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Two Decision Makers' Single Decision over a Back Order EOQ Model with Dense Fuzzy Demand Rate

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Abstract: In this article we develop an economic order quantity (EOQ) model with backlogging where the decision is made jointly from two decision maker supposed to view one of them as the industrialist (developer) and the other one as the responsible manager. The problem is handled under dense fuzzy environment. In fuzzy set theory the concept of dense fuzzy set is quite new which is depending upon the number of negotiations/ turnover made by industrial developers with the supplier of raw materials and/or the customers. Moreover, we have discussed the preliminary concept on dense fuzzy sets with their corresponding membership functions and appropriate defuzzification method. The numerical study explores that the solution under joint decision maker giving the finer optimum of the objective function. A sensitive analysis, graphical illustration and conclusion are made for justification the new approach.

Keywords: Backorder inventory, dense fuzzy set, dense fuzzy lock set, defuzzification, optimization

1. Introduction

The traditional backorder EOQ model has been enriched with the help of modern researchers under different approximations and methodologies of uncertain world. In deterministic world it is quite ancient but in fuzzy environment the problem keeps some new route for final decision making. Zadeh[34] first develop the concept of fuzzy set. Then it has been applied by Bellman and Zadeh[4] in decision making problems. After that, many researchers were being engaged to characterized the actual nature of the fuzzy set Dubois and Prade[23], Kaufmann and Gupta[26], Baez-Sancheza *et al*.[1], Beg and Ashraf[3], Ban and Coroianu[2], Deli and Broumi[7]. The concept of eigen fuzzy number sets was developed nicely by Goetschel and Voxman[24]. Piegat[32] gave a new definition of fuzzy set. Star type fuzzy number was developed by Diamond[20][21]. Compact fuzzy sets were characterized by Diamond and Kloeden[19][22] in which its parameterization into single valued mappings is possible. Heilpern[25] discussed on fuzzy mappings and fixed point theorem. Chutia et al.[5] developed an alternative method of finding the membership of a fuzzy number, by the same time Mahanta et al.[31] were able to construct the structure of fuzzy arithmetic without the help of α – cuts also. The concept of fuzzy complement functional studied by Roychoudhury and Pedrycz[33]. De and Sana[14][15] have developed a backlogging EOQ model under intuitionistic fuzzy set (IFS) using the score function of the objective function. De et al.[11] have applied the IFS technique via interpolating by pass to develop a backorder EOQ model.

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However, in IFS environment De[10] investigated a special type of EOQ model where the natural idle time (general closing time duration per day) has been considered. Das et al.[6] considered a step order fuzzy model for time dependent backlogging over idle time. Also, at the same time De and Sana[13][16] developed a backlogging EOQ model considering pentagonal fuzzy number and for selling price and promotional effort sensitive demand respectively. Recently, De and Sana[17] developed a hill type (p, q, r, l) inventory model for stochastic demand under intuitionistic fuzzy aggregation with Bonferroni mean. Using the learning effect on fuzzy parameters Kazemi et al.[27][28][29][30] developed an EOQ model for imperfect quality items and they incorporated the human forgetting also.

Moreover, as per our concern, in the literature the use of dense fuzzy set in IFS has not yet been done. Though, the concept has already been developed by the researcher De and Beg[8][9]. They developed the triangular dense fuzzy set (TDFS) along with the new defuzzification methods first then applied it in a triangular dense fuzzy neutrosophic set (TDFNS) explicitly. Subsequently, De and Mahata[12] applied this concept with new addition in the name of cloudy fuzzy sets to discuss a backorder EOQ model. In their study they choose a Cauchy sequence which might converges to zero. Using this property they develop the triangular dense fuzzy set where the fuzziness is decreasing with time or gaining learning experiences.

In our present study, we have developed a back order EOQ model under dense fuzzy lock set. Basically, the existing literature over classical EOQ model orients with the single decision maker only. But as the recent business scenario deals with multiple decision maker so we have studied the model with at least two decision makers for making the single decision. First of all we consider the triangular dense fuzzy lock membership functions of the proposed demand rate and we utilize the solution procedure developed by De[18]. Finally, graphical illustration and sensitivity analysis are made followed by a conclusion.

2. Preliminaries[De and Beg[8]]

2.1 Definition

Let $\tilde{A}$ be the fuzzy number whose components are the elements of $R \times N$, R being the set of real numbers and N being the set of natural numbers with the membership grade satisfying the functional relation $\mu: R \times N \rightarrow [0,1]$. Now as $n \rightarrow \infty$ if $\mu(x,n) \rightarrow 1$ for some $x \in R$ and $n \in N$ then we call the set $\tilde{A}$ as dense fuzzy set. If $\tilde{A}$ is triangular then it is called TDFS. Now, if for some n in N, $\mu(x,n)$ attains the highest membership degree 1 then the set itself is called "Normalized Triangular Dense Fuzzy Set" or NTDFS.

Example 1 As per definitions (1) let us assume the NTDFS as follows

$\widetilde{A} = \langle a_2\left(1-\frac{\rho}{1+n}\right), a_2, a_2\left(1+\frac{\sigma}{1+n}\right)\rangle$, for $0<\rho,\sigma<1, n\geq 0$ and the membership function along with graphical illustration (Fig.-1) is given by

$$\mu(x,n) = \begin{cases} 0 \quad if\ x < a_2\left(1-\frac{\rho}{1+n}\right) and\ x > a_2\left(1+\frac{\sigma}{1+n}\right) \\ \left\{\frac{x-a_2\left(1-\frac{\rho}{1+n}\right)}{\frac{\rho a_2}{1+n}}\right\} \quad if\ a_2\left(1-\frac{\rho}{1+n}\right) \leq x \leq a_2 \\ \left\{\frac{a_2\left(1+\frac{\sigma}{1+n}\right)-x}{\frac{\sigma a_2}{1+n}}\right\} \quad if\ a_2 \leq x \leq a_2\left(1+\frac{\sigma}{1+n}\right) \end{cases} \qquad (1)$$

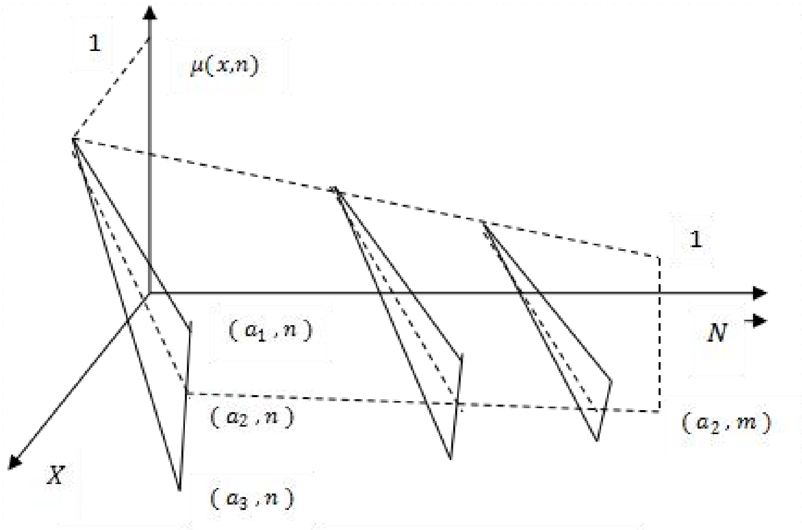


Fig.1: Membership function of NTDFS

Definition 2: [De[18]] Let the TDFS$\widetilde{A} = \langle a\{1-\rho f_n\}, a, a\{1-\sigma g_n\}\rangle$ for $0 < \rho, \sigma \in R$ and

f_n, g_n are two Cauchysequences of functions having converging points resp$\frac{1}{k_1}$ and $\frac{1}{k_2}$, $0 \neq k_1, k_2 \in R$ectively, then the fuzzy set $\widetilde{A}$ is called triangular dense fuzzy lock set with double keys k_1 and k_2, and they depend upon ρ and σ, respectively. The corresponding membership function of $\widetilde{A}$is stated in (2), and its graphical representation is given by Fig. 2.

$$\mu(x,n) = \begin{cases} 0 & if\ x < a(1-\rho f_n)\ and\ x > a(1+\sigma g_n) \\ \frac{x-a(1-\rho f_n)}{a\rho f_n} & if\ a(1-\rho f_n) \le x \le a \\ \frac{a(1+\sigma g_n)-x}{a\sigma g_n} & if\ a \le x \le a(1+\sigma g_n) \end{cases} \quad (2)$$

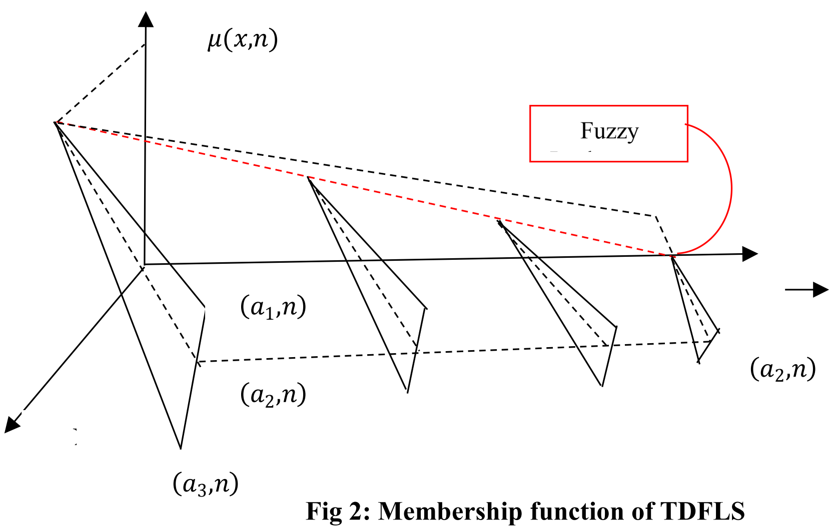


Fig 2: Membership function of TDFLS

2.2 [De[18]] Defuzzification Method Based on α- cuts

We assume, $f_n = \left(\frac{1}{k_1} - \frac{1}{n+1}\right)$ and $g_n = \left(\frac{1}{k_2} - \frac{1}{n+1}\right)$ in the above fuzzy set (2) and we have the left and the right α-cuts of a triangular dense fuzzy number $\widetilde{A} = \mu(x,n)$ as follows:

$$[\mu_L^d, \mu_R^d] = \left[a + a\rho(\alpha - 1)\left(\frac{1}{k_1} - \frac{1}{n+1}\right), a + a\sigma(1 - \alpha)\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right]$$

The corresponding index value is given by

$$I(\tilde{x}) = \frac{1}{2N}\sum_{n=1}^{N}\int_0^1 (\mu_L^d + \mu_R^d)d\alpha = a + \frac{a}{4}\left(\frac{\sigma}{k_2} - \frac{\rho}{k_1}\right) + \frac{a(\rho-\sigma)}{4N}\left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N+1}\right) \quad (3)$$

[for details see (16)]

3. Assumptions and Notations

Notations

C_1: Holding cost per quantity per unit time($)

C_2: Shortage cost per unit quantity per unit time($)

C_3: Set up cost per unit time period per cycle ($)

D_2: Demand in shortage period $(D_2 = D_1 e^{-t_2})$

Q_1: Inventory level within timet_1(days)

Q_2: Shortage quantity during the time t_2(days)

t_1: Inventory run time (days)

t_2: Shortage time (days)

T: Cycle time $(= t_1 + t_2)$(days)

TAC: Total average cost ($)

Assumptions

We have the following assumptions

1. Demand rate is uniform and known
2. Rate of replenishment is finite
3. Lead time is zero/negligible
4. Shortage are allowed and fully backlogged

4. Formulation of Crisp Mathematical Model

Let the inventory starts at time t = 0 with order quantity Q_1 and demand rateD. After time $t = t_1$ the inventory reaches zero level and the shortage starts and it continues up to time $T = t_1 + t_2$). Let Q_2 be the shortage quantity during that time period t_2 . Also, we assume that the shortage time demand rate is depending on the duration of shortage time t_2 .Therefore, the mathematical problem associated to the proposed model is shown in Fig.-4 and the necessary calculations are given below.

$$\text{Inventory holding cost} = \frac{1}{2}C_1Q_1t_1 \quad (4)$$

$$\text{Shortage cost} = \frac{1}{2}C_2Q_2t_2 = \frac{1}{2}C_2D_1(1 - e^{-t_2})t_2 \quad (5)$$

$$\text{Set up cost} = C_3 \quad (6)$$

Where, $Q_1 = D_1 t_1$ (7)

And $Q_2 = \int_0^{t_2} D_2\, dt_2 = D_1 \int_0^{t_2} e^{-t_2} dt_2 = D_1[1 - e^{-t_2}]$ (8)

$T = t_1 + t_2$ (9)

Therefore, the total average inventory cost is given by

$$TAC = \frac{1}{2T} C_1 Q_1 t_1 + \frac{1}{2T} C_2 Q_2 t_2 + \frac{1}{T} C_3$$

$$= \frac{1}{2} C_1 D_1 \frac{t_1^2}{t_1 + t_2} + \frac{1}{2} C_2 D_1 \frac{(1 - e^{-t_2}) t_2}{t_1 + t_2} + \frac{C_3}{t_1 + t_2}$$

$$= D_1 \left\{ \frac{1}{2} C_1 \frac{t_1^2}{t_1+t_2} + \frac{1}{2} C_2 \frac{(1-e^{-t_2})t_2}{t_1+t_2} \right\} + \frac{C_3}{t_1+t_2} \quad (10)$$

Therefore our problem is redefined as

Minimize $TAC = D_1 \psi + \varphi$ (11)

$$\text{where} \begin{cases} \psi = \frac{1}{2} C_1 \frac{t_1^2}{t_1+t_2} + \frac{1}{2} C_2 \frac{(1-e^{-t_2})t_2}{t_1+t_2} \\ \varphi = \frac{C_3}{t_1+t_2} \end{cases} \quad (12)$$

Subject to the conditions (7-9).

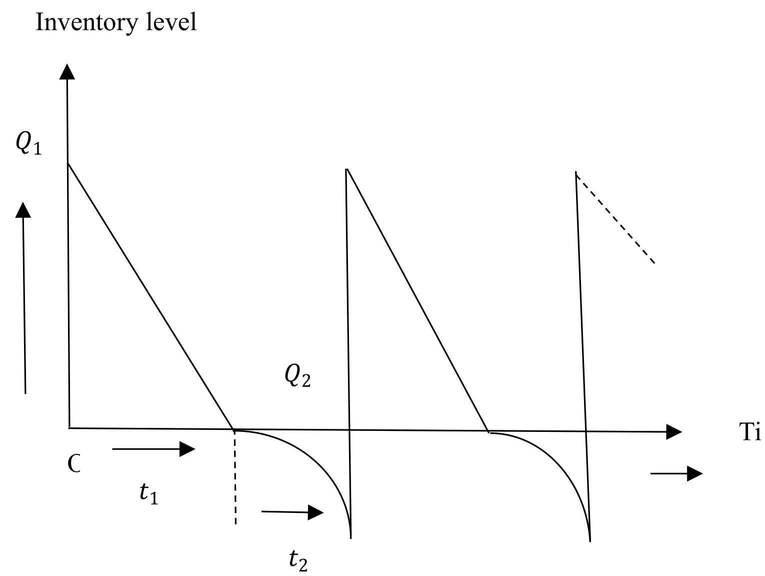


Fig. 3: Back order Inventory

5. Fuzzy Mathematical Model

Since demand rate follows an important role in defining the objective function in an inventory process, so we consider the demand rate assumes flexible values in the propose model and it can be reduce by means of dense fuzzy set. So the objective function ($TAC = Z$) of the crisp model (11) can be written as

$\tilde{Z} = \widetilde{D_1} \psi + \varphi$ (13)

Whereψ andφ are given by (12). Now, (11) can also be written as

$D_1 = \frac{(Z-\varphi)}{\psi}$and its fuzzy equivalent is given by $\widetilde{D_1} \cong \frac{(\tilde{Z}-\varphi)}{\psi}$ (14)

Now, if we think of the demand rate assumes triangular dense fuzzy lock set then as per De[18] the membership function of the demand rate is given by

$$\mu(\widetilde{D_1}) = \begin{cases} \frac{d-d_0\left\{1-\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)\right\}}{d_0\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)}, \text{ if } d_0\left\{1-\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)\right\} \le d \le d_0 \\ \frac{d_0\left\{1+\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\}-d}{d_0\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)}, \text{ if } d_0 \le d \le d_0\left\{1+\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\} \\ 0, \text{ if } d \le d_0\left\{1-\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)\right\} \text{ and } d \ge d_0\left\{1+\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\} \end{cases} \quad (15)$$

The left and right $\alpha-$ cuts of the $\mu(\widetilde{D_1})$ are$\left[\mu_L^d, \mu_R^d\right] = \left[d_0 + d_0\rho(\alpha-1)\left(\frac{1}{k_1}-\frac{1}{n+1}\right), d_0 + d_0\sigma(1-\alpha)\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right]$

The corresponding index value is given by

$$I(\widetilde{D_1}) = \frac{1}{2N}\sum_{n=1}^{N}\int_0^1\left(\mu_L^d + \mu_R^d\right)d\alpha$$

$$= \frac{1}{2N}\sum_{n=1}^{N}\int_0^1\left\{2d_0 + d_0\rho(\alpha-1)\left(\frac{1}{k_1}-\frac{1}{n+1}\right) + d_0\sigma(1-\alpha)\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\}d\alpha$$

$$= \frac{1}{2N}\sum_{n=1}^{N}\left\{2d_0 - \frac{d_0\rho}{2}\left(\frac{1}{k_1}-\frac{1}{n+1}\right) + \frac{d_0\sigma}{2}\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\}$$

$$= \frac{1}{2N}\sum_{n=1}^{N}\left\{2d_0 + \frac{d_0}{2}\left(\frac{\sigma}{k_2}-\frac{\rho}{k_1}\right) + \frac{d_0}{2(n+1)}(\rho-\sigma)\right\}$$

$$= d_0 + \frac{d_0}{4}\left(\frac{\sigma}{k_2}-\frac{\rho}{k_1}\right) + \frac{d_0(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right) \quad (16)$$

Now we obtain the membership function of the fuzzy objective by using (14) in (15)

$$\mu(\tilde{Z}) = \begin{cases} \frac{\frac{Z-\varphi}{\psi}-d_0\left\{1-\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)\right\}}{d_0\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)}, \text{ if } d_0\psi\left\{1-\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)\right\}+\varphi \le Z \le d_0\psi+\varphi \\ \frac{d_0\left\{1+\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\}-\frac{Z-\varphi}{\psi}}{d_0\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)}, \text{ if } d_0\psi+\varphi \le Z \le d_0\psi\left\{1+\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\}+\varphi \\ 0, \quad \text{elsewhere} \end{cases} \quad (17)$$

Now the left and right $\alpha-$ cuts of $\mu(\tilde{Z})$are given by

$$\left[\mu_L^Z, \mu_R^Z\right] = \left[\varphi + d_0\psi + d_0\psi\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)(\alpha-1), \varphi + d_0\psi + d_0\psi\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)(1-\alpha)\right]$$

The corresponding index value is

$$I(\tilde{Z}) = \frac{1}{2N}\sum_{n=1}^{N}\int_0^1\left(\mu_L^Z + \mu_R^Z\right)d\alpha = \frac{1}{2N}\sum_{n=1}^{N}\int_0^1\left\{2(\varphi + d_0\psi) + d_0\psi\rho\left(\frac{1}{k_1}-\frac{1}{n+1}\right)(\alpha-1) + d_0\psi\sigma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)(1-\alpha)\right\}d\alpha = \frac{1}{2N}\sum_{n=1}^{N}\left\{2(\varphi+d_0\psi) - \frac{d_0\psi\rho}{2}\left(\frac{1}{k_1}-\frac{1}{n+1}\right) + \frac{d_0\psi\sigma}{2}\left(\frac{1}{k_2}-\frac{1}{n+1}\right)\right\} = \varphi + d_0\psi - \frac{d_0\psi}{4}\left(\frac{\rho}{k_1}-\frac{\sigma}{k_2}\right) + \frac{d_0\psi(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right) \quad (18)$$

5.1 Particular Cases

1. If we take $k_1 = k_2 = k$ then

2. $I(\tilde{Z}) = \varphi + d_0\psi - \frac{d_0\psi(\rho-\sigma)}{4k} + \frac{d_0\psi(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right)$

$$I(\widetilde{D_1}) = d_0 + \frac{d_0(\sigma-\rho)}{4k} + \frac{d_0(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right) \quad (19)$$

gives the problem of dense fuzzy lock model for single key.

3. If we take $k_1 = k_2 = 1$ then

4. $I(\tilde{Z}) = \varphi + d_0\psi - \frac{d_0\psi(\rho-\sigma)}{4} + \frac{d_0\psi(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right)$

$$I(\widetilde{D_1}) = d_0 + \frac{d_0(\sigma-\rho)}{4} + \frac{d_0(\rho-\sigma)}{4N}\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N+1}\right) \quad (20)$$

gives the problem of dense fuzzy model.

5. If we take $k_1 = k_2 = 1$ and $N \to \infty$ then

$$I(\tilde{Z}) = \phi + d_0\,\psi - \frac{d_0\,\psi\,(\rho-\sigma)}{4} \text{ and } I(\widetilde{D_1}) = d_0 + \frac{d_0(\sigma-\rho)}{4} \quad (21)$$

gives the problem of general fuzzy model.

6. If we take $k_1 = k_2 = 1$ and $N \to \infty$ and $\rho = \sigma$ then

$$I(\tilde{Z}) = \phi + d_0\,\psi \text{ and } I(\widetilde{D_1}) = d_0 \quad (22)$$

Gives the problem of crisp model.

5.2 [De[18]]Rules of Finding Key Values of the Fuzzy Locks

When an uncertainty appears in a parameter, then the expert may not know the exact value but (s)he knows a bound of that parameter. Then (s)he usually fixes a upper limit (a^U, if known), lower limit (a^L, if known) or both for that parameter.For single key, if upper bound is available then the index value of $\tilde{a}$ is given by$I(\tilde{a}) \le a^U$implies $\ge \frac{a(\sigma-\rho)}{4(a^U-a)}$.

If the lower bound is known, then$I(\tilde{a}) \ge a^L$implies $\ge \frac{a(\rho-\sigma)}{4(a-a^L)}$. For double keys the index value of$\widetilde{A}$ is given by

$$I(\widetilde{A}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}d\alpha = \frac{a}{2}\left(1-\frac{\rho}{2k_1}\right) + \frac{a}{2}\left(1+\frac{\sigma}{2k_2}\right),$$

So, $\frac{a}{2}\left(1-\frac{\rho}{2k_1}\right) \ge a^L \Rightarrow k_1 \ge \frac{a\rho}{2(a-2a^L)}$ and $\frac{a}{2}\left(1+\frac{\sigma}{2k_2}\right) \le a^U \Rightarrow k_2 \ge \frac{a\sigma}{2(2a^U-a)}$.

6. Numerical Example 1

Let us consider $C_1 = 2.5, C_2 = 1.8, C_3 = 1200, d_1 = 100, \rho = 0.3, \sigma = 0.2$ then we get the following results.Here we keep the bounds of demand rate stated as under $[d_0^L, d_0^U] = [90, 110]$. Employing the above definition of finding the keys we write,

$$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \ge \begin{bmatrix} \frac{d_0\rho}{2(d_0 - 2d_0^L)} \\ \frac{d_0\sigma}{2(2d_0^U - d_0)} \end{bmatrix} = \begin{bmatrix} \frac{100 \times 0.3}{2(100 - 2 \times 90)} \\ \frac{100 \times 0.2}{2(2 \times 110 - 100)} \end{bmatrix} = \begin{bmatrix} -0.187 \\ 0.083 \end{bmatrix}$$

Let us take $k_1 = 0.47$ and $k_2 = 0.5$ satisfying the above condition and we get the optimal solution stated in table-1.

Table 1:Optimal solution of EOQ model

Model		Time (t_1^*)	Time (t_2^*)	Time (T^*)	Q_1^*	Q_2^*	Minimum cost(Z^*)
Crisp		3.04	3.04	6.09	304.67	95.24	430.21
General fuzzy		3.08	3.08	6.17	301.03	93.05	424.35
Dense fuzzy	N=1	3.06	3.06	6.13	302.86	94.15	427.29
	N=2	3.07	3.07	6.14	302.55	93.96	426.80
	N=3	3.072	3.072	6.145	302.35	93.84	426.47
	N=4	3.074	3.074	6.148	302.20	93.75	426.23
	N=5	3.075	3.075	6.151	302.09	93.69	426.05
	N=6	3.076	3.076	6.153	302.00	93.63	425.91
Dense fuzzy (single key) K=0.5, N=6		3.11	3.11	6.22	298.33	91.43	419.99
Dense fuzzy(Double key) $k_1 = 0.47, k_2 = 0.5$, N=6		3.14	3.14	6.28	296.91	90.58	417.71

From the above Table1 we see that, the minimum objective value came from the dense fuzzy (double key) model having average inventory cost \$ 417.71 for 6.28 days cycle time with 296.91 units of order quantity with respect to the 90.58 units of backorder quantity. The objective values for the other cases are of ascending order. It is also seen that, within 6 days week if we consider the learning experiences with at most 6 times interactions among retailer –supplier then the optimality comes for all the cases of dense fuzzy environment.

6.1 Sensitivity Analysis

Based on the numerical example (Case of dense fuzzy lock sets with double key) considered above for the single production plant model, we now calculate the corresponding outputs for changing inputs parameter one by one. The sensitivity analysis is performed (See Table-2) by changing of each parameter C_1, C_2, C_3, ρ, σ, k_1,k_2 and d_0 by +30%, +20%, -20% and -30% considering one at a time and keeping the remaining parameters as unchanged.

Table 2: Sensitivity analysis with parametric changes from (-30% to +30%)

Parameter	% Change	t_1^* days	t_2^* days	T^* days	Q_1^*	Q_2^*	Minimum costZ^*(\$)	$\frac{Z^* - Z_*}{Z_*}100\%$
C_1	+30	2.74	2.74	5.48	259.86	88.61	469.68	9.17
	+20	2.86	2.86	5.71	270.63	89.26	453.11	5.32
	-20	3.51	3.51	7.02	332.68	91.88	378.49	-12.02
	-30	3.76	3.76	7.52	356.15	92.50	356.99	-17.01
C_2	+30	3.11	3.11	6.23	295.48	90.52	429.94	-0.06
	+20	3.12	3.12	6.25	295.96	90.54	425.86	-1.01
	-20	3.14	3.14	6.29	297.85	90.62	409.56	-4.79
	-30	3.15	3.15	6.30	298.32	90.64	405.48	-5.74
C_3	+30	3.59	3.59	7.18	340.44	92.10	471.20	9.52
	+20	3.45	3.45	6.90	326.58	91.69	454.17	5.57
	-20	2.78	2.78	5.57	263.90	88.86	377.18	-12.32
	-30	2.59	2.59	5.18	245.75	87.63	354.88	-17.51

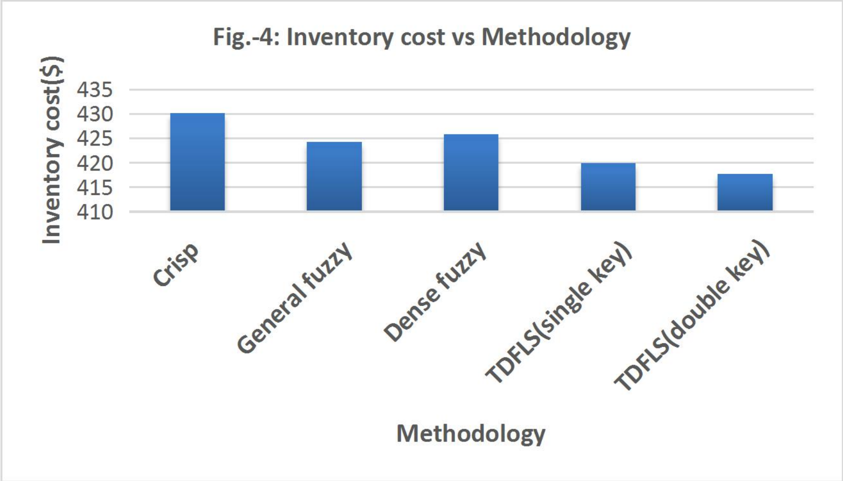
ρ	+30	3.21	3.21	6.42	290.59	86.86	407.60	-5.25
	+20	3.18	3.18	6.36	292.71	88.10	410.99	-4.46
	-20	3.08	3.08	6.17	301.03	93.05	424.34	-1.36
	-30	3.06	3.06	6.12	303.07	94.28	427.63	-0.60
σ	+30	3.09	3.09	6.18	300.75	92.88	423.89	-1.47
	+20	3.10	3.10	6.21	299.48	92.11	421.84	-1.95
	-20	3.16	3.16	6.33	294.31	89.04	413.55	-3.87
	-30	3.18	3.18	6.36	293.00	88.27	411.46	-4.36
d_0	+30	2.72	2.72	5.45	335.98	115.08	481.64	11.95
	+20	2.84	2.84	5.69	323.61	107.05	461.14	7.18
	-20	3.52	3.52	7.04	266.87	73.52	370.22	-13.94
	-30	3.77	3.77	7.54	250.21	64.77	344.50	-19.92
K_1	+30	3.07	3.07	6.14	302.33	93.83	426.44	-0.87
	+20	3.08	3.08	6.17	300.84	92.93	424.03	-1.43
	-20	3.20	3.20	6.41	290.89	87.04	408.09	-5.14
	-30	3.26	3.26	6.52	286.50	84.49	401.10	-6.76
K_2	+30	3.17	3.17	6.35	293.45	88.54	412.17	-4.19
	+20	3.16	3.16	6.32	294.41	89.10	413.71	-3.83
	-20	3.09	3.09	6.18	300.60	92.79	423.65	-1.52
	-30	3.06	3.06	6.12	303.21	94.36	427.85	-0.54

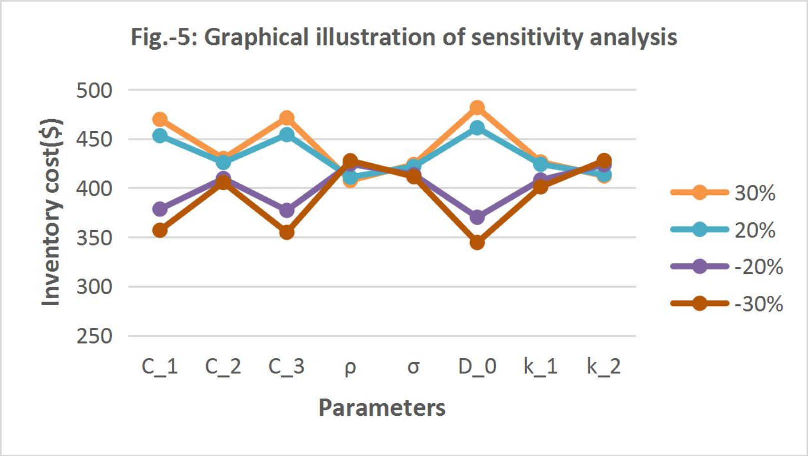
6.2 Discussion on Sensitivity Analysis

Table 2 shows that the shortage cost per unit item C_2 , the fuzzy system parameters ρ , σ and the decision makers' choices K_1 and K_2 are slightly sensitive towards model minimum (unidirectional) with reference to all the changes from -30% to +30 %. For these parameters the objective values (average inventory cost) getting range from \$ 405.48 to \$ 429. 94 for the range of the order quantity 290.59- 303 only. 21 units with maximum cycle time duration 6.52 days only. We also notice that, the order quantity, the backorder quantity and the average cost functions have simple proportional changes. But for the other parameters, all the changes assume similar directional average (considerable) changes. Throughout the whole table we see if we reduce the demand parameter -30% then the average inventory cost reaches value to \$ 344.50. Moreover, if we consider the case of single key then for its -30% change can give the model minimum \$ 401.10 alone. However, if the system itself has double decision maker over single decision, then keeping the first one's perception value fixed and increasing the second other's to +30 % we can arrive at the average inventory cost value to \$ 412.17 exclusively.

6.3 Graphical Illustrations of the Model

We have studied graphically over the numerical outputs (Table- 1) of the model. The Fig.-4 shows the average cost values of the model began to decrease whenever we are going through Crisp environment to dense fuzzy lock rule of double keys. We may note that, the key value of decision maker one corresponds to the choice of perception over the system and that for the other corresponds to the second decision maker also. Although, Fig-5 reveals that, over the changes of the fuzzy system parameters (, σ) the average inventory cost axis revolves around the fuzzy system parameters itself. The unit holding cost, the set up cost and the demand parameters ate most fluctuating parameters getting objective values around 344-481. The variations of the cost functions are negligible for the fuzzy system parameters and the keys also.





7. Conclusion

Here we have discussed a simple backorder EOQ model under double decision maker for single decision under dense fuzzy environment. The novelty of this article is that it expresses the shortage time demand as exponentially decaying with the duration of the shortage time. However, the present article deserve that the concept of single decision maker in a inventory process is vague with certain extent rather it would be beneficial if we consider the multiple decision maker within democratic attitude by means of healthy inter relationship among the hierarchy of the management process over strategic understanding.

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