 3750mm$ and the grid map method is adopted. The environment is divided into 91×75 rectangle cells of the same size. The mobile robot begins to move at the location (1 m, 0.6 m), which faces towards 0 degree. We take the corner of left bottom as the global coordinate origin of the map. Objects/obstacles in rectangular grid map are sketched in Figure 6. Since the environment is small and the robot runs during a quite short period of time, the precision for self-localization of the mobile robot is enough to use the proposed fusion approach. To save the amount of computation, we adopt the arithmetic of restricted spreading. Grid map of belief layout obtained for the perception of objects in Figure 6 with the DST based approach is shown on Figure 8. The result of perception and map building based on DSmT approach is shown on Figure 9.

Analysis of the fusion results:

1) According to Figure 9, the grid map of belief layout based on DSmT shows a clear and precise location of obstacles. This method supplies with a shortcut to do some research on self-localization, path planning and navigation of mobile robot.
2) Though both DSmT and DST can be applied to building grid maps from sonar sensors, DSmT can express more clearly the grid information than DST, which can be seen easily from the comparison between Figure 8 and Figure 9 in terms of the grid map of the original environment as depicted in Figure 6.
3) DSmT can deal with highly conflicting information, while DST cannot do. So when we apply DST to grid map building, it might fail. Under this situation, we sometimes have to abandon this evidence source, and try another one. If so, the total amount of computing will increase, the time to complete the building of the global grid map also increases, which is clearly shown in Table 1 below when the robot runs almost along the same track. Sometimes the global grid map can't be built at all or is

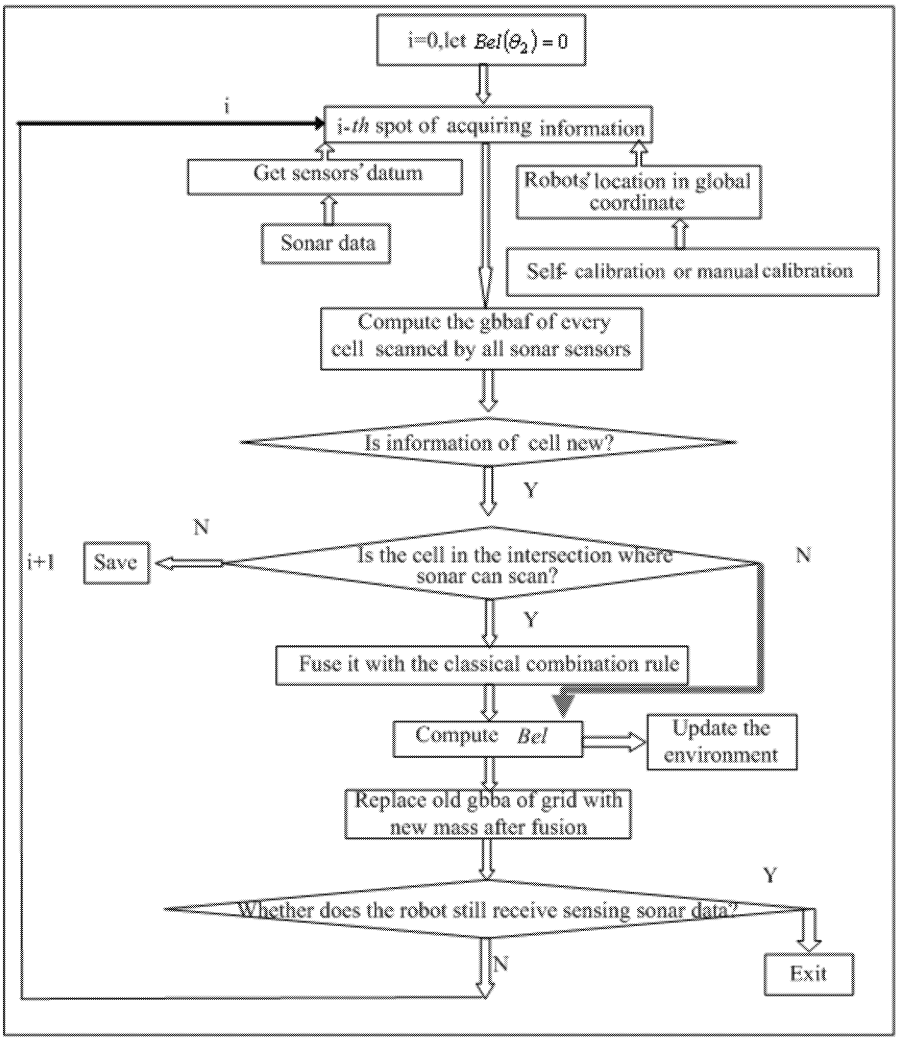

FIGURE 7. Flowchart of the procedure of sonar map building based on DSmT

FIGURE 8. Grid map built with DST-based approach

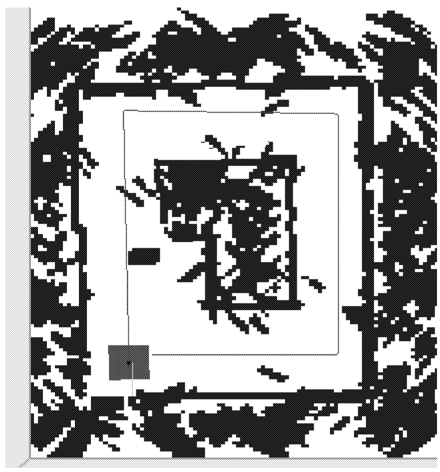

FIGURE 9. Grid map built with DSmT-based approach

totally mistaken, which can be also seen in Table 1, where the level of accuracy for DSmT is higher than DST.

4) In this study, though we define two elementary cell states in either free DSm model or Shafer's model, according to their combination rule, DSmT should have greater amount of computing cost, however, in fact it is inverse. When analyzing carefully the fusion process, it can be shown that it must take much more time for dealing properly with high conflicting information with DST than with DSmT because Dempster's rule does not respond quickly to new contradicting information [7] (chap. 13). If we assume the possibility of further refinement of the frame, in order to start with the same information as DSmT, we must define three hypotheses in Shafer's model, and then the cost of computations is obviously greater than that the one necessary within DSmT approach [6, 7].
5) In Table 1, we provide a comparison of performances (in term of accuracy, quality of grid map building and the time in need of finishing the global map building) between

DSmT and DST approaches. The results indicate clearly the superiority of DSmT over DST.

TABLE 1. Comparison of grid map building with DSmT and DST based approaches

Method/Items	Total time(ms)	Rate of accuracy ($percent$)	Quality of map
DST	36614	93	general
$DSmT$	23632	97	high

6. **Conclusions.** In this paper, we have applied both DSmT and DST approaches for mobile robot's map building in a small static environment. Through the experiment, DSmT proved to be more efficient than DST. When the size of environment becomes very large and complex, or irregular and the robot's position cannot be neglected, the proposed system must also solve in the meantime the self-localization problem. As a concluding remark, it has been proved in this sturdy that DSmT can solve information fusion of multi-source and build helpful grid maps effectively, and improve significantly the performance of the overall perception of the environment. Our approach also provides a useful shortcut for human-computer interface for autonomous exploration of an unknown environment by a mobile robot.

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