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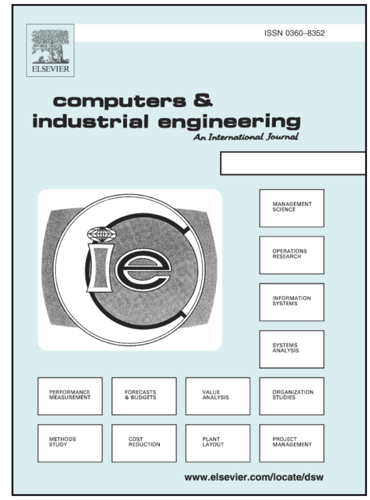

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On the belief universal gravitation (BUG)

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Abstract

In this paper, we proposed a notion of belief universal gravitation (BUG) in the Dempster-Shafer (D-S) evidence theory, of which the notion of mass of a belief function is newly addressed using evidence quality coding (EQC) method. The proposed BUG aims to discuss the process of information fusion from the perspective of Newton's mechanics, which may provide us a new insight to address the issues of D-S evidence theory. A key issue in D-S evidence theory, i.e., conflict management, is solved better than previous methods using the proposed BUG. An application in fault diagnosis is used to illustrate the effectiveness of the proposed BUG. Some further work is also summarized to present the potentials of the proposed BUG.

Keywords: Information fusion, Dempster-Shafer evidence theory, Belief universal gravitation, Evidence quality coding algorithm, Conflict management, Fault diagnosis.

1. Introduction

Dempster-Shafer (D-S) evidence theory was first proposed by Dempster and then further developed by his student Shafer [1, 2]. In the 1970s and 1980s, D-S evidence theory was introduced into the field of artificial intelligence [3]. Like fuzzy sets [4, 5], rough sets [6, 7], Z-number [8, 9, 10], belief structures [11], D numbers [12, 13, 14], soft likelihood functions (SLF) [15, 16, 17], belief entropy [18, 19] and belief function [20] as an uncertainty reasoning method, D-S evidence theory provides a powerful tool for the representation and fusion of decision-level uncertainty information. Up to now, D-S evidence theory has been widely used in information fusion [21, 22], fault diagnosis [23], decision analysis [24, 25], and so on, due to its advantages in dealing with uncertain information.

In particular, Dempster's combination rule is the core part in evidence theory, which is used to fuse evidence information from multiple independent sources. However, the founder of fuzzy mathematics Zadeh found through a counter-example: when using

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it to combine conflicting evidence, the result is contrary to human intuition [26]. For a deeper understanding, we briefly review this example. From Definition 5, it can be clearly observed that there is a clear conflict between the evidence m_1 and m_2. The reason is as follows:

- Proposition A is strongly supported by the m_1, but completely denied by the m_2.
- Proposition C is strongly supported by the m_2, but completely denied by the m_1.

Then the Dempster's combination rule was used to fuse the two pieces of evidence m_1 and m_2, and the fusion result was completely positive for proposition B, which was low in support of original evidence m_1 and m_2. Obviously, this was not intuitive for human beings, and the fusion result was not convincing.

Since then, more and more scholars have questioned the validity of Dempster's combination rules. The research on the conflict between evidences has always been a hot topic in D-S evidence theory. Generally speaking, there are two ways to deal with conflict information. The first way is to modify the combination rule of the classical D-S evidence theory to adapt to the environment of high conflict, that is, to modify the rule. The second idea is to keep the combination rule of the classical evidence theory and preprocess the conflict evidence before fusion. In other words, this type of method is to modify the data model. Where, the viewpoint of modifying rule can be summarized as follows. Conflict management is a key problem to improve and develop evidential reasoning. The use of Dempster's combination rule under high conflict of evidence will produce unreasonable conclusions, which is generated by the normalization step of the rule, so it is necessary to modify Dempster's combination rule. The new combination rule needs to focus on how to redistribute conflicts. The representative of this school is the *"Unified reliability function combination method"* proposed by Lefevre et al. [27]. Other methods under this thinking include: Yager [28, 29], Inagaki [30] et al. introduced the method of average support for propositions, Sun et al. [31], Zhang's method [32], the conditions of Dempster's combination rules [33], the combination rule of *minC* proposed by Dinael et al. [34] refined allocation space and local conflicts and proposed the concept of potential conflicts, Smardndache et al. [35] proposed the conflict proportional allocation rule *PCR*3 and Ma et al. [36] recently proposed the flexible combination rule based on the complete conflict set, and so on. Unlike the first class of methods for modifying combination rule, the second class of methods is based on modifying the original evidence source. The view of this class of methods is that Dempster's combination rules are not inherently wrong. When the evidence is highly conflicted, the conflict evidence should be preprocessed first, and then the Dempster's combination rule should be used. Such methods are represented by Haenni et al. [37]. Other typical methods include Murphy's simple arithmetic average method [38], Deng et al.'s weighted average method [39], et al [40, 41]. For now, researchers lean toward Haenni's point of view, and papers that modify data models dominate. In particular, Liu [42] comprehensively considered the applicability of Dempster's combination rule in the case of conflict, proposed the method of using binary groups of k and $difBetP$ to describe the conflict, and proposed the proposal of using Demspster's combination rule in the authoritative journal $Artificial\ Intelligence$. So far, research on conflicts in evidence theory continues [43, 44, 45].

Above, we briefly reviewed the development history and current status of D-S evidence. Since the proposed method is based on the theory of gravitation, it is necessary to introduce some basic knowledge related to the theory in the following paragraphs.

The law of gravitation was first proposed by Newton in the book *"Mathematical Principles of Natural Philosophy"* published in 1687 [46]. The law of gravitation belongs to the laws of natural science. It shows that any two objects in nature are attracted to each other. The magnitude of gravity is proportional to the product of the masses of two objects and inversely proportional to the square of the distance between them [46]. Subsequently, the law of gravitation has been widely developed and applied in the field of natural science. In recent years, some scholars have proposed a population optimization algorithm based on the idea of the law of gravitation [47], while others have developed a gravitational search algorithm [48], and so on. This novel algorithm combining physical meaning has attracted the attention of many scholars immediately. Since then, more and more scholars have tried to apply gravitation to relevant scientific research fields. Therefore, studies based on gravity have been further developed [49, 50, 51].

Based on the above discussion and knowledge background, inspired by Newton's law of gravitation, in this paper, for the first time, we creatively proposed the belief universal gravitation (BUG) in D-S evidence theory. First, we believed that the essence of evidence information fusion is affected by some potential force. If certain conditions are met, the evidence will be fused. Secondly, in order to quantify the degree of force between the evidences, we proposed the theory of belief universal gravitation (BUG). Among them, the BUG formula is the core part of the theory. The details about it are expressed as follows.

- The evidence obtained by the sensor is abstracted into the quality of logical evidence. The proposed evidence quality coding (EQC) algorithm is used to obtain the quality of evidence for each independent source.
- The evidence distance [52] is used to indicate the spatial distance between two pieces of evidence.
- The evidence gravity parameter G_{ET} is used to distinguish the different discernment frameworks in the system where the same evidence gravity is located.

In addition, by proof, the BUG formula satisfies some basic properties. The relationship between it and other variables is further explained through simulation experiment. Thirdly, based on the BUG theory, we modeled the essence of information fusion. In addition, combined with the clear physical meaning of the BUG formula, we studied its effect in conflict management. Finally, application illustrates the superiority and prospects of the proposed method.

The organizational structure of this paper is as follows. Section 2 introduces some background knowledge required for this paper. In Section 3, the BUG theory is proposed, and some basic properties of the BUG formula are proved. In Section 4, a numerical example is used to model the essence of information fusion in evidence theory. Section 5 illustrates the nature of the proposed method through simulation experiment. In Section 6, the proposed BUG formula is used for the measurement of conflict, and the potential of the proposed method is illustrated by comparison. Section 7 shows the application value of the proposed method through the application of fault diagnosis. Section 8 summarizes and discusses the work done in this paper.

2. Preliminaries

In this section, we briefly review some basic concepts, including D-S evidence theory, evidence distance, and law of gravitation.

2.1. Dempster-Shafer evidence theory

2.1.1. Frame of discernment

DEFINITION 1. *Let Θ be a set of mutually exclusive and collectively exhaustive events defined by [1, 2]*

$$\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\} \tag{1}$$

where the set Θ is called the frame of discernment.

The 2^Θ is the Θ power set, which is expressed as

$$2^\Theta = \{\emptyset, \{\theta_1\}, \cdots \{\theta_N\}, \{\theta_1, \theta_2\}, \cdots \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, \Theta\} \tag{2}$$

and $\emptyset$ is an empty set.

If $A \in 2^\Theta$, A is called a hypothesis or proposition.

2.1.2. Mass function

DEFINITION 2. *For a frame of discernment Θ, a mass function is expressed as a mapping, i.e., from 2^Θ to $[0, 1]$, formally defined by [1, 2]*

$$m : 2^\Theta \to [0, 1] \tag{3}$$

which satisfies the following two attributes

$$m(\emptyset) = 0 \quad and \quad \sum_{\theta \subseteq \Theta} m(\theta) = 1 \tag{4}$$

In D-S evidence theory, m is also called a Basic Probability Assignment (BPA). For exmaple, $m(A)$ is BPA of A, which accurately reflects the extent to which A is supported. If $m(A) > 0$, A is a focal element of the mass function.

2.1.3. Belief and plausibility functions

DEFINITION 3. *From this BPA, a belief function Bel and a plausibility function Pl are defined, respectively, as [1, 2]*

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

and

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{A \bigcap B = \emptyset} m(B) \tag{6}$$

where $\bar{A} = \Theta - A$*,* $Bel : 2^{\Theta} \to [0, 1]$ *and* $Pl : 2^{\Theta} \to [0, 1]$.

The relationship between *Pl* function and *Bel* function is shown in Fig.1.

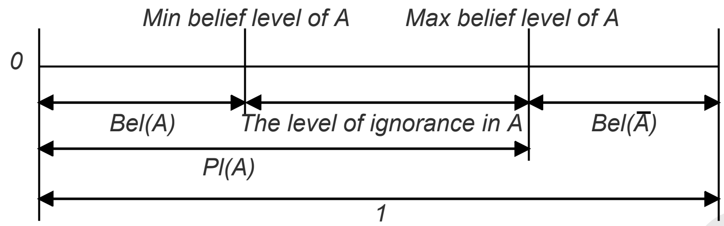

Figure 1: The relationship between *Pl* and *Bel*.

In Fig.1, the quantity $Bel(A)$ can be interpreted as a measure of one's belief that hypothesis A is true. The plausibility $Pl(A)$ can be viewed as the total amount of belief that could be potentially placed in A. The $[Bel(A), Pl(A)]$ indicates the uncertain interval for A.

2.1.4. Dempster's combination rule

The Dempster's combination rule has been widely used to combine multiple independent evidence, and its definition is as follows.

Definition 4. *Suppose the two evidence functions* m_1 *and* m_2 *are on the discernment frame* Θ*, and then the Dempster's combination rule can be defined as follows (*$\oplus$ *represents the orthogonal summation operation.) [1, 2]*

$$[m_1 \oplus m_2](\theta) = \begin{cases} 0 & \theta = \emptyset \\ \frac{\sum_{A_1 \cap A_2 = \theta} m_1(A_1) m_2(A_2)}{1-k} & \theta \neq \emptyset \end{cases} \quad (7)$$

where the conflict coefficient k is defined as follows

$$k = \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1) m_2(A_2) \quad (8)$$

Notably, Dempster's combination rule is useful only under the condition that $k < 1$.

2.2. Zadeh's counter-example

Definition 5. *Suppose* m_1 *and* m_2 *are two BPAs defined on a frame of discernment* $\Theta = \{A, B, C\}$ *with [26]*

$$m_1 : m_1(A) = 0.99, m_1(B) = 0.01, m_1(C) = 0$$

$$m_2 : m_2(A) = 0, m_2(B) = 0.01, m_2(C) = 0.99$$

Then using Dempster's combination rule, the fusion result of proposition B is

$$\begin{aligned}
m_{\oplus\{m_1,m_2\}}(\{B\}) &= \frac{\sum\limits_{X\cap Y=B} m_1(X)m_2(Y)}{1-\sum\limits_{X\cap Y=\emptyset} m_1(X)m_2(Y)} \\
&= \frac{m_1(\{B\})\,m_2(\{B\})}{1-(m_1(\{A\}))\,m_2(\{B\}+m_1(\{A_1\})\,m_2(\{C\})+m_1(\{B\})\,m_2(\{C\}))} \\
&= \frac{0.01\times 0.01}{1-(0.99\times 0.99+0.99\times 0.01+0.01\times 0.99)} \\
&= 1
\end{aligned}$$

2.3. Evidence distance

Jousselme et al. [52] proposed a distance measure for belief functions, the evidence distance is defined as follows.

Definition 6. *Let m_1 and m_2 be two BPAs on the same discernment frame Θ, and the distance between m_1 and m_2 is defined as follows [52]*

$$d(m_1,m_2)=\sqrt{\frac{1}{2}\sum_{\substack{\emptyset\neq A_1\subseteq\Theta\\ \emptyset\neq A_2\subseteq\Theta}}\frac{|A_1\cap A_2|}{|A_1\cup A_2|}(m_1(A_1)-m_2(A_1))(m_1(A_2)-m_2(A_2))} \tag{9}$$

Definition 7. *A metric distance defined on the set $\mathfrak{R}$ is a function. Evidence distance d satisfies the following conditions [52]*

$$\begin{cases} \delta\times\delta\to\mathfrak{R} \\ (A,B)\to d(A,B) \end{cases} \tag{10}$$

which meets the following requirements for any of A and B in $\mathfrak{R}$:

(1) ***Nonnegativity:*** *$d(A,B)\geqslant 0$*

(2) ***Nondegeneracy:*** *$d(A,B)=0 \Leftrightarrow A=B$*

(3) ***Symmetry:*** *$d(A,B)=d(B,A)$*

(4) ***Triangle inequality:*** *$d(A,B)\leq d(A,C)+d(C,B)$ and any $C\in\delta$*

2.4. Law of gravitation

Definition 8. *Let F be the magnitude of the gravitational attraction on the object, G be the universal gravitational constant, $M_1$and M_2 be the masses of any two objects attracted to each other, and R be the distance between the two objects. And then, the formula for gravitation is defined as [46]*

$$F=G\frac{M_1M_2}{R^2} \tag{11}$$

From Eq.(11), we can know that the law of gravitation is expressed as follows. Any two objects in nature are attracted to each other, and the magnitude of gravity is proportional to the mass product of the two objects, and inversely proportional to the square of their distance.

3. Proposed the theory of belief universal gravitation

In this section, inspired by Newton's theory of gravitation, based on evidence distance, we creatively present a novel BUG within the framework of evidence theory.

3.1. Proposed the evidence quality coding algorithm

In order to characterize the quality of evidence obtained from evidence sources, in this subsection, we propose the EQC algorithm to generate evidence quality. The introduction to the method is shown below.

ASSUMPTION 3.1. *Assume that m_1 and m_2 are two pieces of evidence defined on the same discernment frame $\Theta = \{A_1, A_2, A_3\}$, and their BPAs are shown below*

$$m_1 : m_1(A_1) = \vartheta_1,\ m_1(A_2) = \vartheta_2,\ m_1(A_3) = \vartheta_3$$
$$m_2 : m_2(A_1) = \varphi_1,\ m_2(A_2) = \varphi_2,\ m_2(A_3) = \varphi_3$$

where $\vartheta_1 \neq 0$, $\vartheta_2 \neq 0$, $\vartheta_3 \neq 0$, $\varphi_1 \neq 0$, $\varphi_2 \neq 0$ and $\varphi_3 \neq 0$.

Step 1: *Assign each BPA a binary code.* Each BPA was assigned a binary code based on the order of propositions in the discernment frame. The coding principle is as follows: for each BPA, the corresponding bit of the proposition in the discernment framework is labeled 1 and the rest is labeled 0. The binary code of m_2 is shown below

$$m_1(A_1) \rightarrow 100, m_1(A_2) \rightarrow 010, m_1(A_3) \rightarrow 001$$

Similarly, the binary code of m_2 is as is represented as follows

$$m_2(A_1) \rightarrow 100, m_2(A_2) \rightarrow 010, m_2(A_3) \rightarrow 001$$

Step 2: *Convert the binary encoding of each BPA to decimal.* So, m_1 is converted as follows

$$m_1(A_1) \rightarrow 4, m_1(A_2) \rightarrow 2, m_1(A_3) \rightarrow 1$$

Similarly, m_2 is converted as follows

$$m_2(A_1) \rightarrow 4, m_2(A_2) \rightarrow 2, m_2(A_3) \rightarrow 1$$

Step 3: *Generate the quality of each piece of evidence.* The quality of the evidence m_1 is as follows

$$M_{m_1} = \frac{4 \times \vartheta_1 + 2 \times \vartheta_2 + 1 \times \vartheta_3}{n} = \frac{4 \times \vartheta_1 + 2 \times \vartheta_2 + 1 \times \vartheta_3}{3}$$

Similarly, the quality of the evidence m_2 is as follows

$$M_{m_2} = \frac{4 \times \varphi_1 + 2 \times \varphi_2 + 1 \times \varphi_3}{n} = \frac{4 \times \varphi_1 + 2 \times \varphi_2 + 1 \times \varphi_3}{3}$$

where n is the number of focus elements in BPAs.

3.2. Proposed the formula for belief universal gravitation

In the previous subsection, we described the process of generating evidence quality using the proposed EQC algorithm through a model. In this subsection, based on the evidence quality, and the evidence distance, we propose a new BUG formula. The formula is defined as follows.

Definition 9. *Let m_1 and m_2 be two separate and different evidences on the same discernment frame Θ. The BUG formula is defined as*

$$F_{BPA} = G_{ET} \frac{M_{m_1} M_{m_2}}{d^2} \tag{12}$$

where the G_{ET} is defined as

$$G_{ET} = 10^{-\delta|\Theta|} \tag{13}$$

with

$$0 \leqslant \delta \leqslant 1 \tag{14}$$

In Eq.(12), M_{m_1} and M_{m_2} represent the quality of the evidence m_1 and m_2 generated using the proposed EQC algorithm. d represents the distance between evidence from two independent sources. G_{ET} is the evidence gravitation parameter used to distinguish different discernment frames. Where δ is an adjustable parameter that satisfies the constraint of Eq.(14). It is used to dynamically adjust the size of the BUG in a system, so that the BUG can be observed more clearly. Simultaneously, it is important to note that the value of δ in different BUG formulas must be consistent in a definite system (the definite system represents the same environment in which the belief gravitation is used, including the time environment and the space environment.).

The physical meaning of the BUG formula is as follows. F_{BPA} represents the gravitation between two different evidences m_1 and m_2. As can be seen from the Eq.(12), in a definite system, for two different evidences on the same discernment frame, the BUG is proportional to the product of their quality and inversely proportional to the square of the distance between them. Taking this one step further, the smaller the difference between the two pieces of evidence, the greater the gravitation between them. Conversely, the greater the difference between the two pieces of evidence, or even the conflict, the smaller the gravitation between them.

3.3. Basic properties of the proposed belief universal gravitation formula

In this subsection, we discuss and demonstrate some of the basic properties that are satisfied by the BUG formula.

Assumption 3.2. *Assume m_1 and m_2 are two different pieces of evidence on the same discernment frame Θ. In a definite system, F_{BPA} is the gravitational pull between evidence m_1 and m_2, M_{m_1} and M_{m_2} are the quality of the evidence m_1 and m_2 generated using the proposed EQC algorithm, and G_{ET} is the evidence gravitation parameter.*

The BUG formula has the following properties.

(i) ***Non-negative***

Proof. First, from Eq.(9), it can be known that the evidence distance $d > 0$, clearly, $d^2 > 0$. Secondly, through Eq.(13), we can know that $G_e > 0$. Finally, it can be seen from the subsection 3.1 that the quality of evidence generated by EQC algorithm is non-negative, i.e., M_{m_1} and $M_{m_2} > 0$. Based on the above and the Eq.(12), we can observe that F_{BPA} is non-negative. □

(ii) ***Symmetry***

Proof. In Definition 9, the value of δ is determined because of the BUG in a system defined. Based on this, since m_1 and m_2 are on the same discernment frame, the value of G_{ET} is determined. According to the evidence distance symmetry property (see Definition 7), i.e., $d(A, B) = d(B, A)$. In Eq.(12), $M_{m_1} M_{m_2}$ is the product of the masses of evidence m_1 and m_2, hence, which also has symmetry. In general , the BUG formula has a symmetry, i.e., $F_{BPA}(A, B) = F_{BPA}(B, A)$. □

(iii) ***Unbounded***

Proof. Since the BUG is in a definite system, the value of δ is defined. As can be seen from the Eq.(12), the F_{BPA} is proportional to the product of the mass of evidence m_1 and m_2, and inversely proportional to the number of discernment frame $|\Theta|$ and the evidence distance d. It can be seen from Definitions 6 and 7 that the evidence distance d is the scale to measure the difference of evidence, and its value is between 0 and 1. When the discernment frame is infinite, the value of evidence quality will increase exponentially due to the binary encoding used in EQC algorithm. Even if the evidence gravitation parameter G_{ET} weakens the value of BUG to some extent, the value of BUG will approach infinity with the infinite expansion of discernment frame elements. □

4. Model the essence of information fusion

In this section, through an example, we use BUG to model the essence of multi-sensor data information fusion in evidence theory.

EXAMPLE 4.1. *Suppose the discernment frame is* $\Theta = \{A_1, A_2, A_3\}$. *At* $T = 0s$*, sensor 1, sensor 2, and sensor 3 obtained three pieces of evidence, namely* m_1*,* m_2*,* m_3*, respectively. After that, evidence* m_4*,* m_5 *were obtained by sensor 4 and sensor 5 at* $T = 5s$*,* $T = 10s$ *respectively. Their BPAs are shown in Table 1.*

Table 1: The initial evidences obtained by the sensors.

Time	Sensor	$m(\{A_1\})$	$m(\{A_2\})$	$m(\{A_3\})$
$T = 0s$	$S_1 : m_1(\cdot)$	0.99	0.01	0
$T = 0s$	$S_2 : m_2(\cdot)$	0	0.01	0.99
$T = 0s$	$S_3 : m_3(\cdot)$	0.97	0.03	0
$T = 5s$	$S_4 : m_4(\cdot)$	0.95	0.05	0
$T = 10s$	$S_5 : m_5(\cdot)$	0.94	0.06	0

Obviously, the evidence m_2 is a piece of conflict evidence because other evidence supports the proposition A_1 very high, while evidence m_2 has no probability support for it at all. Under the proposed BUG, the fusion results of these five pieces of different evidence information are shown in Fig.2.

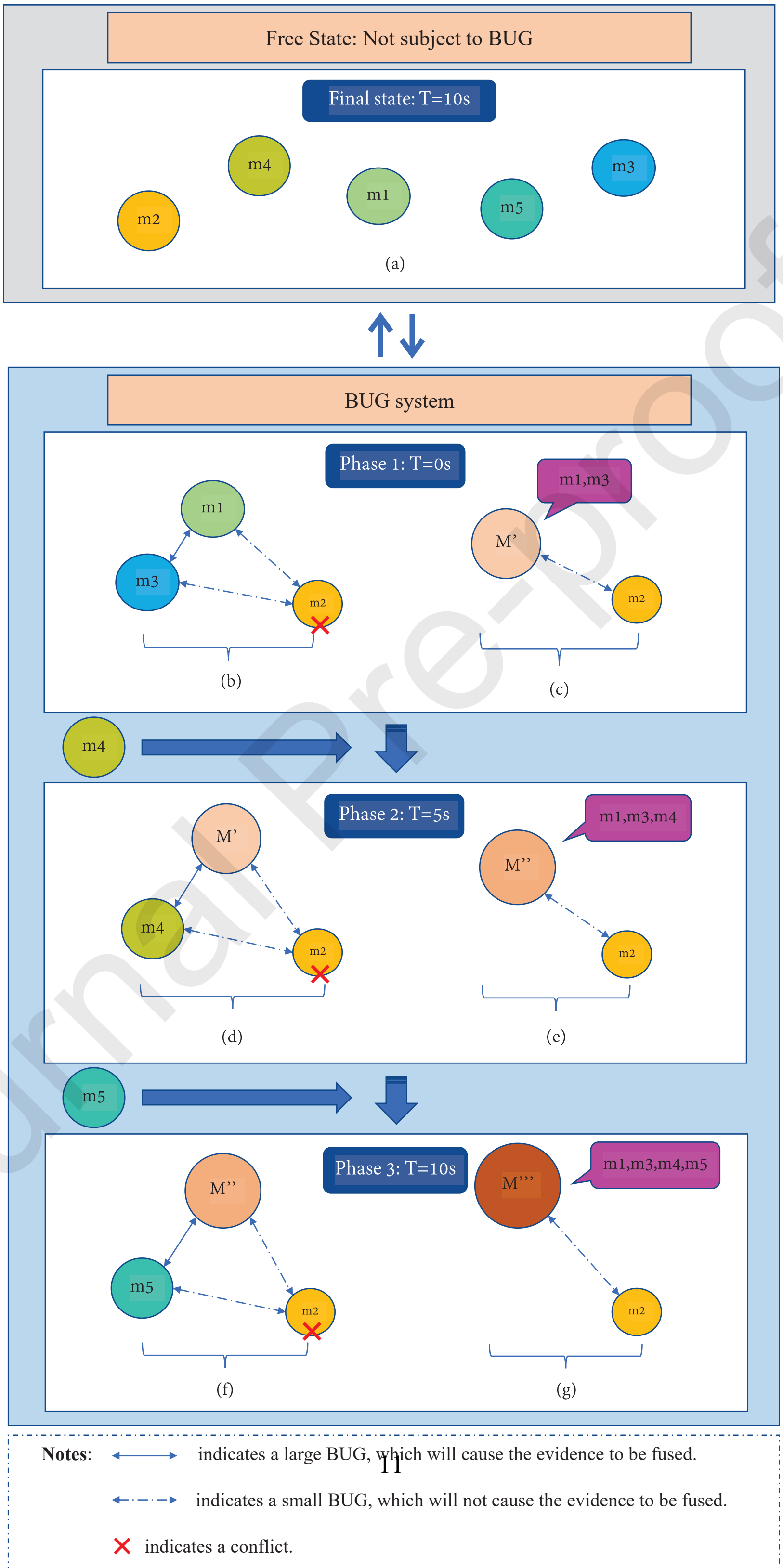

Figure 2: Information fusion modeling of five different evidences based on BUG.

In Fig.2, Fig.(a) shows five pieces of evidence information in the free state. It can be observed that when $T = 10s$, that is, all evidences are obtained, they are also scattered and have no relevance. In this case, information fusion is a simple combination of the five pieces of evidence information. However, the fusion results are not conducive to the final decision, because there is interference information $m2$. If the BUG system is considered, then at each phase of obtaining the evidence, the evidence will gradually merge the potentially fused evidence information under the force of the BUG. In this system, the quality of each piece of evidence information, the distance between them and the size of the BUG between them are different.

In phase 1 ($T = 0s$), for the three pieces of evidence obtained initially, it is clear that $m2$ is a piece of highly conflicting evidence information. Based on the proposed EQC algorithm, the quality of $m2$ is relatively small compared to the other two evidences; based on the evidence distance, the distance between $m1$ and $m3$ is also the closest, as shown in Fig.(b). After that, we assume that under the action of BUG, $m1$ and $m3$ are fused to form a new evidence information M'. Because of the increase of proposition reliability, its quality also becomes larger than the initial evidence $m1$ and $m3$. At this time, there is still a conflict between $m2$ and M', and the BUG between them is not enough to make the two fuse with each other, as shown in Fig.(c). In phase 2 ($T = 5s$), when the evidence $m4$ is added to this system, the force relationship under the action of BUG is shown in Fig.(d). At this time, $m2$ is still a piece of conflicting evidence. The difference is that the distance between M' and $m4$ increases due to the increased reliability of M' for the proposition, but the BUG between them is still relatively large. After that, $m4$ and M' are fused under the action of BUG to form new evidence M'', as shown in Fig.(e). Similarly, in phase 3 ($T = 10s$), Fig.(f) shows the force relationship diagram after the evidence $m5$ is added, and Fig.(g) shows the final state of the system under the action of BUG. On the whole, as the newly formed evidence M', M'' and M''' support for propositions increase, their quality also increases. At the same time, the distances between 1 and $m3$, $m4$ and M', and $m5$ and M'' also gradually increase due to changes in reliability.

Based on the concept of the proposed BUG, in this section we discuss the physical significance of evidence information fusion by multi-source sensors in evidence theory. This provides a new idea for information fusion.

5. Simulation experiment

In this section, the relationship between BUG and other parameter variables is further discussed through simulation experiments.

Example 5.1. *Suppose there are 20 elements in the discernment frame* Θ*, such as* $\Theta = \{1, 2, 3, \cdots, 20\}$. *The BPAs of two different evidences* m_1 *and* m_2 *are defined as follows:*

$$m_1 : m_1(2, 3, 4) = 0.05, m_1(7) = 0.05, m_1(\Theta) = 0.1, m_1(A) = 0.8$$
$$m_2 : m_2(1, 2, 3, 4, 5) = 1$$

Here, assuming that A is not a set of constants, it can be changed by discernment frame. It starts at 1 and ends at 20, increasing by an order of magnitude each time.

Then, we record the data of the mass of evidence m_1 and m_2, the square of the distance of evidence d^2, and the change of the BUG in Table 2.

Table 2: The change data of the parameters in the BUG formula.

A	G_{ET}	M_{m_1}	M_{m_2}	d^2	F_{BPA}
$\{1\}$	10^{-10}	136,908.775	1,015,808	0.6175	22.5226
$\{1,2\}$	10^{-10}	189,337.575	1,015,808	0.4714	40.7982
$\{1,2,3\}$	10^{-10}	215,551.975	1,015,808	0.3255	67.2748
$\{1,2,3,4\}$	10^{-10}	228,659.175	1,015,808	0.1795	129.3848
$\{1,2,3,4,5\}$	10^{-10}	235,212.775	1,015,808	0.0175	1365.1000
$\{1,2,\cdots,6\}$	10^{-10}	238,489.575	1,015,808	0.1509	160.5915
$\{1,2,\cdots,7\}$	10^{-10}	240,127.975	1,015,808	0.2529	96.4475
$\{1,2,\cdots,8\}$	10^{-10}	240,947.175	1,015,808	0.3255	75.2007
$\{1,2,\cdots,9\}$	10^{-10}	241,356.775	1,015,808	0.3828	64.0488
$\{1,2,\cdots,10\}$	10^{-10}	241,561.575	1,015,808	0.4295	57.1251
$\{1,2,\cdots,11\}$	10^{-10}	241,663.975	1,015,808	0.4684	52.4087
$\{1,2,\cdots,12\}$	10^{-10}	241,715.175	1,015,808	0.5015	48.9557
$\{1,2,\cdots,13\}$	10^{-10}	241,740.775	1,015,808	0.5301	46.3212
$\{1,2,\cdots,14\}$	10^{-10}	241,753.575	1,015,808	0.5552	44.2339
$\{1,2,\cdots,15\}$	10^{-10}	241,759.975	1,015,808	0.5774	42.5288
$\{1,2,\cdots,16\}$	10^{-10}	241,763.175	1,015,808	0.5975	41.1001
$\{1,2,\cdots,17\}$	10^{-10}	241,764.775	1,015,808	0.6156	39.8940
$\{1,2,\cdots,18\}$	10^{-10}	241,765.575	1,015,808	0.6322	38.8475
$\{1,2,\cdots,19\}$	10^{-10}	241,765.975	1,015,808	0.6474	37.9356
$\{1,2,\cdots,20\}$	10^{-10}	241,766.175	1,015,808	0.6615	37.1283

[1] In this system, set the value of the adjustable parameter δ to 1/2.

It can be seen from Table 2 that with the increase of the number of propositions in set A, the evidence quality of m_1, i.e., the value of M_{m_1} increases gradually. Since the number of propositions in m_2 remains the same, hence, M_{m_2} is a constant. Further more, the relationships between d^2 and the size of set A, F_{BPA} and the size of set A are shown in Fig.3 and Fig.4, respectively.

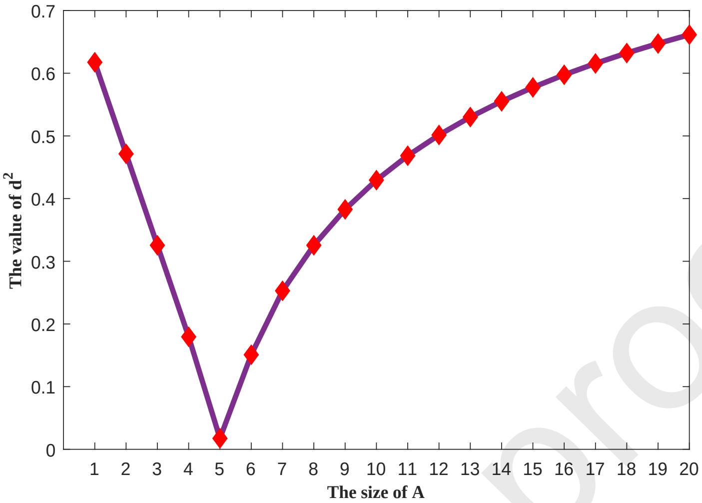

Figure 3: The trend of d^2 with respect to $|A|$.

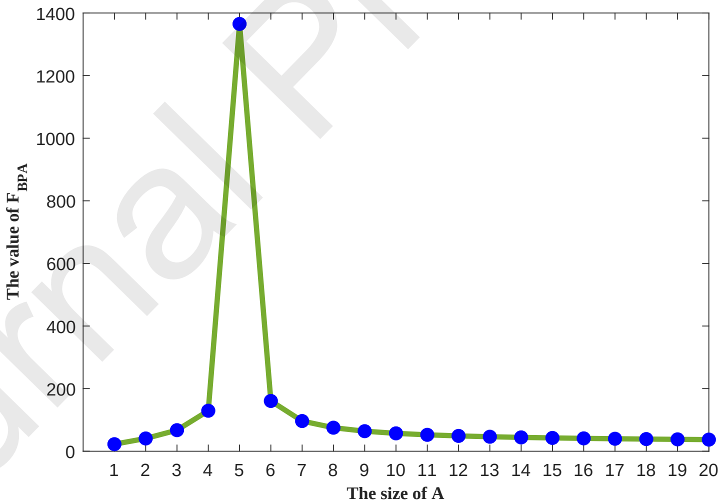

Figure 4: The trend of F_{BPA} with respect to $|A|$.

From Fig.3, we can see that the square of the evidence distance d^2 has the opposite trend of the F_{BPA} with the increase of the number of propositions in set A. More specifically, when set A approaches set $\{1, 2, 3, 4, 5\}$, the value of d^2 tend to be the lowest. Conversely, when the value of set A deviates from the set $\{1,2,3,4,5\}$, the value of F_{BPA} increases. Especially, d^2 shows a decreasing trend until the size of set A is 5. When the size of set A is 5, that is, $A = \{1,2,3,4,5\}$, d^2 reaches the minimum value. And then, as the size of set A increases, d^2 increases, but it's less than 1. It can be seen from Definition 6 that evidence distance d is used to express the similarity between evidences. When the size of set A is 5, that is, $A = \{1,2,3,4,5\}$, m_1 and m_2 have the greatest similarity, that is, d is the smallest, so d^2 is the smallest. After that, the number of propositions in set A increases, so does the difference between m_1 and m_2, so d^2 increases. In addition, from Fig.4, we can also observe that when the value of set A is 5, that is, $A = \{1,2,3,4,5\}$, F_{BPA} reaches the maximum value. In combination with the physical significance of the BUG, the difference between evidence m_1 and m_2 is the smallest, that is, evidence distance d is the smallest. In other words, the similarity between evidence m_1 and m_2 is the highest. Therefore, the universal gravitation generated by m_1 and m_2 is maximized. Thereafter, as the size of set A increases, the similarity between m_1 and m_2 gradually decreases, and the value of the BUG also decreases.

To sum up, through the experimental simulation, we further clarify the relationship between the BUG and its parameters. We can conclude that, in a system, and given the discernment frame, the BUG is inversely proportional to the square of the evidence distance. In other words, when the distance between two pieces of evidence is smaller, at this point, the BUG is greater, the more attractive the two pieces of evidence are to each other.

6. Conflict management based on the proposed belief universal gravitation

How to manage conflicts in D-S evidence theory is a difficult and challenging problem. More crucially, in the management of conflict evidence, how to describe the similarity between the evidence is a key and important problem. In this section, based on the proposed BUG formula and its physical significance, here we creatively use it to describe the similarity between evidence.

Before comparing the proposed approach with the others, let's briefly review some of the work of some scholars on conflict management in the DST. In [52], Jousselme et al. proposed a distance measure for belief function. In [1], the conflict coefficient k was first used to represent the degree of conflict between evidences. However, in [42], Liu pointed out that the k doesn't effectively measure the conflict between two pieces of evidence. A two-dimensional conflict model is then proposed, where the pignistic probability distance [53] and the conflict coefficient k are united to represent the degree of conflict. After that, Daniel [54] defined the plausibility conflict between BPAs. Lefevre and Elouedi [55] put forward a novel called the Combination With Adapted Conflict (CWAC) rule. This rule provided an adaptive weighting between Dempster's rule and conjunctive rule based on Jousselme et al.'s evidence distance. Ma and An [56] considered a method to combine conflict evidence with a probabilistic dissimilarity measure. Song et al. [57] studied the correlation coefficient for the relativity between two BPAs and used it to measure the conflict. Since then, to measure the correlation degree between the two pieces of evidence, Jiang [58] considered a new correlation coefficient that takes into account both the non-intersection and the difference between the focus elements. Recently, a new belief entropy is presented as Deng entropy [59, 60]. Inspired by Deng entropy, Pan et al. [61] proposed a novel association coefficient of belief functions to measure conflicts between evidences.

Here we use the Example 5.1 in Section 5 to compare the degree of conflict between the two groups of BPA with the above-mentioned different methods. The results are shown in Table 3 and Fig.5.

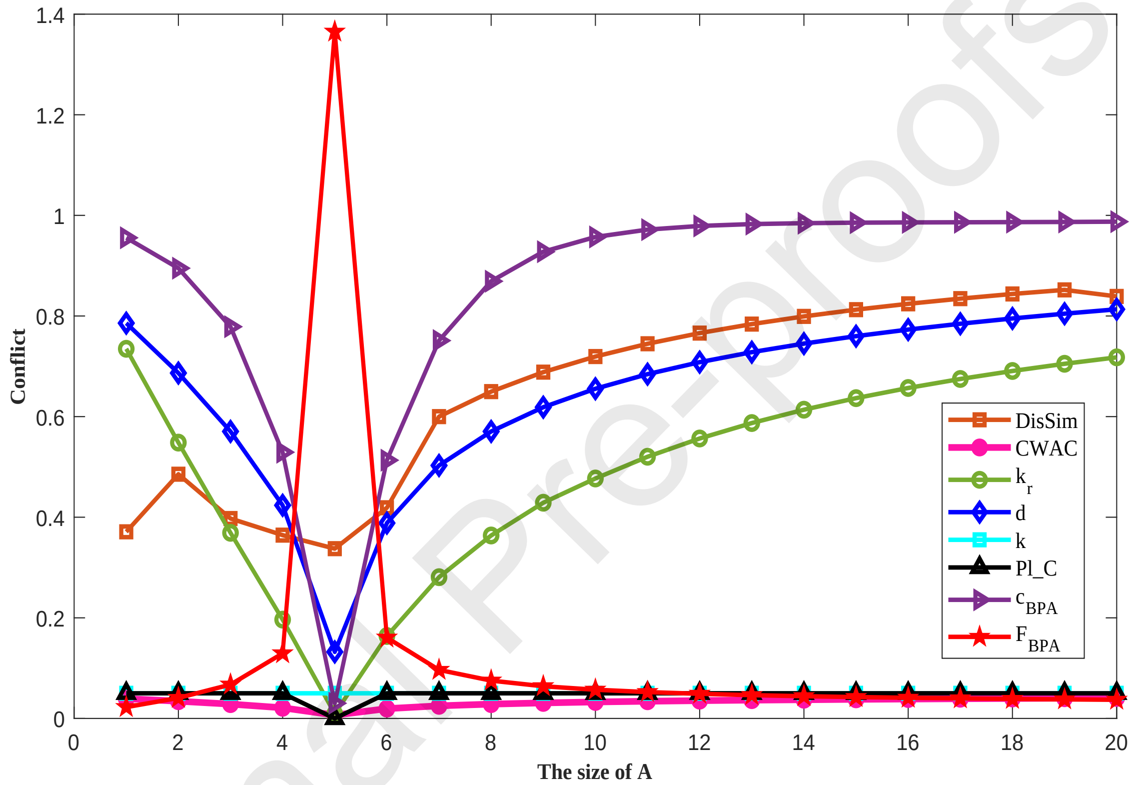

Figure 5: Comparison of conflicts between different parameter values

As shown in Fig.5, more specifically, the d and k_r show a consistent trend for conflict measurement. When the size of set A approaches 5, the measurement value of conflict is the lowest. When the size of set A deviates from 5, the conflicting measurements increase. $DisSim$ is not monotonic until the size of set A is 5, so it is not an effective way to measure conflicts. $CWAC$ is always kept at a low level, and it is insensitive to the conflict change. The classical conflict coefficient k is always maintained at 0.5, which cannot distinguish the variation of evidence m_1. The method of Pl_C takes A value of 0 when set A is 5, which is unacceptable. The k_r method has between 0 and 5 elements in set A, which is more monotone. When the size of set A is 5, the collision value is smaller than the above mentioned methods, which is reasonable. As

Table 3: Comparisons of conflict degree.

A	k[1]	d[52]	$CWAC$[55]	$DisSim$[56]	Pl_C[54]	k_r[58]	c_{BPA}[61]	F_{BPA}
{1}	0.05	0.7858	0.0393	0.3710	0.05	0.7348	0.9555	0.0225
{1, 2}	0.05	0.6866	0.0343	0.4855	0.05	0.5483	0.8950	0.0408
{1, 2, 3}	0.05	0.5705	0.0285	0.3974	0.05	0.3690	0.7787	0.0673
{1, 2, 3, 4}	0.05	0.4237	0.0212	0.3644	0.05	0.1964	0.5292	0.1294
{1, 2, 3, 4, 5}	0.05	0.1323	0.0066	0.3375	0.05	0.0094	0.0302	1.3651
{1, 2, ···, 6}	0.05	0.3884	0.0195	0.4188	0.05	0.1639	0.5133	0.1606
{1, 2, ···, 7}	0.05	0.5029	0.0251	0.6000	0.05	0.2808	0.7511	0.0965
{1, 2, ···, 8}	0.05	0.5705	0.0285	0.6497	0.05	0.3637	0.8691	0.0752
{1, 2, ···, 9}	0.05	0.6187	0.0309	0.6884	0.05	0.4288	0.9278	0.0640
{1, 2, ···, 10}	0.05	0.6554	0.0328	0.7194	0.05	0.4770	0.9571	0.0571
{1, 2, ···, 11}	0.05	0.6844	0.0342	0.7448	0.05	0.5202	0.9717	0.0524
{1, 2, ···, 12}	0.05	0.7082	0.0354	0.7660	0.05	0.5565	0.9790	0.0490
{1, 2, ···, 13}	0.05	0.7281	0.0364	0.7839	0.05	0.5872	0.9827	0.0463
{1, 2, ···, 14}	0.05	0.7451	0.0372	0.7992	0.05	0.6137	0.9845	0.0442
{1, 2, ···, 15}	0.05	0.7599	0.0380	0.8126	0.05	0.6367	0.9855	0.0425
{1, 2, ···, 16}	0.05	0.7730	0.0386	0.8242	0.05	0.6569	0.9860	0.0411
{1, 2, ···, 17}	0.05	0.7846	0.0392	0.8345	0.05	0.6748	0.9863	0.0399
{1, 2, ···, 18}	0.05	0.7951	0.0397	0.8438	0.05	0.6907	0.9866	0.0388
{1, 2, ···, 19}	0.05	0.8046	0.0402	0.8519	0.05	0.7050	0.9869	0.0379
{1, 2, ···, 20}	0.05	0.8133	0.0407	0.8389	0.05	0.7178	0.9875	0.0371

[1] In this system, set the value of the adjustable parameter δ to 1/2.

[2] In this example, in order to facilitate comparison with other methods, we reduce every BUG in the same proportion (i.e., each BUG is divided by 1,000) without affecting the properties of the BUG.

an improved method based on Deng entropy, the change of c_{BPA} is more significant than that of k_r. On the whole, with the increase of the number of elements in set A, the proposed BUG changes in an opposite trend to other methods.

According to the physical meaning of the BUG discussed in subsection 3.2, it describes the magnitude of gravity between the two pieces of evidence. That is to say, the more similar the two evidences support certain propositions, the larger the BUG. Conversely, the greater the difference in support for certain propositions, the smaller the BUG. In this example, although d, k_r and c_{BPA} can also represent the change of conflict, it should be emphasized that the proposed method shows from a mechanical point of view that the BUG can also well reflect the difference of conflict evidence. Fig.5 graphically reflects this change trend. This also shows the potential of BUG for conflict measurement.

7. Application in fault diagnosis

In industrial production, how to effectively integrate multi-source evidence to give decision makers reasonable and accurate machine fault diagnosis results is a challenging problem. In this section, we demonstrate the superiority of the proposed method by applying the BUG to actual fault diagnosis in [62]. A machine fault diagnosis model is shown in Fig.6.

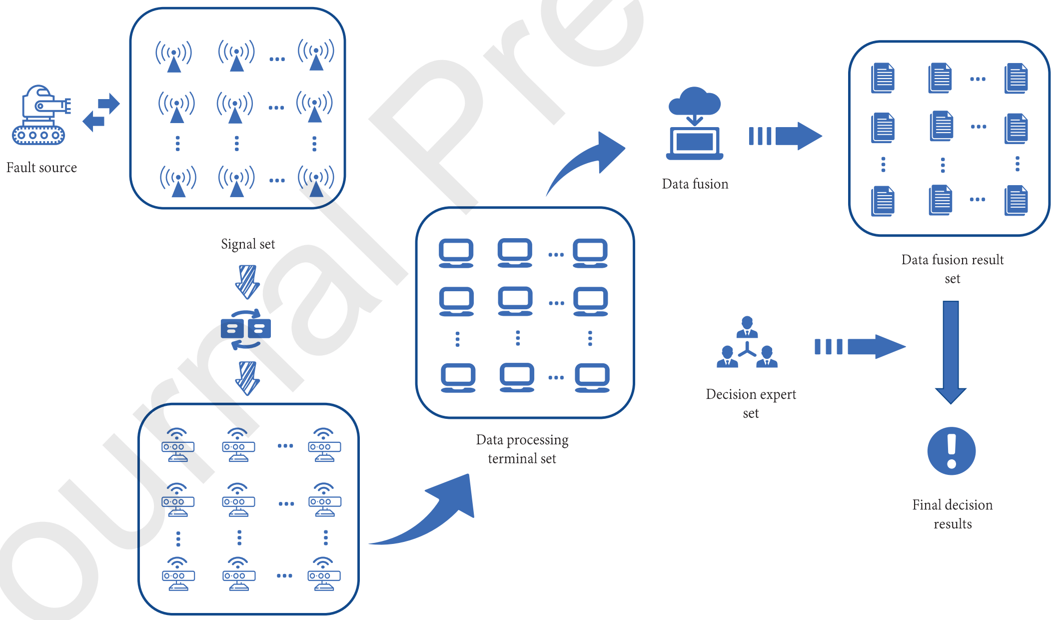

Figure 6: Machine fault diagnosis diagram.

7.1. Application background

In the case of motor rotor fault diagnosis, three kinds of sensors are used to collect the feature information of acceleration, velocity and displacement. The data from these sensors are turned into BPAs, which are listed as follow Table 4.

Table 4: The output of the multi-sensors.

m_i	A_1	A_2	A_3	A_4
m_1	0.06	0.68	0.02	0.04
m_2	0.02	0	0.79	0.05
m_3	0.02	0.58	0.16	0.04

As shown in Table 4, m_1, m_2, and m_3 represent evidence from acceleration sensors, speed sensors, and displacement sensors, respectively. Motor rotors typically have four states, A_1 is normal operation, A_1 is unbalance, A_2 is misalignment, and A_4 is pedestal looseness. Then a discernment frame, i.e., $\Theta = \{A_1, A_2, A_3, A_4\}$ is established. From the information collected by the three acceleration sensors, it is clear that the information obtained by the sensor m_2 is conflicting because it does not support A_2 misalignment, which is highly supported by the sensor m_1 and m_3.

7.2. Fault diagnosis based on the proposed method

In order to fuse the information obtained by the three sensors, we adopt the method proposed in [39]. Note that instead of using evidence distance to represent the similarity between the evidence information obtained by the sensors, we use BUG. The method is briefly expressed as follows.

Step 1: Construct a BUG matrix ($BUGM$), which reflects the degree of similarity between the two evidence bodies. Suppose there are n pieces of evidence, the $BUGM$ as follows:

$$BUGM = \begin{bmatrix} \Im & F_{BPA}(m_1, m_2) & \cdots & F_{BPA}(m_1, m_j) & \cdots & F_{BPA}(m_1, m_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{BPA}(m_i, m_1) & F_{BPA}(m_i, m_2) & \cdots & F_{BPA}(m_i, m_j) & \cdots & F_{BPA}(m_i, m_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{BPA}(m_n, m_1) & F_{BPA}(m_n, m_2) & \cdots & F_{BPA}(m_n, m_j) & \cdots & \Im \end{bmatrix} \quad (15)$$

where $F_{BPA}(m_i, m_j)$ represents the gravitation between the evidence m_i and m_j, and $\Im$ means not defined.

Step 2: Calculate the credibility of all evidence. The degree of support of the body of evidence is expressed as follows:

$$Sup(m_i) = \sum_{j=1, j \neq i}^{n} F_{BPA}(m_i, m_j) \quad (16)$$

and the credibility degree Crd_i of the body of evidence m_i is defined as:

$$Crd_i = \frac{Sup(m_i)}{\sum_{i=1}^{n} Sup(m_i)} \tag{17}$$

Step 3: Get the average weight value of all evidence.

$$MAE(m) = \sum_{i=1}^{n} (Crd_i \times m_i) \tag{18}$$

Step 4: Use the Dempster's combination rule Eq.(7) to fuse the average weight of the evidence $n-1$ times.

More specifically, in the concrete calculation process, using Eq.(17), the credibility of the three pieces of evidence obtained is expressed as $Crd_1 = 0.4872$, $Crd_2 = 0.1088$, $Crd_3 = 0.4041$ respectively. Through Eq.(18), the average value of the three obtained evidences is shown in Table 5.

Table 5: The averaged evidence.

$m(A_1)$	$m(A_2)$	$m(A_3)$	$m(A_4)$	$m(\Theta)$
0.0395	0.5657	0.1604	0.0410	0.1934

Finally, the evidence was fused twice using the Dempster's combination rule, and then the final results are shown in Table 6.

Table 6: The final results after using Dempster's combination rule.

$m(A_1)$	$m(A_2)$	$m(A_3)$	$m(A_4)$	$m(\Theta)$
0.0111	0.8860	0.0763	0.0116	0.0149

7.3. Analysis and discussion

The outcome of a fault diagnosis depends on the probability of supporting a fault event. In this paper, we set the threshold to 0.8 to make a decision. Table 7 shows the initial diagnostic results of three different types of sensors. Table 8 and Fig.7 show the fault diagnosis results after the fusion of fault evidences obtained from three sensors by different methods.

Table 7: The initial output of sensors and diagnosis results.

Sensor type	A_1	A_2	A_3	A_4	Diagnosis result
Acceleration	0.06	0.68	0.02	0.04	uncertainty
Velocity	0.02	0	0.79	0.05	uncertainty
Displacement	0.02	0.58	0.16	0.04	uncertainty

By comparing the fault diagnosis results in Table 7 and Table 8, we can clearly see the advantages of multi-source sensor data fusion. More specifically, in Table 7, if we consider only one feature of the machine's working state, it is difficult to make a correct judgment decision. With a threshold of 0.8, these three single pieces of evidence show that they present uncertain answers before the evidence is combined. In addition, how to deal with the conflicts in the evidence combination is also very important. As shown in Fig.7, if we use Dempster, Murphy and Deng et al.'s method to combine the evidence obtained by the sensors, it is difficult for us to make a decision. Jiang's method and Xiao's method can determine that the failut of the equipment is unbalance, while the proposed method seems to make more accurate judgments.

Table 8: Comparisons of some existing methods.

Method	A_1	A_2	A_3	A_4	Θ	Diagnosis result
Dempster [1]	0.0205	0.5230	0.3933	0.0309	0.0324	uncertainty
Murphy [38]	0.0112	0.6059	0.3508	0.0153	0.0168	uncertainty
Deng et al. [39]	0.0110	0.7730	0.1856	0.0139	0.0165	uncertainty
Jiang [58]	0.0108	0.8063	0.1534	0.0134	0.0162	unbalance
Xiao [63]	0.0146	0.8184	0.1021	0.0187	0.0462	unbalance
Proposed	0.0111	0.8860	0.0763	0.0116	0.0149	unbalance

[1] In this system, set the value of the adjustable parameter δ to 1/4.

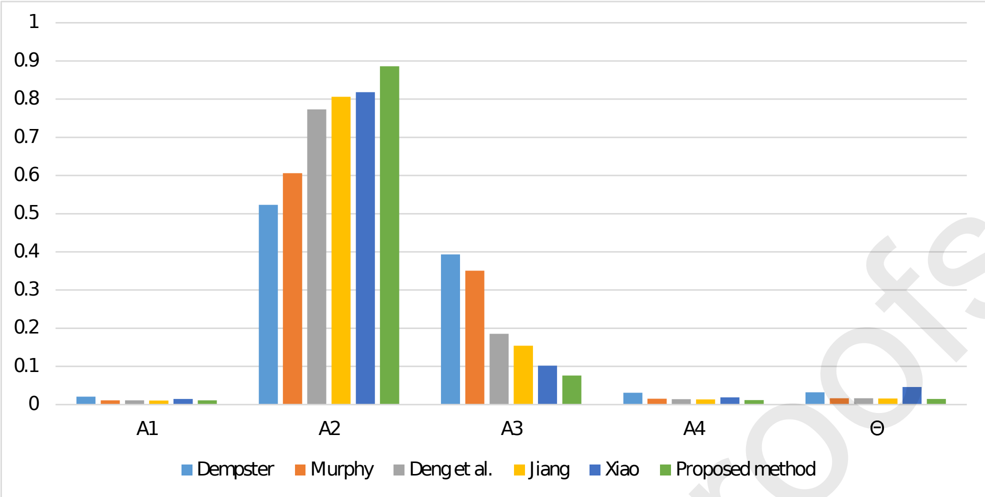

Figure 7: The results of comparison with some existing methods.

To sum up, the example verifies the validity and superiority of BUG in the case of conflict and shows the potential application prospect of proposed method.

8. Conclusion and discussion

In this paper, a new concept of BUG is proposed in the evidence theory system. Overall, the contributions of this paper are as follows. First of all, to the best our knowledge, this is the first time to introduce the law of gravity from natural science into evidence theory. Secondly, according to the knowledge framework of D-S evidence theory, the BUG formula is used to characterize the degree of gravity between evidences. Through the modeling of evidence information fusion, the rationality of the BUG in the fusion process is explained. Finally, by solving the hot issue of conflict measurement in D-S evidence theory, the potential application value of the proposed method is demonstrated. Furthermore, the application also illustrates the effectiveness of the proposed method.

In addition, although we proposed the theory of BUG, there are still some shortcomings that need to be further overcome. For instance, under the BUG, what is the critical point of evidence information fusion? In other words, what conditions are met before the evidence can be fused. As a method of measuring the amount of evidence information, is there a more perfect alternative method than EQC algorithm? how to consider more applications to increase the reliability of BUG, and consider its application to more practical problems, etc. In future work, we will consider introducing speed and acceleration to establish the BUG theory more completely. We also intend to apply the theory of BUG to evidence anti-monitoring, interference interception and measurement of transmission media.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Highlights

1. **A new concept BUG is proposed in the D-S evidence theory.**

2. **BUG provides a new perspective for information fusion in evidence theory.**

3. **BUG formula is based on evidence distance, EQC algorithm, etc.**

Declaration of Interest Statement

The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.