PARTHA PRATIM DEY[1], SURAPATI PRAMANIK[2,*], BIBHAS C. GIRI[3]

1, 3 Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India
*2 Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District –North 24 Parganas, Pin code-743126,West Bengal, India. Corresponding Author's E-mail: sura_pati@yahoo.co.in

Extended Projection Based Models for Solving Multiple Attribute Decision Making Problems with Interval Valued Neutrosophic Information

Abstract

The paper develops two new methods for solving multiple attribute decision making problems with interval – valued neutrosophic assessments. In the decision making situation, the rating of alternatives with respect to the predefined attributes is described by linguistic variables, which can be represented by interval - valued neutrosophic sets. We assume that the weight of the attributes are not equal in the decision making process and they are obtained by using maximizing deviation method. We define weighted projection measure and propose a method to rank the alternatives. Furthermore, we also develop an alternative method to solve multiple attribute decision making problems based on the combination of angle cosine and projection method. Finally, an illustrative numerical example in Khadi institution is solved to verify the effectiveness of the proposed methods.

Keywords

Interval-valued neutrosophic sets; projection measure; weighted projection measure; angle cosine; multiple attribute decision making.

1. Introduction

Multiple attribute decision making (MADM) is one of the most significant parts of modern decision science and it is a well known method for selecting the most desirable alternative from a set of all feasible alternatives with respect to some predefined attributes. However, the information about the attributes is generally incomplete, indeterminate and inconsistent in nature due to the complexity of real world problems. Smarandache [1-4] grounded the concept of neutrosophic sets (NSs) from philosophical point of view by incorporating the degree of indeterminacy or neutrality as independent component to deal with problems involving imprecise, indeterminate and inconsistent information and the concept of NSs has been applied to different fields such as decision sciences, social sciences, humanities, etc. From scientific and realistic point of view, Wang et al. [5] defined single valued NSs (SVNSs) and then presented the set theoretic operators and various properties of SVNSs. Wang et al. [6] also developed the notion of interval neutrosophic sets (INSs) characterized by membership, non-membership and falsity-membership functions, whose values are interval rather than real numbers.

In 2013, Chi and Liu [7] first discussed a novel approach for solving MADM problems based on extended TOPSIS method under interval neutrosophic environment. Zhang et al. [8] defined some operators for INSs and established a multi-criteria decision making method. Broumi and Smarandache [9] defined cosine similarity measure between two INSs and applied the concept to medical diagnosis problem. Ye [10] proposed some similarity measures between two IVNSs based on the relationship of similarity measures and distance measures and utilized the developed method to solve a multi-criteria decision making problem. Sahin and Liu [11] developed maximizing deviation method for solving MADM problems having incomplete weight information. They employed single valued neutrosophic weighted averaging operator and interval neutrosophic weighted averaging operator in order to aggregate the neutrosophic information corresponding to each alternative and the most desirable alternatives are obtained based on score and accuracy functions. Pramanik and Mondal [12] discussed interval neutrosophic MADM based on grey relational analysis (GRA) method where the unknown attribute weights are derived from information entropy method. Later, Dey et al. [13] studied an extended GRA based interval neutrosophic MADM for weaver selection in Khadi institution. Mondal and Pramanik [14] proposed cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set for solving MADM problems. Recently, Dey et al. [15] investigated an extended GRA method for MADM problem with interval neutrosophic uncertain linguistic information.

Projection measure is useful device for solving decision making problems because it takes into account the distance as well as the included angle between points evaluated [16]. Xu and Hu [17] provided projection models for dealing with intuitionistic fuzzy MADM problems. Zeng et al. [18] demonstrated weighted projection algorithms for multiple arttribute group decision problems under intuitionistic fuzzy and interval – valued intuitionistic fuzzy environment. Yue [19-20] presented a projection method to obtain weights of the experts in a group decision making problem. Yue [21] proposed a projection based approach for partner selection in a group decision making problem with linguistic values and intuitionistic fuzzy information. Ju and Wang [22] investigated a methodology to multicriteria group decision problems with incomplete weight information in linguistic setting based on projection method. Yang and Du [23] developed a straightforward method for obtaining the weights of the decision makers based on angle cosine and projection method. Ye [24] discussed a simplified neutrosophic harmonic averaging projection based method to solve MADM problems. Ye [25] provided a decision making method based on credibility-induced interval neutrosophic weighted arithmetic averaging operator and credibility-induced interval neutrosophic weighted geometric averaging operator and the projection measure-based ranking method to solve MADM problems with interval neutrosophic information and credibility information.

In this paper, we define weighted projection measure for interval – neutrosophic information and develop a method for solving MADM problems based on weighted projection method. We also investigate a method for MADM under interval - valued neutrosophic environment based on the combination of angle cosine and projection method.

Rest of the paper is prepared as follows: Sec. 2 presents several definitions. An interval - valued neutrosophic MADM based on weighted projection method is discussed in Sec. 3. Subsection 3.1 presents the algorithm for MADM problems with interval valued neutrosophic information based on weighted projection method. Subsection 3.2 presents the approach for solving interval - valued neutrosophic MADM problems based on angle cosine and projection method. Subsection 3.3 presents the algorithm for MADM problem with interval valued neutrosophic information based on angle cosine and projection method In Sec. 4, we solve a numerical example to show the applicability and feasibility of the proposed method. Sec. 5 provides conslusion and future scope of research.

2. Preliminaries

In this Section, we briefly present some basic definitions which will be useful for the formulation of the paper.

2.1 Neutrosophic set

Definition 2.1.1 [1-4]: Consider U be a universal space of points with generic element in U denoted by x. Then a NS A is defined as follows:

$$A = \{x, \langle \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x) \rangle \mid x \in U\} \tag{2.1}$$

where, $\mathrm{T}_A(x)$, $\mathrm{I}_A(x)$, $\mathrm{F}_A(x)$: $U \to \,]^-0, 1^+[$ are the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively with $^-0 \le \sup \mathrm{T}_A(x) + \sup \mathrm{I}_A(x) + \sup \mathrm{F}_A(x) \le 3^+$.

Definition 2.1.2. [5] Let U be a universal space of points with generic element in U represented by x. Then, a SVNS $S \subset U$ is defined as follows:

$$S = \{x, \langle \mathrm{T}_S(x), \mathrm{I}_S(x), \mathrm{F}_S(x) \rangle \mid x \in U\} \tag{2.2}$$

where $\mathrm{T}_S(x)$, $\mathrm{I}_S(x)$ and $\mathrm{F}_S(x)$ denote truth-membership, indeterminacy-membership and falsity-membership functions, respectively. For each point $x \in U$, we have, $\mathrm{T}_S(x), \mathrm{I}_S(x), \mathrm{F}_S(x)$: $U \to [0, 1]$ and $0 \le \sup \mathrm{T}_S(x) + \sup \mathrm{I}_S(x) + \sup \mathrm{F}_S(x) \le 3$.

Definition 2.1.3. **[6]** Let U be a universe of discourse, with a generic element in U represented by x. An interval valued neutrosophic set N is represented as follows:

$$N = \{x, \langle \mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) \rangle \mid x \in U\} \tag{2.3}$$

where $\mathrm{T}_N(x)$, $\mathrm{I}_N(x)$, $\mathrm{F}_N(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively. For each point $x \in U$, $\mathrm{T}_N(x)$, $\mathrm{I}_N(x)$, $\mathrm{F}_N(x) \subseteq [0, 1]$ and $0 \le \sup \mathrm{T}_N(x) + \sup \mathrm{I}_N(x) + \sup \mathrm{F}_N(x) \le 3$.

For convenience, if $\mathrm{T}_N(x) = [\,\mathrm{T}_N^{\mathrm{L}}(x), \mathrm{T}_N^{\mathrm{U}}(x)\,]; \mathrm{I}_N(x) = [\,\mathrm{I}_N^{\mathrm{L}}(x), \mathrm{I}_N^{\mathrm{U}}(x)\,]; \mathrm{F}_N(x) = [\,\mathrm{F}_N^{\mathrm{L}}(x), \mathrm{F}_N^{\mathrm{U}}(x)\,]$, then

$$N = \{x, \langle [\mathrm{T}_N^{\mathrm{L}}(x), \mathrm{T}_N^{\mathrm{U}}(x)], [\mathrm{I}_N^{\mathrm{L}}(x), \mathrm{I}_N^{\mathrm{U}}(x)], [\mathrm{F}_N^{\mathrm{L}}(x), \mathrm{F}_N^{\mathrm{U}}(x)], \rangle \mid x \in U\} \tag{2.4}$$

with the condition $0 \le \sup \mathrm{T}_N^{\mathrm{U}}(x) + \sup \mathrm{I}_N^{\mathrm{U}}(x) + \sup \mathrm{F}_N^{\mathrm{U}}(x) \le 3$.

For convenience, an interval valued neutrosophic number (IVNN) $\tilde{p}$ is represented by

$$\tilde{p} = \langle [\mathrm{T}^-, \mathrm{T}^+], [\mathrm{I}^-, \mathrm{I}^+], [\mathrm{F}^-, \mathrm{F}^+] \rangle. \tag{2.5}$$

2.2 Projection method

Definition 2.2.1 [16, 26]: Let $e = (\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_\mathrm{q})$ be a vector, then norm of e is defined by

$$\| e \| = \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} \mathrm{e}_\mathrm{j}^2} \tag{2.6}$$

Definition 2.2.2 [16, 26]: Let $e = (\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_\mathrm{q})$ and $f = (\mathrm{f}_1, \mathrm{f}_2, \ldots, \mathrm{f}_\mathrm{q})$ be two vectors, then angle cosine between e and f is defined as follows:

$$\mathrm{Cos}\,(e, f) = \frac{\sum_{\mathrm{j}=1}^{\mathrm{q}} (e_\mathrm{j} f_\mathrm{j})}{\sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} e_\mathrm{j}^2} \times \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} f_\mathrm{j}^2}} \tag{2.7}$$

Obviously, $0 < \mathrm{Cos}\,(e, f) \le 1$, and $\mathrm{Cos}\,(e, f)$ denotes the closeness between e and f only in direction.

Definition 2.2.3 [27]: Consider $p_1 = \langle [\mathrm{T}_1^-, \mathrm{T}_1^+], [\mathrm{I}_1^-, \mathrm{I}_1^+], [\mathrm{F}_1^-, \mathrm{F}_1^+] \rangle$ and $p_2 = \langle [\mathrm{T}_2^-, \mathrm{T}_2^+], [\mathrm{I}_2^-, \mathrm{I}_2^+], [\mathrm{F}_2^-, \mathrm{F}_2^+] \rangle$ be two IVNNs. Then the angle cosine of the included angle between p_1 and p_2 is defined as follows:

$$Cos\,(p_1, p_2) = \frac{(\mathrm{T}_1^-\mathrm{T}_2^- + \mathrm{T}_1^+\mathrm{T}_2^+ + \mathrm{I}_1^-\mathrm{I}_2^- + \mathrm{I}_1^+\mathrm{I}_2^+ + \mathrm{F}_1^-\mathrm{F}_2^- + \mathrm{F}_1^+\mathrm{F}_2^+)}{\sqrt{((\mathrm{T}_1^-)^2 + (\mathrm{T}_1^+)^2 + (\mathrm{I}_1^-)^2 + (\mathrm{I}_1^+)^2 + (\mathrm{F}_1^-)^2 + (\mathrm{F}_1^+)^2)}\sqrt{((\mathrm{T}_2^-)^2 + (\mathrm{T}_2^+)^2 + (\mathrm{I}_2^-)^2 + (\mathrm{I}_2^+)^2 + (\mathrm{F}_2^-)^2 + (\mathrm{F}_2^+)^2)}} \tag{2.8}$$

Definition 2.2.4 [16, 26]: Let $e = (\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_\mathrm{q})$ and $f = (\mathrm{f}_1, \mathrm{f}_2, \ldots, \mathrm{f}_\mathrm{q})$ be two vectors, then the projection of vector e onto vector f can be defined as follows:

$$Proj\,(e)_f = \| e \|\,\mathrm{Cos}\,(e, f) = \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} e_\mathrm{j}^2} \times \frac{\sum_{\mathrm{j}=1}^{\mathrm{q}} (e_\mathrm{j} f_\mathrm{j})}{\sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} e_\mathrm{j}^2} \times \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} f_\mathrm{j}^2}} = \frac{\sum_{\mathrm{j}=1}^{\mathrm{q}} (e_\mathrm{j} f_\mathrm{j})}{\sqrt{\sum_{\mathrm{j}=1}^{\mathrm{q}} f_\mathrm{j}^2}} \tag{2.9}$$

where, $Proj\,(e)_f$ indicates that the closeness of e and f in magnitude.

Definition 2.2.5 [25]: Consider $U = (\mathrm{u}_1, \mathrm{u}_2, \ldots, \mathrm{u}_\mathrm{m})$ be a finite universe of discourse and A, B be two IVNSs in U, then

$$Proj\,(A)_B = \frac{1}{\| B \|} \sum_{\mathrm{j}=1}^{\mathrm{q}} (e_\mathrm{j} f_\mathrm{j}) = \frac{1}{\| B \|} \sum_{\mathrm{j}=1}^{\mathrm{q}} (\mathrm{T}_{\alpha_\mathrm{j}}^- \mathrm{T}_{\beta_\mathrm{j}}^- + \mathrm{T}_{\alpha_\mathrm{j}}^+ \mathrm{T}_{\beta_\mathrm{j}}^+ + \mathrm{I}_{\alpha_\mathrm{j}}^- \mathrm{I}_{\beta_\mathrm{j}}^- + \mathrm{I}_{\alpha_\mathrm{j}}^+ \mathrm{I}_{\beta_\mathrm{j}}^+ + \mathrm{F}_{\alpha_\mathrm{j}}^- \mathrm{F}_{\beta_\mathrm{j}}^{-\,-} + \mathrm{F}_{\alpha_\mathrm{j}}^+ \mathrm{F}_{\beta_\mathrm{j}}^+) \tag{2.10}$$

is called the projection of A on B, where $\alpha_\mathrm{j} = \langle [\mathrm{T}_{\alpha_\mathrm{j}}^-, \mathrm{T}_{\alpha_\mathrm{j}}^+], [\mathrm{I}_{\alpha_\mathrm{j}}^-, \mathrm{I}_{\alpha_\mathrm{j}}^+], [\mathrm{F}_{\alpha_\mathrm{j}}^{-\,-}, \mathrm{F}_{\alpha_\mathrm{j}}^+] \rangle$ and $\beta_\mathrm{j} = \langle [\mathrm{T}_{\beta_\mathrm{j}}^-, \mathrm{T}_{\beta_\mathrm{j}}^+], [\mathrm{I}_{\beta_\mathrm{j}}^-, \mathrm{I}_{\beta_\mathrm{j}}^+], [\mathrm{F}_{\beta_\mathrm{j}}^{-\,-}, \mathrm{F}_{\beta_\mathrm{j}}^+] \rangle$ are the i-th IVNNs of A and B respectively. Especially, when q = 1, we obtain the projection of α_1 on β_1 as follows:

$$Proj\,(\alpha_1)_{\beta_1} = \frac{1}{\| \beta_1 \|} (\mathrm{T}_{\alpha_\mathrm{j}}^- \mathrm{T}_{\beta_\mathrm{j}}^- + \mathrm{T}_{\alpha_\mathrm{j}}^+ \mathrm{T}_{\beta_\mathrm{j}}^+ + \mathrm{I}_{\alpha_\mathrm{j}}^- \mathrm{I}_{\beta_\mathrm{j}}^- + \mathrm{I}_{\alpha_\mathrm{j}}^+ \mathrm{I}_{\beta_\mathrm{j}}^+ + \mathrm{F}_{\alpha_\mathrm{j}}^- \mathrm{F}_{\beta_\mathrm{j}}^{-\,-} + \mathrm{F}_{\alpha_\mathrm{j}}^+ \mathrm{F}_{\beta_\mathrm{j}}^+) \tag{2.11}$$

Definition 2.2.6: Consider U = (u1, u2, …, um) be a finite universe of discourse and A be an IVNS in U, then

$$\| A \| = \sqrt{\sum_{j=1}^{m} \alpha_j^2} \tag{2.12}$$

is called the modulus of A, where $\alpha_j = \left\langle [T^-_{\alpha_j}, T^+_{\alpha_j}], [I^-_{\alpha_j}, I^+_{\alpha_j}], [F^-_{\alpha_j}, F^+_{\alpha_j}] \right\rangle$.

Definition 2.2.7: Consider U = (u1, u2, …, um) be a finite universe of discourse and A be an IVNS in U, then

$$\| A \|_w = \sqrt{\sum_{j=1}^{m} (w_j \alpha_j)^2} \tag{2.13}$$

is said to be the weighted modulus of A, where $\alpha_j = \left\langle [T^-_{\alpha_j}, T^+_{\alpha_j}], [I^-_{\alpha_j}, I^+_{\alpha_j}], [F^-_{\alpha_j}, F^+_{\alpha_j}] \right\rangle$ and $w = \{w_1, w_2, \ldots, w_m\}$ be the weight vector assigned for β_j, where $0 \le w_j \le 1$ with $\sum_{j=1}^{m} w_j = 1$.

Definition 2.2.8: Consider U = (u1, u2, …, um) be a finite universe of discourse and A, B be any two IVNSs in U, then

$$Proj_{\,w}(A)_B = \frac{1}{\| B \|_w} \sum_{j=1}^{q} (e_j f_j) = \frac{1}{\| B \|_w} \sum_{j=1}^{q} (T^-_{\alpha_j} T^-_{\beta_j} + T^+_{\alpha_j} T^+_{\beta_j} + I^-_{\alpha_j} I^-_{\beta_j} + I^+_{\alpha_j} I^+_{\beta_j} + F^-_{\alpha_j} F^-_{\beta_j} + F^+_{\alpha_j} F^+_{\beta_j})$$
(2.14)

is said to be the weighted projection of A on B, where $\alpha_j = \left\langle [T^-_{\alpha_j}, T^+_{\alpha_j}], [I^-_{\alpha_j}, I^+_{\alpha_j}], [F^-_{\alpha_j}, F^+_{\alpha_j}] \right\rangle$ and $\beta_j = \left\langle [T^-_{\beta_j}, T^+_{\beta_j}], [I^-_{\beta_j}, I^+_{\beta_j}], [F^-_{\beta_j}, F^+_{\beta_j}] \right\rangle$ are the i-th IVNNs of A and B respectively. Consider $w = \{w_1, w_2, \ldots, w_q\}$ be the weight vector assigned for β_j, where $0 \le w_j \le 1$ with $\sum_{j=1}^{q} w_j = 1$.

Definition 2.2.7 [28]: Consider $\alpha = ([T^-_1, T^+_1], [I^-_1, I^+_1], [F^-_1, F^+_1])$ and $\beta = ([T^-_2, T^+_2], [I^-_2, I^+_2], [F^-_2, F^+_2])$ be any two IVNNs, then Hamming distance between α and β is defined as follows:

$$\Re_{Ham}(\alpha, \beta) = 1/6(|T^-_1 - T^-_2| + |T^+_1 - T^+_2| + |I^-_1 - I^-_2| + |I^+_1 - I^+_2| + |F^-_1 - F^-_2| + |F^+_1 - F^-_2|). \tag{2.15}$$

Definition 2.2.8 [28]: Consider $\alpha = ([T^-_1, T^+_1], [I^-_1, I^+_1], [F^-_1, F^+_1])$ and $\beta = ([T^-_2, T^+_2], [I^-_2, I^+_2], [F^-_2, F^+_2])$ be any two IVNNs, then the Euclidean distance between α and β is defined as given below.

$$\Re_{Euc}(\alpha, \beta) = \sqrt{1/6\left((T^-_1 - T^-_2)^2 + (T^+_1 - T^+_2)^2 + (I^-_1 - I^-_2)^2 + (I^+_1 - I^+_2)^2 + (F^-_1 - F^-_2)^2 + (F^+_1 - F^+_2)^2\right)}$$
(2.16)

Definition 2.2.9 [28]: Let $A = ([\dot{T}^-_i, \dot{T}^+_i], [\dot{I}^-_i, \dot{I}^+_i], [\dot{F}^-_i, \dot{F}^+_i])$, (i = 1, 2, ..., m) and $B = ([\hat{T}^-_i, \hat{T}^+_i], [\hat{I}^-_i, \hat{I}^+_i], [\hat{F}^-_i, \hat{F}^+_i])$, (i = 1, 2, ..., m) be any two IVNSs, then the Hamming distance between A and B is presented as given below.

$$\Re_{Ham}(A, B) = \frac{1}{6m} \sum_{i=1}^{m} (|\dot{T}^-_i - \hat{T}^-_i| + |\dot{T}^+_i - \hat{T}^+_i| + |\dot{I}^-_i - \hat{I}^-_i| + |\dot{I}^+_i - \hat{I}^+_i| + |\dot{F}^-_i - \hat{F}^-_i| + |\dot{F}^+_i - \hat{F}^+_i|) \tag{2.17}$$

Definition 2.2.10 [28]: Consider A = ([$\dot{T}_i^-, \dot{T}_i^+$], [$\dot{I}_i^-, \dot{I}_i^+$], [$\dot{F}_i^-, \dot{F}_i^+$]), (i = 1, 2, ..., m) and B = ([$\hat{T}_i^-, \hat{T}_i^+$], [$\hat{I}_i^-, \hat{I}_i^+$], [$\hat{F}_i^-, \hat{F}_i^+$]), (i = 1, 2, ..., m) be any two IVNSs, then the Euclidean distance between A and B is defined as given below.

$$\Re_{Euc}(A, B) = \sqrt{1/6m \sum_{i=1}^{m} \left((\dot{T}_i^- - \hat{T}_i^-)^2 + (\dot{T}_i^+ - \hat{T}_i^+)^2 + (\dot{I}_i^- - \hat{I}_i^-)^2 + (\dot{I}_i^+ - \hat{I}_i^+)^2 + (\dot{F}_i^- - \hat{F}_i^-)^2 + (\dot{F}_i^+ - \hat{F}_i^+)^2 \right)} \quad (2.18)$$

2.3 Conversion between linguistic variables and IVNNs

A variable whose values can be represented in terms of words or sentences in a natural language is said to be a linguistic variable. The performance values of the alternatives with respect to attributes can be expressed by linguistic variables such as extreme good, very good, good, and medium good, etc. Linguistic variables can be transformed into IVNNs as given below [15].

Table 1. *Transformation between the linguistic variables and IVNNs*

Linguistic variables	IVNNs
Extreme good (EG)	([0.95, 1], [0.05, 0. 1], [0, 0.1])
Very good (VG)	([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])
Good (G)	([0.6, 0.75], [0.1, 0.2], [0.2, 0.25])
Medium Good (MG)	([0.5, 0.6], [0.2, 0.25], [0.25, 0.35])
Medium (M)	([0.4, 0.5], [0.2, 0.3], [0.35, 0.45])
Medium low (ML)	([0.3, 0.4], [0.15, 0.25], [0.45, 0.5])
Low (L)	([0.2, 0.3], [0.1, 0.2], [0.5, 0.65])
Very low (VL)	([0.05, 0.2], [0.1, 0.15], [0.65, 0.8])
Extreme low (EL)	([0, 0.05], [0.05, 0. 1], [0.8, 0.95])

3. An interval - valued neutrosophic MADM based on weighted projection method

Assume that H = {$h_1, h_2, \ldots, h_m$}, ($m \geq 2$) be a discrete set of alternatives and K = {$k_1, k_2, \ldots, k_n$}, ($n \geq 2$) be a set of attributes under consideration in a MADM problem. The rating of performance value of alternative h_i, i = 1, 2, …, m with respect to the predefined attribute k_j, j = 1, 2, …, n is represented by linguistic variables. The linguistic variables can be expressed by IVNNs. Assume w = {$w_1, w_2, \ldots, w_n$} be the unknown weight vector of the attributes, where $0 \leq w_j \leq 1$ with $\sum_{j=1}^{n} w_j = 1$.

The weighted projection method for solving MADM problem with interval -valued neutrosophic information is described by using the following steps:

Step 1. Formulation of decision matrix with IVNNs

The evaluation value of the alternative h_i, i = 1, 2, …, m with respect to the attribute k_j, j = 1, 2, …, n is presented by the expert in terms of linguistic variables that can be expressed by IVNNs. Therefore, interval – valued neutrosophic decision matrix $D_{\tilde{N}}$ is presented as given below.

$$D_{\tilde{N}} = \left\langle r_{\tilde{N}_{ij}} \right\rangle_{m\times n} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ . & . & \dots & . \\ . & . & \dots & . \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix} \tag{3.1}$$

Here r_{ij} = <[T^-_{ij}, T^+_{ij}], [I^-_{ij}, I^+_{ij}], [F^-_{ij}, F^+_{ij}]>; $T^-_{ij}, T^+_{ij}, I^-_{ij}, I^+_{ij}, F^-_{ij}, F^+_{ij} \in [0, 1]$ and $0 \le \sup T^+_{ij} + \sup I^+_{ij} + \sup F^+_{ij} \le 3$, i = 1, 2, …, m; j = 1, 2, …, n. Here, [T^-_{ij}, T^+_{ij}] represents the degree that the alternative h_i satisfies the attribute k_j. [I^-_{ij}, I^+_{ij}] denotes the degree that the alternative h_i is indeterminacy on the attribute k_j. [F^-_{ij}, F^+_{ij}] indicates the degree that the alternative h_i does not satisfies the attribute k_j.

Step 2. Standardize the decision matrix

Generally, two types of attributes are encountered in practical decision making problems such as benefit type attribute where bigger value of the attribute reflects better alternative and cost type attribute where bigger value of the attribute reflects worse alternative. However, in order to remove the influence of different physical dimensions to decision results, we require to standardize the decision matrix. The standardize decision matrix S = $[s_{ij}]_{m \times n}$ owing to Chi and Liu [7] is formulated as follows:

$$D_{\hat{S}} = \left\langle t_{\hat{S}_{ij}} \right\rangle_{m\times n} = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ . & . & \dots & . \\ . & . & \dots & . \\ t_{m1} & t_{m2} & \dots & t_{mn} \end{bmatrix} \tag{3.2}$$

where t_{ij} = ([$\ddot{T}^-_{ij}, \ddot{T}^+_{ij}$], [$\ddot{I}^-_{ij}, \ddot{I}^+_{ij}$],[$\ddot{F}^-_{ij}, \ddot{F}^+_{ij}$]), (i = 1, 2, ..., m; j = 1, 2, ..., n). Here, we have

$t_{ij} = r_{ij}$, if j is benefit type, (3.3)

$t_{ij} = \breve{r}_{ij}$, if j is cost type (3.4)

where, $\breve{r}_{ij}$ is the complement of t_{ij}.

Step 3. Determination of unknown weights of the attributes

In the decision making environment, we assume that the weights of the attributes are unknown to the expert and generally they are not identical. We use maximizing deviation method of Wang [29] to derive unknown attribute weights. The concept of maximizing deviation method is presented as follows. If an attribute has a small effect on the alternatives then the attribute value should be assigned with a small weight and the attribute which creates bigger deviation should be assigned with a bigger weight. However, if an attribute has very small or no effect on the alternatives then the weight of such attribute may be taken as zero [29].

The deviation values of alternatives h_i to all other alternatives with respect to attribute k_j can be formulated as $\Psi_{ij}(w_j) = \sum_{s=1}^{m} \Re(t_{ij}, t_{sj})\, w_j$, then $\Psi_j(w_j) = \sum_{i=1}^{m} \Re_{ij}(w_j) = \sum_{i=1}^{m}\sum_{s=1}^{m} \Re(t_{ij}, t_{sj}) w_j$ represents total deviation of all alternatives to the other alternatives for the attribute k_j. $\Psi(w_j) = \sum_{j=1}^{n} \Re_j(w_j) =$

$\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{s=1}^{m}\Re(t_{ij},t_{sj})w_j$ denotes the deviation of all attributes for all alternatives to the other alternatives. Now we construct the optimization model [29] as given below.

Maximize $\Psi(w_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{s=1}^{m}\Re(t_{ij},t_{sj})w_j$

Subject to $\sum_{j=1}^{n} w_j^2 = 1$, $w_j \geq 0$, j = 1, 2, …, n. (3.5)

Solving the above model, we obtain attribute weight [29] as follows:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{s=1}^{m}\Re(t_{ij},t_{sj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{s=1}^{m}\Re^2(t_{ij},t_{sj})}},\ j = 1, 2, \ldots, n. \tag{3.6}$$

Then, the normalized attribute weight is obtained as

$$w_j = \frac{\sum_{i=1}^{m}\sum_{s=1}^{m}\Re(t_{ij},t_{sj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{s=1}^{m}\Re(t_{ij},t_{sj})},\ j = 1, 2, \ldots, n. \tag{3.7}$$

Step 4. Determination of interval - valued neutrosophic ideal solution

We determine the interval - valued neutrosophic ideal solution $Z^* = (z_1^*, z_2^*, \ldots, z_n^*)$ [7] as given below.

$$z_j^* = ([1, 1], [0, 0], [0, 0]),\ j = 1, 2, \ldots, n. \tag{3.8}$$

The virtual interval - valued neutrosophic ideal solution $Z^* = (\mu_1^*, \mu_2^*, \ldots, \mu_n^*)$ [7] can also be obtained by identifying the best values for each attribute from all alternatives as shown below.

$$\mu_j^* = (\gamma_j^*, \delta_j^*, \lambda_j^*) \tag{3.9}$$

where, $\gamma_j^* = [\gamma_j^{L*}, \gamma_j^{U*}] = [\underset{i}{Max}\ \ddot{T}_{ij}^-, \underset{i}{Max}\ \ddot{T}_{ij}^+]$; $\delta_j^* = [\delta_j^{L*}, \delta_j^{U*}] = [\underset{i}{Min}\ \ddot{I}_{ij}^-, \underset{i}{Min}\ \ddot{I}_{ij}^+]$; $\lambda_j^* = [\lambda_j^{L*}, \lambda_j^{U*}] = [\underset{i}{Min}\ \ddot{F}_{ij}^-, \underset{i}{Min}\ \ddot{F}_{ij}^+]$.

Step 5. Calculation of the weighted projection

The weighted projection of the alternative h_i (i = 1, 2, …, m) on the ideal solution Z^* is defined as follows:

$$Proj_w(g_i)_{Z^*} = \frac{1}{\|Z^*\|_w}\sum_{j=1}^{n} w_j^2(\ddot{T}_{ij}^-\gamma_j^{L*} + \ddot{T}_{ij}^+\gamma_j^{U*} + \ddot{I}_{ij}^-\delta_j^{L*} + \ddot{I}_{ij}^+\delta_j^{U*} + \ddot{F}_{ij}^-\lambda_j^{L*} + \ddot{F}_{ij}^+\lambda_j^{U*})$$

$$= \frac{\sum_{j=1}^{n} w_j^2(\ddot{T}_{ij}^-\gamma_j^{L*} + \ddot{T}_{ij}^+\gamma_j^{U*} + \ddot{I}_{ij}^-\delta_j^{L*} + \ddot{I}_{ij}^+\delta_j^{U*} + \ddot{F}_{ij}^-\lambda_j^{L*} + \ddot{F}_{ij}^+\lambda_j^{U*})}{\sqrt{\sum_{j=1}^{n} w_j^2((\gamma_j^{L*})^2 + (\gamma_j^{U*})^2 + (\delta_j^{L*})^2 + (\delta_j^{U*})^2 + (\lambda_j^{L*})^2 + (\lambda_j^{U*})^2)}} \tag{3.10}$$

Step 6. Ranking of the alternatives

Rank the alternatives h_i (i = 1, 2, …, m) according to the weighted projection $Proj_w(h_i)_{Z^*}$ and bigger value of $Proj_w(h_i)_{Z^*}$ reflects the better alternative.

3.1 Algorithm 1.

An algorithm for MADM problems with interval valued neutrosophic information based on weighted projection method is provided in the following steps:

Step 1. The expert provides his/ her interval – valued neutrosophic decision matrix $D_{\tilde{N}}$ by Eq. (3.1).

Step 2. The decision matrix $D_{\tilde{N}}$, in Eq. (3.1) is standardized as shown, $D_{\hat{S}} = \left\langle t_{\hat{S}_{ij}} \right\rangle_{m\times n}$ in Eq. (3.2) by using Eqs. (3.3) – (3.4).

Step 3. The unknown weight of the attribute w_j, (j = 1, 2, …, n) is obtained by utilizing Eq. (3.7).

Step 4. The interval – valued neutrosophic ideal solution Z^* is determined from the standardize decision matrix in Eq. (3.2).

Step 5. Determine the weighted projection $Proj\ _w(h_i)_{Z^*}$ using Eq. (3.10).

Step 6. Rank the alternatives h_i (i = 1, 2, …, m) based on $Proj\ _w(h_i)_{Z^*}$ and select the best one.

Step 7. End.

3.2 Extension

An approach for solving interval - valued neutrosophic MADM problems based on angle cosine and projection method

The angle cosine [27] between the alternative h_i (i = 1, 2, …, m) and the ideal solution Z^* is defined as follows:

$$Cos\,(h_i, Z^*) = \frac{\sum_{j=1}^{n}(\ddot{T}_{ij}^{-}\gamma_j^{L*} + \ddot{T}_{ij}^{+}\gamma_j^{U*} + \ddot{I}_{ij}^{-}\delta_j^{L*} + \ddot{I}_{ij}^{+}\delta_j^{U*} + \ddot{F}_{ij}^{-}\lambda_j^{L*} + \ddot{F}_{ij}^{+}\lambda_j^{U*})}{\sqrt{\sum_{j=1}^{n}((\ddot{T}_{ij}^{-})^2 + (\ddot{T}_{ij}^{+})^2 + (\ddot{I}_{ij}^{-})^2 + (\ddot{I}_{ij}^{+})^2 + (\ddot{F}_{ij}^{-})^2 + (\ddot{F}_{ij}^{+})^2)}\ \sqrt{\sum_{j=1}^{n}((\gamma_j^{L*})^2 + (\gamma_j^{U*})^2 + (\delta_j^{L*})^2 + (\delta_j^{U*})^2 + (\lambda_j^{L*})^2 + (\lambda_j^{U*})^2)}} \tag{3.11}$$

Now we propose the direction indicator ξ $(0 \le \xi \le 1)$ to convert the direction closeness and magnitude closeness into relative closeness ρ_i. If the DM gives more interest on direction, then he or she provides bigger value to ξ. Otherwise, smaller value of ξ is provided if the magnitude is much more important to the DM [23].

Therefore, the relative closeness [23] for selecting the best alternative is given as follows:

$$\rho_i = \xi\ Cos\,(h_i, Z^*) + (1 - \xi)\ Proj\ (h_i)_{Z^*} \tag{3.12}$$

The bigger value of ρ_i gives the better alternative.

3.3 Algorithm 2.

An algorithm for MADM problem with interval valued neutrosophic information based on angle cosine and projection method can be demonstrated as follows:

Step 1. The expert presents the decision matrix $D_{\tilde{N}}$ as shown in Eq. (3.1).

Step 2. Utilize Eqs. (3.3) – (3.4) to standardize $D_{\tilde{N}}$ into $D_{\hat{S}} = \left\langle t_{\hat{S}_{ij}} \right\rangle_{m\times n}$.

Step 3. Define the ideal solution Z^*.

Step 4. Determine the angle cosine between the individual decision and the ideal decision Z^* by utilizing Eq. (3.11).

Step 5. Find the projection measure of individual decision and the ideal decision Z^* by using Eq. (3.10).

Step 6. Calculate the relative closeness ρ_i in Eq. (3.12) by combining angle cosine and projection with direction indicator ξ.

Step 7. Rank the alternatives according to the decreasing order of the relative closeness ρ_i and choose the most suitable alternative (s).

Step 8. End.

4. A numerical example

In this Section, we adapt an illustrative example from Dey et al. [13] for weaver selection in Khadi Institution where the information about attributes is expressed by linguistic variables. Consider a Khadi Institution wants to recruit two most competent weavers from a panel of three weavers h_1, h_2, h_3. Seven main attributes for weaver selection are: Skill (k_1); Previous experience (k_2); Honesty (k_3); Physical fitness (k_4); Locality of the weaver (k_5); Personality (k_6); Economic condition of the weaver (k_7) [30]. The Khadi Institution hire a Khadi expert to choose the desirable weavers based on the seven attributes. The evaluation information of an alternative h_i (i = 1, 2, 3) with respect to seven attributes are provided by the Khadi expert in terms of linguistic variables as shown in the Table 2. It is to be noted that the seven attributes are of benefit type and the weights of the attributes are calculated by using maximizing deviation method.

4.1 Method 1

The procedure for weaver selection based on weighted projection method is presented by the following steps:

Step 1: We transform the linguistic decision matrix as shown in Table 2 into interval – valued neutrosophic decision matrix by means of Table 1.

Table 2. *Linguistic decision matrix*

	k_1	k_2	k_3	k_4	k_5	k_6	k_7
h_1	G	G	VG	VG	VG	M	MG
h_2	VG	VG	MG	G	VG	MG	ML
h_3	G	VG	G	MG	G	G	G

Step 2: Then the linguistic decision matrix is transformed into interval – valued neutrosophic decision matrix by using Table 1 as given below (see Table 3).

Table 3. *Interval – valued neutrosophic decision matrix*

C=

$$\left(\begin{array}{llll}
[[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.75,0.95],[0.1,0.15],[0.1,0.2]] & [[0.75,0.95],[0.1,0.15],[0.1,0.2] \\
[[0.75,0.95],[0.1,0.15],[0.1,0.2]] & [[0.75,0.95],[0.1,0.15],[0.1,0.2]] & [[0.5,0.6],[0.2,0.25],[0.25,0.35] & [[0.6,0.75],[0.1,0.2],[0.2,0.25]] \\
[[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.75,0.95],[0.1,0.15],[0.1,0.2]] & [[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.5,0.6],[0.2,0.25],[0.25,0.35]
\end{array}\right.$$

$$\left.\begin{array}{lll}
[[0.75,0.95],[0.1,0.15],[0.1,0.2] & [[0.4,0.5],[0.2,0.3],[0.35,0.45]] & [[0.5,0.6],[[0..2,0.25],[0.25,0..35]] \\
[[0.75,0.95],[0.1,0.15],[0.1,0.2]] & [[0.5,0.6],[0.2,0.25],[0.25,0.35]] & [[0.3,0.4],[0.15,0.2],[0.45,0.55] \\
[[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.6,0.75],[0.1,0.2],[0.2,0.25]] & [[0.6,0.75],[0.1,0.2],[0.2,0.25]]
\end{array}\right)$$

Step 3: We employ Euclidean distance measure to get $\Re(t_{ij}, t_{sj})$, i = t = 1, 2, …, m; j = 1, 2, …, n and the normalized weights of the attributes are obtained as given below.

$w_1 = w_2 = 0.096$, $w_3 = w_4 = 0.176$, $w_5 = 0.096$, $w_6 = 0.151$, $w_7 = 0.207$ such that $\sum_{j=1}^{7} w_j = 1$, $w_j \geq 0$, j = 1, 2, …, 7.

Step 4: The virtual interval - valued neutrosophic ideal solution are obtained as given below.

$\mu_1^+ = ([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])$; $\mu_2^+ = ([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])$; $\mu_3^+ = ([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])$; $\mu_4^+ = ([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])$; $\mu_5^+ = ([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])$; $\mu_6^+ = ([0.6, 0.75], [0.1, 0.2], [0.2, 0.25])$; $\mu_7^+ = ([0.6, 0.75], [0.1, 0.2], [0.2, 0.25])$.

Step 5: The weighted projection $Proj_{\,w}(h_i)_{Z^*}$ of the alternative h_i (i = 1, 2, 3) on Z^* is calculated as follows:

$Proj_{\,w}(h_1)_{Z^*} = 0.4255$, $Proj_{\,w}(h_2)_{Z^*} = 0.3730$, $Proj_{\,w}(h_3)_{Z^*} = 0.3972$.

Step 6: We rank the alternatives (weavers) according to the descending order of $Proj_{\,w}(h_i)_{Z^*}$ (i = 1, 2, 3). Here, we observe that

$$Proj_{\,w}(h_1)_{Z^*} > Proj_{\,w}(h_3)_{Z^*} > Proj_{\,w}(h_2)_{Z^*}$$

Consequently, h_1, h_3 are the most desirable alternatives for the Khadi Institution.

Note 1: We now compare our proposed weighted projection method with the methods investigated by Ye [25], Dey et al. [13], and Chi and Liu [7] and the obtained results are presented in the Table below.

Table Results of different measure methods

Method	Measure value	Ranking order
Proposed method	Proj $_w(h_1)_{Z^*}$ = 0.4255, Proj $_w(h_2)_{Z^*}$ = 0.3730, Proj $_w(h_3)_{Z^*}$ = 0.3972	$h_1 > h_3 > h_2$
Ye [25]	Proj $(h_1)_{Z^*}$ = 2.87, Proj $(h_2)_{Z^*}$ = 2.777, Proj $(h_3)_{Z^*}$ = 2.739	$h_1 > h_2 > h_3$
Dey et al. [13]	R_1 = 0.077209, R_2 = 0.056516, R_3 = 0.056571	$h_1 > h_3 > h_2$
Chi and Liu [7]	RCC_1 = 0.6119, RCC_2 = 0.4231, RCC_3 = 0.4621	$h_1 > h_3 > h_2$

4.2 Method 2

The procedure to get most desirable weaver(s) based on the combination of angle cosine and projection method is described by the following steps:

Step 1: Same as Step 1 of Method 1.
Step 2: Same as Step 2 of Method 1.
Step 3: Same as Step 3 of method 1.
Step 4: Same as Step 4 of method 1.
Step 5: The angle cosine between the alternative h_i (i = 1, 2, 3) and the ideal solution Z^* is calculated using Eq. (3.11) as given below.

$$Cos\,(h_1, Z^*) = 0.981,\ Cos\,(h_1, Z^*) = 0.962,\ Cos\,(h_1, Z^*) = 0.98.$$

Step 6: The projection measure between the alternative h_i (i = 1, 2, 3) and the ideal solution Z^* is calculated as follows.

$$Proj\ (h_1)_{Z^*} = 2.87,\ Proj\ (h_2)_{Z^*} = 2.777,\ Proj\ (h_3)_{Z^*} = 2.739.$$

Step 7. Combining angle cosine and projection measure with direction indicator ξ = 0.5, the relative closeness ρ_i (i = 1, 2, 3) is obtained as

$$\rho_1 = 1.926,\ \rho_2 = 1.87,\ \rho_3 = 1.86.$$

Step 8: The ranking order of the alternatives (weavers) is obtained as given below.

$$\rho_1 > \rho_2 > \rho_3$$

Therefore, h_1, h_2 are the most desirable weavers for Khadi Institution.

Note 2: However, if we take different direction indicators, the ranking order of the alternatives are obtained as given in Table 5.

Table 5. *Ranking order of the alternatives based on different direction indicators*

Alternative	$\xi = 0$		$\xi = 0.25$		$\xi = 0.5$		$\xi = 0.75$		$\xi = 1$	
	ρ_i	Ranking	ρ_i	Ranking	ρ_i	Ranking	ρ_i	Ranking	ρ_i	Ranking
h_1	2.870	1	2.398	1	1.926	1	1.453	1	0.981	1
h_2	2.777	2	2.323	2	1.870	2	1.416	3	0.962	3
h_3	2.739	3	2.299	3	1.860	3	1.420	2	0.980	2

5. Conclusion

The paper is devoted to propose two new models for MADM problems with interval – valued neutrosophic information. In the decision making process, the rating of alternatives with respect to attributes are described by linguistic variables that can be represented by IVNNs. Since the weights of the attributes are fully unknown to the expert, we use maximization deviation method to find them. Then, we determine interval - valued neutrosophic ideal solutions. Finally, we develop weighted projection method to rank the alternatives. In this paper, we also propose an algorithm for MADM problems under interval neutrosophic environment via angle cosine and projection method. An illustrative example for weaver selection is solved to demonstrate the applicability of the proposed models. We also compare the obtained results with other existing approaches. In future, we will extend the concept to solve multi-attribute group decision making problems with interval – valued neutrosophic assessment. The authors hope that the proposed approach can be effective for dealing with diverse practical problems such as medical diagnosis, pattern recognition, management system, school choice, teacher selection, etc.

References

1. F. Smarandache, A unifying field of logics. Neutrosophy: neutrosophic probability, set and logic, Rehoboth, American Research Press, 1998.
2. F. Smarandache, Linguistic paradoxes and tautologies, Libertas Mathematica, University of Texas at Arlington, IX (1999): 143-154.
3. F. Smarandache, Neutrosophic set – a generalization of intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics 24(3) (2005): 287-297.
4. F. Smarandache, Neutrosophic set – a generalization of intuitionistic fuzzy set, Journal of Defence Resources Management 1(1) (2010): 107-116.
5. H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure 4 (2010): 410-413.
6. H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman, Interval Neutrosophic Sets and Logic, Hexis, Arizona, 2005.
7. P. Chi, and P. Liu, An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set, Neutrosophic Sets and Systems 1 (2013): 63-70.
8. H. Zhang, J. Wang, and X. Chen, Interval neutrosophic sets and their application in multicriteria decision making problem, The Scientific World Journal 2014, Article ID 645953, 15 pages, http://dx.doi.org/10.1155/2014/645953 .
9. S. Broumi, and F. Smarandache, Cosine similarity measure of interval neutrosophic sets, Neutrosophic Sets and Systems 5 (2014): 15-21.

10. J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision making, Journal of Intelligent & Fuzzy Systems 26 (2014): 165-172.
11. R. Sahin, and P. Liu, Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information, Neural Computing and Applications (2015), DOI: 10.1007/s00521-015-1995-8.
12. S. Pramanik, and K. Mondal, Interval neutrosophic multi-attribute decision – making based on grey relational analysis, Neutrosophic Sets and Systems 9 (2015): 13-22.
13. P.P. Dey, S. Pramanik, and B.C. Giri, An extended grey relational analysis based interval neutrosophic multi-attribute decision making for weaver selection, Journal of New Theory (9) (2015): 82-93.
14. S. Pramanik, and K. Mondal, Decision making based on some similarity measure under interval rough neutrosophic environment, Neutrosophic Sets and Systems 10 (2015): 46-57.
15. P.P. Dey, S. Pramanik, and B.C. Giri, An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting, Neutrosophic Sets and Systems 11 (2016): 21-30.
16. Z.S. Xu, and Q. L. Da, Projection method for uncertain multi-attribute decision making with preference information on alternatives, International Journal of Information Technology & Decision Making 3 (2004): 429-434.
17. Z. Xu, and H. Hu, Projection models for intuitionistic fuzzy multiple attribute decision making, International Journal of Information Technology & Decision Making 9(2) (2010): 267-280.
18. S. Zeng, T. Baležentis, J. Chen and G. Luo, A projection method for multiple attribute group decision making with intuitionistic fuzzy information, Informatica 24(3) (2013): 485-503.
19. Z. Yue, Approach to group decision making based on determining the weights of experts by using projection method, Applied Mathematical Modelling 36 (2012): 2900-2910.
20. Z. Yue, Application of the projection method to determine weights of decision makers for group decision making, Scientia Iranica 19(3) (2012): 872-878.
21. Z. Yue, An intuitionistic fuzzy projection-based approach for partner selection, Applied Mathematical Modelling 37 (2013): 9538-9551.
22. Y. Ju, and A. Wang, Projection method for multiple criteria group decision making with incomplete weight information in linguistic setting, Applied Mathematical Modelling 37 (2013): 9031-9040.
23. Q. Yang and P.A. Du, A straightforward approach for determining the weights of decision makers based on angle cosine and projection method, International Journal of Social, Behavioral, Educational, Economic, Business and Industrial Engineering 9(10) (2015): 3127-3133.
24. J. Ye, Simplified neutrosophic harmonic averaging projection-based method for multiple attribute decision-making problems, International Journal of Machine Learning & Cybernatics (2015), DOI: 10.1007/s13042-015-0456-0.
25. J. Ye, Interval neutrosophic multiple attribute decision – making method with credibility information, International Journal of Fuzzy Systems (2016), DOI: 10.1007/s40815-015-0122-4.
26. Z.S Xu, Theory method and Applications for multiple attribute decision-making with uncertainty, Tsinghua University Press, Beijing, 2004.
27. J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, International Journal of Fuzzy Systems 16(2) (2014): 204-211.
28. P. Majumder, and S.K. Samanta, On similarity and entropy of neutrosophic sets, Journal of Intelligent and Fuzzy Systems (2013), DOI: 10.3233/IFS-130810.
29. Y.M. Wang, Using the method of maximizing deviations to make decision for multi-indices, System Engineering and Electronics 7 (1998) 24-31.
30. P.P. Dey, S. Pramanik, and B.C. Giri, Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi Institution. Journal of Applied Quantitative Methods 10(4) (2015): 1-14.