Conflict Decision Method based on Quadratic Combination

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Abstract: There are many unsatisfactory situations in the existing improvement methods of evidence theory, such as a large amount of calculation, the normalization process is unreasonable, the evidence combination effect is not ideal in the conflict evidence decision-making process, and so on. This paper proposes a method based on quadratic combination of conflict evidence to improve the above situations. Firstly, a new flow chart of conflict evidence decision method based on quadratic combination is proposed. Secondly, a new multiplicative normalization rule is proposed, and the new rule is analyzed to verify its rationality. Thirdly, the shortcomings of the existing conflict measurement methods are analyzed, a new conflict measurement function is proposed, and the rationality of the new function is analyzed. Finally, through the analysis of the example and comparison with the existing evidence combination rules, the effectiveness of the method of this paper is verified.

1. Introduction

DS evidence theory is proposed and promoted by Dempster-Shafer [1, 2], which can well characterize, integrate and decide uncertain information. At the same time, it is widely used in information fusion, risk assessment, pattern recognition, etc. [3-6]. However, in engineering applications, due to the measurement accuracy of the source and the interference of the external environment, the information that needs to be processed is often conflicting. The Dempster combination rule given in DS evidence theory cannot effectively deal with high conflict information [7,8]. For example, Zadeh gives a classic 0 trust paradox example in [9]. In order to solve this problem, a number of improved algorithms have been proposed by relevant scholars [10,11]. The improvement methods of evidence theory are mainly divided into two categories.

One type of scholars believe that the Dempster combination rule does not deal with conflict evidence well because of the establishment of combination rule. Therefore, this type of scholars have made a lot of improvements to the formulas of combination rules, and proposed many new combination rules. The improvement of the Dempster combination formula is mainly divided into three categories. The first category is to study how to allocate conflict k. For example, Smets [12, 13] proposed a new combination rule, in which k is allocated to the empty set, this rule avoids the normalization process of the evidence theory. The new combination rule proposed by Yager [14, 15] assigns k to the completely uncertain set. PCR1-6 [16-18] focus on how to distribute the conflict amount to each proposition reasonably, PCR2 and PCR5 are more widely used. The second category is to change the recognition framework and extend it to the generalized power set. On this basis, new combination rules are given, such as DSmT combination rules. The third category is to give additive combination rules, such as the Murphy [19] combination rule.

Although the improvement of the combination rules can solve the fusion of conflict evidence, the effect is not very satisfactory. The key to solving the conflict evidence combination problem is to correct the evidence. The research of the revised evidence source is focused on how to obtain the discount weight of the evidence. There are two methods for determining the weight coefficient of evidence discount. One is based on information entropy. U. Höhle [20] et al. proposed Confusion metrics based on the likelihood function. Yager [21] et al. proposed Dissonance metrics based on the plausibility function. Klir [22, 23] et al. proposed Discord metrics and Strife metrics. Deng[24] proposed Deng entropy which can measure the uncertainty of the evidence. The above method can verify the uncertainty of information to some extent. However, the disorder of evidence represented by information entropy cannot reasonably represent the degree of support between information, so it is not suitable as the weight coefficient of discount evidence.

The second is the uncertainty of evidence based on distance. The Jousselme distance proposed by Jousselme [25] is the most widely used evidence distance. Yu [26] et al. proposed the support probability distance, and they thought the discount weight of evidence is determined by calculating the degree of evidence support. Zhou [27] et al. obtained a new weight coefficient by calculating the existing evidence distance. Liu [28] et al. proposed a new evidence distance based on the probability conversion DSmP. Yang [29] used the Tran&Duckstein interval distance to measure the uncertainty of evidence and got good fusion effect.

Although many of the conflict evidence decision methods based on information entropy and distance are introduced above, the process of solving the conflict evidence combination is almost same. They are all first to calculate the degree of support between the evidences, and then calculate the discount coefficient of each piece of evidence through the support matrix. Secondly, discount the evidence and finally combine the discount evidence. The calculation amount of the degree of support between the evidences is relatively large, the normalization process of calculating the support vector is not very reasonable, and the evidence combination effect is not satisfactory. So this paper proposes a process of conflict evidence decision-making based on quadratic combination. Firstly, it calculates the PCR6 combination rule of the evidence group. Secondly, it calculates the distance of the evidence after each evidence distance combination, determines the discount coefficient and discounts the evidence group. Finally, the evidence is combined by using the PCR6 evidence combination rule once again.

2. Preliminaries

X is the recognition object, U is the set of possible values of X, and all the elements in U are incompatible with each other, so U is called a recognition frame.

Suppose U is the recognition framework of X, when function $m:U\to[0,1]$ satisfies the following conditions

$$\begin{cases} m(\phi)=0 \\ \sum_{A\in U} m(A)=1 \end{cases} \tag{1}$$

The function $m(A)$ is called the basic probability assignment (BPA), which indicates the degree of evidence support for the proposition A. Where $m(U)$ is unknown.

Suppose U is a recognition framework, m is BPA, define

$$Bel(A)=\sum_{B\subset A} m(B),(\forall A\subset U)$$

Which is a trust function, and A is the sum of the trusts of all subsets of the decision. Therefore, the trust value for the empty set is $Bel(\phi)=0$, and for the complete set U is $Bel(U)=1$.

Define

$$Pl(A)=\sum_{B\cap A\neq\phi} m(B)$$

Which is a plausibility function, It is not difficult to get the theorem from definition, that is

$$Pl(A) = 1 - Bel(\bar{A})$$

The trust function and the plausibility function satisfy $Bel(A) \le Pl(A)$. Define interval $L = [Bel(A), Pl(A)]$ is the trust interval, and the interval length reflects the uncertainty of the proposition B. When the length of the trust interval becomes 0, that is, $Bel(A) = Pl(A)$, the evidence theory degenerates into probability theory.

Dempster combination rules: Assuming that m_1, m_2 is the basic probability assignment on the recognition framework, then

$$m_{12}(C) = \begin{cases} \dfrac{\sum\limits_{i,j,A_i \cap B_j = C} m_1(A_i) m_2(B_j)}{1-k} & C \ne \phi \\ 0 & C = \phi \end{cases} \tag{2}$$

Where $k = \sum\limits_{i,j,A_i \cap B_j = \phi} m_1(A_i) m_2(B_j) < 1$, and it is the coefficient of conflict.

PCR6 combination rules: Assuming that m_1, m_2 is the basic probability assignment of the recognition framework U , then

$$m_{PCR5}(X) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2) + \sum_{\substack{Y \in 2^\Theta \setminus \{X\} \\ X \cap Y = \phi}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_1(X) + m_2(Y)} \right] \tag{3}$$

3. Conflict Evidence Decision based on Quadratic Combination

3.1 Uncertain Reasoning Model of Discount Evidence Based on Conflict Metrics

Dempster combination rules will produce paradox when dealing with conflict evidence. For this problem, relevant scholars have given a variety of improvement methods, which are mainly divided into two categories. The first category is the combination formula of the original DS evidence theory. Improvement; the second category is the revision of the original evidence. The first category is to improve the original combination formula of DS evidence theory, the second is to correct the original evidence. At present, most scholars accept the second improvement method, and believe that the conflict of evidence is fundamentally derived from the conflict and inaccuracy of obtaining data.

A complete algorithm model has been formed for the improvement of conflict evidence fusion. The model is shown in Figure 1.

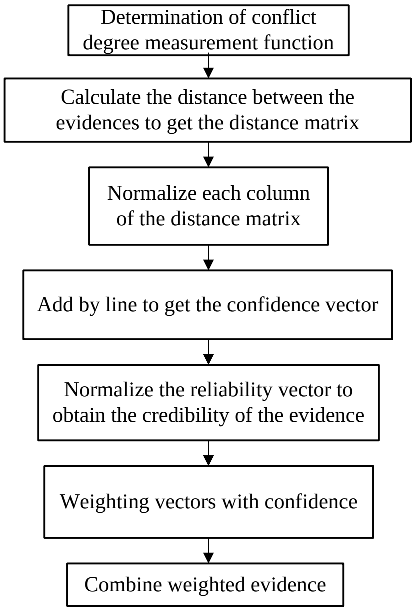

Figure 1. Discount evidence combination model

1) Determine the conflict measure function. This is a critical step. The current improved algorithms for correcting data sources are based on improvements to different conflict metric functions. The conflict metric function not only needs to satisfy the physical meaning, that is, it has a positive correlation with the distance between the evidences, but also needs to satisfy the mathematical conditions, that is, several basic conditions that satisfy the norm.

Suppose the basic probability assignment of the three sets of mappings in the same propositional space are $m_1(\cdot)$, $m_2(\cdot)$ and $m_3(\cdot)$. Then there is a mapping $\mathrm{CM}(\cdot): U \times U \to [0,1]$ that satisfies the following four basic conditions, called the conflict metric function.

a) Symmetry

$\forall m_1(\cdot), m_2(\cdot), \mathrm{CM}(m_1, m_2) = \mathrm{CM}(m_2, m_1)$.

b) Positive definiteness

$\forall m_1(\cdot), m_2(\cdot) \in U$, when $m_1 \neq m_2$, $\mathrm{CM}(m_1, m_2) \geq 0$. When $m_1 = m_2$, $\mathrm{CM}(m_1, m_2) = 0$.

c) Homogeneous

$\mathrm{CM}(k_1 m_1, k_2 m_2) = k_1 k_2 \mathrm{CM}(m_1, m_2)$, $k \in \mathbf{P}$.

d) Triangular inequalities

$\mathrm{CM}(m_1, m_2) \leq \mathrm{CM}\left(m_1, m_3\right) + \mathrm{CM}\left(m_2, m_3\right)$.

The existing conflict metric functions are roughly divided into two categories, one is based on information entropy, and the other is based on the distance between evidences.

2) Calculate the distance between the evidences to obtain the distance matrix D , and the element d_{ij} represents the distance between the i th matrix and the j th evidence. The distance matrix is a symmetric square matrix, and the elements of the diagonal are 0.

3) Normalize each column of the distance matrix, and obtain a vector of the i th column of the matrix which indicates the degree of support of the i th matrix for each piece of evidence, thereby facilitating subsequent operations.

4) Adding each row of the normalized matrix to obtain a vector, the k th element of the vector indicating the total extent that the k th evidence is supported by other evidence.

5) Normalize the vector obtained in the fourth step to obtain the normalized credibility weight.

6) Use the confidence weight to calculate the discount of the evidence. The commonly used discount formula is

$$\begin{cases} m'_i\left(A_j\right)=\alpha_i m_i\left(A_j\right), & \text{for } A_j \neq \Theta \\ m'_i\left(\Theta\right)=1-\sum_j m'_i\left(A_j\right) \end{cases} \tag{4}$$

7) Finally, combine the evidence after the discount to get the final decision evidence.

3.2 Model Establishment of Conflict Evidence Decision based on Quadratic Combination

By analyzing the discount evidence combination model, it can be found that the method of combining the conflict evidence based on revised evidence source needs to calculate the distance between the evidences. For the n evidence, the distance needs to be calculated $\frac{n(n-1)}{2}$ times, and the distance calculation complexity for each time is different according to the different distance formulas. Taking the classic Jousselme distance as an example, the distance formula between the any two evidences is

$$d_J\left(m_1,m_2\right)=\left(\frac{1}{2}\left(m_1-m_2\right)^T D\left(m_1-m_2\right)\right)^{\frac{1}{2}} \tag{5}$$

Where the elements $D_{ij}=\frac{\left|A_i \cap B_j\right|}{\left|A_i \cup B_j\right|}$ in matrix D.

It can be seen that the method of combining conflict evidence for correcting the source of evidence is computationally intensive and not suitable for the combination of multiple pieces of evidence. So next is a conflict evidence decision model based on quadratic combination, as shown in Figure 2.

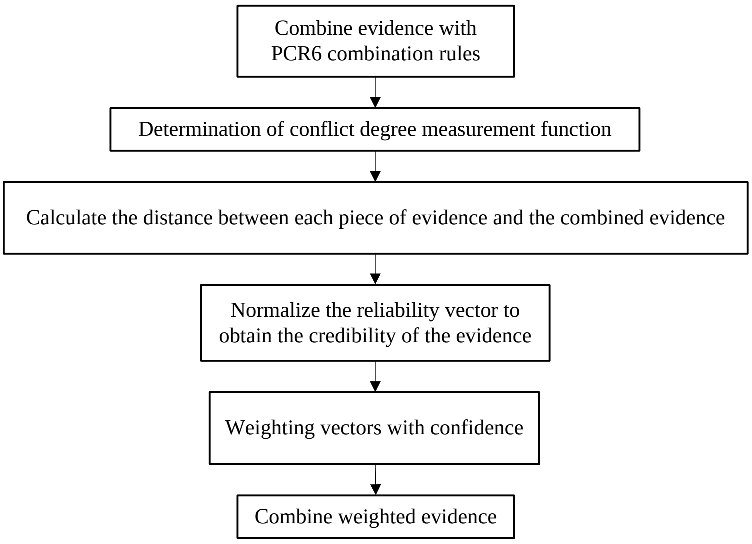

Figure 2. Conflict evidence decision model based on quadratic combination

1) The original evidence is combined by using the PCR6 combination rule to obtain preliminary combination results. The combination of PCR6 is widely used in the combination rules of conflict evidence, and better combination effect can be obtained. The evidence m' obtained after the combination can reflect the overall level of evidence to some extent.

2) Determine the measure function of the degree of conflict. As described above, the measure function needs to satisfy the basic conditions of the norm, and will not be described here.

3) Calculate the distance between each piece of evidence and the combined evidence, and calculate the confidence vector directly.

4) The fourth step normalizes the evidence's confidence vector. In the conventional evidence of conflicting evidence sources, the normalization of evidence is each element divided by the sum of the elements. However, as can be seen from Equations 2 and 3, the elements between the evidences are

multiplied in the combination rule of evidence, that is $m_1(A_i)m_2(A_j)\cdots m_n(A_k)$, the normalization of the summation form does not satisfy the calculation requirements. Normalization should be satisfied

$$\prod_i m'_i(A_j)=\prod_i m_i(A_j) \tag{6}$$

To this end, this paper proposes a new multiplicative normalization calculation formula, namely

$$a'_i=\frac{a_i}{\left(\prod_i a_i\right)^{\frac{1}{n}}} \tag{7}$$

Normalized new weight coefficient, satisfied $\prod_i a'_i=1$, which meet the needs of subsequent multiplicative operations.

5) Discount the evidence, due to the change of the normalization formula, the discount formula needs to be corrected as follows

$$\begin{cases} m'_i(A_j)=\alpha'_i\, m_i(A_j),\ |A_j|=1 \\ m'_i(A_j)=\dfrac{1}{\alpha'_i} m_i(A_j),\ |A_j|\ge 2 \end{cases} \tag{8}$$

where A represents the magnitude of the potential of the element and B represents the set of single elements.

6) Finally, the evidence is combined by using the PCR6 evidence combination rule.

3.3 Validity Analysis of Multiplicative Combination Rules

The normalization formula based on the multiplicative law is proposed above. As shown in Equations 6 and 7, the rationality of the multiplicative normalization rule in the decision theory of conflict evidence is verified by the following example.

Example 1: Assume that four different types of sensors identify target A, the recognition frame is $U=[A,B,C]$, and BPAs are assigned.

$$m_1(A)=0.6, m_1(B)=0.2, m_1(AB)=0.1, m_1(C)=0.1$$

$$m_2(A)=0.6, m_2(B)=0.1, m_2(AB)=0.1, m_2(C)=0.1, m_2(BC)=0.1$$

$$m_3(A)=0.5, m_3(B)=0.1, m_3(AB)=0.1, m_3(C)=0.1, m_3(AC)=0.1, m_3(BC)=0.1$$

$$m_4(A)=0.1, m_4(B)=0.1, m_4(AB)=0.1, m_4(C)=0.5, m_4(AC)=0.1, m_4(BC)=0.1$$

It can be seen from the BPAs that the first three pieces of evidence give higher reliability of A, while the fourth piece of evidence has conflicts, so the fourth piece of evidence has lower credibility, and the first three pieces of evidence have higher credibility. Therefore giving four evidence support vectors as follows

$$w=[0.7 \quad 0.6 \quad 0.65 \quad 0.1]$$

Then calculate the support vector after additive normalization and multiplicative normalization respectively. We can get

$$\begin{cases} w_+=[0.3415 \quad 0.2927 \quad 0.3171 \quad 0.0488] \\ w_\times=[1.7221 \quad 1.4761 \quad 1.5991 \quad 0.2460] \end{cases}$$

The discounted BPAs are calculated by using equations 4 and 8 respectively, and then the evidence is combined by using the PCR6 combination rule. The results are shown in Table 1.

Table 1. Additive and multiplicative normalized discount evidence combination results

$m_i(\bullet)$	A	B	AB	C	AC	BC	ABC
PCR6	0.5436	0.0939	0.0403	0.2544	0.0224	0.0454	0.0000
PCR6_+	0.5386	0.0938	0.0314	0.0588	0.0095	0.0205	0.2475
$\text{PCR6}_\times$	0.6379	0.0751	0.0622	0.0523	0.0641	0.1083	0.0000

In the table, PCR6 indicates the result of the combination of the rules directly by the PCR6. PCR6_+ indicates the result of the combination rules based on the additive normalization rules and the PCR6. $\text{PCR6}_\times$ indicates the result of the combination rules based on the multiplicative normalization rules and the PCR6.

By analyzing the data in Table 1, we can see that the uncertainty assignment $m_i(ABC)$ obtained by the additive PCR6 combination rule is larger, because in equation 4, the additive normalized evidence combination assigns the uncertainty information to the proposition ABC which is not conducive to the subsequent evidence decision. However, the combined result of the multiplicative normalization rule proposed in this paper shows $m(ABC)=0$, and gives the proposition A a large degree of reliability, which is in line with the actual situation, and such normalization is more meaningful.

4. Determination of Conflict Metric Function

4.1 Insufficient of Existing Conflict Measurement Function

There are two types of conflict measurement functions commonly used at present. One is the conflict degree measurement function based on information entropy, and the other is the distance-based conflict measurement function. The distance here is a generalized concept, including vector distance, interval distance, Angle and so on. Information entropy is a parameter to measure the degree of evidence dispersion, which reflects the decision-making ability of evidence to a certain extent, but it does not measure the distance between evidences well. This paper will determine the conflict metric function based on the distance between the evidences.

The most commonly used conflict metric function is based on the Jousselme distance. Jousselme distance can take into account the propositional potential and can effectively measure the degree of acquaintance between BPAs. The definition of Jousselme distance is given below.

Assuming that m_1, m_2 is two BPAs under the same recognition framework, then the Jousselme distance between m_1 and m_2 is defined as

$$d(m_1, m_2) = \left(\frac{1}{2}(m_1 - m_2)^T D(m_1 - m_2)\right)^{\frac{1}{2}} \tag{9}$$

where D is a matrix of $2^{|\Theta|} \times 2^{|\Theta|}$, each of which is

$$D(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i \cup A_j|}$$

Although the Jousselme distance can reflect the distance between BPAs to a certain extent and is widely used, in some special cases, the Jousselme distance can not have a good measurement effect. For example, three sources of evidence give BPAs respectively as follows.

$$m_1(A)=1/3, m_1(B)=1/3, m_1(C)=1/3$$

$$m_2(A)=0.2, m_2(B)=0.2, m_2(C)=0.2, m_2(ABC)=0.4$$

$$m_3(A)=0.2, m_3(B)=0.2, m_3(C)=0.6$$

Calculated by Equation 9, the Jousselme distance between the evidences m_1, m_2 and m_1, m_3 is

$$d_J(m_1,m_2)=d_J(m_1,m_3)=0.231$$

That is, the Jousselme distance between m_1, m_2 and m_1, m_3 is equal. However, through analysis, it can be found that the evidence m_1 represents the most uncertain state, the trust of each element is $1/3$, and the evidence m_3 obviously supports the proposition C, and the evidence m_2 is completely uncertain, so the final result should be shown evidence distance satisfied $d_J(m_1,m_2)<d_J(m_1,m_3)$. In summary, the Jousselme distance does not have a good measurement effect under some special circumstances.

In view of this, this paper will introduce the concept of vector angle to measure the distance between evidences. The angle between vectors can be expressed as

$$d_\theta=\cos(\theta)=\frac{\vec{m}_1\cdot\vec{m}_2}{|\vec{m}_1||\vec{m}_2|} \tag{10}$$

As can be seen from Expression 10, since the assignment of each proposition in the evidence satisfies $0\le m_i(A_j)\le 1$, the product of the evidence satisfies $\vec{m}_1\cdot\vec{m}_2\ge 0$, that is $d_\theta\ge 0$.

However, the evidence is based on the $2^{|\Theta|}$-dimensional vector of the recognition framework Θ. The evidence is simply regarded as a vector, and the angle between the evidences is solved. The difference between the multi-element proposition and the single-element proposition is discarded, and the same weight is given to different propositions. This is unreasonable, for example, for evidence groups

$$m_1(A)=0.1, m_1(B)=0.5, m_1(C)=0.1, m_1(AC)=0.15, m_1(BC)=0.15$$

$$m_2(A)=0.5, m_2(B)=0.1, m_2(C)=0.1, m_2(AC)=0.15, m_2(BC)=0.15$$

$$m_3(A)=0.15, m_3(B)=0.15, m_3(C)=0.1, m_3(AC)=0.1, m_3(BC)=0.5$$

$$m_4(A)=0.15, m_4(B)=0.15, m_4(C)=0.1, m_4(AC)=0.5m_4(BC)=0.1$$

We can get the vector angle cosine between m_1, m_2 and m_3, m_4 is

$$d_\theta(m_1,m_2)=d_\theta(m_3,m_4)=0.508$$

That is, the vector angle cosine between m_1, m_2 and m_3, m_4 is equal. However, through analysis, it can be found that the value of the vector does not change between the evidences m_1 and m_3, but the trust of the propositions A and B exchanged to the trust of the propositions AC and BC. That is, the vector m_1 is rotated to obtain the vector m_3. Similarly, m_2 is rotated to obtain m_4, and their rotation directions and angles are equal. Therefore, from the perspective of the vector, the vector angle cosine of the two is equal.

Because the potential of the evidence propositions are different, the weights of the propositions should be different. The propositions A and B between the evidences m_1 and m_2 are conflicting, and the remaining propositions are not conflicting. The propositions AC and BC between the evidences m_3 and m_4 are conflicting, and the remaining propositions are not conflicting.

The conflict between the two pairs is the same, however, because the credibility of propositions A and B is higher, the reliability of propositions AC and BC is relatively vague, the distance between evidences m_1 and m_2 should be greater than the evidence between m_3 and m_4. However, the angle between the vectors does not fully take into the potential value of the proposition, so the distance between the evidences is not well measured.

In the next section, we will consider the potential value of the proposition and give a conflicting evidence metric function based on the pignistic vector angle.

4.2 Determination of Conflict Evidence Measurement Function Based on Pignistic Vector Angle

This paper combines the concept of pignistic probability transformation and vector angle to propose a conflict evidence metric function based on the pignistic vector angle, ie,

$$d_{Bet-\theta}(m_1,m_2)=\left(1-\left(\frac{BetP_1\cdot BetP_2}{|BetP_1||BetP_2|}\right)^2\right)^{\frac{1}{2}}=\frac{|BetP_1\times BetP_2|}{|BetP_1||BetP_2|} \tag{11}$$

Where $BetP_i$ represents the probability vector after the pignistic probability transformation, and each element in the vector is the probability conversion result $BetP(A_i)$.

$$BetP_k(A_i)=\sum_j m_k(A_j)\frac{|A_i\cap A_j|}{|A_j|} \tag{12}$$

The symmetry, positive definiteness, homogeneity and triangular inequality properties of Equation 11 are proved below.

Proof: 1) symmetry,

$$\begin{aligned}d_{Bet-\theta}(m_1,m_2)&=\frac{|BetP_1\times BetP_2|}{|BetP_1||BetP_2|}=\frac{|BetP_2\times BetP_1|}{|BetP_2||BetP_1|}\\&=d_{Bet-\theta}(m_2,m_1)\end{aligned}$$

2) Positive definiteness,

$$\begin{aligned}&\because m_k(A_i)\ge 0\\&and\ BetP_k(A_i)=\sum_j m_k(A_j)\frac{|A_i\cap A_j|}{|A_j|}\\&\therefore BetP_k(A_i)\ge 0\\&\therefore BetP_1\times BetP_2\ge 0\\&and\ \because \frac{|BetP_1\times BetP_2|}{|BetP_1||BetP_2|}\le 1\\&\therefore 0\le \frac{|BetP_1\times BetP_2|}{|BetP_1||BetP_2|}\le 1\\&\therefore 0\le d_{Bet-\theta}(m_1,m_2)=\frac{|BetP_1\times BetP_2|}{|BetP_1||BetP_2|}\le 1\end{aligned}$$

When $m_1=m_2$

$$\frac{BetP_1 \cdot BetP_2}{|BetP_1||BetP_2|} = 1,$$

$$\therefore d_{Bet-\theta}(m_1, m_2) = \frac{BetP_1 \times BetP_2}{|BetP_1||BetP_2|} = 0$$

3) For the homogeneity of the norm, no proof is given here, because the evidence needs to satisfy the formula 1, so there is no comparison of the distance between $k_1 m_1$ and $k_2 m_2$.

4) Triangle inequality,

According to inequality $\sin(\theta_1) \le \sin(\theta_2) + \sin(\theta_3)$, where $\theta_1, \theta_2, \theta_3$ are the angle between two of the space three vectors, we can get

$$\frac{BetP_1 \times BetP_2}{|BetP_1||BetP_2|} \le \frac{BetP_1 \times BetP_3}{|BetP_1||BetP_3|} + \frac{BetP_3 \times BetP_2}{|BetP_3||BetP_2|}$$

Finished.

5. Computational Amount Analysis of Conflict Evidence Decision based on Quadratic Combination

Firstly, the conflict evidence decision method of the traditional modified data source is analyzed. The evidence distance in Flowchart 1 is exemplified by the classical Jousselme distance. The intersection, union, potential, addition, subtraction, multiplication and division in the operation process are all recorded as one operation. As shown in Equation 5, the Jousselme distance needs to solve the matrix D, and the amount of operations is

$$A_D = 5 \times \frac{2^n(2^n - 1)}{2} = 5 \times 2^{n-1}(2^n - 1)$$

Where n is the potential $|\Theta|$ of the recognition frame Θ, so the calculation of the Jousselme distance between the two evidences is

$$A_{d_J} = A_D + 2^n + 2^n \cdot 2^n + 2^n(2^n - 1) + 2^n + (2^n - 1) + 2 = 9 \times 2^{2n-1} - 2^{n-1} + 1$$

In order to obtain the distance matrix, the evidence distance between two needs to be calculated. If the number of evidences is m, then the amount of computation required by the conflict evidence decision method based on Jousselme distance is

$$A_J = m(m-1)A_{d_J} = m(m-1)(9 \times 2^{2n-1} - 2^{n-1} + 1) \tag{13}$$

Flow chart 2 shows that it is necessary to perform PCR6 combination on m pieces of evidence. By analyzing formula 3, it can be obtained that the combined calculation amount of m pieces of evidence is

$$A_{PCR6} = (m-1)\left(2^n \cdot 2^n + 6 \times \left(\sum_{i=2}^{n-1} C_n^i (2^{n-i} - 1)\right) + 2^n \cdot 2^n - 1\right) = (m-1)\left(2^{2n+1} + 6 \times \left(\sum_{i=2}^{n-1} C_n^i (2^{n-i} - 1)\right) - 1\right) \tag{14}$$

Where $\sum_{i=2}^{n-1} C_n^i (2^{n-i} - 1)$ represents the number of sets $Y \in 2^\Theta \setminus \{X\}, X \cap Y = \phi$, and then the distance between the m pieces of evidence and the combined evidence is solved using Equations 11and 12, the calculated amount is

$$A_{d_{Bet-\theta}} = m\left(5(2^n - n) + \sum_{i=2}^{n} iC_n^i\right) + m(2n + n + 4) = m\left(5(2^n - n) + \sum_{i=2}^{n} iC_n^i + (3n + 4)\right) \tag{15}$$

Where $\sum_{i=2}^{n} iC_n^i$ represents the number of additions in the pignistic probability conversion, and the total amount of the calculation is

$$A_{Bet-\theta} = A_{PCR6} + A_{d_{Bet-\theta}} \tag{16}$$

Through Equation 13-16, the calculation amount of the conflict evidence decision method is analyzed, and the result is shown in Figure 3.

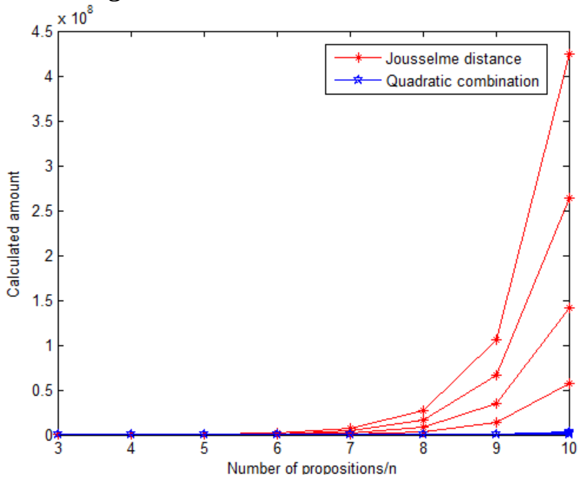

Figure 3. The graph of the calculated amount of conflict evidence decision based on Jousselme distance and quadratic combination

In the figure, the red curve represents the calculation amount of the conflict evidence decision method based on the Jousselme distance, and the four red lines from bottom to top are representing the calculation curve respectively when the evidence is 4-10 (step size is 2). The blue curve represents the calculation amount of the conflict evidence decision method based on the quadratic combination. The four blue lines from bottom to top indicate the calculation curve respectively when the evidence is 4-10 (step size is 2). The abscissa indicates the value of the potential of the recognition frame, and the ordinate indicates the amount of calculation. However, since the blue curve grows slowly, in order to facilitate the observation and analysis, Figure 4 takes the logarithm of the calculated amount and obtains a new calculation amount growth graph.

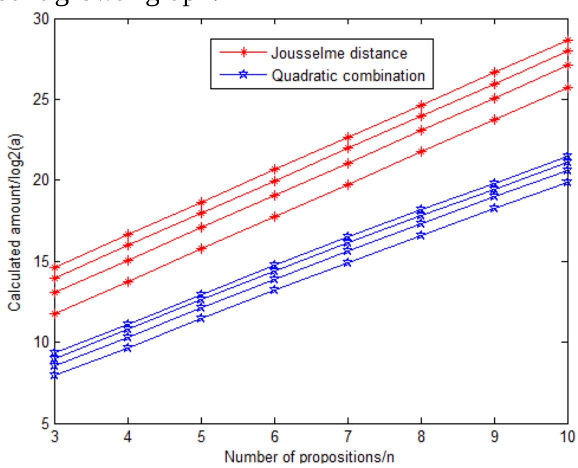

Figure 4. The graph of the logarithm of the calculated amount of conflict evidence based on Jousselme distance and quadratic combination

By analyzing Figure 4, it can be seen that the logarithm of the computational amount based on the Jousselme distance and the conflicting evidence decision method based on the quadratic combination approximate linear growth, so both the original calculation amount are exponentially increasing.

By analysis of Figures 3 and 4, we can know that when the number of evidence is fixed, the logarithm of the computational quantity based on the Jousselme distance and the quadratic combination based conflict evidence decision method increase exponentially, and the calculation amount of the conflict evidence decision method based on Jousselme distance increases by nearly 5dB-10dB faster than the secondary combination. When the potential of the recognition frame is constant, the amount of calculation increases sharply with the amount of evidence.

Based on the above, the calculation amount of the conflict evidence decision method based on quadratic combination proposed in this paper has been greatly improved.

6. Analysis of Conflict Evidence Decision Cases based on Quadratic Combination

Example 1: Consider the identification framework $\Theta=\{A,B,C\}$ and give BBAs of 6 sources, as shown in Table 2.

Table 2. Evidence group BPAs assignment

$m(\bullet)$	A	B	AB	C	AC	BC	ABC
m_1	0.7	0.1	0.1	0.1	0	0	0
m_2	0.6	0.2	0	0.1	0	0	0.1
m_3	0.6	0.05	0	0.05	0	0	0.3
m_4	0.4	0.3	0	0.2	0.1	0	0
m_5	0.1	0.7	0	0.1	0	0	0.1
m_6	0.9	0.05	0	0.05	0	0	0

According to flowchart 2, the evidence is first combined using PCR6 to obtain combined evidence m_{PCR6}.

$$m_{PCR6}(\cdot)=\begin{bmatrix}0.72 & 0.21 & 0 & 0.03 & 0 & 0 & 0.04\end{bmatrix}$$

Using Equations 11 and 12, calculate the distance between each piece of evidence and the combined evidence to obtain a support vector.

$$w=\begin{bmatrix}0.88 & 0.85 & 0.83 & 0.55 & 0.11 & 0.76\end{bmatrix}$$

It can be seen that the fifth piece of evidence has a low degree of support. It can be seen from the Table 3 that the fifth piece of evidence conflicts with the other five pieces of evidence. Then use Equation 7 to normalize the weights and get

$$\bar{w}=\begin{bmatrix}1.59 & 1.54 & 1.50 & 0.99 & 0.20 & 1.38\end{bmatrix}$$

Then, we can obtain a new evidence group by using Equation 8, as shown in Table 3.

Table 3. Discounted evidence group BPAs assignment

$m'(\bullet)$	A	B	AB	C	AC	BC	ABC
m'_1	0.74	0.11	0.04	0.11	0	0	0
m'_2	0.64	0.21	0	0.11	0	0	0.05
m'_3	0.72	0.06	0	0.06	0	0	0.16
m'_4	0.40	0.30	0	0.20	0.10	0	0
m'_5	0.03	0.21	0	0.03	0	0	0.73
m'_6	0.90	0.05	0	0.05	0	0	0

Finally, the discount evidence is combined by using the PCR6 combination rule, and the results are shown in Table 4.

Table 4. Calculation results of different combination rules of the example

$m(\bullet)$	A	B	AB	C	AC	BC	ABC
PCR1	0.57	0.22	0.02	0.10	0.02	0	0.08
PCR2	0.57	0.22	0.02	0.10	0.02	0	0.08
PCR3	0.67	0.24	0	0.06	0	0	0.04
PCR5	0.46	0.35	0.01	0.07	0.01	0	0.10
PCR6	0.72	0.21	0	0.03	0	0	0.04
Method of this paper	0.77	0.07	0	0.02	0	0	0.14

By analyzing the data in Table 2, we can see that the 4th of the evidence gives the BPA of target A and B is 0.4 and 0.3 respectively, so the uncertainty of the fourth evidence is higher. The fifth piece of evidence gives the BPA of target A and B is 0.1 and 0.7, that is, the evidence 5 trust target B, and the remaining four evidences give the target A a higher degree of reliability, so the evidence 5 is a conflicting evidence. It can be known from the analysis that the combined result of the six evidences should be biased toward the target A. By analyzing the data of the experimental results in Table 4, it can be seen that the conflict evidence decision method based on quadratic combination proposed in this paper obtains better combined results and effectively solves the conflict evidence combination problem.

7. Conclusion

This paper proposes a new method based on quadratic combination for conflict evidence decision-making. Firstly, by analyzing the general flow of conflict evidence decision-making methods based on modified data sources, this paper presents a new flow chart of conflict evidence decision-making methods based on quadratic combination, and gives the multiplicative normalization rules. Secondly, by analyzing the shortcomings of the existing conflict metric functions, the paper proposes a conflict evidence metric function based on the pignistic vector angle, and obtains a new evidence metric function which satisfies the symmetry, positive definiteness, homogeneity and triangle Inequality nature of norm theory. Thirdly, the paper also analyzes the calculation amount of conflict evidence decision based on quadratic combination. By comparing with the original evidence theory method, it is found that the new method is nearly 5dB-10dB less than the existing method. Finally, through comparison with different combination rules, it is found that the proposed new method can achieve better combination effect when dealing with the conflict evidence combination.

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