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Fault and Event Tree Analyses for Process Systems Risk Analysis: Uncertainty Handling Formulations

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Quantitative risk analysis (QRA) is a systematic approach for evaluating *likelihood*, consequences, and risk of adverse events. QRA based on event (ETA) and fault tree analyses (FTA) employs two basic assumptions. The first assumption is related to *likelihood* values of input events, and the second assumption is regarding interdependence among the events (for ETA) or basic events (for FTA). Traditionally, FTA and ETA both use crisp probabilities; however, to deal with uncertainties, the probability distributions of input event *likelihoods* are assumed. These probability distributions are often hard to come by and even if available, they are subject to incompleteness (partial ignorance) and imprecision. Furthermore, both FTA and ETA assume that events (or basic events) are independent. In practice, these two assumptions are often unrealistic. This article focuses on handling uncertainty in a QRA framework of a process system. Fuzzy set theory and evidence theory are used to describe the uncertainties in the input event *likelihoods*. A method based on a dependency coefficient is used to express interdependencies of events (or basic events) in ETA and FTA. To demonstrate the approach, two case studies are discussed.

KEY WORDS: Event tree analysis (ETA); fault tree analysis (FTA); interdependence; *likelihoods*; quantitative risk analysis (QRA); uncertainty

SYMBOLS

$m(p_i)/m(c_i)$	Belief mass
m_{1-n}	1 to n numbers of experts' knowledge
n	Number of events/basic events
Subscript (L)	Minimum (left) value of a TFN
Subscript (m)	Most likely value of a TFN
Subscript (U)	Maximum (right) value of a TFN
C_d	Dependency coefficient
N	Number of samples
P_i	Probability of events ($i = 1, 2, \ldots, n$)
$\tilde{P}_i$	Fuzzy probability
λ_i	Frequency
$\tilde{P}_\alpha$	TFN with a α-cut
P_{OR}	"OR" gate operation
P_{AND}	"AND" gate operation
μ	Membership function
α	Membership function at a specific level
Ω	Frame of discernment (FOD)
Φ	Null set
$\cap$	Symbol for intersection
$\subseteq$	Symbol for subsets
Bel, Pl	Belief, plausibility
bpa	Basic probability assignment

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ABBREVIATIONS

E	Events
H	High
I	Independent
P	Perfectly dependent
L	Low
BE	Basic events
MH	Moderately high
ML	Moderately low
RH	Rather high
S	Strong
VH	Very high
VL	Very low
VS	Very strong
VW	Very weak
DS	Dempster & Shafer
W	Weak
T, F	True/false probability
ETA	Event tree analysis
FTA	Fault tree analysis
PDF	Probability density function
MCS	Monte Carlo simulation
TFN	Triangular fuzzy number

1. INTRODUCTION

Process systems in chemical engineering are infamous for fugitive emissions, toxic releases, fire and explosions, and operation disruptions. These incidents have considerable potential to cause an accident and incur environmental and property damage, economic loss, sickness, injury, or death of workers in the vicinity. Quantitative risk analysis (QRA) is a systematic approach that integrates quantitative information about an incident and provides detailed analysis that helps to minimize the *likelihood* of occurrence and reduces its adverse consequences. QRA for process systems is a difficult task as the failures of components and the consequences of an incident are randomly varied from process to process. Furthermore, for a process system comprising thousands of components and steps, it is difficult to acquire the quantitative information for all components.[1] Finally, the interdependencies of various components are not known and are generally assumed to be independent for the purpose of simplicity.

Event tree analysis (ETA) and fault tree analysis (FTA) are two distinct methods for QRA that develop a logical relationship among the events leading to an accident and estimate the risk associated with the accident. The term "event" is frequently used in place of the term "accident" in the analyses of fault trees and event trees for QRA.[2] ETA is a technique used to describe the consequences of an event (initiating event) and estimate the likelihoods (frequency) of possible outcomes of the event. FTA represents basic causes of occurrence of an unwanted event and estimates the likelihood (probability) as well as the contribution of different causes leading to the unwanted event. In FTA, the basic causes are termed basic events, and the unwanted event is called the top event.[3–6] Kumamoto and Henley[7] provide a detailed description of fault tree development and analysis for a process system.

In the event tree, the unwanted event is named as an initiating event, and the follow-up consequences are termed as events or safety barriers.[8] The ETA represents the dichotomous conditions (e.g., success/failure, true/false, or yes/no) of the initiating event until the subsequent events lead to the final outcome events.[8–10] AIChE[8] and Lees[11] provide a detailed procedure for constructing and analyzing the ETA for a process system.

Event and fault trees help to conduct the QRA for process systems based on two major assumptions.[2] First, the *likelihood* of events or basic events is assumed to be exact and precisely known, which is not very often true due to inherent uncertainties in data collection and defining the relationships of events or basic events.[10,12] Moreover, because of variant failure modes, design faults, and poor understanding of failure mechanisms, as well as the vagueness of system phenomena, it is often difficult to predict the acquired probability of basic events/events precisely.[13] Second, the interdependencies of events or basic events in an event tree or fault tree are assumed to be independent, which is often an inaccurate assumption.[14] These two assumptions indeed misrepresent the actual process system behaviors and impart two different types of the uncertainty, namely, *data uncertainty* and *dependency uncertainty*, while performing the QRA using FTA and ETA. In an attempt to circumvent the *data uncertainty* in risk analysis, a number of research works[1,10,13,15–23] have been developed to facilitate the accommodation of expert judgment/knowledge in quantification of the *likelihood* of the basic events/events for QRA. Sadiq *et al.*,[12] Ferson *et al.*,[14] and Li[24] proposed methods to describe the *dependency uncertainty* among the basic events/events.

Fuzzy-based and evidence-theory-based formulations have been proposed and developed to address data and dependency uncertainties in FTA and

ETA. The interdependencies among the events (or basic events) are described by incorporating a dependency coefficient into the fuzzy- and evidence-theory-based formulations for FTA/ETA. Expert judgment/knowledge can be used to quantify the unknown or partially known likelihood and dependency coefficient of the events (or basic events).

2. FTA AND ETA IN PROCESS SYSTEMS

The traditional fault and event trees can be analyzed either deterministically or probabilistically. The deterministic approach uses the crisp probability of events (or basic events) and determines the probability of the top event and the frequency of outcome events in the fault and event trees, respectively. The probabilistic approach treats the crisp probability as a random variable and describes uncertainty using probability density functions (PDF).[10,20,23] Traditionally, the probabilistic approach uses Monte Carlo simulation (MCS) to address the random uncertainty in the inputs (i.e., probability of basic events or events) and propagate the uncertainty for the outputs.[25] The PDFs for the inputs can be derived from historical information, but are often rare, especially when the process system comprises thousands of components.[18]

With an assumption that the events (or basic events) are independent, deterministic and probabilistic approaches use the equations in Table I to analyze the fault and event trees. P_i denotes the probability of ith ($i = 1, 2, 3, \ldots, n$) events (or basic events), P_{OR} and P_{AND}, respectively, denote the "OR" and "AND" gate operations, and λ_i denotes the frequency for the initiating event and the outcome events.

Two examples—an event tree for "LPG release" (Fig. 1) and a fault tree for "runaway reaction" (Fig. 2)—are considered to illustrate the use of deterministic and probabilistic approaches in QRA for the process system. The event and fault trees for these two examples were earlier studied, respectively, in Lees[11] and Skelton.[26] The deterministic approach provides a quick analysis if the probabilities are known accurately.[10] Based on assigned probabilities (Fig. 1 and Table II), the frequency of outcome events for "liquefied petroleum gas (LPG) release event tree" and the probability of top event for "runaway reaction fault tree" are calculated as crisp values (Table III). In the probabilistic approach, triangular PDFs are assumed to perform MCS ($N = 5{,}000$ iterations) and the PDFs for the outcome events' frequency and the top event probability are determined based on this assumption. The 90% confidence intervals for the outcome events of the "LPG event tree" and top event of "runaway reaction fault tree" are summarized in Table IV and Fig. 3, respectively.

Table I. Equations Used in Traditional Fault Tree Analyses and Event Tree Analyses

QRA Method	Equation
ETA	$\lambda_i = \lambda \times \prod_{i=1}^{n} P_i$
FTA	$P_{OR} = 1 - \prod_{i=1}^{n} (1 - P_i)$
	$P_{AND} = \prod_{i=1}^{n} P_i$

3. UNCERTAINTY IN FTA AND ETA

FTA and ETA require probability data of events (or basic events) as inputs to conduct a comprehensive QRA for a process system. Since most of the time the crisp data as well as PDFs are rarely available for all events and basic events, experts' judgment/knowledge are often employed as an alternative to the objective data.[13] Two types of uncertainties, namely, *aleatory and epistemic uncertainties*, are usually addressed while using the expert's knowledge in QRA.[10,27,28] *Aleatory uncertainty* is a natural variation, randomness, or heterogeneity of a physical system. It can be well described using probabilistic methods if enough experimental data are available to support the analysis.[29] *Epistemic uncertainty* means ambiguity and vagueness, ignorance, knowledge deficiency, or imprecision in system behaviors.

In QRA, it is important to characterize, represent, and propagate the uncertainty accurately in order to get a reliable analysis. However, when the input PDFs are "reasonably known," MCS can be used to estimate and propagate the uncertainties, especially two dimensional MCS, which can effectively deal with both *aleatory* and *epistemic uncertainties* (not discussed here).[30] If knowledge is limited for definition of the PDFs, probabilistic approaches might not be the best choice to handle the uncertainty in QRA.[31] In addition, the independence assumption of events (or basic events) might be convenient to simplify the FTA or ETA; however, it is not always true for all cases.[14] This assumption in fact is

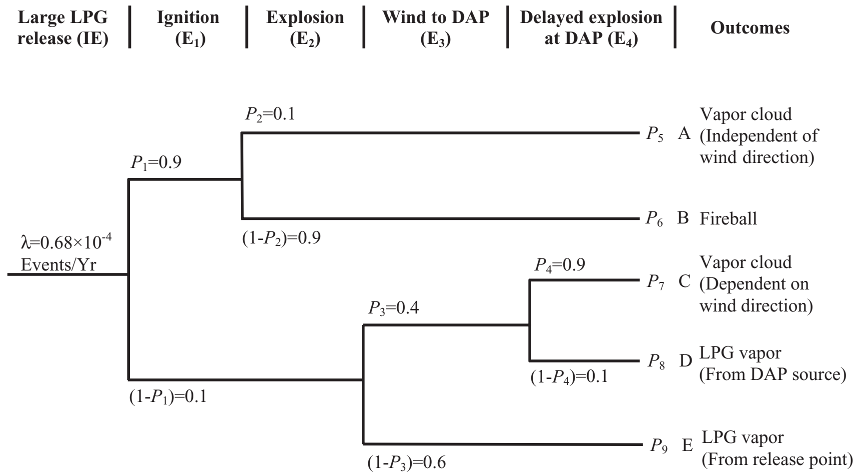

Fig. 1. Event tree for LPG release.

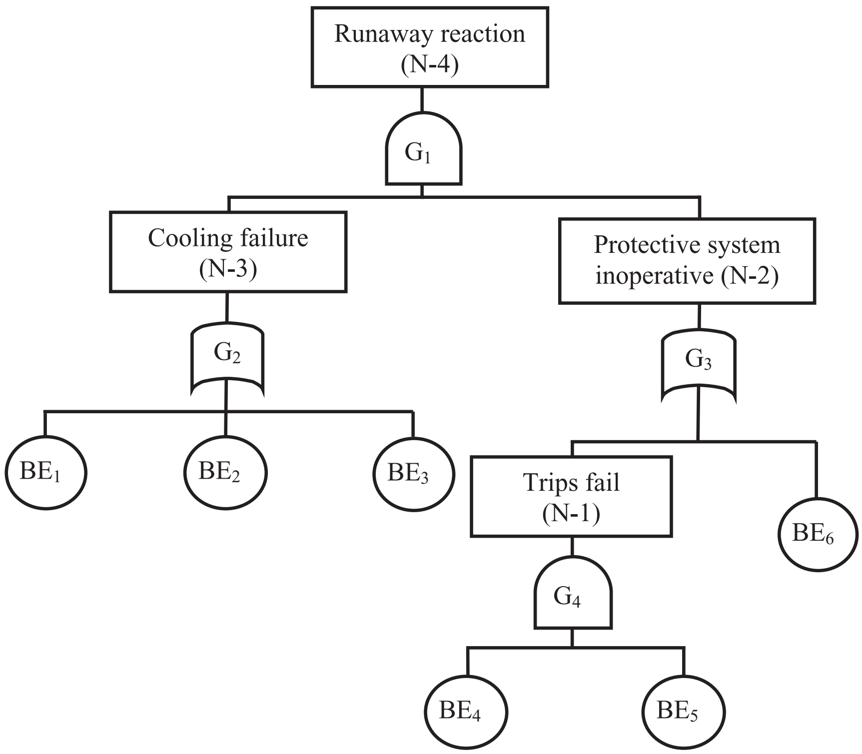

Fig. 2. Fault tree for runaway reaction in a reactor.

Table II. Basic Events Causing the Runaway Reaction

Symbol	Basic-Event	Probability of Basic Event
BE_1	Pump fails	0.2
BE_2	Line block	0.01
BE_3	No cooling water	0.1
BE_4	Low coolant flow	0.01
BE_5	High temp	0.01
BE_6	Dump valve fails	0.001

adding other kind of uncertainty, that is, the *dependency uncertainty,* during the analyses. Vesely *et al.*[3] show several cases of FTA where the independent assumptions of basic events are not valid.

Fuzzy set and evidence theories have recently been used in many engineering applications where expert knowledge is employed as an alternative to crisp data or PDFs.[12,23,28,29,32] Fuzzy set theory is used to address the subjectivity in expert judgment, whereas the evidence theory is more promptly

Table III. Deterministic Results for Fault Tree Analyses and Event Tree Analyses

LPG release event tree	Frequency of outcome events (Events/Yr)	A 6.1E-06	B 5.5E-05	C 2.4E-06	D 2.7E-07	E 4.1E-06
Runaway reaction fault tree	Probability of top event			$P_{Top} = 3.16\text{E-}04$		

Table IV. Frequency Determination of Outcome Events Using Monte Carlo Simulation

	90% Confidence Interval		
Outcome Events	Lower Bound (5th Percentile)	Upper Bound (95th Percentile)	Median (50th Percentile)
A	1.958E-06	1.024E-05	6.100E-06
B	4.935E-05	6.090E-05	5.512E-05
C	7.353E-07	4.151E-06	2.443E-06
D	3.057E-10	5.467E-07	2.736E-07
E	1.300E-06	6.850E-06	4.080E-06

employed in handling the uncertainty that arises due to ignorance, conflict, and incomplete information. In addition, to describe the *dependency uncertainty* among the basic events in FTA, Ferson *et al.*[14] described the Frank copula and Frechet's limit. For known dependency, the Pearson correlation in the Frank copula describes the full range of dependencies; that is, from perfect dependence to opposite dependence.[12] Li[24] proposed a dependency factor-based fuzzy approach to address the dependencies in performing risk analysis. Li[24] uses fuzzy numbers to define the dependency factor among basic events.

In this article, the probabilities of events (or basic events) and their dependency coefficients are treated as fuzzy numbers or basic probability assignments (*bpas*), which are derived through expert knowledge. Fuzzy set and evidence theories along with dependency coefficients, are used to explore the *data* and *dependency uncertainty* in ETA/FTA. The fuzzy numbers in fuzzy set theory describe linguistic and subjective uncertainty while *bpa*s in evidence theory are used to handle ignorance, incompleteness, and inconsistency in expert knowledge. A generic framework is shown in Fig. 4 illustrating the use of fuzzy set theory and evidence theory to handle two different kinds of *uncertainties* in FTA and ETA. The following sections describe the fuzzy set theory and the evidence theory with respect to handling uncertainties.

4. FUZZY SET THEORY

Zadeh[33] introduced fuzzy sets, which have recently been applied where probability theory alone was found insufficient to represent all types of uncertainties. Fuzzy set theory is flexible in describing linguistic terms as fuzzy sets, hedges, predicates, and quantifiers.[34] Fuzzy set theory is an extension of

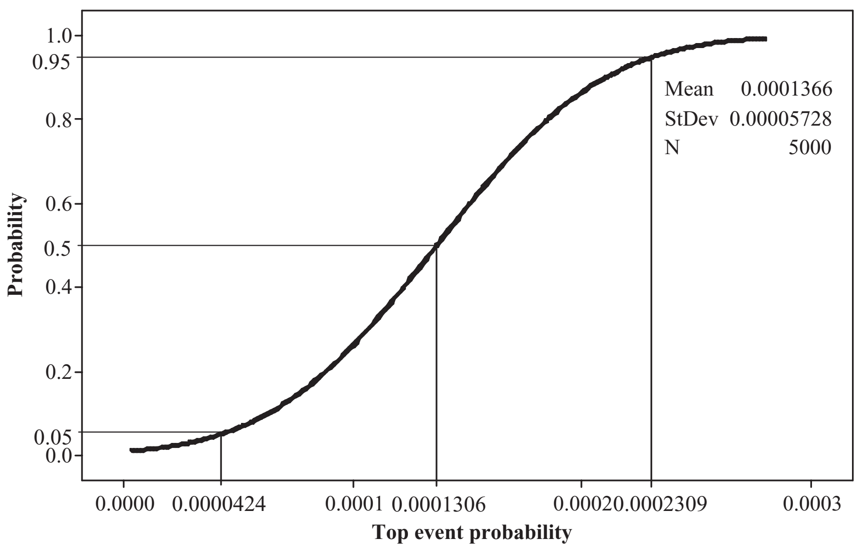

Fig. 3. 90% confidence interval for top event probability.

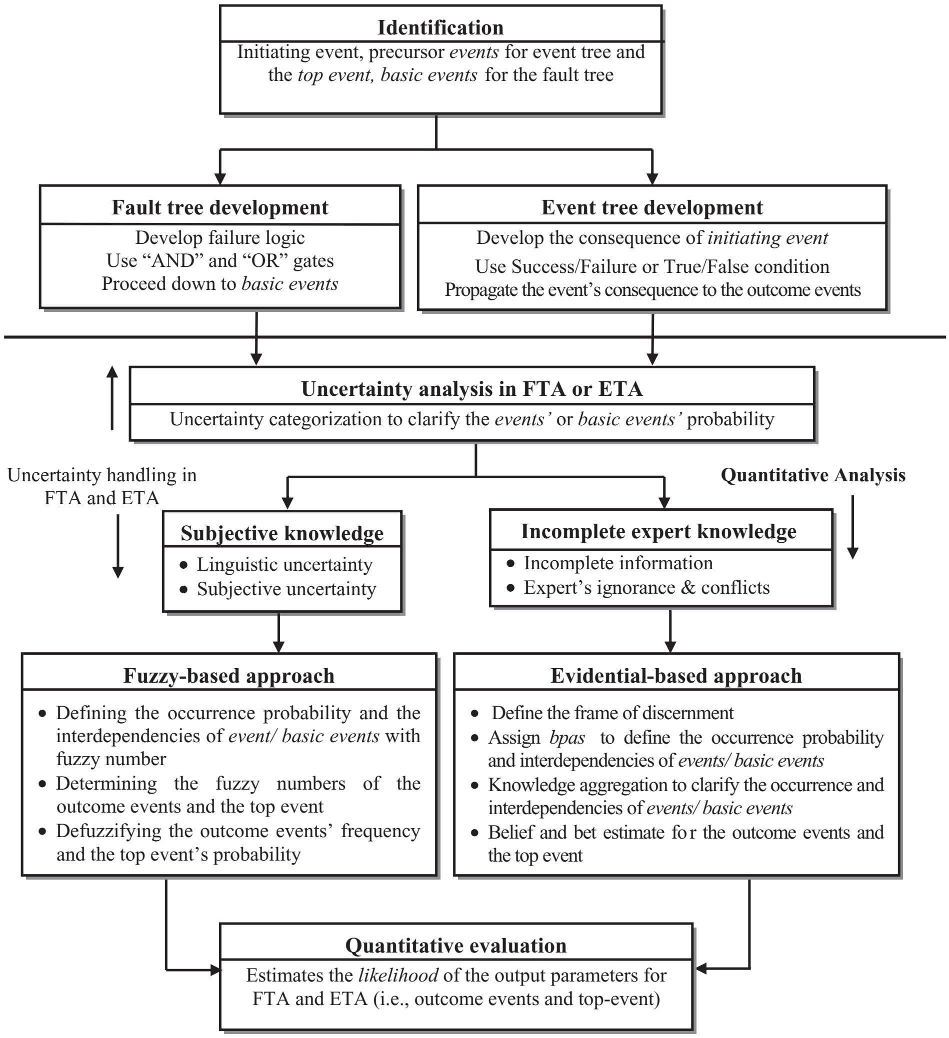

Fig. 4. Framework for fault tree analyses and event tree analyses under uncertainty.

traditional set theory, which represents imprecise values as fuzzy numbers and characterizes the uncertainty using a continuous membership function (μ).

4.1. Fundamentals

Fuzzy numbers are used to describe the vagueness and subjectivity in expert judgment through a relationship between the uncertain quantity p (e.g., event or basic event probability) and a membership function μ that may range between 0 and 1. Any shape of a fuzzy number is possible, but the selected shape should be justified by available information (but it should be normal, bounded, and convex). Generally, triangular or trapezoidal fuzzy numbers (TFN or ZFN) are used for representing linguistic variables.[18,21,35,36] In this study, we used triangular fuzzy numbers (TFN) in which the fuzzy intervals are derived using α-cuts. Fig. 5 shows a TFN in which fuzzy intervals are estimated using Equation (1). The values p_L, p_m, and p_R below represent the minimum, most likely, and maximum values, respectively, in an interval $\tilde{P}_\alpha$:

$$\tilde{P}_\alpha = [p_L + \alpha(p_m - p_L),\, p_R - \alpha(p_R - p_m)]. \quad (1)$$

Fuzzy set theory uses the fuzzy arithmetic operations based on α-cut formulation to manipulate fuzzy numbers.[24,37,38] Traditional fuzzy arithmetic operations assume that the events (or basic events) are

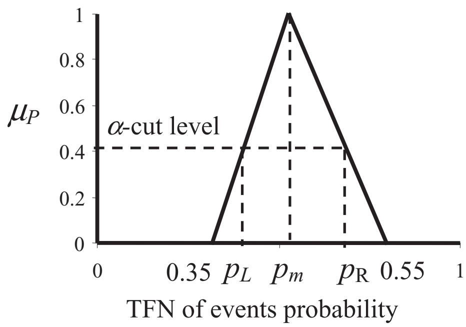

Fig. 5. Triangular fuzzy numberto represent the probability of events (or basic events).

independent and use equations in Table V for FTA and ETA.(15–21,23,37)

4.2. Fuzzy-Based Approach for FTA/ETA

In the proposed fuzzy-based approach, the probability of events (or basic events) can be defined linguistically and described using TFN. The interdependence of events (or basic events) is defined linearly using a dependency coefficient (C_d) that can also be described using a TFN. Fuzzy probability and dependency coefficients are used to determine the probability of top event and the frequency of outcome events in fuzzy terms. The fuzzy-based approach comprises the following three steps:

(1) Definition of input probability and dependency coefficient using TFN.
(2) Determination of *likelihood* of outcome events (ETA) and top event (FTA) as a TFN.
(3) Defuzzification.

4.2.1. Definition of Input Probability and Dependency Coefficient Using TFN

Experts are more comfortable using linguistic expression rather than numerical judgment when they are asked to define an uncertain quantity like the probability of occurrence of events (or basic events) and dependency coefficients.(28) In order to capture these linguistic expressions, eight linguistic grades are defined in the proposed approach (Fig. 6). They include: *Very Highly* (VH), *Very Low* (VL), *Moderately High* (MH), *Moderately Low* (ML), *Low* (L), *Moderate* (M), *High* (H), and *Rather High* (RH). These grades can be used to assign the probability of events (or basic events) for ETA (or FTA).

As mentioned earlier, the traditional methods of FTA and ETA assume that the events in an event tree and the basic events in a fault tree are independent. However, in practice, the interdependencies among the events (or basic events) could be ranged from *perfectly dependent* to *oppositely dependent*. A scalar quantity $\in [+1, -1]$ may describe the dependency between two events, where the scalar quantity $+1$ refers to *perfect dependence* and -1 refers to *opposite dependence*.(14) More specifically, the positive dependence belongs to an interval $[0, +1]$, whereas the negative dependence belongs to an interval $[-1, 0]$. However, various levels of dependency are possible in between the events (or basic events). This work explores only the positive dependence of events (or basic events) at each node in FTA (or ETA). Six linguistic grades are used in this study to describe the different levels of interdependencies among the events and basic events, which include: *Perfectly Dependent* (P), *Very Strong* (VS), *Strong* (S), *Weak* (W), *Very Weak* (VW), and *Independent* (I). The left bound (C_{dL}) and the right bound (C_{dR}) in Table VI

Table V. Traditional α-Cut-Based Fuzzy Arithmetic Operations

Method	Operation	α-Cut Formulation
ETA	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$
	$\tilde{P}_1 \times \tilde{P}_2$	$p_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}; p_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$
	$\tilde{P}_1 + \tilde{P}_2$	$p_L^{\alpha} = \sum_{i=1}^{n} p_{iL}^{\alpha}; p_R^{\alpha} = \sum_{i=1}^{n} p_{iR}^{\alpha}$
FTA	"OR" gate	$p_L^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iL}^{\alpha}); p_R^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iR}^{\alpha})$
	"AND" gate	$p_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}; p_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$

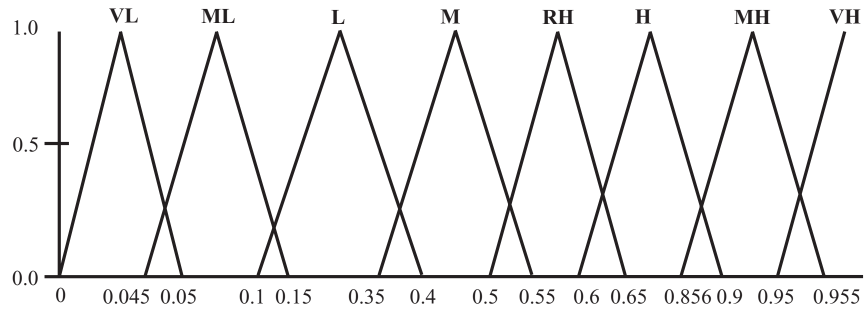

Fig. 6. Mapping linguistic grades for fault tree analyses and event tree analyses.

Table VI. Scale to Categorize the Interdependence Among the Basic Events/Events

Linguistic Grade	Description	Minimum (C_{dL})	Maximum Bound (C_{dU})
P	Perfect dependence between the events	1.000	1.000
VS	Very strong dependence, but not fully dependent	0.800	0.995
S	Strong dependence, but not too strong	0.450	0.850
W	Weak dependence, but not too weak	0.150	0.500
VW	Very weak dependence, but not fully independent	0.005	0.200
I	Perfect independence between the events	0.000	0.000

are representing the TFNs boundary for the dependency coefficients.

4.2.2. Determination of Likelihood of Outcome Event and Top Event as a TFN

The dependency coefficient C_d defines the dependence of the events (or basic events) at each node of a fault and event tree (Table VI). The modified fuzzy arithmetic with the empirical relation for FTA and ETA are described in Table VII, where $C_d \approx 1$ refers to perfect dependence and $C_d \approx 0$ refers to complete independence among the event (or basic events).

4.2.3. Defuzzification

Defuzzification transforms the fuzzy number into a crisp value.[39] The crisp value is useful in evaluating the rank of outcome events' frequency for ETA and calculating the contribution of basic events leading to the top event FTA. A number of defuzzification methods, including max membership principle, centroid method, weighted average method, mean max membership, center of sums, center of largest area, and first (or last) of maxima, are available in the literature.[39–41] The weighted average method is comparatively easy and computationally efficient to implement.[34,41] The following equation for the weighted average method is used to defuzzify the obtained fuzzy numbers for the event tree and fault tree outputs:[41]

$$P_{out} = \frac{\sum \mu_P(\tilde{P}) \cdot \tilde{P}}{\sum \mu_P(\tilde{P})}. \tag{2}$$

5. EVIDENCE THEORY (EVIDENTIAL REASONING)

Multisource knowledge can provide more reliable information about the probability of events (or basic events) than a single source. Knowledge can never be absolute as it is socially constructed and negotiated and often suffers incompleteness and conflict.[42] Evidence theory has alternatively been used in many applications, especially when the uncertainty is due to ignorance and incomplete knowledge.[43,44] The main advantages of evidence theory are:

(1) Individual belief, including complete ignorance, can be assigned.
(2) An interval probability can be obtained for each uncertain parameter.
(3) Multisource information can be combined that helps to avoid bias due to some specific source.[45]

5.1. Fundamentals

Evidence theory was first proposed by Dempster and later extended by Shafer. This theory is

Table VII. Modified α-Cut-Based Fuzzy Arithmetic Operations

Method	Operation	α-Cut Formulation
ETA	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$
	$\tilde{P}_1 \times \tilde{P}_2$	$p_L^{\alpha} = [1 - (1 - C_{dL}^{\alpha})(1 - p_{1L}^{\alpha})] \times p_{2L}^{\alpha}$ $p_R^{\alpha} = [1 - (1 - C_{dR}^{\alpha})(1 - p_{1R}^{\alpha})] \times p_{2R}^{\alpha}$
FTA	$\tilde{P}_1$"OR"$\tilde{P}_2$	$p_L^{\alpha} = \{1 - (1 - p_{1L}^{\alpha}) \times [1 - (1 - C_{dL}^{\alpha}) \times p_{2L}^{\alpha}]\}$ $p_R^{\alpha} = \{1 - (1 - p_{1R}^{\alpha}) \times [1 - (1 - C_{dR}^{\alpha}) \times p_{2R}^{\alpha}]\}$
	$\tilde{P}_1$"AND"$\tilde{P}_2$	$p_L^{\alpha} = \{[1 - (1 - C_{dL}^{\alpha}) \times p_{1L}^{\alpha}] \times p_{2L}^{\alpha}\}$ $p_R^{\alpha} = \{[1 - (1 - C_{dR}^{\alpha}) \times p_{1R}^{\alpha}] \times p_{2R}^{\alpha}\}$

also known as Dempster-Shafer Theory (DST).[24,45] DST uses three basic parameters, that is, *basic probability assignment* (*bpa*), *belief measure* (*Bel*), and *plausibility measure* (*Pl*) to characterize the uncertainty in a belief structure.[32,36,46] The belief structure represents a continuous interval [*belief*, *plausibility*] for the uncertain quantities in which the true probability may lie. Narrow belief structures are representative of more precise probabilities. The main contribution of DST is a scheme for the aggregation of multisource knowledge based on individual degrees of belief.

5.1.1. Frame of Discernment (FOD)

FOD Ω is defined as a set of mutually exclusive elements that allows having a total of $2^{|\Omega|}$ subsets in a power set (P), where $|\Omega|$ is the *cardinality* of an FOD. For example, if $\Omega = \{T, F\}$, then the power set (P) includes four subsets, that is, $\{\Phi$ (a null set), $\{T\}$, $\{F\}$, and $\{T, F\}\}$, as the cardinality is two.

5.1.2. Basic Probability Assignment

The *bpa*, also known as belief mass, is denoted by $m(p_i)$. The *bpa* represents the portion of total knowledge assigned to the proposition of the power set (P) such that the sum of the proposition is 1. The focal elements, that is, $p_i \subseteq P$ with $m(p_i) > 0$, collectively represent the evidence. The *bpa* can be characterized by the following equation:

$$m(p_i) \to [0, 1]; \quad m(\Phi) \to 0; \quad \sum_{p_i \subseteq P} m(p_i) = 1. \quad (3)$$

For example, suppose an expert reports that the occurrence probability of an event in ETA is 80% true and 10% false. For this example, the *baps* of every subset of $m(p_i)$ can be written as $m(T) = 0.8$, and $m(F) = 0.1$. The unassigned *bpa* is referred to the set $m(\Omega) = m(T, F) = 0.1$. This is because, the unassigned *bpa* is taken as ignorance, which is usually represented by the subset $m\{\Omega\}$.[43]

5.1.3. Belief Measure

The *belief* (*Bel*) *measure*, sometimes termed as the lower bound for a set p_i, is defined as the sum of all the *bpa*s of the proper subsets p_k of the set of interest p_i, that is, $p_k \subseteq p_i$. The relationship between *bpa* and *belief measure* is written as:

$$Bel(p_i) = \sum_{p_k \subseteq p_i} m(p_k). \quad (4)$$

The *belief measures* in the above example are given by:

$$Bel(T) = m(T) = 0.8; \quad Bel(F) = m(F) = 0.1$$

and

$$Bel(T, F) = m(T) + m(F) + m(T, F) = 1.0.$$

5.1.4. Plausibility Measure

The upper bound, that is, the *plausibility (Pl) measure*, for a set p_i is the summation of *bpa*s of the sets p_k that intersect with the set of interest p_i, that is, $p_k \cap p_i \neq \Phi$. Therefore, the relationship can be written as:

$$Pl(p_i) = \sum_{p_k \cap p_i \neq \Phi} m(p_k). \quad (5)$$

The *plausibility measures* for the above example are given by:

$$Pl(T) = m(T) + m(T, F) = 0.8 + 0.01 = 0.9;$$

$$Pl(F) = m(F) + m(T, F) = 0.1 + 0.1 = 0.2; \quad \text{and}$$

$$Pl(T, F) = 1.0.$$

5.1.5. *Rule of Combination for Inference*

The combination rules allow aggregating the individual beliefs of experts and provide a combined belief structure. The Dempster & Shafer (DS) combination rule is the fundamental for all combination rules. Many modifications of the DS rule of combination have been reported. The most common modifications include those by Yager, Smets, Inagaki, Dubois and Prade, Zhang, Murphy, and more recently by Dezert and Smarandache.(43) Detailed discussions on these rules can be found in Dezert and Smarandache.(47) In this study, DS and Yager combination rules are discussed in detail and used in developing the evidence-theory-based approach for FTA and ETA. *DS combination rule:* The DS combination rule uses a normalizing factor $(1-k)$ to develop an agreement among the acquired knowledge from multiple sources, and ignores all conflicting evidence through *normalization.* Assuming that the knowledge sources are independent, this combination rule uses the AND-type operator (product) for aggregation.(43) For example, if the m_1 (p_a) and m_2 (p_b) are two sets of evidence for the same event collected from two different experts, the DS combination rule uses the following relation to combine the evidence:

$$m_{1-2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \dfrac{\sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b)}{1-k} & \text{for } p_i \neq \Phi \end{cases}. \quad (6)$$

In the above equation, m_{1-2} (p_i) denotes the combined knowledge of two experts for the event, and k measures the *degree of conflict* between the two experts, which is determined by the factor k:

$$k = \sum_{p_a \cap p_b = \Phi} m_1(p_a) \times m_2(p_b). \quad (7)$$

Yager combination rule: Zadeh in 1984(48) pointed out that the DS combination rule yields counterintuitive results and exhibits numerical instability if conflict is large among the sources.(45) To resolve this issue, Yager in 1987(49) proposed an extension, which is similar to the DS combination rule except that it does not allow normalization of joint evidence with the normalizing factor $(1-k)$. The total degree of conflict (k) is assigned to be part of ignorance Ω.(43) However, in a non- (or less) conflicting case, the Yager combination rule exhibits similar results as the DS combination rule. For high conflict

Table VIII. Determination of Belief Structure of an Event Using Different Combination Rules

m_1	m_2	{T} 0.60	{F} 0.30	{T, F} 0.10
{T}	0.80	{T} = 0.48	{Φ} = 0.24	{T} = 0.08
{F}	0.10	{Φ} = 0.06	{F} = 0.03	{F} = 0.01
{T, F}	0.10	{T} = 0.06	{F} = 0.03	{T, F} = 0.01
k			0.30	
$\sum_{p_a \cap p_b = p_i} m_1(p_a)m_2(p_b)$		0.62	0.07	0.01
m_{1-2} (DS)		0.89	0.1	0.014
m_{1-2} (Yager)		0.62	0.07	0.31

Rules of Combination	Belief Structure			
	Bel (T)	*Pl* (T)	*Bel* (F)	*Pl* (F)
DS rule	0.89	0.90	0.10	0.11
Yager rule	0.62	0.93	0.07	0.38

cases (i.e., higher k value), it provides more stable and robust results than the DS combination rule:(10)

$$m_{1-2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) & \text{for } p_i \neq \Omega \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) + k & \text{for } p_i = \Omega \end{cases}. \quad (8)$$

In the above example, if we assume another expert reports new evidence for the same event: m ({T}) = 0.6, m ({F}) = 0.3 and m ({T, F}) = 0.1. Both bodies of evidence are combined using DS and Yager combination rules. The aggregation of the knowledge is performed using Equations (6) and (8). Equations (3) and (4) are used to obtain the combined belief structure of the event (Table VIII).

5.2. Evidence Theory-Based Approach for FTA/ETA

Expert knowledge is used to define the probability of occurrence and dependency coefficient of events (or basic events). Each expert may have their own belief or knowledge that may be incomplete and that may be in conflict with the others. In an evidential reasoning framework, the ignorance in an evidence is assigned to a subset $m(\Omega)$. The conflict among the sources is dealt with using combination rules as discussed above. The following sections describe the steps of the evidence-theory-based approach to analyze the event tree/fault tree under uncertainties.

5.2.1. Definition of FODs

Three different FODs for three uncertain quantities in FTA and ETA, including the probability of events, the probability of basic events, and the dependency coefficient (C_d), are used to acquire the belief masses from different experts. The subsets for the FODs are generated based on the cardinality of each FOD (Ω).

Traditionally, the consequences of events in ETA are dichotomous, that is, {T} and {F}. Therefore, the FOD for ETA is defined as Ω {T, F}, which leads to four subsets in a power set (P) that includes {Φ, {T}, {F}, {T, F}}.

The operational state of a system is usually defined on the basis of evaluating the success (S) or failure (F) state of basic components.[3,5,6] Hence, the occurrence probability of a basic event in FTA can be described using the FOD $\Omega = \{S, F\}$. As the cardinality is two for the FOD, the power set (P) of each event comprises four subsets, which includes {Φ, {S}, {F}, {S, F}}.

Six qualitative grades of dependency are categorized in this study to describe interdependences through dependency coefficients for FTA or ETA. The notations of these grades are: Independent (I); Very Weak (M); Weak (W); Strong (S); Very Strong (VS); and Perfect dependence (P). The FOD for this case consists of six cardinal elements, which is represented by $\Omega = \{P, VS, S\ W, VW, I\}$.

5.2.2. Assignment of bpas to Basic Events/Events

The *bpa*s or belief masses for the events (or basic events) and the dependency coefficients (C_d) are elicited using the experts' knowledge. Assuming that the knowledge sources are independent, the *bpas* are assigned to particular subsets of each FOD. However, for the dependency coefficient, experts' knowledge is collected only for the subsets {P}, {VS}, {S}, {W}, {VW}, {I}, and {Ω}. The *bpa*s for each subset individually represent the degree of belief of each expert, and implicitly, it represents the total evidences that support the probability of occurrence of an event (or a basic event) and a dependency coefficient (C_d).

5.2.3. Belief Structure and Bet Estimation

The combination rules allow merging the knowledge from different sources as coherent evidence. These rules help to account for ignorance in knowledge and resolve conflicts among the sources. The DS (Equation (6)) or Yager (Equation (8)) combination rules are used in this study to aggregate collected knowledge from different sources. Equations (4) and (5) are then used to derive the *belief* and *plausibility* measure for the probability and dependency coefficients of events (or basic events). The *belief* and *plausibility* measure for six kinds of dependencies (in each node of FTA or ETA) are normalized to attain a generalized belief structure. Information in Table VI, which represents the *belief* and *plausibility* for each kind of dependency, is used for normalizing the belief structure of dependency coefficient for each node. Subsequently, equations shown in Table IX are used to estimate the *likelihoods* of outcome events and top event for the ETA and FTA, respectively.

"*Bet*" provides a point estimate in belief structure (similar to defuzzification), which is often used to represent the crisp value of the final events. It is estimated based on the following equation:

$$Bet(P) = \sum_{p_i \subseteq P} \frac{m(p_i)}{|p_i|}, \tag{9}$$

where $|p_i|$ is the *cardinality* in the set p_i. For example, the "*Bet*" estimate for the belief structure obtained

Method	Operation	Formulation
ETA	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} [Bel(P_i), Pl(P_i)]$
	$P_1 \times P_2$	$Bel(P_{out}) = \left[1 - \{(1 - Bel(C_d)\} \times \{(1 - Bel(P_1)\}\right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{(1 - Pl(C_d)\} \times \{(1 - Pl(P_1)\}\right] \times Pl(P_2)$
FTA	P_1"OR"P_2	$Bel(P_{out}) = 1 - \{(1 - Bel(P_1)\} \times \left[\{1 - (1 - Bel(C_d)\} \times Bel(P_2)\right]$
		$Pl(P_{out}) = 1 - \{(1 - Pl(P_1)\} \times \left[\{1 - (1 - Pl(C_d)\} \times Pl(P_2)\right]$
	P_1"AND"P_2	$Bel(P_{out}) = \left[1 - \{(1 - Bel(C_d)\} \times \{(1 - Bel(P_1)\}\right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{(1 - Pl(C_d)\} \times \{(1 - Pl(P_1)\}\right] \times Pl(P_2)$

Table IX. Equations to Analyze the Event and Fault Trees using the DS combination rule is calculated as:

$$Bet(P_T) = \frac{m(T)}{1} + \frac{m(T, F)}{2} = \frac{0.89}{1} + \frac{0.01}{2} = 0.895.$$

The denominators "1" and "2" represent the cardinality in the respective subsets.

6. APPLICATION OF DEVELOPED APPROACHES

The same examples ("LPG release event tree" and "runaway reaction fault tree") discussed earlier in Section 2 are studied in detail here using both fuzzy-based and evidence theory-based approaches.

6.1. LPG Release—Event Tree Analysis

6.1.1. Fuzzy-Based Approach

The revised event tree with fuzzy probabilities is illustrated in Fig. 7. In the fuzzy-based approach, the probability of events (or basic events) and their dependency coefficients (C_d) are defined using TFNs. The frequency for the outcome events are then estimated using the α-cut-based fuzzy formulations developed in Table VII. For example, the path leading to the outcome event "A" is followed by the two events. The probability and the coefficient of dependency (C_d) of these two events are linguistically expressed, which are, respectively, assumed to be "Moderately Low (ML)," "Moderately High (MH)," and "Strong (S)." The assigned linguistic expressions for these two events are converted into TFNs (based on Fig. 6 and Table VI). The TFN for the outcome event "A" (shown in Fig. 8) is derived using the empirical equations described in Table VII. Using numerous trials for event dependency at each node of the LPG event tree, the uncertainty ranges (i.e., fuzzy interval) for the outcome event "A" are estimated (shown in Fig. 9). It can be observed that the uncertainty ranges are varied according to the change of event dependency at each node of the event tree.

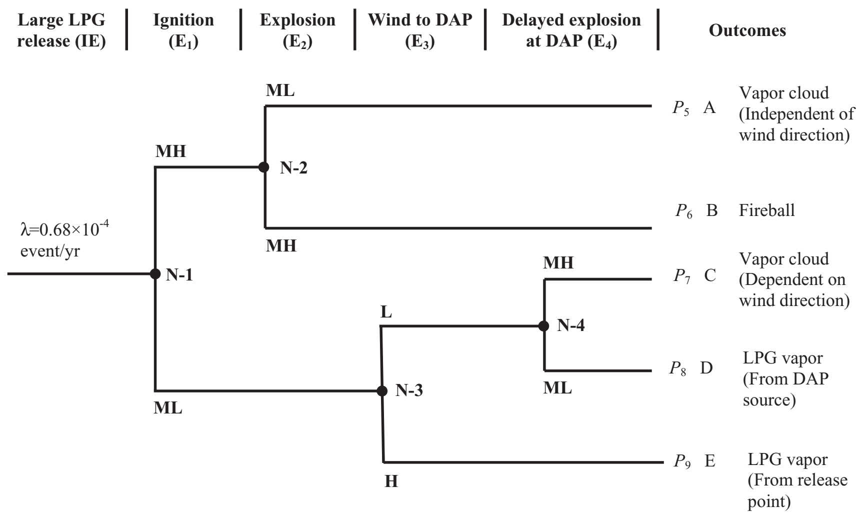

Fig. 7. Event tree with fuzzy linguistic grades.

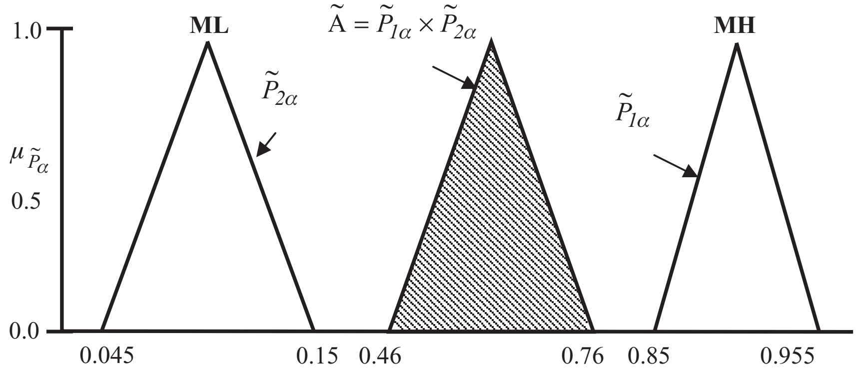

Fig. 8. Triangular fuzzy numbers of outcome event "A" with "strong" dependency.

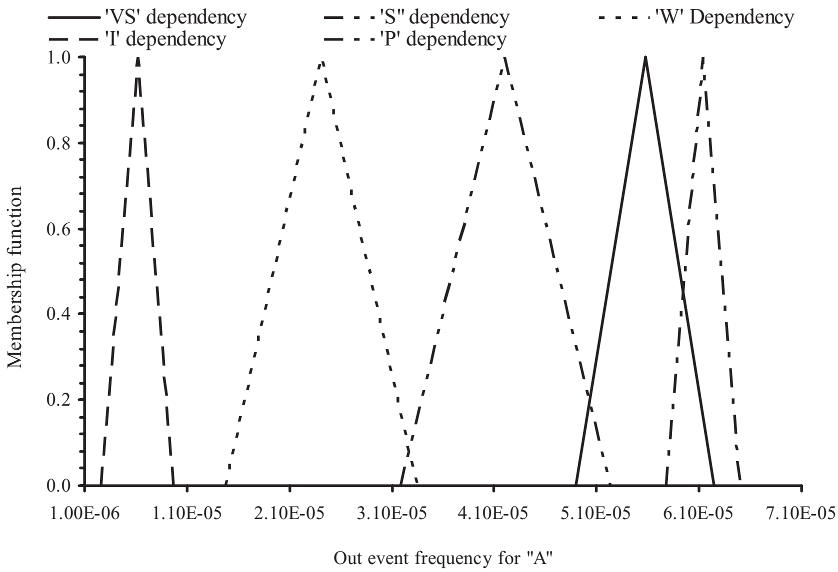

Fig. 9. Uncertainty in outcome event's frequency "A."

6.1.2. Evidence-Theory-Based Approach

To demonstrate the evidence-theory-based approach, experts' knowledge from two unbiased and independent sources is considered for determining the probability as well as the dependency coefficients of events for ETA. The elicited knowledge from the sources is shown in Tables X(a) and X(b).

DS and Yager combination rules are used to aggregate and determine the belief structures of probability and dependency coefficients of events for the ETA. Table XI lists the belief structures of events and dependency coefficients for the LPG release event tree (Fig. 1). These belief structures and the equations in Table IX are used to derive the belief structures for the outcome events of LPG release. Two different kinds of dependence, that is, independent and dependent, are considered while estimating the belief structures for the outcome events. The

Table X. Experts' Knowledge on **(a)** the Probability of Events

		Expert 1 (m_1)			Expert 2 (m_2)		
Symbol	Events' Name	{T}	{F}	{T, F}	{T}	{F}	{T, F}
E_1	Ignition	0.80	0.10	0.10	0.60	0.30	0.10
E_2	Explosion	0.10	0.80	0.10	0.05	0.80	0.15
E_3	Wind to DAP	0.60	0.20	0.20	0.50	0.40	0.10
E_4	Ignition Explosion at DAP	0.85	0.10	0.05	0.80	0.10	0.10

(b) Interdependence of Events at Different Nodes

Number of Experts	Node (N)	{*P}	{VS}	{S}	{W}	{VW)	{I}	{Ω}
Expert 1 (m_1)	N-1	0.15	0.00	0.30	0.10	0.00	0.00	0.45
	N-2	0.00	0.30	0.20	0.00	0.10	0.00	0.40
	N-3	0.40	0.00	0.20	0.00	0.00	0.20	0.20
	N-4	0.50	0.20	0.00	0.00	0.00	0.20	0.10
Expert 2 (m_2)	N-1	0.30	0.00	0.20	0.15	0.00	0.00	0.35
	N-2	0.20	0.30	0.00	0.00	0.00	0.15	0.35
	N-3	0.00	0.20	0.40	0.00	0.20	0.00	0.20
	N-4	0.00	0.30	0.40	0.00	0.20	0.00	0.10
	N-1	0.30	0.00	0.20	0.15	0.00	0.00	0.35

Table XI. Belief Structures for the Probability and Interdependence of Events

Reference in the Event Tree		DS Rule of Combination *Bel*	DS Rule of Combination *Pl*	Yager Rule of Combination *Bel*	Yager Rule of Combination *Pl*
E_1	T	0.8857	0.9000	0.6200	0.9300
	F	0.1000	0.1143	0.0700	0.3800
E_2	T	0.0284	0.0455	0.0250	0.1600
	F	0.9545	0.9716	0.8400	0.9750
E_3	T	0.6970	0.7273	0.4600	0.8200
	F	0.2727	0.3030	0.1800	0.5400
E_4	T	0.9641	0.9701	0.8050	0.9750
	F	0.0299	0.0359	0.0250	0.1950
*N-1		0.2549	0.7450	0.1605	0.8395
N-2		0.2755	0.7245	0.1526	0.8474
N-3		0.3150	0.6849	0.0769	0.9230
N-4		0.4013	0.5987	0.0512	0.9488

		P	VS	S	W	VW	I
N-1	*Bel*	0.305	0.000	0.334	0.154	0.000	0.000
	Pl	0.511	0.207	0.541	0.361	0.207	0.207

$$Bel(C_d) \quad \frac{(0.305 \times 1 + 0 \times 0.80 + 0.334 \times 0.45 + 0.154 \times 0.15 + 0 \times 0.005 + 0 \times 0)}{(0.305 + 0 \times 0.80 + 0.334 \times 0.45 + 0.154 \times 0.15 + 0 \times 0.005) + (0.551 + 0.207 \times 0.995 + 0.541 \times 0.85 + 0.361 \times 0.5 + 0.207 \times 0.2)} = 0.255$$

$$Pl(C_d) \quad \frac{(0.511 \times 1 + 0.207 \times 0.995 + 0.541 \times 0.85 + 0.361 \times 0.5 + 0.207 \times 0.2 + 0 \times 0.207)}{(0.305 + 0 \times 0.80 + 0.334 \times 0.45 + 0.154 \times 0.15 + 0 \times 0.005) + (0.551 + 0.207 \times 0.995 + 0.541 \times 0.85 + 0.361 \times 0.5 + 0.207 \times 0.2)} = 0.745$$

*Normalization of belief structure at N-1 for DS rule of combination.
T = true; F = false.

Table XII. Outcome Events Frequency for Two Kinds of Interdependence of Events

Outcome Events	Independent: Belief Structures (DS Rule of Combination) *Bel*	Independent: Belief Structures (DS Rule of Combination) *Pl*	Independent *Bet*	Dependent: Belief Structures (Yager Rule of Combination) *Bel*	Dependent: Belief Structures (Yager Rule of Combination) *Pl*	Dependent *Bet*
A	1.054E-06	1.012E-05	*5.586E-06	1.153E-06	1.076E-05	5.958E-06
B	3.541E-05	6.166E-05	4.854E-05	3.873E-05	6.559E-05	5.216E-05
C	1.763E-06	2.066E-05	1.121E-05	6.186E-06	6.556E-05	3.587E-05
D	5.474E-08	4.132E-06	2.093E-06	1.921E-07	1.311E-05	6.652E-06
E	8.568E-07	1.395E-05	7.405E-06	1.733E-06	3.497E-05	1.835E-05

*Belief structure of outcome event "A" is [1.054E-06, 1.012E-05]. So, $m\,(T) = 1.054\text{E-}06$, and $m(T, F) = 9.064\text{E-}06$

$$Bet(\mathrm{A}) = \frac{m(T)}{1} + \frac{m(T,F)}{2} = \frac{1.054\text{E-}06}{1} + \frac{9.064\text{E-}06}{2} = 5.586\text{E-}06$$

results are presented in Table XII. An order of magnitude difference is observed in the "*Bet*" estimation for the outcome event "E." This difference signifies the importance of defining the dependency relationships in ETA.

The difference of using the DS and Yager combination rules is shown in Fig. 10. In the figure, different kinds of dependencies are labeled on the x-axis. The *belief* and *plausibility* measure for each kind of dependency is plotted on the y-axis. The minimum

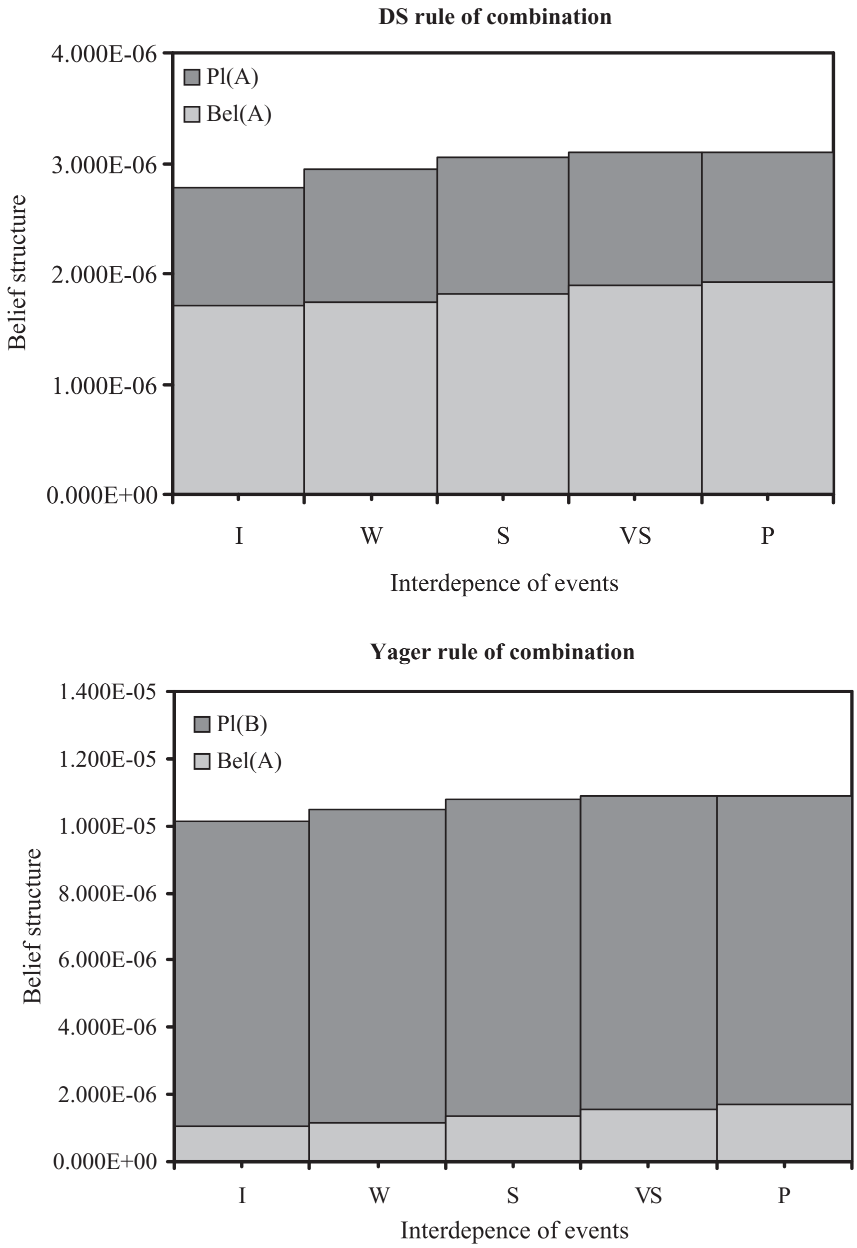

Fig. 10. Belief structure representing the frequency for outcome event "A."

and maximum values presented in Table VI are considered as the belief structure of dependency coefficient for each kind of dependency. The shaded areas in Fig. 10 represent the *belief* and *plausibility* measures for the outcome event "A." These areas show that the Yager combination rule measures a large belief structure in comparison to the DS combination rule. Hence, an interpretation can be made that the Yager combination rule yields more conservative results (i.e., a larger belief structure) in the context of existing high conflicts in the sources.

6.2. Runaway Reaction—FTA

6.2.1. Fuzzy-Based Approach

To demonstrate the fuzzy-based approach for FTA, the probability of basic events and their dependencies are defined using expert linguistic expressions. The linguistic expressions are converted into TFNs. The linguistic expressions and the corresponding TFNs are given in Table XIII. A total of seven different trials and the fuzzy arithmetic operations (described in Table VII) are used to evaluate the

Table XIII. Expert's Knowledge on the Probability of Basic Events

Event	Linguistic Variable	TFNs
BE_1	L	(0.1, 0.25, 0.4)
BE_2	VL	(0, 0.025, 0.05)
BE_3	ML	(0.045, 0.0975, 0.15)
BE_4	VL	(0, 0.025, 0.05)
BE_5	VL	(0, 0.025, 0.05)
BE_6	VL	(0, 0.025, 0.05)

TFN for the top event. The trials are categorized based on different assumptions of dependencies at each node of the fault tree. The TFNs of the top event for the different trials are shown in Fig. 11. In trial 7, when perfect dependencies are assumed, the top event probability bears the maximum uncertainty. Contrary to trial 1, when the events are assumed independent, the top event probability bears the smallest uncertainty.

6.2.2. *Evidence-Theory-Based Approach*

The fault tree for the runaway reaction as shown in Fig. 2 is studied to demonstrate the application of evidence-theory-based approach in FTA. The probability of basic events and the dependency coefficients for the fault tree are obtained from two independent sources. Tables X(b) and XIV show the experts' knowledge for defining the probability and dependency coefficients of basic events for the FTA.

DS and Yager combination rules are used to aggregate the knowledge and estimate the belief structures for the basic events and dependency coefficients. The belief structure of the top event is

	Dependency of basic events in different nodes (N)				TFNs of top event probability
Trials (T)	N-1	N-2	N-3	N-4	*(P_L,P_m,P_R)*
1	I	I	I	I	(0, 0.013, 0.027)
2	VW	VW	VW	VW	(0, 0.061, 0.122)
3	W	W	W	W	(0, 0.122, 0.244)
4	VS	S	W	W	(0, 0.121, 0.243)
5	S	S	S	S	(0, 0.179, 0.359)
6	VS	VS	VS	VS	(0, 0.199, 0.399)
7	P	P	P	P	(0, 0.200, 0.400)

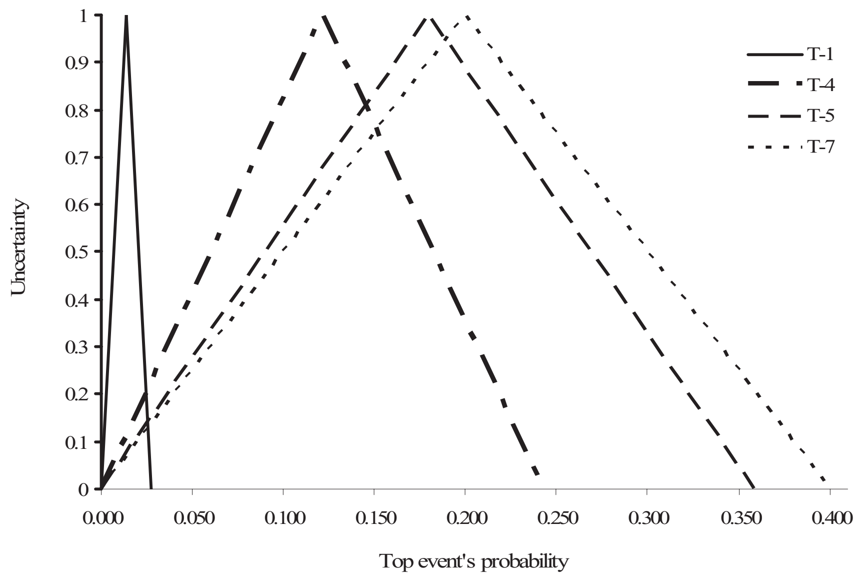

Fig. 11. Uncertainty representation for top event using different trials.

Table XIV. Multisource Knowledge for the Probability of Basic Events

Basic Event	Expert 1 (m_1) {F}	{S}	{SF}	Expert 2 (m) {F}	{S}	{SF}
BE_1	0.150	0.750	0.100	0.250	0.650	0.100
BE_2	0.020	0.800	0.180	0.015	0.900	0.085
BE_3	0.200	0.700	0.100	0.100	0.800	0.100
BE_4	0.015	0.950	0.035	0.025	0.950	0.025
BE_5	0.015	0.900	0.085	0.010	0.980	0.010
BE_6	0.002	0.950	0.048	0.001	0.940	0.059

S = success; F = failure.

then calculated by using the equations in Table IX. Table XV shows the belief structure and the *"Bet"* estimate of the top event for two combination rules. A total of seven trials are performed using different assumptions of interdependence between the basic events. The belief structure for each kind of dependence is defined in Table VI. Table XV indicates that the uncertainty in calculating the belief structure and "*Bet"* estimate varies accordingly with the change of interdependence at different nodes.

7. UNCERTAINTY-BASED FORMULATIONS FOR FTA AND ETA: A COMPARISON

The level of uncertainty associated with a system is proportional to its complexity, which arises as a result of vaguely known relationships among various entities, and randomness in the mechanisms governing the domain. Sadiq *et al.*[50] described complex systems such as environmental, sociopolitical, engineering, or economic systems, which involve human interventions, and where vast arrays of inputs and outputs could not all possibly be captured analytically or controlled in any conventional sense. Moreover, relationships between causes and effects in these systems are often not well understood but can be expressed empirically. Typical complex systems consist of numerous interacting factors or concepts. These systems are highly nonlinear in behavior and the combined effects of contributing factors are often subadditive or superadditive. The modeling of complex dynamic systems requires methods that combine human knowledge and experience as well as expert judgment. When significant historical data exist, model-free methods such as artificial neural networks (ANN) can provide insights into cause-effect relationships and uncertainties through learning from data.[4] But, if historical data are scarce and/or available information is ambiguous and imprecise, soft computing techniques can provide an appropriate framework to handle such relationships and uncertainties. Such techniques include probabilistic and evidential reasoning (Dempster-Shafer Theory), fuzzy logic, and evolutionary algorithms.[50] Table XVI provides a qualitative comparison between five soft computing techniques, including ANN, decision trees (DT), fuzzy rule-based models (FRBM), Bayesian networks (BN), and cognitive maps/fuzzy cognitive maps (CM/FCM). Central to this comparison is an assessment of how each technique treats inherent uncertainties and its ability to handle interacting factors that encompass issues specific to engineering systems.[50]

Qualitative and quantitative comparisons have been performed in this section to investigate the features and uncertainty handling abilities of different tools and the proposed approaches for FTA and ETA. The qualitative comparison presented in Table XVII illustrates that most of the tools such

Table XV. Belief Structures and "*Bet*" Estimations of Top Event for Different Trails

Trials (T)	Dependency of Basic Events at Different Nodes: N-1	N-2	N-3	N-4	Belief Structure of Top Event's Probability: DS Rule *Bel*	DS Rule *Pl*	DS Rule *Bet*	Yager Rule *Bel*	Yager Rule *Pl*	Yager Rule *Bet*
1	I	I	I	I	3.440E-05	6.234E-04	3.289E-04	2.579E-05	3.942E-03	1.984E-03
2	VW	VW	VW	VW	3.595E-05	3.865E-02	1.934E-02	2.721E-05	1.133E-01	5.666E-02
3	W	W	W	W	9.650E-05	8.294E-02	4.152E-02	8.555E-05	2.441E-01	1.221E-01
4	VS	S	W	W	1.998E-04	8.292E-02	4.156E-02	1.772E-04	2.483E-01	1.242E-01
5	S	S	S	S	2.506E-04	1.151E-01	5.768E-02	2.440E-04	3.458E-01	1.730E-01
6	VS	VS	VS	VS	3.167E-04	1.222E-01	6.126E-02	3.160E-04	3.718E-01	1.860E-01
7	P	P	P	P	2.000E-04	1.224E-01	6.130E-02	2.000E-04	3.725E-01	1.864E-01

P = perfect dependence; VS = very strong; S = strong; W = weak; VW = very weak; I = independent.

Table XVI. Comparison of Various Techniques for Complex Systems[50]

Attributes	Soft Computing Techniques: Decision Tree	Fuzzy Rule-Based Models	Artificial Neural Networks	Bayesian Networks	Cognitive Maps/Fuzzy Cognitive Maps
Network capability	N[a]	L[b]	N	H[c]	VH[d]
Ability to express causality	H	M	N	H	VH
Formulation transparency	H	H	N[e]	H	VH
Ease in model development	H	M	M	M	VH
Ability to model complex systems	M	H	VH	H	VH
Ability to handle qualitative inputs	H	H	N	H	VH
Scalability and modularity	VL	L	VL[f]	H	VH[g]
Data requirements	H	L	VH	M	L[h]
Difficulty in modification	VH	H	M	L	N
Interpretability of results	VH	VH	VH	VH	H
Learning/training capability	H	M[i]	VH[j]	H[k]	H[l]
Time required for simulation	L	L	H	L	L
Maturity of science	VH	H	H	VH	M
Ability to handle dynamic data	L	H	H	H	M
Examples of hybrid models (ability to combine with other approaches)	H	VH[m]	VH[m]	H	H[n]

Ratings: N = no or negligible; VL = very low; L = low; M = medium; H = high; VH = very high.
[a]Structure is hierarchical.
[b]Dimensionality is a major problem and formulation becomes complicated for network systems.
[c]Can manage networks but cannot handle feedback loops; therefore referred to as directed acyclic graphs (DAG).
[d]Can handle feedback loops.
[e]Generally, referred to as black-box models.
[f]ANN needs to be retrained for new set of conditions.
[g]Very easy to expand because algorithm is in the form of matrix algebra.
[h]Minimal data requirement because causal relationships are generally soft in nature.
[i]Clustering techniques, e.g., fuzzy C-means.
[j]Algorithms, e.g., Hebbian learning.
[k]Algorithms, e.g., evolutionary algorithms and Markov chain Monte Carlo.
[l]Training algorithms are available, which have been successful in training ANNs.
[m]Examples are available in the literature to develop models using hybrid techniques, e.g., neuro-fuzzy models.
[n]Has a potential to be used with other soft techniques.

as Relex V7.7,[51] RAM Commander 7.7[52], and PROFAT[53] are unable to handle *dependency uncertainty*. Except for PROFAT, the other tools cannot handle subjective uncertainty in the fault and event trees for a system. PROFAT[53] is a fuzzy-based tool that can handle subjective uncertainty; however, it fails to account for *epistemic uncertainty* owing to ignorance or incompleteness of an expert's knowledge.

Another type of uncertainty arises due to lack of information on dependencies among events. Traditional FTA uses a default assumption of "independence" among the risk events to determine the joint probability (risk) of a parent event. This

Table XVII. Qualitative Comparisons of Proposed Approach with Available Fault Tree Analyses/Event Tree Analyses Tools

Uncertainty	Relax V7.7[(51)a]	RAM Commander 7.7[(52)a]	PROFAT (Khan and Abbasi, 1999)[(53)]	Proposed Approach
Subjective (fuzzy-based)	NC[b]	NC	C[c]	C
Incompleteness (evidence based)	NC	NC	NC	C
Dependency	NC	NC	NC	C

[a]A commercial software.
[b]Not considered.
[c]Considered.

assumption simplifies the analysis, but may not be very realistic. The relationship between risk events may be positively or negatively correlated (or independent). In the case of two independent events X and Y, the joint probability of their conjunction is simply a product of their individual probabilities.[(12,14)] There exist many different methods to express correlation (dependence) but the Frank model (copula) is the most common.

Simple dependency coefficient-based empirical relations (similar to Li approach[(24)]) embedded within the proposed approach can concurrently handle the *dependency uncertainty* in FTA and ETA. The proposed approach successfully accounts for the subjective uncertainty using a membership function and evaluates the uncertainty range as a fuzzy interval. The evidence-theory-based approach can describe the *epistemic* and *aleatory uncertainties* in experts' knowledge using *bpa* and is able to provide a measure of uncertainty using belief structures.

Relax V7.7[(51)] and RAM Commander 7.7[(52)] are two useful tools for reliability and safety engineering. The probability of top event for the "runaway reaction fault tree" and the frequency of outcome events for the "LPG release event tree" have been analyzed using these tools for the same input (Fig. 1 and Table II). Results (Table XVIII) show that by introducing 10% uncertainty into the input data, these two tools accumulated about 19% and 9% of uncertainty on the calculated top event's probability and outcome event's frequency of "B." The original input data (Fig. 1 and Table II) are reduced by 10% to introduce the uncertainty into the analysis. The traditional (probabilistic) methods used predefined PDFs to describe the uncertainty in the input data (i.e., the probability of basic events or events in FTA

Table XVIII. Quantitative Comparison of Fault Tree Analyses/Event Tree Analyses Tools

Determination of Probability of Top Event for "Runaway Reaction Fault Tree"

Commercial Packages: Relex V7.7[(51)]		Commercial Packages: RAM Commander 7.7[(52)]		Fuzzy-Based Defuzzified Value		Evidence-Theory-Based *Bet* Estimation	
No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty
3.16E-04	2.55E-04	3.41E-04	2.74E-04	8.71E-03	8.76E-03	3.16E-04	2.86E-04

Determination of Frequency of Outcome Events for LPG Release

	Relex V7.7, RAM Commander 7.7[(51,52)]		Fuzzy-Based Defuzzified Value		Evidence-Theory-Based *Bet* Estimation	
Outcome Events	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty
A	6.12E-06	4.95E-06	5.98E-06	5.99E-06	6.12E-06	8.36E-06
B	5.51E-05	5.01E-05	5.54E-05	5.54E-05	5.51E-05	5.05E-05
C	2.45E-06	3.76E-06	1.50E-06	1.58E-06	2.45E-06	3.60E-06
D	2.72E-07	8.84E-07	1.62E-07	1.71E-07	2.72E-07	6.64E-07
E	4.08E-06	8.27E-06	4.97E-06	4.98E-06	4.08E-06	5.79E-06

or ETA). When the crisp data or the PDFs for the input data are not known or limited (a very common situation in process systems), the FTA or ETA are highly dependent on expert knowledge. In these situations, traditional tools and probabilistic approaches are not helpful. This makes the FTA/ETA less credible. Both fuzzy set theory and evidence theory are not limited by availability of detailed data. The results using both approaches are presented in Table XVIII. An expert knowledge and assumption of independence among events (or basic events) are used in calculating the top event probability and outcome events frequency. In fuzzy-based approach, the uncertainty is assigned using the membership function. The TFNs corresponding to 90% membership are considered as input data for the analysis. In evidence-theory-based approach, the uncertainty is allocated through the unassigned mass (as ignorance) of the power set. For the 10% uncertainty in the basic event probabilities, the evidence-theory-based approach estimates about 9% and 8% uncertainties in the response, that is, [2.55×10^{-4}, 3.16×10^{-4}] and [4.46×10^{-5}, 5.63×10^{-5}], respectively. Similarly, the fuzzy-based approach measures less than 1% uncertainty results in the response (top event's probability as well as outcome event's frequency "B.") with corresponding fuzzy intervals of [7.38×10^{-3}, 1.01×10^{-2}] and [5.47×10^{-5}, 5.60×10^{-5}].

8. RESULTS AND DISCUSSIONS

Two types of uncertainty, namely, *data uncertainty* and *dependency uncertainty*, were explored. Expert knowledge in terms of fuzzy linguistic grades and *bpa*s was used instead of assigning the *likelihood* and interdependencies of basic events/events as crisp probabilities for FTA/ETA. The dependency coefficient in each node of the fault tree and event tree addressed the *dependency uncertainty* and described the relationships among the basic events/events. Fuzzy linguistic grades were assigned to TFNs and α-cut-based fuzzy empirical relations of the fuzzy-based approach were used to handle the linguistic and subjective uncertainty in expert knowledge. For multisource knowledge, the incomplete and inconsistent *bap*s were combined by using combination rules. The dependency coefficients in evidence-theory-based empirical relations were used to describe the *dependency uncertainty* and analyze the fault tree and event tree under uncertainty due to inconsistent, incomplete, and partial ignorance of multisource knowledge.

The developed approaches were applied to two case studies: "LPG release event tree" and "runaway reaction fault tree." The interdependencies among the events (or basic events) were varied in each node of the fault tree or event tree. The impacts of the interdependencies were observed so as to understand the effects of the dependencies of events (or basic events) in FTA/ETA for process systems. For two dependence cases of basic events/events, independent and perfectly dependent, the output results for the FTA and ETA are provided in Tables XIX and XX, respectively. It can be observed in the first three rows of Table XIX that the results remain almost the same. However, when dependency was considered (fourth row in Table XIX), the results varied by an order of magnitude. This highlights the importance of dependencies in ETA.

The results in Table XX are inconsistent mainly because of different types of uncertainties modeled in the different approaches. The perfectly dependent case in FTA determines the probability range for the top event as [0, 0.400], which is a maximum in comparison to the independent case for representing the uncertainty. It can also be observed in Table XX that when the basic events are perfectly dependent, the point estimate (defuzzified value) of the top event exhibits a higher ordered magnitude in comparison to the deterministic approach and MCS-based

Table XIX. Summary of Event Tree Analyses Results

Approach		Dependency of Events	Frequency of Outcome Event "A"
Deterministic approach		Independent	6.12E-06
MCS-based approach	90% confidence interval	Independent	(1.96E-05, 1.02E-04)
	Median		6.10E-06
Fuzzy-based approach	Fuzzy interval	Independent	(2.60E-06, 9.74E-06)
	Defuzzified value		6.17E-06
Fuzzy-based approach	Fuzzy interval	Perfectly	(5.78E-05, 6.49E-05)
	Defuzzified value	dependent	6.14E-05

Table XX. Summary of Fault Tree Analyses Results

Approach		Dependency of Basic Events	Top Event's Probability (P_{Top})
Deterministic approach		Independent	3.16E-04
MCS-based approach	90% Confidence interval	Independent	(4.24E-05, 2.31E-04)
	Median		1.30E-04
Fuzzy-based approach	Fuzzy interval	Independent	(0, 2.70E-02)
	Defuzzified value		1.35E-02
Fuzzy-based approach	Fuzzy interval	Perfectly dependent	(0, 4.00E-01)
	Defuzzified value		2.00E-01

approach. This confirms the significance of including the dependencies of the basic events in FTA. Similar observations of using the evidence-theory-based approach for FTA/ETA (Tables XII and XV) confirm that a reliable and robust result cannot be attained without considering the interdependencies of events (or basic events).

9. CONCLUSIONS

FTA and ETA are two fairly established techniques; however, the uncertainty in defining the probabilities and the relationships of events (or basic events) can lead to questionable results for QRA. The traditional approaches require the known probability and the independence assumption of events (or basic events), which are rare and often unrealistic for process systems. Two different approaches to handle these types of uncertainties in FTA and ETA are derived in this study by combining expert knowledge with fuzzy set theory and evidence theory. The application of these approaches to two different case studies shows the proposed approaches are more robust to handle the uncertainty in QRA for the process systems in the following ways:

(1) Fuzzy-based approaches and evidence-theory-based approaches properly address the uncertainties in expert knowledge and analyze the event trees or fault trees associated with different kinds of uncertainties in expert knowledge.
(2) Introduction of a dependency coefficient in the fuzzy- and evidence-theory-based approaches describes interdependencies among the events (or basic events) in a fault tree/event tree.
(3) The proposed approaches can be applied to FTA/ETA for any process systems that have *data* and *dependency uncertainties*.

Including the negative dependencies of events (or basic events), and combining the subjectivity (using fuzzy-based approach) and incompleteness (evidence theory) into a single approach, for example, Fuzzy-Dempster-Shafer, may offer additional future improvement to the approaches developed here.

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