NSS

Filters via Neutrosophic Crisp Sets

A. A. Salama* & Florentin Smarandache**

*Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University, Egypt, E-mail: drsalama44@gmail.com
**University of New Mexico, Math & Science Division, 705 Gurley Ave., Gallup, NM 87301, USA, E-mail: smarand@unm.edu

Abstract. In this paper we introduce the notion of filter on the neutrosophic crisp set, then we consider a generalization of the filter's studies. Afterwards, we present the important neutrosophic crisp filters. We also study several relations between different neutrosophic crisp filters and neutrosophic topologies. Possible applications to database systems are touched upon.

Keywords: Filters; Neutrosophic Sets; Neutrosophic crisp filters; Neutrosophic Topology; Neutrosophic Crisp Ultra Filters; Neutrosophic Crisp Sets.

1 Introduction

The fundamental concept of neutrosophic set, introduced by Smarandache in [6, 7, 8] and studied by Salama in [1, 2, 3, 4, 5, 9, 10], provides a groundwork to mathematically act towards the neutrosophic phenomena which exists pervasively in our real world and expand to building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts, such as the neutrosophic crisp set theory.

2 Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [6, 7, 8] and Salama et al. [1, 2, 3, 4, 5, 9, 10]. Smarandache introduced the neutrosophic components T, I, and F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0,1^+[$ is the non- standard unit interval.

3 Neutrosophic Crisp Filters

3.1 Definition 1

First we recall that a neutrosophic crisp set A is an object of the form A = <A_1, A_2, A_3>, where A_1, A_2, A_3 are subsets of X, and

$$A_1 \cap A_2 = \phi, A_1 \cap A_3 = \phi, A_2 \cap A_3 = \phi.$$

Let Ψ be a neutrosophic crisp set in the set X. We call Ψ a neutrosophic crisp filter on X if it satisfies the following conditions:

(N_1) Every neutrosophic crisp set in X, containing a member of Ψ , belongs to Ψ .

(N_2) Every finite intersection of members of Ψ belongs to Ψ .

(N_3) ϕ_N is not in Ψ .

In this case, the pair (X, Ψ) is neutrosophically filtered by Ψ .

It follows from (N_2) and (N_3) that every finite intersection of members of Ψ is not ϕ_N (not empty). We obtain the following results.

3.2 Proposition 1

The conditions (N_2) and *(N₁)* are equivalent to the following two conditions:

(N_{2a}) The intersection of two members of Ψ belongs to Ψ .

(N_{1a}) X_N belongs to Ψ .

3.3 Proposition 1.2

Let Ψ be a non-empty neutrosophic subsets in X satisfying (N_1) .

Then,

(1) $X_N \in \Psi$ iff $\Psi \neq \phi_N$;

(2) $\phi_N \notin \Psi$ iff $\Psi \neq$ all neutrosophic crisp subsets of X.

From the above Propositions (1) and (2),   we can characterize the concept of neutrosophic crisp filter.

3.4 Theorem 1.1

Let Ψ be a neutrosophic crisp subsets in a set X. Then Ψ is neutrosophic crisp filter on X, if and only if it satisfies the following conditions:

(i)   Every neutrosophic crisp set in X, containing a member of Ψ , belongs to Ψ .

(ii) If $A, B \in \Psi$, then $A \cap B \in \Psi$.

(iii) $\Psi^X \neq \Psi \neq \phi_N$.

Proof: It's clear.

3.5 Theorem 1.2

Let $X \neq \phi$. Then the set $\{X_N\}$ is a neutrosophic crisp filter on X. Moreover if A is a non-empty neutrosophic crisp set in X, then $\{B \in \Psi^X : A \subseteq B\}$ is a neutrosophic crisp filter on X.

Proof: Let $N = \{B \in \Psi^X : A \subseteq B\}$. Since $X_N \in \Psi$ and $\phi_N \notin \Psi$, $\phi_N \neq \Psi \neq \Psi^X$. Suppose $U, V \in \Psi$, then $A \subseteq U, A \subseteq V$. Thus $A_1 \subseteq U_1 \cap V_1$, $A_2 \subseteq U_2 \cap V_2$ or $A_2 \subseteq U_2 \cup V_2$, and $A_3 \subseteq U_3 \cup V_3$ for all $x \in X$. So $A \subseteq U \cap V$ and hence $U \cap V \in N$.

4 Comparison of Neutrosophic Crisp Filters

4.1 Definition 2

Let Ψ_1 and Ψ_2 be two neutrosophic crisp filters on a set X. We say that Ψ_2 is finer than Ψ_1, or Ψ_1 is coarser than Ψ_2, if $\Psi_1 \subset \Psi_2$.

If also $\Psi_1 \neq \Psi_2$, then we say that Ψ_2 is strictly finer than Ψ_1, or Ψ_1 is strictly coarser than Ψ_2.

We say that two neutrosophic crisp filters are comparable if one is finer than the other. The set of all neutrosophic crisp filters on X is ordered by the relation: Ψ_1 coarser than Ψ_2, this relation inducing the inclusion relation in Ψ^X .

4.2 Proposition 2

Let $(\Psi_j)_{j \in J}$ be any non-empty family of neutrosophic crisp filters on X. Then $\Psi = \cap_{j \in J} \Psi_j$ is a neutrosophic crisp filter on X. In fact, Ψ is the greatest lower bound of the neutrosophic crisp set $(\Psi_j)_{j \in J}$ in the ordered set of all neutrosophic crisp filters on X.

4.3 Remark 2

The neutrosophic crisp filter induced by the single neutrosophic set X_N is the smallest element of the ordered set of all neutrosophic crisp filters on X.

4.4 Theorem 2

Let A be a neutrosophic set in X. Then there exists a neutrosophic filter $\Psi(A)$ on X containing A if for any given finite subset $\{S_1, S_2, ..., S_n\}$ of A, the intersection $\cap_{i=1} S_i \neq \phi_N$. In fact $\Psi(A)$ is the coarsest neutrosophic crisp filter containing A .

Proof ($\Rightarrow$) Suppose there exists a neutrosophic filter $\Psi(A)$ on X containing A . Let B be the set of all finite intersections of members of A . Then by axiom (N_2), $B \subset \Psi(A)$. By axiom (N_3), $\phi_N \notin \Psi(A)$. Thus for each member B of B, we get that the necessary condition holds

($\Leftarrow$) Suppose the necessary condition holds.

Let $\Psi(A) = \{A \in \Psi^X : A$ contains a member of B$\}$, where B is the family of all finite intersections of members of A. Then we can easily check that $\Psi(A)$ satisfies the conditions in Definition 1. We say that the neutrosophic crisp filter $\Psi(A)$ defined above is generated by A, and A is called a *sub-base* of $\Psi(A)$.

4.5 Corollary 2.1

Let Ψ be a neutrosophic crisp filter in a set X, and A a neutrosophic set. Then there is a neutrosophic crisp filter Ψ' which is finer than Ψ and such that $A \in \Psi'$ if and A is a neutrosophic set. Then there is a neutrosophic crisp filter Ψ' which is finer than Ψ and such that $A \in \Psi'$ iff $A \cap U \neq \phi_N$ for each $U \in \Psi$.

4.6 Corollary 2.2

A set φ_N of a neutrosophic crisp filter on a non-empty set X, has a least upper bound in the set of all neutrosophic crisp filters on X if for all finite sequence $(\Psi_j)_{j \in J}, 0 \leq j \leq n$ of elements of φ_N and all $A_j \in \Psi_j (1 \leq j \leq n)$, $\cap_{j=1} A_j \neq \phi_N$.

4.7 Corollary 2.3

The ordered set of all neutrosophic crisp filters on a non-empty set X is inductive.

If Λ is a sub-base of a neutrosophic filter N on X, then Ψ is not in general the set of neutrosophic sets in X containing an element of Λ; for Λ to have this property it is necessary and sufficient that every finite intersection of members of Λ should contain an element of Λ. Hence, we have the following results.

4.8 Theorem 3

Let β be a set of neutrosophic crisp sets on a set X. Then the set of neutrosophic crisp sets in X containing an element of β is a neutrosophic crisp filter on X if β possesses the following two conditions:

(β_1) The intersection of two members of β contain a member of β .

(β_2) $\beta \neq \phi_N$ and $\phi_N \notin \beta$.

4.9 Definition 3

Let Λ and β be two neutrosophic sets on X satisfying conditions (β_1) and (β_2) . We call them bases of neutrosophic crisp filters they generate. We consider two neutrosophic bases equivalent, if they generate the same neutrosophic crisp filter.

4.10 Remark 3

Let Λ be a sub-base of neutrosophic filter Ψ . Then the set β of finite intersections of members of Λ is a base of a neutrosophic filter Ψ .

4.11 Proposition 3.1

A subset β of a neutrosophic crisp filter Ψ on X is a base of Ψ if every member of Ψ contains a member of β .

Proof $(\Rightarrow)$ Suppose β is a base of N . Then clearly, every member of Ψ contains an element of β . $(\Leftarrow)$ Suppose the necessary condition holds. Then the set of neutrosophic sets in X containing a member of β coincides with Ψ by reason of $(\Psi_j)_{j \in J}$.

4.12 Proposition 3.2

On a set X, a neutrosophic crisp filter Ψ' with base β' is finer than a neutrosophic crisp filter Ψ with base β if every member of β contains a member of β' .

Proof: This is an immediate consequence of Definitions 2 and 3.

4.13 Proposition 3.3

Two neutrosophic crisp filters bases β and β' on a set X are equivalent if every member of β contains a member of β' and every member of β' and every member of β' contains a member of β .

5 Neutrosophic Crisp Ultrafilters

5.1 Definition 4

A neutrosophic ultrafilter on a set X is a neutrosophic crisp filter Ψ such that there is no neutrosophic crisp filter on X which is strictly finer than Ψ (in other words, a maximal element in the ordered set of all neutrosophic crisp filters on X).

Since the ordered set of all neutrosophic crisp filters on X is inductive, Zorn's lemma shows that:

5.2 Theorem 4

Let Ψ be any neutrosophic ultrafilter on a set X; then there is a neutrosophic ultrafilter other than Ψ .

5.3 Proposition 4

Let Ψ be a neutrosophic ultrafilter on a set X. If A and B are two neutrosophic subsets such that $A \cup B \in \Psi$, then $A \in \Psi$ or $B \in \Psi$.

Proof: Suppose not. Then there are neutrosophic sets A and B in X such that $A \notin \Psi, B \notin \Psi$ and $A \cup B \in \Psi$ Let $\Lambda = \{M \in \Psi^X : A \cup M \in \Psi\}$. It is straightforward to check that Λ is a neutrosophic crisp filter on X, and Λ is strictly finer than Ψ , since $B \in \Lambda$ .This contradiction proves the hypothesis that Ψ is a neutrosophic crisp ultrafilter.

5.4 Corollary 4

Let Ψ be a neutrosophic crisp ultrafilter on a set X and let $(\Psi_j)_{1 \leq j \leq n}$ be a finite sequence of neutrosophic crisp sets in X. If $\bigcup_{j=1} \Psi_j \in \Psi$, then at least one of the Ψ_j belongs to Ψ .

5.5 Definition 5

Let A be a neutrosophic crisp set in a set X. If U is any neutrosophic crisp set in X, then the neutrosophic crisp set $A \cap U$ is called trace of U on A, and it is denoted by U_A . For all neutrosophic crisp sets U and V in X, we have $(U \cap V)_A = U_A \cap V_A$.

5.6 Definition 6

Let A be a neutrosophic crisp set in a set X. Then the set Λ_A of traces $A \in \Psi^X$ of members of Λ is called the trace of Λ on A .

5.7 Proposition 5

Let Ψ be a neutrosophic crisp filter on a set X and $A \in \Psi^X$. Then the trace Ψ_A of Ψ on A is a neutrosophic crisp filter if each member of Ψ intersects with A .

Proof: The result in Definition 6 shows that Ψ_A satisfies (N_2) . If $M \cap A \subset P \subset A$, then $P = (M \cup P) \cap A$. Thus Ψ_A satisfies (N_1) . Hence Ψ_A is a neutrosophic crisp filter if it satisfies (N_3) , i.e. if each member of Ψ intersects with A .

5.8 Definition 7

Let Ψ be a neutrosophic crisp filter on a set X and $A \in \Psi^X$. If the trace is Ψ_A of Ψ on A, then Ψ_A is said to be induced by Ψ and A.

5.9 Proposition 6

Let Ψ be a neutrosophic crisp filter on a set X inducing a neutrosophic filter N_A on $A \in \Psi^X$. Then the trace β_A on A of a base β of Ψ is a base of Ψ_A.

References

[1] A.A. Salama and S.A. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces", in Journal computer Sci. Engineering, Vol. (2) No. (7) (2012).

[2] A.A. Salama and S.A. Albolwi, "Neutrosophic set and Neutrosophic topological space", in ISORJ. Mathematics, Vol.(3), Issue(4), pp-31-35 (2012).

[3] A.A. Salama and S.A. Albalwi, "Intuitionistic Fuzzy Ideals Topological Spaces", Advances in Fuzzy Mathematics, Vol.(7), Number 1, pp. 51- 60, (2012).

[4] A.A.Salama, and H.Elagamy, "Neutrosophic Filters", in International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), Vol.3, Issue(1),Mar 2013, pp 307-312 (2013).

[5] S. A. Albowi, A. A. Salama & Mohmed Eisa, "New Concepts of Neutrosophic Sets", in International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol.3, Issue 4, Oct 2013, 95-102 (2013).

[6] Florentin Smarandache, "Neutrosophy and Neutrosophic Logic", First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA (2002).

[7] Florentin Smarandache, "An introduction to the Neutrosophy probability applied in Quantum Physics", International Conference on Introduction Neutrosophic Physics, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA 2-4 December (2011).

[8] F. Smarandache. "A Unifying Field in Logics: Neutrosophic Logic". Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.

[9] I. Hanafy, A.A. Salama and K. Mahfouz, "Correlation of Neutrosophic Data", in International Refereed Journal of Engineering and Science (IRJES) , Vol.(1), Issue 2 PP.39-43 (2012).

[10] I.M. Hanafy, A.A. Salama and K.M. Mahfouz, "Neutrosophic Crisp Events and Its Probability", in International Journal of Mathematics and Computer Applications Research (IJMCAR) Vol. (3), Issue 1, Mar 2013, pp 171-178 (2013).