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Fusion of ESM allegiance reports using DSmT

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Abstract: *Electronic Support Measures consist of passive receivers which can identify emitters coming from a small bearing angle, which, in turn, can be related to platforms that belong to 3 classes: either Friend, Neutral, or Hostile. Decision makers prefer results presented in STANAG 1241 allegiance form, which adds 2 new classes: Assumed Friend, and Suspect. Dezert-Smarandache theory (DSmT) is particularly suited to this problem, since it allows for intersections between the original 3 classes. In this way, an intersection of Friend and Neutral can lead to an Assumed Friend, and an intersection of Hostile and Neutral can lead to a Suspect. Results are presented showing that the theory can be successfully applied to the problem of associating ESM reports to established tracks, and its results identify when miss-associations have occurred and to what extent. Results are also compared to Dempster-Shafer theory (DST) which can only reason on the original 3 classes. Thus decision makers are offered STANAG 1241 allegiance results in a timely manner, with quick allegiance change when appropriate and stability in allegiance declaration otherwise.*

19.1 Background

Electronic Support Measures (ESM) consists of passive receivers which can identify emitters coming from a small bearing angle, but cannot determine range (although some are in development to provide a rough measure of range). The detected emitters can be related to platforms that belong to 3 classes: either Friend ($F = 1$), Neutral ($N = 2$) or Hostile ($H = 3$), heretofore called ESM-allegiance, within that bearing angle.

In the case of dense targets, ESM allegiance can fluctuate wildly due to miss-associations of an ESM report to established track. Hence, decision makers would like the target platforms to be identified on a more refined basis, belonging to 5 classes: Hostile (or Foe), Suspect (S), Neutral, Assumed Friend (AF), and Friend, since they realize that no fusion algorithm can be perfect and would prefer some stability in an allegiance declaration, rather than oscillations between extremes. This will heretofore be referred to as STANAG 1241 allegiance (or STANAG-allegiance for short).

With this more refined STANAG-allegiance, a decision maker would probably take no aggressive action against either a friend or an assumed friend (although he would monitor an assumed friend more closely). Similarly a decision maker would probably take aggressive action against a foe and send a reconnaissance force (or a warning salvo) towards a suspect. Neutral platforms would correspond to countries not involved in the current conflict, or to commercial airliners.

All incoming sensor declarations correspond to a frame of discernment of 3 classes, and several theories exist to treat a series of such declarations to obtain a fused result in the same frame of discernment, like Bayesian reasoning and Dempster-Shafer (DS) reasoning (often called evidence theory). However, when the output frame of discernment is larger that the input frame of discernment, an interpretation has to be made as to what this could mean, or how that could be generated. This is the subject of the next sub-section.

19.1.1 An interpretation of STANAG 1241

Both Bayes and Dempster-Shafer assume that the universe of discourse remains fixed (at 3 singletons “Hostile”, “Neutral”, and “Friend”), and is the same for the input declarations and the fused output results, after repeated use of their respective combining rules.

However, there exists a new theory called Dezert-Smarandache theory which can coherently, with well-defined fusion rules, lead to an output amongst 5 classes, even though the input classes number only 3, because the theory allows for intersections. For example, “Suspect” might be the result obtained after fusing “Hostile” with “Neutral” (although other possibilities also exist), and “Assumed Friend” might be the result obtained after fusing “Friend” with “Neutral” (although again other possibilities also exist).

This is illustrated in the Venn diagram of Figure 19.1.

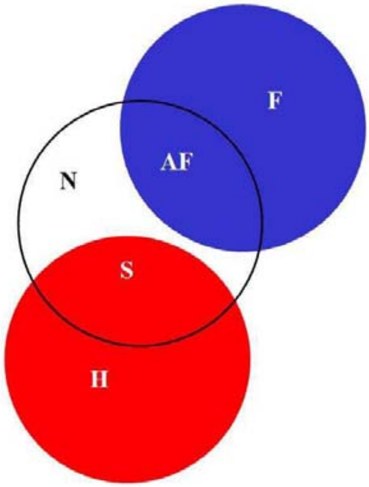

Figure 19.1: Venn diagram for the STANAG allegiances.

19.1.2 Another interpretation of STANAG 1241

The interpretation in the preceding sub-section is a conservative one, namely that there is only one easy way to become suspect. This could correspond to a decision maker being in a non-threatening situation due to the choice of mission, e.g. peace-keeping. There could be situations where there is a need for a more aggressive response. In the case of a combat mission for example, the appropriate Venn diagram might be the one of Figure 19.2, where there are many more ways to become suspect, namely all the intersections bordering Hostile.

Note that for Figure 19.1, the intersection of Friend $= \theta_1$ and Hostile $= \theta_3$ is empty (i.e. not allowed, or $\theta_1 \cap \theta_3 = \emptyset$, the null set), and this corresponds to an interesting constraint situation in Dezert-Smarandache theory, as we shall see. It also corresponds to a more likely mission for Canadian Forces (CF), namely peace-keeping, or general surveillance.

On the other hand, Figure 19.2 corresponds to a combat situation more appropriate for the U.S.A., or to the CF as long as they play an active role in the Kandahar region of Afghanistan. For these reasons, and also because all of the features of Dezert-Smarandache theory will be exercised, without the additional complexity of keeping all the intersections of Figure 19.2, the situation of Figure 19.1 will correspond to the one implemented in this chapter.

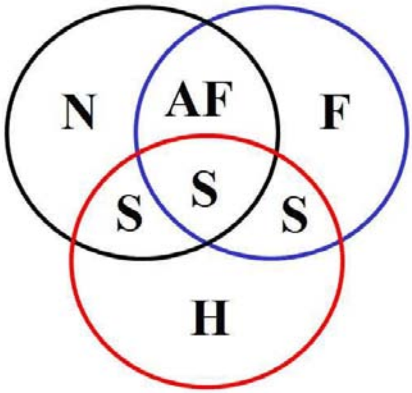

Figure 19.2: Another possible Venn diagram for the STANAG allegiances.

19.2 Dezert-Smarandache theory

19.2.1 Formulae for DST and DSmT

Since Dempster-Shafer (DS) Theory (DST) has been in use for over 40 years, the reader is assumed to be familiar with it. Only a brief review will be provided here, in order to stress the difference between it and Dezert-Smarandache (DSm) Theory (DSmT). DSmT encompasses DST as a special case, namely when all intersections are null. Both use the language of masses assigned to each declaration from a sensor (in our case the ESM sensor). A declaration is a set made up of singletons of the frame of discernment Θ , and all sets that can be made from them through unions are allowed (this is referred to as the power set 2^Θ). In DSmT, all unions and intersections are allowed for a declaration, thus forming the much larger hyper-power set D^Θ. For our special case of cardinality 3, $\Theta = \{\theta_1, \theta_2, \theta_3\}$, with $|\Theta| = 3$, D^Θ is still of manageable size:

$$\begin{aligned} D^\Theta\,(|\Theta| = 3) \equiv \{ & \{\emptyset\}, \{\theta_1\}, \{\theta_2\}, \{\theta_3\}, \{\theta_1 \cup \theta_2\}, \{\theta_1 \cup \theta_3\}, \{\theta_2 \cup \theta_3\}, \\ & \{\theta_1 \cap \theta_2\}, \{\theta_1 \cap \theta_3\}, \{\theta_2 \cap \theta_3\}, \{(\theta_1 \cup \theta_2) \cap \theta_3\}, \{(\theta_1 \cup \theta_3) \cap \theta_2\}, \\ & \{(\theta_2 \cup \theta_3) \cap \theta_1\}, \{(\theta_1 \cap \theta_2) \cup \theta_3\}, (\{\theta_1 \cap \theta_3) \cup \theta_2\}, \{(\theta_2 \cap \theta_3 \cup \theta_1)\}, \\ & \{\theta_1 \cap \theta_2 \cap \theta_3\}, \{\theta_1 \cup \theta_2 \cup \theta_3\}, \{(\theta_1 \cap \theta_2) \cup (\theta_1 \cap \theta_3) \cup (\theta_2 \cap \theta_3)\}\}. \end{aligned}$$

For larger cardinalities, the hyper-power set makes computations prohibitively expensive (in CPU time) as the following table summarizes (the cardinal of D^Θ follows the Dedekind sequence, and both 2^Θ and D^Θ include the null set):

Cardinal of Θ	2	3	4	5	6
Cardinal of 2^{Θ}	4	8	16	32	64
Cardinal of D^{Θ}	5	19	167	7580	7828353

Table 19.1: Cardinalities for DST vs DSmT.

This is one of the reasons why this application is well suited to DSmT, because a low cardinality of Θ (three) generates a cardinality in DSmT which is computationally feasible (nineteen).

In DST, a combined "fused" mass is obtained by combining the previous (presumably the results of previous fusion steps) $m_1(A)$ with a new $m_2(B)$ to obtain a new fused result as follows:

$$(m_1 \oplus m_2)(C) = \frac{1}{1 - K_{\cap}} \sum_{A \cap B = C} m_1(A) m_2(B) \qquad \forall C \subseteq \Theta \tag{19.1}$$

The renormalization step using the conflict $K_{\cap}$, corresponding to the sum of all masses for which the set intersection yields the null set, is a critical feature of DST, and allows for it to be associative, whereas a multitude of alternate ways of redistributing the conflict (proposed by numerous authors) lose this property. The associativity within the DST is key when the time tags of the sensor reports are unreliable, since associative rules are impervious to a different order of reports coming in, but all others rules can be extremely sensitive to the order of reports. This is the main reason we concentrate only on DST vs. DSmT, but another reason is the ridiculous proliferation of alternatives to DST.

In DSmT, the hybrid rule (called DSmH in [4]) appropriate for constraints such as described previously (corresponding to Figure 19.1) turns out to be much more complicated:

$$m_{\mathcal{M}(\Theta)}(X) \triangleq \phi(X)\Big[S_1(X) + S_2(X) + S_3(X)\Big] \tag{19.2}$$

where all sets involved in formulas are in canonical form and where $\phi(X)$ is the characteristic non-emptiness function of a set X, i.e. $\phi(X) = 1$ if $X \notin \boldsymbol{\emptyset}$ and $\phi(X) = 0$ otherwise, where $\boldsymbol{\emptyset} \triangleq \{\boldsymbol{\emptyset}_{\mathcal{M}}, \emptyset\}$. $\boldsymbol{\emptyset}_{\mathcal{M}}$ is the set of all elements of D^{Θ} which have been forced to be empty through the constraints of the model $\mathcal{M}$ and $\emptyset$ is the classical/universal empty set. $S_1(X)$, $S_2(X)$ and $S_3(X)$ are defined by

$$S_1(X) \triangleq \sum_{\substack{X_1, X_2 \in D^{\Theta} \\ X_1 \cap X_2 = X}} \prod_{i=1}^{2} m_i(X_i) \tag{19.3}$$

$$S_2(X) \triangleq \sum_{\substack{X_1, X_2 \in \boldsymbol{\emptyset} \\ [\mathcal{U}=X] \vee [(\mathcal{U} \in \boldsymbol{\emptyset}) \wedge (X = I_t)]}} \prod_{i=1}^{2} m_i(X_i) \tag{19.4}$$

$$S_3(X) \triangleq \sum_{\substack{X_1, X_2 \in D^\Theta \\ X_1 \cup X_2 = X \\ X_1 \cap X_2 \in \boldsymbol{\emptyset}}} \prod_{i=1}^{2} m_i(X_i) \tag{19.5}$$

with $\mathcal{U} \triangleq u(X_1) \cup u(X_2) \cup \ldots \cup u(X_s)$ where $u(X)$ is the union of all θ_i that compose X and $I_t \triangleq \theta_1 \cup \theta_2 \cup \ldots \cup \theta_n$ is the total ignorance. The reader is referred to a series of books on DSmT [4, 5] for lengthy descriptions of the meaning of this formula. A three-step approach [3] is proposed in the second of these books [4, 5], which is used in this chapter. From now on, the term "hybrid" will be dropped for simplicity.

If the incoming sensor reports are in DST-space Friend ($F = 1$), Neutral ($N = 2$) or Hostile ($H = 3$), then Figure 19.1 has the interpretation in DSmT fused space (allowing intersections) is:

$$\begin{aligned}
\{\theta_1 - \theta_1 \cap \theta_2\} &= \quad \text{Friend} \\
\{\theta_3 - \theta_3 \cap \theta_2\} &= \quad \text{Hostile} \\
\{\theta_1 \cap \theta_2\} &= \quad \text{Assumed Friend} \\
\{\theta_2 \cap \theta_3\} &= \quad \text{Suspect} \\
\{\theta_2 - \theta_1 \cap \theta_2 - \theta_2 \cap \theta_3\} &= \quad \text{Neutral}
\end{aligned}$$

As expected, all STANAG-allegiances (masses assigned to the sets mentioned above) sum up to 1. Hence the first line of eq.(19.6)[1], which is the sum for all 5 considered classes of STANAG 1241, yields the second line after using the DSmT cardinality criterion (with a multiplying factor of -1 for each non-null intersection) and since $\theta_1 \cap \theta_3 = \emptyset$ by construction of Figure 19.1.

$$\begin{aligned}
&\theta_1 - \theta_1 \cap \theta_2 + \theta_3 - \theta_3 \cap \theta_2 + \theta_1 \cap \theta_2 + \theta_2 \cap \theta_3 + \theta_2 - \theta_1 \cap \theta_2 - \theta_3 \cap \theta_2 \\
&\qquad = \theta_1 + \theta_2 + \theta_3 - \theta_1 \cap \theta_2 - \theta_3 \cap \theta_2 = 1
\end{aligned} \tag{19.6}$$

19.2.2 A typical simulation scenario

In order to compare DST with DSmT, one must list the pre-requisites that the scenario must address. It must:

1. be able to adequately represent the known ground truth,

[1] In eq. (19.6), we use a concise notation for the masses, i.e. θ_1 means $m(\theta_1)$, etc.

2. contain sufficient countermeasures (or miss-associations) to be realistic and to test the robustness of the theories,
3. only provide partial knowledge about the ESM sensor declaration, which therefore contains uncertainty,
4. be able to show stability under countermeasures, yet
5. be able to switch allegiance when the ground truth does so.

The following scenario parameters have therefore been chosen accordingly:

1. Ground truth is FRIEND for the first 50 iterations of the scenario and HOSTILE for the last 50.
2. The number of correct associations is 80%, corresponding to countermeasures appearing 20% of the time, in a randomly selected sequence.
3. The ESM declaration has a mass (confidence value in Bayesian terms) of 0.7, with the rest (0.3) being assigned to the ignorance (the full set of elements, namely Θ).

Items 4 and 5 of the first list would translate into stability (item 4) for the first 50 iterations and eventual stability (item 4) for the last 50 iterations after the allegiance switch at iteration 50 (item 5).

This scenario will be the one addressed in the next section, while a Monte-Carlo study is described in the subsequent section. Each Monte-Carlo run corresponds to a different realization using the above scenario parameters, but with a different random seed.

The scenario chosen is depicted in Figure 19.3 below.

Roughly 80% of the time the ESM declares the correct allegiance according to ground truth, and the remaining 20% is roughly equally split between the other two allegiances. There is an allegiance switch at the 50^{th} iteration, and the selected randomly selected seed in the above generated scenario generates a rather unusual sequence of 4 false Friend declarations starting at iteration 76 (when actually Hostile is the ground truth), which will be very challenging for the theories.

19.3 Results for the simulated scenario

Before presenting the results for DST, it should be noted that the original form of DST tends to be overly optimistic. Given enough evidence concerning an allegiance, it will be very hard for it to change allegiances at iteration 50. This is a well-known problem, and a well-known *ad hoc* solution exists, and consists in renormalizing after each fusion step by giving a value to the complete ignorance which can never be below

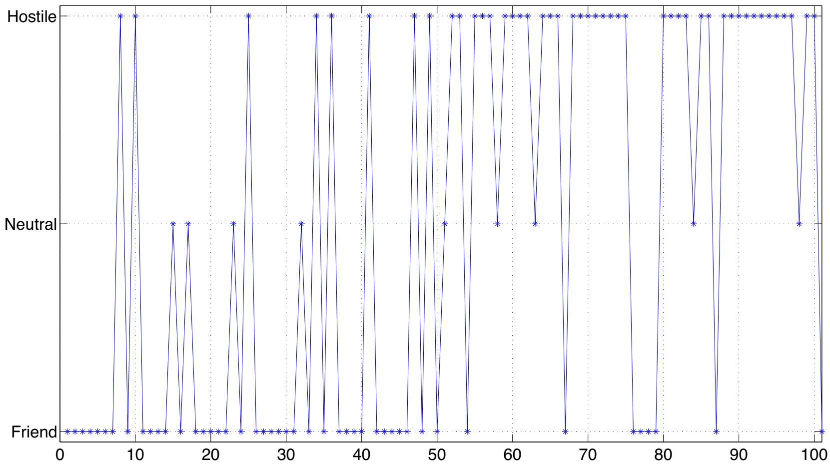

Figure 19.3: Chosen scenario.

a certain factor (chosen here to be 0.02). A comparison will be made with DSmH and the Proportional Conflict Redistribution rule number 5 (PCR5) preferred by Dezert and Smarandache.

19.3.1 DST results

The result for DST is shown in Figure 19.4 below.

DST never becomes confused, reaches the ESM-allegiance quickly and maintains it until iteration 50. It then reacts reasonably rapidly and takes about 6 reports before switching allegiance as it should. Furthermore after being confused for an iteration around the sequence of 4 Friend reports starting at iteration 76, it quickly reverts to the correct Hostile status.

Note that a decision maker could look at this curve and see an oscillation pointing to miss-associations without being able to clearly distinguish between a miss-association with the other two possible allegiances. This fairly quick reaction is due to the 0.02 assigned to the ignorance, which translates to DST never being more than 98% sure of an ESM-allegiance, as can be seen by the curve topping out at 0.98. The Figure 19.4 shows the mass, which is also the pignistic probability for this case, with the latter being normally used to make a decision.

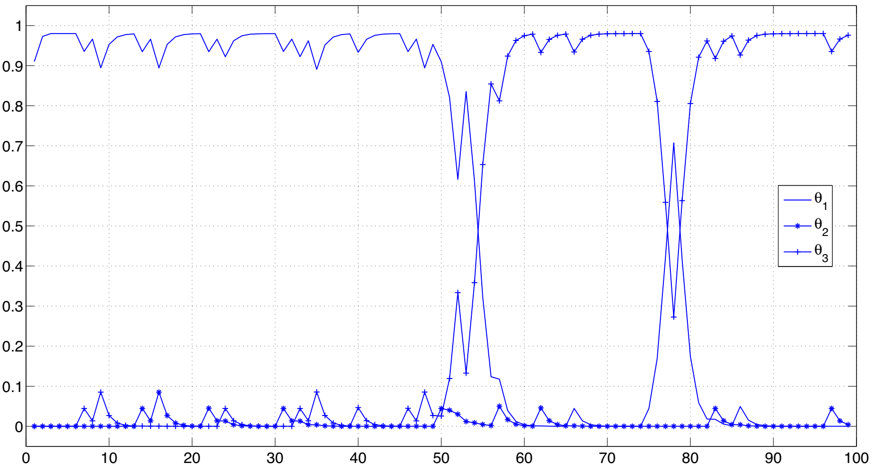

Figure 19.4: DST result for the chosen scenario. Masses in function of time.

19.3.2 DSmH results

For the hybrid rule of the DSmT, it was suggested to use the Generalized Pignistic Probability [4] in order to make a decision on a singleton belonging to the input ESM-allegiance. This seems to cause problems [1]. Since the whole idea behind using DSmT was to present the results to the decision maker in the STANAG-allegiance format, the result of Figure 19.5 would be shown to the decision maker.

The decision maker would clearly be informed that miss-associations have occurred, since Assumed Friend dominates for the first 50 iterations and Suspect for the latter 50. DSmH is more susceptible to miss-associations than DST (the dips are more pronounced), but it has the advantage of giving extra information to the decision maker, namely that the fusion algorithm is having difficulty with associating ESM reports to established tracks.

Just like DST, the 4 Friend declarations starting at iteration 76 cause confusion, as it should. The change in allegiance at iteration 50 is detected nearly as fast as DST. What is even more important is that F and AF are clearly preferred for the first 50 iterations and S and H for the last 50, as they should.

19.3.3 PCR5 results

PCR5 shows a similar behaviour, but is much less sure of what's going on (the peaks are not as pronounced), as seen in Figure 19.6. Again, F and AF are clearly preferred for the first 50 iterations and S and H for the last 50, as they should.

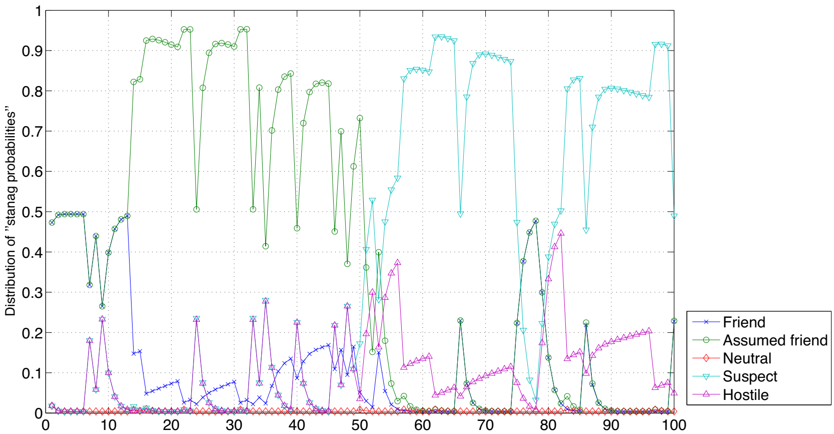

Figure 19.5: DSmH result for the chosen scenario.

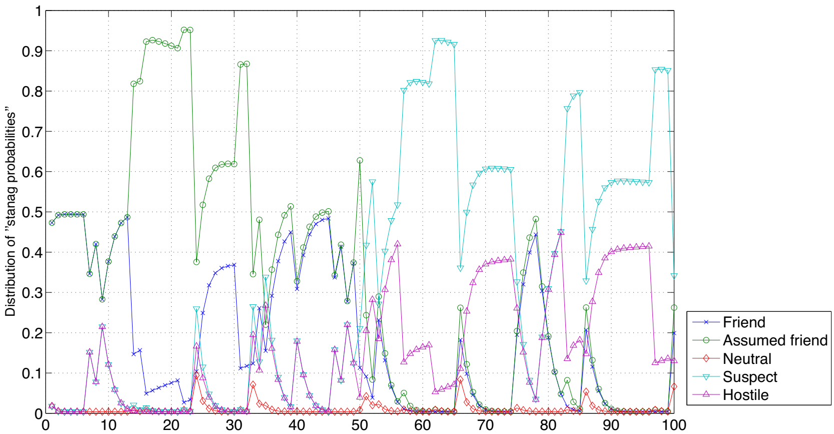

Figure 19.6: PCR5 result for the chosen scenario.

19.3.4 Decision-making threshold

Because of the sometimes oscillatory nature of some combination rules, one has to ask oneself when to make a decision or recommend one to the commander. This is illustrated in figure 19.7 for DST although the same is applicable for all the others. A threshold at a very secure 90% would result in a longer time for allegiance change, and result in a longer period of indecision around iteration 76, compared to one at 70%.

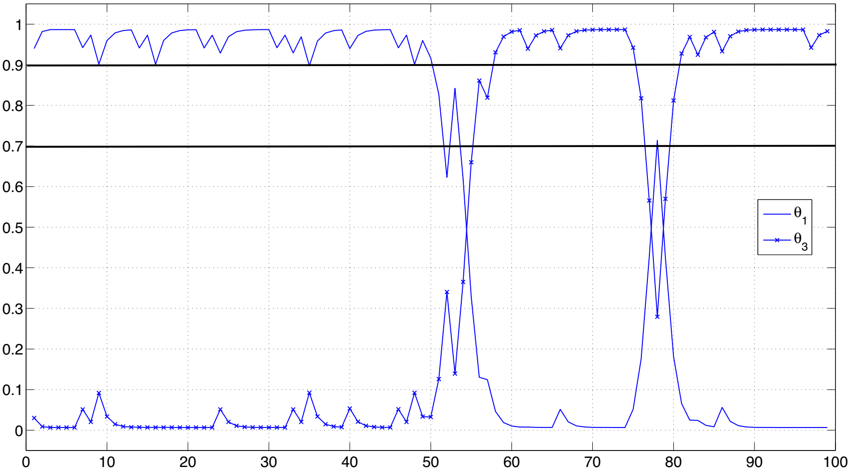

Figure 19.7: Decision thresholds. Masses in function of time.

19.4 Monte-Carlo results

Although a special case such as the one described in the previous section offers valuable insight, one might question if the conclusions from that one scenario pass the test of multiple Monte-Carlo scenarios. This question is answered in this section.

In order to sample the parameter space in a different way, the simulations below correspond to 90% correct associations (higher than the previous 80%), an ESM confidence at 60% (lower than the previous 70%) and an ignorance threshold at 0.02 as before. The number of Monte-Carlo runs was set to 100.

19.4.1 DST results

The result for DST is shown in Figure 19.8. As expected, since DST reasons over the 3 input classes, Suspect and Assumed Friend are not involved. Naturally, since Assumed Friend and Suspect do not exist in DST, these are calculated as zero. Friend, Neutral and Hostile have the expected behaviour. One sees the same response times, after an average over 100 runs, as was seen in the selected scenario of the previous section.

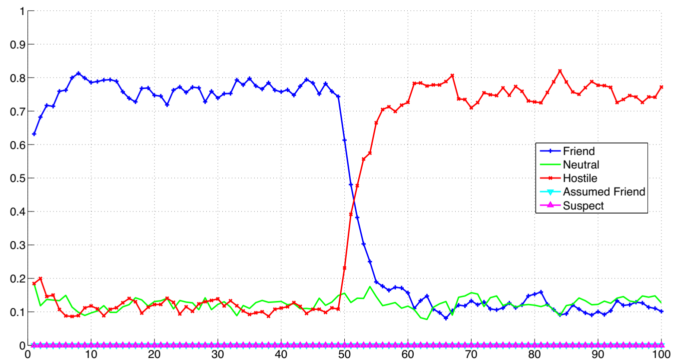

Figure 19.8: DST result after 100 Monte-Carlo runs. *Stanag* probabilities in function of time.

19.4.2 DSmH results

The similar result for DSmH is shown in Figure 19.9. In this case, AF dominates for the first 50 iteration, on average (over 100 runs) and S for the last 50, confirming that the chosen scenario was representative of the behaviour of DSmH. The response times are similar on average also. DSmH is slightly less sure (plateau at 70%) than DST (plateau at 80%), but this can be adjusted by lowering the decision threshold accordingly.

19.4.3 PCR5 results

Finally, the PCR5 result is shown in Figure 19.10. In this case also, AF dominates for the first 50 iterations, on average (over 100 runs), and S for the last 50, confirming that the chosen scenario was representative of the behaviour of PCR5. The response times are similar on average also. PCR5 is slightly less sure (plateau at 60%) than DST (plateau at 80%) or DSmH (plateau at 70%).

19.4.4 Effect of varying the ESM parameters

In order to study the effects of varying the ESM parameters, the simulations below correspond to an ESM confidence at 80% (higher than the previous 60%) and an ignorance threshold at 0.05 (higher than the 0.02 used previously). The number of

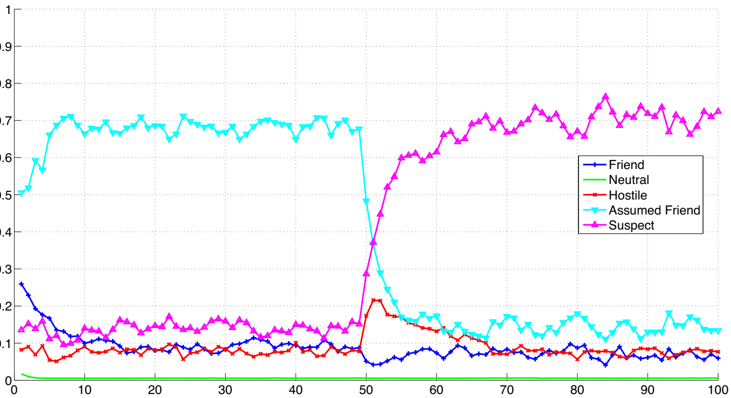

Figure 19.9: DSmH result after 100 Monte-Carlo runs. *Stanag* probabilities in function of time.

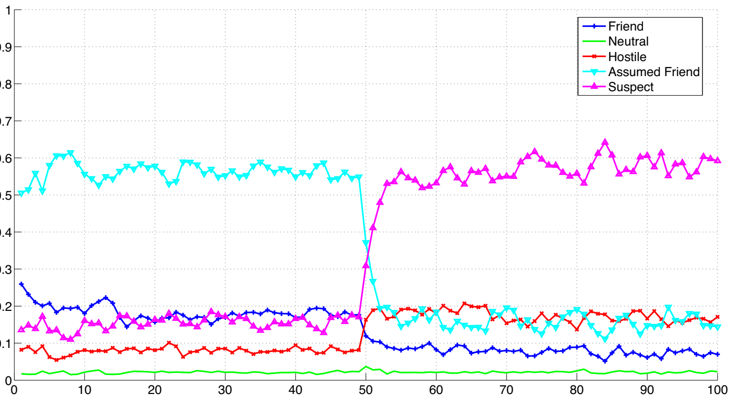

Figure 19.10: PCR5 result after 100 Monte-Carlo runs. *Stanag* probabilities in function of time.

Monte-Carlo runs was again set to 100.

A filter was also applied to the input ESM declarations over a window of 4 iterations then assigns lesser confidence to ESM reports which are not well represented in the window. The results are shown in Figure 19.11 for DST, Figure 19.12 for DSmH and Figure 19.13 for PCR5. From these figures, one can see the smoothing effect of the filter, but more importantly all of the conclusions of the previous Monte-Carlo runs, as well as the selected scenario of the previous section hold in their totality.

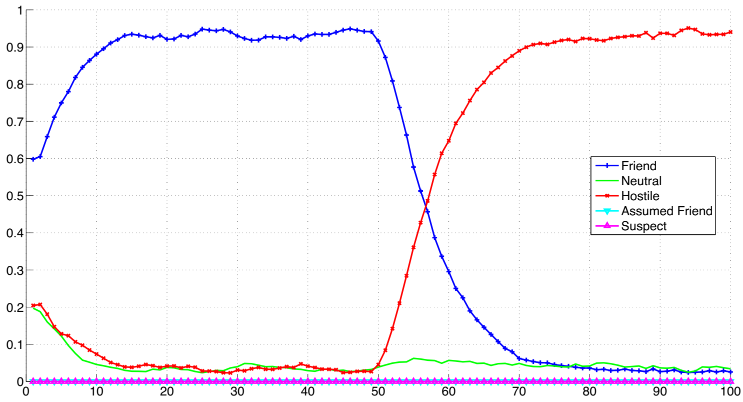

Figure 19.11: DST result after 100 Monte-Carlo runs and input filter. *Stanag* probabilities in function of time.

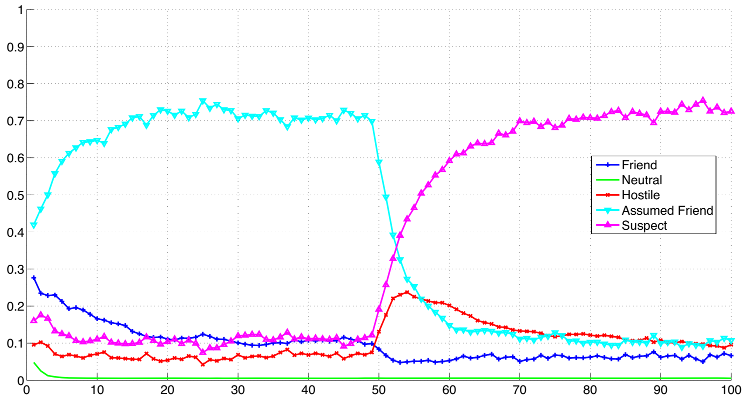

Figure 19.12: DSmH result after 100 Monte-Carlo runs and input filter. *Stanag* probabilities in function of time.

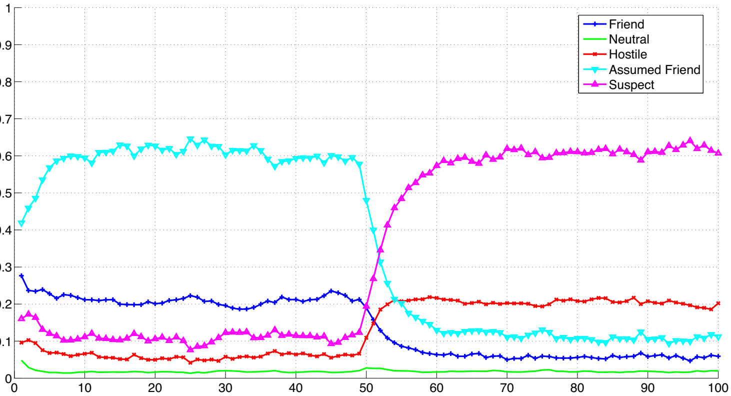

Figure 19.13: PCR5 result after 100 Monte-Carlo runs and input filter. *Stanag* probabilities in function of time.

19.5 Conclusions

Because of the nature of Electronic Support Measures which consist of passive receivers that can identify emitters coming from a small bearing angle, and which, in turn, can be related to platforms that belong to 3 classes: either Friend, Neutral, or Hostile, and to the fact that decision makers would prefer results presented in STANAG 1241 allegiance form, which adds 2 new classes: Assumed Friend, and Suspect, Dezert-Smarandache theory was used instead, but also compared to Dempster-Shafer theory. In DSmT an intersection of Friend and Neutral can lead to an Assumed Friend, and an intersection of Hostile and Neutral can lead to a Suspect. Recent results were presented showing that the theory can be successfully applied to the problem of associating ESM reports to established tracks confirming the work published in [2]. Results are also compared to Dempster-Shafer theory which can only reason on the original 3 classes. Thus decision makers are offered STANAG 1241 allegiance results in a timely manner, with quick allegiance change when appropriate and stability in allegiance declaration otherwise. In more details, results were presented for a typical scenario and for Monte-Carlo runs with the same conclusions, namely that Dempster-Shafer works well over the original 3 classes, if a minimum to the ignorance is applied. The same can be said for Dezert-Smarandache hybrid rule, and to a lesser extent for a popular Proportional Conflict Redistribution rule, but with the added benefit that Dezert-Smarandache theory identifies when miss-associations occur, and to what extent.

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