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AUTHOR
Prof. Dr. Florentin Smarandache
Prof. Dr. Muhammad Saeed
Muhammad Saqlain
Dr. Mohamed Abdel-Baset

Theory and Application of Hypersoft Set

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(Editors)

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Table of Contents

Aims and Scope

Florentin Smarandache generalize the soft set to the hypersoft set by transforming the function F into a multi-argument function. This extension reveals that the hypersoft set with neutrosophic, intuitionistic, and fuzzy set theory will be very helpful to construct a connection between alternatives and attributes. Also, the hypersoft set will reduce the complexity of the case study. The Book "Theory and Application of Hypersoft Set" focuses on theories, methods, algorithms for decision making and also applications involving neutrosophic, intuitionistic, and fuzzy information. Our goal is to develop a strong relationship with the MCDM solving techniques and to reduce the complexion in the methodologies. It is interesting that the hypersoft theory can be applied on any decision-making problem without the limitations of the selection of the values by the decision-makers. Some topics having applications in the area: Multi-criteria decision making (MCDM), Multi-criteria group decision making (MCGDM), shortest path selection, employee selection, e-learning, graph theory, medical diagnosis, probability theory, topology, and some more.

(Editors)

Chapter 1

An Inclusive Study on Fundamentals of Hypersoft Set

Muhammad Saeed[1], Atiqe Ur Rahman[2*], Muhammad Ahsan[3], Florentin Smarandache[4]

[1,2,3] University of Management and Technology, Lahore, Pakistan.

E-mail: muhammad.saeed@umt.edu.pk, E-mail: aurkhb@gmail.com, E-mail: ahsan1826@gmail.com

[4] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA. E-mail: smarand@unm.edu

Abstract: Smarandache developed hypersoft set theory as an extension of soft set theory, to adequate the existing concepts for multi-attribute function. In this study, essential elementary properties e.g. not set, subset, absolute set, and aggregation operations e.g. union, intersection, complement, AND, OR, restricted union, extended intersection, relevant complement, restricted difference, restricted symmetric difference, are characterized under hypersoft set environment with illustrated examples. New notions of relation, function and their basic properties are also discussed for hypersoft sets. Moreover, matrix representation of hypersoft set is presented along with different operations.

Keywords: *Hypersoft Set, Hypersoft Relation, Hypersoft Function, Hypersoft Matrix.*

1. Introduction

The theories like theory of probability, theory of fuzzy sets, and the interval mathematics, are considered as mathematical means to tackle many intricate problems involving various uncertainties in different fields of mathematical sciences. These theories have their own complexities which restrain them to solve these problems successfully. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. A mathematical tool is needed for dealing with uncertainties which should be free of all such impediments. In 1999, Molodtsov [1] introduced a mathematical tool called soft sets in literature as a new parameterized family of subsets of the universe of discourse. In 2003, Maji *et al.* [2] extended the concept and introduced some fundamental terminologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR and also the operations of union and intersection. They verified De Morgan's laws and a number of other results. In 2005, Pei *et al*. [3] discussed the relationship between soft sets and information systems. They showed the soft sets as a class of special information systems. In 2009, Ali *et al.* [4] pointed out several assertions in previous work of Maji et al. and proposed new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. In 2010, 2011, Babitha *et. al.* [5,6] introduced concept of soft set relation as a sub soft set of the Cartesian product of the soft sets and also discussed many related concepts such as equivalent soft set relation, partition, composition and function. In 2011, Sezgin et al. [7], Ge et al. [8], Fuli [9] gave some modifications in the work of Maji et al. and also established some new results. In 2020, Saeed et al. [10] performed an extensive inspection of the concept of soft elements and soft members in soft sets. Many researchers [11–22] developed certain hybrids with soft sets to get more generalized results for implementation in decision making and other related disciplines. In 2018, Smarandache [23] introduced the concept of hypersoft set as a generalization of soft set.

In this paper, some essential fundamentals (i.e. elementary properties, set theoretic operations, basic laws, set relations, set function and matrix representation) are conceptualized under hypersoft set environment. The rest of this article is structured as follows: Section 2 gives some basic definitions and results on hyper soft sets. Section 3 presents elementary properties of hypersoft sets. Section 4 describes set theoretic operations of hypersoft sets. Section 5 provides some basic properties, results and laws on hypersoft sets. Section 6 discusses hypersoft relations and hypersoft functions. Section 7 presents the matrix representation of hypersoft sets with some operations. Section 8 presents some hybrids of hypersoft sets and then last section 9 concludes the paper.

2. Preliminaries

Here we recall some basic terminologies regarding soft set and hypersoft set. Throughout the paper, $\mathcal{U}$ denotes the universe of discourse.

Definition 2.1. [1]
A pair (ζ_S, Λ) is called a *soft set* over $\mathcal{U}$, where $\zeta_S : \Lambda \to P(\mathcal{U})$ and Λ be a set of attributes of $\mathcal{U}$.

Definition 2.2. [2]
A soft set (ζ_{S_1}, Λ_1) is a *soft subset* of another soft set (ζ_{S_2}, Λ_2) if

(i) $\Lambda_1 \subseteq \Lambda_2$, and
(ii) $\zeta_{S_1}(\omega) \subseteq \zeta_{S_2}(\omega)$ for all $\omega \in \Lambda_1$.

Definition 2.3. [2]
Union of two soft sets (ζ_{S_1}, Λ_1) and (ζ_{S_2}, Λ_2) is a soft set (ζ_{S_3}, Λ_3) with $\Lambda_3 = \Lambda_1 \cup \Lambda_2$ and for $\omega \in \Lambda_3$,

$$\zeta_{S_3}(\omega) = \begin{cases} \zeta_{S_1}(\omega) & \omega \in (\Lambda_1 \setminus \Lambda_2) \\ \zeta_{S_2}(\omega) & \omega \in (\Lambda_2 \setminus \Lambda_1) \\ \zeta_{S_1}(\omega) \cup \zeta_{S_2}(\omega) & \omega \in (\Lambda_1 \cap \Lambda_2) \end{cases}$$

Definition 2.4. [2]
Intersection of two soft sets (ζ_{S_1}, Λ_1) and (ζ_{S_2}, Λ_2) is a soft set (ζ_{S_3}, Λ_3) with $\Lambda_3 = \Lambda_1 \cap \Lambda_2$ and for $\omega \in \Lambda_3$,

$$\zeta_{S_3}(\omega) = \zeta_{S_1}(\omega) \cap \zeta_{S_2}(\omega)$$

For more details on soft set can be found in [1–9]

3. Hypersoft Set (HS-set)

In this section, some fundamentals of hypersoft set are presented. Some of the definitions given in [24] are modified.

Definition 3.1. [21]
The pair (Ψ, G) is called a *hypersoft set* over $\mathcal{U}$, where G is the cartesian product of n disjoint attribute-valued sets $G_1, G_2, G_3, ..., G_n$ corresponding to n distinct attributes $g_1, g_2, g_3, ..., g_n$ respectively and $\Psi : G \to P(\mathcal{U})$. The collection of all hypersoft sets is denoted by $\Omega_{(\Psi,G)}$.

Example 3.2. Suppose that Mr. X wants to buy a mobile from a mobile market. There are eight kinds of mobiles (options) which form the set of discourse $\mathcal{U} = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$. The best selection may be evaluated by observing the attributes i.e. $a_1 =$ Company, $a_2 =$ Camera Resolution, $a_3 =$ Size, $a_4 =$ RAM, and $a_5 =$ Battery power. The attribute-valued sets corresponding to these attributes are:
$B_1 = \{b_{11}, b_{12}\}$
$B_2 = \{b_{21}, b_{22}\}$
$B_3 = \{b_{31}, b_{32}\}$
$B_4 = \{b_{41}, b_{42}\}$
$B_5 = \{b_{51}\}$
then $G = B_1 \times B_2 \times B_3 \times B_4 \times B_5$
$G = \{g_1, g_2, g_3, g_4, ..., g_{16}\}$ where each $g_i, i = 1, 2, ..., 16$, is a 5-tuple element.
The hypersoft set (Ψ, G) is given as

$$(\Psi, G) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2, m_3, m_4\}), (g_4, \{m_4, m_5, m_6\}), \\ (g_5, \{m_6, m_7, m_8\}), (g_6, \{m_2, m_3, m_4, m_7\}), (g_7, \{m_1, m_3, m_5, m_6\}), \\ (g_8, \{m_2, m_3, m_6, m_7\}), (g_9, \{m_2, m_3, m_6, m_7, m_8\}), (g_{10}, \{m_1, m_3, m_6, m_7, m_8\}), \\ (g_{11}, \{m_2, m_4, m_6, m_7, m_8\}), (g_{12}, \{m_1, m_2, m_3, m_6, m_7, m_8\}), \\ (g_{13}, \{m_2, m_3, m_5, m_7, m_8\}), (g_{14}, \{m_1, m_3, m_5, m_7, m_8\}), \\ (g_{15}, \{m_1, m_2, m_3, m_5, m_7, m_8\}), (g_{16}, \{m_4, m_5, m_6, m_7, m_8\}) \end{array} \right\}$$

Definition 3.3. Let $\mathcal{F}(\mathcal{U})$ be the collection of all fuzzy sets over $\mathcal{U}$. Let $a_1, a_2, a_3, ..., a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $G_1, G_2, G_3, ..., G_n$, with $G_i \cap G_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$. Then a *fuzzy hypersoft set* (Ψ_{fhs}, G) over $\mathcal{U}$ is defined by the set of ordered pairs as follows,

$$(\Psi_{fhs}, G) = \{(g, \Psi_{fhs}(g)) : g \in G, \Psi_{fhs}(g) \in \mathcal{F}(\mathcal{U})\}$$

where $\Psi_{fhs} : G \to \mathcal{F}(\mathcal{U})$ and for all $g \in G = G_1 \times G_2 \times G_3 \times ..... \times G_n$

$$\Psi_{fhs}(g) = \{\mu_{\Psi_{fhs}(g)}(u)/u : u \in \mathcal{U}, \mu_{\Psi_{fhs}(g)}(u) \in [0, 1]\}$$

is a fuzzy set over $\mathcal{U}$.

Above definition is a modified version of fuzzy hypersoft set given in [21] and [23].

Example 3.4. Considering the example 3.2, we have
Fuzzy hypersoft set (Ψ_{fhs}, G) is given as

$$(\Psi_{fhs}, G) = \left\{ \begin{array}{l} (g_1, \{0.1/m_1, 0.2/m_2\}), (g_2, \{0.1/m_1, 0.2/m_2, 0.3/m_3\}), \\ (g_3, \{0.2/m_2, 0.3/m_3, 0.4/m_4\}), \\ (g_4, \{0.4/m_4, 0.5/m_5, 0.6/m_6\}), \\ (g_5, \{0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_6, \{0.2/m_2, 0.3/m_3, 0.4/m_4, 0.7/m_7\}), \\ (g_7, \{0.1/m_1, 0.3/m_3, 0.5/m_5, 0.6/m_6\}), \\ (g_8, \{0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7\}), \\ (g_9, \{0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{10}, \{0.1/m_1, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{11}, \{0.2/m_2, 0.4/m_4, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{12}, \{0.1/m_1, 0.2/m_2, 0.3/m_3, 0.6/m_6, 0.7/m_7, 0.8/m_8\}), \\ (g_{13}, \{0.2/m_2, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{14}, \{0.1/m_1, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{15}, \{0.1/m_1, 0.2/m_2, 0.3/m_3, 0.5/m_5, 0.7/m_7, 0.8/m_8\}), \\ (g_{16}, \{0.4/m_4, 0.5/m_5, 0.6/m_6, 0.7/m_7, 0.8/m_8\}) \end{array} \right\}$$

Definition 3.5. Let $(\Psi_1, G_1), (\Psi_2, G_2) \in \Omega_{(\Psi, G)}$ then (Ψ_1, G_1) is said to be *hypersoft subset* of (Ψ_2, G_2) if

(i) $G_1 \subseteq G_2$

(ii) $\forall g \in G_1, \Psi_1(g) \subseteq \Psi_2(g)$

Example 3.6. Considering example 3.2, if

$$(\Psi_1, G_1) = \left\{ (g_1, \{m_1\}), (g_2, \{m_1, m_2\}), (g_3, \{m_2, m_3\}) \right\}$$

$$(\Psi_2, G_2) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), \\ (g_3, \{m_2, m_3, m_4\}), (g_4, \{m_4, m_5, m_6\}) \end{array} \right\}$$

then

$(\Psi_1, G_1) \subseteq (\Psi_2, G_2)$

Definition 3.7. A set $G = G_1 \times G_2 \times G_3 \times ..... \times G_n$ in hypersoft set (Ψ, G) is said to be *Not set* if it has the representation as

$$\ltimes G = \{\ltimes g_1, \ltimes g_2, \ltimes g_3, \ltimes g_4, ..., \ltimes g_m\}$$

where $m = \prod_{i=1}^{n} |G_i|$, each $\ltimes g_i, i = 1, 2, ..., m$, is a *Not* n-tuple element.

Example 3.8. Taking sets $G_1, G_2, G_3, G_4, , G_5$ from example 3.2, we have

$$\ltimes G = \{\ltimes g_1, \ltimes g_2, \ltimes g_3, \ltimes g_4, ..., \ltimes g_{16}\}$$

where each $\ltimes g_i, i = 1, 2, ..., 16$, is a *Not* 5-tuple element.

Definition 3.9. A hypersoft set (Ψ, G_1) is called a *relative null hypersoft set* w.r.t $G_1 \subseteq G$, denoted by $(\Psi, G_1)_\Phi$, if $\Psi(g) = \emptyset, \forall g \in G_1$.

Example 3.10. Considering example 3.2, if

$$(\Psi, G_1)_\Phi = \left\{ \ (g_1, \emptyset), (g_2, \emptyset), (g_3, \emptyset) \ \right\}$$

where $G_1 \subseteq G$.

Definition 3.11. A hypersoft set (Ψ, G_1) is called a *relative whole hypersoft set* w.r.t $G_1 \subseteq G$, denoted by $(\Psi, G_1)_\mathcal{U}$, if $\Psi(g) = \mathcal{U}, \forall g \in G_1$.

Example 3.12. Considering example 3.2, if

$$(\Psi, G_1)_\mathcal{U} = \left\{ \ (g_1, \mathcal{U}), (g_2, \mathcal{U}), (g_3, \mathcal{U}) \ \right\}$$

where $G_1 \subseteq G$.

Definition 3.13. A hypersoft set (Ψ, G) is called a *absolute whole hypersoft set* over $\mathcal{U}$, denoted by $(\Psi, G)_\mathcal{U}$, if $\Psi(g) = \mathcal{U}, \forall g \in G$.

Example 3.14. Considering example 3.2, if

$$(\Psi, G)_\mathcal{U} = \left\{ \begin{array}{l} (g_1, \mathcal{U}), (g_2, \mathcal{U}), (g_3, \mathcal{U}), (g_4, \mathcal{U}), \\ (g_5, \mathcal{U}), (g_6, \mathcal{U}), (g_7, \mathcal{U}), (g_8, \mathcal{U}), \\ (g_9, \mathcal{U}), (g_{10}, \mathcal{U}), (g_{11}, \mathcal{U}), (g_{12}, \mathcal{U}), \\ (g_{13}, \mathcal{U}), (g_{14}, \mathcal{U}), (g_{15}, \mathcal{U}), (g_{16}, \mathcal{U}) \end{array} \right\}$$

Proposition 3.15. *Let* $(\Psi_1, G_1), (\Psi_2, G_2), (\Psi_3, G_3) \in \Omega_{(\Psi,G)}$ *with* $G_1, G_2, G_3 \subseteq G$ *then*

(i) $(\Psi_1, G_1) \subseteq (\Psi_1, G_1)_\mathcal{U}$

(ii) $(\Psi_1, G_1)_\Phi \subseteq (\Psi_1, G_1)$

(iii) $(\Psi_1, G_1) \subseteq (\Psi_1, G_1)$

(iv) *If* $(\Psi_1, G_1) \subseteq (\Psi_2, G_2)$ *and* $(\Psi_2, G_2) \subseteq (\Psi_3, G_3)$ *then* $(\Psi_1, G_1) \subseteq (\Psi_3, G_3)$

(v) *If* $(\Psi_1, G_1) = (\Psi_2, G_2)$ *and* $(\Psi_2, G_2) = (\Psi_3, G_3)$ *then* $(\Psi_1, G_1) = (\Psi_3, G_3)$

Definition 3.16. The *complement* of a hypersoft set (Ψ, G), denoted by $(\Psi, G)^{\ominus}$, is defined as

$$(\Psi, G)^{\ominus} = (\Psi^{\ominus}, \ltimes G)$$

where

$$\Psi^{\ominus} : \ltimes G \to P(\mathcal{U})$$

with

$$\Psi^{\ominus}(\ltimes g) = \mathcal{U} \setminus \Psi(g), \forall g \in G$$

Example 3.17. Assuming data from example 3.2, we have

$$(\Psi, G)^{\ominus} = \left\{ \begin{array}{l} (\ltimes g_1, \{m_3, m_4, m_5, m_6, m_7, m_8\}), (\ltimes g_2, \{m_4, m_5, m_6, m_7, m_8\}), \\ (\ltimes g_3, \{m_1, m_5, m_6, m_7, m_8\}), (\ltimes g_4, \{m_1, m_2, m_3, m_7, m_8\}), \\ (\ltimes g_5, \{m_1, m_2, m_3, m_4, m_5\}), (\ltimes g_6, \{m_2, m_3, m_4, m_7\}), \\ (\ltimes g_7, \{m_2, m_4, m_7, m_8\}), (\ltimes g_8, \{m_1, m_4, m_5, m_8\}), \\ (\ltimes g_9, \{m_1, m_4, m_5\}), (\ltimes g_{10}, \{m_2, m_4, m_5\}), \\ (\ltimes g_{11}, \{m_1, m_3, m_5\}), (\ltimes g_{12}, \{m_4, m_5\}), \\ (\ltimes g_{13}, \{m_1, m_4, m_6\}), (\ltimes g_{14}, \{m_2, m_4, m_6\}), \\ (\ltimes g_{15}, \{m_4, m_6\}), (\ltimes g_{16}, \{m_1, m_2, m_3\}) \end{array} \right\}$$

Definition 3.18. The *relative complement* of a hypersoft set (Ψ, G), denoted by $(\Psi, G)^{\circledast}$, is defined as

$$(\Psi, G)^{\circledast} = (\Psi^{\circledast}, G)$$

where

$$\Psi^{\circledast} : G \to P(\mathcal{U})$$

with

$$\Psi^{\circledast}(g) = \mathcal{U} \setminus \Psi(g), \forall g \in G$$

Example 3.19. Assuming data from example 3.2, we have

$$(\Psi, G)^{\circledast} = \left\{ \begin{array}{l} (g_1, \{m_3, m_4, m_5, m_6, m_7, m_8\}), (g_2, \{m_4, m_5, m_6, m_7, m_8\}), \\ (g_3, \{m_1, m_5, m_6, m_7, m_8\}), (g_4, \{m_1, m_2, m_3, m_7, m_8\}), \\ (g_5, \{m_1, m_2, m_3, m_4, m_5\}), (g_6, \{m_2, m_3, m_4, m_7\}), \\ (g_7, \{m_2, m_4, m_7, m_8\}), (g_8, \{m_1, m_4, m_5, m_8\}), \\ (g_9, \{m_1, m_4, m_5\}), (g_{10}, \{m_2, m_4, m_5\}), \\ (g_{11}, \{m_1, m_3, m_5\}), (g_{12}, \{m_4, m_5\}), \\ (g_{13}, \{m_1, m_4, m_6\}), (g_{14}, \{m_2, m_4, m_6\}), \\ (g_{15}, \{m_4, m_6\}), (g_{16}, \{m_1, m_2, m_3\}) \end{array} \right\}$$

Proposition 3.20. *Let* $(\Psi, G) \in \Omega_{(\Psi,G)}$ *then*

(i) $((\Psi, G)^{\ominus})^{\ominus} = (\Psi, G)$

(ii) $((\Psi, G)^{\circledast})^{\circledast} = (\Psi, G)$

(iii) $((\Psi_1, G_1)_{\mathcal{U}})^{\ominus} = (\Psi_1, G_1)_{\Phi} = ((\Psi_1, G_1)_{\mathcal{U}})^{\circledast}$ *where* $G_1 \subseteq G$

(iv) $((\Psi_1, G_1)_{\Phi})^{\ominus} = (\Psi_1, G_1)_{\mathcal{U}} = ((\Psi_1, G_1)_{\Phi})^{\circledast}$ *where* $G_1 \subseteq G$

4. Set Theoretic Operations on Hypersoft Set

In this section, set theoretic operations i.e. union, intersection, difference, AND, OR etc., are discussed under hypersoft set environment.

Definition 4.1. *Union of two hypersoft sets* (π, G_1) and (λ, G_2) , denoted by $(\pi, G_1) \cup (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cup G_2$ and for $g \in G_3$,

$$\mu(g) = \begin{cases} \pi(g) & g \in (G_1 \setminus G_2) \\ \lambda(g) & g \in (G_2 \setminus G_1) \\ \pi(g) \cup \lambda(g) & g \in (G_1 \cap G_2) \end{cases}$$

Example 4.2. Let
$(\pi, G_1) = \left\{ (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2, m_3, m_4\}) \right\}$
$(\lambda, G_2) = \left\{ (g_3, \{m_1, m_2\}), (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \right\}$
then

$$(\mu, G_3) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_1, m_2, m_3, m_4\}), \\ (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \end{array} \right\}$$

Definition 4.3. *Intersection of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cap (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) \cap \lambda(g).$$

Example 4.4. Consider the example 4.2, we have

$$(\mu, G_3) = \left\{ (g_3, \{m_2\}) \right\}$$

Definition 4.5. *Extended Intersection of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cap_{\varepsilon} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cup G_2$ and for $g \in G_3$,

$$\mu(g) = \begin{cases} \pi(g) & g \in (G_1 \setminus G_2) \\ \lambda(g) & g \in (G_2 \setminus G_1) \\ \pi(g) \cap \lambda(g) & g \in (G_1 \cap G_2) \end{cases}$$

Example 4.6. Assuming sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \begin{array}{l} (g_1, \{m_1, m_2\}), (g_2, \{m_1, m_2, m_3\}), (g_3, \{m_2\}), \\ (g_4, \{m_4, m_5, m_6\}), (g_5, \{m_2, m_4, m_6\}) \end{array} \right\}$$

Definition 4.7. *AND-operation of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \bigwedge (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \times G_2$ and for $(g_i, g_j) \in G_3, g_i \in G_1, g_j \in G_2$,

$$\mu(g_i, g_j) = \pi(g_i) \cup \lambda(g_j).$$

Example 4.8. Consider the example 4.2, we have

$$G_1 \times G_2 = \left\{ \begin{array}{l} h_1 = (g_1, g_3), h_2 = (g_1, g_4), h_3 = (g_1, g_5), h_4 = (g_2, g_3), h_5 = (g_2, g_4), \\ h_6 = (g_2, g_5), h_7 = (g_3, g_3), h_8 = (g_3, g_4), h_9 = (g_3, g_5) \end{array} \right\}$$

then

$$(\mu, G_3) = \left\{ \begin{array}{l} (h_1, \{m_1, m_2\}), (h_2, \{m_1, m_2, m_4, m_5, m_6\}), \\ (h_3, \{m_1, m_2, m_4, m_6\}), (h_4, \{m_1, m_2, m_3\}), \\ (h_5, \{m_1, m_2, m_3, m_4, m_5, m_6\}), (h_6, \{m_1, m_2, m_3, m_4, m_6\}), \\ (h_7, \{m_1, m_2, m_3, m_4\}), (h_8, \{m_2, m_3, m_4, m_5, m_6\}), \\ (h_9, \{m_2, m_3, m_4, m_6\}), \end{array} \right\}$$

Definition 4.9. *OR-operation of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \bigvee (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \times G_2$ and for $(g_i, g_j) \in G_3, g_i \in G_1, g_j \in G_2$,

$$\mu(g_i, g_j) = \pi(g_i) \cap \lambda(g_j).$$

Example 4.10. Considering data from examples 4.2 and 4.10, we have

$$(\mu, G_3) = \left\{ \begin{array}{l} (h_1, \{m_1, m_2\}), (h_2, \{\}), (h_3, \{m_2\}), (h_4, \{m_1, m_2\}), \\ (h_5, \{\}), (h_6, \{m_2\}), (h_7, \{m_2\}), (h_8, \{m_4\}), (h_9, \{m_2, m_4\}), \end{array} \right\}$$

Definition 4.11. *Restricted Union of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \cup_{\mathcal{R}} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) \cup \lambda(g).$$

Example 4.12. For sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \ (g_3, \{m_1, m_2, m_3, m_4\}) \ \right\}$$

Definition 4.13. *Restricted Difference of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)$, is a hypersoft set (μ, G_3) with $G_3 = G_1 \cap G_2$ and for $g \in G_3$,

$$\mu(g) = \pi(g) - \lambda(g).$$

Example 4.14. For sets given in example 4.2, we have

$$(\mu, G_3) = \left\{ \ (g_3, \{m_3, m_4\}) \ \right\}$$

Definition 4.15. *Restricted Symmetric Difference of two hypersoft sets* (π, G_1) and (λ, G_2), denoted by $(\pi, G_1)\blacktriangle(\lambda, G_2)$, is a hypersoft set (μ, G_3) defined by

$$(\mu, G_3) = \left\{ \ ((\pi, G_1) \cup_{\mathcal{R}} (\lambda, G_2)) \setminus_{\mathcal{R}} ((\pi, G_1) \cap (\lambda, G_2)) \ \right\}$$

or

$$(\mu, G_3) = \left\{ \ ((\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)) \cup_{\mathcal{R}} ((\lambda, G_2) \setminus_{\mathcal{R}} (\pi, G_1)) \ \right\}$$

Example 4.16. For sets given in example 4.2, we have

$$((\pi, G_1) \setminus_{\mathcal{R}} (\lambda, G_2)) = \left\{ \ (g_3, \{m_3, m_4\}) \ \right\}$$

and

$$((\lambda, G_2) \setminus_{\mathcal{R}} (\pi, G_1)) = \left\{ \ (g_3, \{m_1\}) \ \right\}$$

then

$$(\mu, G_3) = \left\{ \ (g_3, \{m_1, m_3, m_4\}) \ \right\}$$

5. Basic Properties and Laws of Hypersoft Set Operations

In this section, some basic properties and laws are discussed for hypersoft set theoretic operations. All hypersoft sets in $\Omega_{(\psi,\mathcal{G})}$ satisfy the following properties, results and laws.

(a) **Idempotent Laws**
- (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G}) = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})$
- (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G}) = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})$

(b) **Identity Laws**
- (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})_{\Phi}$
- (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})_{\mathcal{U}}$
- (iii) $(\psi, \mathcal{G}) \setminus_{\mathcal{R}} (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) = (\psi, \mathcal{G})\blacktriangle(\psi, \mathcal{G})_{\Phi}$
- (iv) $(\psi, \mathcal{G}) \setminus_{\mathcal{R}} (\psi, \mathcal{G}) = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G})\blacktriangle(\psi, \mathcal{G})$

(c) **Domination Laws**
- (i) $(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})_{\mathcal{U}}$
- (ii) $(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})_{\Phi}$

(d) **Property of Exclusion**

$$(\psi, \mathcal{G}) \cup (\psi, \mathcal{G})^{\circledast} = (\psi, \mathcal{G})_{\mathcal{U}} = (\psi, \mathcal{G}) \cup_{\mathcal{R}} (\psi, \mathcal{G})^{\circledast}$$

(e) **Property of Contradiction**

$$(\psi, \mathcal{G}) \cap (\psi, \mathcal{G})^{\circledast} = (\psi, \mathcal{G})_{\Phi} = (\psi, \mathcal{G}) \cap_{\varepsilon} (\psi, \mathcal{G})^{\circledast}$$

(f) **Absorption Laws**
- (i) $(\zeta, \mathcal{G}_1) \cup ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$
- (ii) $(\zeta, \mathcal{G}_1) \cap ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$

(iii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$

(iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) = (\zeta, \mathcal{G}_1)$

(g) **Commutative Laws**

(i) $(\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cup (\zeta, \mathcal{G}_1)$

(ii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\zeta, \mathcal{G}_1)$

(iii) $(\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cap (\zeta, \mathcal{G}_1)$

(iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \cap_{\varepsilon} (\zeta, \mathcal{G}_1)$

(v) $(\zeta, \mathcal{G}_1) \blacktriangle (\xi, \mathcal{G}_2) = (\xi, \mathcal{G}_2) \blacktriangle (\zeta, \mathcal{G}_1)$

(h) **Associative Laws**

(i) $(\zeta, \mathcal{G}_1) \cup ((\xi, \mathcal{G}_2) \cup (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) \cup (\psi, \mathcal{G}_3)$

(ii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)$

(iii) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cap (\psi, \mathcal{G}_3)$

(iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\xi, \mathcal{G}_2) \cap_{\varepsilon} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) \cap_{\varepsilon} (\psi, \mathcal{G}_3)$

(v) $(\zeta, \mathcal{G}_1) \bigvee ((\xi, \mathcal{G}_2) \bigvee (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2)) \bigvee (\psi, \mathcal{G}_3)$

(vi) $(\zeta, \mathcal{G}_1) \bigwedge ((\xi, \mathcal{G}_2) \bigwedge (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2)) \bigwedge (\psi, \mathcal{G}_3)$

(i) **De Morgans Laws**

(i) $((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \cap_{\varepsilon} (\xi, \mathcal{G}_2)^{\ominus}$

(ii) $((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \cup (\xi, \mathcal{G}_2)^{\ominus}$

(iii) $((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \cap (\xi, \mathcal{G}_2)^{\circledast}$

(iv) $((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)^{\circledast}$

(v) $((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \bigwedge (\xi, \mathcal{G}_2)^{\ominus}$

(vi) $((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2))^{\ominus} = (\zeta, \mathcal{G}_1)^{\ominus} \bigvee (\xi, \mathcal{G}_2)^{\ominus}$

(vii) $((\zeta, \mathcal{G}_1) \bigvee (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \bigwedge (\xi, \mathcal{G}_2)^{\circledast}$

(viii) $((\zeta, \mathcal{G}_1) \bigwedge (\xi, \mathcal{G}_2))^{\circledast} = (\zeta, \mathcal{G}_1)^{\circledast} \bigvee (\xi, \mathcal{G}_2)^{\circledast}$

(j) **Distributive Laws**

(i) $(\zeta, \mathcal{G}_1) \cup ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup (\xi, \mathcal{G}_2)) \cap ((\zeta, \mathcal{G}_1) \cup (\psi, \mathcal{G}_3))$

(ii) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cup (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cup ((\zeta, \mathcal{G}_1) \cap (\psi, \mathcal{G}_3))$

(iii) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cap_{\varepsilon} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cap_{\varepsilon} ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3))$

(iv) $(\zeta, \mathcal{G}_1) \cap_{\varepsilon} ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap_{\varepsilon} (\psi, \mathcal{G}_3))$

(v) $(\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} ((\xi, \mathcal{G}_2) \cap (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\xi, \mathcal{G}_2)) \cap ((\zeta, \mathcal{G}_1) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3))$

(vi) $(\zeta, \mathcal{G}_1) \cap ((\xi, \mathcal{G}_2) \cup_{\mathcal{R}} (\psi, \mathcal{G}_3)) = ((\zeta, \mathcal{G}_1) \cap (\xi, \mathcal{G}_2)) \cup_{\mathcal{R}} ((\zeta, \mathcal{G}_1) \cap (\psi, \mathcal{G}_3))$

6. Relations and Functions on Hypersoft Sets

Here we present the notions of relations and functions for hypersoft sets.

Definition 6.1. *Cartesian Product* of two hypersoft sets (Ψ_1, G_1) and (Ψ_2, G_2), denoted by $(\Psi_1, G_1) \times (\Psi_2, G_2)$, is a hypersoft (Ψ_3, G_3) where $G_3 = G_1 \times G_2$ and

$$\Psi_3 : G_3 \to P(\mathcal{U} \times \mathcal{U})$$

defined by

$$\Psi_3(g_i, g_j) = \Psi_1(g_i) \times \Psi_2(g_j) \ \forall \ (g_i, g_j) \in G_3$$

that is

$$\Psi_3(g_i, g_j) = \{(h_i, h_j) : h_i \in \Psi_1(g_i), h_j \in \Psi_2(g_j)\}$$

Definition 6.2. If $(\Psi_1, G_1), (\Psi_2, G_2) \in \Omega_{(\Psi,G)}$ then a relation from (Ψ_1, G_1) to (Ψ_2, G_2) is called *hypersoft set relation* $(\mathfrak{R}, G_4)$ (simply $\mathfrak{R}$) which is the hypersoft subset of $(\Psi_1, G_1) \times (\Psi_2, G_2)$ where $G_4 \subseteq G_1 \times G_2$ and $\forall \ (h_1, h_2) \in G_4, \mathfrak{R}(h_1, h_2) = \Psi_3(h_1, h_2)$, where $(\Psi_3, G_3) = (\Psi_1, G_1) \times (\Psi_2, G_2)$.

Definition 6.3. Let $\mathfrak{R}$ be a hypersoft set relation from (Ψ_1, G_1) to (Ψ_2, G_2) such that $(\Psi_3, G_3) = (\Psi_1, G_1) \times (\Psi_2, G_2)$. Then

(i) Domain of $\mathfrak{R}$ (Dom $\mathfrak{R}$) is a hypersoft set $(\psi, K) \subset (\Psi_1, G_1)$ where

$$K = \{g_i \in G_1 : \Psi_3(g_i, g_j) \in \mathfrak{R} \ \ forsome \ \ g_j \in G_2\}$$

and

$$\psi(g_1) = \Psi_1(g_1), \forall \ g_1 \in K$$

(ii) Range of $\mathfrak{R}$ (Range $\mathfrak{R}$) is a hypersoft set $(\xi, L) \subset (\Psi_2, G_2)$ where $L \subset G_2$ and

$$L = \{g_j \in G_2 : \Psi_3(g_i, g_j) \in \mathfrak{R} \ \ forsome \ \ g_i \in G_1\}$$

and

$$\xi(g_2) = \Psi_1(g_2), \forall \ g_2 \in L$$

(iii) The inverse of $\mathfrak{R}$ $(\mathfrak{R}^{-1})$ is a hypersoft set relation from (Ψ_2, G_2) to (Ψ_1, G_1) defined by

$$\mathfrak{R}^{-1} = \{\Psi_2(q_j) \times \Psi_1(q_i) : \Psi_1(q_i)\mathfrak{R}\Psi_2(q_j)\}$$

Example 6.4. Let

$$(\Psi_1, G_1) = \left\{ \ \Psi_1(g_1), \Psi_1(g_2), \Psi_1(g_3) \ \right\}, (\Psi_2, G_2) = \left\{ \ \Psi_2(g_4), \Psi_2(g_5), \Psi_2(g_6) \ \right\}$$

$$(\Psi_1, G_1) \times (\Psi_2, G_2) = \left\{ \begin{array}{l} (\Psi_1(g_1) \times \Psi_2(g_4)), (\Psi_1(g_1) \times \Psi_2(g_5)), (\Psi_1(g_1) \times \Psi_2(g_6)), \\ (\Psi_1(g_2) \times \Psi_2(g_4)), (\Psi_1(g_2) \times \Psi_2(g_5)), (\Psi_1(g_2) \times \Psi_2(g_6)), \\ (\Psi_1(g_3) \times \Psi_2(g_4)), (\Psi_1(g_3) \times \Psi_2(g_5)), (\Psi_1(g_3) \times \Psi_2(g_6)) \end{array} \right\}$$

then

$$\mathfrak{R} = \left\{ \ (\Psi_1(g_1) \times \Psi_2(g_4)), (\Psi_1(g_1) \times \Psi_2(g_6)), (\Psi_1(g_2) \times \Psi_2(g_6)), (\Psi_1(g_3) \times \Psi_2(g_6)) \ \right\}$$

(i) $Dom\,\mathfrak{R} = (\psi, K)$ where $K = \{g_1, g_2, g_3\} \subseteq G_1$ and $\psi(g_i) = \Psi_1(g_i) \forall\ g_i \in K$

(ii) $Range\,\mathfrak{R} = (\xi, L)$ where $L = \{g_4, g_6\} \subset G_2$ and $\xi(g_j) = \Psi_2(g_j) \forall\ g_j \in L$

(iii) $\mathfrak{R}^{-1} = \left\{\ (\Psi_2(g_4) \times \Psi_1(g_1)), (\Psi_2(g_6) \times \Psi_1(g_1)), (\Psi_2(g_6) \times \Psi_1(g_2)), (\Psi_2(g_6) \times \Psi_1(g_3))\ \right\}$

Definition 6.5. Let $\mathfrak{R}$ and $\mathfrak{S}$ are two hypersoft set relations on hypersoft set (Ψ, K), then we have

(i) $\mathfrak{R} \subset \mathfrak{S}$, if for all $u, v \in K, \Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $\Psi(u) \times \Psi(v) \in \mathfrak{S}$

(ii) The Complement of $\mathfrak{R}$, denoted by $\mathfrak{R}^{©}$, is defined as

$$\mathfrak{R}^{©} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \notin \mathfrak{R}, \forall\ u, v \in K\}$$

(iii) The union of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \cup \mathfrak{S}$, defined as

$$\mathfrak{R} \cup \mathfrak{S} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \in \mathfrak{R}\ or\ \Psi(u) \times \Psi(v) \in \mathfrak{S}, \forall\ u, v \in K\}$$

(iv) The intersection of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \cap \mathfrak{S}$, defined as

$$\mathfrak{R} \cap \mathfrak{S} = \{\Psi(u) \times \Psi(v) : \Psi(u) \times \Psi(v) \in \mathfrak{R}\ and\ \Psi(u) \times \Psi(v) \in \mathfrak{S}, \forall\ u, v \in K\}$$

Example 6.6. Let

$$(\Psi, K) = \left\{\ \Psi(g_1), \Psi(g_2), \Psi(g_3)\ \right\}$$

$$(\Psi, K) \times (\Psi, K) = \left\{ \begin{array}{l} (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), \\ (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_3)), \\ (\Psi(g_3) \times \Psi(g_1)), (\Psi(g_3) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \end{array} \right\}$$

then we have

$$\mathfrak{R} = \left\{\ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_3))\ \right\}$$

and

$$\mathfrak{S} = \left\{\ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_2))\ \right\}$$

now

(1)

$$\mathfrak{R}^{©} = \left\{\ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_1)), (\Psi(g_3) \times \Psi(g_2))\ \right\}$$

$$\mathfrak{S}^{©} = \left\{\ (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_3))\ \right\}$$

(2)

$$\mathfrak{R} \cup \mathfrak{S} = \left\{ \begin{array}{l} (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_2)), \\ (\Psi(g_2) \times \Psi(g_3)), (\Psi(g_3) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \end{array} \right\}$$

(3)

$$\mathfrak{R} \cap \mathfrak{S} = \left\{\ (\Psi(g_1) \times \Psi(g_1))\ \right\}$$

Definition 6.7. Let $\mathfrak{R}$ be a hypersoft set relation on (Ψ, K), then

(i) $\mathfrak{R}$ is *reflexive* if $\Psi(u) \times \Psi(u) \in \mathfrak{R}$ for all $u \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)) \ \Big\}$$

(ii) $\mathfrak{R}$ is *symmetric* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $\Psi(v) \times \Psi(u) \in \mathfrak{R}$ for all $u, v \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)) \ \Big\}$$

(iii) $\mathfrak{R}$ is *transitive* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ and $\Psi(v) \times \Psi(w) \in \mathfrak{R}$ then $\Psi(u) \times \Psi(w) \in \mathfrak{R}$ for all $u, v, w \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_1) \times \Psi(g_3)), (\Psi(g_2) \times \Psi(g_3)) \ \Big\}$$

(iv) $\mathfrak{R}$ is called *equivalence relation* if it is reflexive, symmetric and transitive. e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_1) \times \Psi(g_2)), (\Psi(g_2) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)) \ \Big\}$$

(v) $\mathfrak{R}$ is called *identity* if $\Psi(u) \times \Psi(v) \in \mathfrak{R}$ then $u = v$ for all $u, v \in K$, e.g.

$$\mathfrak{R} = \Big\{ \ (\Psi(g_1) \times \Psi(g_1)), (\Psi(g_2) \times \Psi(g_2)), (\Psi(g_3) \times \Psi(g_3)) \ \Big\}$$

Definition 6.8. If $\mathfrak{R}$ is a hypersoft set relation from (Ψ_1, G_1) to (Ψ_2, G_2) and $\mathfrak{S}$ is a hypersoft set relation from (Ψ_2, G_2) to (Ψ_3, G_3) then composition of $\mathfrak{R}$ and $\mathfrak{S}$, denoted by $\mathfrak{R} \circ \mathfrak{S}$, is also a hypersoft set relation $\mathfrak{T}$ from (Ψ_1, G_1) to (Ψ_3, G_3) defined as
if $\Psi_1(u) \in (\Psi_1, G_1)$ and $\Psi_3(w) \in (\Psi_3, G_3)$ then $\Psi_1(u) \times \Psi_3(w) \in \mathfrak{R} \circ \mathfrak{S}$
i.e.
$\Psi_1(u) \times \Psi_3(w) \in \mathfrak{R} \circ \mathfrak{S}$ iff $\Psi_1(u) \times \Psi_2(v) \in \mathfrak{R}$ and $\Psi_2(v) \times \Psi_3(w) \in \mathfrak{R}$

Example 6.9. Let

$$\mathfrak{R} = \Big\{ \ (\Psi_1(g_1) \times \Psi_2(g_1)), (\Psi_1(g_1) \times \Psi_2(g_3)), (\Psi_1(g_2) \times \Psi_2(g_3)), (\Psi_1(g_3) \times \Psi_2(g_3)) \ \Big\}$$

and

$$\mathfrak{S} = \Big\{ \ (\Psi_2(g_1) \times \Psi_3(g_1)), (\Psi_2(g_1) \times \Psi_3(g_2)), (\Psi_2(g_2) \times \Psi_3(g_2)), (\Psi_2(g_3) \times \Psi_3(g_2)) \ \Big\}$$

then

$$\mathfrak{R} \circ \mathfrak{S} = \Big\{ \ (\Psi_1(g_1) \times \Psi_3(g_1)), (\Psi_1(g_1) \times \Psi_3(g_2)), (\Psi_1(g_2) \times \Psi_3(g_2)), (\Psi_1(g_3) \times \Psi_3(g_2)) \ \Big\}$$

Definition 6.10. A hypersoft set relation $\mathfrak{F}$ from (Ψ_1, G_1) to (Ψ_2, G_2), represented by $\mathfrak{F} : (\Psi_1, G_1) \to (\Psi_2, G_2)$, is said to be hypersoft function if

(i) domain of $\mathfrak{F} = G_1$

(ii) there is no repetition of elements in $Dom\, \mathfrak{F}$

(iii) there is a unique element in $Range\, \mathfrak{F}$ corresponding to every element in $Dom\, \mathfrak{F}$.
i.e. if $\Psi_1(u)\, \mathfrak{F}\, \Psi_2(v)$ (or $\Psi_1(u) \times \Psi_2(v) \in \mathfrak{F}$) then $\mathfrak{F}(\Psi_1(u)) = \Psi_2(v)$.

Example 6.11. Let $G_1 = \{u_1, u_2, u_3\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$(\Psi_1, G_1) = \left\{ \Psi_1(u_1), \Psi_1(u_2), \Psi_1(u_3) \right\}$
$(\Psi_2, G_2) = \left\{ \Psi_2(v_1), \Psi_2(v_2), \Psi_2(v_3), \Psi_2(v_4) \right\}$
so hypersoft functions is

$$\mathfrak{F} = \left\{ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)) \right\}$$

Definition 6.12. A hypersoft function $\mathfrak{F} : (\Psi_1, G_1) \to (\Psi_2, G_2)$ is said to be

(i) *into-hypersoft function* if *Range* $\mathfrak{F} \subset G_2$.
e.g. Let $G_1 = \{u_1, u_2, u_3\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$\mathfrak{F} = \left\{ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)) \right\}$

(ii) *into-hypersoft function (or surjective hypersoft function)* if *Range* $\mathfrak{F} = G_2$.
e.g. Let $G_1 = \{u_1, u_2, u_3, u_4\}$ and $G_2 = \{v_1, v_2, v_3, v_4\}$ then
$\mathfrak{F} = \left\{ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_3)), (\Psi_1(u_3) \times \Psi_2(v_4)), (\Psi_1(u_4) \times \Psi_2(v_2)) \right\}$

(iii) *one-to-one hypersoft function (or injective hypersoft function)* if $\Psi_1(u_1) \neq \Psi_1(u_2)$ then $\mathfrak{F}(\Psi_1(u_1)) \neq \mathfrak{F}(\Psi_1(u_2))$.
e.g.
$\mathfrak{F} = \left\{ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_4)), (\Psi_1(u_3) \times \Psi_2(v_2)), (\Psi_1(u_4) \times \Psi_2(v_3)) \right\}$

(iv) *bijective hypersoft function (or one-to-one hypersoft correspondence)* if it is both injective and surjective.
e.g.
$\mathfrak{F} = \left\{ (\Psi_1(u_1) \times \Psi_2(v_1)), (\Psi_1(u_2) \times \Psi_2(v_2)), (\Psi_1(u_3) \times \Psi_2(v_3)), (\Psi_1(u_4) \times \Psi_2(v_4)) \right\}$

Definition 6.13. The identity hypersoft set function on hypersoft soft set (Ψ, L) is defined by $\mathfrak{I} : (\Psi, L) \to (\Psi, L)$ such that $\mathfrak{I}(\Psi(l)) = \Psi(l) \;\; \forall \; \Psi(l) \in (\Psi, L)$.
e.g. Let $L = \{l_1, l_2, l_3, l_4\}$ then
$\mathfrak{I} = \left\{ (\Psi(l_1) \times \Psi(l_1)), (\Psi(l_2) \times \Psi(l_2)), (\Psi(l_3) \times \Psi(l_3)), (\Psi(l_4) \times \Psi(l_4)) \right\}$

7. Matrix Representation of Hypersoft Set

In this section, matrix representation of hypersoft set is presented.

Definition 7.1.

(i) Let (ζ, H) be a hypersoft set over $\mathcal{U}$. A subset $\mathbb{R}_H$ of $\mathcal{U} \times H$ is said to be relation form of (ζ, H) if it is uniquely represented as

$$\mathbb{R}_H = \left\{ (u, h) : h \in H, u \in \zeta(h) \right\}.$$

(ii) *The characteristic function* $\mathcal{X}_{\mathbb{R}_H}$ is defined by $\mathcal{X}_{\mathbb{R}_H} : \mathcal{U} \times H \to \{0, 1\}$, where

$$\mathcal{X}_{\mathbb{R}_H}(u, h) = \begin{cases} 1 & ; \;\; (u, h) \in \mathbb{R}_H \\ 0 & ; \;\; (u, h) \notin \mathbb{R}_H \end{cases}$$

(iii) If $|\mathcal{U}| = m$ and $|H| = n$ then hypersoft set (ζ, H) can be represented by a matrix (α_{ij}) called an $m \times n$ hypersoft matrix of (ζ, H) over $\mathcal{U}$ as given below

$$(\alpha_{ij})_{m\times n} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots\cdot & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots\cdot & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots\cdot & \alpha_{mn} \end{pmatrix}$$

Note: The collection of all $m \times n$ hypersoft matrices over $\mathcal{U}$ is denoted by $HSM(\mathcal{U})_{m\times n}$.

Example 7.2. Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ and $H = \{h_1, h_2, h_3, h_4, h_5\}$ where each h_i is a i^{th} tuple, for $i =$ number of attribute-valued sets. Then

$$\zeta(h_1) = \{u_1, u_2\}, \ \ \zeta(h_2) = \emptyset, \ \ \zeta(h_3) = \{u_4, u_5\}, \ \ \zeta(h_4) = \{u_2, u_3, u_4, \}, \ \ \zeta(h_5) = \emptyset,$$

therefore we have

$$(\zeta, H) = \Big\{ \ (h_1, \{u_1, u_2\}), (h_3, \{u_4, u_5\}), (h_4, \{u_2, u_3, u_4, \}) \ \Big\}$$

and

$$\mathbb{R}_H = \Big\{ \ (u_1, h_1), (u_2, h_1), (u_4, h_3), (u_5, h_3), (u_2, h_4), (u_3, h_4), (u_4, h_4) \ \Big\}.$$

Hence hypersoft matrix is given as

$$(\alpha_{ij})_{5\times 5} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} i, j = 1, 2, 3, 4, 5.$$

Definition 7.3. Let $(\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$ then $(\alpha_{ij})_{m\times n}$ is said to be:

(i) *A zero (or null) hypersoft matrix*, denoted by $(0)_{m\times n}$, if $\alpha_{ij} = 0 \ \forall \ i, j$ e.g.

$$(0)_{5\times 5} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} i, j = 1, 2, 3, 4, 5.$$

(ii) An H_1*-universal hypersoft matrix*, denoted by $(\alpha_{ij})^{H_1}_{m\times n}$, if $\alpha_{ij} = 1, \forall j \in J_{H_1} = \{j : h_j \in H_1\}$ and i.
e.g. Let H be as given in 7.2 and $H_1 = \{h_2, h_4, h_5\} \subseteq H$ with $\zeta(h_2) = \zeta(h_4) = \zeta(h_5) =$

$\mathcal{U}$ then

$$(\alpha_{ij})^{H_1}_{5\times 5} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} i,j = 1,2,3,4,5.$$

(iii) A *universal hypersoft matrix*, denoted by $(\alpha_{ij})^{\mathcal{U}}_{m\times n}$, if $\alpha_{ij} = 1, \forall\ i,j$.

e.g. Let H be as given in 7.2 with $\zeta(h_1) = \zeta(h_2) = \zeta(h_3) = \zeta(h_4) = \zeta(h_5) = \mathcal{U}$ then

$$(\alpha_{ij})^{\mathcal{U}}_{5\times 5} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} i,j = 1,2,3,4,5.$$

Definition 7.4. Let $L_1 = (\alpha_{ij})_{m\times n}, L_2 = (\beta_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$ then

(i) L_1 is said to be *hypersoft sub-matrix* of L_2, denoted by $L_1 \subseteq L_2$ if $\alpha_{ij} \leq \beta_{ij}$ e.g.

$$L_1 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \text{ and } L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Note: We may also say that L_1 is dominated by L_2 or L_2 dominates L_1.

(ii) L_1 and L_2 are said to be comparable, denoted by $L_1 \parallel L_2$, if $L_1 \subseteq L_2$ or $L_2 \subseteq L_1$.

(iii) L_1 is said to be *proper hypersoft sub-matrix* of L_2, denoted by $L_1 \subset L_2$ if for atleast one term $\alpha_{ij} \leq \beta_{ij}$ e.g. $L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ and $L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Note: We may also say that L_1 is dominated properly by L_2 or L_2 properly dominates L_1.

(iv) L_1 is said to be *strictly hypersoft sub-matrix* of L_2, denoted by $L_1 \subsetneqq L_2$ if for each term $\alpha_{ij} \leq \beta_{ij}$ e.g. $L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ and $L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Note: We may also say that L_1 is dominated strictly by L_2 or L_2 strictly dominates L_1

(v) *union* of L_1 and L_2, denoted by $L_1 \cup L_2$, is also a hypersoft matrix $L_3 = (\delta_{ij})_{m\times n}$ if $\delta_{ij} = max\{\alpha_{ij}, \beta_{ij}\}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$$L_3 = L_1 \cup L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(vi) *intersection* of L_1 and L_2, denoted by $L_1 \cap L_2$, is also a hypersoft matrix $L_3 = (\delta_{ij})_{m\times n}$ if $\delta_{ij} = min\{\alpha_{ij}, \beta_{ij}\}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$$L_3 = L_1 \cap L_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

(vii) *complement* of $L = (\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$, denoted by $L^{©}\,(\mu_{ij})_{m\times n}$, is also a hypersoft matrix if $\mu_{ij} = 1 - \alpha_{ij}\ \forall\ i, j$ e.g.

$$\text{Let}\quad L = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{then} \quad L^{©} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

(viii) *difference* of L_1 from L_2, denoted by $L_2 \setminus L_1$, is also a hypersoft matrix L_3 such that $L_3 = L_2 \cap L_1^{©}$ e.g.

$$L_1 = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad L_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{then}$$

$L_3 = L_2 \cap L_1^{©}$

$$L_3 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cap \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Proposition 7.5. *For $X = (\alpha_{ij})_{m\times n}, Y = (\beta_{ij})_{m\times n}, Z = (\alpha_{ij})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$, we have following characteristics properties, operations and laws:*

(i) $X \cup X = X,\ \ X \cap X = X$

(ii) $X \cup (0)_{m\times n} = X,\ \ X \cap (\alpha_{ij})^{\mathcal{U}}_{m\times n} = X$

(iii) $X \cap (0)_{m\times n} = (0)_{m\times n},\ \ X \cup (\alpha_{ij})^{\mathcal{U}}_{m\times n} = (\alpha_{ij})^{\mathcal{U}}_{m\times n}$

(iv) $((0)_{m\times n})^{©} = (\alpha_{ij})^{\mathcal{U}}_{m\times n},\ \ ((\alpha_{ij})^{\mathcal{U}}_{m\times n})^{©} = (0)_{m\times n}$

(v) $X \cup X^{©} = (\alpha_{ij})^{\mathcal{U}}_{m\times n},\ \ X \cap X^{©} = (0)_{m\times n}$

(vi) $(X \cup Y)^{©} = X^{©} \cap Y^{©},\ \ (X \cap Y)^{©} = X^{©} \cup Y^{©}$

(vii) $(X^{©})^{©} = X$

(viii) $X \cup Y = Y \cup X,\ \ X \cap Y = Y \cap X$

(ix) $X \cup (Y \cup Z) = (X \cup Y) \cup Z,\ \ X \cap (Y \cap Z) = (X \cap Y) \cap Z$

(x) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z),\ \ X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Definition 7.6. Let $P = (p_{ij})_{m\times n},\ \ Q = (q_{ik})_{m\times n} \in HSM(\mathcal{U})_{m\times n}$, then

(i) *AND- product* of P and Q, denoted by $P \wedge Q$, is defined as $\wedge : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \wedge (q_{ik}) = (r_{il})$ where $r_{il} = min\{p_{ij}, q_{ik}\}$ and $l = n(j-1)+k$.

(ii) *OR- product* of P and Q, denoted by $P \vee Q$, is defined as $\vee : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \vee (q_{ik}) = (r_{il})$ where $r_{il} = max\{p_{ij}, q_{ik}\}$ and $l = n(j-1)+k$.

(iii) *AND - NOT - product* of P and Q, denoted by $P \barwedge Q$, is defined as $\barwedge : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \barwedge (q_{ik}) = (r_{il})$ where $r_{il} = min\{p_{ij}, 1-q_{ik}\}$ and $l = n(j-1)+k$.

(iv) *OR - NOT - product* of P and Q, denoted by $P \veebar Q$, is defined as $\veebar : HSM(\mathcal{U})_{m\times n} \times HSM(\mathcal{U})_{m\times n} \to HSM(\mathcal{U})_{m\times n^2}$ with $(p_{ij}) \veebar (q_{ik}) = (r_{il})$ where $r_{il} = max\{p_{ij}, 1-q_{ik}\}$ and $l = n(j-1)+k$.

Example 7.7. Let $P = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ then

$$\text{(i)}\ \ P \wedge Q = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\text{(ii)}\ \ P \vee Q = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\text{(iii)}\ \ P \,\overline{\wedge}\, Q = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \wedge \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{(iv)}\ \ P \,\underline{\vee}\, Q = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \vee \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

8. **Hybrids of Hypersoft Sets**

Smarandache [23] defined some hybrids of hypersoft set. Here we give some more hybrids of hypersoft set. In this section $\mathcal{J} = \mathcal{J}_1 \times \mathcal{J}_2 \times ..... \times \mathcal{J}_m$ with $\mathcal{J}_p \cap \mathcal{J}_q = \emptyset \ \forall \ p, q = 1, 2, ..., m$ where $\mathcal{J}_p$ are attribute-valued sets corresponding to m distinct attributes $j_1, j_2, ..., j_m$ respectively.

Definition 8.1. Let $\mathcal{F}_{ivf}(\mathcal{U})$ be a collection of interval-valued fuzzy sets over $\mathcal{U}$ then a *interval-valued fuzzy hypersoft set*$(ifhs - set)$ $(\Gamma, \mathcal{J})$ over $\mathcal{U}$ is defined as,

$$(\Gamma, \mathcal{J}) = \left\{ \ (j, \Gamma(j)) : j \in \mathcal{J}, \Gamma(j) \in \mathcal{F}_{ivf}(\mathcal{U}) \ \right\}$$

where $\Gamma : \mathcal{J} \to \mathcal{F}_{ivf}(\mathcal{U})$ and

$$\Gamma(j) = \left\{ \ \mu_{\Gamma(j)}(u)/u : u \in \mathcal{U}, \mu_{\Gamma(j)}(u) \in [0, 1] \ \right\}$$

is an interval-valued fuzzy set over $\mathcal{U}$.

Example 8.2. Let $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and $\mathcal{J} = \{j_1, j_2, j_3, j_4, j_5, j_6, j_7, j_8\}$ where each j_i is r^{th}-tuple, r is the product of orders of $\mathcal{J}_i$, we have
Interval-Valued fuzzy hypersoft set $(\Gamma, \mathcal{J})$ is given as

$$(\Gamma, \mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{[0.1, 0.2]/u_1, [0.2, 0.3]/u_2, [0.4, 0.5]/u_4, [0.5, 0.6]/u_5\}), \\ (j_2, \{[0.1, 0.3]/u_1, [0.2, 0.4]/u_2, [0.3, 0.4]/u_3, [0.6, 0.8]/u_6\}), \\ (j_3, \{[0.2, 0.3]/u_2, [0.3, 0.4]/u_3, [0.4, 0.5]/u_4, , [0.5, 0.7]/u_5\}), \\ (j_4, \{[0.4, 0.5]/u_4, [0.5, 0.6]/u_5, [0.6, 0.7]/u_6, [0.7, 0.8]/u_7\}), \\ (j_5, \{[0.3, 0.6]/u_3, [0.6, 0.7]/u_6, [0.7, 0.8]/u_7, [0.8, 0.9]/u_8\}), \\ (j_6, \{[0.2, 0.4]/u_2, [0.3, 0.5]/u_3, [0.4, 0.6]/u_4, [0.7, 0.8]/u_7\}), \\ (j_7, \{[0.1, 0.4]/u_1, [0.3, 0.4]/u_3, [0.5, 0.7]/u_5, [0.6, 0.8]/u_6\}), \\ (j_8, \{[0.2, 0.5]/u_2, [0.3, 0.6]/u_3, [0.6, 0.8]/u_6, [0.7, 0.8]/u_7\}) \end{array} \right\}$$

Definition 8.3. A *fuzzy parameterized hypersoft set* ($fphs$-set) $(\mathcal{D}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{D}, \mathcal{J}) = \left\{ \ (\zeta_{\mathcal{F}}(j)/j, \psi_{\mathcal{F}}(j)) : j \in \mathcal{J}, \ \psi_{\mathcal{F}}(j) \in P(\mathcal{U}), \ \zeta_{\mathcal{F}}(j) \in [0, 1] \ \right\}$$

where $\mathcal{F}$ is a fuzzy set with $\zeta_{\mathcal{F}} : \mathcal{J} \to [0, 1]$ as membership function of $fphs$-set and $\psi_{\mathcal{F}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.4. From example 8.2, we have

$$(\mathcal{D}, \mathcal{J}) = \left\{ \begin{array}{l} (0.1/j_1, \{u_1, u_2\}), (0.2/j_2, \{u_1, u_2, u_3\}), (0.3/j_3, \{u_2, u_3, u_4\}), \\ (0.4/j_4, \{u_4, u_5, u_6\}), (0.5/j_5, \{u_6, u_7, u_8\}), (0.6/j_6, \{u_2, u_3, u_4, u_7\}), \\ (0.7/j_7, \{u_1, u_3, u_5, u_6\}), (0.8/j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.5. An *interval-valued fuzzy parameterized hypersoft set* (iv-$fphs$-set) $(\mathcal{E}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{E}, \mathcal{J}) = \left\{ \ (\eta_{\mathcal{F}_{iv}}(j)/j, \gamma_{\mathcal{F}_{iv}}(j)) : j \in \mathcal{J}, \ \gamma_{\mathcal{F}_{iv}}(j) \in P(\mathcal{U}), \ \eta_{\mathcal{F}_{iv}}(j) \in [0, 1] \ \right\}$$

where $\mathcal{F}_{iv}$ is an interval-valued fuzzy set with $\eta_{\mathcal{F}_{iv}} : \mathcal{J} \to [0, 1]$ as membership function of $fphs$-set and $\gamma_{\mathcal{F}_{iv}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.6. From example 8.2, we have

$$(\mathcal{E}, \mathcal{J}) = \left\{ \begin{array}{l} ([0.1, 0.2]/j_1, \{u_1, u_2\}), ([0.2, 0.3]/j_2, \{u_1, u_2, u_3\}), ([0.3, 0.4]/j_3, \{u_2, u_3, u_4\}), \\ ([0.4, 0.5]/j_4, \{u_4, u_5, u_6\}), ([0.5, 0.6]/j_5, \{u_6, u_7, u_8\}), ([0.6, 0.7]/j_6, \{u_2, u_3, u_4, u_7\}), \\ ([0.7, 0.8]/j_7, \{u_1, u_3, u_5, u_6\}), ([0.8, 0.9]/j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.7. An *intuitionistic fuzzy parameterized hypersoft set* ($ifphs$-set) $(\mathcal{H}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{H}, \mathcal{J}) = \left\{ \ (< \eta_T(j), \eta_F(j) > /j, \gamma_{\mathcal{IF}}(j)) : j \in \mathcal{J}, \ \gamma_{\mathcal{IF}}(j) \in P(\mathcal{U}), \ \eta_T(j) \in [0, 1], \eta_F(j) \in [0, 1] \ \right\}$$

where $\mathcal{IF}$ is an intuitionistic fuzzy set with $\eta_T(j), \eta_F(j) : \mathcal{J} \to [0, 1]$ as truth and falsity membership functions of $ifphs$-set and $\gamma_{\mathcal{IF}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.8. From example 8.2, we have

$$(\mathcal{H}, \mathcal{J}) = \left\{ \begin{array}{l} (< 0.1, 0.2 > /j_1, \{u_1, u_2\}), (< 0.2, 0.3 > /j_2, \{u_1, u_2, u_3\}), \\ (< 0.3, 0.4 > /j_3, \{u_2, u_3, u_4\}), (< 0.4, 0.5 > /j_4, \{u_4, u_5, u_6\}), \\ (< 0.5, 0.6 > /j_5, \{u_6, u_7, u_8\}), (< 0.6, 0.7 > /j_6, \{u_2, u_3, u_4, u_7\}), \\ (< 0.7, 0.8 > /j_7, \{u_1, u_3, u_5, u_6\}), (< 0.8, 0.9 > /j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.9. A *neutrosophic parameterized hypersoft set* (*nphs*-set) $(\mathcal{N}, \mathcal{J})$ over $\mathcal{U}$ is defined as

$$(\mathcal{N}, \mathcal{J}) = \left\{ \begin{array}{l} (< \eta_T(j), \eta_I(j), \eta_F(j) > /j, \gamma_{\mathcal{N}}(j)) : j \in \mathcal{J}, \gamma_{\mathcal{N}}(j) \in P(\mathcal{U}), \\ \eta_T(j) \in [0,1], \eta_I(j) \in [0,1], \eta_F(j) \in [0,1] \end{array} \right\}$$

where $\mathcal{IF}$ is an intuitionistic fuzzy set with $\eta_T(j), \eta_I(j), \eta_F(j) : \mathcal{J} \to [0,1]$ as truth, indeterminacy and falsity membership functions of *nphs*-set and $\gamma_{\mathcal{N}} : \mathcal{J} \to P(\mathcal{U})$ is called approximate function.

Example 8.10. From example 8.2, we have

$$(\mathcal{N}, \mathcal{J}) = \left\{ \begin{array}{l} (< 0.1, 0.2, 0.2 > /j_1, \{u_1, u_2\}), (< 0.2, 0.3, 0.3 > /j_2, \{u_1, u_2, u_3\}), \\ (< 0.3, 0.4, 0.4 > /j_3, \{u_2, u_3, u_4\}), (< 0.4, 0.5, 0.5 > /j_4, \{u_4, u_5, u_6\}), \\ (< 0.5, 0.6, 0.6 > /j_5, \{u_6, u_7, u_8\}), (< 0.6, 0.7, 0.7 > /j_6, \{u_2, u_3, u_4, u_7\}), \\ (< 0.7, 0.5, 0.8 > /j_7, \{u_1, u_3, u_5, u_6\}), (< 0.8, 0.4, 0.9 > /j_8, \{u_2, u_3, u_6, u_7\}) \end{array} \right\}$$

Definition 8.11. A hypersoft set $(\mathcal{B}, \mathcal{J})$ is said to be *bijective hypersoft set* (*bhs*-set) over $\mathcal{U}$ if

(i) $\bigcup_{j \in \mathcal{J}} \mathcal{B}(j) = \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}(j_p) \cap \mathcal{B}(j_q) = \emptyset$

Example 8.12. Taking data from example 8.2, we have

$$(\mathcal{B}, \mathcal{J}) = \left\{ (j_1, \{u_1\}), (j_2, \{u_2\}), (j_3, \{u_3\}), (j_4, \{u_4\}), (j_5, \{u_5\}), (j_6, \{u_6\}), (j_7, \{u_7\}), (j_8, \{u_8\}) \right\}$$

Definition 8.13. A fuzzy hypersoft set $(\mathcal{B}_f, \mathcal{J})$ is said to be *bijective fuzzy hypersoft set* (*bfhs*-set) over $\mathcal{U}$ if

(i) $\bigcup_{\substack{j \in \mathcal{J} \\ u \in \mathcal{U}}} \mathcal{B}_f(j) = \mathcal{U}$ with $\sum_{u \in \mathcal{U}} \mu_f(u) \in [0, 1]$ where $\mu_f(u)$ is a fuzzy membership for each

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_f(j_p) \cap \mathcal{B}_f(j_q) = \emptyset$

Example 8.14. Assuming example 8.2, we have

$$(\mathcal{B}_f, \mathcal{J}) = \left\{ \begin{array}{l} (j_1, \{0.1/u_1\}), (j_2, \{0.2/u_2\}), (j_3, \{0.13/u_3\}), (j_4, \{0.14/u_4\}), \\ (j_5, \{0.05/u_5\}), (j_6, \{0.06/u_6\}), (j_7, \{0.07/u_7\}), (j_8, \{0.08/u_8\}) \end{array} \right\}$$

Definition 8.15. An interval-valued fuzzy hypersoft set $(\mathcal{B}_{ivf}, \mathcal{J})$ is said to be *bijective interval-valued fuzzy hypersoft set* (*biv-fhs*-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{ivf}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} Sup(\mu_f(u)) \in [0,\,1]$ where $\mu_f(u)$ is an interval-valued fuzzy membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{ivf}(j_p) \cap \mathcal{B}_{ivf}(j_q) = \emptyset$

Example 8.16. Suppose sets given in example 8.2, we have

$$(\mathcal{B}_{ivf},\mathcal{J}) = \left\{ \begin{array}{l} (j_1,\{[0.01,0.1]/u_1\}),(j_2,\{[0.02,0.2]/u_2\}),(j_3,\{[0.03,0.13]/u_3\}),(j_4,\{[0.04,0.14]/u_4\}), \\ (j_5,\{[0.03,0.05]/u_5\}),(j_6,\{[0.02,0.06]/u_6\}),(j_7,\{[0.03,0.07]/u_7\}),(j_8,\{[0.04,0.08]/u_8\}) \end{array} \right\}$$

Definition 8.17. An intuitionistic fuzzy hypersoft set $(\mathcal{B}_{if},\mathcal{J})$ is said to be *bijective intuitionistic fuzzy hypersoft set* ($bifhs$-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{if}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} T_{if}(u) \in [0,\,1]$ and $\sum\limits_{u\in\mathcal{U}} F_{if}(u) \in [0,\,1]$ where $T_{if}(u)$ and $F_{if}(u)$ are truth and false membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{if}(j_p) \cap \mathcal{B}_{if}(j_q) = \emptyset$

Example 8.18. Let the sets provided in example 8.2, we have

$$(\mathcal{B}_{if},\mathcal{J}) = \left\{ \begin{array}{l} (j_1,\{<0.01,0.1>/u_1\}),(j_2,\{<0.02,0.2>/u_2\}),(j_3,\{<0.03,0.13>/u_3\}), \\ (j_4,\{<0.04,0.14>/u_4\}),(j_5,\{<0.03,0.05>/u_5\}),(j_6,\{<0.02,0.06>/u_6\}), \\ (j_7,\{<0.03,0.07>/u_7\}),(j_8,\{<0.04,0.08>/u_8\}) \end{array} \right\}$$

Definition 8.19. An neutrosophic hypersoft set $(\mathcal{B}_{\mathcal{N}},\mathcal{J})$ is said to be *bijective neutrosophic hypersoft set* ($bnhs$-set) over $\mathcal{U}$ if

(i) $\bigcup\limits_{j\in\mathcal{J}} \mathcal{B}_{\mathcal{N}}(j) = \mathcal{U}$ with $\sum\limits_{u\in\mathcal{U}} T_{\mathcal{N}}(u) \in [0,\,1]$, $\sum\limits_{u\in\mathcal{U}} I_{\mathcal{N}}(u) \in [0,\,1]$ and $\sum\limits_{u\in\mathcal{U}} F_{\mathcal{N}}(u) \in [0,\,1]$ where $T_{\mathcal{N}}(u), I_{\mathcal{N}}(u)$ and $F_{\mathcal{N}}(u)$ are truth, indeterminacy and false membership for each $u \in \mathcal{U}$

(ii) for $j_p, j_q \in \mathcal{J}, p \neq q, \mathcal{B}_{\mathcal{N}}(j_p) \cap \mathcal{B}_{\mathcal{N}}(j_q) = \emptyset$

Example 8.20. Considering example 8.2, we have

$$(\mathcal{B}_{\mathcal{N}},\mathcal{J}) = \left\{ \begin{array}{l} (j_1,\{<0.01,0.02,0.1>/u_1\}),(j_2,\{<0.02,0.03,0.2>/u_2\}), \\ (j_3,\{<0.03,0.04,0.13>/u_3\}),(j_4,\{<0.04,0.05,0.14>/u_4\}), \\ (j_5,\{<0.03,0.04,0.05>/u_5\}),(j_6,\{<0.02,0.05,0.06>/u_6\}), \\ (j_7,\{<0.03,0.04,0.07>/u_7\}),(j_8,\{<0.04,0.05,0.08>/u_8\}) \end{array} \right\}$$

9. Conclusions

In this study, fundamental properties, aggregation operations, basic set laws, relations and functions are characterized under hypersoft set environment. Moreover, essential concepts of matrices and their basic operations are discussed for hypersoft sets. Future work may include the development of hybrids of hypersoft set with fuzzy set, rough set, expert set, cubic set etc. and algebraic structures like hypersoft topological spaces, hypersoft functional spaces, hypersoft groups, hypersoft vector spaces, hypersoft ring, hypersoft measure etc.

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Chapter 2

Hybrid set structures under uncertainly parameterized hypersoft sets: Theory and applications

Irfan Deli

Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey. E-mail: irfandeli@kilis.edu.tr

Abstract: In this paper, we firstly introduced a hybrid set structure with hypersoft sets under uncertainty, vagueness and indeterminacy, called neutrosophic valued n−attribute neutrosophic hypersoft set, which is a combination of neutrosophic sets and hypersoft sets. The neutrosophic valued n−attribute neutrosophic hypersoft set (n − NNHS−set) generalizes the following hybrid sets as; n−attribute Hypersoft Set, n−attribute Fuzzy Hypersoft Set, n−attribute Intuitionistic Fuzzy Hypersoft Set, n−attribute Neutrosophic Hypersoft Set , Fuzzy Valued n−attribute Hypersoft Set, Fuzzy Valued n−attribute Fuzzy Hypersoft Set, Fuzzy Valued n−attribute Intuitionistic Fuzzy Hypersoft Set, Fuzzy Valued n−attribute Neutrosophic Hypersoft Set, Intuitionistic Fuzzy Valued n−attribute Hypersoft Set, Intuitionistic Fuzzy Valued n−attribute Fuzzy Hypersoft Set, Intuitionistic Fuzzy Valued n−attribute Intuitionistic Fuzzy Hypersoft Set, Intuitionistic Fuzzy Valued n−attribute Neutrosophic Hypersoft Set, Neutrosophic Valued n−attribute Hypersoft Set, Neutrosophic Valued n−attribute Fuzzy Hypersoft Set, Neutrosophic Valued n−attribute Intuitionistic Fuzzy Hypersoft Set, neutrosophic parameterized neutrosophic soft set, and so on. Then, we introduce some definitions and operations on n − NNHS−set and some properties of the sets which are connected to operations. Finally, we proposed the decision-making method on the n−NNHS−set and presented a numerical example to show that this method can be successfully applied.

Keywords: *Fuzzy set, Intuitionistic Fuzzy set, Neutrosophic set, Soft set, hypersoft sets, n − NNHS−set, decision making*

1. Introduction

To handle uncertainties or indeterminate information, fuzzy sets [34] in [0,1], intuitionistic fuzzy sets [2] in [0,1]×[0,1], neutrosophic sets [25,32] in [0,1]×[0,1]×[0,1] and soft sets [22] in parameterize set and universe set are consistently being defined as efficient mathematical tools. Recently, soft sets have been developed by embedding the idea of fuzzy set, intuitionistic fuzzy set, neutrosophic set to expand the field of application of soft sets. For example, fuzzy soft sets [19], intuitionistic fuzzy soft sets [6], neutrosophic soft sets [8], fuzzy parameterized soft sets [5], fuzzy parameterized fuzzy soft sets [12], fuzzy parameterized intuitionistic fuzzy soft sets [10], fuzzy parameterized neutrosophic soft sets (Special version of [13]), intuitionistic fuzzy parameterized soft sets [9], intuitionistic fuzzy parameterized fuzzy soft sets [33], intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets [11], neutrosophic parameterized soft sets [12] and neutrosophic parameterized neutrosophic soft sets [13], are some of the studies. Also, soft sets generalized to the hypersoft set [31] by using a multi-argument function and n distinct attribute sets. Also, some studies have made by the many authors, for example, on basic operations on hypersoft sets and hypersoft point [1], on m-Polar and m-Polar interval valued neutrosophic hypersoft sets [26], on aggregate operators of neutrosophic hypersoft set [27], on decision making based on TOPSIS under neutrosophic soft set [28]- [24], on generalized aggregate operators on neutrosophic hypersoft set [35], on generalization of TOPSIS for neutrosophic hypersoft set [29], on single and multi-valued neutrosophic hypersoft set [30], on multi-valued interval neutrosophic linguistic soft set [15], and son on.

In this chapter, we introduced hybrid set structure with hypersoft sets under uncertainty, vagueness and indeterminacy, called neutrosophic valued $n-$attribute neutrosophic hypersoft set, which is a combination of neutrosophic sets [25] and hypersoft sets [31]. The neutrosophic valued $n-$attribute neutrosophic hypersoft set generalizes the following sets:

(1) $n-$attribute Hypersoft Set [31],
(2) $n-$attribute Fuzzy Hypersoft Set [31],
(3) $n-$attribute Intuitionistic Fuzzy Hypersoft Set [31],
(4) $n-$attribute Neutrosophic Hypersoft Set [31],
(5) Fuzzy Valued $n-$attribute Hypersoft Set (Special version of this study),
(6) Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set (Special version of this study),
(7) Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set (Special version of this study),
(8) Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set (Special version of this study),
(9) Intuitionistic Fuzzy Valued $n-$attribute Hypersoft Set (Special version of this study),
(10) Intuitionistic Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set (Special version of this study),
(11) Intuitionistic Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set (Special version of this study),
(12) Intuitionistic Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set (Special version of this study),
(13) Neutrosophic Valued $n-$attribute Hypersoft Set (Special version of this study),
(14) Neutrosophic Valued $n-$attribute Fuzzy Hypersoft Set (Special version of this study),
(15) Neutrosophic Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set (Special version of this study),
(16) Soft sets [4,22],
(17) fuzzy soft sets [19],
(18) intuitionistic fuzzy soft sets [6,18,21],
(19) neutrosophic soft sets [8,16,17,20],
(20) fuzzy parameterized soft sets [5],
(21) fuzzy parameterized fuzzy soft sets [12],

(22) fuzzy parameterized intuitionistic fuzzy soft sets [10],

(23) fuzzy parameterized neutrosophic soft sets(Special version of [13]),

(24) intuitionistic fuzzy parameterized soft sets [9],

(25) intuitionistic fuzzy parameterized fuzzy soft sets [33],

(26) intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets [11],

(27) intuitionistic fuzzy parameterized neutrosophic soft sets(Special version of [13]),

(28) neutrosophic parameterized soft sets [3],

(29) neutrosophic parameterized fuzzy soft sets (Special version of [13]),

(30) neutrosophic parameterized intuitionistic fuzzy soft sets(Special version of [13]),

(31) neutrosophic parameterized neutrosophic soft sets [13],

Therefore, the neutrosophic valued $n-$attribute neutrosophic hypersoft set is a powerful general formal framework and is most comprehensive of above sets to handle the indeterminate information and inconsistent information which exist commonly in real situations. Researchers, can be just study on neutrosophic valued $n-$attribute neutrosophic hypersoft sets which are the most general form of the above hybrid sets in (1-31). Thus, instead of working separately, it will save time and it will be easier to research and find solutions of problems which contain uncertainties or indeterminate information. To do this, the rest of this paper is structured as follows: In section 2, we give the basic definitions and results of fuzzy set theory, intuitionistic fuzzy set theory, neutrosophic set theory, soft set theory, neutrosophic parameterized neutrosophic soft set theory and hypersoft set theory. In section 3, we develop the Hybrid Set Structure with Hypersoft Sets under Uncertainty, Vagueness and Indeterminacy, called neutrosophic valued $n-$attribute neutrosophic hypersoft sets, including some special hybrid set structures, NPNSS-aggregation operator, decision making algorithm. In section 4, we present a section of conclusions.

2. **Preliminary**

In this section, we give the basic definitions and results of fuzzy set theory, intuitionistic fuzzy set theory, neutrosophic set theory, soft set theory, neutrosophic parameterized neutrosophic soft set theory and hypersoft set theory that are useful for subsequent discussions. For more details, the reader could refer to [2,8,9,13,14,16,17,20,22,23,25,31,32,34].

Definition 2.1. [34] Let U be a space of points (objects), with a generic element in U denoted by u. A fuzzy set F in U is characterized by a membership function $\mu_F : U \to [0,1]$. It can be written as

$$F = \{< u, (\mu_F(u)) >: u \in U,\ \mu_F(u) \in [0,1]\}.$$

Definition 2.2. [14] t-norms are associative, monotonic and commutative two valued functions t that map from $[0,1] \times [0,1]$ into $[0,1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in [0,1]$,

(1) $t(0,0) = 0$ and $t(a,1) = t(1,a) = a$,

(2) If $a \leq c$ and $b \leq d$, then $t(a,b) \leq t(c,d)$

(3) $t(a,b) = t(b,a)$

(4) $t(a, t(b,c)) = t(t(a,b,c))$

Definition 2.3. [14] t-conorms (s-norm) are associative, monotonic and commutative two placed functions s which map from $[0,1] \times [0,1]$ into $[0,1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in [0,1]$,

(1) $s(1,1) = 1$ *and* $s(a,0) = s(0,a) = a$,

(2) if $a \leq c$ and $b \leq d$, then $s(a,b) \leq s(c,d)$

(3) $s(a,b) = s(b,a)$

(4) $s(a, s(b,c)) = s(s(a,b,c)$

t-norm and t-conorm are related in a sense of logical duality. Typical dual pairs of non parametrized t-norm and t-conorm are complied below:

(1) Drastic product:
$$t_w(a,b) = \begin{cases} min\{a,b\}, & max\{ab\} = 1 \\ 0, & otherwise \end{cases}$$

(2) Drastic sum:
$$s_w(a,b) = \begin{cases} max\{a,b\}, & min\{ab\} = 0 \\ 1, & otherwise \end{cases}$$

(3) Bounded product:
$$t_1(a,b) = max\{0, a+b-1\}$$

(4) Bounded sum:
$$s_1(a,b) = min\{1, a+b\}$$

(5) Einstein product:
$$t_{1.5}(a,b) = \frac{a.b}{2-[a+b-a.b]}$$

(6) Einstein sum:
$$s_{1.5}(a,b) = \frac{a+b}{1+a.b}$$

(7) Algebraic product:
$$t_2(a,b) = a.b$$

(8) Algebraic sum:
$$s_2(a,b) = a+b-a.b$$

(9) Hamacher product:
$$t_{2.5}(a,b) = \frac{a.b}{a+b-a.b}$$

(10) Hamacher sum:
$$s_{2.5}(a,b) = \frac{a+b-2.a.b}{1-a.b}$$

(11) Minumum:

$$t_3(a, b) = min\{a, b\}$$

(12) Maximum:

$$s_3(a, b) = max\{a, b\}$$

Definition 2.4. [2] Let U be a space of points (objects), with a generic element in U denoted by u. A intuitionistic fuzzy set (IF-set) IF in U is characterized by a membership function $\mu_{IF} : U \to [0, 1]$ and a non-membership function $\nu_{IF} : U \to [0, 1]$. It can be written as

$$IF = \{< u, (\mu_{IF}(u), \nu_{IF}(u)) >: u \in U, \ \mu_{IF}(u), \mu_{IF}(u) \in [0, 1]\}.$$

where $0 \leq \mu_{IF}(u) + \nu_{IF}(u) \leq 1$.

Definition 2.5. [32] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set N in U is characterized by a truth-membership function $T_N : U \to [0, 1]$, a indeterminacy-membership function $I_N : U \to [0, 1]$ and a falsity-membership function $F_N : U \to [0, 1]$. It can be written as

$$N = \{< u, (T_N(u), I_N(u), F_N(u)) >: u \in U\}.$$

There is no restriction on the sum of $T_N(u)$; $I_N(u)$ and $F_N(u)$, so $0 \leq T_N(u) + I_N(u) + F_N(u) \leq 3$. Also, for special cases, we have in the following hybrid set structures:

(1) A neutrosophic set for $I_N(u) = 0$ and $F_N(u) = 1 - T_N(u)$ reduced to fuzzy set [34] as;

$$N = \{< u, (T_N(u), 0, 1 - T_N(u))) >: u \in U\}.$$

(2) A neutrosophic set for $I_N(u) = 0$ and $0 \leq F_N(u) + T_N(u) \leq 1$ reduced to intuitionistic fuzzy set [2] as;

$$N = \{< u, (T_N(u), 0, F_N(u))) >: u \in U\}.$$

Definition 2.6. [22] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters. Then a soft set S over U is a set defined by a function representing a mapping $f_S : E \to P(U)$ Here, f_X is called approximate function of the soft set S, and the value $f_S(x)$ is a set called x*-element* of the soft set for all $x \in E$. Thus, a soft set over U can be represented by the set of ordered pairs

$$S = \{(x, f_S(x)) : x \in E, f_S(x) \in P(U)\}$$

Definition 2.7. [31] Let U be a universe, E be a set of attributes that are describe the elements of U, $E_i \subseteq E(i \in \{1, 2, ..., n\})$ be i set of attributes such that $E_i \cap E_j \neq \emptyset$, for all $i \neq j$, and $i, j \in \{1, 2, ..., n\}$.

Then, an $n-$attribute Hypersoft Set($n-HS-$set) S_{n-HS} over U is a set defined by a set valued function $f_{S_{n-HS}}$ representing a mapping

$$f_{S_{n-HS}} : E_1 \times E_2 \times ... \times E_n \to P(U)$$

where $f_{S_{n-HS}}$ is called approximate function of the S_{n-HS}. For $(x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n$, the set $f_{S_{n-HS}}(x_1, x_2, ...x_n)$ is called $(x_1, x_2, ...x_n)-$approximation of the S_{n-HS}. It can be written as a set of ordered pairs,

$$S_{n-HS} = \Big\{((x_1, x_2, ...x_n), f_{S_{n-HS}}(x_1, x_2, ...x_n)) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n, \\ f_{S_{n-HS}}(x_1, x_2, ...x_n) \in U\Big\}$$

Definition 2.8. [13] Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of parameters that are describe the elements of U and K be a neutrosophic set over E. Then, a neutrosophic parameterized neutrosophic soft set($npn-$soft set) S_{NpN} over U is a set defined by a set valued function $f_{S_{NpN}}$ representing a mapping

$$f_{S_{NpN}} : K \to N(U)$$

where $f_{S_{NpN}}$ is called approximate function of the $npn-$soft set S_{NpN}. For $x \in E$, the set $f_{S_{NpN}}(x)$ is called x-approximation of the $npn-$soft set S_{NpN}. It can be written a set of ordered pairs,

$$S_{NpN} = \Big\{(< x, T_{S_{NpN}}(x), I_{S_{NpN}}(x), F_{S_{NpN}}(x) >, \\ \{< u, T_{f_{S_{NpN}}(x)}(u), I_{f_{S_{NpN}}(x)}(u), F_{f_{S_{NpN}}(x)}(u) >: u \in U\}) : x \in E\Big\}$$

where

$$F_{S_{NpN}}(x), I_{S_{NpN}}(x), T_{S_{NpN}}(x), T_{S_{NpN}}(u), I_{S_{NpN}}(u), F_{S_{NpN}}(u) \in [0, 1].$$

3. Hybrid Set Structure with Hypersoft Sets under Uncertainty, Vagueness and Indeterminacy

In this section, we combined the concept of Hypersoft set [31] and neutrosophic set [32], for generalizing $npn-$soft set [13], by introducing a new hybrid set structure called neutrosophic valued $n-$attribute neutrosophic Hypersoft set($n-NNHS-$set). Then, we introduce some definitions and operations on $n-NNHS-$sets and some properties of the $n-NNHS-$sets which are connected to operations have been proposed. Some of it is quoted or inspired or generalized from [8, 9, 13, 16, 17, 20, 23, 25, 31].

3.1. *Neutrosophic Valued $n-$attribute Neutrosophic Hypersoft Sets*

Definition 3.1. Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of attributes that are described the elements of U, $E_i \subseteq E(i \in \{1, 2, ..., n\})$ be i set of attributes such that $E_i \cap E_j \neq \emptyset$, for all $i \neq j$, and $i, j \in \{1, 2, ..., n\}$ and K_i be a neutrosophic set over E_i. Then, a

Neutrosophic Valued $n-$attribute Neutrosophic Hypersoft Set($n - NNHS-$set) S_{n-NNHS} over U is a set defined by a set valued function $f_{S_{n-NNHS}}$ representing a mapping

$$f_{S_{n-NNHS}} : K_1 \times K_2 \times ... \times K_n \to N(U)$$

where $f_{S_{n-NNHS}}$ is called approximate function of the S_{n-NNHS}. For $(x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n$, the set $f_{S_{n-NNHS}}(x_1, x_2, ...x_n)$ is called $(x_1, x_2, ...x_n)-$approximation of the S_{n-NNHS}. It can be written as a set of ordered pairs,

$$S_{n-NNHS} = \Bigg\{ \Big(\quad (< x_1, T_{S_{n-NNHS}}(x_1), I_{S_{n-NNHS}}(x_1), F_{S_{n-NNHS}}(x_1) >, < x_2, T_{S_{n-NNHS}}(x_2), \\ I_{S_{n-NNHS}}(x_2), F_{S_{n-NNHS}}(x_2) >, ..., < x_n, T_{S_{n-NNHS}}(x_n), I_{S_{n-NNHS}}(x_n), \\ F_{S_{n-NNHS}}(x_n) >), \{< u, T_{S_{n-NNHS(x_1,x_2,...x_n)}}(u), I_{S_{n-NNHS(x_1,x_2,...x_n)}}(u), \\ F_{S_{n-NNHS(x_1,x_2,...x_n)}}(u) >: u \in U\}\Big) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Bigg\}$$

where

$T_{S_{n-NNHS}}(x_i)(i \in \{1, 2, ..., n\})$, $I_{S_{n-NNHS}}(x_i)(i \in \{1, 2, ..., n\})$, $F_{S_{n-NNHS}}(x_i) \in [0, 1](i \in \{1, 2, ..., n\})$,

and

$T_{S_{n-NNHS(x_1,x_2,...x_n)}}(u)$, $I_{S_{n-NNHS(x_1,x_2,...x_n)}}(u)$, $F_{S_{n-NNHS(x_1,x_2,...x_n)}}(u) \in [0, 1]$

such that

$0 \le T_{S_{n-NNHS}}(x_i) + I_{S_{n-NNHS}}(x_i), F_{S_{n-NNHS}}(x_i) \le 3(i \in \{1, 2, ..., n\})$

and

$0 \le T_{S_{n-NNHS(x_1,x_2,...x_n)}}(u) + I_{S_{n-NNHS(x_1,x_2,...x_n)}}(u) + F_{S_{n-NNHS(x_1,x_2,...x_n)}}(u) \le 3.$

Note that if the set of attributes E_i is single then the $n-NNHS-$sets reduced to $npn-$soft sets [13]. Therefore, the $n - NNHS-$sets generalizes the following sets:

(1) Soft sets [22],

(2) fuzzy soft sets [19],

(3) intuitionistic fuzzy soft sets [6],

(4) neutrosophic soft sets [8],

(5) fuzzy parameterized soft sets [5],

(6) fuzzy parameterized fuzzy soft sets [12],

(7) fuzzy parameterized intuitionistic fuzzy soft sets [10],

(8) fuzzy parameterized neutrosophic soft sets(Special version of [13]),

(9) intuitionistic fuzzy parameterized soft sets [9],

(10) intuitionistic fuzzy parameterized fuzzy soft sets [33],

(11) intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets [11],

(12) intuitionistic fuzzy parameterized neutrosophic soft sets(Special version of [13]),

(13) neutrosophic parameterized soft sets [3],

(14) neutrosophic parameterized fuzzy soft sets (Special version of [13]),

(15) neutrosophic parameterized intuitionistic fuzzy soft sets(Special version of [13]),

(16) neutrosophic parameterized neutrosophic soft sets [13],

Definition 3.2. Let S_{n-NNHS} be an $n-NNHS-$set. Then, the complement of S_{n-NNHS} denoted by S^c_{n-NNHS} and is defined by

$$S^c_{n-NNHS}=\Big\{\big(\quad (<x_1, F_{S_{n-NNHS}}(x_1), 1-I_{S_{n-NNHS}}(x_1), T_{S_{n-NNHS}}(x_1)>, <x_2, F_{S_{n-NNHS}}(x_2), 1-I_{S_{n-NNHS}}(x_2), T_{S_{n-NNHS}}(x_2)>, ..., <x_n, F_{S_{n-NNHS}}(x_n), 1-I_{S_{n-NNHS}}(x_n), T_{S_{n-NNHS}}(x_n)>), \{<u, F_{S_{n-NNHS(x_1,x_2,...x_n)}}(u), 1-I_{S_{n-NNHS(x_1,x_2,...x_n)}}(u), T_{S_{n-NNHS(x_1,x_2,...x_n)}}(u)>: u\in U\}) : (x_1,x_2,...x_n)\in E_1\times E_2\times ... \times E_n\Big\}$$

Definition 3.3. Let S^1_{n-NNHS} and S^2_{n-NNHS} be two $n-NNHS-$sets. Then, S^1_{n-NNHS} is said to be $n-NNHS-$subset of S^2_{n-NNHS}, is denoted by $S^1_{n-NNHS}\tilde{\subseteq}S^2_{n-NNHS}$, if

$$T_{S^1_{n-NNHS}}(x_i)\le T_{S^2_{n-NNHS}}(x_i), T_{S^1_{n-NNHS(x_1,x_2,...x_n)}}(u)\le T_{S^2_{n-NNHS(x_1,x_2,...x_n)}}(u),$$

$$I_{S^1_{n-NNHS}}(x_i)\ge I_{S^2_{n-NNHS}}(x_i), I^1_{S_{n-NNHS(x_1,x_2,...x_n)}}(u)\ge I_{S^2_{n-NNHS(x_1,x_2,...x_n)}}(u),$$

$$F_{S^1_{n-NNHS}}(x_i)\ge F_{S^2_{n-NNHS}}(x_i), F_{S^1_{n-NNHS(x_1,x_2,...x_n)}}(u)\ge F_{S^2_{n-NNHS(x_1,x_2,...x_n)}}(u).$$

for all $i\in\{1,2,...,n\}$.

Also, If S^1_{n-NNHS} is $n-NNHS-$subset of S^2_{n-NNHS} and S^2_{n-NNHS} is $n-NNHS-$subset of S^1_{n-NNHS}, then S^2_{n-NNHS} and S^2_{n-NNHS} is equal and we denote it with $S^1_{n-NNHS}=S^1_{n-NNHS}$.

Proposition 3.4. *Let S^1_{n-NNHS}, S^2_{n-NNHS} and S^3_{n-NNHS} be any three $n-NNHS-$sets. Then,*

(1) $S^1_{n-NNHS}\tilde{\subseteq}S^1_{n-NNHS}$

(2) $S^1_{n-NNHS}\tilde{\subseteq}S^2_{n-NNHS}$ *and* $S^2_{n-NNHS}\tilde{\subseteq}S^3_{n-NNHS}\Rightarrow S^1_{n-NNHS}\tilde{\subseteq}S^3_{n-NNHS})$

(3) $S^1_{n-NNHS}=S^2_{n-NNHS}$ *and* $S^2_{n-NNHS}=S^3_{n-NNHS}\Rightarrow S^1_{n-NNHS}=S^3_{n-NNHS})$

Definition 3.5. Let S^1_{n-NNHS} be an $n-NNHS-$set. Then, S^1_{n-NNHS} is called null $n-NNHS-$set, denoted by $S_\emptyset$, if

$$T_{S^1_{n-NNHS}}(x_i)=0, T_{S^1_{n-NNHS(x_1,x_2,...x_n)}}(u)=0,$$

$$I_{S^1_{n-NNHS}}(x_i)=1, I_{S^1_{n-NNHS(x_1,x_2,...x_n)}}(u)=1,$$

$$F_{S^1_{n-NNHS}}(x_i)=1, F_{S^1_{n-NNHS(x_1,x_2,...x_n)}}(u)=1.$$

for all $i\in\{1,2,...,n\}$

(or

$$T_{S^1_{n-NNHS}}(x_i) = 0, T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 0,$$

$$I_{S^1_{n-NNHS}}(x_i) = 0, I_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 0,$$

$$F_{S^1_{n-NNHS}}(x_i) = 1, F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1.$$

for all $i \in \{1, 2, ..., n\}$.(The definition do not use in this study))

Definition 3.6. Let S^1_{n-NNHS} be an $n - NNHS-$set. Then, S^1_{n-NNHS} is called universal $n - NNHS-$set, denoted by S_U, if

$$T_{S^1_{n-NNHS}}(x_i) = 1, T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1,$$

$$I_{S^1_{n-NNHS}}(x_i) = 0, I^1_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) = 0,$$

$$F_{S^1_{n-NNHS}}(x_i) = 0, F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 0.$$

(or

$$T_{S^1_{n-NNHS}}(x_i) = 1, T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1,$$

$$I_{S^1_{n-NNHS}}(x_i) = 1, I^1_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1,$$

$$F_{S^1_{n-NNHS}}(x_i) = 0, F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u) = 0.$$

for all $i \in \{1, 2, ..., n\}$.(The definition do not use in this study))

Proposition 3.7. *Let S^1_{n-NNHS}, S^2_{n-NNHS} and S^3_{n-NNHS} be any three $n - NNHS-$sets. Then,*

(1) $(S_U)^c = S_\emptyset$
(2) $(S_\emptyset)^c = S_U$
(3) $S^1_{n-NNHS} \tilde{\subseteq} S_U$
(4) $((S^1_{n-NNHS})^c)^c = S^1_{n-NNHS}$
(5) $S_\emptyset \tilde{\subseteq} S^1_{n-NNHS}$

Definition 3.8. Let S^1_{n-NNHS} and S^2_{n-NNHS} be two $n - NNHS-$sets. Then, the union(or sum) of S^1_{n-NNHS} and S^2_{n-NNHS} is denoted by $S^3_{n-NNHS} = S^1_{n-NNHS} \tilde{\cup} S^2_{n-NNHS}$(or $S^3_{n-NNHS} = S^1_{n-NNHS} \tilde{+} S^2_{n-NNHS}$) and is defined by

$$S^3_{n-NNHS} = \Big\{ (\quad (< x_1, T_{S^3_{n-NNHS}}(x_1), I_{S^3_{n-NNHS}}(x_1), F_{S^3_{n-NNHS}}(x_1) >, < x_2, T_{S^3_{n-NNHS}}(x_2), \\ I_{S^3_{n-NNHS}}(x_2), F_{S^3_{n-NNHS}}(x_2) >, ..., < x_n, T_{S^3_{n-NNHS}}(x_n), I_{S^3_{n-NNHS}}(x_n), \\ F_{S^3_{n-NNHS}}(x_n) >), \{< u, T_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u), \\ F_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\}$$

where

$$T_{S^3_{n-NNHS}}(x_i) = s(T_{S^1_{n-NNHS}}(x_i), T_{S^2_{n-NNHS}}(x_i)),$$

$$T_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u) = s(T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), T_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)),$$

$$I_{S^3_{n-NNHS}}(x_i) = t(I_{S^1_{n-NNHS}}(x_i), I_{S^2_{n-NNHS}}(x_i)),$$

$$I_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u) = t(I_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)),$$

$$F_{S^3_{n-NNHS}}(x_i) = t(F_{S^1_{n-NNHS}}(x_i), F_{S^2_{n-NNHS}}(x_i)),$$

$$F_{S^3_{n-NNHS}(x_1,x_2,...x_n)}(u) = t(F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), F_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)).$$

for all $i \in \{1, 2, ..., n\}$.

Proposition 3.9. *Let S^1_{n-NNHS}, S^2_{n-NNHS} and S^3_{n-NNHS} be any three $n-NNHS-$sets. Then,*

(1) $S^1_{n-NNHS}\widetilde{\cup}S_U = S_U$
(2) $S^1_{n-NNHS}\widetilde{\cup}S_\emptyset = S^1_{n-NNHS}$
(3) $S^1_{n-NNHS}\widetilde{\cup}(S^1_{n-NNHS} = (S^1_{n-NNHS}$
(4) $S^1_{n-NNHS}\widetilde{\cup}S^2_{n-NNHS} = S^2_{n-NNHS}\widetilde{\cup}S^1_{n-NNHS}$
(5) $S^1_{n-NNHS}\widetilde{\cup}(S^2_{n-NNHS}\widetilde{\cup}S^3_{n-NNHS}) = (S^1_{n-NNHS}\widetilde{\cup}S^2_{n-NNHS})\widetilde{\cup}S^3_{n-NNHS}$

Definition 3.10. Let S^1_{n-NNHS} and S^2_{n-NNHS} be two $n-NNHS-$sets. Then, the intersection(or product) of S^1_{n-NNHS} and S^2_{n-NNHS} is denoted by $S^4_{n-NNHS} = S^1_{n-NNHS}\tilde{\cap}S^2_{n-NNHS}$ (or $S^4_{n-NNHS} = S^1_{n-NNHS}\tilde{\times}S^2_{n-NNHS}$) and is defined by

$$S^4_{n-NNHS} = \Big\{ (\quad (< x_1, T_{S^4_{n-NNHS}}(x_1), I_{S^4_{n-NNHS}}(x_1), F_{S^4_{n-NNHS}}(x_1) >, < x_2, T_{S^4_{n-NNHS}}(x_2), I_{S^4_{n-NNHS}}(x_2), F_{S^4_{n-NNHS}}(x_2) >, ..., < x_n, T_{S^4_{n-NNHS}}(x_n), I_{S^4_{n-NNHS}}(x_n), F_{S^4_{n-NNHS}}(x_n) >), \{< u, T_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u), F_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\}$$

where

$$T_{S^4_{n-NNHS}}(x_i) = t(T_{S^1_{n-NNHS}}(x_i), T_{S^2_{n-NNHS}}(x_i)),$$

$$T_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u) = t(T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), T_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)),$$

$$I_{S^4_{n-NNHS}}(x_i) = s(I_{S^1_{n-NNHS}}(x_i), I_{S^2_{n-NNHS}}(x_i)),$$

$$I_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u) = s(I_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)),$$

$$F_{S^4_{n-NNHS}}(x_i) = s(F_{S^1_{n-NNHS}}(x_i), F_{S^2_{n-NNHS}}(x_i)),$$

$$F_{S^4_{n-NNHS}(x_1,x_2,...x_n)}(u) = s(F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u), F_{S^2_{n-NNHS}(x_1,x_2,...x_n)}(u)).$$

for all $i \in \{1, 2, ..., n\}$.

Proposition 3.11. *Let S^1_{n-NNHS}, S^2_{n-NNHS} and S^3_{n-NNHS} be any three $n-NNHS-$sets. Then,*

(1) $S^1_{n-NNHS} \widetilde{\cap} S_U = S^1_{n-NNHS}$
(2) $S^1_{n-NNHS} \widetilde{\cap} S_\emptyset = S_\emptyset$
(3) $S^1_{n-NNHS} \widetilde{\cap} S^1_{n-NNHS} = S^1_{n-NNHS}$
(4) $S^1_{n-NNHS} \widetilde{\cap} S^2_{n-NNHS} = S^2_{n-NNHS} \widetilde{\cap} S^1_{n-NNHS}$
(5) $S^1_{n-NNHS} \widetilde{\cap} (S^2_{n-NNHS} \widetilde{\cap} S^3_{n-NNHS}) = (S^1_{n-NNHS} \widetilde{\cap} S^2_{n-NNHS}) \widetilde{\cap} S^3_{n-NNHS}$

Definition 3.12. Let S^1_{n-NNHS} be an $n-NNHS-$set and $\gamma \neq 0$. Then, $S^5_{n-NNHS} = \gamma(S^1_{n-NNHS})$ is defined as;

$$S^5_{n-NNHS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S^5_{n-NNHS}}(x_1), I_{S^5_{n-NNHS}}(x_1), F_{S^5_{n-NNHS}}(x_1) >, < x_2, T_{S^5_{n-NNHS}}(x_2), \\ \quad I_{S^5_{n-NNHS}}(x_2), F_{S^5_{n-NNHS}}(x_2) >, ..., < x_n, T_{S^5_{n-NNHS}}(x_n), I_{S^5_{n-NNHS}}(x_n), \\ \quad F_{S^5_{n-NNHS}}(x_n) >), \{< u, T_{S^5_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^5_{n-NNHS}(x_1,x_2,...x_n)}(u), \\ \quad F_{S^5_{n-NNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$$T_{S^5_{n-NNHS}}(x_i) = 1 - (1 - T_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$T_{S^5_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1 - (1 - T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma,$$

$$I_{S^5_{n-NNHS}}(x_i) = (I_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$I^5_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) = (I_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma,$$

$$F_{S^5_{n-NNHS}}(x_i) = (F_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$F_{S^5_{n-NNHS}(x_1,x_2,...x_n)}(u) = (F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma.$$

for all $i \in \{1, 2, ..., n\}$.

Definition 3.13. Let S^1_{n-NNHS} be an $n-NNHS-$set and $\gamma \neq 0$. Then, $S^6_{n-NNHS} = (S^1{}_{n-NNHS})^\gamma$ is defined as;

$$S^6_{n-NNHS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S^6_{n-NNHS}}(x_1), I_{S^6_{n-NNHS}}(x_1), F_{S^6_{n-NNHS}}(x_1) >, < x_2, T_{S^6_{n-NNHS}}(x_2), \\ I_{S^6_{n-NNHS}}(x_2), F_{S^6_{n-NNHS}}(x_2) >, ..., < x_n, T_{S^6_{n-NNHS}}(x_n), I_{S^6_{n-NNHS}}(x_n), \\ F_{S^6_{n-NNHS}}(x_n) >), \{< u, T_{S^6_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S^6_{n-NNHS}(x_1,x_2,...x_n)}(u), \\ F_{S^6_{n-NNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$$T_{S^6_{n-NNHS}}(x_i) = (T_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$T_{S^6_{n-NNHS}(x_1,x_2,...x_n)}(u) = (T_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma,$$

$$I_{S^6_{n-NNHS}}(x_i) = 1 - (1 - I_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$I^6_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1 - (1 - I_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma,$$

$$F_{S^6_{n-NNHS}}(x_i) = 1 - (1 - F_{S^1_{n-NNHS}}(x_i))^\gamma,$$

$$F_{S^6_{n-NNHS}(x_1,x_2,...x_n)}(u) = 1 - (1 - F_{S^1_{n-NNHS}(x_1,x_2,...x_n)}(u))^\gamma.$$

for all $i \in \{1, 2, ..., n\}$.

Proposition 3.14. *Let S^1_{n-NNHS}, S^2_{n-NNHS} and S^3_{n-NNHS} be any three $n-NNHS-$set and $\gamma_1, \gamma_2, \gamma \neq 0$. Then,*

(1) $S^1_{n-NNHS}\widetilde{\cup}(S^2_{n-NNHS}\widetilde{\cap}S^3_{n-NNHS}) = (S^1_{n-NNHS}\widetilde{\cup}S^2_{n-NNHS})\widetilde{\cap}(S^1_{n-NNHS}\widetilde{\cup}S^3_{n-NNHS})$
(2) $S^1_{n-NNHS}\widetilde{\cap}(S^2_{n-NNHS}\widetilde{\cup}S^3_{n-NNHS}) = (S^1_{n-NNHS}\widetilde{\cap}S^2_{n-NNHS})\widetilde{\cup}(S^1_{n-NNHS}\widetilde{\cap}S^3_{n-NNHS})$
(3) $(S^1_{n-NNHS}\widetilde{\cap}S^2_{n-NNHS})^c = (S^1_{n-NNHS})^c\widetilde{\cup}(S^2_{n-NNHS})^c$
(4) $(S^1_{n-NNHS}\widetilde{\cup}S^2_{n-NNHS})^c = (S^1_{n-NNHS})^c\widetilde{\cap}(S^2_{n-NNHS})^c$

Note that the proofs of Proposition 3.4-3.14 can be easily obtained according to Definition 3.1-3.13 and properties of t-norm function and s-norm functions in Definition 2.2-2.3.

Example 3.15. Let $U = \{u_1, u_2, u_3\}$ be a universe, $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a set of attributes that are describe the elements of U, $E_i \subseteq E(i \in \{1, 2, 3\})$ be i set of attributes such that $E_1 = \{x_1, x_2, x_3\}$, $E_2 = \{x_4, x_5\}$,$E_3 = \{x_6\} \subseteq E$ and us consider the t-norm $t(a, b) = a.b$ and s-norm $s(a, b) = a + b - a.b$. Then, S^1_{3-NNHS} and S^2_{3-NNHS} over U given as;

$$\begin{aligned}
S^1_{3-NNHS} = \Big\{ & ((<x_1,(0.1,0.5,0.4)>,<x_4,(0.2,0.8,0.9)>,<x_6,(0.1,0.2,0.7)>);\\
& \{<u_1,(0.2,0.7,0.4)>,<u_2,(0.7,0.2,0.3)>,<u_3,(0.5,0.6,0.2)>\}),\\
& ((<x_1,(0.4,0.8,0.9)>,<x_5,(0.4,0.8,0.7)>,<x_6,(0.7,0.4,0.9)>);\\
& \{<u_1,(0.5,0.4,0.7)>,<u_2,(0.2,0.3,0.7)>,<u_3,(0.4,0.3,0.7)>\}),\\
& ((<x_2,(0.7,0.8,0.4)>,<x_4,(0.3,0.5,0.1)>,<x_6,(0.4,0.5,0.1)>);\\
& \{<u_1,(0.4,0.5,0.3)>,<u_2,(0.2,0.2,0.2)>,<u_3,(0.8,0.7,0.4)>\}),\\
& ((<x_2,(0.4,0.4,0.9)>,<x_5,(0.4,0.5,0.2)>,<x_6,(0.2,0.5,0.8)>);\\
& \{<u_1,(0.5,0.6,0.7)>,<u_2,(0.8,0.5,0.5)>,<u_3,(0.5,0.3,0.2)>\}),\\
& ((<x_3,(0.6,0.7,0.8)>,<x_4,(0.5,0.7,0.1)>,<x_6,(0.4,0.1,0.8)>);\\
& \{<u_1,(0.1,0.2,0.8)>,<u_2,(0.5,0.3,0.3)>,<u_3,(0.5,0.4,0.6)>\}),\\
& ((<x_3,(0.7,0.4,0.1)>,<x_5,(0.9,0.7,0.5)>,<x_6,(0.5,0.2,0.2)>);\\
& \{<u_1,(0.8,0.3,0.7)>,<u_2,(0.6,0.7,0.1)>,<u_3,(0.7,0.5,0.7)>\})\Big\}
\end{aligned}$$

and

$$\begin{aligned}
S^2_{3-NNHS} = \Big\{ & ((<x_1,(0.2,0.3,0.4)>,<x_4,(0.3,0.1,0.9)>,<x_6,(0.2,0.5,0.5)>);\\
& \{<u_1,(0.5,0.2,0.3)>,<u_2,(0.7,0.4,0.1)>,<u_3,(0.2,0.5,0.3)>\}),\\
& ((<x_1,(0.5,0.8,0.8)>,<x_5,(0.5,0.9,0.7)>,<x_6,(0.7,0.5,0.9)>);\\
& \{<u_1,(0.5,0.5,0.7)>,<u_2,(0.6,0.7,0.7)>,<u_3,(0.5,0.7,0.7)>\}),\\
& ((<x_2,(0.7,0.8,0.5)>,<x_4,(0.7,0.5,0.1)>,<x_6,(0.5,0.5,0.1)>);\\
& \{<u_1,(0.5,0.5,0.7)>,<u_2,(0.6,0.6,0.6)>,<u_3,(0.9,0.7,0.5)>\}),\\
& ((<x_2,(0.5,0.5,0.9)>,<x_5,(0.5,0.5,0.6)>,<x_6,(0.6,0.5,0.9)>);\\
& \{<u_1,(0.5,0.6,0.7)>,<u_2,(0.9,0.5,0.5)>,<u_3,(0.5,0.7,0.6)>\}),\\
& ((<x_3,(0.6,0.7,0.9)>,<x_4,(0.5,0.7,0.1)>,<x_6,(0.5,0.1,0.9)>);\\
& \{<u_1,(0.1,0.6,0.8)>,<u_2,(0.5,0.7,0.7)>,<u_3,(0.5,0.5,0.6)>\}),\\
& ((<x_3,(0.7,0.5,0.1)>,<x_5,(0.9,0.7,0.5)>,<x_6,(0.5,0.6,0.6)>);\\
& \{<u_1,(0.8,0.7,0.7)>,<u_2,(0.6,0.7,0.1)>,<u_3,(0.7,0.5,0.7)>\})\Big\}
\end{aligned}$$

Therefore,

(1) S^c_{n-NNHS} is computed as;

$$
\begin{aligned}
{S^1}^c_{n-NNHS} = \Big\{ & ((< x_1, (0.4, 0.5, 0.1) >, < x_4, (0.9, 0.2, 0.2) >, < x_6, (0.7, 0.8, 0.1) >); \\
& \{< u_1, (0.4, 0.3, 0.2) >, < u_2, (0.3, 0.8, 0.7) >, < u_3, (0.2, 0.4, 0.5) >\}), \\
& ((< x_1, (0.9, 0.2, 0.4) >, < x_5, (0.7, 0.2, 0.4) >, < x_6, (0.9, 0.6, 0.7) >); \\
& \{< u_1, (0.7, 0.6, 0.5) >, < u_2, (0.7, 0.7, 0.2) >, < u_3, (0.7, 0.7, 0.4) >\}), \\
& ((< x_2, (0.4, 0.2, 0.7) >, < x_4, (0.1, 0.5, 0.3) >, < x_6, (0.1, 0.5, 0.4) >); \\
& \{< u_1, (0.3, 0.5, 0.4) >, < u_2, (0.2, 0.8, 0.2) >, < u_3, (0.4, 0.3, 0.8) >\}), \\
& ((< x_2, (0.9, 0.6, 0.4) >, < x_5, (0.2, 0.5, 0.4) >, < x_6, (0.8, 0.5, 0.2) >); \\
& \{< u_1, (0.7, 0.4, 0.5) >, < u_2, (0.5, 0.5, 0.8) >, < u_3, (0.2, 0.7, 0.5) >\}), \\
& ((< x_3, (0.8, 0.3, 0.6) >, < x_4, (0.1, 0.3, 0.5) >, < x_6, (0.8, 0.9, 0.4) >); \\
& \{< u_1, (0.8, 0.8, 0.1) >, < u_2, (0.3, 0.7, 0.5) >, < u_3, (0.6, 0.6, 0.5) >\}), \\
& ((< x_3, (0.1, 0.6, 0.7) >, < x_5, (0.5, 0.3, 0.9) >, < x_6, (0.2, 0.8, 0.5) >); \\
& \{< u_1, (0.7, 0.7, 0.8) >, < u_2, (0.1, 0.3, 0.6) >, < u_3, (0.7, 0.5, 0.7) >\}) \Big\}
\end{aligned}
$$

(2) $S^3_{n-NNHS} = S^1_{n-NNHS} \tilde{\cup} S^2_{n-NNHS}$ is computed as;

$$
\begin{aligned}
S^3_{n-NNHS} = \Big\{ & ((< x_1, (0.28, 0.15, 0.24) >, < x_4, (0.44, 0.08, 0.81) >, < x_6, (0.28, 0.10, 0.35) >); \\
& \{< u_1, (0.60, 0.14, 0.12) >, < u_2, (0.91, 0.08, 0.03) >, < u_3, (0.60, 0.30, 0.06) >\}), \\
& ((< x_1, (0.70, 0.64, 0.72) >, < x_5, (0.70, 0.72, 0.49) >, < x_6, (0.91, 0.20, 0.81) >); \\
& \{< u_1, (0.75, 0.20, 0.49) >, < u_2, (0.68, 0.21, 0.49) >, < u_3, (0.70, 0.21, 0.49) >\}), \\
& ((< x_2, (0.91, 0.64, 0.20) >, < x_4, (0.79, 0.25, 0.01) >, < x_6, (0.70, 0.25, 0.01) >); \\
& \{< u_1, (0.70, 0.25, 0.21) >, < u_2, (0.68, 0.12, 0.12) >, < u_3, (0.98, 0.49, 0.20) >\}), \\
& ((< x_2, (0.70, 0.20, 0.81) >, < x_5, (0.70, 0.25, 0.12) >, < x_6, (0.68, 0.25, 0.72) >); \\
& \{< u_1, (0.75, 0.36, 0.49) >, < u_2, (0.98, 0.25, 0.25) >, < u_3, (0.75, 0.21, 0.12) >\}), \\
& ((< x_3, (0.84, 0.49, 0.72) >, < x_4, (0.75, 0.49, 0.01) >, < x_6, (0.70, 0.01, 0.72) >); \\
& \{< u_1, (0.19, 0.12, 0.64) >, < u_2, (0.75, 0.21, 0.21) >, < u_3, (0.75, 0.20, 0.36) >\}), \\
& ((< x_3, (0.91, 0.20, 0.01) >, < x_5, (0.99, 0.49, 0.25) >, < x_6, (0.75, 0.12, 0.12) >); \\
& \{< u_1, (0.96, 0.21, 0.49) >, < u_2, (0.84, 0.49, 0.01) >, < u_3, (0.91, 0.25, 0.49) >\}) \Big\}
\end{aligned}
$$

(3) $S^4_{n-NNHS} = S^1_{n-NNHS} \tilde{\cap} S^2_{n-NNHS}$ is computed as;

$$
\begin{aligned}
S^4_{n-NNHS} = \ & \Big\{((< x_1,(0.02,0.65,0.76)>, < x_4,(0.06,0.82,0.99)>, < x_6,(0.02,0.60,0.85)>);\\
& \{< u_1,(0.10,0.76,0.58)>, < u_2,(0.49,0.52,0.37)>, < u_3,(0.10,0.80,0.44)>\}),\\
& ((< x_1,(0.20,0.96,0.98)>, < x_5,(0.20,0.98,0.91)>, < x_6,(0.49,0.70,0.99)>);\\
& \{< u_1,(0.25,0.70,0.91)>, < u_2,(0.12,0.79,0.91)>, < u_3,(0.20,0.79,0.91)>\}),\\
& ((< x_2,(0.49,0.96,0.70)>, < x_4,(0.21,0.75,0.19)>, < x_6,(0.20,0.75,0.19)>);\\
& \{< u_1,(0.20,0.75,0.79)>, < u_2,(0.12,0.68,0.68)>, < u_3,(0.72,0.91,0.70)>\}),\\
& ((< x_2,(0.20,0.70,0.99)>, < x_5,(0.20,0.75,0.68)>, < x_6,(0.12,0.75,0.98)>);\\
& \{< u_1,(0.25,0.84,0.91)>, < u_2,(0.72,0.75,0.75)>, < u_3,(0.25,0.79,0.68)>\}),\\
& ((< x_3,(0.36,0.91,0.98)>, < x_4,(0.25,0.91,0.19)>, < x_6,(0.20,0.19,0.98)>);\\
& \{< u_1,(0.01,0.68,0.96)>, < u_2,(0.25,0.79,0.79)>, < u_3,(0.25,0.70,0.84)>\}),\\
& ((< x_3,(0.49,0.70,0.19)>, < x_5,(0.81,0.91,0.75)>, < x_6,(0.25,0.68,0.68)>);\\
& \{< u_1,(0.64,0.79,0.91)>, < u_2,(0.36,0.91,0.19)>, < u_3,(0.49,0.75,0.91)>\})\Big\}
\end{aligned}
$$

(4) $S^5_{n-NNHS} = 2(S^1_{n-NNHS})$ is computed as;

$$
\begin{aligned}
S^5_{3-NNHS} = \ & \Big\{((< x_1,(0.19,0.25,0.36)>, < x_4,(0.36,0.64,0.81)>, < x_6,(0.19,0.04,0.49)>);\\
& \{< u_1,(0.36,0.49,0.16)>, < u_2,(0.91,0.04,0.09)>, < u_3,(0.75,0.36,0.04)>\}),\\
& ((< x_1,(0.64,0.64,0.81)>, < x_5,(0.64,0.64,0.49)>, < x_6,(0.91,0.16,0.81)>);\\
& \{< u_1,(0.75,0.16,0.49)>, < u_2,(0.36,0.09,0.49)>, < u_3,(0.64,0.09,0.49)>\}),\\
& ((< x_2,(0.91,0.64,0.16)>, < x_4,(0.51,0.25,0.01)>, < x_6,(0.64,0.25,0.01)>);\\
& \{< u_1,(0.64,0.25,0.09)>, < u_2,(0.36,0.04,0.04)>, < u_3,(0.96,0.49,0.16)>\}),\\
& ((< x_2,(0.64,0.16,0.81)>, < x_5,(0.64,0.25,0.04)>, < x_6,(0.36,0.25,0.64)>);\\
& \{< u_1,(0.75,0.36,0.49)>, < u_2,(0.96,0.25,0.25)>, < u_3,(0.75,0.09,0.04)>\}),\\
& ((< x_3,(0.75,0.49,0.01)>, < x_4,(0.75,0.49,0.01)>, < x_6,(0.64,0.01,0.64)>);\\
& \{< u_1,(0.19,0.04,0.64)>, < u_2,(0.75,0.09,0.09)>, < u_3,(0.75,0.16,0.36)>\}),\\
& ((< x_3,(0.91,0.16,0.01)>, < x_5,(0.99,0.49,0.25)>, < x_6,(0.75,0.04,0.04)>);\\
& \{< u_1,(0.96,0.09,0.49)>, < u_2,(0.84,0.49,0.01)>, < u_3,(0.91,0.25,0.49)>\})\Big\}
\end{aligned}
$$

(5) $S^6_{n-NNHS} = (S^1{}_{n-NNHS})^2$ is computed as;

$$S^6_{3-NNHS} = \Big\{((< x_1, (0.01, 0.75, 0.84) >, < x_4, (0.04, 0.96, 0.99) >, < x_6, (0.01, 0.36, 0.91) >);$$
$$\{< u_1, (0.04, 0.91, 0.64) >, < u_2, (0.49, 0.36, 0.51) >, < u_3, (0.25, 0.84, 0.36) >\}),$$
$$((< x_1, (0.16, 0.96, 0.99) >, < x_5, (0.16, 0.96, 0.91) >, < x_6, (0.49, 0.64, 0.99) >);$$
$$\{< u_1, (0.25, 0.64, 0.91) >, < u_2, (0.04, 0.51, 0.91) >, < u_3, (0.16, 0.51, 0.91) >\}),$$
$$((< x_2, (0.49, 0.96, 0.64) >, < x_4, (0.09, 0.75, 0.19) >, < x_6, (0.16, 0.75, 0.19) >);$$
$$\{< u_1, (0.16, 0.75, 0.51) >, < u_2, (0.04, 0.36, 0.36) >, < u_3, (0.64, 0.91, 0.64) >\}),$$
$$((< x_2, (0.16, 0.64, 0.99) >, < x_5, (0.16, 0.75, 0.36) >, < x_6, (0.04, 0.75, 0.96) >);$$
$$\{< u_1, (0.25, 0.84, 0.91) >, < u_2, (0.64, 0.75, 0.75) >, < u_3, (0.25, 0.51, 0.36) >\}),$$
$$((< x_3, (0.36, 0.91, 0.96) >, < x_4, (0.25, 0.91, 0.19) >, < x_6, (0.16, 0.19, 0.96) >);$$
$$\{< u_1, (0.01, 0.36, 0.96) >, < u_2, (0.25, 0.51, 0.51) >, < u_3, (0.25, 0.64, 0.84) >\}),$$
$$((< x_3, (0.49, 0.64, 0.19) >, < x_5, (0.81, 0.91, 0.75) >, < x_6, (0.25, 0.36, 0.36) >);$$
$$\{< u_1, (0.64, 0.51, 0.91) >, < u_2, (0.36, 0.91, 0.19) >, < u_3, (0.49, 0.75, 0.91) >\})\Big\}$$

3.2. *Hybrid set structures*

Let S_{n-NNHS} an $n-NNHS-$set over U as;

$$S_{n-NNHS} = \Big\{(\quad (< x_1, T_{S_{n-NNHS}}(x_1), I_{S_{n-NNHS}}(x_1), F_{S_{n-NNHS}}(x_1) >, < x_2, T_{S_{n-NNHS}}(x_2),$$
$$I_{S_{n-NNHS}}(x_2), F_{S_{n-NNHS}}(x_2) >, ..., < x_n, T_{S_{n-NNHS}}(x_n), I_{S_{n-NNHS}}(x_n),$$
$$F_{S_{n-NNHS}}(x_n) >), \{< u, T_{S_{n-NNHS}(x_1,x_2,...x_n)}(u), I_{S_{n-NNHS}(x_1,x_2,...x_n)}(u),$$
$$F_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n\Big\}$$

where

$T_{S_{n-NNHS}}(x_i)(i \in \{1, 2, ..., n\})$, $I_{S_{n-NNHS}}(x_i)(i \in \{1, 2, ..., n\})$, $F_{S_{n-NNHS}}(x_i)(i \in \{1, 2, ..., n\}) \in [0, 1]$,

and

$T_{S_{n-NNHS}(x_1,x_2,...x_n)}(u)$, $I_{S_{n-NNHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) \in [0, 1]$

such that

$0 \leq T_{S_{n-NNHS}}(x_i) + I_{S_{n-NNHS}}(x_i), F_{S_{n-NNHS}}(x_i) \leq 3(i \in \{1, 2, ..., n\})$

and

$0 \leq T_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) + I_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) \leq 3.$

Then, we have in the following hybrid set structures

(1) an $n-$attribute Hypersoft Set($n - HS-$set) S_{n-HS} over U can be written as a set of ordered pairs as;

$$S_{n-HS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S_{n-HS}}(x_1), 0, 1 - T_{S_{n-HS}}(x_1) >, < x_2, T_{S_{n-HS}}(x_2), \\ 0, 1 - T_{S_{n-HS}}(x_2) >, ..., < x_n, T_{S_{n-HS}}(x_n), 0, \\ 1 - T_{S_{n-HS}}(x_n) >), \{< u, T_{S_{n-HS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-HS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$T_{S_{n-HS}}(x_i)(i \in \{1, 2, ..., n\}) \in \{0, 1\}$ and $T_{S_{n-HS}(x_1,x_2,...x_n)}(u) \in \{0, 1\}$

(2) an $n-$attribute Fuzzy Hypersoft Set($n - FHS-$set) S_{n-FHS} over U can be written as a set of ordered pairs as;

$$S_{n-FHS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S_{n-FHS}}(x_1), 0, 1 - T_{S_{n-FHS}}(x_1) >, < x_2, T_{S_{n-FHS}}(x_2), \\ 0, 1 - T_{S_{n-FHS}}(x_2) >, ..., < x_n, T_{S_{n-FHS}}(x_n), 0, \\ 1 - T_{S_{n-FHS}}(x_n) >), \{< u, T_{S_{n-FHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-FHS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$T_{S_{n-FHS}}(x_i)(i \in \{1, 2, ..., n\}) \in \{0, 1\}$ and $T_{S_{n-FHS}(x_1,x_2,...x_n)}(u) \in [0, 1]$.

(3) an $n-$attribute Intuitionistic Fuzzy Hypersoft Set($n - IFHS-$set) S_{n-IFHS} over U can be written as a set of ordered pairs as;

$$S_{n-IFHS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S_{n-IFHS}}(x_1), 0, 1 - T_{S_{n-IFHS}}(x_1) >, < x_2, T_{S_{n-IFHS}}(x_2), \\ 0, 1 - T_{S_{n-IFHS}}(x_2) >, ..., < x_n, T_{S_{n-IFHS}}(x_n), 0, \\ 1 - T_{S_{n-IFHS}}(x_n) >), \{< u, T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u), 0, F_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$T_{S_{n-IFHS}}(x_i)(i \in \{1, 2, ..., n\}) \in \{0, 1\}$, and $T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) \in [0, 1]$

such that $0 \leq T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) \leq 1$.

(4) an $n-$attribute Neutrosophic Hypersoft Set($n - NHS-$set) S_{n-NHS} over U can be written as a set of ordered pairs as;

$$S_{n-NHS} = \left\{ \begin{array}{l} (\quad (< x_1, T_{S_{n-NHS}}(x_1), 0, 1 - T_{S_{n-NHS}}(x_1) >, < x_2, T_{S_{n-NHS}}(x_2), 0, \\ 1 - T_{S_{n-NHS}}(x_2) >, ..., < x_n, T_{S_{n-NHS}}(x_n), 0, 1 - T_{S_{n-NHS}}(x_n) >), \\ \{< u, T_{S_{n-NHS}(x_1,x_2,...x_n)}(u), I_{S_{n-NHS}(x_1,x_2,...x_n)}(u), F_{S_{n-NHS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \end{array} \right\}$$

where

$T_{S_{n-NHS}}(x_i)(i \in \{1,2,...,n\}) \in \{0,1\}$

and

$T_{S_{n-NHS}(x_1,x_2,...x_n)}(u)$, $I_{S_{n-NHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-NHS}(x_1,x_2,...x_n)}(u) \in [0,1]$

such that

$0 \leq T_{S_{n-NHS}(x_1,x_2,...x_n)}(u) + I_{S_{n-NHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-NHS}(x_1,x_2,...x_n)}(u) \leq 3.$

(5) a Fuzzy Valued $n-$attribute Hypersoft Set($n-FHS-$set) S_{n-FHS} over U can be written as a set of ordered pairs as;

$$\begin{aligned} S_{n-FHS} = \Big\{ (\quad & (< x_1, T_{S_{n-FHS}}(x_1), 0, 1 - T_{S_{n-FHS}}(x_1) >, < x_2, T_{S_{n-FHS}}(x_2), \\ & 0, 1 - T_{S_{n-FHS}}(x_2) >, ..., < x_n, T_{S_{n-FHS}}(x_n), 0, \\ & 1 - T_{S_{n-FHS}}(x_n) >), \{< u, T_{S_{n-FHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-FHS}(x_1,x_2,...x_n)}(u) >: \\ & u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\} \end{aligned}$$

where

$T_{S_{n-FHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$ and $T_{S_{n-FHS}(x_1,x_2,...x_n)}(u) \in \{0,1\}$

(6) a Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set($n-FFHS-$set) S_{n-FFHS} over U can be written as a set of ordered pairs as;

$$\begin{aligned} S_{n-FFHS} = \Big\{ (\quad & (< x_1, T_{S_{n-FFHS}}(x_1), 0, 1 - T_{S_{n-FFHS}}(x_1) >, < x_2, T_{S_{n-FFHS}}(x_2), \\ & 0, 1 - T_{S_{n-FFHS}}(x_2) >, ..., < x_n, T_{S_{n-FFHS}}(x_n), 0, \\ & 1 - T_{S_{n-FFHS}}(x_n) >), \{< u, T_{S_{n-FFHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-FFHS}(x_1,x_2,...x_n)}(u) >: \\ & u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\} \end{aligned}$$

where

$T_{S_{n-FFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$ and $T_{S_{n-FFHS}(x_1,x_2,...x_n)}(u) \in [0,1]$.

(7) a Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set($n-FIFHS-$set) $S_{n-FIFHS}$ over U can be written as a set of ordered pairs as;

$$\begin{aligned} S_{n-FIFHS} = \Big\{ (\quad & (< x_1, T_{S_{n-FIFHS}}(x_1), 0, 1 - T_{S_{n-FIFHS}}(x_1) >, < x_2, T_{S_{n-FIFHS}}(x_2), \\ & 0, 1 - T_{S_{n-FIFHS}}(x_2) >, ..., < x_n, T_{S_{n-FIFHS}}(x_n), 0, \\ & 1 - T_{S_{n-FIFHS}}(x_n) >), \{< u, T_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u), 0, F_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u) >: \\ & u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\} \end{aligned}$$

where

$T_{S_{n-FIFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u) \in [0,1]$

such that

$0 \leq T_{S_{n-FIFHS}}(x_i) \leq 1 (i \in \{1, 2, ..., n\})$

and

$0 \leq T_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-FIFHS}(x_1,x_2,...x_n)}(u) \leq 1.$

(8) a Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set($n - FNHS-$set) S_{n-FNHS} over U can be written as a set of ordered pairs as;

$$S_{n-FNHS} = \Big\{ (\quad (< x_1, T_{S_{n-FNHS}}(x_1), 0, 1 - T_{S_{n-FNHS}}(x_1) >, < x_2, T_{S_{n-FNHS}}(x_2), 0, 1 - T_{S_{n-FNHS}}(x_2) >, ..., < x_n, T_{S_{n-FNHS}}(x_n), 0, 1 - T_{S_{n-FNHS}}(x_n) >), \{< u, T_{S_{n-FNHS}(x_1,x_2,...x_n)}(u), I_{S_{n-FNHS}(x_1,x_2,...x_n)}(u), F_{S_{n-FNHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\}$$

where

$T_{S_{n-FNHS}}(x_i)(i \in \{1, 2, ..., n\}) \in [0, 1]$

and

$T_{S_{n-FNHS}(x_1,x_2,...x_n)}(u),\ I_{S_{n-FNHS}(x_1,x_2,...x_n)}(u),\ F_{S_{n-FNHS}(x_1,x_2,...x_n)}(u) \in [0, 1]$

such that

$0 \leq T_{S_{n-FNHS}}(x_i) \leq 1 (i \in \{1, 2, ..., n\})$

and

$0 \leq T_{S_{n-FNHS}(x_1,x_2,...x_n)}(u) + I_{S_{n-FNHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-FNHS}(x_1,x_2,...x_n)}(u) \leq 3.$

(9) an Intuitionistic Fuzzy Valued $n-$attribute Hypersoft Set($n - IFHS-$set) S_{n-IFHS} over U can be written as a set of ordered pairs as;

$$S_{n-IFHS} = \Big\{ (\quad (< x_1, T_{S_{n-IFHS}}(x_1), 0, F_{S_{n-IFHS}}(x_1) >, < x_2, T_{S_{n-IFHS}}(x_2), 0, F_{S_{n-IFHS}}(x_2) >, ..., < x_n, T_{S_{n-IFHS}}(x_n), 0, F_{S_{n-IFHS}}(x_n) >), \{< u, T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\}$$

where

$T_{S_{n-IFHS}}(x_i)(i \in \{1, 2, ..., n\})$, $F_{S_{n-IFHS}}(x_i)(i \in \{1, 2, ..., n\}) \in [0, 1]$, and $T_{S_{n-IFHS}(x_1,x_2,...x_n)}(u) \in \{0, 1\}$

such that

$0 \leq T_{S_{n-IFHS}}(x_i) + F_{S_{n-IFHS}}(x_i) \leq 1 (i \in \{1, 2, ..., n\}).$

(10) an Intuitionistic Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set($n - IFFHS-$set) $S_{n-IFFHS}$ over U can be written as a set of ordered pairs as;

$$S_{n-IFFHS}=\Bigg\{\Big(\quad (<x_1,T_{S_{n-IFFHS}}(x_1),0,F_{S_{n-IFFHS}}(x_1)>,<x_2,T_{S_{n-IFFHS}}(x_2),$$
$$0,F_{S_{n-IFFHS}}(x_2)>,...,<x_n,T_{S_{n-IFFHS}}(x_n),0,$$
$$F_{S_{n-IFFHS}}(x_n)>),\{<u,T_{S_{n-IFFHS}(x_1,x_2,...x_n)}(u),0,1-T_{S_{n-IFFHS}(x_1,x_2,...x_n)}(u)>:$$
$$u\in U\}):(x_1,x_2,...x_n)\in E_1\times E_2\times...\times E_n\Bigg\}$$

where

$T_{S_{n-IFFHS}}(x_i)(i \in \{1,2,...,n\})$, $F_{S_{n-IFFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-IFFHS}(x_1,x_2,...x_n)}(u)\in[0,1]$

such that

$0\le T_{S_{n-IFFHS}}(x_i)+F_{S_{n-IFFHS}}(x_i)\le 1(i\in\{1,2,...,n\})$

and

$0\le T_{S_{n-IFFHS}(x_1,x_2,...x_n)}(u)\le 1.$

(11) an Intuitionistic Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set($n-IFIFHS-$set) $S_{n-IFIFHS}$ over U can be written as a set of ordered pairs as;

$$S_{n-IFIFHS}=\Bigg\{\Big(\quad (<x_1,T_{S_{n-IFIFHS}}(x_1),0,F_{S_{n-IFIFHS}}(x_1)>,<x_2,T_{S_{n-IFIFHS}}(x_2),$$
$$0,F_{S_{n-IFIFHS}}(x_2)>,...,<x_n,T_{S_{n-IFIFHS}}(x_n),0,$$
$$F_{S_{n-IFIFHS}}(x_n)>),\{<u,T_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u),0,F_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u)>:$$
$$u\in U\}):(x_1,x_2,...x_n)\in E_1\times E_2\times...\times E_n\Bigg\}$$

where

$T_{S_{n-IFIFHS}}(x_i)(i \in \{1,2,...,n\})$, $F_{S_{n-IFIFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u)\in[0,1]$

such that

$0\le T_{S_{n-IFIFHS}}(x_i)+F_{S_{n-IFIFHS}}(x_i)\le 1(i\in\{1,2,...,n\})$

and

$0\le T_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u)+F_{S_{n-IFIFHS}(x_1,x_2,...x_n)}(u)\le 1.$

(12) an Intuitionistic Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set($n-IFNHS-$set) $S_{n-IFNHS}$ over U can be written as a set of ordered pairs as;

$$S_{n-IFNHS}=\Bigg\{\Big(\quad (<x_1,T_{S_{n-IFNHS}}(x_1),0,F_{S_{n-IFNHS}}(x_1)>,<x_2,T_{S_{n-IFNHS}}(x_2),0,$$
$$F_{S_{n-IFNHS}}(x_2)>,...,<x_n,T_{S_{n-IFNHS}}(x_n),0,F_{S_{n-IFNHS}}(x_n)>),$$
$$\{<u,T_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u),I_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u),F_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u)>:$$
$$u\in U\}):(x_1,x_2,...x_n)\in E_1\times E_2\times...\times E_n\Bigg\}$$

where

$T_{S_{n-IFNHS}}(x_i)(i\in\{1,2,...,n\})$, $F_{S_{n-IFNHS}}(x_i)(i\in\{1,2,...,n\})\in[0,1]$, and

$T_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u)$, $I_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u) \in [0,1]$

such that

$0 \leq T_{S_{n-IFNHS}}(x_i) + F_{S_{n-IFNHS}}(x_i) \leq 1(i \in \{1,2,...,n\})$

and

$0 \leq T_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u) + I_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-IFNHS}(x_1,x_2,...x_n)}(u) \leq 3.$

(13) a Neutrosophic Valued $n-$attribute Hypersoft Set($n-NHS-$set) S_{n-NHS} over U can be written as a set of ordered pairs as;

$$S_{n-NHS} = \Bigg\{ \Big(\begin{array}{l} (< x_1, T_{S_{n-NHS}}(x_1), I_{S_{n-NHS}}(x_1), F_{S_{n-NHS}}(x_1) >, < x_2, T_{S_{n-NHS}}(x_2), \\ I_{S_{n-NHS}}(x_2), F_{S_{n-NHS}}(x_2) >, ..., < x_n, T_{S_{n-NHS}}(x_n), I_{S_{n-NHS}}(x_n), \\ F_{S_{n-NHS}}(x_n) >), \{< u, T_{S_{n-NHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-NHS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Bigg\} \end{array}$$

where

$T_{S_{n-NHS}}(x_i)(i \in \{1,2,...,n\})$, $I_{S_{n-NHS}}(x_i)(i \in \{1,2,...,n\})$, $F_{S_{n-NHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-NHS}(x_1,x_2,...x_n)}(u) \in \{0,1\}$

such that

$0 \leq T_{S_{n-NHS}}(x_i) + I_{S_{n-NHS}}(x_i), F_{S_{n-NHS}}(x_i) \leq 3(i \in \{1,2,...,n\}).$

(14) a Neutrosophic Valued $n-$attribute Fuzzy Hypersoft Set($n-NFHS-$set) S_{n-NFHS} over U can be written as a set of ordered pairs as;

$$S_{n-NFHS} = \Bigg\{ \Big(\begin{array}{l} (< x_1, T_{S_{n-NFHS}}(x_1), I_{S_{n-NFHS}}(x_1), F_{S_{n-NFHS}}(x_1) >, < x_2, T_{S_{n-NFHS}}(x_2), \\ I_{S_{n-NFHS}}(x_2), F_{S_{n-NFHS}}(x_2) >, ..., < x_n, T_{S_{n-NFHS}}(x_n), I_{S_{n-NFHS}}(x_n), \\ F_{S_{n-NFHS}}(x_n) >), \{< u, T_{S_{n-NFHS}(x_1,x_2,...x_n)}(u), 0, 1 - T_{S_{n-NFHS}(x_1,x_2,...x_n)}(u) >: \\ u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Bigg\} \end{array}$$

where

$T_{S_{n-NFHS}}(x_i)(i \in \{1,2,...,n\})$, $I_{S_{n-NFHS}}(x_i)(i \in \{1,2,...,n\})$, $F_{S_{n-NFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-NFHS}(x_1,x_2,...x_n)}(u) \in [0,1]$

such that

$0 \leq T_{S_{n-NFHS}}(x_i) + I_{S_{n-NFHS}}(x_i), F_{S_{n-NFHS}}(x_i) \leq 3(i \in \{1,2,...,n\})$

and

$0 \leq T_{S_{n-NFHS}(x_1,x_2,...x_n)}(u) \leq 1.$

(15) a Neutrosophic Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set($n-NIFHS-$set) $S_{n-NIFHS}$ over U can be written as a set of ordered pairs as;

$$S_{n-NIFHS} = \Big\{ \big(\; (< x_1, T_{S_{n-NIFHS}}(x_1), I_{S_{n-NIFHS}}(x_1), F_{S_{n-NIFHS}}(x_1) >, < x_2, T_{S_{n-NIFHS}}(x_2), I_{S_{n-NIFHS}}(x_2), F_{S_{n-NIFHS}}(x_2) >, ..., < x_n, T_{S_{n-NIFHS}}(x_n), I_{S_{n-NIFHS}}(x_n), F_{S_{n-NIFHS}}(x_n) >), \{< u, T_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u), 0, F_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u) >: u \in U\}) : (x_1, x_2, ...x_n) \in E_1 \times E_2 \times ... \times E_n \Big\}$$

where

$T_{S_{n-NIFHS}}(x_i)(i \in \{1,2,...,n\})$, $I_{S_{n-NIFHS}}(x_i)(i \in \{1,2,...,n\})$, $F_{S_{n-NIFHS}}(x_i)(i \in \{1,2,...,n\}) \in [0,1]$, and $T_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u)$, $F_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u) \in [0,1]$

such that

$0 \le T_{S_{n-NIFHS}}(x_i) + I_{S_{n-NIFHS}}(x_i), F_{S_{n-NIFHS}}(x_i) \le 3(i \in \{1,2,...,n\})$

and

$0 \le T_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u) + F_{S_{n-NIFHS}(x_1,x_2,...x_n)}(u) \le 1.$

3.3. *NPNSS-aggregation operator*

Definition 3.16. Let U be a universe, $NS(U)$ be the set of all neutrosophic soft sets on U, E be a set of attributes that are describe the elements of U, $E_i \subseteq E(i \in \{1,2,...,n\})$ be i set of attributes such that $E_i \cap E_j \neq \emptyset$, for all $i \neq j$, and $i, j \in \{1,2,...,n\}$ and $N(E_i)$ be the set of all neutrosophic set on E_i. Also, let S_{n-NNHS} be any a $n-NNHS-$set. Then NNHS-aggregation operator of S_{n-NNHS}, denoted by $NNHS_{agg}$, is defined by

$$NNHS_{agg} : (N(E_1) \times N(E_2) \times ... \times N(E_n)) \times NS(U) \to F(U)$$

$$NPNSS_{agg}(N(E_1) \times N(E_2) \times ... \times N(E_n), NS(U)) = S^*$$

where

$$S^* = \{\mu_{S^*}(u)/u : u \in U\}$$

which is a fuzzy set over U. The set S^* is called aggregate fuzzy set of the S_{n-NNHS}.

In here, the membership degree $\mu_{S^*}(u)$ of u is computed as follows

$$\mu_{S^*}(u) = \frac{1}{2.|E_1 \times E_2 \times ... \times E_n|} \sum_{(x_1,x_2,...x_n) \in E_1 \times E_2 \times ... \times E_n} Sc(N(x_1, x_2, ...x_n)).Sc(N_{(x_1,x_2,...x_n)}(u))$$

where

$Sc(N(x_1, x_2, ...x_n)) = (|T_{S_{n-NNHS}}(x_1, x_2, ...x_n) + I_{S_{n-NNHS}}(x_1, x_2, ...x_n) - F_{S_{n-NNHS}}(x_1, x_2, ...x_n)|)$,

$Sc(N_{(x_1,x_2,...x_n)}(u)) = (|T_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) + I_{S_{n-NNHS}(x_1,x_2,...x_n)}(u) - F_{S_{n-NNHS}(x_1,x_2,...x_n)}(u)|)$,

$|E_1 \times E_2 \times ... \times E_n|$ is the cardinality of $E_1 \times E_2 \times ... \times E_n$

and where

$T_{S_{n-NNHS}}(x_1, x_2, ...x_n) = s(T_{S_{n-NNHS}}(x_1), T_{S_{n-NNHS}}(x_2), ..., T_{S_{n-NNHS}}(x_n))$,

$I_{S_{n-NNHS}}(x_1, x_2, ...x_n) = t(I_{S_{n-NNHS}}(x_1), I_{S_{n-NNHS}}(x_2), ..., I_{S_{n-NNHS}}(x_n))$

and

$F_{S_{n-NNHS}}(x_1, x_2, ...x_n) = t(F_{S_{n-NNHS}}(x_1), F_{S_{n-NNHS}}(x_2), ..., F_{S_{n-NNHS}}(x_n))$

3.4. *Algorithm*

Let U be a universe set and E be a attributes set such that $E_i \subseteq E(i = 1, 2, ..., n)$ that are described by the elements of U. The algorithm for the solution is given below;

Step 1: Construct feasible an $n - NNHS-$set S_{n-NNHS} over U,

Step 2: Find the aggregate fuzzy set S^* of S_{n-NNHS},

Step 3: Find the largest membership grade $max\{\mu_{S^*_{n-NNHS}}(u) : u \in U\}$.

Example 3.17. Let $U = \{u_1, u_2, u_3\}$ be a universe, $E_1, E_2, E_3 \subseteq E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be attributes sets such that

$$\begin{aligned}
E_1 = &\quad \text{Foreign language} = \{x_1 = \text{English}, x_2 = \text{French}, x_3 = \text{German}\} \\
E_2 = &\quad \text{Work experience} = \{x_4 = \textit{computer}, x_5 = \text{E-marketing}\} \\
E_3 = &\quad \text{Nationality} = \{x_6 = \text{Turkish}\}
\end{aligned}$$

and let us consider the t-norm $t(a, b) = a.b$ and s-norm $s(a, b) = a + b - a.b$. Then, suppose that a company's board of directors plans to get an expert personal for online trading the during COVID-19 pandemic. The process is summarized based on $n - NNHS-$sets as follows;

Step 1: We constructed feasible a $3 - NNHS-$set S_{3-NNHS} over U as,

$$\begin{aligned}
S_{3-NNHS} = \quad & \Big\{((< x_1, (0.1, 0.5, 0.4) >, < x_4, (0.2, 0.8, 0.9) >, < x_6, (0.1, 0.2, 0.7) >); \\
& \{< u_1, (0.2, 0.7, 0.4) >, < u_2, (0.7, 0.2, 0.3) >, < u_3, (0.5, 0.6, 0.2) >\}), \\
& ((< x_1, (0.4, 0.8, 0.9) >, < x_5, (0.4, 0.8, 0.7) >, < x_6, (0.7, 0.4, 0.9) >); \\
& \{< u_1, (0.5, 0.4, 0.7) >, < u_2, (0.2, 0.3, 0.7) >, < u_3, (0.4, 0.3, 0.7) >\}), \\
& ((< x_2, (0.7, 0.8, 0.4) >, < x_4, (0.3, 0.5, 0.1) >, < x_6, (0.4, 0.5, 0.1) >); \\
& \{< u_1, (0.4, 0.5, 0.3) >, < u_2, (0.2, 0.2, 0.2) >, < u_3, (0.8, 0.7, 0.4) >\}), \\
& ((< x_2, (0.4, 0.4, 0.9) >, < x_5, (0.4, 0.5, 0.2) >, < x_6, (0.2, 0.5, 0.8) >); \\
& \{< u_1, (0.5, 0.6, 0.7) >, < u_2, (0.8, 0.5, 0.5) >, < u_3, (0.5, 0.3, 0.2) >\}), \\
& ((< x_3, (0.6, 0.7, 0.8) >, < x_4, (0.5, 0.7, 0.1) >, < x_6, (0.4, 0.1, 0.8) >); \\
& \{< u_1, (0.1, 0.2, 0.8) >, < u_2, (0.5, 0.3, 0.3) >, < u_3, (0.5, 0.4, 0.6) >\}), \\
& ((< x_3, (0.7, 0.4, 0.1) >, < x_5, (0.9, 0.7, 0.5) >, < x_6, (0.5, 0.2, 0.2) >); \\
& \{< u_1, (0.8, 0.3, 0.7) >, < u_2, (0.6, 0.7, 0.1) >, < u_3, (0.7, 0.5, 0.7) >\})\Big\}
\end{aligned}$$

Step 2:We found the aggregate fuzzy set S^* of S_{n-NNHS} as,

$$S^*_{n-NNHS} = \{u_1/0.1581, u_2/0.2139, u_3/0.2001\}$$

Step 3: Finally, since the u_2 has the largest membership grade, it is the best expert personal for online trading on U.

4. Conclusion

In this study, hybrid set structure with hypersoft sets under uncertainty, vagueness and indeterminacy, called neutrosophic valued $n-$attribute neutrosophic hypersoft set, which is a combination of neutrosophic sets [25] and hypersoft sets [31] was presented. The neutrosophic valued $n-$attribute neutrosophic hypersoft set ($n-NNHS-$set) generalizes the following hybrid sets as; $n-$attribute Hypersoft Set, $n-$attribute Fuzzy Hypersoft Set, $n-$attribute Intuitionistic Fuzzy Hypersoft Set, $n-$attribute Neutrosophic Hypersoft Set , Fuzzy Valued $n-$attribute Hypersoft Set, Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set, Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set, Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set, Intuitionistic Fuzzy Valued $n-$attribute Hypersoft Set, Intuitionistic Fuzzy Valued $n-$attribute Fuzzy Hypersoft Set, Intuitionistic Fuzzy Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set, Intuitionistic Fuzzy Valued $n-$attribute Neutrosophic Hypersoft Set, Neutrosophic Valued $n-$attribute Hypersoft Set, Neutrosophic Valued $n-$attribute Fuzzy Hypersoft Set, Neutrosophic Valued $n-$attribute Intuitionistic Fuzzy Hypersoft Set, neutrosophic parameterized neutrosophic soft sets [13], and so on. Then, some definitions and operations on $n-NNHS-$set and some properties of the sets which are connected to operations have been given. Finally, a decision making method on the $n-NNHS-$set to show that this method can be successfully worked was developed. In the future works, $n-NNHS-$sets can be expanded with new research subjects as; Matrices, neutrosophic numbers and arithmetical operations, relational structures, relational equations, entropy, similarity measure, distance measuress, orderings, probability, logical operations, programming, implicators, multi-valued mappings, mathematical morphology, algebraic structures, models, topology, cognitive maps, matrix, graph, fusion rules, relational maps, relational databases, image processing, linguistic variables, decision making based on VIKOR, TOPSIS, AHP and ELECTRE I-II-III, preference structures, expert systems, reliability theory, soft computing techniques, game theory and so on.

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Chapter 3

Fuzzy Hypersoft Sets and It's Application to Decision-Making

Adem Yolcu [1,*], Taha Yasin Ozturk [2]

[1] Department of Mathematic, Kafkas University, Kars, Turkey. E-mail: yolcu.adem@gmail.com

[2] Department of Mathematic, Kafkas University, Kars, Turkey. E-mail: taha36100@hotmail.com

Abstract: Multi-criteria decision-making (MCDM) is concerned with coordinating and taking care of matters of preference and preparation, including multi-criteria. Fuzzy soft set environments cannot be used to solve certain types of problems if attributes are more than one and further bifurcated. Therefore, there was a serious need to identify a new approach to solve such problems, so a new setting, namely the Fuzzy Hypersoft Sets (FHSS), is established for this reason. In this paper, we introduced some notions like union, intersection, subset, equal set, complement, null set, absolute set etc. of Fuzzy Hypersoft sets. The study has been enriched with many suitable examples to support the accuracy of the defined concepts. We also present an object recognition system from an imprecise data on a multiobserver. Decision-making system involves the construct of Comparison Table from a fuzzy hypersoft set in a parametric sense for decision-making.

Keywords: *Soft sets, fuzzy sets, Hypersoft sets, Fuzzy hypersoft sets, Decision-making problem.*

1. Introduction

Numerous complicated problems include unclear data in social sciences, economics, medical sciences, engineering and other fields. These problems, with which one is faced in life, cannot be solved classical mathematical tools. In classical mathematics, a model is designed and the exact solution of this model is calculated. However, if the given situation contains uncertainty, solving this model with classical mathematical method is very complex. Fuzzy set theory [1], rough set theory [2] and other theories have been described to solve situations involving uncertainty. The fuzzy set theory introduced by Zadeh has become very popular for uncertainty problems and has been a suitable construct for representing uncertain concepts as it allows for the partial membership function. Mathematicians and computer scientists have worked on fuzzy sets and over the years many useful applications of fuzzy set structure have emerged such as fuzzy control systems, fuzzy automata, fuzzy logic, fuzzy topology.

some structural difficulties of fuzzy set theory and other theories, as Molodtsov [3] introduced in 1999 when he presented the idea of soft set theory which is a completely new approach to modeling uncertainty. Further generalizations and extensions have been the subject of many efforts of the soft sets of Molodtsov. Maji et al. [4] by combining the fuzzy set and soft set structure, it has created the fuzzy soft cluster structure, which is a hybrid structure. In other words, a degree is added to the parameterisation of fuzzy sets when defining a fuzzy soft set. The fuzzy soft set structure, which is a combination of soft set structure and fuzzy set structure, has been actively used by researchers and many studies have been added to the literature [5–8]. As a result of the intense interest of researchers in this subject, important developments have been realized regarding the use of fuzzy soft set structure in decision-making problems. Because the definition of unreal objects in soft sets is not restricted, researchers can choose the parameter format they need, which greatly simplifies the decision-making process and makes it more efficient when partial information is missing. Soft sets used Maji and Roy [9] for the first time in decision-making problems. Chen [10] described the reduction of the parameterisation of the soft set and discussed its application of the problem of decision-making. Cagman and Enginoglu [11, 12] discussed theory of soft matrix and uni-int decision-making, choosing a collection of optimal elements from various alternatives. Studies on decision-making problems have taken place widely in the literature and have been studied by many researchers [13–23].

Smarandache [24] introduced new technique handling uncertainty. By transforming the functionality into a multi-decision function, he generalized the soft set to hypersoft set. Although Hypersoft set theory is more recent, it has attracted great attention from researchers [25–30].

In this paper, we have defined some basic concepts such as subset, equal set, union, intersection, complement, null set, absolute set, AND, OR operations on the fuzzy hypersoft set structure. We also applied fuzzy hypersoft sets to solve the decision-making problem. We have presented an appropriate decision problem by applying Roy and Maji's method [15] to fuzzy hypersoft sets. This study has the basic study feature in which the fuzzy hypersoft set structure is introduced. Therefore plays an important role for many subsequent studies.

2. **Preliminaries**

Definition 2.1. [1] Let U be a initial universe. A fuzzy set Λ in U, $\Lambda = \{(u, \mu_\Lambda(u)) : u \in U\}$, where $\mu_\Lambda : U \to [0,1]$ is the membership function of the fuzzy set Λ; $\mu_\Lambda(u) \in [0,1]$ is the membership $u \in U$ in Λ. The set of all fuzzy sets ove U will be denoted by $FP(U)$.

Definition 2.2. [3] Let U be an initial universe and E be a set of parameters. A pair (F, E) is called a soft set over U, where F is a mapping $F : E \to \mathcal{P}(U)$. In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.3. [4]Let U be a initial universe, E be a set of parameters and $FP(U)$ be the set of all fuzzy sets in U. Then a pair (f, E) is called a fuzzy soft set over U, where $f : E \to FP(U)$ is a mapping.

Definition 2.4. [24] Let U be the universal set and $P(U)$ be the power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are resspectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, then the pair $(\Theta, E_1 \times E_2 \times ... \times E_n)$ is said to be Hypersoft set over U where $\Theta : E_1 \times E_2 \times ... \times E_n \to P(U)$.

3. **Fuzzy Hypersoft Sets**

Definition 3.1. Let U be the universal set and $FP(U)$ be a family of all fuzzy set over U and $E_1, E_2, ..., E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each $i = 1, 2, ..., n$. A fuzzy hypersoft set defined as the pair $(\Theta, A_1 \times A_2 \times ... \times A_n)$ where; $\Theta : A_1 \times A_2 \times ... \times A_n \to FP(U)$ and

$$\Theta(A_1 \times A_2 \times ... \times A_n) = \{< u, \Theta(\alpha)(u) >: u \in U, \alpha \in A_1 \times A_2 \times ... \times A_n \subseteq E_1 \times E_2 \times ... \times E_n\}$$

For sake of simplicity, we write the symbols Σ for $E_1 \times E_2 \times ... \times E_n$, Γ for $A_1 \times A_2 \times ... \times A_n$ and α for an element of the set Γ. The set of all fuzzy hypersoft sets over U will be denoted by $FHS(U, \Sigma)$. Here after, FHS will be used for short instead of fuzzy hypersoft sets.

Definition 3.2. i) A fuzzy hypersoft set (Θ, Γ) over the universe U is said to be null fuzzy hypersoft set and denoted by $0_{(U_{FH}, \Sigma)}$ if for all $u \in U$ and $\varepsilon \in \Gamma$, $\Theta(\varepsilon)(u) = 0$.

ii) A fuzzy hypersoft set (Θ, Γ) over the universe U is said to be absolute fuzzy hypersoft set and denoted by $1_{(U_{FH}, \Sigma)}$ if for all $u \in U$ and $\varepsilon \in \Gamma$, $\Theta(\varepsilon)(u) = 1$.

Example 3.3. Let U be the set of computers given as $U = \{u_1, u_2, u_3\}$ also consider the set of attributes given as;

$$\begin{aligned} E_1 &= \text{CPU Type} = \{\text{Amd}(\alpha_1), \text{Intel}(\alpha_2)\} \\ E_2 &= \text{Case Size} = \{\text{Mid Tower}(\beta_1), \text{Full Tower}(\beta_2), \text{Compact Case}(\beta_3)\} \\ E_3 &= \text{Hard Drive} = \{\text{1TB}(\gamma_1), \text{512GB}(\gamma_2), \text{256GB}(\gamma_3)\} \end{aligned}$$

Suppose that

$$\begin{aligned} A_1 &= \{\alpha_2\},\ A_2 = \{\beta_2, \beta_3\}, A_3 = \{\gamma_1, \gamma_2\} \\ B_1 &= \{\alpha_1, \alpha_2\},\ B_2 = \{\beta_1, \beta_2\}, B_3 = \{\gamma_1\} \end{aligned}$$

are subset of E_i for each $i = 1, 2, 3$. Then the fuzzy hypersoft sets (Θ_1, Γ_1) and (Θ_2, Γ_2) defined as follows;

$$(\Theta_1,\Gamma_1)=\left\{\begin{array}{c}<(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0,3},\frac{u_2}{0,4}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0,2},\frac{u_2}{0,5},\frac{u_3}{0,1}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0,6},\frac{u_3}{0,7}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0,4},\frac{u_3}{0,5}\}>\end{array}\right\},$$

$$(\Theta_2,\Gamma_2)=\left\{\begin{array}{c}<(\alpha_1,\beta_1,\gamma_1),\{\frac{u_1}{0,4},\frac{u_3}{0,7}\}>,<(\alpha_1,\beta_2,\gamma_1),\{\frac{u_2}{0,3},\frac{u_3}{0,6}\}>,\\ <(\alpha_2,\beta_1,\gamma_1),\{\frac{u_1}{0,7},\frac{u_3}{0,8}\}>,<(\alpha_2,\beta_2,\gamma_1),\{\frac{u_2}{0,5},\frac{u_3}{0,7}\}>\end{array}\right\}$$

Corollary 3.4. *It is clear that each fuzzy hypersoft set is also fuzzy soft set. An example of this situation is provided below.*

Example 3.5. We consider that Example-3.3. If we select the parameters from a single attribute set such as E_1 while creating the fuzzy hypersoft set, then the resulting set becomes the fuzzy soft set. Therefore, it is clear that each fuzzy hypersoft set is also fuzzy soft set.That is, the fuzzy hypersoft set structure is the generalized version of the fuzzy soft sets.

Definition 3.6. Let U be an initial universe set $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be two fuzzy hypersoft sets over the universe U. We say that (Θ_1,Γ_1) is a fuzzy hypersoft subset of (Θ_2,Γ_2) and denote $(\Theta_1,\Gamma_1)\widetilde{\subseteq}(\Theta_2,\Gamma_2)$ if

i) $\Gamma_1\subseteq\Gamma_2$

ii) For any $\varepsilon\in\Gamma_1$, $\Theta_1(\varepsilon)\subseteq\Theta_2(\varepsilon)$.

Definition 3.7. Let U be an initial universe set $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be two fuzzy hypersoft sets over the universe U. We say that (Θ_1,Γ_1) is fuzzy hypersoft equal to (Θ_2,Γ_2) and denote $(\Theta_1,\Gamma_1)\widetilde{=}(\Theta_2,\Gamma_2)$ if (Θ_1,Γ_1) is a fuzzy hypersoft subset of (Θ_2,Γ_2) and (Θ_2,Γ_2) is a fuzzy hypersoft subset of (Θ_1,Γ_1).

Theorem 3.8. *Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)$ be fuzzy hypersoft sets over the universe U. Then,*

i) $(\Theta,\Gamma_1)\widetilde{\subseteq}1_{(U_{FH},\Sigma)}$,

ii) $0_{(U_{FH},\Sigma)}\widetilde{\subseteq}(\Theta,\Gamma_1)$,

iii) $(\Theta_1,\Gamma_1)\widetilde{\subseteq}(\Theta_2,\Gamma_2)$ *and* $(\Theta_2,\Gamma_2)\widetilde{\subseteq}(\Theta_3,\Gamma_3)\Rightarrow(\Theta_1,\Gamma_1)\widetilde{\subseteq}(\Theta_3,\Gamma_3)$.

Proof. Straightforward. □

Definition 3.9. The complement of fuzzy hypersoft set (Θ,Γ) over the universe U is denoted by $(\Theta,\Gamma)^c$ and defined as $(\Theta,\Gamma)^c=(\Theta^c,\Gamma)$, where $\Theta^c(\varepsilon)$ is complement of the set $\Theta(\varepsilon)$, for $\varepsilon\in\Gamma$.

Example 3.10. According to Example-3.3, consider the fuzzy hypersoft set (Θ_1,Γ_1) over the universe $U=\{u_1,u_2,u_3\}$.

$$(\Theta_1,\Gamma_1)=\left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}3},\frac{u_2}{0{,}4}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}2},\frac{u_2}{0{,}5},\frac{u_3}{0{,}1}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}6},\frac{u_3}{0{,}7}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0{,}4},\frac{u_3}{0{,}5}\}> \end{array}\right\},$$

Then the complement of (Θ_1,Γ_1) is written as;

$$(\Theta_1,\Gamma_1)^c=\left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}7},\frac{u_2}{0{,}6},\frac{u_3}{1}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}8},\frac{u_2}{0{,}5},\frac{u_3}{0{,}9}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}4},\frac{u_2}{1}\frac{u_3}{0{,}3}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_1}{1},\frac{u_2}{0{,}6},\frac{u_3}{0{,}5}\}> \end{array}\right\},$$

Theorem 3.11. *Let (Θ,Γ) be any fuzzy hypersoft set over the universe U. Then,*

i) $((\Theta,\Gamma)^c)^c=(\Theta,\Gamma)$

ii) $0^c_{(U_{FH},\Sigma)}=1_{(U_{FH},\Sigma)}$

iii) $1^c_{(U_{FH},\Sigma)}=0_{(U_{FH},\Sigma)}$

Proof. Proofs are trivial. □

Definition 3.12. Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be two fuzzy hypersoft sets over the universe U. The union of (Θ_1,Γ_1) and (Θ_2,Γ_2) is denoted by $(\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2)=(\Theta_3,\Gamma_3)$ where $\Gamma_3=\Gamma_1\cup\Gamma_2$ and

$$\Theta_3(\varepsilon)=\begin{cases}\Theta_1(\varepsilon) & \text{if}\quad \varepsilon\in\Gamma_1-\Gamma_2\\ \Theta_2(\varepsilon) & \text{if}\quad \varepsilon\in\Gamma_2-\Gamma_1\\ \max\{\ \Theta_1(\varepsilon),\Theta_2(\varepsilon)\} & \quad\text{if}\ \ \varepsilon\in\Gamma_1\cap\Gamma_2\end{cases}$$

Theorem 3.13. *Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)$ be two fuzzy hypersoft sets over the universe U Then;*

i) $(\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_1,\Gamma_1)=(\Theta_1,\Gamma_1)$

ii) $0_{(U_{FH},\Sigma)}\ \tilde{\cup}(\Theta_1,\Gamma_1)=(\Theta_1,\Gamma_1)$

iii) $(\Theta_1,\Gamma_1)\tilde{\cup}1_{(U_{FH},\Sigma)}=1_{(U_{FH},\Sigma)}$

iv) $(\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2)=(\Theta_2,\Gamma_2)\tilde{\cup}(\Theta_1,\Gamma_1)$

v) $((\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2))\,\tilde{\cup}(\Theta_3,\Gamma_3)=(\Theta_1,\Gamma_1)\tilde{\cup}\,((\Theta_2,\Gamma_2)\tilde{\cup}(\Theta_3,\Gamma_3))$

Proof. Proofs are trivial. □

Definition 3.14. Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be fuzzy hypersoft sets over the universe U. The intersection of (Θ_1,Γ_1) and (Θ_2,Γ_2) is denoted by $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)=(\Theta_3,\Gamma_3)$ where $\Gamma_3=\Gamma_1\cap\Gamma_2$ and each $\varepsilon\in\Gamma_3,\ \Theta_3(\varepsilon)(u)=\min\{\ \Theta_1(\varepsilon)(u),\Theta_2(\varepsilon)(u)\}$.

Theorem 3.15. *Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)$ be fuzzy hypersoft sets over the universe U. Then;*

i) $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_1,\Gamma_1)=(\Theta_1,\Gamma_1)$

ii) $0_{(U_{FH},\Sigma)}\ \tilde{\cap}(\Theta_1,\Gamma_1) = 0_{(U_{FH},\Sigma)}$
iii) $(\Theta_1,\Gamma_1)\tilde{\cap}1_{(U_{FH},\Sigma)} = (\Theta_1,\Gamma_1)$
iv) $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2) = (\Theta_2,\Gamma_2)\tilde{\cap}(\Theta_1,\Gamma_1)$
v) $((\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2))\,\tilde{\cap}(\Theta_3,\Gamma_3) = (\Theta_1,\Gamma_1)\tilde{\cap}\,((\Theta_2,\Gamma_2)\tilde{\cap}(\Theta_3,\Gamma_3))$

Proof. Proofs are trivial. □

Definition 3.16. Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be two fuzzy hypersoft sets over the universe U. The difference of (Θ_1,Γ_1) and (Θ_2,Γ_2) is denoted by $(\Theta_1,\Gamma_1)\widetilde{\setminus}(\Theta_2,\Gamma_2) = (\Theta_3,\Gamma_3)$ and is defined by $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)^c = (\Theta_3,\Gamma_3)$ where $\Gamma_3 = \Gamma_1\cup\Gamma_2$

Example 3.17. We consider that attributes in Example-3.3. Then the fuzzy hypersoft sets (Θ_1,Γ_1) and (Θ_1,Γ_1) defined as follows;

$$(\Theta_1,\Gamma_1) = \left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}3},\frac{u_2}{0{,}4}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}2},\frac{u_2}{0{,}5},\frac{u_3}{0{,}1}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}6},\frac{u_3}{0{,}7}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0{,}4},\frac{u_3}{0{,}5}\}> \end{array}\right\},$$

$$(\Theta_2,\Gamma_2) = \left\{\begin{array}{c} <(\alpha_1,\beta_1,\gamma_1),\{\frac{u_1}{0{,}4},\frac{u_3}{0{,}7}\}>,<(\alpha_1,\beta_2,\gamma_1),\{\frac{u_2}{0{,}3},\frac{u_3}{0{,}6}\}>,\\ <(\alpha_2,\beta_1,\gamma_1),\{\frac{u_1}{0{,}7},\frac{u_3}{0{,}8}\}>,<(\alpha_2,\beta_2,\gamma_1),\{\frac{u_2}{0{,}5},\frac{u_3}{0{,}7}\}> \end{array}\right\}$$

The union, intersection and difference operator of above IFHSS is written as;

$$(\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2) = \left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}3},\frac{u_2}{0{,}5},\frac{u_3}{0{,}7}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}2},\frac{u_2}{0{,}5},\frac{u_3}{0{,}1}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}6},\frac{u_3}{0{,}7}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0{,}4},\frac{u_3}{0{,}5}\}>,\\ <(\alpha_1,\beta_1,\gamma_1),\{\frac{u_1}{0{,}4},\frac{u_3}{0{,}7}\}>,<(\alpha_1,\beta_2,\gamma_1),\{\frac{u_2}{0{,}3},\frac{u_3}{0{,}6}\}>,\\ <(\alpha_2,\beta_1,\gamma_1),\{\frac{u_1}{0{,}7},\frac{u_3}{0{,}8}\}> \end{array}\right\}$$

$$(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2) = \left\{<(\alpha_2,\beta_2,\gamma_1),\{\frac{u_2}{0,4}\}>\right\}$$

$$(\Theta_1,\Gamma_1)\widetilde{\setminus}(\Theta_2,\Gamma_2) = \left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}3},\frac{u_2}{0{,}4}\}>,<(\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}2},\frac{u_2}{0{,}5},\frac{u_3}{0{,}1}\}>,\\ <(\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}6},\frac{u_3}{0{,}7}\}>,<(\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0{,}4},\frac{u_3}{0{,}5}\}> \end{array}\right\}$$

Theorem 3.18. *Let U be an initial universe set $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be fuzzy hypersoft sets over the universe U. Then De-Morgan Laws are hold.*
i) $((\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2))^c = (\Theta_1,\Gamma_1)^c\tilde{\cap}(\Theta_2,\Gamma_2)^c$
ii)$((\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2))^c = (\Theta_1,\Gamma_1)^c\tilde{\cup}(\Theta_2,\Gamma_2)^c$

Proof. We only prove $((\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2))^c = (\Theta_1,\Gamma_1)^c\tilde{\cap}(\Theta_2,\Gamma_2)^c$. The other properties can be similarly proved. Suppose that $((\Theta_1,\Gamma_1)\tilde{\cup}(\Theta_2,\Gamma_2))^c = (K,\Gamma_1\cup\Gamma_2)$ and $(\Theta_1,\Gamma_1)^c\tilde{\cap}(\Theta_2,\Gamma_2)^c = (I,\Gamma_1\cup\Gamma_2)$. For any $\varepsilon\in\Gamma_1\cap\Gamma_2$, we consider the following cases.

Case 1: $\varepsilon\in\Gamma_1-\Gamma_2$. Then $K(\varepsilon) = \Theta_1^c(\varepsilon) = I(\varepsilon)$.

Case 2: $\varepsilon \in \Gamma_2 - \Gamma_1$. Then $K(\varepsilon) = \Theta_2^c(\varepsilon) = I(\varepsilon)$.

Case 3: $\varepsilon \in \Gamma_1 \cap \Gamma_2$. Then $K(\varepsilon) = (\sigma_{\Theta_1(\varepsilon)}(u) \cap \sigma_{\Theta_1(\varepsilon)}(u), \theta_{\Theta_2(\varepsilon)}(u) \cup \theta_{\Theta_2(\varepsilon)}(u)) = \Theta_1^c(\varepsilon) \cap \Theta_2^c(\varepsilon) = I(\varepsilon)$.

Therefore, K and I are same operators, and so $((\Theta_1, \Gamma_1)\tilde{\cup}(\Theta_2, \Gamma_2))^c = (\Theta_1, \Gamma_1)^c \tilde{\cap} (\Theta_2, \Gamma_2)^c$.

□

Definition 3.19. Let U be an initial universe set and $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2)$ be fuzzy hypersoft sets over the universe U. The "AND" operation on them is denoted by $(\Theta_1, \Gamma_1) \wedge (\Theta_2, \Gamma_2) = (\Upsilon, \Gamma_1 \times \Gamma_2)$ is given as;

$$(\Upsilon, \Gamma_1 \times \Gamma_2) = \{(\varepsilon_1, \varepsilon_2), < u, \Upsilon(\varepsilon_1, \varepsilon_2)(u) >: u \in U, (\varepsilon_1, \varepsilon_2) \in \Gamma_1 \times \Gamma_2\}$$

where

$$\Upsilon(\varepsilon_1, \varepsilon_2)(u) = \{< u, \min\{\Theta_1(\varepsilon_1)(u), \Theta_2(\varepsilon_2)(u)\} >\}$$

Definition 3.20. Let U be an initial universe set and (Θ_1, Γ_1),(Θ_2, Γ_2) be fuzzy hypersoft sets over the universe U. The "OR" operation on them is denoted by $(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2) = (\Upsilon, \Gamma_1 \times \Gamma_2)$ is given as;

$$(\Upsilon, \Gamma_1 \times \Gamma_2) = \{(\varepsilon_1, \varepsilon_2), < u, \Upsilon(\varepsilon_1, \varepsilon_2)(u) >: u \in U, (\varepsilon_1, \varepsilon_2) \in \Gamma_1 \times \Gamma_2\}$$

where

$$\Upsilon(\varepsilon_1, \varepsilon_2)(u) = \{< u, \max\{\Theta_1(\varepsilon_1)(u), \Theta_2(\varepsilon_2)(u)\} >\}$$

Theorem 3.21. *Let U be an initial universe set and $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2), (\Theta_3, \Gamma_3)$ be fuzzy hypersoft sets over the universe U. Then;*

i) $(\Theta_1, \Gamma_1) \vee [(\Theta_2, \Gamma_2) \vee (\Theta_3, \Gamma_3)] = [(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2)] \vee (\Theta_3, \Gamma_3)$

ii) $(\Theta_1, \Gamma_1) \wedge [(\Theta_2, \Gamma_2) \wedge (\Theta_3, \Gamma_3)] = [(\Theta_1, \Gamma_1) \wedge (\Theta_2, \Gamma_2)] \wedge (\Theta_3, \Gamma_3)$

Proof. Straightforward. □

Theorem 3.22. *Let U be an initial universe set and (Θ_1, Γ_1), (Θ_2, Γ_2) be fuzzy hypersoft sets over the universe U Then;*

i) $[(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2)]^c = (\Theta_1, \Gamma_1)^c \wedge (\Theta_2, \Gamma_2)^c$

ii) $[(\Theta_1, \Gamma_1) \wedge (\Theta_2, \Gamma_2)]^c = (\Theta_1, \Gamma_1)^c \vee (\Theta_2, \Gamma_2)^c$.

Proof. We only prove (i). The other properties can be similarly proved.

For all $(\varepsilon_1, \varepsilon_2) \in \Gamma_1 \times \Gamma_2$ and $u \in U$,

$$\begin{aligned}
(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2) &= \{< u, \max\{\Theta_1(\varepsilon_1)(u), \Theta_2(\varepsilon_2)(u)\} >\}, \\
[(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2)]^c &= \{< u, 1 - \max\{\Theta_1(\varepsilon_1)(u), \Theta_2(\varepsilon_2)(u)\} >\} \\
&= \{< u, \min\{1 - \Theta_1(\varepsilon_1)(u), 1 - \Theta_2(\varepsilon_2)(u)\} >\}
\end{aligned}$$

On the other hand,

$$\begin{aligned}(\Theta_1,\Gamma_1)^c &= \{< u, 1-\Theta_1(\varepsilon_1)(u) >: u\in U, \varepsilon_1\in\Gamma_1\}\\ (\Theta_2,\Gamma_2)^c &= \{< u, 1-\Theta_2(\varepsilon_2)(u) >: u\in U, \varepsilon_2\in\Gamma_2\}\end{aligned}$$

Then,

$$\begin{aligned}(\Theta_1,\Gamma_1)^c\wedge(\Theta_2,\Gamma_2)^c &= \{< u, \min\{1-\Theta_1(\varepsilon_1)(u), 1-\Theta_2(\varepsilon_2)(u)\} >\}\\ &= [(\Theta_1,\Gamma_1)\vee(\Theta_2,\Gamma_2)]^c\end{aligned}$$

Hence, $[(\Theta_1,\Gamma_1)\vee(\Theta_2,\Gamma_2)]^c = (\Theta_1,\Gamma_1)^c\wedge(\Theta_2,\Gamma_2)^c$ is obtained. □

Example 3.23. We consider that attributes in Example-3.3. Then the fuzzy hypersoft sets (Θ_1,Γ_1) and (Θ_2,Γ_2) defined as follows;

$$(\Theta_1,\Gamma_1)=\left\{\begin{array}{c}< (\alpha_2,\beta_2,\gamma_1),\{\frac{u_1}{0{,}3},\frac{u_2}{0{,}4}\} >, < (\alpha_2,\beta_2,\gamma_2),\{\frac{u_1}{0{,}2},\frac{u_2}{0{,}5},\frac{u_3}{0{,}1}\} >,\\ < (\alpha_2,\beta_3,\gamma_1),\{\frac{u_1}{0{,}6},\frac{u_3}{0{,}7}\} >, < (\alpha_2,\beta_3,\gamma_2),\{\frac{u_2}{0{,}4},\frac{u_3}{0{,}5}\} >\end{array}\right\},$$

$$(\Theta_2,\Gamma_2)=\left\{\begin{array}{c}< (\alpha_1,\beta_1,\gamma_1),\{\frac{u_1}{0{,}4},\frac{u_3}{0{,}7}\} >, < (\alpha_1,\beta_2,\gamma_1),\{\frac{u_2}{0{,}3},\frac{u_3}{0{,}6}\} >,\\ < (\alpha_2,\beta_1,\gamma_1),\{\frac{u_1}{0{,}7},\frac{u_3}{0{,}8}\} >, < (\alpha_2,\beta_2,\gamma_1),\{\frac{u_2}{0{,}5},\frac{u_3}{0{,}7}\} >\end{array}\right\}$$

Let's assume $(\alpha_2,\beta_2,\gamma_1)=\eta_1$, $(\alpha_2,\beta_2,\gamma_2)=\eta_2$,$(\alpha_2,\beta_3,\gamma_1)=\eta_3$,$(\alpha_2,\beta_3,\gamma_2)=\eta_4$ in (Θ_1,Γ_1) and $(\alpha_1,\beta_1,\gamma_1)=k_1$,$(\alpha_1,\beta_2,\gamma_1)=k_2$,$(\alpha_2,\beta_1,\gamma_1)=k_3$, $(\alpha_2,\beta_2,\gamma_1)=k_4$ in (Θ_2,Γ_2) for easier operation. The tabular forms of these sets are as follows.

(Θ_1,Γ_1)	u_1	u_2	u_3
η_1	$0,3$	$0,4$	0
η_2	$0,2$	$0,5$	$0,1$
η_3	$0,6$	0	$0,7$
η_4	0	$0,4$	$0,5$

Table 1: Tabular form of FHSS (Θ_1,Γ_1)

and

(Θ_2,Γ_2)	u_1	u_2	u_3
k_1	$0,4$	0	$0,7$
k_2	0	$0,3$	$0,6$
k_3	$0,7$	0	$0,8$
k_4	0	$0,5$	$0,7$

Table 2: Tabular form of FHSS (Θ_2,Γ_2)

Then the "AND" and "OR" operations of these sets are given as below.

$(\Theta_1, \Gamma_1) \wedge (\Theta_2, \Gamma_2)$	u_1	u_2	u_3
$\eta_1 \times k_1$	$0,3$	0	0
$\eta_1 \times k_2$	0	$0,3$	0
$\eta_1 \times k_3$	$0,3$	0	0
$\eta_1 \times k_4$	0	$0,4$	0
$\eta_2 \times k_1$	$0,2$	0	$0,1$
$\eta_2 \times k_2$	0	$0,3$	$0,1$
$\eta_2 \times k_3$	$0,2$	0	$0,1$
$\eta_2 \times k_4$	0	$0,5$	$0,1$
$\eta_3 \times k_1$	$0,4$	0	$0,7$
$\eta_3 \times k_2$	0	0	$0,6$
$\eta_3 \times k_3$	$0,6$	0	$0,7$
$\eta_3 \times k_4$	0	0	$0,7$
$\eta_4 \times k_1$	0	0	$0,5$
$\eta_4 \times k_2$	0	$0,3$	$0,5$
$\eta_4 \times k_3$	0	0	$0,5$
$\eta_4 \times k_4$	0	$0,4$	$0,5$

Table 3: Tabular forn of FHSS $(\Theta_1, \Gamma_1) \wedge (\Theta_2, \Gamma_2)$

$(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2)$	u_1	u_2	u_3
$\eta_1 \times k_1$	$0,4$	$0,4$	$0,7$
$\eta_1 \times k_2$	$0,3$	$0,4$	$0,6$
$\eta_1 \times k_3$	$0,7$	$0,4$	$0,8$
$\eta_1 \times k_4$	$0,3$	$0,5$	$0,7$
$\eta_2 \times k_1$	$0,4$	$0,5$	$0,7$
$\eta_2 \times k_2$	$0,2$	$0,5$	$0,6$
$\eta_2 \times k_3$	$0,7$	$0,5$	$0,8$
$\eta_2 \times k_4$	$0,2$	$0,5$	$0,7$
$\eta_3 \times k_1$	$0,6$	0	$0,7$
$\eta_3 \times k_2$	$0,6$	$0,3$	$0,7$
$\eta_3 \times k_3$	$0,7$	0	$0,8$
$\eta_3 \times k_4$	$0,6$	$0,5$	$0,7$
$\eta_4 \times k_1$	$0,4$	$0,4$	$0,7$
$\eta_4 \times k_2$	0	$0,4$	$0,6$
$\eta_4 \times k_3$	$0,7$	$0,4$	$0,8$
$\eta_4 \times k_4$	0	$0,5$	$0,7$

Table 4: Tabular form of FHSS $(\Theta_1, \Gamma_1) \vee (\Theta_2, \Gamma_2)$

4. Application in Decision-Making Problem

The comparison table of fuzzy hypersoft set (Θ, Γ) is a square table in which the number of rows and number of columns are equal, rows and columns both are labelled by the objects names $u_1, u_2, ..., u_n$ of the universe U and the entries c$_{ij}$ $(i, j = 1, 2, ..., n)$ is the number of parameters for which the membership value of u_i exceed or equal to the membership value of u_j. Clearly $0 \leq c_{ij} \leq k$ where k is the number of parameters in a fuzzy hypersoft set. Thus, c_{ij} indicates a numerical measure and integer number which is u_i dominates u_j in c_{ij} number of parameters out of k parameters.

4.1. *Column Sum, Row Sum and Score value of an Object*

a: The row sum of object u_i is denoted by r_i and calculated by following formula,

$$r_i = \sum_{j=1}^{n} c_{ij}$$

It is clear that r_i indicates the total number of parametersin which u_i dominates all the members of U.

b: The column sum of object u_j is denoted by t_i and calculated bu following formula,

$$t_i = \sum_{i=1}^{n} c_{ij}$$

It is clear that t_j indicates the total number of parametersin which u_j dominates all the members of U.

c: The score value of u_i is S_i and calculated by following formula,

$$S_i = r_i - t_i$$

4.2. *Algorithm*

The problem here is to select an object from the set of objects given based on a set of parameters Γ. We now present an object recognition algorithm, based on input data from multiobservers defined by age, foreign language knowledge and education.

(1) Input the fuzzy hypersoft sets $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2), (\Theta_3, \Gamma_3)$.
(2) Input the parameter set Σ as observed by the decision makers.
(3) Compute the corresponding resultant fuzzy hypersoft set from the fuzzy hypersoft sets $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2), (\Theta_3, \Gamma_3)$ and place the tabular form.
(4) Construct the Comparison Table of the fuzzy hypersoft set and compute row sum r_i and column sum t_i for u_i.
(5) Compute the Score value of u_i.
(6) The decision is $S_k = \max S_i$

(7) If k has more than one value then any one of u_k may be chosen.

4.3. *Application*

Consider the problem of selecting the most suitable staff from the set of staff with respect to a set of choice parameters. Let U be the set of staff given as $U = \{u_1, u_2, u_3, u_4, u_5\}$ also consider the set of attributes as E_1 = Age, E_2 = Foreign language knowledge, E_3 = Education, and their respective attributes are given as

$$\begin{aligned} E_1 &= \text{Age} = \{\text{18-25}(\alpha_1),\ \text{25-30}(\alpha_2),\ \text{30-35}(\alpha_3)\} \\ E_2 &= \text{Foreign language knowledge} = \{\text{English}(\beta_1),\ \text{Turkish}(\beta_2),\ \text{German}(\beta_3)\} \\ E_3 &= \text{Education} = \{\text{BSc}(\gamma_1),\ \text{MSc}(\gamma_2),\ \text{PhD}(\gamma_3)\} \end{aligned}$$

Suppose that

$$\begin{aligned} A_1 &= \{\alpha_3\},\ A_2 = \{\beta_1, \beta_2\}, A_3 = \{\gamma_2, \gamma_3\} \\ B_1 &= \{\alpha_1, \alpha_2\},\ B_2 = \{\beta_2, \beta_3\}, B_3 = \{\gamma_3\} \\ C_1 &= \{\alpha_2, \alpha_3\},\ A_2 = \{\beta_1, \beta_3\}, A_3 = \{\gamma_1\} \end{aligned}$$

are subset of E_i for each $i = 1, 2, 3$. Then the fuzzy hypersoft sets (Θ_1, Γ_1) and (Θ_2, Γ_2) defined as follows;

$$(\Theta_1, \Gamma_1) = \left\{ \begin{array}{l} <(\alpha_3, \beta_1, \gamma_2), \{\frac{u_1}{0{,}4}, \frac{u_2}{0{,}5}, \frac{u_3}{0{,}6}, \frac{u_4}{0{,}1}, \frac{u_5}{0{,}8}\}>, <(\alpha_3, \beta_1, \gamma_3), \{\frac{u_1}{0{,}4}, \frac{u_2}{0{,}2}, \frac{u_3}{0{,}6}, \frac{u_4}{0{,}1}, \frac{u_5}{0{,}3}\}>, \\ <(\alpha_3, \beta_2, \gamma_2), \{\frac{u_1}{0{,}2}, \frac{u_2}{0{,}7}, \frac{u_3}{0{,}4}, \frac{u_4}{0{,}4}, \frac{u_5}{0{,}5}\}>, <(\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{0{,}7}, \frac{u_2}{0{,}1}, \frac{u_3}{0{,}4}, \frac{u_4}{0{,}2}, \frac{u_5}{0{,}6}\}> \end{array} \right\},$$

$$(\Theta_2, \Gamma_2) = \left\{ \begin{array}{l} <(\alpha_1, \beta_2, \gamma_3), \{\frac{u_1}{0{,}8}, \frac{u_2}{0{,}4}, \frac{u_3}{0{,}7}, \frac{u_4}{0{,}3}, \frac{u_5}{0{,}8}\}>, <(\alpha_1, \beta_3, \gamma_3), \{\frac{u_1}{0{,}4}, \frac{u_2}{0{,}5}, \frac{u_3}{0{,}6}, \frac{u_4}{0{,}1}, \frac{u_5}{0{,}8}\}>, \\ <(\alpha_2, \beta_2, \gamma_3), \{\frac{u_1}{0{,}2}, \frac{u_2}{0{,}2}, \frac{u_3}{0{,}6}, \frac{u_4}{0{,}3}, \frac{u_5}{0{,}1}\}>, <(\alpha_2, \beta_3, \gamma_3), \{\frac{u_1}{0{,}5}, \frac{u_2}{0{,}5}, \frac{u_3}{0{,}1}, \frac{u_4}{0{,}1}, \frac{u_5}{0{,}7}\}> \end{array} \right\}$$

$$(\Theta_3, \Gamma_3) = \left\{ \begin{array}{l} <(\alpha_2, \beta_1, \gamma_1), \{\frac{u_1}{0{,}2}, \frac{u_2}{0{,}3}, \frac{u_3}{0{,}1}, \frac{u_4}{0{,}7}, \frac{u_5}{0{,}6}\}>, <(\alpha_2, \beta_3, \gamma_1), \{\frac{u_1}{0{,}6}, \frac{u_2}{0{,}2}, \frac{u_3}{0{,}3}, \frac{u_4}{0{,}3}, \frac{u_5}{0{,}1}\}>, \\ <(\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{0{,}7}, \frac{u_2}{0{,}5}, \frac{u_3}{0{,}8}, \frac{u_4}{0{,}4}, \frac{u_5}{0{,}8}\}>, <(\alpha_3, \beta_3, \gamma_1), \{\frac{u_1}{0{,}3}, \frac{u_2}{0{,}4}, \frac{u_3}{0{,}1}, \frac{u_4}{0{,}6}, \frac{u_5}{0{,}5}\}> \end{array} \right\}$$

The tabular representions of $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2), (\Theta_3, \Gamma_3)$ are shown in Table 5-7.

(Θ_1, Γ_1)	u_1	u_2	u_3	u_4	u_5
$(\alpha_3, \beta_1, \gamma_2) = \eta_1$	$0,4$	$0,5$	$0,6$	$0,1$	$0,8$
$(\alpha_3, \beta_1, \gamma_3) = \eta_2$	$0,4$	$0,2$	$0,6$	$0,1$	$0,3$
$(\alpha_3, \beta_2, \gamma_2) = \eta_3$	$0,2$	$0,7$	$0,4$	$0,4$	$0,5$
$(\alpha_3, \beta_2, \gamma_3) = \eta_4$	$0,7$	$0,1$	$0,4$	$0,2$	$0,6$

Table-5: Tabular form of FHSS (Θ_1, Γ_1)

(Θ_2,Γ_2)	u_1	u_2	u_3	u_4	u_5
$(\alpha_1,\beta_2,\gamma_3)=k_1$	$0,8$	$0,4$	$0,7$	$0,3$	$0,8$
$(\alpha_1,\beta_3,\gamma_3)=k_2$	$0,4$	$0,5$	$0,6$	$0,1$	$0,8$
$(\alpha_2,\beta_2,\gamma_3)=k_3$	$0,2$	$0,2$	$0,6$	$0,3$	$0,1$
$(\alpha_2,\beta_3,\gamma_3)=k_4$	$0,5$	$0,5$	$0,1$	$0,1$	$0,7$

Table-6: Tabular form of FHSS (Θ_2,Γ_2)

(Θ_3,Γ_3)	u_1	u_2	u_3	u_4	u_5
$(\alpha_2,\beta_1,\gamma_1)=m_1$	$0,2$	$0,3$	$0,1$	$0,7$	$0,6$
$(\alpha_2,\beta_3,\gamma_1)=m_2$	$0,6$	$0,2$	$0,3$	$0,3$	$0,1$
$(\alpha_3,\beta_1,\gamma_1)=m_3$	$0,7$	$0,5$	$0,8$	$0,4$	$0,8$
$(\alpha_3,\beta_3,\gamma_1)=m_4$	$0,3$	$0,4$	$0,1$	$0,6$	$0,5$

Table-7: Tabular form of FHSS (Θ_3,Γ_3)

Now let's see how the original problem can be solved using the algorithm. Consider the fuzzy hypersoft sets $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ as defined above. If we perform $(\Theta_1,\Gamma_1)\wedge(\Theta_2,\Gamma_2)$ then we will have $4\times 4=16$ parameters. If we require fuzzy hypersoft set for the parameters $R=\{\eta_1\times k_1,\eta_2\times k_4,\eta_3\times k_2,\eta_3\times k_3,\eta_4\times k_1\}$, then we obtain the resultant fuzzy hypersoft for the fuzzy hypersoft sets $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$. So, after performing $(\Theta_1,\Gamma_1)\wedge(\Theta_2,\Gamma_2)$ for some parameters the tabular form of the resultant fuzzy hypersoft set (K,R) will take the form as Table-8,

(K,R)	u_1	u_2	u_3	u_4	u_5
$\eta_1\times k_1$	$0,4$	$0,4$	$0,6$	$0,1$	$0,8$
$\eta_2\times k_4$	$0,4$	$0,2$	$0,1$	$0,1$	$0,3$
$\eta_3\times k_2$	$0,2$	$0,5$	$0,4$	$0,1$	$0,5$
$\eta_3\times k_3$	$0,2$	$0,2$	$0,4$	$0,3$	$0,1$
$\eta_4\times k_1$	$0,7$	$0,1$	$0,4$	$0,2$	$0,6$

Table-8: Tabular form of FHSS (K,R)

Consider the fuzzy hypersoft sets $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ and (Θ_3,Γ_3) as defined above.Suppose that P be set of choice parameters of a decision maker. On the basis of this parameter we have to take the decision from the availability set U . The tabular representation of resultant fuzzy hypersoft set (S,P) will be as. (Table 19).

(S,P)	u_1	u_2	u_3	u_4	u_5
$(\eta_1\times k_1)\times m_1$	$0,2$	$0,3$	$0,1$	$0,1$	$0,6$
$(\eta_2\times k_4)\times m_3$	$0,4$	$0,2$	$0,1$	$0,1$	$0,3$
$(\eta_3\times k_2)\times m_2$	$0,2$	$0,2$	$0,3$	$0,1$	$0,1$
$(\eta_3\times k_3)\times m_4$	$0,2$	$0,2$	$0,1$	$0,3$	$0,1$
$(\eta_4\times k_1)\times m_3$	$0,7$	$0,1$	$0,4$	$0,2$	$0,6$

Table-9: Tabular form of FHSS (S,P)

The comparison table of the above resultant fuzzy hypersoft set is as below,

	u_1	u_2	u_3	u_4	u_5
u_1	5	4	4	4	4
u_2	3	5	3	3	2
u_3	1	2	5	4	2
u_4	1	2	3	5	2
u_5	1	3	4	4	5

Table-10: The comparison table of the above resultant fuzzy hypersoft set

Next we compute the column sum, row sum and score value for each u_i as shown following,

	Row Sum(r_i)	*Column Sum*(t_i)	*Score*(S_i)
u_1	21	11	10
u_2	16	16	-0
u_3	14	19	-5
u_4	13	20	-7
u_5	17	15	2

Table-11: The row sum, column sum and score of u_i

From the above score table, it is clear that the maximum score is 10, scored by u_1. Therefore the decision is in favour of selecting u_1.

5. **Conclusion**

In the present paper, Fuzzy Hypersoft set's operations such as like subset, union, intersection, equal set, complement, AND and OR are introduced. The validity and complementarity of the proposed operations and meanings is checked by presenting the correct example. Fuzzy hypersoft set NHSS would be a new method for correct selection in decision taking problems. we give also an application of fuzzy hypersoft theory in object recognition problem. The recognition strategy is based on collection of data parameters for multiobserver inputs. The algorithm involves constructing the Comparison Table from the resulting fuzzy hypersoft set and making the final decision based on the maximum score determined from the Comparison Table (Tables 10 and 11). In order to expand our work, more research should be undertaken to study the issues of reducing fuzzy hypersoft sets parameterization, and to explore the possibilities of using the fuzzy hypersoft sets method to solve real-world problems such as decision-making, forecasting, and data analysis. Matrices, similarity measure, single and multi-valued, interval valued, functions, distance measures, algorithms: score function, VIKOR, TOPSIS, AHP of Intutionistic Hypersoft set will be future work. Also, in the topological field, topics such as fuzzy hypersoft topological spaces, separation axioms, compactness, connectedness, continuity can be studied in the future.

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Chapter 4

Matrix Theory for Intuitionistic Fuzzy Hypersoft Sets and its application in Multi-Attributive Decision-Making Problems

Muhammad Naveed Jafar[1], Muhammad Saeed[1*], Muhammad Haseeb[2], Ahtasham Habib[3]

[1] Department of Mathematics University of Management and Technology, Johar Town C-Block 54000, Pakistan, E-Mail: 15009265004@umt.edu.pk

[1*] Department of Mathematics University of Management and Technology, Johar Town C-Block 54000, Pakistan, E-Mail: muhammad.saeed@umt.edu.pk

[2-3] Department of Mathematics Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan,

Correspondence: muhammad.saeed@umt.edu.pk

Abstract: The main aspect of this study is to broaden the concept of intuitionistic fuzzy hypersoft set (IFHSS) and apply it as intuitionistic fuzzy hypersoft matrix (IFHSM). Fundamental workings of intuitionistic hypersoft matrices have been elaborated through suitable examples. Analytical study of common operations of IFHSM has been devised. A new algorithm, based on score function, has been constituted to represent decision-making issues and these issues can now be defined as numerical values to elaborate the hiring of employees for a private firm.

Keywords: *MCDM, Hypersoft Matrix, Score Function, Intuitionistic Fuzzy Hypersoft Matrix*.

1. Introduction

Zadeh L.A introduced the concept 'Fuzzy sets' to deal with the problem of uncertainty in 1965 [1]. Many problems arising due to uncertainty in many applications have been successfully dealt by using fuzzy sets and fuzzy logic. In 1975, Zadeh [2] introduced the interval valued fuzzy sets. This concept emphasized on the degree of membership of an element on closed subinterval of [0,1] rather than an element of [0,1]. Fuzzy set theory can better handle the problem of uncertainty that is due to unclarity or incomplete belonging of an element in a set, but it has its limitations when dealing with various real physical problems or incomplete data.

Atanassov [12] in 1986 worked on the generalization of fuzzy set known as intuitionistic fuzzy set (IFS). For the first time, the concept of natural hesitation occurring in human beings' brain was awarded a membership or non-membership value in closed interval, so that the impact and effect of uncertainty could be portrayed realistically as happening in real world during decision making processes. It has been seen that situations where insufficient data is present, membership values may not be possible up to the level of our satisfaction. The same happens with the non-membership values as allocating them values is not sometimes possible and degree of hesitation remains. In such scenarios, fuzzy set theory cannot be used effectively and this is where intuitionistic fuzzy set theory plays its role. In fact, the problems for which fuzzy set theory is used, these problems can be solved by intuitionistic fuzzy set theory as well and it is well suited for solving complicated problems. Hence, IFS has been found quite effective in dealing and solving the issue of uncertainty. [12,13]

In 1999, Molodtsov introduced a new mathematical tool know as soft set to deal with the complexities arising due to uncertainties. He demonstrated the way in which soft sets could be used. Their application can be used in game theory, operation research, Peron integration, Riemman-integration, probability, theory of management etc. Presently, much progress is being made in the working and applications of the soft set theory. [25,27]

The parameters in soft set theory are fuzzy concepts taken from the fuzzy set theory. Hence, set soft is a specified classification of subsets of the universe. Results have been projected by the application of using fuzzy soft sets in decision making in the works of Roy and Maji [32]. Measures showing similarity in fuzzy soft sets were proposed by Majumdar and Samanta [24]. By minimization of fuzzy soft sets, Yang et al. [37] did an analysis of decision-making problems using fuzzy soft sets. The idea of soft number, soft integral and soft derivative was developed through an analysis relying on the theory of soft sets. The problems of soft optimization were solved through this manner by Kovkov *et al.* [21]. For the forecasting of the volume of export and import in international trade, using fuzzy soft sets of soft set theory were introduced by Xiao et al. [34]. For the studying of business competitive capacity evaluation, Xiao et al. [35] also introduced soft set theory.

Solutions regarding data analysis due to the conditions of incomplete information by using soft sets were introduced by Zou and Xiao [38]. These solutions, by using soft sets, reflect the accurate state of incomplete data. Problems arising with the classifications of natural textures were dealt by a new algorithm by Mushrif et al. [28]. Analysis of various operations of soft sets was introduced by Ali et al. [14].

The idea of intuitionistic fuzzy soft set theory made up of intuitionistic fuzzy set and soft set models was introduced by Maji et al. [41-42]. Interval-valued fuzzy soft sets containing the combination of interval-valued fuzzy set and soft set models was worked upon by Yang et al. [42]

Importance of matrices cannot be ignored in the fields of science and engineering. But due to uncertainties arising due to imprecision in the nature of our environment, classic matrix theory cannot meet the demand of solving the problem of uncertainty. However, fuzzy soft sets represented as matrices were successfully used in the process known as fuzzy soft matrix to deal with the problems of decision making by Yong Tang and Chenli [39]. Jafar *et al* [30] worked on IFSS in Selection of Laptop.

Recently in 2018, another tool was created by Smarandache [43] in diversifying the modes available in overcoming the problems related to uncertainty. He generalized soft set theory to hyper soft set theory.

The propose of this research work is to introduce a new tool of dealing with uncertainty by combining intuitionist fuzzy set theory with hyper soft set theory. This combination of the two theories will make a new tool by the name of 'Intuitionistic fuzzy hyper soft set'. Later on, it would be converted in the form of intuitionistic fuzzy hyper soft matrix.

2. Preliminaries

Definition 2.1: Fuzzy set

A pair ($\mathcal{U}$,e) is said to be fuzzy set if there exist a function $e: \mathcal{U} \to [0,1]$ where $\mathcal{U}$ is set. The function $e = \mu_{\ddot{A}}$ is called membership function of the fuzzy set $\ddot{A} = (\mathcal{U}, e)$.

Definition 2.2: Soft set

Let $\mathcal{U}$ be the universal set and $\mathrm{p}(\mathcal{U})$ be the power set of $\mathcal{U}$. Let $\check{A}$ be the set of attributes then the pair (F, $\mathcal{U}$) is said to be soft set over $\mathcal{U}$ if $\mathrm{F}: \check{A} \to \mathrm{p}(\mathcal{U})$.

Definition 2.3: Hypersoft set

Let $\mathcal{U}$ be the universal set and $P(\mathcal{U})$be the power set of $\mathcal{U}$. Suppose $h_1, h_2, h_3 \ldots h_n$ where $n \geq 1$ be n distinct attributes whose corresponding attributive values respectively the sets $H_1, H_2, H_3 \ldots H_n$ with $H_i \cap H_j = \emptyset, i \neq j$ and $i, j \epsilon \{0,1,2,3 \ldots n\}$ then the pair $(F, H_1 \times H_2 \times H_3 \times \ldots \times H_n)$,where $F: H_1 \times H_2 \times H_3 \times \ldots \times H_n \to P(\mathcal{U})$ is called a hypersoft set over $\mathcal{U}$.

Definition 2.4: Intuitionistic fuzzy soft set

Consider $\mathcal{U}$ is a universal set and Ĕ be the set of parameters. Let $P(\mathcal{U})$ denote the set of all intuitionistic fuzzy sets of $\mathcal{U}$. Let $\bar{A} \subseteq$ Ĕ. A pair (F, Ĕ) is an intuitionistic fuzzy soft set over $\mathcal{U}$, where is mapping given by $\mathrm{F}: \bar{A} \to P(\mathcal{U})$.

Definition 2.4: Intuitionistic fuzzy Hypersoft set (IFHSS)

Let E be the initial universe of discourse and P(E) is the set of all possibilities of E. suppose $h_1, h_2, h_3 \ldots h_n$ where $n \geq 1$ be n distinct attributes whose corresponding attributive values respectively the sets $H_1, H_2, H_3 \ldots H_n$with $H_i \cap H_j = \emptyset, i \neq j$ and $i, j \epsilon \{0,1,2,3, \ldots, n\}$ then the relation $H_1 \times H_2 \times H_3 \times \ldots \times H_n = \alpha$ then the pair (F, α) is said to be Intuitionistic fuzzy hypersoft set (IFHSS). $F: H_1 \times H_2 \times H_3 \times \ldots \times H_n \to P(E)$ and $F(H_1 \times H_2 \times H_3 \times \ldots \times H_n) = \{< x, \mu(F(\alpha)), \gamma(F(\alpha)) >, x \epsilon E\}$ where μ is the value of membership and γ is the value of non-membership such that $\mu: E \to [0,1]$, $\gamma: E \to [0,1]$ and also

$$0 < \mu(F(\alpha)) + \gamma(F(\alpha)) < 2$$

3. Calculations

Definition 3.1: Intuitionistic fuzzy Hyper Soft Matrix (IFHSM): Let $\digamma = \{\mathfrak{f}^1, \mathfrak{f}^2, \dots, \mathfrak{f}^n\}$ be the universal set and $P(\digamma)$ be the power sets of $\digamma$. Suppose $\daleth_1, \daleth_2, \dots, \daleth_m$ where $\alpha \geq 1$, α be the well-defined attributes , whose corresponding attributive values are $\daleth_1^a, \daleth_2^b, \dots, \daleth_m^z$ and their relation is $\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z$ where $\boldsymbol{a, b, c, \dots, z = 1, 2, \dots, n}$ then the pair, $(F, \daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z)$ is said to be Intuitionistic fuzzy Hyper Soft set over $\digamma$ where $F: (\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z) \to P(\digamma)$ and it is define as $F(\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z) = \{< \mathfrak{f}, T_{\pounds}(\mathfrak{f}), F_{\pounds}(\mathfrak{f}) > \mathfrak{f} \in \digamma, \pounds \in (\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z)\}$. Let $D_{\pounds} = \daleth_1^a \times \daleth_2^b \times \dots \times \daleth_\alpha^z$ be the relation and its characteristic function is $Y_{D_{\pounds}}: (\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z) \to P(\digamma)$ and it is define as $Y_{D_{\pounds}} = \{< \mathfrak{f}, T_{\pounds}(\mathfrak{f}), F_{\pounds}(\mathfrak{f}) > \mathfrak{f} \in \digamma, \pounds \in (\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z)\}$. Then the Tabular form of $D_{\pounds}$ is given as

Table 1: Tabular form of $D_{\pounds}$

	$\daleth_1^a$	$\daleth_2^b$	$\cdots$	$\daleth_m^z$
$\mathfrak{f}^1$	$Y_{D_{\pounds}}(\mathfrak{f}^1, \daleth_1^a)$	$Y_{D_{\pounds}}(\mathfrak{f}^1, \daleth_2^b)$	$\cdots$	$Y_{D_{\pounds}}(\mathfrak{f}^1, \daleth_m^z)$
$\mathfrak{f}^2$	$Y_{D_{\pounds}}(\mathfrak{f}^2, \daleth_1^a)$	$Y_{D_{\pounds}}(\mathfrak{f}^2, \daleth_2^b)$	$\cdots$	$Y_{D_{\pounds}}(\mathfrak{f}^2, \daleth_m^z)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathfrak{f}^n$	$Y_{D_{\pounds}}(\mathfrak{f}^n, \daleth_1^a)$	$Y_{D_{\pounds}}(\mathfrak{f}^n, \daleth_2^b)$	$\cdots$	$Y_{D_{\pounds}}(\mathfrak{f}^n, \daleth_m^z)$

If $Q_{ij} = Y_{D_{\pounds}}(\mathfrak{f}^i, \daleth_j^k)$ where i=1,2,3…n , j=1,2,3…m , k=a,b,c,…,z

Then a matrix is defining as

$$[Q_{ij}]_{n\times m} = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1m} \\ Q_{21} & Q_{22} & \cdots & Q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{n1} & Q_{n2} & \cdots & Q_{nm} \end{bmatrix}$$

Where $Q_{ij} = \left(T_{\daleth_j^k}(\mathfrak{f}_i), F_{\daleth_j^k}(\mathfrak{f}_i),\ \mathfrak{f}_i \in \digamma, \daleth_j^k \in (\daleth_1^a \times \daleth_2^b \times \dots \times \daleth_m^z) \right) = (T_{ijk}^Q, F_{ijk}^Q)$

Thus, we can represent any Intuitionistic Fuzzy Hyper Soft Set in term of Intuitionistic Fuzzy Hyper Soft matrix.

Example 3.1.1:

To illustrate the working of this theory, a need arises for a company to hire an employee to fill one of its vacant spaces. A total of five promising applicants apply to fill up the vacant space. From the human resources department of the company, a decision maker (DM) has been tasked for selection.

Let $U = \{f^1, f^2, f^3, f^4, f^5\}$ be the set of five applicants also consider the set of attributes as

$\daleth_1 =$ Qualification, $\daleth_2$ =Exprince, $\daleth_3 =$ Age, $\daleth_4$ =Gender

And their respective attributes are

$\daleth_1^a$ =Qualification = {BS Hons., MS, Ph.D., Post Doctorate}

$\daleth_2^b$ =Exprince= {5yr, 7yr, 10yr, 15yr}

$\daleth_3^c$ =Age= {Less than thirty, Greater than thirty}

$\daleth_4^d$ =Gender= {Male, Female}

Let the function be F: $\daleth_1^a \times \daleth_2^b \times \daleth_3^c \times \daleth_4^d \rightarrow P(U)$

Blew are the Intuitionistic values table from different dissuasion maker

Table 2: Decision maker Intuitionistic values for Qualification

$\daleth_1^a$**(Qualification)**	f^1	f^2	f^3	f^4	f^5
BS Hons.	(0.5,0.7)	(0.3,0.5)	(0.5,1.0)	(0,0.9)	(0.6,0.5)
MS	(0.3,0.8)	(0.1,0.6)	(0.1,0.3)	(0.2,0.5)	(0.3,0.9)
Ph.D.	(0.9,1)	(0.4,0.6)	(0.6,0.8)	(0.1,0.9)	(0.1,0.7)
Post Doctorate	(0.2,0.3)	(0.3,0.4)	(0.4,0.5)	(0.5,0.6)	(0.6,0.7)

Table 3: Decision maker Intuitionistic values for Experience

$\daleth_2^b$**(Experience)**	f^1	f^2	f^3	f^4	f^5
5yr	(0.4,0.7)	(0.1,0.3)	(0.2,0.4)	(0.3,0.5)	(0.4,0.6)
7yr	(0.5,0.7)	(0.6,0.8)	(0.7,0.9)	(0.8,1)	(0.9,0)
10yr	(0.4,0.7)	(0.5,0.8)	(0.6,0.9)	(0.7,1)	(0.8,0.3)
15yr	(0.1,0.5)	(0.2,0.6)	(0.3,0.7)	(0.4,0.8)	(0.5,0.9)

Table 4: Decision maker Intuitionistic values for Age

$\daleth_3^c$**(Age)**	f^1	f^2	f^3	f^4	f^5
Less than thirty	(0.6,0.8)	(0.6,0.9)	(0.3,0.5)	(0.4,0.8)	(0.5,0.9)
Greater than thirty	(0.7,1)	(0.5,0.7)	(0.3,0.7)	(0.5,0.6)	(0.9,1)

Table 5: Decision maker Intuitionistic values for Gender

$Ʈ_4^d$(Gender)	f^1	f^2	f^3	f^4	f^5
Male	(0.5,0.7)	(0.1,0.5)	(0.6,0.9)	(0.4,0.8)	(0.5,0.9)
Female	(0.3,0.7)	(0.2,0.6)	(0.2,0.4)	(0.7,1)	(0.9,0)

Intuitionistic fuzzy Hyper Soft set is defining as

$$\mathrm{F}: (Ʈ_1^a \times Ʈ_2^b \times Ʈ_3^c \times Ʈ_4^d) \rightarrow P(U)$$

Let's assume that $\mathrm{F}(Ʈ_1^a \times Ʈ_2^b \times Ʈ_3^c \times Ʈ_4^d) = \mathrm{F}(\mathrm{MS}, 7\mathrm{yr}, \text{Greater than thirty}, \mathrm{Male}) = (f^1, f^2, f^3, f^5)$

Then Intuitionistic Fuzzy Hyper Soft set of above relation

$$\mathrm{F}(Ʈ_1^a \times Ʈ_2^b \times Ʈ_3^c \times Ʈ_4^d) =$$

$$\{< f^1, ((MS(0.3{,}0.8), 7yr(0.5{,}0.7), \text{Greater than thirty}(0.7{,}1), \text{Male}(0.5{,}0.7)) >$$

$$< f^2, (MS(0.1{,}0.6), 7yr(0.6{,}0.8), \text{Greater than thirty}(0.5{,}0.7), \text{Male}(0.1{,}0.5)) >$$

$$< f^3, (MS(0.1{,}0.3), 7yr(0.7{,}0.9), \text{Greater than thirty}(0.3{,}0.7), \text{Male}(0.6{,}0.9)) >$$

$$< f^5, (MS(0.3{,}0.9), 7yr(0.9{,}0.), \text{Greater than thirty}(0.9{,}1), \text{Male}(0.5{,}0.9)) >\}$$

The above relation can be written in the form of

Table 6: Tabular Representation of above relation

	$Ʈ_1^a$	$Ʈ_2^b$	$Ʈ_3^c$	$Ʈ_4^d$
f^1	$(MS(0.3{,}0.8))$	$(7yr(0.5{,}0.7))$	(Greater than thirty(0.7,1))	(Male(0.5,0.7))
f^2	$(MS(0.1{,}0.6))$	$(7yr(0.6{,}0.8))$	(Greater than thirty(0.5,0.7))	(Male(0.1,0.5)
f^3	$(MS(0.1{,}0.3))$	$(7yr(0.7{,}0.9))$	(Greater than thirty(0.3,0.7))	(Male(0.6,0.9))
f^5	$(MS(0.3{,}0.9))$	$(7yr(0.9{,}0))$	(Greater than thirty(0.9,1))	(Male(0.5,0.9))

And its matrix form is defined as

$$[Q]_{4\times 4} = \begin{bmatrix} (MS,(0.3{,}0.8)) & (7yr,(0.5{,}0.7)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.5{,}0.7)) \\ (MS,(0.1{,}0.6)) & (7yr,(0.6{,}0.8)) & (\text{Greater than thirty},(0.5{,}0.7)) & (\text{Male},(0.1{,}0.5)) \\ (MS,(0.1{,}0.3)) & (7yr,(0.7{,}0.9)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.6{,}0.9)) \\ (MS,(0.3{,}0.9)) & (7yr,(0.9{,}0)) & (\text{Greater than thirty},(0.9{,}1)) & (\text{Male},(0.5{,}0.9)) \end{bmatrix}$$

Definition 3.2: Square Intuitionistic Fuzzy Hyper Soft Matrix

Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$ where $Q_{ij} = \left(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}\right)$, then Q is said to be square IFHSM if n= m . It means that if a IFHSM have same number of columns (alternatives) and rows (attributes) then it's called square IFHSM.

Example 3.2.2:

$$[Q]_{4\times4} = \begin{bmatrix} (MS,(0.3{,}0.8)) & (7yr,(0.5{,}0.7)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.5{,}0.7)) \\ (MS,(0.1{,}0.6)) & (7yr,(0.6{,}0.8)) & (\text{Greater than thirty},(0.5{,}0.7)) & (\text{Male},(0.1{,}0.5)) \\ (MS,(0.1{,}0.3)) & (7yr,(0.7{,}0.9)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.6{,}0.9)) \\ (MS,(0.3{,}0.9)) & (7yr,(0.9{,}0)) & (\text{Greater than thirty},(0.9{,}1)) & (\text{Male},(0.5{,}0.9)) \end{bmatrix}$$

Definition 3.3: Transpose of square IFHSM

Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$ where $Q_{ij} = \left(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}\right)$ then Q^t is said to be transpose of square IFHSM if rows interchange with column (column interchange with rows) of Q. It is denoted as

$$Q^t = [Q_{ij}]^t = \left(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}\right)^t = \left(T^{Q}_{jki}, \mathrm{F}^{Q}_{jki}\right) = [Q_{ji}]$$

Example 3.3.1

$$[Q]_{4\times4} = \begin{bmatrix} (MS,(0.3{,}0.8)) & (7yr,(0.5{,}0.7)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.5{,}0.7)) \\ (MS,(0.1{,}0.6)) & (7yr,(0.6{,}0.8)) & (\text{Greater than thirty},(0.5{,}0.7)) & (\text{Male},(0.1{,}0.5)) \\ (MS,(0.1{,}0.3)) & (7yr,(0.7{,}0.9)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.6{,}0.9)) \\ (MS,(0.3{,}0.9)) & (7yr,(0.9{,}0)) & (\text{Greater than thirty},(0.9{,}1)) & (\text{Male},(0.5{,}0.9)) \end{bmatrix}$$

Transpose of the upper Metrix is defining as

$$[Q]^t{}_{4\times4} =$$

$$\begin{bmatrix} (MS,(0.3{,}0.8)) & (MS,(0.1{,}0.6)) & (MS,(0.1{,}0.3)) & (MS,(0.3{,}0.9)) \\ (7yr,(0.5{,}0.7)) & (7yr,(0.6{,}0.8)) & (7yr,(0.7{,}0.9)) & (7yr,(0.9{,}0)) \\ (\text{Greater than thirty},(0.7{,}1)) & (\text{Greater than thirty},(0.5{,}0.7)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Greater than thirty},(0.9{,}1)) \\ (\text{Male},(0.5{,}0.7)) & (\text{Male},(0.1{,}0.5) & (\text{Male},(0.6{,}0.9)) & (\text{Male},(0.5{,}0.9)) \end{bmatrix}$$

Definition 3.4: Symmetric IFHSM

Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$ where $Q_{ij} = \left(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}\right)$ then Q is said to be symmetric IFHSM if $Q^t = \mathrm{Q}$ i.e $\left(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}\right) = \left(T^{Q}_{jki}, \mathrm{F}^{Q}_{jki}\right)$

Definition 3.5: Scalar multiplication of IFHSM

Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$ where $Q_{ij} = (T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk})$ and μ be any scalar. Then the product of scalar μ and matrix Q is defined by multiplying each element Q by μ. It is denoted as $\mu Q = [\mu Q_{ij}]$ where$0 \leq \mu \leq 1$.

Example 3.5.1: Consider a IFHSM $[Q]_{4\times 4}$ and 0.3 is a scalar.

$$[Q]_{4\times4} = \begin{bmatrix} (MS,(0.3{,}0.8)) & (7yr,(0.5{,}0.7)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.5{,}0.7)) \\ (MS,(0.1{,}0.6)) & (7yr,(0.6{,}0.8)) & (\text{Greater than thirty},(0.5{,}0.7)) & (\text{Male},(0.1{,}0.5) \\ (MS,(0.1{,}0.3)) & (7yr,(0.7{,}0.9)) & (\text{Greater than thirty},(0.7{,}1)) & (\text{Male},(0.6{,}0.9)) \\ (MS,(0.3{,}0.9)) & (7yr,(0.9{,}0)) & (\text{Greater than thirty},(0.9{,}1)) & (\text{Male},(0.5{,}0.9)) \end{bmatrix}$$

Then scalar multiplication of IFHSM $[Q]_{4\times 4}$ is shown as

$$[(0.3)Q]_{4\times4} =$$

$$\begin{bmatrix} (MS,(0.09{,}0.24)) & (7yr,(0.15{,}0.21)) & (\text{Greater than thirty},(0.21{,}0.3)) & (\text{Male},(0.15{,}0.21)) \\ (MS,(0.03{,}0.18)) & (7yr,(0.18{,}0.24)) & (\text{Greater than thirty},(0.15{,}0.21)) & (\text{Male},(0.03{,}0.15)) \\ (MS,(0.03{,}0.09)) & (7yr,(0.21{,}0.27)) & (\text{Greater than thirty},(0.21{,}0.3)) & (\text{Male},(0.18{,}0.27)) \\ (MS,(0.09{,}0.27)) & (7yr,(0.27{,}0)) & (\text{Greater than thirty},(0.27{,}0.3)) & (\text{Male},(0.15{,}0.27)) \end{bmatrix}$$

Proposition 3.6

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be the two IFHSM, Where $Q_{ij} = (T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk})$ and $R_{ij} = (T^{R}_{ijk}, \mathrm{F}^{R}_{ijk})$ for two scalars *α*, β ∈ [0,1], then

I. $\alpha\ (\beta Q) = (\alpha\beta)Q$.

II. If $\alpha < \beta$ then $\alpha Q < \beta Q$.

III. If $Q \subseteq R$ then $\alpha Q \subseteq \alpha R$.

Proof

I. $\alpha(\beta Q) = \alpha(\beta Q_{ij}) = \alpha(\beta T^{Q}_{ijk}, \beta \mathrm{F}^{Q}_{ijk}) = (\alpha\beta T^{Q}_{ijk}, \alpha\beta \mathrm{F}^{Q}_{ijk})$

$$= [\alpha\beta(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk})] = \alpha\beta[Q_{ij}] = (\alpha\beta)Q$$

II. Since $(T^{Q}_{ijk}, \mathrm{F}^{Q}_{ijk}) \in [0,1]$ so $\alpha T^{Q}_{ijk} \leq \beta T^{Q}_{ijk}, \alpha \mathrm{F}^{Q}_{ijk} \leq \beta \mathrm{F}^{Q}_{ijk}$

Now αQ=$[\alpha Q_{ij}]$=$[(\alpha T^{Q}_{ijk}, \alpha \mathrm{F}^{Q}_{ijk})] \leq [(\beta T^{Q}_{ijk}, \beta \mathrm{F}^{Q}_{ijk})] = [\beta Q_{ij}] = \beta Q$

III. $Q \subseteq R \Rightarrow [Q_{ij}] \subseteq [R_{ij}]$

$$\Rightarrow T^{Q}_{ijk} \leq T^{R}_{ijk}, \mathrm{F}^{Q}_{ijk} \geq \mathrm{F}^{R}_{ijk}$$

$$\Rightarrow \alpha T^{Q}_{ijk} \leq \alpha T^{R}_{ijk}, \alpha \mathrm{F}^{Q}_{ijk} \geq \alpha \mathrm{F}^{R}_{ijk}$$

$$\Rightarrow \alpha[Q_{ij}] \subseteq \alpha[R_{ij}]$$

$$\Rightarrow \alpha Q \subseteq \alpha R$$

Theorem 3.7

Let $Q = [Q_{ij}]$ be the IFHSM of order n×m , where $Q_{ij} = (T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q})$, then

I. $(\alpha Q)^t = \alpha Q^t$ where $\alpha \in [0,1]$.

II. $(Q^t)^t = Q$.

Proof

I. Here $(\alpha Q)^t$, $(\alpha\ Q)^t \in \mathrm{IFHSM}_{\mathrm{n\times m}}$ so

$$(\alpha Q)^t = \left[(\alpha T_{ijk}^{Q}, \alpha \mathrm{F}_{ijk}^{Q})\right]^t$$

$$= \left[(\alpha T_{jki}^{Q}, \alpha \mathrm{F}_{jki}^{Q})\right]$$

$$= \alpha\left[(T_{jki}^{Q}, \mathrm{F}_{jki}^{Q})\right]$$

$$= \alpha\left[(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q})\right]^t = \alpha Q^t$$

II. Since $Q^t \in IFHSM_{n\times m}$ so $(Q^t)^t \in \mathrm{IFHSM}_{\mathrm{n\times m}}$

Now

$$(Q^t)^t = \left(\left[(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q})\right]^t\right)^t$$

$$= \left(\left[(T_{jki}^{Q}, \mathrm{F}_{jki}^{Q})\right]\right)^t$$

$$= \left[(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q})\right] = Q$$

Definition 3.8: Trace of IFHSM

Let $Q = [Q_{ij}]$ be the square IFHSM of order n×m , where $Q_{ij} = (T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q})$ and $n = m$ then trace of IFHSM is written as tr(Q) and define as $tr(Q) = \sum_{i=1,k=a}^{m,z}[T_{ijk}^{Q} - \mathrm{F}_{ijk}^{Q}]$

Example 3.8.1: Let us consider a IFHSM $[Q]_{4\times 4}$

$$[Q]_{4\times 4} = \begin{bmatrix} (MS,(0.3,0.8)) & (7yr,(0.5,0.7)) & (\text{Greater than thirty},(0.7,1)) & (\text{Male},(0.5,0.7)) \\ (MS,(0.1,0.6)) & (7yr,(0.6,0.8)) & (\text{Greater than thirty},(0.5,0.7)) & (\text{Male},(0.1,0.5)) \\ (MS,(0.1,0.3)) & (7yr,(0.7,0.9)) & (\text{Greater than thirty},(0.7,1)) & (\text{Male},(0.6,0.9)) \\ (MS,(0.3,0.9)) & (7yr,(0.9,0)) & (\text{Greater than thirty},(0.9,1)) & (\text{Male},(0.5,0.9)) \end{bmatrix}$$

Then tr(Q) =$(0.3 - 0.8) + (0.6 - 0.8) + (0.7 - 1) + (0.5 - 0.9) = -1.4$

Proposition 3.9

Let $Q = [Q_{ij}]$ be the square IFHSM of order $m \times n$, where $Q_{ij} = (T^Q_{ijk}, \mathrm{F}^Q_{ijk})$ and $n = m$. α be any scalar then $tr(\alpha Q) = \alpha\, tr(Q)$.

Proof:

$$tr(\alpha Q) = \sum_{i=1,k=a}^{m,z} [\alpha T^Q_{ijk} - \alpha \mathrm{F}^Q_{ijk}] = \alpha \sum_{i=1,k=a}^{m,z} [T^Q_{ijk} - \mathrm{F}^Q_{ijk}] = \alpha tr(Q)$$

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be two IFHSM, where $Q_{ij} = (T^Q_{ijk}, \mathrm{F}^Q_{ijk})$ and $R_{ij} = (T^R_{ijk}, \mathrm{F}^R_{ijk})$

Definition 3.10: Max-Min Product of IFHSM

Let $Q = [Q_{ij}]$ and $R = [R_{jt}]$ be two IFHSM, where $Q_{ij} = (T^Q_{ijk}, \mathrm{F}^Q_{ijk})$ and $R_{jt} = (T^R_{jkt}, \mathrm{F}^R_{jkt})$. Then, Q and R are said to be conformable if their dimensions are equal to each other (number of columns of Q is equal to number of rows of R). If $Q = [Q_{ij}]_{n\times m}$ and $R = [R_{jt}]_{m\times o}$ then $Q \otimes R = [\mathcal{S}_{it}]_{n\times o}$ where

$$[\mathcal{S}_{it}] = (max_{jk} \min(T^Q_{ijk}, T^R_{jkt}), min_{jk} \max(\mathrm{F}^Q_{ijk}, \mathrm{F}^R_{jkt}))$$

Theorem 3.11

Let $Q = [Q_{ij}]$ and $R = [R_{jt}]$ be two IFHSM, where $Q_{ij} = (T^Q_{ijk}, \mathrm{F}^Q_{ijk})$ and $R_{jt} = (T^R_{jkt}, \mathrm{F}^R_{jkt})$. Then, $(Q \otimes R)^t = R^t \otimes Q^t$

Proof:

Let $Q \otimes R = [\mathcal{S}_{it}]_{n\times o}$, then $(Q \otimes R)^t = [\mathcal{S}_{ti}]_{o\times n}$, $Q^t = [Q_{ji}]_{m\times n}$, $R^t = [R_{tj}]_{o\times m}$

Now $(Q \otimes R)^t = (T^{\mathcal{S}}_{kti}, F^{\mathcal{S}}_{kti})_{o\times n}$

$$= (max_{jk} \min(T^R_{tjk}, T^Q_{jki}), min_{jk} \max(F^R_{tjk}, F^Q_{jki}))_{o\times n}$$

$$= (T^R_{tjk}, F^R_{tjk})_{o\times m} \otimes (T^Q_{jki}, F^Q_{jki})_{m\times n} = R \otimes Q^t$$

4. Operators of IFHSMs

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be the two IFHSM, where $Q_{ij} = (T^Q_{ijk}, \mathrm{F}^Q_{ijk})$ and $R_{ij} = (T^R_{ijk}, \mathrm{F}^R_{ijk})$. Then,

i. **Union:**

$Q \cup R = S$ where $T^S_{ijk} = \max(T^Q_{ijk}, T^R_{ijk})$, $F^S_{ijk} = \min(F^Q_{ijk}, F^R_{ijk})$.

ii. **Intersection:**

$Q \cap R = S$ where $T_{ijk}^{S} = \min\,(T_{ijk}^{Q}, T_{ijk}^{R}),\ F_{ijk}^{S} = \max\,(F_{ijk}^{Q}, F_{ijk}^{R})$.

iii. **Arithmetic Mean:**

$Q \oplus R = S$ where $T_{ijk}^{S} = \frac{\left(T_{ijk}^{Q} + T_{ijk}^{R}\right)}{2}$, $F_{ijk}^{S} = \frac{\left(F_{ijk}^{Q} + F_{ijk}^{R}\right)}{2}$.

iv. **Weighted Arithmetic Mean:**

$Q \oplus^{w} R = S$ where $T_{ijk}^{S} = \frac{\left(w^{1} T_{ijk}^{Q} + w^{2} T_{ijk}^{R}\right)}{w^{1} + w^{2}}$, $F_{ijk}^{S} = \frac{\left(w^{1} F_{ijk}^{Q} + w^{2} F_{ijk}^{R}\right)}{w^{1} + w^{2}}$ $. w^{1}, w^{2} > 0$

v. **Geometric Mean:**

$Q \odot R = S$ where $T_{ijk}^{S} = \sqrt{T_{ijk}^{Q} \,.\, T_{ijk}^{R}}$, $F_{ijk}^{S} = \sqrt{F_{ijk}^{Q} \,.\, F_{ijk}^{R}}$.

vi. **Weighted Geometric Mean:**

$Q \odot^{w} R = S$ where $T_{ijk}^{S} = \sqrt[w^{1}+w^{2}]{(T_{ijk}^{Q})^{w^{1}} . \,(T_{ijk}^{R})^{w^{2}}}$,

$F_{ijk}^{S} = \sqrt[w^{1}+w^{2}]{(F_{ijk}^{Q})^{w^{1}} . \,(F_{ijk}^{R})^{w^{2}}}$ and $w^{1}, w^{2} > 0$

vii. **Harmonic Mean:**

$Q \oslash R = S$ where $T_{ijk}^{S} = \frac{2 T_{ijk}^{Q}\, T_{ijk}^{R}}{T_{ijk}^{Q} + T_{ijk}^{R}}$, $F_{ijk}^{S} = \frac{2 F_{ijk}^{Q}\, F_{ijk}^{R}}{F_{ijk}^{Q} + F_{ijk}^{R}}$.

viii. **Weighted Harmonic Mean:**

$Q \oslash^{w} R = S$ where $T_{ijk}^{S} = \frac{w^{1} + w^{2}}{\frac{w^{1}}{T_{ijk}^{Q}} + \frac{w^{2}}{T_{ijk}^{R}}}$, $F_{ijk}^{S} = \frac{w^{1} + w^{2}}{\frac{w^{1}}{F_{ijk}^{Q}} + \frac{w^{2}}{F_{ijk}^{R}}}$

Proposition 4.1

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be two IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$ and $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$. Then,

i. $(Q \cup R)^{t} = Q^{t} \cup R^{t}$

ii. $(Q \cap R)^{t} = Q^{t} \cap R^{t}$

iii. $(Q \oplus R)^{t} = Q^{t} \oplus R^{t}$

iv. $(Q \oplus^{w} R)^{t} = Q^{t} \oplus^{w} R^{t}$

v. $(Q \odot R)^{t} = Q^{t} \odot R^{t}$

vi. $(Q \odot^{w} R)^{t} = Q^{t} \odot^{w} R^{t}$

vii. $(Q \oslash R)^{t} = Q^{t} \oslash R^{t}$

viii. $(Q \oslash^{w} R)^{t} = Q^{t} \oslash^{w} R^{t}$

Proof:

i. $(Q \cup R)^{t} = \left[\left(\max\left(T_{ijk}^{Q}, T_{ijk}^{R}\right), \min\left(F_{ijk}^{Q}, F_{ijk}^{R}\right)\right)\right]^{t}$

$$= \left[\left(\max\left(T_{jki}^{Q}, T_{jki}^{R}\right), \min\left(F_{jki}^{Q}, F_{jki}^{R}\right)\right)\right]$$

$$= \left[\left(T_{jki}^{Q}, \quad \mathcal{F}_{jki}^{Q}\right)\right] \cup \left[\left(T_{jki}^{R}, F_{jki}^{R}\right)\right]$$

$$= \left[\left(T_{ijk}^{Q}, F_{ijk}^{Q}\right)\right]^{t} \cup \left[\left(T_{ijk}^{R}, F_{ijk}^{R}\right)\right]^{t} = Q^{t} \cup R^{t}$$

Proposition 4.2

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be the two-upper triangular IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$ and $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$. Then, $(Q \cup R)$, $(Q \cap R)$, $(Q \oplus R)$, $(Q \oplus^{w} R)$, $(Q \odot R)$ and $(Q \odot^{w} R)$ are all upper triangular IFHSM and vice versa.

Theorem 4.3:

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be the two IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$ and $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$. Then,

i. $(Q \cup R)^{\circ} = Q^{\circ} \cap R^{\circ}$
ii. $(Q \cap R)^{\circ} = Q^{\circ} \cup R^{\circ}$
iii. $(Q \oplus R)^{\circ} = Q^{\circ} \oplus R^{\circ}$
iv. $(Q \oplus^{w} R)^{\circ} = Q^{\circ} \oplus^{w} R^{\circ}$
v. $(Q \odot R)^{\circ} = Q^{\circ} \odot R^{\circ}$
vi. $(Q \odot^{w} R)^{\circ} = Q^{\circ} \odot^{w} R^{\circ}$
vii. $(Q \oslash R)^{\circ} = Q^{\circ} \oslash R^{\circ}$
viii. $(Q \oslash^{w} R)^{\circ} = Q^{\circ} \oslash^{w} R^{\circ}$

Proof:

i. $$(Q \cup R)^{\circ} = \left[\left(\max\left(T_{ijk}^{Q}, T_{ijk}^{R}\right), , \min\left(F_{ijk}^{Q}, F_{ijk}^{R}\right)\right)\right]^{\circ}$$

$$= \left[\left(\min\left(F_{ijk}^{Q}, F_{ijk}^{R}\right), \max\left(T_{ijk}^{Q}, T_{ijk}^{R}\right)\right)\right]$$

$$= \left(F_{ijk}^{Q}, T_{ijk}^{Q}\right) \cap \left(F_{ijk}^{R}, T_{ijk}^{R}\right)$$

$$= \left(T_{ijk}^{Q}, F_{ijk}^{Q}\right)^{\circ} \cap \left(T_{ijk}^{R}, \quad F_{ijk}^{R}\right)^{\circ}$$

$$= Q^{\circ} \cap R^{\circ}$$

Theorem 4.4

Let $Q = [Q_{ij}]$ and $R = [R_{ij}]$ be the two IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$ and $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$. Then,

i. $(Q \cup R) = (R \cup Q)$
ii. $(Q \cap R) = (R \cap Q)$
iii. $(Q \oplus R) = (R \oplus Q)$

iv. $(Q \oplus^{w} R) = (R \oplus^{w} Q)$

v. $(Q \odot R) = (R \odot Q)$

vi. $(Q \odot^{w} R) = (R \odot^{w} Q)$

vii. $(Q \oslash R) = (R \oslash Q)$

viii. $(Q \oslash^{w} R) = (R \oslash^{w} Q)$

Proof:

i. $$(Q \cup R) = \left[\left(\max\left(T_{ijk}^{Q}, T_{ijk}^{R}\right), \min\left(F_{ijk}^{Q}, F_{ijk}^{R}\right)\right)\right]$$
$$= \left[\left(\max\left(T_{ijk}^{R}, T_{ijk}^{Q}\right), \min\left(F_{ijk}^{R}, F_{ijk}^{Q}\right)\right)\right]$$
$$= \left[\left(T_{ijk}^{R},\ F_{ijk}^{R}\right)\right] \cup \left[\left(T_{ijk}^{Q}, F_{ijk}^{Q}\right)\right]$$
$$= (R \cup Q)$$

Theorem 4.5

Let $= [Q_{ij}]$, $R = [R_{ij}]$ and $S = [S_{ij}]$ be the two IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$, $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$ and $S_{ij} = \left(T_{ijk}^{S}, \mathrm{F}_{ijk}^{S}\right)$, then

i. $(Q \cup R) \cup S = Q \cup (R \cup S)$

ii. $(Q \cap R) \cap S = Q \cap (R \cap S)$

iii. $(Q \oplus R) \oplus S \neq Q \oplus (R \oplus S)$

iv. $(Q \odot R) \odot S \neq Q \odot (R \odot S)$

v. $(Q \oslash R) \oslash S \neq Q \oslash (R \oslash S)$

Proof:

i. $$(Q \cup R) \cup S = \left[\left(\max\left(T_{ijk}^{Q}, T_{ijk}^{R}\right), \min\left(F_{ijk}^{Q}, F_{ijk}^{R}\right)\right)\right] \cup \left[\left(T_{ijk}^{S}, F_{ijk}^{S}\right)\right]$$
$$= \left[\left(\max\left(T_{ijk}^{Q}, T_{ijk}^{R}, T_{ijk}^{S}\right), \min\left(F_{ijk}^{Q}, F_{ijk}^{R}, F_{ijk}^{S}\right)\right)\right]$$
$$= \left[\left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)\right] \cup \left[\left(\max\left(T_{ijk}^{R}, T_{ijk}^{S}\right), \min\left(F_{ijk}^{R}, F_{ijk}^{S}\right)\right)\right]$$
$$= \left[\left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)\right] \cup \left[\left(\left(T_{ijk}^{R}, F_{ijk}^{R}\right) \cup \left(T_{ijk}^{S}, F_{ijk}^{S}\right)\right)\right]$$
$$= Q \cup (R \cup S)$$

Theorem 4.6

Let $= [Q_{ij}]$, $R = [R_{ij}]$ and $S = [S_{ij}]$ be the two IFHSM, where $Q_{ij} = \left(T_{ijk}^{Q}, \mathrm{F}_{ijk}^{Q}\right)$, $R_{ij} = \left(T_{ijk}^{R}, \mathrm{F}_{ijk}^{R}\right)$ and $S_{ij} = \left(T_{ijk}^{S}, \mathrm{F}_{ijk}^{S}\right)$ Then,

i. $Q \cap (R \oplus S) = (Q \cap R) \oplus (Q \cap S)$

ii. $(Q \oplus R) \cap S = (Q \cap S) \oplus (R \cap S)$

iii. $Q \cup (R \oplus S) = (Q \cup R) \oplus (Q \cup S)$

iv. $(Q \oplus R) \cup S = (Q \cup S) \oplus (R \cup S)$

Proof:

i. $$Q \cap (R \oplus S) = \left(T_{ijk}^{Q},, F_{ijk}^{Q}\right) \cap \left[\left(\frac{\left(T_{ijk}^{R}+T_{ijk}^{S}\right)}{2}, \frac{\left(F_{ijk}^{R}+F_{ijk}^{S}\right)}{2}\right)\right]$$

$$= \left[\left(\min\left(T_{ijk}^{Q}, \frac{\left(T_{ijk}^{R}+T_{ijk}^{S}\right)}{2}\right), \max\left(F_{ijk}^{Q}, \frac{\left(F_{ijk}^{R}+F_{ijk}^{S}\right)}{2}\right)\right)\right]$$

$$= \left[\left(\min\left(\frac{\left(T_{ijk}^{Q}+T_{ijk}^{R}\right)}{2}, \frac{\left(T_{ijk}^{Q}+T_{ijk}^{S}\right)}{2}\right), \max\left(\frac{\left(F_{ijk}^{Q}+F_{ijk}^{R}\right)}{2}, \frac{\left(F_{ijk}^{Q}+F_{ijk}^{S}\right)}{2}\right)\right)\right]$$

$$= \left[\left(\min\left(T_{ijk}^{Q}, T_{ijk}^{R}\right), \max\left(F_{ijk}^{Q}, F_{ijk}^{R}\right)\right)\right] \oplus \left[\left(\min\left(T_{ijk}^{Q}, T_{ijk}^{S}\right), \max\left(F_{ijk}^{Q}, F_{ijk}^{S}\right)\right)\right]$$

$$= \left[\left(T_{ijk}^{Q}, F_{ijk}^{Q}\right) \cap \left(T_{ijk}^{R}, F_{ijk}^{R}\right)\right] \oplus \left[\left(T_{ijk}^{Q}, \quad F_{ijk}^{Q}\right) \cap \left(T_{ijk}^{S}, \quad F_{ijk}^{S}\right)\right]$$

$$= (Q \cap R) \oplus (Q \cap S)$$

5. Applications

Intuitionistic fuzzy Hypersoft Matrix (IFHSM) in Decision Making Using Score Function

For example, a task of selection befalls a group of decision makers who have been tasked to select from n number of objects. The objects are further presented on the basis of their m number of attributes. IFHSM can be used to show their relation with each other. These attributes are allocated Intuitionistic values by the decision-makers and are portrayed by IFHSM of order $n \times m$. We can get score matrix by using IFHSM to calculate value matrices. In this way, total score of each object from score matrix can be determined.

Value matrices are considered to be real matrices because they abide by the properties of real matrices. Score function also serves as a real matrix because we acquire it from two or more value matrices.

Definition 5.1

Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$, where $Q_{ij} = \left(T_{ijk}^{Q}, F_{ijk}^{Q}\right)$, then the value of matrix Q is denoted as $\Upsilon(Q)$ and it is defined as $\mathcal{V}(Q) = \left[\mathcal{V}_{ij}^{Q}\right]$ of order $n \times m$, where $\Upsilon_{ij}^{Q} = T_{ijk}^{Q} - F_{ijk}^{Q}$. The Score of two IFHSM, $Q = [Q_{ij}]$ and $R = [R_{ij}]$ of order $n \times m$ is given as $\zeta(Q, R) = \Upsilon(Q) + \Upsilon(R)$ and $\zeta(Q, R) = [\zeta_{ij}]$ where $\zeta_{ij} = \Upsilon_{ij}^{Q} + \Upsilon_{ij}^{R}$. The total score of each object in universal set is $\left|\sum_{j=1}^{n} \zeta_{ij}\right|$.

5.2 Algorithm

Step 1: Construct a IFHSM as define in 3.1.

Step 2: Calculate the value matrix from IFHSM. Let $Q = [Q_{ij}]$ be the IFHSM of order $n \times m$, where $Q_{ij} = (T^{Q}_{ijk}, F^{Q}_{ijk})$ then the value of matrix Q is denoted as $\Upsilon(Q)$ and it is defined as $\Upsilon(Q) = [\Upsilon^{Q}_{ij}]$ of order $n \times m$, where$\Upsilon^{Q}_{ij} = T^{Q}_{ijk} - F^{Q}_{ijk}$.

Step 3: Compute score matrix with the help of value matrices. The Score of two IFHSM $Q = [Q_{ij}]$ and $R = [R_{ij}]$ of order $n \times m$ is given as $\zeta(Q,R) = \Upsilon(Q) + \Upsilon(R)$ and $\zeta(Q,R) = [\zeta_{ij}]$ where $\zeta_{ij} = \Upsilon^{Q}_{ij} + \Upsilon^{R}_{ij}$.

Step 4: Compute total score from score matrix. The total score of each object in universal set is $\left|\sum_{j=1}^{n} \zeta_{ij}\right|$.

Step 5: Find optimal solution by selecting an object of maximum score from total score matrix.

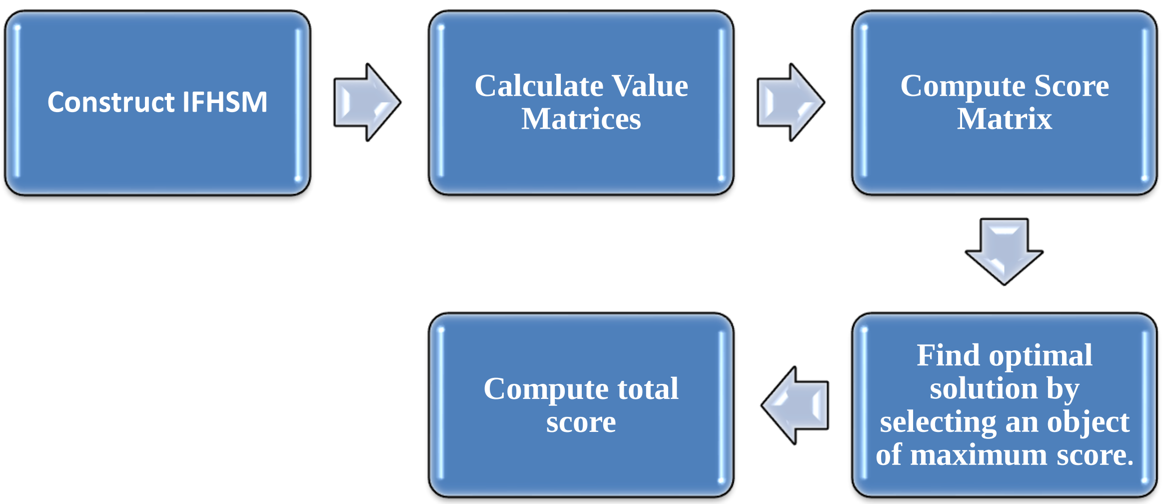

Figure 1: Algorithm design for the proposed technique

5.3 Numerical Example

To illustrate the working of this theory, a need arises for a company to hire an employee to fill one of its vacant spaces. A total of fifteen promising applicants apply to fill up the vacant space. From the human resources department of the company, a decision maker (DM) has been tasked for selection.

The decision maker finds it quite hard and a time-consuming task to interview all of them to fill up its vacant seat in the company. But by the help of the theory Intuitionistic fuzzy hypersoft matrix he would be able to narrow down the criteria for selection of the best possible candidate to just one candidate. Let us assume that $\digamma$ be the set of all candidates

$$\digamma = \{\mathfrak{f}^1, \mathfrak{f}^2, \mathfrak{f}^3, \mathfrak{f}^4, \mathfrak{f}^5, \mathfrak{f}^6, \mathfrak{f}^7, \mathfrak{f}^8, \mathfrak{f}^9, \mathfrak{f}^{10}, \mathfrak{f}^{11}, \mathfrak{f}^{12}, \mathfrak{f}^{13}, \mathfrak{f}^{14}, \mathfrak{f}^{15}\}$$

And the selection criteria set by the company is given in the form of attributes as

$\mathrm{T}_1 =$ Qualification, T_2 =Exprince, $\mathrm{T}_3 =$ Age, T_4 =Gender

Also, these attributes are further classified as

T_1^a =Qualification ={BS Hons. , MS , Ph.D. , Post Doctorate}

T_2^b =Exprince= {5yr, 7yr, 10yr, 15yr}

T_3^c =Age= {Less than thirty, Greater than thirty}

T_4^d =Gender= {Male, Female}

The function $\mathrm{F}: \mathrm{T}_1^a \times \mathrm{T}_2^b \times \mathrm{T}_3^c \times \mathrm{T}_4^d \to P(\digamma)$ is

Let's assume the relation

$\mathrm{F}((\mathrm{T}_1^a \times \mathrm{T}_2^b \times \mathrm{T}_3^c \times \mathrm{T}_4^d) = \mathrm{F}(\text{MS, 7yr, Greater than thirty, Male})$ is the actual requirement of company for the selection of candidates?

Four candidates $\{\mathfrak{f}^4, \mathfrak{f}^8, \mathfrak{f}^{10}, \mathfrak{f}^{13}\}$ are shortlisted on the basis of assumed relation i.e. $(\text{MS, 7yr, Greater than thirty, Male})$.

Decision makers $\{\mathbb{A}, \mathbb{B}\}$ are set for the selection of short-listed candidates. These decision makers give their valuable opinion in the form of IFHSSs separately as

$$\mathbb{A} = \mathrm{F}(\text{MS, 7yr, Greater than thirty, Male}) = \{< \mathfrak{f}^2, (\text{MS}\{0.5, 0.6\}, 7yr\{0.3, 0.7\},$$

$$\text{Greater than thirty}\{0.5, 0.9\}, \text{Male}\{0.6, 0.5\}) >, < \mathfrak{f}^6, (\text{MS}\{0.3, 0.1\}, 7yr\{0.6, 0.3\},$$

$$\text{Greater than thirty}\{0.7, 0.3\}, \text{Male}\{0.7, 0.3\}) >, < \mathfrak{f}^8(\text{MS}\{0.7, 0.6\}, 7yr\{0.6, 0.8\},$$

$$\text{Greater than thirty}\{0.8, 0.4\}, \text{Male}\{0.6, 0.1\}) >, < \mathfrak{f}^{14}, (\text{MS}\{0.5, 0.5\}, 7yr\{0.3, 0.7\},$$

$$\text{Greater than thirty}\{0.9, 0.1\}, \text{Male}\{0.4, 0.3\}) >\}$$

$$\mathbb{B} = \mathcal{F}(\text{MS}, \text{7yr}, \text{Greater than thirty}, \text{Male}) = \{< \mathcal{T}^2, (\text{MS}\{0.8, 0.2\}, 7yr\{0.7, 0.3\},$$

$$\text{Greater than thirty}\{0.4, 0.3\}, \text{Male}\{0.5, 0.5\}) >, < \mathcal{T}^6, (\text{MS}\{0.8, 0.1\}, 7yr\{0.7, 0.3\},$$

$$\text{Greater than thirty}\{0.8, 0.1\}, \text{Male}\{0.9, 0.2\}) >, < \mathcal{T}^8(\text{MS}\{0.5, 0.4\}, 7yr\{0.7, 0.2\},$$

$$\text{Greater than thirty}\{0.9, 0.1\}, \text{Male}\{0.4, 0.7\}) >, < \mathcal{T}^{14}, (\text{MS}\{0.7, 0.2\}, 7yr\{0.2, 0.7\},$$

$$\text{Greater than thirty}\{0.7, 0.1\}, \text{Male}\{0.6, 0.4\}) >, \}$$

Let's apply the above define algorithm for the calculation of total score

Step I: The above two NHSSs are given in the form of IFHSMs as

$$[\mathbb{A}] = \begin{bmatrix} (\text{MS}, (0.5, 0.6)) & (7yr, (0.3, 0.7)) & (\text{Greater than thirty}, (0.5, 0.9)) & (\text{Male}, (0.6, 0.5)) \\ (\text{MS}, (0.3, 0.1)) & (7yr, (0.6, 0.3)) & (\text{Greater than thirty}, (0.7, 0.3)) & (\text{Male}, (0.7, 0.3)) \\ (\text{MS}, (0.7, 0.6)) & (7yr, (0.6, 0.8)) & (\text{Greater than thirty}, (0.8, 0.4)) & (\text{Male}, (0.6, 0.1)) \\ (\text{MS}, (0.5, 0.5)) & (7yr, (0.3, 0.7)) & (\text{Greater than thirty}, (0.9, 0.1)) & (\text{Male}, (0.4, 0.3)) \end{bmatrix}$$

$$[\mathbb{B}] = \begin{bmatrix} (\text{MS}, (0.4, 0.5)) & (7yr, (0.2, 0.5)) & (\text{Greater than thirty}, (0.2, 0.4)) & (\text{Male}, (0.5, 0.5)) \\ (\text{MS}, (0.4, 0.2)) & (7yr, (0.7, 0.4)) & (\text{Greater than thirty}, (0.8, 0.9)) & (\text{Male}, (0.8, 0.9)) \\ (\text{MS}, (0.5, 0.2)) & (7yr, (0.2, 0.3)) & (\text{Greater than thirty}, (0.5, 0.5)) & (\text{Male}, (0.7, 0.2)) \\ (\text{MS}, (0.4, 0.3)) & (7yr, (0.2, 0.7)) & (\text{Greater than thirty}, (0.1, 0.1)) & (\text{Male}, (0.4, 0.4)) \end{bmatrix}$$

Step II: Now calculate the value matrices of IFHSMs define in Step I**.**

$$[\Upsilon(\mathbb{A})] = \begin{bmatrix} (\text{MS}, (-0.1)) & (7yr, (-0.4)) & (\text{Greater than thirty}, (-0.4)) & (\text{Male}, (0.1)) \\ (\text{MS}, (0.2)) & (7yr, (0.3)) & (\text{Greater than thirty}, (0.4)) & (\text{Male}, (0.4)) \\ (\text{MS}, (0.1)) & (7yr, (-0.2)) & (\text{Greater than thirty}, (0.4)) & (\text{Male}, (0.5)) \\ (\text{MS}, (0)) & (7yr, (-0.4)) & (\text{Greater than thirty}, (-0.8)) & (\text{Male}, (-0.1)) \end{bmatrix}$$

$$[\Upsilon(\mathbb{B})] = \begin{bmatrix} (\text{MS}, (-0.1)) & (7yr, (-0.3)) & (\text{Greater than thirty}, (-0.2)) & (\text{Male}, (0)) \\ (\text{MS}, (0.2)) & (7yr, (0.3)) & (\text{Greater than thirty}, (-0.1)) & (\text{Male}, (-0.1)) \\ (\text{MS}, (0.3)) & (7yr, (-0.1)) & (\text{Greater than thirty}, (0)) & (\text{Male}, (0.5)) \\ (\text{MS}, (0.1)) & (7yr, (-0.5)) & (\text{Greater than thirty}, (0)) & (\text{Male}, (0)) \end{bmatrix}$$

Step III: Now compute score matrix by adding value matrices obtained in Step II.

$$[\zeta(\mathbb{A}, \mathbb{B})] = \begin{bmatrix} (\text{MS}, (-0.2)) & (7yr, (-0.7)) & (\text{Greater than thirty}, (-0.6)) & (\text{Male}, (0.1)) \\ (\text{MS}, (0.4)) & (7yr, (0.6)) & (\text{Greater than thirty}, (0.3)) & (\text{Male}, (0.3)) \\ (\text{MS}, (0.4)) & (7yr, (-0.3)) & (\text{Greater than thirty}, (0.4)) & (\text{Male}, (1)) \\ (\text{MS}, (0.1)) & (7yr, (-0.9)) & (\text{Greater than thirty}, (0.8)) & (\text{Male}, (-0.1)) \end{bmatrix}$$

Step IV: Now total score of score matrix is given as;

$$Total\ score = \begin{bmatrix} 1.4 \\ 1.6 \\ 1.5 \\ 0.1 \end{bmatrix}$$

Step V: The candidate f^8 will be selected for vacant spaces as the total score of f^8 is highest among the rest of the total score of candidates.

6. Conclusions

In this study, new operations and definitions of IFHSM have been put forward and their workings have been illustrated by using a numerical example. Utility of IFHSM, concerning decision-making troubles, has been portrayed using a score matrix. Its usefulness is effective and helps us get accurate results. In future, this study will further help researchers in decision-making related calculations like TOPSIS, AHP and VIKOR.

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Conflicts of Interest:

The authors declare that they have no conflict of interest.

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A Development of Pythagorean fuzzy hypersoft set with basic operations and decision-making approach based on the correlation coefficient

Rana Muhammad Zulqarnain[1], Xiao Long Xin[1, 2 *], Muhammad Saeed[3]

[1] School of Mathematics, Northwest University Xi'an, China. E-mail: ranazulqarnain7777@gmail.com
[2*] School of Science, Xi'an Polytechnic University, Xi'an 710048, China; xlxin@nwu.edu.cn
[3] School of Science, Department of Mathematics, University of Management and Technology, Lahore, Pakistan. E-mail: muhammad.saeed@umt.edu.pk
[2*] Correspondence: E-mail: xlxin@nwu.edu.cn

Abstract: The idea of the Pythagorean fuzzy hypersoft set is a generalization of the intuitionistic fuzzy hypersoft set, which is used to express insufficient evaluation, uncertainty, and anxiety in decision-making. Compared with the intuitionistic fuzzy hypersoft set, the Pythagorean fuzzy hypersoft set can accommodate more uncertainty, which is the most important strategy for analyzing fuzzy information in the decision-making process. The most important determination of the present research is to perform basic operations under the Pythagorean fuzzy hypersoft set (PFHSS) with their mandatory properties. In it, we establish logical operators and propose the idea of necessity and possibility operations under PFHSS. In the following research under PFHSS, some desirable properties are proposed by using the proposed operations. We also introduce the correlation coefficient under the PFHSS structure and develop an algorithm for decision-making by using the developed correlation coefficient. Furthermore, a case study on decision-making difficulties proves the application of the proposed algorithm. Finally, a comparative analysis with the advantages, effectiveness, flexibility, and numerous existing studies demonstrates this method's effectiveness.

Keywords: *Hypersoft set, intuitionistic fuzzy set, Pythagorean fuzzy soft set, Pythagorean fuzzy hypersoft set, correlation coefficient.*

1. Introduction

Imprecision performs a dynamic part in many facets of life (such as modeling, medicine, engineering, etc.). However, people have raised a general question, that is, how can we express and use the concept of uncertainty in mathematical modeling. Many researchers in the world have proposed and recommended different methods of using uncertainty theory. First, Zadeh stepped forward the theory of fuzzy set (FS) [1] to resolve the problem of uncertainty and ambiguity.

In some cases, we need to investigate membership as a non-membership value to properly interpret objects that FS cannot handle. To overcome the above-mentioned issues, Atanasov proposed the idea of intuitionistic fuzzy sets (IFS) [2]. Researchers have also used several other theories, such as cubic intuitionistic fuzzy sets [3], interval value IFS [4], linguistic interval-valued IFS [5], etc. After carefully considering the above theories, the experts considered the essence, and the sum of its two membership values and non-membership values cannot exceed one.

Atanassov's intuitionistic fuzzy sets only deal with insufficient data due to membership and non-membership values, but IFS cannot deal with incompatible and imprecise information. Molodtsov [6] proposed a general mathematical tool to deal with uncertain, ambiguous, and uncertain matters, called soft set (SS). Maji et al. [7] extended the concept of SS and developed some operations with properties and used the established concepts for decision-making [8]. Maji et al. [9] proposed the concept of a fuzzy soft set (FSS) by combining FS and SS. They also proposed an Intuitionistic Fuzzy Soft Set (IFSS) with basic operations and properties [10]. Yang et al. [11] proposed the concept of interval-valued fuzzy soft sets with operations (IVFSS) and proved some important results by combining IVFS and SS, and they also used the developed concepts for decision-making. Jiang et al. [12] proposed the concept of interval-valued intuitionistic fuzzy soft sets (IVIFSS) by extending IVIFS. They also introduced the necessity and possibility operators of IVIFSS and their properties.

Garg and Arora [13] progressed the generalized version of the IFSS with weighted averaging and geometric aggregation operators and built a decision-making technique to resolve complications beneath an intuitionistic fuzzy environment. Garg [14] developed some improved score functions to analyze the ranking of the normal intuitionistic and interval-valued intuitionistic sets and established the new methodologies to solve multi-attribute decision making (MADM) problems. The idea of entropy measure and TOPSIS under the correlation coefficient (CC) has been developed by using complex q-rung orthopair fuzzy information and used the established strategies for decision making [15]. The authors [16] developed the aggregate operators by using dual hesitant fuzzy soft numbers and utilized the proposed operators to solve multi-criteria decision making (MCDM) problems. To measure the relationship among dual hesitant fuzzy soft set Arora and Garg [17] introduced the CC and developed a decision-making approach under the presented environment to solve the MCDM approach, they also used the proposed methodology for decision making, medical diagnoses, and pattern recognition. They also developed the operational laws and presented some prioritized aggregation operators under linguistic IFS environment [18] and extended the Maclaurin symmetric mean (MSM) operators to IFSS based on Archimedean T-conorm and T-norm [19].

As the above work is considered an environment where linear inequalities have been examined between membership degree (MD) as well as non-membership degree (NMD). However, if the decision-maker goes steady with object MD = 0.7 and NDM = 0.6, then $\mathbf{0.7+0.6 \nleq 1}$. We can see that; it cannot be handled by the above studied IFS theories. To overcome the above-mentioned limitations, Yager [20, 21] prolonged the IFS to Pythagorean fuzzy sets (PFSs) by modifying the condition $\boldsymbol{\mathcal{T}+\ \mathcal{J} \leq 1}$ to $\boldsymbol{\mathcal{T}^2+\ \mathcal{J}^2 \leq 1}$. Zhang and Xu [22] defined some operational laws and extended the TOPSIS technique to solve MCDM problems under PFSs environment. Many researchers used the TOPSIS method for medical diagnoses, pattern recognition, and decision-making, etc. to find the positive ideal alternative in different structures [23-31]. Wei and Lu [32] presented several Pythagorean fuzzy power aggregation operators with their properties and proposed the decision-making approaches to solving MADM problems based on developed operators. Wang and Li [33] proposed the Pythagorean fuzzy interaction operational laws and power Bonferroni mean operators. Then, they discussed some specific cases of established operators and considered their properties. Zhang [34] established a novel decision-making technique based on Pythagorean fuzzy numbers (PFNs) to solve multiple criteria group decision making (MCGDM) problems. He also developed the accuracy function for the ranking of PFNs and similarity measures under a PFSs environment with some desirable properties. Guleria and Bajaj [35] introduced a Pythagorean fuzzy soft matrix and its various possible types and binary operations with their properties. Further, they used the proposed Pythagorean fuzzy soft matrices for decision making by developing a new algorithm by using a choice matrix and weighted choice matrix. They also presented some noel information measures to solve MCDM problems [36]. Bajaj and Guleria [37] proposed the notion of object-oriented Pythagorean fuzzy soft matrix and the parameter-oriented Pythagorean fuzzy soft matrix has been utilized to outline an algorithm for the dimensionality reduction in the process of decision making. The authors developed the new (R, S)-norm discriminant measure of PFSs has been proposed along with its various properties and proposed a decision-making approach to solving MCDM problems [38]. Zulqarnain et al. [39] proposed the aggregation operators for Pythagorean fuzzy soft sets and established the decision-making approach using their developed technique. The authors of [40] extended the TOPSIS technique under Pythagorean fuzzy soft sets and used their proposed method for supplier selection in green supply chain management.

Recently, Smarandache [41] extended the concept of soft sets to hypersoft sets (HSS) by replacing the one-parameter function F with a multi-parameter (sub-attribute) function defined on the Cartesian product of n different attributes. The established HSS is more flexible than soft sets and is more suitable for the decision-making environment. He also introduced the further extension of HSS, such as crisp HSS, fuzzy HSS, intuitionistic fuzzy HSS, neutrosophic HSS, and plithogenic HSS. Nowadays, HSS theory and its extensions are developing rapidly. Many researchers have developed different operators and properties based on HSS and its extensions [42-46]. Zulqarnain et al. [47] introduced the TOPSIS technique and aggregation operators for PFHSS. They also utilized their developed technique for the selection of anti-virus face mask. Abdel-Basset [48] uses plithogenic set theory to resolve uncertain information and evaluate the financial performance of manufacturing. Then, they use VIKOR and TOPSIS methods to find the weight vector of financial ratios using the AHP method to achieve this goal. Abdel basset, etc. [49] proposed an effective combination of plithogenic aggregation operations and quality feature deployment methods. The advantage of this combination is that it can improve accuracy and thus evaluate decision-makers.

The following research is organized as follows: In Section 2, we review some basic definitions used in the following sequels, such as SS, FSS, IFS, IFSS, and IFHSS, etc. In Section 3, we propose some operations with their necessary properties such as union, intersection, restricted union, and extended intersection, etc. under PFHSS. We develop the AND operator, OR operator, necessity operation, and possibility operation with their several desirable properties in section 4. In section 5, the idea of correlation coefficient in PFHSS structure is introduced, and develop the decision-making technique based on the presented CC. We also used the developed approach to solve decision making problems in an uncertain environment. Furthermore, we use some existing techniques to present comparative studies between our proposed methods. Likewise, present the advantages, naivety, flexibility as well as effectiveness of the planned algorithms. We organized a brief discussion and a comparative analysis of the recommended approach and the existing techniques in section 6.

2. Preliminaries

In this section, we recollect some basic definitions which are helpful to build the structure of the following manuscript such as soft set, hypersoft set, fuzzy hypersoft set, and intuitionistic fuzzy hypersoft set.

Definition 2.1 [6]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \rightarrow \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \notin \mathcal{A}\}$$

Definition 2.2 [9]

$\mathcal{F}(\mathcal{U})$ be a collection of all fuzzy subsets over $\mathcal{U}$ and $\mathcal{E}$ be a set of attributes. Let $\mathcal{A} \subseteq \mathcal{E}$, then a pair $(\mathcal{F}, \mathcal{A})$ is called FSS over $\mathcal{U}$, where $\mathcal{F}$ is a mapping such as $\mathcal{F}: \mathcal{A} \rightarrow F(\mathcal{U})$.

Definition 2.3 [41]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, ..., k_n\}$, (n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of multi-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}})$ is said to be HSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \rightarrow \mathcal{P}(\mathcal{U})$.

It is also defined as

$(\mathcal{F}, \ddot{\mathcal{A}}) = \{\breve{a}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}): \breve{a} \in \ddot{\mathcal{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}) \in \mathcal{P}(\mathcal{U})\}$

Definition 2.4 [2]

An IFS is an object of the form $\mathcal{A} = \{\langle(\delta_i, \sigma_{\mathcal{A}}(\delta_i), \tau_{\mathcal{A}}(\delta_i)) \mid \delta_i \in \mathcal{U}\rangle\}$ on a universe $\mathcal{U}$, where $\sigma_{\mathcal{A}}$ and $\tau_{\mathcal{A}}: \mathcal{U} \rightarrow [0,1]$ represents the degree of membership and non-membership respectively of any element $\delta_i \in \mathcal{U}$, to set $\mathcal{A}$ with the following condition $0 \leq \sigma_{\mathcal{A}}(\delta_i) + \tau_{\mathcal{A}}(\delta_i) \leq 1$.

Definition 2.5 [10]

A mapping $\mathcal{F}: \mathcal{A} \rightarrow F(\mathcal{U})$ is known as an IFSS and defined as $\mathcal{F}_{\delta_i}(e) = \{(\delta_i, \sigma_{\mathcal{A}}(\delta_i), \tau_{\mathcal{A}}(\delta_i)) \mid \delta_i \in \mathcal{U}\}$, where $\sigma_{\mathcal{A}}(\delta_i)$ and $\tau_{\mathcal{A}}(\delta_i)$ are the degree of acceptance and rejection respectively for all $\delta_i \in \mathcal{U}$ and $0 \leq \sigma_{\mathcal{A}}(\delta_i), \tau_{\mathcal{A}}(\delta_i), \sigma_{\mathcal{A}}(\delta_i) + \tau_{\mathcal{A}}(\delta_i) \leq 1$.

Definition 2.6 [41]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, ..., k_n\}$, (n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. "Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$ and $\mathbb{F}^{\mathcal{U}}$ be a collection of all fuzzy subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}})$ is said to be FHSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \rightarrow \mathbb{F}^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{\mathcal{A}}) = \{(\breve{a}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a})): \breve{a} \in \ddot{\mathcal{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}) \in \mathbb{F}^{\mathcal{U}} \in [0,1]\}$

Example 2.7

Consider the universe of discourse $\mathcal{U} = \{\delta_1, \delta_2\}$ and $\mathfrak{L} = \{\ell_1 = Teaching\ methdology, \ell_2 = Subjects, \ell_3 = Classes\}$ be a collection of attributes with following their corresponding attribute values are given as teaching methodology = L_1 = $\{a_{11} = project\ base, a_{12} = \ class\ discussion\}$, Subjects = L_2 = $\{a_{21} = Mathematics, a_{22} = \ Computer\ Science, a_{23} = \ Statistics\}$, and Classes = L_3 = $\{a_{31} = Masters, a_{32} = \ Doctorol\}$. Let $\ddot{\mathcal{A}}$ = $L_1 \times L_2 \times L_3$ be a set of attributes

$\ddot{\mathcal{A}}$ = $L_1 \times L_2 \times L_3$ = $\{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$

$$= \left\{\begin{matrix}(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{23}, a_{31}), (a_{12}, a_{23}, a_{32}),\end{matrix}\right\}$$

$\ddot{\mathcal{A}}$ = $\{\breve{\alpha}_1, \breve{\alpha}_2, \breve{\alpha}_3, \breve{\alpha}_4, \breve{\alpha}_5, \breve{\alpha}_6, \breve{\alpha}_7, \breve{\alpha}_8, \breve{\alpha}_9, \breve{\alpha}_{10}, \breve{\alpha}_{11}, \breve{\alpha}_{12}\}$

Then the FHSS over $\mathcal{U}$ is given as follows

$$(\mathcal{F}, \ \ddot{\mathcal{A}}) = \left\{\begin{matrix}(\breve{\alpha}_1, (\delta_1, .6), (\delta_2, .3)), (\breve{\alpha}_2, (\delta_1, .7), (\delta_2, .5)), (\breve{\alpha}_3, (\delta_1, .8), (\delta_2, .3)), (\breve{\alpha}_4, (\delta_1, .2), (\delta_2, .8)), \\ (\breve{\alpha}_5, (\delta_1, .4), (\delta_2, .3)), (\breve{\alpha}_6, (\delta_1, .2), (\delta_2, .5)), (\breve{\alpha}_7, (\delta_1, .6), (\delta_2, .9)), (\breve{\alpha}_8, (\delta_1, .2), (\delta_2, .3)), \\ (\breve{\alpha}_9, (\delta_1, .4), (\delta_2, .7)), (\breve{\alpha}_{10}, (\delta_1, .1), (\delta_2, .7)), (\breve{\alpha}_{11}, (\delta_1, .4), (\delta_2, .6)), (\breve{\alpha}_5, (\delta_1, .2), (\delta_2, .7))\end{matrix}\right\}$$

Definition 2.8 [46]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and k = $\{k_1, \ k_2, \ k_3, \ldots, k_n\}$,(n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ \ldots \ n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}}$ = $\{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \ \beta$, and $\gamma \in \mathbb{N}$ and $IFS^{\mathcal{U}}$ be a collection of all intuitionistic fuzzy subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, \ K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}})$ is said to be IFHSS over $\mathcal{U}$ and its mapping is defined as $\mathcal{F}\colon K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \rightarrow IFS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ \ddot{\mathcal{A}}) = \{(\breve{\alpha}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}))\colon \breve{\alpha} \in \ddot{\mathcal{A}}, \ \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}) \ \in \ IFS^{\mathcal{U}} \in \ [0,1]\}$, where $\mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta)\rangle\colon \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta)$ and $\tau_{\mathcal{F}(\breve{\alpha})}(\delta)$ represents the membership and non-membership values of the attributes such as $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \ \tau_{\mathcal{F}(\breve{\alpha})}(\delta) \ \in \ [0,1]$, and $0 \leq \sigma_{\mathcal{F}(\breve{\alpha})}(\delta) + \tau_{\mathcal{F}(\breve{\alpha})}(\delta) \leq 1$.

Simply an intuitionistic fuzzy hypersoft number (IFHSN) can be expressed as $\mathcal{F}$ = $\left\{\left(\sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta)\right)\right\}$, where $0 \leq \sigma_{\mathcal{F}(\breve{\alpha})}(\delta) + \tau_{\mathcal{F}(\breve{\alpha})}(\delta) \leq 1$.

3. Basic Operations and properties on a Pythagorean fuzzy hypersoft set

In this section, we introduce PFHSS and some basic operations with their properties under the Pythagorean fuzzy hypersoft environment.

Definition 3.1

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and k = $\{k_1, \ k_2, \ k_3, \ldots, k_n\}$,(n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ \ldots \ n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}}$ = $\{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \ \beta$, and $\gamma \in \mathbb{N}$ and $PFS^{\mathcal{U}}$ be a collection of all Pythagorean fuzzy subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, \ K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}})$ is said to be PFHSS over $\mathcal{U}$ and its mapping is defined as $\mathcal{F}\colon K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \rightarrow PFS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ \ddot{\mathcal{A}}) = \{(\breve{\alpha}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}))\colon \breve{\alpha} \in \ddot{\mathcal{A}}, \ \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}) \ \in \ PFS^{\mathcal{U}} \in \ [0,1]\}$, where $\mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta)\rangle\colon \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta)$ and $\tau_{\mathcal{F}(\breve{\alpha})}(\delta)$ represents the membership and non-membership values of the attributes such as $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \ \tau_{\mathcal{F}(\breve{\alpha})}(\delta) \ \in \ [0,1]$, and $0 \leq \left(\sigma_{\mathcal{F}(\breve{\alpha})}(\delta)\right)^2 + \left(\tau_{\mathcal{F}(\breve{\alpha})}(\delta)\right)^2 \leq 1$.

Simply a Pythagorean fuzzy hypersoft number (PFHSN) can be expressed as $\mathcal{F} = \left\{\left(\sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right)\right\}$, where $0 \leq \left(\sigma_{\mathcal{F}(\breve{a})}(\delta)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a})}(\delta)\right)^2 \leq 1$.

Example 3.2

Consider the universe of discourse $\mathcal{U} = \{\delta_1, \delta_2\}$ and $\mathfrak{L} = \{\ell_1 = Teaching\ methdology, \ell_2 = Subjects, \ell_3 = Classes\}$ be a collection of attributes with following their corresponding attribute values are given as teaching methodology = $L_1 = \{a_{11} = project\ base, a_{12} = class\ discussion\}$, Subjects = $L_2 = \{a_{21} = Mathematics, a_{22} = Computer\ Science, a_{23} = Statistics\}''$, and Classes = $L_3 = \{a_{31} = Masters, a_{32} = Doctorol\}$. Let $\ddot{\mathcal{A}} = L_1 \times L_2 \times L_3$ be a set of attributes

$\ddot{\mathcal{A}} = L_1 \times L_2 \times L_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$

$$= \begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{23}, a_{31}), (a_{12}, a_{23}, a_{32}), \end{Bmatrix}$$

$\ddot{\mathcal{A}} = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8, \breve{a}_9, \breve{a}_{10}, \breve{a}_{11}, \breve{a}_{12}\}$

Then the PFHSS over $\mathcal{U}$ is given as follows

$(\mathcal{F}, \ddot{\mathcal{A}}) =$

$$\begin{Bmatrix} (\breve{a}_1, (\delta_1, (.6, .3)), (\delta_2, (.5, .7))), (\breve{a}_2, (\delta_1, (.6, .7)), (\delta_2, (.7, .5))), (\breve{a}_3, (\delta_1, (.4, .8)), (\delta_2, (.3, .7))), \\ \left(\breve{a}_4, \left(\delta_1, (.6, .5)\right), \left(\delta_2, (.5, .6)\right)\right), \left(\breve{a}_5, \left(\delta_1, (.7, .3)\right), \left(\delta_2, (.4, .8)\right)\right), \left(\breve{a}_6, \left(\delta_1, (.5, .4)\right), \left(\delta_2, (.6, .5)\right)\right), \\ (\breve{a}_7, (\delta_1, (.5, .6)), (\delta_2, (.4, .5))), (\breve{a}_8, (\delta_1, (.2, .5)), (\delta_2, (.3, .9))), (\breve{a}_9, (\delta_1, (.4, .6)), (\delta_2, (.8, .5))), \\ (\breve{a}_{10}, (\delta_1, (.7, .4)), (\delta_2, (.7, .2))), (\breve{a}_{11}, (\delta_1, (.4, .5)), (\delta_2, (.5, .3))), (\breve{a}_{12}, (\delta_1, (.5, .7)), (\delta_2, (.4, .7))) \end{Bmatrix}$$

Definition 3.3

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then $(\mathcal{F}, \ddot{\mathcal{A}})$ is said to be a Pythagorean fuzzy hypersoft subset of $(\mathcal{G}, \ddot{\mathfrak{B}})$, if

1. $\ddot{\mathcal{A}} \subseteq \ddot{\mathfrak{B}}$
2. $\mathcal{F}_{\ddot{\mathcal{A}}(\breve{a})}(\delta) \subseteq \mathcal{G}_{\ddot{\mathfrak{B}}(\breve{a})}(\delta)$ for all $\delta \in \mathcal{U}$.

Where $\sigma_{\mathcal{F}(\breve{a})}(\delta) \leq \sigma_{\mathcal{G}(\breve{a})}(\delta)$, and $\tau_{\mathcal{F}(\breve{a})}(\delta) \geq \tau_{\mathcal{G}(\breve{a})}(\delta)$: $\delta \in \mathcal{U}$.

Definition 3.4

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then $(\mathcal{F}, \ddot{\mathcal{A}}) = (\mathcal{G}, \ddot{\mathfrak{B}})$, if $(\mathcal{F}, \ddot{\mathcal{A}}) \subseteq (\mathcal{G}, \ddot{\mathfrak{B}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) \subseteq (\mathcal{F}, \ddot{\mathcal{A}})$.

Definition 3.5

Let $\mathcal{U}$ be a universe of discourse and $\ddot{\mathcal{A}}$ be a set of attributes, then a pair $(\emptyset, \ddot{\mathcal{A}})$ is said to be empty PFHSS, if $\sigma_{\mathcal{F}(\breve{a})}(\delta) = 0$, and $\tau_{\mathcal{F}(\breve{a})}(\delta) = 1$ for all $\breve{a} \in \ddot{\mathcal{A}}$ and $\delta \in \mathcal{U}$. It can be represented by $\emptyset_{\mathcal{F}(\breve{a})}(\delta)$ and defined as follows

$(\emptyset, \ddot{\mathcal{A}}) = \{\breve{a}, (\delta, (0, 1)): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$.

Definition 3.6

Let $\mathcal{U}$ be a universe of discourse and $\ddot{\mathcal{A}}$ be a set of attributes, then a pair $(\emptyset, \ddot{\mathcal{A}})$ is said to be universal PFHSS, if $\sigma_{\mathcal{F}(\breve{a})}(\delta) = 1$, and $\tau_{\mathcal{F}(\breve{a})}(\delta) = 0$ for all $\breve{a} \in \ddot{\mathcal{A}}$ and $\delta \in \mathcal{U}$. It can be represented by $\mathbb{E}_{\mathcal{F}(\breve{a})}(\delta)$ and defined as follows

$(\mathbb{E}, \ddot{\mathcal{A}}) = \{\breve{a}, (\delta, (1, 0)): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$.

Definition 3.7

Let $(\mathcal{F}, \ddot{\mathcal{A}}) = \{(\breve{a}, \langle\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle: \delta \in \mathcal{U}): \breve{a} \in \ddot{\mathcal{A}}\}$ be a PFHSS over $\mathcal{U}$, then its complement is denoted by $(\mathcal{F}, \ddot{\mathcal{A}})^c$, and is defined as follows $(\mathcal{F}, \ddot{\mathcal{A}})^c = \{(\breve{a}, \langle\delta, \tau_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{F}(\breve{a})}(\delta)\rangle: \delta \in \mathcal{U}): \breve{a} \in \ddot{\mathcal{A}}\}$.

Proposition 3.8

If $(\mathcal{F},\ \ddot{\mathcal{A}})$ be a PFHSS, then

1. $(\mathcal{F}^c, \ddot{\mathcal{A}})^c = (\mathcal{F},\ \ddot{\mathcal{A}})$
2. $(\emptyset^c, \ddot{\mathcal{A}}) = (\mathbb{E}, \ddot{\mathcal{A}})$
3. $(\mathbb{E}^c, \ddot{\mathcal{A}}) = (\emptyset, \ddot{\mathcal{A}})$

Proof 1

Let $(\mathcal{F},\ \ddot{\mathcal{A}}) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$ be a PFHSS over $\mathcal{U}$, then by using definition 3.7. we have

$(\mathcal{F}^c, \ddot{\mathcal{A}}) = \{(\breve{a}, \langle \delta, \tau_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$. Again, "by using definition 3.7

$(\mathcal{F}^c, \ddot{\mathcal{A}})^c = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$

Hence,

$(\mathcal{F}^c, \ddot{\mathcal{A}})^c = (\mathcal{F},\ \ddot{\mathcal{A}})$.

Similarly, we can prove 2 and 3.

Definition 3.9

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$ and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their union is defined as

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{G},\ \ddot{\mathfrak{B}}) = \{\delta, (max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}) : \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$.

Proposition 3.10

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$, $(\mathcal{G},\ \ddot{\mathfrak{B}})$, and $(\mathcal{H},\ \ddot{\mathrm{C}})$ be three PFHSS over $\mathcal{U}$. Then

1. $(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{F},\ \ddot{\mathcal{A}}) = (\mathcal{F},\ \ddot{\mathcal{A}})$
2. $(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\emptyset, \ddot{\mathcal{A}}) = (\mathcal{F},\ \ddot{\mathcal{A}})$
3. $(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathbb{E}, \ddot{\mathcal{A}}) = (\mathbb{E}, \ddot{\mathcal{A}})$
4. $(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{G},\ \ddot{\mathfrak{B}}) = (\mathcal{G},\ \ddot{\mathfrak{B}}) \cup (\mathcal{F},\ \ddot{\mathcal{A}})$
5. $((\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{G},\ \ddot{\mathfrak{B}})) \cup (\mathcal{H},\ \ddot{\mathrm{C}}) = (\mathcal{F},\ \ddot{\mathcal{A}}) \cup ((\mathcal{G}, \ddot{\mathfrak{B}}) \cup (\mathcal{H},\ \ddot{\mathrm{C}}))$

Proof 1 As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ be an PFHSS, then

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{F},\ \ddot{\mathcal{A}}) = \{\delta, (max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{F}(\breve{a})}(\delta)\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\}) : \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{F},\ \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$

Hence

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\mathcal{F},\ \ddot{\mathcal{A}}) = (\mathcal{F},\ \ddot{\mathcal{A}})$.

Proof 2 As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ be a PFHSS, and $(\emptyset, \ddot{\mathcal{A}}) = \{\breve{a}, (\delta, (0, 1)) : \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$ be an empty PFHSS. Then,

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\emptyset, \ddot{\mathcal{A}}) = \{\delta, (max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), 0\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), 1\}) : \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\emptyset, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$.

$(\mathcal{F},\ \ddot{\mathcal{A}}) \cup (\emptyset, \ddot{\mathcal{A}}) = (\mathcal{F},\ \ddot{\mathcal{A}})$.

Proof 3 As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ be a PFHSS, and $\left(\mathbb{E}, \ddot{\mathcal{A}}\right) = \{\breve{a}, \left(\delta, (1,0)\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$ be an empty PFHSS. Then,

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup \left(\mathbb{E}, \ddot{\mathcal{A}}\right) = \{\delta, \left(max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), 1\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), 0\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup \left(\mathbb{E}, \ddot{\mathcal{A}}\right) = \{\breve{a}, \left(\delta, (1,0)\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}.$

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup \left(\mathbb{E}, \ddot{\mathcal{A}}\right) = \left(\mathbb{E}, \ddot{\mathcal{A}}\right).$

Proof 4 As

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ be two PFHSS, then

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}}) = \{\delta, \left(max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}} \cup \ddot{\mathfrak{B}}\}$

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}}) = \{\delta, \left(max\{\sigma_{\mathcal{G}(\breve{a})}(\delta), \sigma_{\mathcal{F}(\breve{a})}(\delta)\}, min\{\tau_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}} \cup \ddot{\mathfrak{B}}\}$

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}}) = (\mathcal{G}, \ddot{\mathfrak{B}}) \cup (\mathcal{F}, \ddot{\mathcal{A}}).$

Similarly, we can prove 5.

Definition 3.11

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their intersection is defined as follows:

$(\mathcal{F}, \ddot{\mathcal{A}}) \cap (\mathcal{G}, \ddot{\mathfrak{B}}) = \{\delta, \left(min\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}, max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}.$

Proposition 3.12

Let $(\mathcal{F}, \ddot{\mathcal{A}})$, $(\mathcal{G}, \ddot{\mathfrak{B}})$, and $(\mathcal{H}, \ddot{\mathrm{C}})$ be three PFHSS over $\mathcal{U}$. Then

1. $(\mathcal{F}, \ddot{\mathcal{A}}) \cap (\mathcal{F}, \ddot{\mathcal{A}}) = (\mathcal{F}, \ddot{\mathcal{A}})$
2. $(\mathcal{F}, \ddot{\mathcal{A}}) \cap \left(\emptyset, \ddot{\mathcal{A}}\right) = (\mathcal{F}, \ddot{\mathcal{A}})$
3. $(\mathcal{F}, \ddot{\mathcal{A}}) \cap \left(\mathbb{E}, \ddot{\mathcal{A}}\right) = \left(\mathbb{E}, \ddot{\mathcal{A}}\right)$
4. $(\mathcal{F}, \ddot{\mathcal{A}}) \cap (\mathcal{G}, \ddot{\mathfrak{B}}) = (\mathcal{G}, \ddot{\mathfrak{B}}) \cap (\mathcal{F}, \ddot{\mathcal{A}})$
5. $((\mathcal{F}, \ddot{\mathcal{A}}) \cap (\mathcal{G}, \ddot{\mathfrak{B}})) \cap (\mathcal{H}, \ddot{\mathrm{C}}) = (\mathcal{F}, \ddot{\mathcal{A}}) \cap ((\mathcal{G}, \ddot{\mathfrak{B}}) \cap (\mathcal{H}, \ddot{\mathrm{C}}))$

Proof By using Definition 3.11 we can prove easily.

Proposition 3.13

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be three PFHSS, then

1. $\left((\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}})\right)^c = \left(\mathcal{F}, \ddot{\mathcal{A}}\right)^c \cap \left(\mathcal{G}, \ddot{\mathfrak{B}}\right)^c$
2. $\left((\mathcal{F}, \ddot{\mathcal{A}}) \cap (\mathcal{G}, \ddot{\mathfrak{B}})\right)^c = \left(\mathcal{F}, \ddot{\mathcal{A}}\right)^c \cup \left(\mathcal{G}, \ddot{\mathfrak{B}}\right)^c$

Proof 1

As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ be two PFHSS, then by using Definition 3.9

$(\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}}) = \{\delta, \left(max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}", min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}.$

By using definition 3.7, we have

$\left((\mathcal{F}, \ddot{\mathcal{A}}) \cup (\mathcal{G}, \ddot{\mathfrak{B}})\right)^c = \{\delta, \left(min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}, max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}\right): \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$

Now

$\left(\mathcal{F}, \ddot{\mathcal{A}}\right)^c = \left\{\langle\left(\delta, \tau_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\rangle\right\}$ and $\left(\mathcal{G}, \ddot{\mathfrak{B}}\right)^c = \left\{\langle\left(\delta, \tau_{\mathcal{G}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\rangle\right\}.$

By using Definition 3.11

$(\mathcal{F},\ddot{\mathcal{A}})^c \cap (\mathcal{G},\ddot{\mathfrak{B}})^c = \{\delta,(min\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\},max\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\}):\delta \in \mathcal{U},\breve{a} \in \ddot{\mathcal{A}}\}$

So

$((\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}}))^c = (\mathcal{F},\ddot{\mathcal{A}})^c \cap (\mathcal{G},\ddot{\mathfrak{B}})^c$

Proof 2

As we know that

$(\mathcal{F},\ddot{\mathcal{A}}) = \{\langle(\delta,\sigma_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{F}(\breve{a})}(\delta)) \mid \delta \in \mathcal{U}\rangle\}$ and $(\mathcal{G},\ddot{\mathfrak{B}}) = \{\langle(\delta,\sigma_{\mathcal{G}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)) \mid \delta \in \mathcal{U}\rangle\}$ be two PFHSS, then by using Definition 3.11

$(\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}}) = \{\delta,(min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\},max\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\}):\delta \in \mathcal{U},\breve{a} \in \ddot{\mathcal{A}}\}$.

By using Definition 3.7, we have

$((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}}))^c = \{\delta,(max\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\},min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\}):\delta \in \mathcal{U},\breve{a} \in \ddot{\mathcal{A}}\}$

Now

$(\mathcal{F},\ddot{\mathcal{A}})^c = \{(\delta,\ \tau_{\mathcal{F}(\breve{a})}(\delta),\ \sigma_{\mathcal{F}(\breve{a})}(\delta)) \mid \delta \in \mathcal{U}\}$ and $(\mathcal{G},\ddot{\mathfrak{B}})^c = \{(\delta,\tau_{\mathcal{G}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)) \mid \delta \in \mathcal{U}\}$.

By using Definition 3.9

$(\mathcal{F},\ddot{\mathcal{A}})^c \cup (\mathcal{G},\ddot{\mathfrak{B}})^c = \{\delta,(max\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\},min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\}):\delta \in \mathcal{U},\breve{a} \in \ddot{\mathcal{A}}\}$

So

$((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}}))^c = (\mathcal{F},\ddot{\mathcal{A}})^c \cup (\mathcal{G},\ddot{\mathfrak{B}})^c$

Definition 3.14

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$ and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their restricted union is defined as

$$\sigma(\mathcal{F},\ \ddot{\mathcal{A}}) \cup_R (\mathcal{G},\ \ddot{\mathfrak{B}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau(\mathcal{F},\ \ddot{\mathcal{A}}) \cup_R (\mathcal{G},\ \ddot{\mathfrak{B}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Definition 3.15

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$ and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their extended intersection is defined as

$$\sigma(\mathcal{F},\ \ddot{\mathcal{A}}) \cap_\varepsilon (\mathcal{G},\ \ddot{\mathfrak{B}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau(\mathcal{F},\ \ddot{\mathcal{A}}) \cap_\varepsilon (\mathcal{G},\ \ddot{\mathfrak{B}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Proposition 3.16

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$, and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then

1. $(\mathcal{F},\ddot{\mathcal{A}}) \cup ((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}})) = (\mathcal{F},\ddot{\mathcal{A}})$
2. $(\mathcal{F},\ddot{\mathcal{A}}) \cap ((\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}})) = (\mathcal{F},\ddot{\mathcal{A}})$
3. $(\mathcal{F},\ddot{\mathcal{A}}) \cup_R ((\mathcal{F},\ddot{\mathcal{A}}) \cap_\varepsilon (\mathcal{G},\ddot{\mathfrak{B}})) = (\mathcal{F},\ddot{\mathcal{A}})$

4. $(\mathcal{F},\ddot{\mathcal{A}}) \cap_{\varepsilon} \big((\mathcal{F},\ddot{\mathcal{A}}) \cup_{R} (\mathcal{G},\ddot{\mathfrak{B}})\big) = (\mathcal{F},\ddot{\mathcal{A}})$

Proof 1 Consider

$(\mathcal{F}, \ddot{\mathcal{A}}) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}},\}$, and $(\mathcal{G},\ddot{\mathfrak{B}}) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathfrak{B}},\}$ are two PFHSS over the universe of discourse $\mathcal{U}$

$(\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}}) = \{\delta, (min\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}, max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}) : \delta \in \mathcal{U}\}$

$(\mathcal{F},\ddot{\mathcal{A}}) \cup \big((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}})\big) =$

$$\left\{\delta, \left(max\left\{\sigma_{\mathcal{F}(\breve{a})}(\delta), min\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}\right\}, min\left\{\tau_{\mathcal{F}(\breve{a})}(\delta), max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}\right\}\right) : \delta \in \mathcal{U}\right\}$$

$(\mathcal{F},\ddot{\mathcal{A}}) \cup \big((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}})\big) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$.

Therefore,

$(\mathcal{F},\ddot{\mathcal{A}}) \cup \big((\mathcal{F},\ddot{\mathcal{A}}) \cap (\mathcal{G},\ddot{\mathfrak{B}})\big) = (\mathcal{F},\ddot{\mathcal{A}})$.

Proof 2 Consider

$(\mathcal{F}, \ddot{\mathcal{A}}) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$, and $(\mathcal{G},\ddot{\mathfrak{B}}) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathfrak{B}}\}$ are two PFHSS over the universe of discourse $\mathcal{U}$

$(\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}}) = \{\delta, (max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}, min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}) : \delta \in \mathcal{U}\}$

$(\mathcal{F},\ddot{\mathcal{A}}) \cap \big((\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}})\big) =$

$$\left\{\delta, \left(min\left\{\sigma_{\mathcal{F}(\breve{a})}(\delta), max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\}\right\}, max\left\{\tau_{\mathcal{F}(\breve{a})}(\delta), min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\}\right\}\right) : \delta \in \mathcal{U}\right\}$$

$(\mathcal{F},\ddot{\mathcal{A}}) \cap \big((\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}})\big) = \{(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}) : \breve{a} \in \ddot{\mathcal{A}}\}$.

Therefore,

$(\mathcal{F},\ddot{\mathcal{A}}) \cap \big((\mathcal{F},\ddot{\mathcal{A}}) \cup (\mathcal{G},\ddot{\mathfrak{B}})\big) = (\mathcal{F},\ddot{\mathcal{A}})$.

Similarly, we can prove 3 and 4.

4. Logical operators and Necessity and Possibility operators under the Pythagorean fuzzy hypersoft set.

In this section, we propose the idea of AND-operator, OR-operator, Necessity operator, and possibility operator under the PFHSS with their several desirable properties. We also introduce the correlation coefficient under the PFHSS environment.

Definition 4.1

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their OR-operator is represented by $(\mathcal{F}, \ddot{\mathcal{A}}) \vee (\mathcal{G}, \ddot{\mathfrak{B}})$ and defined as follows

$(\mathcal{F}, \ddot{\mathcal{A}}) \vee (\mathcal{G}, \ddot{\mathfrak{B}}) = (\curlywedge, \ddot{\mathcal{A}} \times \ddot{\mathfrak{B}})$, where $\curlywedge(\breve{a}_1 \times \breve{a}_2) = \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}_1) \cup \mathcal{G}_{\ddot{\mathfrak{B}}}(\breve{a}_2)$ for all $(\breve{a}_1 \times \breve{a}_2) \in \ddot{\mathcal{A}} \times \ddot{\mathfrak{B}}$.

$\curlywedge(\breve{a}_1 \times \breve{a}_2) = \{max\{\sigma_{\mathcal{F}(\breve{a}_1)}(\delta), \sigma_{\mathcal{G}(\breve{a}_2)}(\delta)\}, min\{\tau_{\mathcal{F}\breve{a}_1}(\delta), \tau_{\mathcal{G}(\breve{a}_2)}(\delta)\} : \delta \in \mathcal{U}, \breve{a} \in \ddot{\mathcal{A}}\}$

Definition 4.2

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS over $\mathcal{U}$, then their AND-operator is represented by $(\mathcal{F}, \ddot{\mathcal{A}}) \wedge (\mathcal{G}, \ddot{\mathfrak{B}})$ and defined as follows

$(\mathcal{F}, \ddot{\mathcal{A}}) \wedge (\mathcal{G}, \ddot{\mathfrak{B}}) = (\curlywedge, \ddot{\mathcal{A}} \times \ddot{\mathfrak{B}})$, where $\curlywedge(\breve{a}_1 \times \breve{a}_2) = \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}_1) \cap \mathcal{G}_{\ddot{\mathfrak{B}}}(\breve{a}_2)$ for all $(\breve{a}_1 \times \breve{a}_2) \in \ddot{\mathcal{A}} \times \ddot{\mathfrak{B}}$.

$\curlywedge(\breve{a}_1 \times \breve{a}_2) = \{min\{\sigma_{\mathcal{F}(\breve{a}_1)}(\delta), \sigma_{\mathcal{G}(\breve{a}_2)}(\delta)\}, max\{\tau_{\mathcal{F}(\breve{a}_1)}(\delta), \tau_{\mathcal{G}(\breve{a}_2)}(\delta)\} : \delta \in \mathcal{U}\}$

Proposition 4.3

Let $(\mathcal{F}, \ddot{\mathcal{A}})$, $(\mathcal{G}, \ddot{\mathfrak{B}})$, and $(\mathcal{H}, \ddot{\mathrm{C}})$ be three PFHSS over $\mathcal{U}$. Then

1. $(\mathcal{F}, \ddot{\mathcal{A}}) \vee (\mathcal{G}, \ddot{\mathfrak{B}}) = (\mathcal{G}, \ddot{\mathfrak{B}}) \vee (\mathcal{F}, \ddot{\mathcal{A}})$
2. $(\mathcal{F}, \ddot{\mathcal{A}}) \wedge (\mathcal{G}, \ddot{\mathfrak{B}}) = (\mathcal{G}, \ddot{\mathfrak{B}}) \wedge (\mathcal{F}, \ddot{\mathcal{A}})$
3. $(\mathcal{F}, \ddot{\mathcal{A}}) \vee ((\mathcal{G}, \ddot{\mathfrak{B}}) \vee (\mathcal{H}, \ddot{\mathrm{C}})) = ((\mathcal{F}, \ddot{\mathcal{A}}) \vee (\mathcal{G}, \ddot{\mathfrak{B}})) \vee (\mathcal{H}, \ddot{\mathrm{C}})$
4. $(\mathcal{F}, \ddot{\mathcal{A}}) \wedge ((\mathcal{G}, \ddot{\mathfrak{B}}) \wedge (\mathcal{H}, \ddot{\mathrm{C}})) = ((\mathcal{F}, \ddot{\mathcal{A}}) \wedge (\mathcal{G}, \ddot{\mathfrak{B}})) \wedge (\mathcal{H}, \ddot{\mathrm{C}})$
5. $\left((\mathcal{F}, \ddot{\mathcal{A}}) \vee (\mathcal{G}, \ddot{\mathfrak{B}})\right)^c = \mathcal{F}^c(\ddot{\mathcal{A}}) \wedge \wp^c(\ddot{\mathfrak{B}})$
6. $\left((\mathcal{F}, \ddot{\mathcal{A}}) \wedge (\mathcal{G}, \ddot{\mathfrak{B}})\right)^c = \mathcal{F}^c(\ddot{\mathcal{A}}) \vee \wp^c(\ddot{\mathfrak{B}})$

Proof 1 By using Definitions 4.1, 4.2 we can prove easily.

Definition 4.4

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ be a PFHSS, then necessity operation on PFHSS represented by $\oplus\,(\mathcal{F}, \ddot{\mathcal{A}})$ and defined as follows

$\oplus\,(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\breve{a}, \langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), 1 - \sigma_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}\right) : \breve{a} \in \ddot{\mathcal{A}}, \right\}$

Definition 4.5

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ be a PFHSS, then possibility operation on PFHSS represented by $(\mathcal{F}, \ddot{\mathcal{A}})$ and defined as follows

$\otimes\,(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\breve{a}, \langle \delta, 1 - \tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}\right) : \breve{a} \in \ddot{\mathcal{A}}, \right\}$

Proposition 4.6

Let $(\mathcal{F}, \ddot{\mathcal{A}})$ and $(\mathcal{G}, \ddot{\mathfrak{B}})$ be two PFHSS, then

1. $\oplus\left((\mathcal{F}, \ddot{\mathcal{A}}) \cup_R (\mathcal{G}, \ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{G}, \ddot{\mathfrak{B}}) \cup_R \oplus\,(\mathcal{F}, \ddot{\mathcal{A}})$
2. $\oplus\left((\mathcal{F}, \ddot{\mathcal{A}}) \cap_\varepsilon (\mathcal{G}, \ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{G}, \ddot{\mathfrak{B}}) \cap_\varepsilon \oplus\,(\mathcal{F}, \ddot{\mathcal{A}})$

Proof 1

As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ are two PFHSS.

Let $\left((\mathcal{F}, \ddot{\mathcal{A}}) \cup_R (\mathcal{G}, \ddot{\mathfrak{B}})\right) = (\mathcal{H}, \ddot{\mathrm{C}})$

$$\sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\left\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\right\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\left\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

By using Definition 4.4

$$\oplus\,\sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\left\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\right\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\oplus\,\tau(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} 1 - \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ 1 - \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\left\{1 - \sigma_{\mathcal{F}(\breve{a})}(\delta), 1 - \sigma_{\mathcal{G}(\breve{a})}(\delta)\right\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Assume $\oplus(\mathcal{F}, \ddot{\mathcal{A}}) \cup_R \oplus(\mathcal{G}, \ddot{\mathfrak{B}}) = \aleph$, where $\oplus(\mathcal{F}, \ddot{\mathcal{A}})$ and $\oplus(\mathcal{G}, \ddot{\mathfrak{B}})$ are given as follows by using the definition of necessity operation.

$\oplus\,(\mathcal{F},\ddot{\mathcal{A}}) = \left\{\left(\delta,\sigma_{\mathcal{F}(\breve{a})}(\delta),1-\ \sigma_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta\in\ \mathcal{U}\right\}$ and $\oplus\,(\mathcal{G},\ddot{\mathfrak{B}}) = \left\{\left(\delta,\sigma_{\mathcal{G}(\breve{a})}(\delta),1-\sigma_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta\in \mathcal{U}\right\}$. By using Definition 3.14

$$\sigma\aleph = \begin{cases}\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ max\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

$$\tau\aleph = \begin{cases}1-\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ 1-\sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ min\{1-\sigma_{\mathcal{F}(\breve{a})}(\delta),1-\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

Consequently $\oplus\,(\mathcal{H},\ \ddot{\mathrm{C}})$ and $\aleph$ are the same, so

$\oplus\left((\mathcal{F},\ddot{\mathcal{A}})\ \cup_R\ (\mathcal{G},\ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{G},\ \ddot{\mathfrak{B}})\ \cup_R\ \oplus\,(\mathcal{F},\ \ddot{\mathcal{A}})$.

Proof 2

Let $\left((\mathcal{F},\ddot{\mathcal{A}})\ \cap_\varepsilon (\mathcal{G},\ddot{\mathfrak{B}})\right) = (\mathcal{H},\ \ddot{\mathrm{C}})$

$$\sigma(\mathcal{H},\ \ddot{\mathrm{C}}) = \begin{cases}\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

$$\tau(\mathcal{H},\ \ddot{\mathrm{C}}) = \begin{cases}\tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ max\{\tau_{\mathcal{F}(\breve{a})}(\delta),\tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

By using Definition 4.4

$$\oplus\,\sigma(\mathcal{H},\ \ddot{\mathrm{C}}) = \begin{cases}\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

$$\oplus\,\tau(\mathcal{H},\ \ddot{\mathrm{C}}) = \begin{cases}1-\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ 1-\sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ max\{1-\sigma_{\mathcal{F}(\breve{a})}(\delta),1-\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

Assume $\oplus\,(\mathcal{G},\ \ddot{\mathfrak{B}})\ \cap_\varepsilon\ \oplus\,(\mathcal{F},\ \ddot{\mathcal{A}}) = \aleph$, where $\oplus(\mathcal{F},\ddot{\mathcal{A}})$ and $\oplus(\mathcal{G},\ddot{\mathfrak{B}})$ are given as follows by using the definition of necessity operation.

$\oplus\,(\mathcal{F},\ddot{\mathcal{A}}) = \left\{\left(\delta,\sigma_{\mathcal{F}(\breve{a})}(\delta),1-\ \sigma_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta\in\ \mathcal{U}\right\}$ and $\oplus\,(\mathcal{G},\ddot{\mathfrak{B}}) = \left\{\left(\delta,\sigma_{\mathcal{G}(\breve{a})}(\delta),1-\sigma_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta\in \mathcal{U}\right\}$. By using Definition 3.15

$$\sigma\aleph = \begin{cases}\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ min\{\sigma_{\mathcal{F}(\breve{a})}(\delta),\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

$$\tau\aleph = \begin{cases}1-\sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathcal{A}}-\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\\ 1-\sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a}\in\ddot{\mathfrak{B}}-\ddot{\mathcal{A}},\delta\in\ \mathcal{U}\\ max\{1-\sigma_{\mathcal{F}(\breve{a})}(\delta),1-\sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a}\in\ddot{\mathcal{A}}\cap\ddot{\mathfrak{B}},\delta\in\ \mathcal{U}\end{cases}$$

Consequently $\oplus\,(\mathcal{H},\ \ddot{\mathrm{C}})$ and $\aleph$ are the same, so

$\oplus\left((\mathcal{F},\ddot{\mathcal{A}})\ \cap_\varepsilon\ (\mathcal{G},\ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{G},\ \ddot{\mathfrak{B}})\ \cap_\varepsilon\ \oplus\,(\mathcal{F},\ \ddot{\mathcal{A}})$.

Proposition 4.7

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$ and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS, then

1. $\otimes\ \left((\mathcal{F},\ddot{\mathcal{A}})\ \cup_R\ (\mathcal{G},\ddot{\mathfrak{B}})\right) = \otimes\ (\mathcal{G},\ \ddot{\mathfrak{B}})\ \cup_R\ \otimes\ (\mathcal{F},\ \ddot{\mathcal{A}})$
2. $\otimes\ \left((\mathcal{F},\ddot{\mathcal{A}})\ \cap_\varepsilon\ (\mathcal{G},\ddot{\mathfrak{B}})\right) = \otimes\ (\mathcal{G},\ \ddot{\mathfrak{B}})\ \cap_\varepsilon\ \otimes\ (\mathcal{F},\ \ddot{\mathcal{A}})$

Proof 1

As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ are two PFHSS.

Let $\left((\mathcal{F}, \ddot{\mathcal{A}}) \cup_R (\mathcal{G}, \ddot{\mathfrak{B}})\right) = (\mathcal{H}, \ddot{\mathrm{C}})$

$$\sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

By using Definition 4.5

$$\otimes\, \sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} 1 - \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ 1 - \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{1 - \tau_{\mathcal{F}(\breve{a})}(\delta), 1 - \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\otimes\, \tau(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Assume $\otimes (\mathcal{F}, \ddot{\mathcal{A}}) \cup_R \otimes (\mathcal{G}, \ddot{\mathfrak{B}}) = \aleph$, where $\otimes (\mathcal{F}, \ddot{\mathcal{A}})$ and $\otimes (\mathcal{G}, \ddot{\mathfrak{B}})$ are given as follows by using the definition of necessity operation.

$\otimes (\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, 1 - \tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $\otimes (\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, 1 - \tau_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$. By using Definition 3.14

$$\sigma\aleph = \begin{cases} 1 - \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ 1 - \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{1 - \tau_{\mathcal{F}(\breve{a})}(\delta), 1 - \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau\aleph = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Consequently $\otimes\ (\mathcal{H}, \ddot{\mathrm{C}})$ and $\aleph$ are the same, so

$\otimes \left((\mathcal{F}, \ddot{\mathcal{A}}) \cup_R (\mathcal{G}, \ddot{\mathfrak{B}})\right) = \otimes\ (\mathcal{G}, \ddot{\mathfrak{B}}) \cup_R \otimes\ (\mathcal{F}, \ddot{\mathcal{A}})$.

Proof 2

As we know that

$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta, \sigma_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ are two PFHSS.

Let $\left((\mathcal{F}, \ddot{\mathcal{A}}) \cap_\varepsilon (\mathcal{G}, \ddot{\mathfrak{B}})\right) = (\mathcal{H}, \ddot{\mathrm{C}})$

$$\sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \sigma_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \sigma_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{\sigma_{\mathcal{F}(\breve{a})}(\delta), \sigma_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

By using Definition 4.5

$$\otimes\, \sigma(\mathcal{H}, \ddot{\mathrm{C}}) = \begin{cases} 1 - \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ 1 - \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{1 - \tau_{\mathcal{F}(\breve{a})}(\delta), 1 - \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\otimes\, \tau(\mathcal{H},\ \ddot{C}) = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Assume $\otimes\,(\mathcal{F},\ddot{\mathcal{A}}) \cap_{\varepsilon} \otimes\,(\mathcal{G},\ddot{\mathfrak{B}}) = \aleph$, where $\otimes\,(\mathcal{F},\ddot{\mathcal{A}})$ and $\otimes\,(\mathcal{G},\ddot{\mathfrak{B}})$ are given as follows by using the definition of necessity operation.

$\otimes\,(\mathcal{F},\ddot{\mathcal{A}}) = \left\{\left(\delta, 1-\tau_{\mathcal{F}(\breve{a})}(\delta),\ \tau_{\mathcal{F}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$ and $\otimes\,(\mathcal{G},\ddot{\mathfrak{B}}) = \left\{\left(\delta, 1-\tau_{\mathcal{G}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\right) \mid \delta \in \mathcal{U}\right\}$. By using Definition 3.14

$$\sigma\aleph = \begin{cases} 1-\tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ 1-\tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ min\{1-\tau_{\mathcal{F}(\breve{a})}(\delta), 1-\tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

$$\tau\aleph = \begin{cases} \tau_{\mathcal{F}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathcal{A}} - \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \\ \tau_{\mathcal{G}(\breve{a})}(\delta) & if\ \breve{a} \in \ddot{\mathfrak{B}} - \ddot{\mathcal{A}}, \delta \in \mathcal{U} \\ max\{\tau_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{G}(\breve{a})}(\delta)\} & if\ \breve{a} \in \ddot{\mathcal{A}} \cap \ddot{\mathfrak{B}}, \delta \in \mathcal{U} \end{cases}$$

Consequently $\otimes\ (\mathcal{H},\ \ddot{C})$ and $\aleph$ are the same, so

$\otimes\left((\mathcal{F},\ddot{\mathcal{A}}) \cap_{\varepsilon} (\mathcal{G},\ddot{\mathfrak{B}})\right) = \otimes\ (\mathcal{G},\ \ddot{\mathfrak{B}}) \cap_{\varepsilon} \otimes\ (\mathcal{F},\ \ddot{\mathcal{A}})$.

Proposition 4.8

Let $(\mathcal{F},\ \ddot{\mathcal{A}})$ and $(\mathcal{G},\ \ddot{\mathfrak{B}})$ be two PFHSS, then

1. $\oplus\left((\mathcal{F},\ddot{\mathcal{A}}) \wedge (\mathcal{G},\ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{F},\ \ddot{\mathcal{A}}) \wedge \oplus\,(\mathcal{G},\ \ddot{\mathfrak{B}})$
2. $\oplus\left((\mathcal{F},\ddot{\mathcal{A}}) \vee (\mathcal{G},\ddot{\mathfrak{B}})\right) = \oplus\,(\mathcal{F},\ \ddot{\mathcal{A}}) \vee \oplus\,(\mathcal{G},\ \ddot{\mathfrak{B}})$
3. $\otimes\left((\mathcal{F},\ddot{\mathcal{A}}) \wedge (\mathcal{G},\ddot{\mathfrak{B}})\right) = \otimes\,(\mathcal{F},\ddot{\mathcal{A}}) \wedge \otimes\,(\mathcal{G},\ddot{\mathfrak{B}})$
4. $\otimes\left((\mathcal{F},\ddot{\mathcal{A}}) \vee (\mathcal{G},\ddot{\mathfrak{B}})\right) = \otimes\,(\mathcal{F},\ddot{\mathcal{A}}) \vee \otimes\,(\mathcal{G},\ddot{\mathfrak{B}})$

Proof 1 Proof is straight forward.

5. Application of Correlation Coefficient for Decision Making Under PFHSS Environment

In this section, we present the correlation coefficient under the PFHSS environment and establish an algorithm based on the proposed CC under PFHSS and utilize the proposed approach for decision making in real-life problems.

Definition 5.1

Let $(\mathcal{F},\ddot{\mathcal{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathfrak{B}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two PFHSSs defined over a universe of discourse $\mathcal{U}$. Then, their informational energies of $(\mathcal{F},\ddot{\mathcal{A}})$ and $(\mathcal{G},\ddot{\mathfrak{B}})$ can be described as follows:

$$\varsigma_{PFHSS}(\mathcal{F},\ddot{\mathcal{A}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^4 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^4\right)$$

$$\boldsymbol{\varsigma_{PFHSS}(\mathcal{G},\ddot{\mathfrak{B}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^4 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^4\right)}.$$

Definition 5.2

Let $(\mathcal{F},\ddot{\mathcal{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathfrak{B}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two PFHSSs defined over a universe of discourse $\mathcal{U}$. Then, their correlation measure between $(\mathcal{F},\ddot{\mathcal{A}})$ and $(\mathcal{G},\ddot{\mathfrak{B}})$ can be described as follows:

$$\mathcal{C}_{PFHSS}((\mathcal{F},\ddot{\mathcal{A}}),(\mathcal{G},\ddot{\mathfrak{B}})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right).$$

Definition 5.3

Let $(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\mathfrak{B}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two PFHSSs, then correlation coefficient between them given as $\delta_{IFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}}))$ and expressed as follows:

$$\delta_{PFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}})) = \frac{\mathcal{C}_{PFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}}))}{\sqrt{\varsigma_{PFHSS}(\mathcal{F}, \ddot{\mathcal{A}})} * \sqrt{\varsigma_{PFHSS}(\mathcal{G}, \ddot{\mathfrak{B}})}}$$

$$\boldsymbol{\delta_{IFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}}))} = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^4 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^4\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^4 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^4\right)}}$$

5.1 Algorithm for Correlation Coefficient under PFHSS

Step 1. Pick out the set containing sub-attributes of parameters.

Step 2. Construct the PFHSS according to experts in form of PFHSNs.

Step 3. Find the informational energies of PFHSS.

Step 4. Calculate the correlation between PFHSSs by using the following formula

$$\mathcal{C}_{PFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 * \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)$$

Step 5. Calculate the CC between PFHSSs by using the following formula

$$\delta_{PFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}})) = \frac{\mathcal{C}_{PFHSS}((\mathcal{F}, \ddot{\mathcal{A}}), (\mathcal{G}, \ddot{\mathfrak{B}}))}{\sqrt{\varsigma_{PFHSS}(\mathcal{F}, \ddot{\mathcal{A}})} * \sqrt{\varsigma_{PFHSS}(\mathcal{G}, \ddot{\mathfrak{B}})}}$$

Step 6. Choose the alternative with a maximum value of CC.

Step 7. Analyze the ranking of the alternatives.

A flowchart of the above-presented algorithm can be seen in Figure 1.

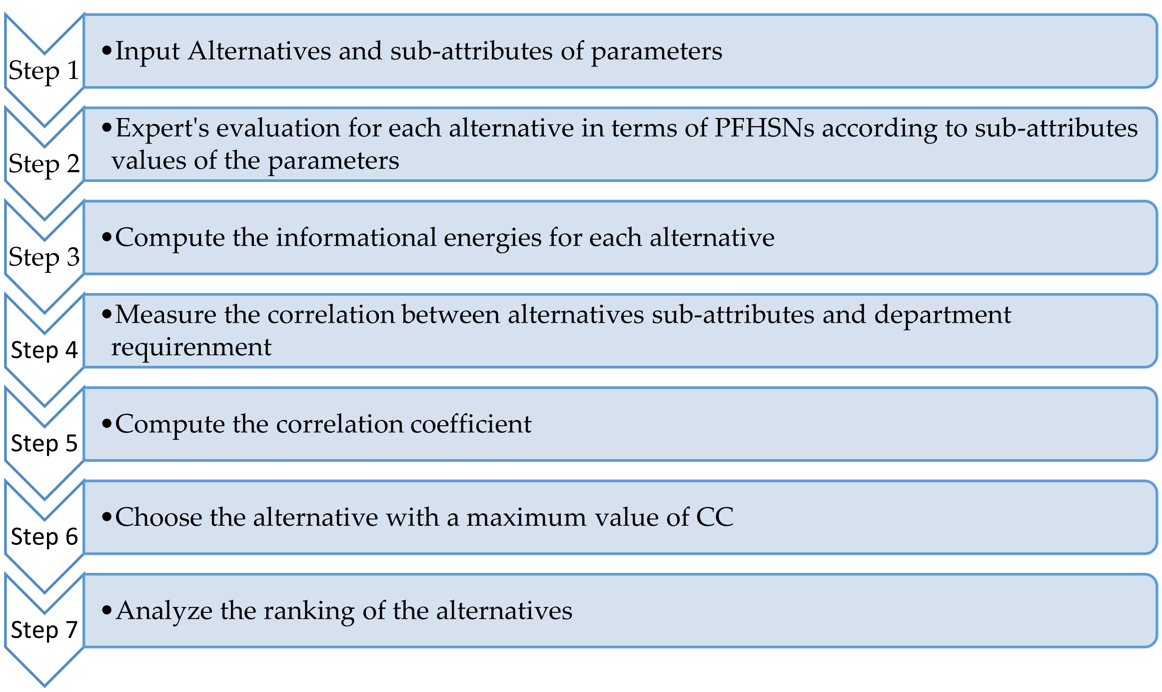

Figure 1: Flowchart for correlation coefficient under PFHSS

5.2 Problem Formulation and Application of PFHSS For Decision Making

Department of the scientific discipline of some university $\mathcal{U}$ will have one scholarship for the position of the doctoral degree. Several students apply to get a scholarship but referable probabilistic along with CGPA (cumulative grade points average), simply four students call for enrolled for undervaluation such as $\mathcal{T} = \{\mathcal{T}^1, \mathcal{T}^2, \mathcal{T}^3, \mathcal{T}^4\}$ be a set of selected students call for the interview. The president of the university hires a committee of four decision-makers (DM) $\mathcal{U} = \{\partial_1, \partial_2, \partial_3, \partial_4\}$ for the selection of doctoral degree student. The team of DM decides the criteria (attributes) for the selection of doctoral degree such as $\mathfrak{L} = \{\ell_1 = \text{Publications}, \ell_2 = \text{Subjects}, \ell_3 = \text{IF}\}$ be a collection of attributes and their corresponding sub-attribute are given as Publications = $\ell_1 = \{a_{11} = more\ than\ 10, a_{12} = \ less\ than\ 10\}$, Subjects = ℓ_2 = $\{a_{21} = Mathematics, a_{22} = Computer\ Science\}$, and IF = $\ell_3 = \{a_{31} = 45, a_{32} = 47\}$. Let $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes

$$\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\}$$

$= \begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}) \end{Bmatrix}$, $\mathfrak{L}' = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8\}$ be a set of all multi sub-attributes. Each DM will evaluate the ratings of each alternative in the form of PFHSNs under the considered multi sub-attributes. The developed method to find the best alternative is as follows.

5.3 Application of PFHSS For Decision Making

Assume $\mathcal{T} = \{\mathcal{T}^1, \mathcal{T}^2, \mathcal{T}^3, \mathcal{T}^4\}$ be a set of alternatives who are shortlisted for interview and $\mathfrak{L} = \{\ell_1 = \text{Publications}, \ell_2 = \text{Subjects}, \ell_3 = \text{Qualification}\}$ be a set of parameters for the selection of scholarship positions. Let the corresponding sub-attribute are given as Publications = $\ell_1 = \{a_{11} = more\ than\ 10, a_{12} = \ less\ than\ 10\}$, Subjects = ℓ_2 = $\{a_{21} = Mathematics, a_{22} = Computer\ Science\}$, and IF = $\ell_3 = \{a_{31} = 45, a_{32} = 47\}$. Let $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes. The development of the decision matrix according to the requirement of the scientific discipline department in terms of PFHSNs is given in Table 1.

Table 1. Decision Matrix for Concerning Department

$\wp$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
∂_1	(.3,.8)	(.7 .3)	(.6,.7)	(.5,.4)	(.2,.4)	(.4,.6)	(.5,.8)	(.9,.3)
∂_2	(.6,.7)	(.4,.6)	(.3,.4)	(.9,.2)	(.3,.8)	(.2,.4)	(.7,.5)	(.4,.5)
∂_3	(.7,.3)	(.2,.5)	(.1,.6)	(.3,.4)	(.4 .6)	(.8,.4)	(.6,.7)	(.2,.5)
∂_4	(.8,.4)	(.2,.9)	(.2,.4)	(.4,.6)	(.6,.5)	(.5,.6)	(.4,.5)	(.8,.3)

Develop the decision matrices for each alternative in terms of PFHSNs by considering their sub-attributes values of given attributes can be seen in Table 2- Table 5.

Table 2. Decision Matrix for Alternative $\mathcal{T}^{(1)}$

$\mathcal{T}^{(1)}$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
∂_1	(.7,.6)	(.3,.4)	(.6,.5)	(.3,.9)	(.5,.4)	(.4,.6)	(.7,.5)	(.4,.8)

$\boldsymbol{\partial_2}$	(.8,.5)	(.7,.4)	(.9,.2)	(.7,.4)	(.4,.5)	(.9,.3)	(.2,.7)	(.3,.8)
$\boldsymbol{\partial_3}$	(.3,.7)	(.4,.5)	(.4,.8)	(.3,.4)	(.6,.7)	(.3,.4)	(.9,.2)	(.7,.2)
$\boldsymbol{\partial_4}$	(.5,.4)	(.7,.6)	(.9,.3)	(.8,.5)	(.9,.2)	(.2,.4)	(.4,.6)	(.6,.5)

Table 3. Decision Matrix for Alternative $\mathcal{T}^{(2)}$

$\boldsymbol{\mathcal{T}^{(2)}}$	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
$\boldsymbol{\partial_1}$	(.6,.5)	(.3,.8)	(.4,.5)	(.7,.4)	(.6,.4)	(.4,.5)	(.3,.4)	(.7,.5)
$\boldsymbol{\partial_2}$	(.8,.4)	(.9,.3)	(.1,.8)	(.1,.2)	(.4,.6)	(.3,.7)	(.6,.8)	(.8,.4)
$\boldsymbol{\partial_3}$	(.6,.7)	(.7,.4)	(.7,.5)	(.3,.4)	(.9,.2)	(.6,.5)	(.3,.5)	(.6,.7)
$\boldsymbol{\partial_4}$	(.5,.4)	(.4,.8)	(.5,.6)	(.3,.4)	(.7,.6)	(.7,.5)	(.4,.9)	(.5,.2)

Table 4. Decision Matrix for Alternative $\mathcal{T}^{(3)}$

$\boldsymbol{\mathcal{T}^{(3)}}$	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
$\boldsymbol{\partial_1}$	(.5,.7)	(.8,.5)	(.7,.4)	(.4,.3)	(.4,.9)	(.2,.4)	(.8,.4)	(.7,.5)
$\boldsymbol{\partial_2}$	(.8,.5)	(.7,.4)	(.8,.5)	(.5,.2)	(.5,.7)	(.7,.5)	(.7,.6)	(.6,.4)
$\boldsymbol{\partial_3}$	(.6,.8)	(.4,.5)	(.6,.5)	(.6,.4)	(.7,.5)	(.8,.4)	(.5,.8)	(.4 .5)
$\boldsymbol{\partial_4}$	(.5,.7)	(.9,.3)	(.3,.5)	(.5,.7)	(.3,.5)	(.8,.5)	(.7,.5)	(.2,.5)

Table 5. Decision Matrix for Alternative $\mathcal{T}^{(5)}$

$\boldsymbol{\mathcal{T}^{(4)}}$	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
$\boldsymbol{\partial_1}$	(.6,.5)	(.3,.8)	(.4,.5)	(.7,.4)	(.6,.4)	(.4,.5)	(.3,.4)	(.7,.5)
$\boldsymbol{\partial_2}$	(.8,.4)	(.9,.3)	(.1,.8)	(.1,.2)	(.4,.6)	(.3,.7)	(.6,.8)	(.8,.4)
$\boldsymbol{\partial_3}$	(.6,.7)	(.7,.4)	(.7,.5)	(.3,.4)	(.9,.2)	(.6,.5)	(.3,.5)	(.6,.7)
$\boldsymbol{\partial_4}$	(.5,.4)	(.4,.8)	(.5,.6)	(.3,.4)	(.7,.6)	(.7,.5)	(.4,.9)	(.5,.2)

By using Tables 1-5, compute the correlation coefficient between $\delta_{PFHSS}(\wp,\mathcal{T}^{(1)})$, $\delta_{PFHSS}(\wp,\mathcal{T}^{(2)})$, $\delta_{PFHSS}(\wp,\mathcal{T}^{(3)})$, $\delta_{PFHSS}(\wp,\mathcal{T}^{(4)})$ by using Definition 5.3 given as follows:
$\delta_{PFHSS}(\wp,\mathcal{T}^{(1)}) = .99658$, $\delta_{PFHSS}(\wp,\mathcal{T}^{(2)}) = .99732$, $\delta_{PFHSS}(\wp,\mathcal{T}^{(3)}) = .99894$, and $\delta_{PFHSS}(\wp,\mathcal{T}^{(4)}) = .99669$. This shows that $\delta_{PFHSS}(\wp,\mathcal{T}^{(3)}) > \delta_{PFHSS}(\wp,\mathcal{T}^{(2)}) > \delta_{PFHSS}(\wp,\mathcal{T}^{(4)}) > \delta_{PFHSS}(\wp,\mathcal{T}^{(1)})$. It can be seen from this ranking alternative $\mathcal{T}^{(3)}$ is the most suitable alternative. Therefore $\mathcal{T}^{(3)}$ is the best alternative, the ranking of other alternatives given as $\mathcal{T}^{(3)} > \mathcal{T}^{(2)} > \mathcal{T}^{(4)} > \mathcal{T}^{(1)}$. Graphical results of alternatives ratings can be seen in Figure 2.

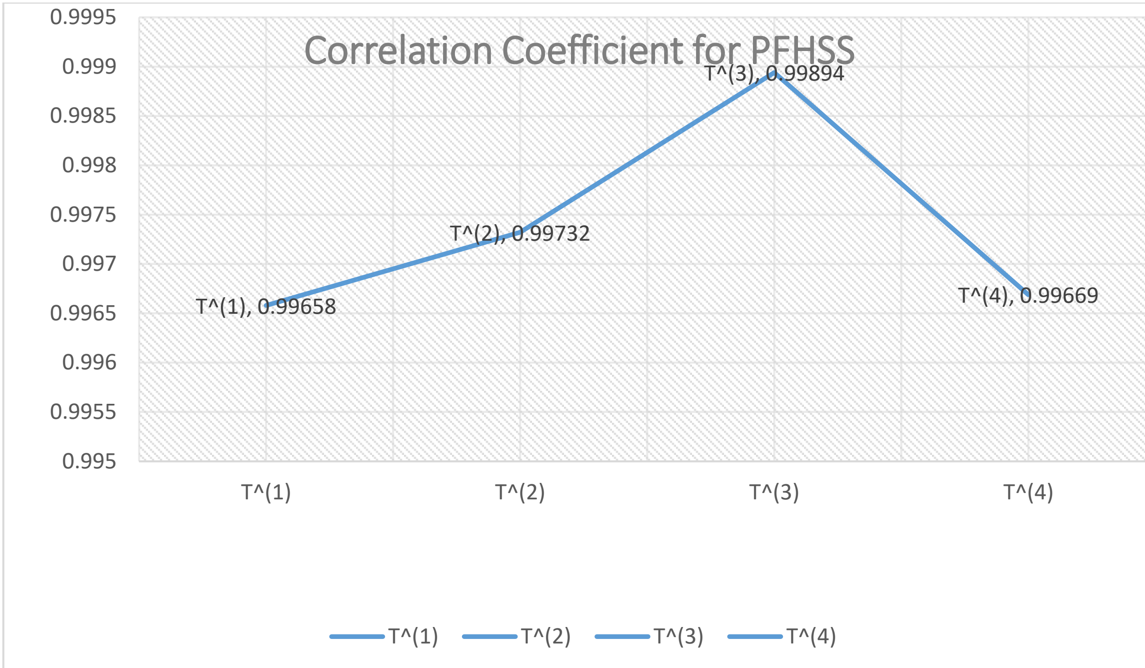

Figure 2: Alternatives rating based on correlation coefficient under PFHSS

6. Discussion and Comparative Analysis

In the following section, we are going to debate the effectivity, naivety, flexibility as well as favorable position of the suggested method along with the algorithm. We also organized a brief comparative analysis of the following: The suggested method along with the prevailing approaches.

6.1 Superiority of the Proposed Method

Through this research and comparative analysis, we have concluded that the proposed methods' results are more general than prevailing techniques. However, in the decision-making process, compared with the existing decision-making methods, it contains more information to deal with the data's uncertainty. Moreover, many of FS's mixed structure has become a special case of PFHSS, add some suitable conditions. In it, the information related to the object can be expressed more accurately and empirically, so it is a convenient tool for combining inaccurate and uncertain information in the decision-making process. Therefore, our proposed method is effective, flexible, simple, and superior to other hybrid structures of fuzzy sets.

Table 6: Comparative analysis between some existing techniques and the proposed approach

	Set	**Truthiness**	**Falsity**	**Attributes**	**Multi sub-attributes**	Advantages
Zadeh [1]	FS	✓	×	✓	×	Deals uncertainty by using fuzzy interval
Atanassov [2]	IFS	✓	✓	✓	×	Deals uncertainty by using MD and NMD

Yager [21]	PFS	✓	✓	✓	×	Deals more uncertainty by using MD and NMD comparative to IFS
Zulqarnain et al. [46]	IFHSS	✓	✓	✓	✓	Deals uncertainty of multi sub-attributes
Proposed approach	PFHSS	✓	✓	✓	✓	Deals more uncertainty comparative to IFHSS

6.2 Comparative Analysis

By using the technique of Zadeh [1], we deal with the true information of the alternatives, but this method cannot deal with falsity objects and multi sub-attributes of the alternatives. By utilizing the Atanassov [2], and Yager [21] methodologies, we cannot deal with the alternatives' multi-sub-attribute information. But our proposed method can easily solve these obstacles and provide more effective results for the MCDM problem. Zulqarnain et al. [46] presented IFHSS deals with the uncertainty by using the MD and NMD of the sub-attributes of a set of parameters, but the sum of MD and NMD of sub-attributes cannot exceed 1. In some cases the sum of MD and NMD exceeds 1, then existing IFHSS fails to deal with such situations. Instead of this, our developed method is an advanced technique that can handle alternatives with multi-sub-attributes information when the sum of MD and NMD exceeds 1. A comparison can be seen in the above-listed Table 6. However, on the other hand, the methodology we have established deals with the truthiness and falsity of alternatives with multi sub-attributes. Therefore, the technique we developed is more efficient and can provide better results for decision-makers through a variety of information comparative to existing techniques.

7. Conclusion

The Pythagorean fuzzy hypersoft set is a novel concept that is an extension of the intuitionistic fuzzy soft set and generalization of the intuitionistic fuzzy hypersoft set. In this work, we studied some basic concepts and developed some basic operations for PFHSS with their properties. We proposed the AND-operation and OR-operation under the PFHSS environment with their desirable properties in the following research. The idea of necessity and possibility operations with numerous properties under the Pythagorean fuzzy hypersoft set is also presented in it. Furthermore, the concept of CC is also established in this research with its decision-making methodology. We used the developed methodology to solve decision-making problems to ensure the validity and applicability of the developed decision-making methodology. Furthermore, A comparative analysis is presented to verify the validity and demonstration of the proposed method. Finally, based on the results obtained, it is concluded that the suggested techniques showed higher stability and practicality for decision-makers in the decision-making process. In the future, anyone can extend the PFHSS to interval-valued PFHSS, aggregation operators, and TOPSIS technique under PFHSS.

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Chapter 6

Development of TOPSIS using Similarity Measures and Generalized weighted distances for Interval Valued Neutrosophic Hypersoft Matrices along with Application in MAGDM Problems

Adeel Saleem[1], Muhammad Saqlain[1*], Sana Moin[2]

[2]Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan. E-mail: adeelsaleem1992@hotmail.com

[1*]Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan. E-mail: msaqlain@lgu.edu.pk

[2]Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan. E-mail: moinsana64@gmail.com

Abstract: In this chapter, we stretch out the TOPSIS technique to illuminate MAGDM issues in the interval valued neutrosophic hypersoft set environment. To build up the proposed TOPSIS procedures, distance measures, similarity measures and the concept of interval valued neutrosophic hypersoft matrices (IVNHSM) is introduced along with definitions, theorems, propositions and examples. To exhibit dependability and suitability of the proposed TOPSIS technique, we solve a numerical example for the selection of the best marriage proposal. TOPSIS is a fascinating tool for dealing with MCDM. The Future directions and the limitations of the proposed algorithm are also presented.

Keywords: *Interval Valued Neutrosophic Hypersoft Set (IVNHSS), MCDM, Neutrosophic Hypersoft Set (NHSS), Neutrosophic Hypersoft Matrix (NHSM), TOPSIS, MAGDM, Similarity Measures, Distance Measures.*

1.Introduction

In our daily life every being has to decide something and it is very common that most of us faces problem in decision making. To make decision making easy different researchers and mathematicians invent many techniques. Multi criteria decision making is also one of the decision-making techniques. MADM is a procedure of finding an ideal alternative that is most suitable to fulfill the need from a lot of options associated with various different attributes. A huge range of strategies has been adapted for managing MADM issues, such as, VIKOR [1], AHP [2], TOPSIS [3,4] and etc. In MADM problems, evaluation of attributes can't be constantly given in crisp values due to uncertain and unpredictable nature of the characteristics in real life. Fuzzy set [5] introduced by Zadeh is equipped for managing uncertainty in scientific structure. Thus MADM [6-9] can be demonstrated very well by utilizing the fuzzy set theory into the field of developing decision-making techniques. Fuzzy set are progressing rapidly but the mathematicians faces problem in handling uncertainty because fuzzy set can just concentrate on the participation degree and it neglects to consider non-participation degree and indeterminacy level of imprecise information. Atanassov [10] presented Intuitionistic Fuzzy set (IFS) for dealing with uncertainty and unpredictability in the more accurate manner by considering the truth membership and falsity membership values. Working on intuitionistic fuzzy set it is noticed that some of the incomplete or indeterminate information do exist. Hence, IFS can't deal with uncertainty in the appropriate manner in MADM [11-13] and MAGDM [14-16] issues in which some problems arise due to indeterminate information. Smarandache [17] supports the idea of Neutrosophic set that is a scientific gadget to overcome issued like indeterminant, uncertain and contradictory information. Neutrosophic set displays real membership value, indeterminacy membership value and falsity membership value. Such an idea is of great significance in numberless applications because indeterminacy is checked extraordinarily and truth membership values, indeterminacy membership values and falsity values are independent. Molodtsov [18] highlights the idea of soft set to manage issues Indefinite circumstances. It was said that soft set was parameterized family of subsets of universal sets. Soft sets have their own paramount importance in the fields of artificial insight, game hypothesis and fundamental decision-making issues., it also helps us to find out various functions for different parameters and benefit values against the accepted and established parameters. We come to know that the fundamentals of soft set theories have been mediated and pondered over by different learned people for the last two years. For example, Maji et al [19,20] presented a hypothetical analysis of soft sets and upper set of soft sets, equality of soft sets and operation on soft sets, for example, union, intersection AND, and OR operation between different sets. Ali at el [21] new operations in soft theory that covers restricted union, intersection and difference. Cagman and Egniloglus [22,23] support soft set theory that empowers itself a very important measurement while looking after issues to make different choices. Onyeozili [24] presented research which claims that soft set is equal to corresponding soft matrices. From Molodtsov [25] to present different beneficial applications identified with soft set theory have been presented and annexed to numberless fields of science and data innovation. Maji [26] presented Neutrosophic soft set exhibited by truth, indeterminacy and falsity membership values that are independent in nature. Neutrosophic soft set can manage inadequate, uncertain and inconstant data while intuitionistic fuzzy soft set can only deal with partial data. Smarandache [27] came up with new plans to cope with uncertainly. Soft sets were generalized to hypersoft set by altering the function into multi decision function. When features and attributes are more than one and further diverge, Neutrosophic soft set environment can't help to cope with such kind of issues owing to soft sets as soft sets deal with sole argument function. In order to overcome such problem, Neutrosophic hypersoft set [28] was introduced. There are various MADM methodologies are accessible in the literature. Among them TOPSIS is extremely mainstream to manage MADM.TOPSIS strategy offered by Hwang and Yoon [3] can be called one of the strategies which can provide suitable solutions. The central idea of TOPSIS is that the best alternative must keep the shortest distance from the positive ideal solution (PIS)and the farther distance from the negative ideal solution (NIS) simultaneously. TOPSIS is very renowned and well known to deal with MADM. Behzaian et al. [4] offered a detailed survey on TOPSIS applications in various fields. Many researchers proposed TOPSIS technique to solve MCDM and MAGDM problems. [29-35]. Multi-Criteria decision problems (MCDM) consist of several attributes and indeterminacy, to deal with such types neutrosophic sets (N's) and interval valued neutrosophic sets (IVN's) are used because (N's) fully deal with indeterminacy whereas to deal with vagueness and uncertainty neutrosophic soft sets (NS's) are used. But when attributes are more than one and further bifurcated the concept of neutrosophic soft set (NS's) cannot be used to tackle such type of issues so, there was a dire need to define the new environment, for this purpose the concept of neutrosophic hypersoft set (NHSS) was proposed by [28][36] Matrix notations on neutrosophic hypersoft set is proposed in []. This concept was extended to IVNHSS [37] with the generalization of definition and operators. To solve MCDM and MAGDM problems in IVNHSS environment it is necessary to propose any technique. So, in this chapter the basic operation like; interval valued neutrosophic hypersoft Matrix (IVNHSM), Generalized weighted distance for IVNHSM and IVNHSS the similarity measure for IVNHSM and IVNHSS are proposed. By using these operations, the algorithm of TOPSIS is extended to IVNHSS TOPSIS.

1.1 The Chapter Presentation:

After introduction rest of the chapter is contain section 2 of some basic preliminaries. Then we have Section 3, in which definition of IVNHSM along with some operations are defined. Section 4 consists of generalized weighted similarity measure for both IVNHSS and IVNHSM. Section 6, have distance measures for the both IVNHSS and IVNHSM. Section 7, consist of the TOPSIS algorithm which is proposed using the similarity measure, distance measure and IVNHSM. In section 7, a case study is presented. Finally, Sect. 8 presents some concluding remarks and future scope of research. Figure: 1 represents the chapter presentation graphically.

2.Preliminaries

This section consists of some basic definitions which leads us to neutrosophic hypersoft sets and will be helpful in rest of the article.

Definition 2.1: Neutrosophic Soft Set

Let $\mathbb{U}$ be the universal set and $\mathbb{E}$ be the set of attributes with respect to $\mathbb{U}$. Let $\mathbb{P}(\mathbb{U})$ be the set of Neutrosophic values of $\mathbb{U}$ and $\mathbb{A} \subseteq \mathbb{E}$. A pair (F, $\mathbb{A}$) is called a Neutrosophic soft set over $\mathbb{U}$ and its mapping is given as

$$Ꞙ: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U})$$

Definition 2.2: Hyper Soft Set:

Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$ be the power set of $\mathbb{U}$. Consider $l^1, l^2, l^3 \ldots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \ldots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$, then the pair $(Ꞙ, L^1 \times L^2 \times L^3 \ldots L^n)$ is said to be Hypersoft set over $\mathbb{U}$ where $Ꞙ: L^1 \times L^2 \times L^3 \ldots L^n \rightarrow \mathbb{P}(\mathbb{U})$

Definition 2.3: Neutrosophic Hypersoft Set [28]

Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$be the power set of $\mathbb{U}$. Consider $\mathrm{l}^1, \mathrm{l}^2, \mathrm{l}^3 \dots \mathrm{l}^\mathrm{n}$ for $\mathrm{n} \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $\mathrm{L}^1, \mathrm{L}^2, \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n}$ with $\mathrm{L}^\mathrm{i} \cap \mathrm{L}^\mathrm{j} = \emptyset$, for $\mathrm{i} \neq \mathrm{j}$ and $\mathrm{i}, \mathrm{j} \epsilon \{1,2,3 \dots \mathrm{n}\}$ and their relation $\mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n} = \$$, then the pair $(₣, \$)$ is said to be Neutrosophic Hypersoft set (NHSS) over $\mathbb{U}$ where

$₣: \mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n} \rightarrow \mathbb{P}(\mathbb{U})$and

$₣(\mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n}) = \{< \mathrm{x}, \mathrm{T}(₣(\$)), \mathrm{I}(₣(\$)), \mathrm{F}(₣(\$)) >, \mathrm{x} \in \mathbb{U}\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $\mathrm{T}, \mathrm{I}, \mathrm{F}: \mathbb{U} \rightarrow [0,1]$ also $0 \leq \mathrm{T}(₣(\$)) + \mathrm{I}(₣(\$)) + \mathrm{F}(₣(\$)) \leq 3$.

Definition 2.4: Interval Valued Neutrosophic Hypersoft Set (IVNHSS) [37]

Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$be the power set of $\mathbb{U}$. Consider $\mathrm{l}^1, \mathrm{l}^2, \mathrm{l}^3 \dots \mathrm{l}^\mathrm{n}$ for $\mathrm{n} \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $\mathrm{L}^1, \mathrm{L}^2, \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n}$ with $\mathrm{L}^\mathrm{i} \cap \mathrm{L}^\mathrm{j} = \emptyset$, for $\mathrm{i} \neq \mathrm{j}$ and $\mathrm{i}, \mathrm{j} \epsilon \{1,2,3 \dots \mathrm{n}\}$ and their relation $\mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n} = \$$, then the pair $(₣, \$)$ is said to be Interval Valued Neutrosophic Hypersoft set (IVNHSS) over $\mathbb{U}$ where

$₣: \mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n} \rightarrow \mathbb{P}(\mathbb{U})$and

$₣(\mathrm{L}^1 \times \mathrm{L}^2 \times \mathrm{L}^3 \dots \mathrm{L}^\mathrm{n}) = \{< \mathrm{x}, [\mathrm{T}^\mathrm{L}(₣(\$)), \mathrm{T}^\mathrm{U}(₣(\$))], [\mathrm{I}^\mathrm{L}(₣(\$)), \mathrm{I}^\mathrm{U}(₣(\$))], [\mathrm{F}^\mathrm{L}(₣(\$)), \mathrm{F}^\mathrm{U}(₣(\$))] >, \mathrm{x} \in \mathbb{U}\}$ where $[\mathrm{T}^\mathrm{L}(₣(\$))]$ is the lower membership value of truthiness, $\mathrm{I}^\mathrm{L}(₣(\$))$ is the lower membership value of indeterminacy and $\mathrm{F}^\mathrm{L}(₣(\$))$ is the lower membership value of falsity. Similarly, $[\mathrm{T}^\mathrm{U}(₣(\$))]$ is the upper membership value of truthiness, $\mathrm{I}^\mathrm{U}(₣(\$))$ is the upper membership value of indeterminacy and $\mathrm{F}^\mathrm{U}(₣(\$))$ is the upper membership value of falsity. Such that

$[\mathrm{T}^\mathrm{L}(₣(\$)), \mathrm{T}^\mathrm{U}(₣(\$))], [\mathrm{I}^\mathrm{L}(₣(\$)), \mathrm{I}^\mathrm{U}(₣(\$))], [\mathrm{F}^\mathrm{L}(₣(\$)), \mathrm{F}^\mathrm{U}(₣(\$))]: \mathbb{U} \rightarrow [0,1]$.

Also $0 \leq \sup\mathrm{T}(₣(\$)) + \sup \mathrm{I}(₣(\$)) + \sup\mathrm{F}(₣(\$)) \leq 3$.

Definition 2.5: Neutrosophic Hypersoft Matrix (NHSM) [38]

Let $\mathbb{U} = \{u^1, u^2, \dots u^a\}$ and $\mathbb{P}(\mathbb{U})$ be the universal set and power set of universal set respectively, also consider $\mathbb{L}_1, \mathbb{L}_2, \dots \mathbb{L}_{\mathscr{b}}$ for $\mathscr{b} \geq 1$, $\mathscr{b}$ well defined attributes, whose corresponding attributive values are respectively the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \dots \mathbb{L}_{\mathscr{b}}^z$ and their relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z$ where $a, b, c, \dots z = 1,2, \dots n$ then the pair $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z)$ is said to be Neutrosophic Hypersoft set over $\mathbb{U}$ where $\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z) \rightarrow \mathbb{P}(\mathbb{U})$ and it is define as

$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z) = \{< u, \mathbb{T}_\ell(u), \mathbb{I}_\ell(u), \mathbb{F}_\ell(u) > u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z)\}$

Let $\mathbb{R}_\ell = (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z)$ be the relation and its characteristic function is $\mathcal{X}_{\mathbb{R}_\ell}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z) \rightarrow \mathbb{P}(\mathbb{U})$ and it is define as $\mathcal{X}_{\mathbb{R}_\ell} = \{< u, \mathbb{T}_\ell(u), \mathbb{I}_\ell(u), \mathbb{F}_\ell(u) > u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \mathbb{L}_{\mathscr{b}}^z)\}$.

The tabular representation of $\mathbb{R}_\ell$ is given as

	$\mathbb{L}_1^a$	$\mathbb{L}_1^b$	…	$\mathbb{L}_{\mathscr{b}}^z$
u^1	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^b)$	…	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_{\mathscr{b}}^z)$
u^2	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^b)$	…	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_{\mathscr{b}}^z)$
⋮	⋮	⋮	⋱	⋮
u^a	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^b)$	…	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_{\mathscr{b}}^z)$

If $A_{ij} = \mathcal{X}_{\mathbb{R}_\ell}(u^i, \mathbb{L}_j^k)$, where $i = 1,2,3 \ldots a$, $j = 1,2,3, \ldots b$, $k = a, b, c, \ldots z$, then a matrix is defined as

$$[A_{ij}]_{a\times b} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1b} \\ A_{21} & A_{22} & \cdots & A_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ A_{a1} & A_{a2} & \cdots & A_{ab} \end{pmatrix}$$

Where $A_{ij} = \left(\mathbb{T}_{\mathbb{L}_j^k}(u_i), \mathbb{I}_{\mathbb{L}_j^k}(u_i), \mathbb{F}_{\mathbb{L}_j^k}(u_i), u_i \in \mathbb{U}, \mathbb{L}_j^k \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_b^z)\right) = \left(\mathbb{T}_{ijk}^A, \mathbb{I}_{ijk}^A, \mathbb{F}_{ijk}^A\right)$

Thus, we can represent any Neutrosophic hypersoft set in term of Neutrosophic hypersoft matrix (NHSM), it means that they are interchangeable. Its generalized form is given as

$$[A_{ij}]_{a\times b} = \begin{bmatrix} \mathbb{T}_{\mathbb{L}_1^a}(u_1), \mathbb{I}_{\mathbb{L}_1^a}(u_1), \mathbb{F}_{\mathbb{L}_1^a}(u_1) & \mathbb{T}_{\mathbb{L}_2^b}(u_1), \mathbb{I}_{\mathbb{L}_2^b}(u_1), \mathbb{F}_{\mathbb{L}_2^b}(u_1) & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_1), \mathbb{I}_{\mathbb{L}_b^z}(u_1), \mathbb{F}_{\mathbb{L}_b^z}(u_1) \\ \mathbb{T}_{\mathbb{L}_1^a}(u_2), \mathbb{I}_{\mathbb{L}_1^a}(u_2), \mathbb{F}_{\mathbb{L}_1^a}(u_2) & \mathbb{T}_{\mathbb{L}_2^b}(u_2), \mathbb{I}_{\mathbb{L}_2^b}(u_2), \mathbb{F}_{\mathbb{L}_2^b}(u_2) & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_2), \mathbb{I}_{\mathbb{L}_b^z}(u_2), \mathbb{F}_{\mathbb{L}_b^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}_{\mathbb{L}_1^a}(u_a), \mathbb{I}_{\mathbb{L}_1^a}(u_a), \mathbb{F}_{\mathbb{L}_1^a}(u_a) & \mathbb{T}_{\mathbb{L}_2^b}(u_a), \mathbb{I}_{\mathbb{L}_2^b}(u_a), \mathbb{F}_{\mathbb{L}_2^b}(u_a) & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_a), \mathbb{I}_{\mathbb{L}_b^z}(u_a), \mathbb{F}_{\mathbb{L}_b^z}(u_a) \end{bmatrix}$$

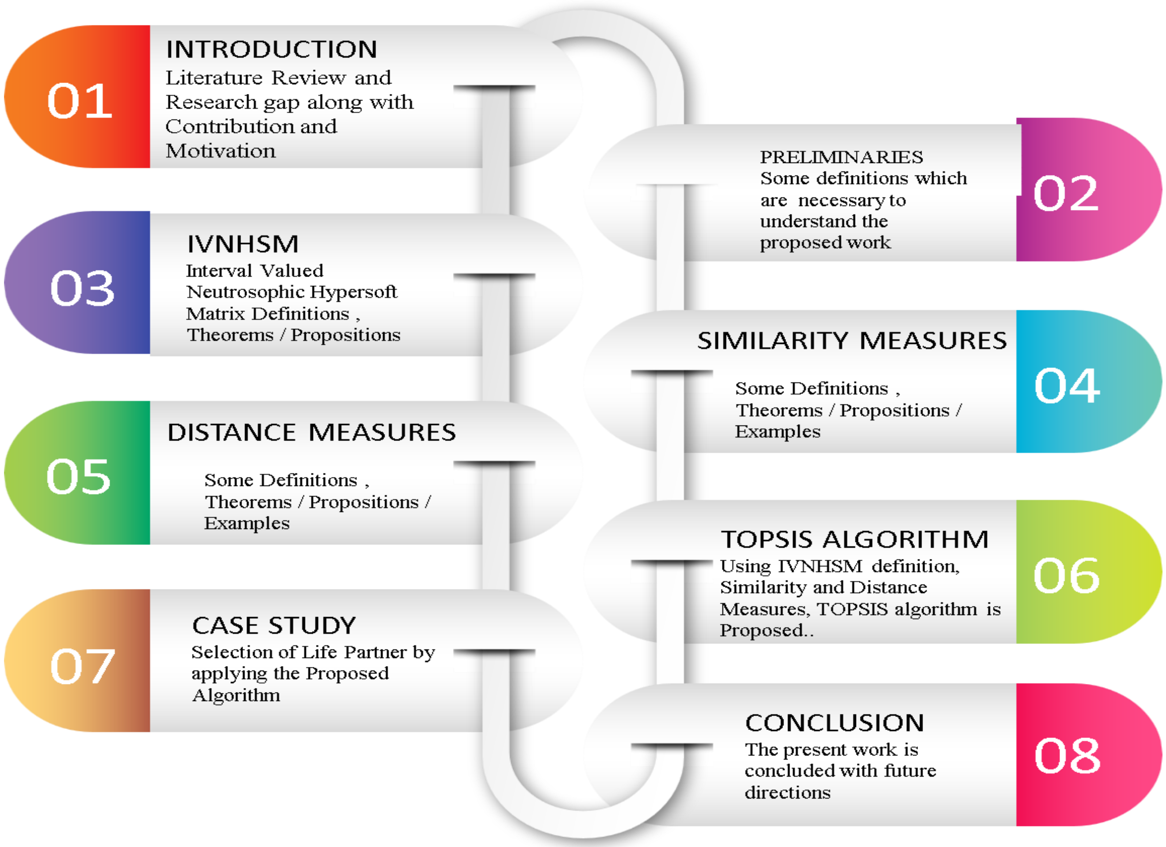

Figure: 1 The chapter presentation

3. Interval Valued Neutrosophic Hypersoft Matrix (IVNHSM)

In this Section IVNHSM, theorems and propositions with examples are defined.

Definition 3.1: Interval Valued Neutrosophic Hypersoft matrix (IVNHSM)

Let $\mathbb{U} = \{u^1, u^2, \ldots u^a\}$ and $\mathbb{P}(\mathbb{U})$ be the universal set and power set of universal set respectively, also consider $\mathbb{L}_1, \mathbb{L}_2, \ldots \mathbb{L}_b$ for $b \geq 1$, b well defined attributes, whose corresponding attributive values are respectively the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \ldots \mathbb{L}_{\mathscr{b}}^z$ and their relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z$ where $a, b, c, \ldots z = 1,2, \ldots n$ then the pair $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z)$ is said to be Neutrosophic Hypersoft set over $\mathbb{U}$ where $\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z) \to \mathbb{P}(\mathbb{U})$ and it is define as

$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z) = \{< u, ([\mathbb{T}_\ell^L(u), \mathbb{T}_\ell^U(u)], [\mathbb{I}_\ell^L(u), \mathbb{I}_\ell^U(u)], [\mathbb{F}_\ell^L(u), \mathbb{F}_\ell^U(u)]) > u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z)\}$

Let $\mathbb{R}_\ell = (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z)$ be the relation and its characteristic function is $\mathcal{X}_{\mathbb{R}_\ell}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z) \to \mathbb{P}(\mathbb{U})$ and it is define as $\mathcal{X}_{\mathbb{R}_\ell} = \{< u, ([\mathbb{T}_\ell^L(u), \mathbb{T}_\ell^U(u)], [\mathbb{I}_\ell^L(u), \mathbb{I}_\ell^U(u)], [\mathbb{F}_\ell^L(u), \mathbb{F}_\ell^U(u)]) > u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z)\}$. The tabular representation of $\mathbb{R}_\ell$ is given as

	$\mathbb{L}_1^a$	$\mathbb{L}_1^b$	...	$\mathbb{L}_{\mathscr{b}}^z$
u^1	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_{\mathscr{b}}^z)$
u^2	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_{\mathscr{b}}^z)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u^a	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_{\mathscr{b}}^z)$

If $A_{ij} = \mathcal{X}_{\mathbb{R}_\ell}(u^i, \mathbb{L}_j^k)$, where $i = 1,2,3 \ldots a$, $j = 1,2,3, \ldots \mathscr{b}$, $k = a, b, c, \ldots z$, then a matrix is define as

$$[\bar{\bar{A}}_{ij}]_{a \times \mathscr{b}} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1\mathscr{b}} \\ A_{21} & A_{22} & \cdots & A_{2\mathscr{b}} \\ \vdots & \vdots & \ddots & \vdots \\ A_{a1} & A_{a2} & \cdots & A_{a\mathscr{b}} \end{pmatrix}$$

Where $\bar{\bar{A}}_{ij} = \left(\left[\mathbb{T}^L_{\mathbb{L}_j^k}(u_i), \mathbb{T}^U_{\mathbb{L}_j^k}(u_i)\right], \left[\mathbb{I}^L_{\mathbb{L}_j^k}(u_i), \mathbb{I}^U_{\mathbb{L}_j^k}(u_i)\right], \left[\mathbb{F}^L_{\mathbb{L}_j^k}(u_i), \mathbb{F}^U_{\mathbb{L}_j^k}(u_i)\right], u_i \in \mathbb{U}, \mathbb{L}_j^k \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathscr{b}}^z)\right) = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$

Thus, we can represent any Interval valued neutrosophic hypersoft set in term of Interval valued neutrosophic hypersoft matrix (IVNHSM), it means that they are interchangeable. Its generalized form is given as;

$$[\bar{\bar{A}}_{ij}]_{a \times \mathscr{b}} = \begin{bmatrix} \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_1^a}(u_1), \mathbb{T}^U_{\mathbb{L}_1^a}(u_1)], \\ [\mathbb{I}^L_{\mathbb{L}_1^a}(u_1), \mathbb{I}^U_{\mathbb{L}_1^a}(u_1)], \\ [\mathbb{F}^L_{\mathbb{L}_1^a}(u_1), \mathbb{F}^U_{\mathbb{L}_1^a}(u_1)] \end{pmatrix} & \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_2^b}(u_1), \mathbb{T}^U_{\mathbb{L}_2^b}(u_1)], \\ [\mathbb{I}^L_{\mathbb{L}_2^b}(u_1), \mathbb{I}^U_{\mathbb{L}_2^b}(u_1)], \\ [\mathbb{F}^L_{\mathbb{L}_2^b}(u_1), \mathbb{F}^U_{\mathbb{L}_2^b}(u_1)] \end{pmatrix} & \cdots & \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_1), \mathbb{T}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_1)], \\ [\mathbb{I}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_1), \mathbb{I}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_1)], \\ [\mathbb{F}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_1), \mathbb{F}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_1)] \end{pmatrix} \\ \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_1^a}(u_2), \mathbb{T}^U_{\mathbb{L}_1^a}(u_2)], \\ [\mathbb{I}^L_{\mathbb{L}_1^a}(u_2), \mathbb{I}^U_{\mathbb{L}_1^a}(u_2)], \\ [\mathbb{F}^L_{\mathbb{L}_1^a}(u_2), \mathbb{F}^U_{\mathbb{L}_1^a}(u_2)] \end{pmatrix} & \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_2^b}(u_2), \mathbb{T}^U_{\mathbb{L}_2^b}(u_2)], \\ [\mathbb{I}^L_{\mathbb{L}_2^b}(u_2), \mathbb{I}^U_{\mathbb{L}_2^b}(u_2)], \\ [\mathbb{F}^L_{\mathbb{L}_2^b}(u_2), \mathbb{F}^U_{\mathbb{L}_2^b}(u_2)] \end{pmatrix} & \cdots & \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_2), \mathbb{T}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_2)], \\ [\mathbb{I}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_2), \mathbb{I}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_2)], \\ [\mathbb{F}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_2), \mathbb{F}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_2)] \end{pmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_1^a}(u_a), \mathbb{T}^U_{\mathbb{L}_1^a}(u_a)], \\ [\mathbb{I}^L_{\mathbb{L}_1^a}(u_a), \mathbb{I}^U_{\mathbb{L}_1^a}(u_a)], \\ [\mathbb{F}^L_{\mathbb{L}_1^a}(u_a), \mathbb{F}^U_{\mathbb{L}_1^a}(u_a)] \end{pmatrix} & \begin{pmatrix} [\mathbb{T}^L_{\mathbb{L}_2^b}(u_a), \mathbb{T}^U_{\mathbb{L}_2^b}(u_a)], \\ [\mathbb{I}^L_{\mathbb{L}_2^b}(u_a), \mathbb{I}^U_{\mathbb{L}_2^b}(u_a)], \\ [\mathbb{F}^L_{\mathbb{L}_2^b}(u_a), \mathbb{F}^U_{\mathbb{L}_2^b}(u_a)] \end{pmatrix} & \cdots & \begin{pmatrix} [\mathbb{T}^L{}_{\mathbb{L}_{\mathscr{b}}^z}(u_a), \mathbb{T}^U{}_{\mathbb{L}_{\mathscr{b}}^z}(u_a)], \\ [\mathbb{I}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_a), \mathbb{I}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_a)], \\ [\mathbb{F}^L_{\mathbb{L}_{\mathscr{b}}^z}(u_a), \mathbb{F}^U_{\mathbb{L}_{\mathscr{b}}^z}(u_a)] \end{pmatrix} \end{bmatrix}$$

Definition 3.2: Row IVNHSM Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ be the IVIVNHSM of order $a \times \mathscr{b}$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ then A is said to be Row IVNHSM if $a = 1$. It means that if an IVNHSM contain only one attribute i.e. $\mathbb{U} = \{u^1\}$ then it is a Row IVIVNHSM.

Definition 3.3: Column IVNHSM Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ be the IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ then $\bar{\bar{A}}$ is said to be Column IVNHSM if $b = 1$. It means that if an IVNHSM contain only one alternative i.e. $\ell = \{\mathbb{L}^a_1\}$ then it is a Column IVNHSM.

Definition 3.4: Zero IVNHSM Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ be the IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ then $\bar{\bar{A}}$ is said to be Zero IVNHSM if $\bar{\bar{A}}_{ij} = ([0,0],[1,1],[1,1])$.

Definition 3.5: Universal IVNHSM Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ be the IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ then $\bar{\bar{A}}$ is said to be Universal IVNHSM if $\bar{\bar{A}}_{ij} = ([1,1],[0,0],[0,0])$.

Definition 3.6: Interval Valued Neutrosophic Hypersoft Submatrix (IVNHSSM)

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^B_{ijk}, \bar{\bar{\mathbb{I}}}^B_{ijk}, \bar{\bar{\mathbb{F}}}^B_{ijk})$ then$[\bar{\bar{A}}_{ij}]$is said to be IVNHSSM of $[\bar{\bar{B}}_{ij}]$ if $\left[{\mathbb{T}^{\mathrm{L}}}^A_{ijk} \leq {\mathbb{T}^{\mathrm{L}}}^B_{ijk}, {\mathbb{T}^{\mathrm{U}}}^A_{ijk} \leq {\mathbb{T}^{\mathrm{U}}}^B_{ijk}\right], \left[{\mathbb{I}^{\mathrm{L}}}^A_{ijk} \leq {\mathbb{I}^{\mathrm{L}}}^B_{ijk}, {\mathbb{I}^{\mathrm{U}}}^A_{ijk} \leq {\mathbb{I}^{\mathrm{U}}}^B_{ijk}\right], \left[{\mathbb{F}^{\mathrm{L}}}^A_{ijk} \geq {\mathbb{F}^{\mathrm{L}}}^B_{ijk},\ {\mathbb{F}^{\mathrm{U}}}^A_{ijk} \geq {\mathbb{F}^{\mathrm{U}}}^B_{ijk}\right]$.

Definition 3.7: Interval Valued Neutrosophic Hypersoft Equal Matrix (IVNHSEM)

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^B_{ijk}, \bar{\bar{\mathbb{I}}}^B_{ijk}, \bar{\bar{\mathbb{F}}}^B_{ijk})$ then$[\bar{\bar{A}}_{ij}]$is said to be IVNHSEM of $[\bar{\bar{B}}_{ij}]$ if $\left[{\mathbb{T}^{\mathrm{L}}}^A_{ijk} = {\mathbb{T}^{\mathrm{L}}}^B_{ijk}, {\mathbb{T}^{\mathrm{U}}}^A_{ijk} = {\mathbb{T}^{\mathrm{U}}}^B_{ijk}\right], \left[{\mathbb{I}^{\mathrm{L}}}^A_{ijk} = {\mathbb{I}^{\mathrm{L}}}^B_{ijk}, {\mathbb{I}^{\mathrm{U}}}^A_{ijk} = {\mathbb{I}^{\mathrm{U}}}^B_{ijk}\right], \left[{\mathbb{F}^{\mathrm{L}}}^A_{ijk} = {\mathbb{F}^{\mathrm{L}}}^B_{ijk},\ {\mathbb{F}^{\mathrm{U}}}^A_{ijk} = {\mathbb{F}^{\mathrm{U}}}^B_{ijk}\right]$.

Definition 3.8: AND Operation

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^B_{ijk}, \bar{\bar{\mathbb{I}}}^B_{ijk}, \bar{\bar{\mathbb{F}}}^B_{ijk})$ then$[\bar{\bar{A}}_{ij}] \wedge [\bar{\bar{B}}_{ij}]$ is given as

$$[\bar{\bar{A}}_{ij}] \wedge [\bar{\bar{B}}_{ij}] = \begin{pmatrix} \left[min\left({\mathbb{T}^{\mathrm{L}}}^A_{ijk}, {\mathbb{T}^{\mathrm{L}}}^B_{ijk}\right), \min\left({\mathbb{T}^{U}}^A_{ijk}, {\mathbb{T}^{U}}^B_{ijk}\right)\right], \\ \left[\dfrac{\left({\mathbb{I}^{\mathrm{L}}}^A_{ijk} . {\mathbb{I}^{\mathrm{L}}}^B_{ijk}\right)}{n}, \dfrac{\left({\mathbb{I}^{\mathrm{U}}}^A_{ijk} . {\mathbb{I}^{\mathrm{U}}}^B_{ijk}\right)}{n}\right], \\ \left[max\left({\mathbb{F}^{\mathrm{L}}}^A_{ijk}, {\mathbb{F}^{\mathrm{L}}}^B_{ijk}\right), \max\left({\mathbb{F}^{U}}^A_{ijk}, {\mathbb{F}^{U}}^B_{ijk}\right)\right] \end{pmatrix}$$

Definition 3.9: OR Operation

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^A_{ijk}, \bar{\bar{\mathbb{I}}}^A_{ijk}, \bar{\bar{\mathbb{F}}}^A_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^B_{ijk}, \bar{\bar{\mathbb{I}}}^B_{ijk}, \bar{\bar{\mathbb{F}}}^B_{ijk})$ then$[\bar{\bar{A}}_{ij}] \vee [\bar{\bar{B}}_{ij}]$ is given as

$$[\bar{\bar{A}}_{ij}] \vee [\bar{\bar{B}}_{ij}] = \begin{pmatrix} \left[max\left(\mathbb{T}^{\mathrm{L}A}_{ij\hbar}, \mathbb{T}^{\mathrm{L}B}_{ij\hbar}\right), \max\left(\mathbb{T}^{UA}_{ij\hbar}, \mathbb{T}^{UB}_{ij\hbar}\right)\right], \\ \left[\frac{\left(\mathbb{I}^{\mathrm{L}A}_{ij\hbar}.\mathbb{I}^{\mathrm{L}B}_{ij\hbar}\right)}{n}, \frac{\left(\mathbb{I}^{\mathrm{U}A}_{ij\hbar}.\mathbb{I}^{\mathrm{U}B}_{ij\hbar}\right)}{n}\right], \\ \left[min\left(\mathbb{F}^{\mathrm{L}A}_{ij\hbar}, \mathbb{F}^{\mathrm{L}B}_{ij\hbar}\right), \min\left(\mathbb{F}^{UA}_{ij\hbar}, \mathbb{F}^{UB}_{ij\hbar}\right)\right] \end{pmatrix}$$

Definition 3.10: Sum of IVNHSM

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{A}_{ij\hbar}, \bar{\bar{\mathbb{I}}}^{A}_{ij\hbar}, \bar{\bar{\mathbb{F}}}^{A}_{ij\hbar}\right)$ and $\bar{\bar{B}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{B}_{ij\hbar}, \bar{\bar{\mathbb{I}}}^{B}_{ij\hbar}, \bar{\bar{\mathbb{F}}}^{B}_{ij\hbar}\right)$ then$[\bar{\bar{A}}_{ij}] + [\bar{\bar{B}}_{ij}]$ is given as

$$\mathbb{T}\left(\left[[\bar{\bar{A}}_{ij}] + [\bar{\bar{B}}_{ij}]\right]\right) = \left(\frac{\mathbb{T}^{\mathrm{L}A}_{ij\hbar} + \mathbb{T}^{\mathrm{L}B}_{ij\hbar}}{n}, \frac{\mathbb{T}^{UA}_{ij\hbar}, \mathbb{T}^{UB}_{ij\hbar}}{n}\right)$$

$$\mathbb{I}\left(\left[[\bar{\bar{A}}_{ij}] + [\bar{\bar{B}}_{ij}]\right]\right) = \left(\frac{\left(\mathbb{I}^{\mathrm{L}A}_{ij\hbar} + \mathbb{I}^{\mathrm{L}B}_{ij\hbar}\right)}{n}, \frac{\left(\mathbb{I}^{\mathrm{U}A}_{ij\hbar} + \mathbb{I}^{\mathrm{U}B}_{ij\hbar}\right)}{n}\right)$$

$$\mathbb{F}\left(\left[[\bar{\bar{A}}_{ij}] + [\bar{\bar{B}}_{ij}]\right]\right) = \left(\frac{\mathbb{F}^{\mathrm{L}A}_{ij\hbar} + \mathbb{F}^{\mathrm{L}B}_{ij\hbar}}{n}, \frac{\mathbb{F}^{UA}_{ij\hbar}, \mathbb{F}^{UB}_{ij\hbar}}{n}\right)$$

4. Similarity Measures for Interval Valued Neutrosophic Hypersoft Matrix (IVNHSM)

In this section similarity measures for Interval Valued Neutrosophic Hypersoft Matrices (IVNHSM) and Generalized weighted Similarity measure for NHSS's are defined.

Proposition 4.1: Similarity Axioms

Let $\mathcal{A} = \mathcal{A}_i$, $\mathcal{B} = \mathcal{B}_i$ and $\mathcal{C} = \mathcal{C}_i$ be the three IVNHSs where $\mathcal{A}_i = \left([\mathbb{T}^{\mathcal{A}}_i, \mathbb{I}^{\mathcal{A}}_i, \mathbb{F}^{\mathcal{A}}_i\right)$, $\mathcal{B}_i = \left(\mathbb{T}^{\mathcal{B}}_i, \mathbb{I}^{\mathcal{B}}_i, \mathbb{F}^{\mathcal{B}}_i\right)$ and $\mathcal{C}_i = \left(\mathbb{T}^{\mathcal{C}}_i, \mathbb{I}^{\mathcal{C}}_i, \mathbb{F}^{\mathcal{C}}_i\right)$ and $i = \{1,2,3 \ldots \mathrm{n}\}$ then it satisfies the following axioms

1. $0 \le \mathcal{S}(\mathcal{A}, \mathcal{B}) \le 1$
2. $\mathcal{S}(\mathcal{A}, \mathcal{B}) = 1 \text{ if and only if } \mathcal{A} = \mathcal{B}$.
3. $\mathcal{S}(\mathcal{A}, \mathcal{B}) = \mathcal{S}(\mathcal{B}, \mathcal{A})$
4. If $A \subset B \subset \mathrm{C}$ then $\mathcal{S}(\mathcal{A}, \mathcal{C}) \le \mathcal{S}(\mathcal{A}, \mathcal{B})$ and $\mathcal{S}(\mathcal{A}, \mathcal{C}) \le \mathcal{S}(\mathcal{B}, \mathcal{C})$.

Definition 4.2: Generalized weighted similarity measure for IVNHSS.

Let $\mathcal{A} = \mathcal{A}_i$ and , $\mathcal{B} = \mathcal{B}_i$ be the two IVNHSs where $\mathcal{A}_i = \left([\mathbb{T}^{\mathcal{A}}_i, \mathbb{I}^{\mathcal{A}}_i, \mathbb{F}^{\mathcal{A}}_i\right)$ and $\mathcal{B}_i = \left(\mathbb{T}^{\mathcal{B}}_i, \mathbb{I}^{\mathcal{B}}_i, \mathbb{F}^{\mathcal{B}}_i\right)$, and $i = \{1,2,3 \ldots \mathrm{n}\}$ then the generalized weighted similarity measure is given as

$$\mathcal{S}_\lambda(\mathcal{A}, \mathcal{B}) = 1 - \left[\frac{1}{6n}\sum_i^n w_i \left(\left|\mathbb{T}^{A^L}_i - \mathbb{T}^{B^L}_i\right|^\lambda + \left|\mathbb{T}^{A^U}_i - \mathbb{T}^{B^U}_i\right|^\lambda + \left|\mathbb{I}^{A^L}_i - \mathbb{I}^{B^L}_i\right|^\lambda + \left|\mathbb{I}^{A^U}_i - \mathbb{I}^{B^U}_i\right|^\lambda + \left|\mathbb{F}^{A^L}_i - \mathbb{F}^{B^L}_i\right|^\lambda + \left|\mathbb{F}^{A^U}_i - \mathbb{F}^{B^U}_i\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$$

Where $\lambda > 0$.

Definition 4.3: Similarity measure for IVNHSM

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^{A}_{ijk}, \bar{\bar{\mathbb{I}}}^{A}_{ijk}, \bar{\bar{\mathbb{F}}}^{A}_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^{B}_{ijk}, \bar{\bar{\mathbb{I}}}^{B}_{ijk}, \bar{\bar{\mathbb{F}}}^{B}_{ijk})$ then the similarity measure between $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ is given as

$$\mathcal{S}(\bar{\bar{A}}, \bar{\bar{B}}) = 1 - \frac{1}{6ab}\sum_{i}^{a}\sum_{j}^{b}\left(\left|\mathbb{T}^{L}{}^{A}_{ijk} - \mathbb{T}^{L}{}^{B}_{ijk}\right| + \left|\mathbb{T}^{U}{}^{A}_{ijk} - \mathbb{T}^{U}{}^{B}_{ijk}\right| + \left|\mathbb{I}^{L}{}^{A}_{ijk} - \mathbb{I}^{L}{}^{B}_{ijk}\right| + \left|\mathbb{I}^{U}{}^{A}_{ijk} - \mathbb{I}^{U}{}^{B}_{ijk}\right| + \left|\mathbb{F}^{L}{}^{A}_{ijk} - \mathbb{F}^{L}{}^{B}_{ijk}\right| + \left|\mathbb{F}^{U}{}^{A}_{ijk} - \mathbb{F}^{U}{}^{B}_{ijk}\right|\right)$$

Example: Let $\bar{\bar{A}} =$
$[samsung\{[0.6, 0.7], [0.4,0.5], [0.5, 0.6]\} \quad 6\ GB\{[0.6, 0.7], [0.1, 0.2], [0.2,0.3]\} \quad Dual\{[0.7,0.8], [0.1,0.2], [0.05,0.1]\}]$
and $\bar{\bar{B}} =$
$[samsung\{[0.7,0.8], [0.05,0.1], [0.1,0.2]\} \quad 6\ GB\{[0.5,0.6], [0.05, 0.1], [0.1, 0.2]\} \quad Dual\{[0.2, 0.3], [0.5, 0.6], [0.3,0.4]\}]$
be the two IVNHSM then the similarity measure is given as;

$$\mathcal{S}(\bar{\bar{A}}, \bar{\bar{B}}) = 1 - \frac{1}{6(1)(3)}(|0.6 - 0.7| + |0.7 - 0.8| + |0.4 - 0.05| + |0.5 - 0.1| + |0.5 - 0.1| + |0.6 - 0.2| + |0.6 - 0.5| + |0.7 - 0.6| + |0.1 - 0.05| + |0.2 - 0.1| + |0.2 - 0.1| + |0.3 - 0.2| + |0.7 - 0.2| + |0.8 - 0.3| + |0.1 - 0.5| + |0.2 - 0.6| + |0.05 - 0.3| + |0.1 - 0.4|)$$

$$\mathcal{S}(\bar{\bar{A}}, \bar{\bar{B}}) = 0.7417$$

Definition 4.4: Generalized weighted similarity measure for IVNHSM

Let $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = (\bar{\bar{\mathbb{T}}}^{A}_{ijk}, \bar{\bar{\mathbb{I}}}^{A}_{ijk}, \bar{\bar{\mathbb{F}}}^{A}_{ijk})$ and $\bar{\bar{B}}_{ij} = (\bar{\bar{\mathbb{T}}}^{B}_{ijk}, \bar{\bar{\mathbb{I}}}^{B}_{ijk}, \bar{\bar{\mathbb{F}}}^{B}_{ijk})$ then the generalized weighted similarity measure between $\bar{\bar{A}} = [\bar{\bar{A}}_{ij}]$ and $\bar{\bar{B}} = [\bar{\bar{B}}_{ij}]$ is given as

$$\mathcal{S}_{\lambda}(\bar{\bar{A}}, \bar{\bar{B}}) = 1 - \left[\frac{1}{6ab}\sum_{i}^{a}\sum_{j}^{b} w_i\left(\left|\mathbb{T}^{L}{}^{A}_{ijk} - \mathbb{T}^{L}{}^{B}_{ijk}\right|^{\lambda} + \left|\mathbb{T}^{U}{}^{A}_{ijk} - \mathbb{T}^{U}{}^{B}_{ijk}\right|^{\lambda} + \left|\mathbb{I}^{L}{}^{A}_{ijk} - \mathbb{I}^{L}{}^{B}_{ijk}\right|^{\lambda} + \left|\mathbb{I}^{U}{}^{A}_{ijk} - \mathbb{I}^{U}{}^{B}_{ijk}\right|^{\lambda} + \left|\mathbb{F}^{L}{}^{A}_{ijk} - \mathbb{F}^{L}{}^{B}_{ijk}\right|^{\lambda} + \left|\mathbb{F}^{U}{}^{A}_{ijk} - \mathbb{F}^{U}{}^{B}_{ijk}\right|^{\lambda}\right)\right]^{\frac{1}{\lambda}}$$

5. Distance for Interval Valued Neutrosophic Hypersoft Matrix (IVNHSM)

In this section distance measures for Interval Valued Neutrosophic Hypersoft Matrices (IVNHSM) and Generalized weighted distance measure for NHSS's are defined.

Proposition 5.1: Distance Axioms

Let $\mathcal{A} = \mathcal{A}_i$, $\mathcal{B} = \mathcal{B}_i$ and $\mathcal{C} = \mathcal{C}_i$ be the three IVNHSs where $\mathcal{A}_i = \left([\mathbb{T}_i^{\mathcal{A}}, \mathbb{I}_i^{\mathcal{A}}, \mathbb{F}_i^{\mathcal{A}}\right)$, $\mathcal{B}_i = \left(\mathbb{T}_i^{\mathcal{B}}, \mathbb{I}_i^{\mathcal{B}}, \mathbb{F}_i^{\mathcal{B}}\right)$ and $\mathcal{C}_i = \left(\mathbb{T}_i^{\mathcal{C}}, \mathbb{I}_i^{\mathcal{C}}, \mathbb{F}_i^{\mathcal{C}}\right)$ and $i = \{1,2,3 \ldots \mathrm{n}\}$ then it satisfies the following axioms

1. $\mathcal{D}(\mathcal{A}, \mathcal{B}) \geq 0$
2. $\mathcal{D}(\mathcal{A}, \mathcal{B}) = \mathcal{D}(\mathcal{B}, \mathcal{A})$
3. $\mathcal{D}(\mathcal{A}, \mathcal{B}) = 0 \; iff \; \mathcal{A} = \mathcal{B}$
4. $\mathcal{D}(\mathcal{A}, \mathcal{B}) + \mathcal{D}(\mathcal{B}, \mathcal{C}) \geq \mathcal{D}(\mathcal{A}, \mathcal{C})$

Definition 5.2: Generalized weighted distance for IVNHSS

Let $\mathcal{A} = \mathcal{A}_i$ and , $\mathcal{B} = \mathcal{B}_i$ be the two IVNHSs where $\mathcal{A}_i = \left([\mathbb{T}_i^{\mathcal{A}}, \mathbb{I}_i^{\mathcal{A}}, \mathbb{F}_i^{\mathcal{A}}\right)$ and $\mathcal{B}_i = \left(\mathbb{T}_i^{\mathcal{B}}, \mathbb{I}_i^{\mathcal{B}}, \mathbb{F}_i^{\mathcal{B}}\right)$, and $i = \{1,2,3 \ldots \mathrm{n}\}$ then the generalized weighted distance is given as

$$\mathcal{D}_\lambda(\mathcal{A}, \mathcal{B}) = \left[\frac{1}{6n}\sum_i^n w_i\left(\left|\mathbb{T}_i^{A^L} - \mathbb{T}_i^{B^L}\right|^\lambda + \left|\mathbb{T}_i^{A^U} - \mathbb{T}_i^{B^U}\right|^\lambda + \left|\mathbb{I}_i^{A^L} - \mathbb{I}_i^{B^L}\right|^\lambda + \left|\mathbb{I}_i^{A^U} - \mathbb{I}_i^{B^U}\right|^\lambda + \left|\mathbb{F}_i^{A^L} - \mathbb{F}_i^{B^L}\right|^\lambda + \left|\mathbb{F}_i^{A^U} - \mathbb{F}_i^{B^U}\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$$

Where $\lambda > 0$.

Definition 5.3: Normalized hamming distance for IVNHSM

Let $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = \left(\bar{\bar{\mathbb{T}}}_{ijk}^A, \bar{\bar{\mathbb{I}}}_{ijk}^A, \bar{\bar{\mathbb{F}}}_{ijk}^A\right)$ and $\bar{\bar{B}}_{ij} = \left(\bar{\bar{\mathbb{T}}}_{ijk}^B, \bar{\bar{\mathbb{I}}}_{ijk}^B, \bar{\bar{\mathbb{F}}}_{ijk}^B\right)$ then the normalized hamming distance between $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ is given as

$$\mathcal{D}\left(\bar{\bar{A}}, \bar{\bar{B}}\right) = \frac{1}{6ab}\sum_i^a\sum_j^b\left(\left|\mathbb{T}^{L}{}_{ijk}^{A} - \mathbb{T}^{L}{}_{ijk}^{B}\right| + \left|\mathbb{T}^{U}{}_{ijk}^{A} - \mathbb{T}^{U}{}_{ijk}^{B}\right| + \left|\mathbb{I}^{L}{}_{ijk}^{A} - \mathbb{I}^{L}{}_{ijk}^{B}\right| + \left|\mathbb{I}^{U}{}_{ijk}^{A} - \mathbb{I}^{U}{}_{ijk}^{B}\right| + \left|\mathbb{F}^{L}{}_{ijk}^{A} - \mathbb{F}^{L}{}_{ijk}^{B}\right| + \left|\mathbb{F}^{U}{}_{ijk}^{A} - \mathbb{F}^{U}{}_{ijk}^{B}\right|\right)$$

Example: Let $\bar{\bar{A}} =$
$[samsung\{[0.6, 0.7], [0.4,0.5], [0.5, 0.6]\} \quad 6\,GB\{[0.6, 0.7], [0.1, 0.2], [0.2,0.3]\} \quad Dual\{[0.7,0.8], [0.1,0.2], [0.05,0.1]\}]$
and $\bar{\bar{B}} =$
$[samsung\{[0.7,0.8], [0.05,0.1], [0.1,0.2]\} \quad 6\,GB\{[0.5,0.6], [0.05, 0.1], [0.1, 0.2]\} \quad Dual\{[0.2,0.3], [0.5, 0.6], [0.3,0.4]\}]$
be the two IVNHSM then the normalized hamming distance is given as

$$\mathcal{D}\left(\bar{\bar{A}}, \bar{\bar{B}}\right) = \frac{1}{6(1)(3)}\left(|0.6 - 0.7| + |0.7 - 0.8| + |0.4 - 0.05| + |0.5 - 0.1| + |0.5 - 0.1| + |0.6 - 0.2| + |0.6 - 0.5| + |0.7 - 0.6| + |0.1 - 0.05| + |0.2 - 0.1| + |0.2 - 0.1| + |0.3 - 0.2| + |0.7 - 0.2| + |0.8 - 0.3| + |0.1 - 0.5| + |0.2 - 0.6| + |0.05 - 0.3| + |0.1 - 0.4|\right)$$

$$\mathcal{D}\left(\bar{\bar{A}},\bar{\bar{B}}\right) = 0.2583$$

Definition 5.4: Normalized Euclidean distance for IVNHSM

Let $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{A}_{ijk}, \bar{\bar{\mathbb{I}}}^{A}_{ijk}, \bar{\bar{\mathbb{F}}}^{A}_{ijk}\right)$ and $\bar{\bar{B}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{B}_{ijk}, \bar{\bar{\mathbb{I}}}^{B}_{ijk}, \bar{\bar{\mathbb{F}}}^{B}_{ijk}\right)$ then the normalized Euclidean distance between $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ is given as

$$\mathcal{D}(\bar{\bar{A}},\bar{\bar{B}}) = \sqrt{\frac{\sum_{i}^{a}\sum_{j}^{b}\left(\left|\mathbb{T}^{LA}_{ijk} - \mathbb{T}^{LB}_{ijk}\right|^{2} + \left|\mathbb{T}^{UA}_{ijk} - \mathbb{T}^{UB}_{ijk}\right|^{2} + \left|\mathbb{I}^{LA}_{ijk} - \mathbb{I}^{LB}_{ijk}\right|^{2} + \left|\mathbb{I}^{UA}_{ijk} - \mathbb{I}^{UB}_{ijk}\right|^{2} + \left|\mathbb{F}^{LA}_{ijk} - \mathbb{F}^{LB}_{ijk}\right|^{2} + \left|\mathbb{F}^{UA}_{ijk} - \mathbb{F}^{UB}_{ijk}\right|^{2}\right)}{6ab}}$$

Example: Let $\bar{\bar{A}} =$

$[samsung\{[0.6, 0.7], [0.4,0.5], [0.5, 0.6]\} \quad 6\ GB\{[0.6, 0.7], [0.1, 0.2], [0.2,0.3]\} \quad Dual\{[0.7,0.8], [0.1,0.2], [0.05,0.1]\}]$

and $\bar{\bar{B}} =$

$[samsung\{[0.7,0.8], [0.05,0.1], [0.1,0.2]\} \quad 6\ GB\{[0.5,0.6], [0.05, 0.1], [0.1, 0.2]\} \quad Dual\{[0.2,0.3], [0.5, 0.6], [0.3,0.4]\}]$

be the two IVNHSM then the normalized Euclidean distance is given as;

𝒟(A ̅, B ̅) =√ (((|0.6-0.7|^2+|0.7-0.8|^2+|0.4-0.05|^2+|0.5-0.1|^2+|0.5-0.1|^2+|0.6-0.2|^2+| 0.6-0.5|^2+|0.7-0.6|^2+|0.1-0.05|^2+|0.2-0.1|^2+|0.2-0.1|^2+|0.3-0.2|^2+|0.7-0.2|^2+|0.8-0.3|^2+|0.1-0.5|^2+|0.2-0.6|^2+|0.05-0.3|^2+|0.1-0.4|^2))/(6(1)(3)))

$$\mathcal{D}\left(\bar{\bar{A}},\bar{\bar{B}}\right) = 0.3025$$

Definition 5.5: Generalized weighted distance for IVNHSM

Let $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ be the two IVNHSM of order $a \times b$, where $\bar{\bar{A}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{A}_{ijk}, \bar{\bar{\mathbb{I}}}^{A}_{ijk}, \bar{\bar{\mathbb{F}}}^{A}_{ijk}\right)$ and $\bar{\bar{B}}_{ij} = \left(\bar{\bar{\mathbb{T}}}^{B}_{ijk}, \bar{\bar{\mathbb{I}}}^{B}_{ijk}, \bar{\bar{\mathbb{F}}}^{B}_{ijk}\right)$ then the generalized weighted distance between $\bar{\bar{A}} = \left[\bar{\bar{A}}_{ij}\right]$ and $\bar{\bar{B}} = \left[\bar{\bar{B}}_{ij}\right]$ is given as

$$\mathcal{D}_{\lambda}\left(\bar{\bar{A}},\bar{\bar{B}}\right) = \left[\frac{1}{6ab}\sum_{i}^{a}\sum_{j}^{b} w_i\left(\left|\mathbb{T}^{LA}_{ijk} - \mathbb{T}^{LB}_{ijk}\right|^{\lambda} + \left|\mathbb{T}^{UA}_{ijk} - \mathbb{T}^{UB}_{ijk}\right|^{\lambda} + \left|\mathbb{I}^{LA}_{ijk} - \mathbb{I}^{LB}_{ijk}\right|^{\lambda} + \left|\mathbb{I}^{UA}_{ijk} - \mathbb{I}^{UB}_{ijk}\right|^{\lambda} + \left|\mathbb{F}^{LA}_{ijk} - \mathbb{F}^{LB}_{ijk}\right|^{\lambda} + \left|\mathbb{F}^{UA}_{ijk} - \mathbb{F}^{UB}_{ijk}\right|^{\lambda}\right)\right]^{\frac{1}{\lambda}}$$

3. TOPSIS for Multi Attribute Group Decision Making (MAGDM)

TOPSIS (The Technique for Order Preference by Similarly to Ideal Solution) is the suitable approach to deal with Multi Attribute Group Decision Making problems. TOPSIS technique (Shown in Figure:2) is consist of following steps.

1. Compose a decision matrix.
2. Normalizing decision matrix.
3. Determine the weighted normalized decision matrix.
4. Calculate the positive and negative ideal solution.
5. Calculate the distance of each alternative to the positive and negative ideal solution.
6. Calculate the relative closeness coefficients.
7. Rank the alternatives.

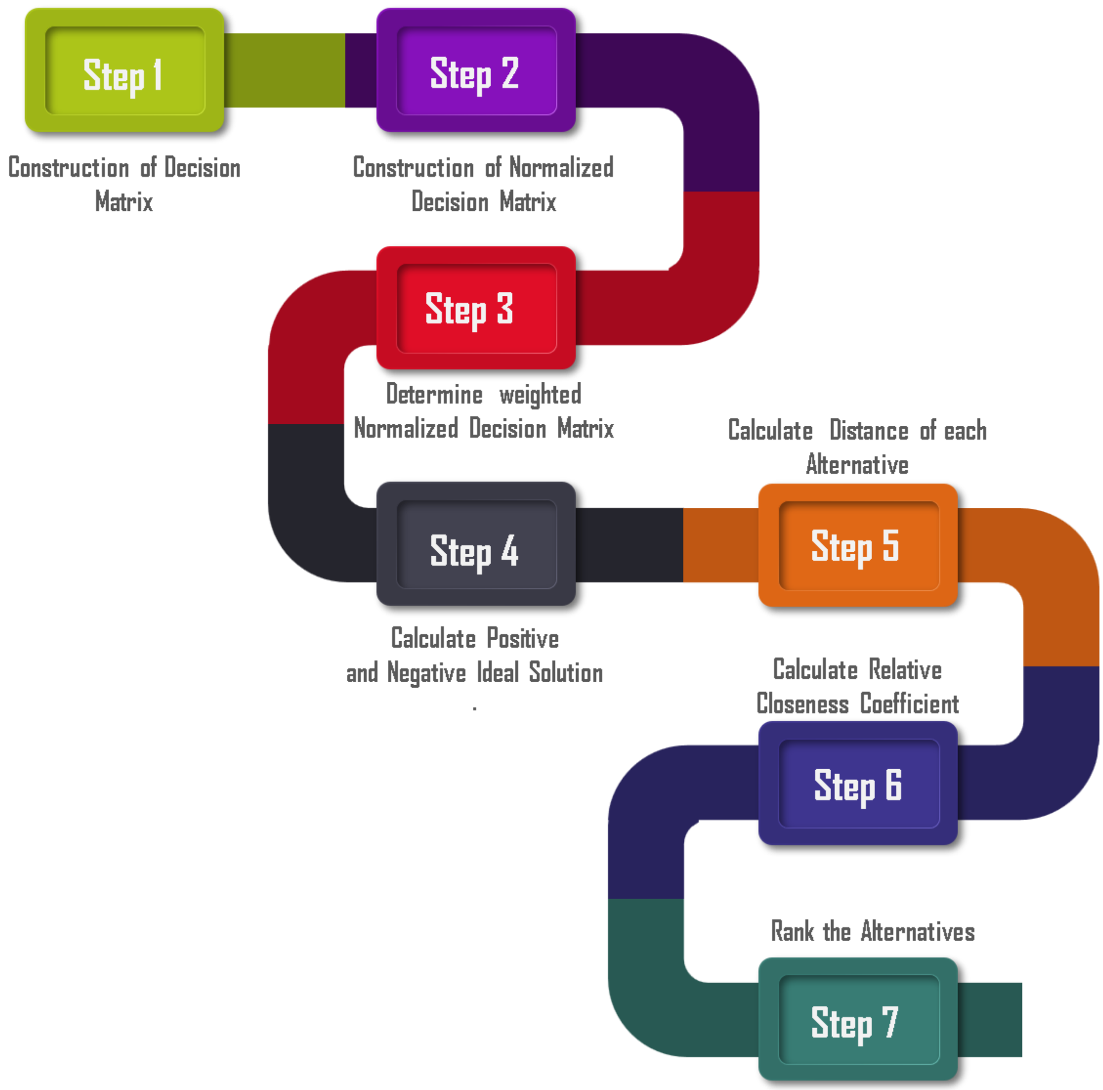

Figure: 2 Proposed TOPSIS Algorithm for IVNHSS environment

4. TOPSIS Technique for MAGDM With Interval Valued Neutrosophic Hypersoft Set

Consider a Multi Attribute Group Decision Making (MAGDM) problem based on interval valued neutrosophic hypersoft set in which $\mathbb{U} = \{u^1, u^2, \dots u^a\}$ be the set of alternatives and $\mathbb{L}_1, \mathbb{L}_2, \dots \mathbb{L}_{\ell}$ be the sets of attributes and their corresponding attributive values are respectively the set

$\mathbb{L}_1^a, \mathbb{L}_2^b, \dots \mathbb{L}_{\ell}^z$ where $a, b, c, \dots z = 1,2, \dots n$. Let $\mathbb{w}^j$ be the weight of attributes $\mathbb{L}_j^z, j = 1,2 \dots \ell$, where $0 \leq \mathbb{w}^j \leq 1$ and $\sum_{j=1}^{\ell} \mathbb{w}^j = 1$. Suppose that $\mathbb{D} = (\mathbb{D}_1, \mathbb{D}_2, \dots \mathbb{D}_t)$ be the set of t decision makers and Δ^x be the weight of t decision makers with $0 \leq \Delta^x \leq 1$ and $\sum_{x=1}^{t} \Delta^x = 1$.

Let $[\bar{\bar{A}}_{ij}^x]_{a \times \ell}$ be the decision matrix where $\bar{\bar{A}}_{ij}^x = (\bar{\bar{\mathbb{T}}}_{ijk}^x, \bar{\bar{\mathbb{I}}}_{ijk}^x, \bar{\bar{\mathbb{F}}}_{ijk}^x) = \left((\mathbb{T}_{ijk}^{x\mathbb{L}}, \mathbb{T}_{ijk}^{x\mathbb{U}}), (\mathbb{I}_{ijk}^{x\mathbb{L}}, \mathbb{I}_{ijk}^{x\mathbb{U}}), (\mathbb{F}_{ijk}^{x\mathbb{L}}, \mathbb{F}_{ijk}^{x\mathbb{U}})\right)$, $i = 1,2,3 \dots a$, $j = 1,2,3, \dots \ell$, $k = a, b, c, \dots z$ and $\bar{\bar{\mathbb{T}}}_{ijk}^x = [\mathbb{T}_{ijk}^{x\mathbb{L}}, \mathbb{T}_{ijk}^{x\mathbb{U}}], \bar{\bar{\mathbb{I}}}_{ijk}^x = [\mathbb{I}_{ijk}^{x\mathbb{L}}, \mathbb{I}_{ijk}^{x\mathbb{U}}], \bar{\bar{\mathbb{F}}}_{ijk}^x = [\mathbb{F}_{ijk}^{x\mathbb{L}}, \mathbb{F}_{ijk}^{x\mathbb{U}}] \in [0,1]$, $0 \leq sup\, \bar{\bar{\mathbb{T}}}_{ijk}^x + sup\, \bar{\bar{\mathbb{I}}}_{ijk}^x + sup\, \bar{\bar{\mathbb{F}}}_{ijk}^x \leq 3$.

Utilizing the following steps, the determination strategy for the selection of alternatives is given as follow:

Step 1: Determine the Weight of Decision Makers

Let $[\bar{\bar{A}}_{ij}^x]_{a \times \ell}$ be the decision matrix where

$$[\bar{\bar{A}}_{ij}^x]_{a \times \ell} = \begin{bmatrix} \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^x(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^x(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^x(u_1) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^x(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^x(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^x(u_1) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^x(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^x(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^x(u_1) \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^x(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^x(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^x(u_2) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^x(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^x(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^x(u_2) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^x(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^x(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^x(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^x(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^x(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^x(u_a) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^x(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^x(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^x(u_a) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^x(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^x(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^x(u_a) \end{bmatrix}$$

To find the ideal matrix we will average all the individual decision matrix $\bar{\bar{A}}_{ij}^x$ *where* $x = 1,2 \dots t$ as

$$[\bar{\bar{A}}_{ij}^{\star`}]_{a \times \ell} = \begin{bmatrix} \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^\star(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^\star(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^\star(u_1) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^\star(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^\star(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^\star(u_1) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^\star(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^\star(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^\star(u_1) \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^\star(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^\star(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^\star(u_2) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^\star(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^\star(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^\star(u_2) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^\star(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^\star(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^\star(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}^\star(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}^\star(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}^\star(u_a) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}^\star(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}^\star(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}^\star(u_a) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_{\ell}^z}^\star(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_{\ell}^z}^\star(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_{\ell}^z}^\star(u_a) \end{bmatrix}$$

where

$$\bar{\bar{A}}_{ij}^\star = \left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j^k}^\star(u_i), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_j^k}^\star(u_i), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_j^k}^\star(u_i)\right)$$

$$= \left(\begin{matrix} \left[1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\frac{1}{t}}, 1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\frac{1}{t}}\right], \\ \left[\prod_{x=1}^{t}\left(\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\frac{1}{t}}\right], \left[\prod_{x=1}^{t}\left(\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\frac{1}{t}}\right] \end{matrix} \right)$$

$for\ i = 1,2,3 \dots a$, $j = 1,2,3, \dots \ell$, $k = a, b, c, \dots z$ and $a, b, c, \dots z = 1,2, \dots n$.

To determine the weights of the decision makers, first we will find the similarity measure between each decision matrix and the ideal matrix as

$$\mathbb{S}(\bar{\bar{A}}_{ij}^x, \bar{\bar{A}}_{ij}^\star) = 1 - \frac{1}{6a\ell}\sum_{i}^{a}\sum_{j}^{\ell}\left(\left|\mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i) - \mathbb{T}_{\mathbb{L}_j^k}^{\star\mathbb{L}}(u_i)\right| + \left|\mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i) - \mathbb{T}_{\mathbb{L}_j^k}^{\star\mathbb{U}}(u_i)\right| + \left|\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i) - \mathbb{I}_{\mathbb{L}_j^k}^{\star\mathbb{L}}(u_i)\right| \right.$$

$$\left. + \left|\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i) - \mathbb{I}_{\mathbb{L}_j^k}^{\star\mathbb{U}}(u_i)\right| + \left|\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i) - \mathbb{F}_{\mathbb{L}_j^k}^{\star\mathbb{L}}(u_i)\right| + \left|\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i) - \mathbb{F}_{\mathbb{L}_j^k}^{\star\mathbb{U}}(u_i)\right|\right)$$

Now we calculate the weight $\Delta^x (x = 1,2, \dots t)$ of t decision makers using above equation

$$\Delta^x = \frac{\mathbb{S}(\bar{\bar{A}}_{ij}^x, \bar{\bar{A}}_{ij}^\star)}{\sum_{x=1}^{t}\mathbb{S}(\bar{\bar{A}}_{ij}^x, \bar{\bar{A}}_{ij}^\star)} \text{ Where } 0 \leq \Delta^x \leq 1 \text{ and } \sum_{x=1}^{t}\Delta^x = 1.$$

Step 2: Aggregate Neutrosophic Hypersoft Decision Matrices

By accumulating all the individual decision matrices, we construct an aggregated neutrosophic hypersoft decision matrix to obtain one group decision. Aggregated neutrosophic hypersoft decision matrix is denoted as $\bar{\bar{A}}_{ij}$ and it is given as

$$[\bar{\bar{A}}_{ij}]_{a\times b}=\begin{bmatrix} \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_1),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_1),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_1) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_1),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_1),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_1) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_1),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_1),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_1) \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_2),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_2),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_2) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_2),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_2),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_2) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_2),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_2),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_a),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_a),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_a) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_a),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_a),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_a) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_a),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_a),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_a) \end{bmatrix}$$

The elements of $\bar{\bar{A}}_{ij}$in the matrix $[\bar{\bar{A}}_{ij}]_{a\times b}$ is calculated as

$$[\bar{\bar{A}}_{ij}]_{a\times b} = \left(\left[1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}, 1 - \prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\Delta^x}\right]\right)$$

Where $i = 1,2,3 \ldots a\,, j = 1,2,3, \ldots b$ *and* $x = 1,2, \ldots t$

Step 3: Determine the weight of attributes

In decision making procedure, decision makers may perceive that all attribute are not similarly significant. In this manner, each decision maker may have their own one opinion regarding attribute weights. To acquire the gathering assessment of the picked attributes, all the decision makers' opinions for the importance of each attribute need to be aggregated.

For this purpose, weight $\bar{\bar{\mathbb{w}}}^j$ of attributes $\mathbb{L}_j^z, j = 1,2 \ldots b$is calculated as

$$\bar{\bar{\mathbb{w}}}^j = \left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j}, \bar{\bar{\mathbb{I}}}_{\mathbb{L}_j}, \bar{\bar{\mathbb{F}}}_{\mathbb{L}_j}\right)$$

$$= \left(\left[1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x}, 1 - \prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right]\right)$$

Step 4: Calculate the weighted aggregated decision matrix

After finding the weight of individual attributes, we apply these weights to each row of aggregated decision matrix (step 2) as

$$\left[\bar{\bar{A}}_{ij}^{\omega}\right]_{a\times b}=\left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j^{k}}^{\omega}(u_i),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_j^{k}}^{\omega}(u_i),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right)$$

$$=\left(\left[\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j},\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j}\right],\left[\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}(u_i)+\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j}-\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j},\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}(u_i)\right.\right.$$

$$\left.+\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j}-\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j}\right],\left[\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}(u_i)+\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j}-\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j},\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}(u_i)+\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j}\right.$$

$$\left.\left.-\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}(u_i).\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j}\right]\right)$$

And then we get a weighted aggregated decision matrix.

Step 5: Determine the ideal solution

In real life we deal with two type of attributes, one is benefit type attributes and other is cost type attribute.

In our MAGDM problem we also deal with these two types of attributes. Let $\mathbb{C}_1$ be the benefit type attributes and $\mathbb{C}_2$ be the cost type attributes.

Neutrosophic hypersoft positive ideal solution is given as

$$\bar{\bar{A}}_j^{\omega+}=\left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right)$$

$$=\begin{cases}\left[\overset{i}{\max}\left\{\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\min}\left\{\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\min}\left\{\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],j\in\mathbb{C}_1\\\left[\overset{i}{\min}\left\{\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\max}\left\{\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\max}\left\{\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],j\in\mathbb{C}_2\end{cases}$$

Similarly, neutrosophic hypersoft negative ideal solution is given as

$$\bar{\bar{A}}_j^{\omega-}=\left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i),\bar{\bar{\mathbb{I}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i),\bar{\bar{\mathbb{F}}}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right)$$

$$=\begin{cases}\left[\overset{i}{\min}\left\{\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\max}\left\{\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\max}\left\{\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],j\in\mathbb{C}_1\\\left[\overset{i}{\max}\left\{\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\min}\left\{\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],\left[\overset{i}{\min}\left\{\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)\right\}\right],j\in\mathbb{C}_2\end{cases}$$

Step 6: Calculate the distance measure

Now we will find the normalized hamming distance between the alternatives and positive ideal solution as;

$$\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega},\bar{\bar{A}}_j^{\omega+}\right)=\frac{1}{6b}\sum_{j=1}^{b}\left(\left|\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|+\left|\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|+\left|\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\right.\right.$$

$$\left.\left.\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|+\left|\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|+\left|\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|+\left|\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{+}(u_i)\right|\right)$$

Similarly, we will find the normalized hamming distance between the alternatives and positive ideal solution as; $\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega},\bar{\bar{A}}_j^{\omega-}\right)=\frac{1}{6b}\sum_{j=1}^{b}\left(\left|\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|+\left|\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|+\right.$

$$\left.\left|\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|+\left|\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|+\left|\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|+\left|\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}(u_i)-\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j^{k}}^{\omega}{}^{-}(u_i)\right|\right)$$

Step 7: Calculate the relative closeness co-efficient

Relative closeness index is used to rank the alternatives and it is calculated as

$$\mathbb{RC}^{i} = \frac{\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^{-}}\right)}{\max\limits_{i}\left\{\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^{-}}\right)\right\}} - \frac{\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^{+}}\right)}{\min\limits_{i}\left\{\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^{+}}\right)\right\}}$$

Where $\imath = 1,2,3 \ldots \alpha$.

The set of selected alternatives are ranked according to the descending order of relative closeness index.

5. Case study

Marriage is one of the most significant social organizations and today it is similarly as important as it was many years ago. With the time, the standards of adoration, steadfastness and responsibility has changes, which are the crucial segments of a solid society. We know that marriage brings steadiness and it ties us together. It helps make our families more grounded. A significant part of the quality of marriage lies in its capacity to change with the occasions. As society has changed, so marriage has changed, and become accessible to an undeniably expansive scope of people. This coordinate creation process has been messing up best choices.

Objective:

The primary objective of this case study is to provide excellent matchmaking experience by exploring the opportunities and resources to meet true potential partner. Keeping the objective in mind, we can select the best proposal to fulfill the current demand in marriage for eligible bachelors. For this purpose, let $\mathbb{R} = \{\mathbb{R}^1, \mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^4, \mathbb{R}^5, \mathbb{R}^6, \mathbb{R}^7, \mathbb{R}^8\}$ be the set of different bachelors for Marriage Proposal.

Following are the attributes for respective bachelors.

$$\mathbb{A}_1 = Fraternity$$

$$\mathbb{A}_2 = Occupation$$

$$\mathbb{A}_3 = Nature$$

$$\mathbb{A}_4 = Family\ Size$$

These attributes are further characterized as

$$\mathbb{A}_1^a = Fraternity = \{suni, shia, deobandi, bralevi\}$$

Where $a = 1,2,3,4$.

$$\mathbb{A}_2^b = Occupation = \{doctor, engineer, businesman, bankers, merchant\}$$

Where $b = 1,2,3,4,5$.

$$\mathbb{A}_3^c = Nature = \{conservative, broadminded, respecting, supportive\}$$

Where $c = 1,2,3,4$.

$$\mathbb{A}_4^d = Family\ Size = \{\ joint\ family\ system, nuclear\ family\ system\}$$

Where $d = 1,2$.

Let's assume the relation for the function $\mathcal{F}: \mathbb{A}_1^a \times \mathbb{A}_2^b \times \mathbb{A}_3^c \times \mathbb{A}_4^d \to P(\mathbb{S})$ as $\mathcal{F}(\mathbb{A}_1^a \times \mathbb{A}_2^b \times \mathbb{A}_3^c \times \mathbb{A}_4^d) = (\mathbb{A}_1^2, \mathbb{A}_2^2, \mathbb{A}_3^1, \mathbb{A}_4^2) =$

$\left(\ Suni(S), Businesman\ (BM), Broadminded\ (B), Nuclear\ family\ system(NFS)\right)$

is the actual requirement of a family for marriage proposal.

Four proposals $\{\mathbb{R}^2, \mathbb{R}^4, \mathbb{R}^5, \mathbb{R}^6\}$ are selected on the basis of assumed relation i.e.$\big(Suni(S), Businesman\ (BM), Broadminded\ (B), Nuclear\ family\ system(NFS)\big)$.

Four decision makers from the family members $\{\mathbb{D}^1, \mathbb{D}^2, \mathbb{D}^3, \mathbb{D}^4\}$ are intended to select the most suitable proposal. These decision makers give their valuable opinion in the form of IVNHSM separately as.

$$[\mathbb{D}^1]_{4\times 4} = \begin{bmatrix} \left(S,\begin{pmatrix}{[0.8, 0.9]},\\ {[0.1, 0.2]},\\ {[0.2, 0.1]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.2, 0.3]},\\ {[0.2, 0.3]},\\ {[0.6, 0.7]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.5,0.6]},\\ {[0.3,0.4]},\\ {[0.1,0.2]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.6,0.7]},\\ {[0.05,0.1]},\\ {[0.2,0.3]}\end{pmatrix}\right) \\ \begin{pmatrix}S,({[0.7, 0.8]},\\ {[0.2, 0.3]},\\ {[0.1, 0.2]})\end{pmatrix} & \left(BM,\begin{pmatrix}{[0.5, 0.6]},\\ {[0.1, 0.2]},\\ {[0.5, 0.6]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.7,0.8]},\\ {[0.2,0.3]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.1,0.2]},\\ {[0.5,0.6]},\\ {[0.7,0.8]}\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}{[0.5, 0.6]},\\ {[0.05, 0.1]},\\ {[0.2, 0.3]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.7, 0.8]},\\ {[0.05, 0.1]},\\ {[0.1, 0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.7,0.8]},\\ {[0.1,0.2]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.5,0.6]},\\ {[0.2,0.3]},\\ {[0.3,0.4]}\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}{[0.8, 0.9]},\\ {[0.05, 0.1]},\\ {[0.05, 0.1]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8, 0.9]},\\ {[0.05, 0.1]},\\ {[0.05, 0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.7,0.8]},\\ {[0.05,0.1]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.8,0.9]},\\ {[0.05,0.1]},\\ {[0.1,0.2]}\end{pmatrix}\right) \end{bmatrix}$$

$$[\mathbb{D}^2]_{4\times 4} = \begin{bmatrix} \left(S,\begin{pmatrix}{[0.2,0.3]},\\ {[0.2,0.3]},\\ {[0.6,0.7]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8,0.9]},\\ {[0.1,0.2]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.5,0.6]},\\ {[0.05,0.1]},\\ {[0.2,0.3]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.2,0.3]},\\ {[0.5,0.6]},\\ {[0.1,0.2]}\end{pmatrix}\right) \\ \begin{pmatrix}S,({[0.7,0.8]},\\ {[0.1,0.2]},\\ {[0.05,0.1]})\end{pmatrix} & \left(BM,\begin{pmatrix}{[0.7,0.8]},\\ {[0.2,0.3]},\\ {[0.1,0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.8,0.9]},\\ {[0.05,0.1]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.7,0.8]},\\ {[0.2,0.3]},\\ {[0.05,0.1]}\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}{[0.5,0.6]},\\ {[0.2,0.3]},\\ {[0.3,0.4]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.7,0.8]},\\ {[0.05,0.1]},\\ {[0.1,0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.8,0.9]},\\ {[0.05,0.1]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.1,0.2]},\\ {[0.2,0.3]},\\ {[0.7,0.8]}\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}{[0.8,0.9]},\\ {[0.05,0.1]},\\ {[0.1,0.2]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.7,0.8]},\\ {[0.05,0.1]},\\ {[0.05,0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.6,0.7]},\\ {[0.05,0.1]},\\ {[0.2,0.3]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.5,0.6]},\\ {[0.2,0.3]},\\ {[0.3,0.4]}\end{pmatrix}\right) \end{bmatrix}$$

$$[\mathbb{D}^3]_{4\times 4}=\begin{bmatrix}
\left(S,\begin{pmatrix}{[0.5,0.6]},\\{[0.2,0.3]},\\{[0.3,0.4]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.1,0.2]},\\{[0.2,0.3]},\\{[0.7,0.8]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.2,0.3]},\\{[0.5,0.6]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.2,0.3]},\\{[0.5,0.6]},\\{[0.1,0.2]}\end{pmatrix}\right)\\
\left(S,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.7,0.8]},\\{[0.2,0.3]},\\{[0.05,0.1]}\end{pmatrix}\right)\\
\left(S,\begin{pmatrix}{[0.7,0.8]},\\{[0.2,0.3]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8,0.9]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.1,0.2]},\\{[0.2,0.3]},\\{[0.7,0.8]}\end{pmatrix}\right)\\
\left(S,\begin{pmatrix}{[0.2,0.3]},\\{[0.2,0.3]},\\{[0.6,0.7]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.6,0.7]},\\{[0.05,0.1]},\\{[0.2,0.3]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.5,0.6]},\\{[0.2,0.3]},\\{[0.3,0.4]}\end{pmatrix}\right)
\end{bmatrix}$$

$$[\mathbb{D}^4]_{4\times 4}=\begin{bmatrix}
\left(S,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.7,0.8]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.2,0.3]},\\{[0.5,0.6]},\\{[0.1,0.2]}\end{pmatrix}\right)\\
\begin{pmatrix}S,({[0.7,0.8]},\\{[0.1,0.2]},\\{[0.05,0.1]})\end{pmatrix} & \left(BM,\begin{pmatrix}{[0.7,0.8]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.5,0.6]},\\{[0.2,0.3]},\\{[0.3,0.4]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.7,0.8]},\\{[0.2,0.3]},\\{[0.1,0.2]}\end{pmatrix}\right)\\
\left(S,\begin{pmatrix}{[0.7,0.8]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.7,0.8]},\\{[0.05,0.1]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.2,0.3]},\\{[0.5,0.6]},\\{[0.1,0.2]}\end{pmatrix}\right)\\
\left(S,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(BM,\begin{pmatrix}{[0.2,0.3]},\\{[0.2,0.3]},\\{[0.6,0.7]}\end{pmatrix}\right) & \left(B,\begin{pmatrix}{[0.7,0.8]},\\{[0.2,0.3]},\\{[0.1,0.2]}\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)
\end{bmatrix}$$

Importance of selected attributes by each decision maker is given as

$$\mathbb{D}^1\longrightarrow\left(S,\begin{pmatrix}{[0.8,0.9]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}{[0.6,0.7]},\\{[0.1,0.2]},\\{[0.1,0.2]}\end{pmatrix}\right)\ \left(B,\begin{pmatrix}{[0.7,0.8]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}{[0.5,0.6]},\\{[0.2,0.3]},\\{[0.4,0.5]}\end{pmatrix}\right)$$

$$\mathbb{D}^2\longrightarrow\left(S,\begin{pmatrix}{[0.7,0.8]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}{[0.5,0.6]},\\{[0.2,0.3]},\\{[0.3,0.4]}\end{pmatrix}\right)\ \left(B,\begin{pmatrix}{[0.4,0.5]},\\{[0.1,0.2]},\\{[0.3,0.4]}\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}{[0.6,0.7]},\\{[0.1,0.2]},\\{[0.1,0.2]}\end{pmatrix}\right)$$

$$\mathbb{D}^3\longrightarrow\left(S,\begin{pmatrix}{[0.4,0.5]},\\{[0.1,0.2]},\\{[0.3,0.4]}\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}{[0.7,0.8]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(B,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.1,0.2]}\end{pmatrix}\right)$$

$$\mathbb{D}^4\longrightarrow\left(S,\begin{pmatrix}{[0.8,0.9]},\\{[0.05,0.1]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}{[0.8,0.9]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(B,\begin{pmatrix}{[0.8,0.9]},\\{[0.1,0.2]},\\{[0.05,0.1]}\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}{[0.6,0.7]},\\{[0.05,0.1]},\\{[0.2,0.3]}\end{pmatrix}\right)$$

Step 1: Determine the Weight of Decision Makers

To find the ideal matrix we will average all the individual decision matrix $\mathbb{D}^1,\mathbb{D}^2,\mathbb{D}^3,\mathbb{D}^4$using

$$\left[\bar{\bar{A}}^{\star`}_{i j}\right]_{a\times b} = \begin{bmatrix} \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_1^a}(u_1), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_1^a}(u_1), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_1^a}(u_1) & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_2^b}(u_1), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_2^b}(u_1), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_2^b}(u_1) & \cdots & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_b^z}(u_1), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_b^z}(u_1), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_b^z}(u_1) \\ \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_1^a}(u_2), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_1^a}(u_2), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_1^a}(u_2) & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_2^b}(u_2), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_2^b}(u_2), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_2^b}(u_2) & \cdots & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_b^z}(u_2), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_b^z}(u_2), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_b^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_1^a}(u_a), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_1^a}(u_a), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_1^a}(u_a) & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_2^b}(u_a), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_2^b}(u_a), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_2^b}(u_a) & \cdots & \overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_b^z}(u_a), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_b^z}(u_a), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_b^z}(u_a) \end{bmatrix}$$

where

$$\bar{\bar{A}}^{\star}_{i j} = \left(\overline{\overline{\mathbb{T}}}^{\star}_{\mathbb{L}_j^k}(u_i), \overline{\overline{\mathbb{I}}}^{\star}_{\mathbb{L}_j^k}(u_i), \overline{\overline{\mathbb{F}}}^{\star}_{\mathbb{L}_j^k}(u_i)\right)$$

$$= \left(\begin{array}{c} \left[1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}, 1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}\right], \\ \left[\prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}\right], \left[\prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{L}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{U}}_{\mathbb{L}_j^k}(u_i)\right)^{\frac{1}{t}}\right] \end{array}\right)$$

$for\ i = 1,2,3 \ldots a\ , j = 1,2,3, \ldots b, k = a, b, c, \ldots z$ and $a = 1, b = 3, c = 2, d = 2$

By averaging all decision matrices, we get

$[\bar{\bar{S}}^{\star}]$

$$= \begin{bmatrix} \left(S, \begin{pmatrix} [0.6443,0.7700], \\ [0.1189, 0.2060], \\ [0.2060,0.2300] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.5880,0.7264], \\ [0.1189,0.2060], \\ [0.2141,0.3253] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.5051,0.6131], \\ [0.1655,0.2632], \\ [0.1,0.1861] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.3273,0.4336], \\ [0.2812,0.3834], \\ [0.1189, 0.2213] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.7289,0.8318], \\ [0.1, 0.1861], \\ [0.0595,0.1189] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.6920,0.800], \\ [0.1, 0.1861], \\ [0.1507,0.1861] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.7217,0.8318], \\ [0.1, 0.1732], \\ [0.0783, 0.1414] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [06052,0.7172], \\ [0.2515,0.3568], \\ [0.1150, 0.200] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.6127,0.7172], \\ [0.1, 0.1732], \\ [0.1316, 0.2213] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.7289, ,0.8318], \\ [0.0595, 0.1189], \\ [0.1107, 0.1682] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.7787,0.8811], \\ [0.0595, 0.1189], \\ [0.05, 0.100] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.2455,0.3494], \\ [0.2515, 0.3568], \\ [0.3482, 0.4757] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.7172,0.8373], \\ [0.0707, 0.1316], \\ [0.1316, 0.2300] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.6870,0.8066], \\ [0.0707, 0.1316], \\ [0.1107,0.1934] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.6536,0.7551], \\ [0.0707,0.1316], \\ [0.1189, 0.2060] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.6838,0.800], \\ [0.1,0.1732], \\ [0.1456,0.2378] \end{pmatrix}\right) \end{bmatrix}$$

One calculation is provided for the convenience of reader

for $i = 1\ , j = 1, k = a = 1$

$$\bar{\bar{\mathcal{S}}}^{\star}_{11}=\left(\bar{\bar{\mathbb{T}}}^{\star}_{\mathbb{L}^1_1}(\mathcal{s}_1),\bar{\bar{\mathbb{I}}}^{\star}_{\mathbb{L}^1_1}(\mathcal{s}_1),\bar{\bar{\mathbb{F}}}^{\star}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)$$

$$=\begin{pmatrix}\begin{pmatrix}1-\left(1-\mathbb{T}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}},\\ 1-\left(1-\mathbb{T}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(1-\mathbb{T}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\end{pmatrix},\\ \begin{pmatrix}\left(\mathbb{I}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}},\\ \left(\mathbb{I}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\end{pmatrix},\\ \begin{pmatrix}\left(\mathbb{F}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}},\\ \left(\mathbb{F}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\frac{1}{4}}\end{pmatrix}\end{pmatrix}$$

$$\bar{\bar{\mathcal{S}}}^{\star}_{11}=\begin{pmatrix}\begin{pmatrix}1-(1-0.8)^{\frac{1}{4}}(1-0.2)^{\frac{1}{4}}(1-0.5)^{\frac{1}{4}}(1-0.8)^{\frac{1}{4}},\\ 1-(1-0.9)^{\frac{1}{4}}(1-0.3)^{\frac{1}{4}}(1-0.6)^{\frac{1}{4}}(1-0.9)^{\frac{1}{4}}\end{pmatrix},\\ \begin{pmatrix}(0.1)^{\frac{1}{4}}(0.2)^{\frac{1}{4}}(0.2)^{\frac{1}{4}}(0.05)^{\frac{1}{4}},\\ (0.2)^{\frac{1}{4}}(0.3)^{\frac{1}{4}}(0.3)^{\frac{1}{4}}(0.1)^{\frac{1}{4}}\end{pmatrix}\\ \begin{pmatrix}(0.2)^{\frac{1}{4}}(0.6)^{\frac{1}{4}}(0.3)^{\frac{1}{4}}(0.05)^{\frac{1}{4}},\\ (0.1)^{\frac{1}{4}}(0.7)^{\frac{1}{4}}(0.4)^{\frac{1}{4}}(0.1)^{\frac{1}{4}}\end{pmatrix}\end{pmatrix}$$

$$\bar{\bar{\mathcal{S}}}^{\star}_{11}=([0.6443,0.7700],[0.1189,0.2060],[0.2060,0.2300])$$

To determine the weights of the decision makers, first we will find the similarity measure between each decision matrix $\mathbb{D}^1,\mathbb{D}^2,\mathbb{D}^3,\mathbb{D}^4$and the ideal matrix $\mathcal{S}^{\star}$ using

$$\mathbb{S}\left(\bar{\bar{A}}^{x}_{ij},\bar{\bar{A}}^{\star}_{ij}\right)=1-\frac{1}{6ab}\sum_{i}^{a}\sum_{j}^{b}\left(\left|\mathbb{T}^{x\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{T}^{\star\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)\right|+\left|\mathbb{T}^{x\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{T}^{\star\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)\right|+\left|\mathbb{I}^{x\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{I}^{\star\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)\right|\right.$$
$$\left.+\left|\mathbb{I}^{x\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{I}^{\star\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)\right|+\left|\mathbb{F}^{x\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{F}^{\star\mathbb{L}}_{\mathbb{L}^{k}_{j}}(u_i)\right|+\left|\mathbb{F}^{x\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)-\mathbb{F}^{\star\mathbb{U}}_{\mathbb{L}^{k}_{j}}(u_i)\right|\right)$$

So,

$\mathbb{S}\left(\bar{\bar{\mathcal{S}}}^{1},\bar{\bar{\mathcal{S}}}^{\star}\right)=0.8774,$

$\mathbb{S}\left(\bar{\bar{\mathcal{S}}}^{2},\bar{\bar{\mathcal{S}}}^{\star}\right)=0.9133,$

$\mathbb{S}\left(\bar{\bar{\mathcal{S}}}^{3},\bar{\bar{\mathcal{S}}}^{\star}\right)=0.8834,$

$\mathbb{S}\left(\bar{\bar{\mathcal{S}}}^{4},\bar{\bar{\mathcal{S}}}^{\star}\right)=0.9101$

Now we calculate the weight $\Delta^x\ for\ (x=1,2,3,4)$ of each decision makers using

$$\Delta^x=\frac{\mathbb{S}\left(\bar{\bar{A}}^{x}_{ij},\bar{\bar{A}}^{\star}_{ij}\right)}{\sum_{x=1}^{t}\mathbb{S}\left(\bar{\bar{A}}^{x}_{ij},\bar{\bar{A}}^{\star}_{ij}\right)}$$

$$\Delta^1=\frac{0.8774}{(0.8774+0.9133+0.8834+0.9101)}=0.2448$$

$$\Delta^2 = \frac{0.9133}{(0.8774 + 0.9133 + 0.8834 + 0.9101)} = 0.2548$$

$$\Delta^3 = \frac{0.8834}{(0.8774 + 0.9133 + 0.8834 + 0.9101)} = 0.2465$$

$$\Delta^4 = \frac{0.9101}{(0.8774 + 0.9133 + 0.8834 + 0.9101)} = 0.2539$$

Where $0 \leq \Delta^x \leq 1$ and $\sum_{x=1}^{t} \Delta^x = 1$.

Step 2: Aggregate Neutrosophic Hypersoft Decision Matrices

Now we construct an aggregated neutrosophic hypersoft decision matrix to obtain one group decision. Aggregated neutrosophic hypersoft decision matrix is denoted as $\bar{\bar{A}}_{ij}$ and it is given as

$$[\bar{\bar{A}}_{ij}]_{a\times b} = \begin{bmatrix} \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_1) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_1) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_1), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_1), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_1) \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_2) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_2) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_2), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_2), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\bar{\mathbb{T}}}_{\mathbb{L}_1^a}(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_1^a}(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_1^a}(u_a) & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_2^b}(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_2^b}(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_2^b}(u_a) & \cdots & \bar{\bar{\mathbb{T}}}_{\mathbb{L}_b^z}(u_a), \bar{\bar{\mathbb{I}}}_{\mathbb{L}_b^z}(u_a), \bar{\bar{\mathbb{F}}}_{\mathbb{L}_b^z}(u_a) \end{bmatrix}$$

The elements of $\bar{\bar{A}}_{ij}$ in the matrix $[\bar{\bar{A}}_{ij}]_{a\times b}$ is calculated as

$$[\bar{\bar{A}}_{ij}]_{a\times b} = \left(\left[1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\Delta^x}, 1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\Delta^x}\right], \left[\prod_{x=1}^{t}\left(\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{L}}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}_{\mathbb{L}_j^k}^{x\mathbb{U}}(u_i)\right)^{\Delta^x}\right]\right)$$

Where $i = 1,2,3 \dots a$, $j = 1,2,3, \dots b$ and $x = 1,2, \dots t$.

After calculations, the aggregated neutrosophic hypersoft decision matrices is

$$[\bar{\bar{S}}] = \begin{bmatrix} \left(S, \begin{pmatrix} [0.6431,0.7691], \\ [0.1187,0.2057], \\ [0.2055,0.2309] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.5932,0.7303], \\ [0.1179,0.2051], \\ [0.2099, 0.3212] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.5069,0.6146], \\ [0.1631,0.2606], \\ [0.1007, 0.1862] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.3249,0.4322], \\ [0.2846, 0.3854], \\ [0.1185,0.2211] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.7285,0.8313], \\ [0.0999, 0.1864], \\ [0.0592, 0.1187] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.6924,0.7998], \\ [0.1006, 0.1870], \\ [0.1048, 0.1857] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.7213,0.8316], \\ [0.0998, 0.1731], \\ [0.0788,0.1419] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.6074,0.7183], \\ [0.2503, 0.3560], \\ [0.1138,0.1991] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.6128,0.7168], \\ [0.1002, 0.1733], \\ [0.1315, 0.2215] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.7285,0.8318], \\ [0.0593, 0.1186], \\ [0.0843, 0.1687] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.7791,0.8813], \\ [0.0592, 0.1187], \\ [0.0500, 0.100] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.2436,0.3482], \\ [0.2524,0.3573], \\ [0.3471, 0.4752] \end{pmatrix}\right) \\ \left(S, \begin{pmatrix} [0.7185,0.8386], \\ [0.0704, 0.1310], \\ [0.1313, 0.2293] \end{pmatrix}\right) & \left(BM, \begin{pmatrix} [0.6847,0.8051], \\ [0.0711, 0.1319], \\ [0.1115, 0.1937] \end{pmatrix}\right) & \left(B, \begin{pmatrix} [0.6535,0.7550], \\ [0.0711, 0.1319], \\ [0.1195, 0.2065] \end{pmatrix}\right) & \left(NFS, \begin{pmatrix} [0.6834,0.7998], \\ [0.1002, 0.1733], \\ [0.1455, 0.2376] \end{pmatrix}\right) \end{bmatrix}$$

One calculation is provided for the convenience of reader

for $i = 1$, $j = 1, k = a = 1$

$$\bar{\bar{\mathcal{S}}}_{11}=\begin{pmatrix}\begin{pmatrix}\left(1-\left(1-\mathbb{T}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(1-\mathbb{T}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(1-\mathbb{T}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(1-\mathbb{T}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4}\right),\\ \left(1-\left(1-\mathbb{T}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(1-\mathbb{T}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(1-\mathbb{T}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(1-\mathbb{T}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4}\right)\end{pmatrix},\\ \begin{pmatrix}\left(\mathbb{I}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(\mathbb{I}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(\mathbb{I}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(\mathbb{I}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4},\\ \left(\mathbb{I}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(\mathbb{I}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(\mathbb{I}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(\mathbb{I}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4}\end{pmatrix},\\ \begin{pmatrix}\left(\mathbb{F}^{1\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(\mathbb{F}^{2\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(\mathbb{F}^{3\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(\mathbb{F}^{4\mathbb{L}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4},\\ \left(\mathbb{F}^{1\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^1}\left(\mathbb{F}^{2\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^2}\left(\mathbb{F}^{3\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^3}\left(\mathbb{F}^{4\mathbb{U}}_{\mathbb{L}^1_1}(\mathcal{s}_1)\right)^{\Delta^4}\end{pmatrix}\end{pmatrix}$$

$$\bar{\bar{\mathcal{S}}}_{11}=\begin{pmatrix}\begin{pmatrix}1-(1-0.8)^{0.2448}(1-0.2)^{0.2548}(1-0.5)^{0.2465}(1-0.8)^{0.2539},\\ 1-(1-0.9)^{0.2448}(1-0.3)^{0.2548}(1-0.6)^{0.2465}(1-0.9)^{0.2539}\end{pmatrix},\\ \begin{pmatrix}(0.1)^{0.2448}(0.2)^{0.2548}(0.2)^{0.2465}(0.05)^{0.2539},\\ (0.2)^{0.2448}(0.3)^{0.2548}(0.3)^{0.2465}(0.1)^{0.2539}\end{pmatrix}\\ \begin{pmatrix}(0.2)^{0.2448}(0.6)^{0.2548}(0.3)^{0.2465}(0.05)^{0.2539},\\ (0.1)^{0.2448}(0.7)^{0.2548}(0.4)^{0.2465}(0.1)^{0.2539}\end{pmatrix}\end{pmatrix}$$

$$\bar{\bar{\mathcal{S}}}_{11}=([0.6431,0.7691],[0.1187,0.2057],[0.2057,0.2309])$$

Step 3: Determine the weight of attributes

Weight $\bar{\bar{\mathbb{w}}}^j$ of attributes $\mathbb{L}_j, j=1,2\ldots\mathcal{b}$ is calculated using

$$\bar{\bar{\mathbb{w}}}^j=\left(\bar{\bar{\mathbb{T}}}_{\mathbb{L}_j},\bar{\bar{\mathbb{I}}}_{\mathbb{L}_j},\bar{\bar{\mathbb{F}}}_{\mathbb{L}_j}\right)$$

$$=\left(\left[1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x},1-\prod_{x=1}^{t}\left(1-\mathbb{T}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right],\left[\prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x},\prod_{x=1}^{t}\left(\mathbb{I}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right],\left[\prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{L}}_{\mathbb{L}_j}\right)^{\Delta^x},\prod_{x=1}^{t}\left(\mathbb{F}^{x\mathbb{U}}_{\mathbb{L}_j}\right)^{\Delta^x}\right]\right)$$

To calculate the weight of attributes, we use importance of selected attributes by each decision maker

$$\mathbb{D}^1\longrightarrow\left(S,\begin{pmatrix}[0.8,0.9],\\ [0.1,0.2],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}[0.6,0.7],\\ [0.1,0.2],\\ [0.1,0.2]\end{pmatrix}\right)\ \left(B,\begin{pmatrix}[0.7,0.8],\\ [0.05,0.1],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}[0.5,0.6],\\ [0.2,0.3],\\ [0.4,0.5]\end{pmatrix}\right)$$

$$\mathbb{D}^2\longrightarrow\left(S,\begin{pmatrix}[0.7,0.8],\\ [0.05,0.1],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}[0.5,0.6],\\ [0.2,0.3],\\ [0.3,0.4]\end{pmatrix}\right)\ \left(B,\begin{pmatrix}[0.4,0.5],\\ [0.1,0.2],\\ [0.3,0.4]\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}[0.6,0.7],\\ [0.1,0.2],\\ [0.1,0.2]\end{pmatrix}\right)$$

$$\mathbb{D}^3\longrightarrow\left(S,\begin{pmatrix}[0.4,0.5],\\ [0.1,0.2],\\ [0.3,0.4]\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}[0.7,0.8],\\ [0.05,0.1],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(B,\begin{pmatrix}[0.8,0.9],\\ [0.05,0.1],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}[0.8,0.9],\\ [0.05,0.1],\\ [0.1,0.2]\end{pmatrix}\right)$$

$$\mathbb{D}^4 \longrightarrow \left(S,\begin{pmatrix}[0.8,0.9],\\ [0.05,0.1],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(BM,\begin{pmatrix}[0.8,0.9],\\ [0.1,0.2],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(B,\begin{pmatrix}[0.8,0.9],\\ [0.1,0.2],\\ [0.05,0.1]\end{pmatrix}\right)\ \left(NFS,\begin{pmatrix}[0.6,0.7],\\ [0.05,0.1],\\ [0.2,0.3]\end{pmatrix}\right)$$

Using this importance of attributes, we get

$$\mathbb{w}^1 = ([0.7092,0.8228],[0.0703,0.1407],[0.0778,0.1406])$$

$$\mathbb{w}^2 = ([0.6692,0.7785],[0.1006,0.1870],[0.0935,0.1424])$$

$$\mathbb{w}^3 = ([0.7078,0.8212],[0.0711,0.1421],[0.0789,0.1424])$$

$$\mathbb{w}^4 = ([0.6439,0.7542],[0.0838,0.1565],[0.1674,0.2778])$$

One calculation is provided for the convenience of reader

for $j = 1$

$$\mathbb{w}^1 = \begin{pmatrix} \begin{pmatrix} \left(1-\left(1-\mathbb{T}^{1\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(1-\mathbb{T}^{2\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(1-\mathbb{T}^{3\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(1-\mathbb{T}^{4\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4}\right), \\ \left(1-\left(1-\mathbb{T}^{1\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(1-\mathbb{T}^{2\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(1-\mathbb{T}^{3\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(1-\mathbb{T}^{4\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4}\right) \end{pmatrix}, \\ \begin{pmatrix} \left(\mathbb{I}^{1\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(\mathbb{I}^{2\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(\mathbb{I}^{3\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(\mathbb{I}^{4\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4}, \\ \left(\mathbb{I}^{1\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(\mathbb{I}^{2\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(\mathbb{I}^{3\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(\mathbb{I}^{4\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4} \end{pmatrix}, \\ \begin{pmatrix} \left(\mathbb{F}^{1\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(\mathbb{F}^{2\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(\mathbb{F}^{3\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(\mathbb{F}^{4\mathbb{L}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4}, \\ \left(\mathbb{F}^{1\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^1}\left(\mathbb{F}^{2\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^2}\left(\mathbb{F}^{3\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^3}\left(\mathbb{F}^{4\mathbb{U}}_{\mathbb{L}_1}(s_1)\right)^{\Delta^4} \end{pmatrix} \end{pmatrix}$$

$$\mathbb{w}^1 = \begin{pmatrix} \begin{pmatrix} 1-(1-0.8)^{0.2448}(1-0.7)^{0.2548}(1-0.4)^{0.2465}(1-0.8)^{0.2539}, \\ 1-(1-0.9)^{0.2448}(1-0.8)^{0.2548}(1-0.5)^{0.2465}(1-0.9)^{0.2539} \end{pmatrix}, \\ \begin{pmatrix} (0.1)^{0.2448}(0.05)^{0.2548}(0.1)^{0.2465}(0.05)^{0.2539}, \\ (0.2)^{0.2448}(0.1)^{0.2548}(0.2)^{0.2465}(0.1)^{0.2539} \end{pmatrix} \\ \begin{pmatrix} (0.05)^{0.2448}(0.05)^{0.2548}(0.3)^{0.2465}(0.05)^{0.2539}, \\ (0.1)^{0.2448}(0.1)^{0.2548}(0.4)^{0.2465}(0.1)^{0.2539} \end{pmatrix} \end{pmatrix}$$

$$\mathbb{w}^1 = ([0.7092,0.8228],[0.0703,0.1407],[0.0778,0.1406])$$

Step 4: Calculate the weighted aggregated decision matrix

After finding the weight of individual attributes, we apply these weights to each row of aggregated decision matrix using

$$\left[\bar{\bar{A}}^{\omega}_{ij}\right]_{a\times b} = \left(\bar{\bar{\mathbb{T}}}^{\omega}_{\mathbb{L}^{k}_{j}}(u_i), \bar{\bar{\mathbb{I}}}^{\omega}_{\mathbb{L}^{k}_{j}}(u_i), \bar{\bar{\mathbb{F}}}^{\omega}_{\mathbb{L}^{k}_{j}}(u_i)\right)$$

$$= \Big(\left[\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_j}, \mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_j}\right], \Big[\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}^{k}_{j}}(u_i)+\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j}-\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_j}, \mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}^{k}_{j}}(u_i)$$

$$+\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j}-\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_j}\Big], \Big[\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}^{k}_{j}}(u_i)+\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j}-\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_j}, \mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}^{k}_{j}}(u_i)+\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j}$$

$$-\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}^{k}_{j}}(u_i).\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_j}\Big]\Big)$$

And we get a weighted aggregated decision matrix as

$$[\bar{\bar{\mathcal{S}}}^{\omega}]$$

$$=\begin{bmatrix} \left(S,\begin{pmatrix}[0.4562, 0.6328],\\ [0.1807, 0.3175],\\ [0.2675, 0.3390]\end{pmatrix}\right) & \left(BM,\begin{pmatrix}[0.3960, 0.5685],\\ [0.2066, 0.3537],\\ [0.2838, 0.4179]\end{pmatrix}\right) & \left(B,\begin{pmatrix}[0.3588, 0.5047],\\ [0.2226, 0.3657],\\ [0.1703, 0.3021]\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}[0.2092, 0.326],\\ [0.3446, 0.4816],\\ [0.2661, 0.4375]\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}[0.5167, 0.6840],\\ [0.1632, 0.3009],\\ [0.1324, 0.2426]\end{pmatrix}\right) & \left(BM,\begin{pmatrix}[0.4634, 0.6226],\\ [0.1911, 0.3390],\\ [0.1885, 0.3017]\end{pmatrix}\right) & \left(B,\begin{pmatrix}[0.5106, 0.6829],\\ [0.1638, 0.2906],\\ [0.1507, 0.2641]\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}[0.3911, 0.5417],\\ [0.3131, 0.4568],\\ [0.2622, 0.4216]\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}[0.4347, 0.5898],\\ [0.1635, 0.2896],\\ [0.1991, 0.331]\end{pmatrix}\right) & \left(BM,\begin{pmatrix}[0.4875, 0.6476],\\ [0.1539, 0.2834],\\ [0.1699, 0.2871]\end{pmatrix}\right) & \left(B,\begin{pmatrix}[0.5515, 0.7237],\\ [0.1261, 0.2439],\\ [0.1241, 0.2282]\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}[0.1569, 0.2626],\\ [0.3150, 0.4579],\\ [0.4564, 0.621]\end{pmatrix}\right) \\ \left(S,\begin{pmatrix}[0.5096, 0.69],\\ [0.1358, 0.2533],\\ [0.1989, 0.3377]\end{pmatrix}\right) & \left(BM,\begin{pmatrix}[0.4582, 0.6268],\\ [0.1645, 0.2942],\\ [0.1946, 0.3085]\end{pmatrix}\right) & \left(B,\begin{pmatrix}[0.4626, 0.62],\\ [0.1371, 0.2551],\\ [0.1882, 0.3195]\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}[0.4400, 0.6032],\\ [0.1756, 0.3027],\\ [0.2885, 0.4494]\end{pmatrix}\right) \end{bmatrix}$$

One calculation is provided for the convenience of reader

for $\imath = 1\,, \jmath = 1, \hbar = a = 1$

$$\bar{\bar{\mathcal{S}}}^{\omega}_{11} = \Big([\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{T}^{\mathbb{L}}{}_{\mathbb{L}_1}, \mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{T}^{\mathbb{U}}{}_{\mathbb{L}_1}]\,, \Big[\Big(\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_1^1}(u_1) + \mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_1} - \mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{I}^{\mathbb{L}}{}_{\mathbb{L}_1}\Big), \Big(\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_1^1}(u_1) + \mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_1} - \mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{I}^{\mathbb{U}}{}_{\mathbb{L}_1}\Big)\Big], \Big[\Big(\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_1^1}(u_1) + \mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_1} - \mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{F}^{\mathbb{L}}{}_{\mathbb{L}_1}\Big), \Big(\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_1^1}(u_1) + \mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_1} - \mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_1^1}(u_1).\,\mathbb{F}^{\mathbb{U}}{}_{\mathbb{L}_1}\Big)\Big]\Big)$$

$$\bar{\bar{\mathcal{S}}}^{\omega}_{11} = \Big(\big((0.6431)(0.7092), (0.7691)(0.8228)\big)\,, \Big(\big(0.1187 + 0.0703 - (0.1187)(0.0703)\big), \big(0.2057 + 0.1407 - (0.2057)(0.1407)\big)\Big)\,, \Big(\big(0.2057 + 0.0778 - (0.2057)(0.0778)\big), \big(0.2309 + 0.1406 - (0.2309)(0.1406)\big)\Big)\Big)$$

$$\bar{\bar{\mathcal{S}}}^{\omega}_{11} = ([0.4562, 0.6328], [\,0.1807, 0.3175], [0.2675, 0.3390])$$

Step 5: Determine the ideal solution

Since we are dealing with benefit type ($\mathbb{C}_1$) attributes so Neutrosophic hypersoft positive ideal solution is calculated using

$$\bar{\bar{A}}_{\jmath}^{\omega^+} = \left(\bar{\bar{\mathbb{T}}}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}{}^{+}(u_i), \bar{\bar{\mathbb{I}}}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}{}^{+}(u_i), \bar{\bar{\mathbb{F}}}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}{}^{+}(u_i)\right)$$

$$= \left[\overset{i}{\max}\left\{\mathbb{T}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}, \overset{i}{\max}\left\{\mathbb{T}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}\right], \left[\overset{i}{\min}\left\{\mathbb{I}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}, \overset{i}{\min}\left\{\mathbb{I}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}\right], \left[\overset{i}{\min}\left\{\mathbb{F}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}, \overset{i}{\min}\left\{\mathbb{F}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_{\jmath}^{\hbar}}(u_i)\right\}\right], \jmath \in \mathbb{C}_1$$

$$\bar{\bar{\mathcal{S}}}^{\omega^+}$$

$$= \begin{bmatrix} \left(S,\begin{pmatrix}[0.5167, 0.69],\\ [0.1358, 0.2533],\\ [0.1324, 0.2426]\end{pmatrix}\right) & \left(BM,\begin{pmatrix}[0.4875, 0.6476],\\ [0.1539, 0.2834],\\ [0.1699, 0.2871]\end{pmatrix}\right) & \left(B,\begin{pmatrix}[0.5515, 0.7237],\\ [0.1261, 0.3439],\\ [0.1507, 0.2282]\end{pmatrix}\right) & \left(NFS,\begin{pmatrix}[0.4400, 0.6032],\\ [0.1756, 0.3027],\\ [0.2622, 0.4216]\end{pmatrix}\right) \end{bmatrix}$$

One calculation is provided for the convenience of reader for $i = 1,2,3,4,\ j = 1, k = a = 1$

$$\bar{\bar{S}}_1^{\omega^+} = \begin{pmatrix} (max\{0.4562, 0.5167, 0.4347, 0.5096\}, max\{0.6328, 0.6840, 0.5898, 0.69\}), \\ min\{0.1807,0.1632,0.1635,0.1358\}, min\{0.3175, 0.3009, 0.2896, 0.2533\}), \\ (min\{0.2675, 0.1324, 0.1991, 0.1989\}, min\{0.3390, 0.2426, 0.331, 0.3377\}) \end{pmatrix}$$

$$\bar{\bar{S}}_1^{\omega^+} = \left(S, ([0.5167, 0.69], [0.1358, 0.2533], [0.1324, 0.2426])\right)$$

Similarly, neutrosophic hypersoft negative ideal solution is given as

$$\bar{\bar{A}}_j^{\omega^-} = \left(\bar{\bar{\mathbb{T}}}^{\omega}_{\mathbb{L}_j^k}{}^-(u_i), \bar{\bar{\mathbb{I}}}^{\omega}_{\mathbb{L}_j^k}{}^-(u_i), \bar{\bar{\mathbb{F}}}^{\omega}_{\mathbb{L}_j^k}{}^-(u_i)\right)$$

$$= \left[\overset{i}{\min}\left\{\mathbb{T}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}, \overset{i}{\min}\left\{\mathbb{T}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}\right], \left[\overset{i}{\max}\left\{\mathbb{I}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}, \overset{i}{\max}\left\{\mathbb{I}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}\right], \left[\overset{i}{\max}\left\{\mathbb{F}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}, \overset{i}{\max}\left\{\mathbb{F}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i)\right\}\right], j \in \mathbb{C}_1$$

$$\bar{\bar{S}}^{\omega^-} = \left[\left(S, \begin{pmatrix}[0.4347, 0.5898], \\ [0.1807, 0.3175], \\ [0.2675, 0.3390]\end{pmatrix}\right) \left(BM, \begin{pmatrix}[0.3960, 0.5685], \\ [0.2066, 0.3537], \\ [0.2838, 0.4179]\end{pmatrix}\right) \left(B, \begin{pmatrix}[0.3588, 0.5047], \\ [0.2226, 0.3657], \\ [0.1882, 0.3195]\end{pmatrix}\right) \left(NFS, \begin{pmatrix}[0.1569, 0.2626], \\ [0.3446, 0.4816], \\ [0.4564, 0.621]\end{pmatrix}\right)\right]$$

One calculation is provided for the convenience of reader

for $i = 1,2,3,4,\ j = 1, k = a = 1$

$$\bar{\bar{S}}_1^{\omega^-} = \begin{pmatrix} (min\{0.4562, 0.5167, 0.4347, 0.5096\}, min\{0.6328, 0.6840, 0.5898, 0.69\}), \\ (max\{0.1807,0.1632,0.1635,0.1358\}, max\{0.3175, 0.3009, 0.2896, 0.2533\}), \\ (max\{0.2675, 0.1324, 0.1991, 0.1989\}, max\{0.3390, 0.2426, 0.331, 0.3377\}) \end{pmatrix}$$

$$\bar{\bar{S}}_1^{\omega^-} = \left(S, ([0.4347, 0.5898], [0.1807, 0.3175], [0.2675, 0.3390])\right)$$

Step 6: Calculate the distance measure

Now we will find the normalized hamming distance between the alternatives and positive ideal solution using

$$\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_j^{\omega^+}\right) = \frac{1}{6\ell}\sum_{j=1}^{\ell}\left(\left|\mathbb{T}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{T}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right| + \left|\mathbb{T}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{T}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right| + \left|\mathbb{I}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{I}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right| + \left|\mathbb{I}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{I}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right| + \left|\mathbb{F}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{F}^{\mathbb{L}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right| + \left|\mathbb{F}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}(u_i) - \mathbb{F}^{\mathbb{U}}{}^{\omega}_{\mathbb{L}_j^k}{}^+(u_i)\right|\right)$$

We get,

$$\mathbb{D}^{1+}\left(\bar{\bar{S}}_1^{\omega}, \bar{\bar{S}}^{\omega^+}\right) = 0.1081$$
$$\mathbb{D}^{2+}\left(\bar{\bar{S}}_2^{\omega}, \bar{\bar{S}}^{\omega^+}\right) = 0.0361$$
$$\mathbb{D}^{3+}\left(\bar{\bar{S}}_3^{\omega}, \bar{\bar{S}}^{\omega^+}\right) = 0.0723$$
$$\mathbb{D}^{4+}\left(\bar{\bar{S}}_4^{\omega}, \bar{\bar{S}}^{\omega^+}\right) = 0.0283$$

One calculation is provided for the convenience of reader

for $i = 1$ and

When $j = 1 \Rightarrow k = a = 1$,

When $j = 2 \Rightarrow k = b = 3$,

When $j = 3 \Rightarrow k = c = 2$,

When $j = 4 \Rightarrow k = d = 2$,

$$\mathbb{D}^{1+}\left(\bar{\bar{S}}_1^{\omega},\bar{\bar{S}}^{\omega^+}\right)=\frac{1}{6(4)}\Bigg(\Bigg(\left|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{T}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_1^1}\right|+\left|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{T}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_1^1}\right|+\left|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{I}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_1^1}\right|$$

$$+\left|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{I}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_1^1}\right|+\left|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{F}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_1^1}\right|+\left|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1)-\mathbb{F}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_1^1}\right|\Bigg)$$

$$+\Bigg(\left|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{T}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_2^3}\right|+\left|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{T}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_2^3}\right|+\left|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{I}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_2^3}\right|$$

$$+\left|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{I}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_2^3}\right|+\left|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{F}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_2^3}\right|\Bigg)+\left|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1)-\mathbb{F}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_2^3}\right|$$

$$+\Bigg(\left|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{T}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_3^2}\right|+\left|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{T}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_3^2}\right|+\left|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{I}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_3^2}\right|$$

$$+\left|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{I}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_3^2}\right|+\left|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{F}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_3^2}\right|\Bigg)+\left|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1)-\mathbb{F}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_3^2}\right|$$

$$+\Bigg(\left|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{T}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_4^2}\right|+\left|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{T}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_4^2}\right|+\left|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{I}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_4^2}\right|$$

$$+\left|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{I}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_4^2}\right|+\left|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{F}^{\mathbb{L}^{\omega^+}}_{\mathbb{L}_4^2}\right|\Bigg)+\left|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1)-\mathbb{F}^{\mathbb{U}^{\omega^+}}_{\mathbb{L}_4^2}\right|\Bigg)$$

$$\mathbb{D}^{1+}\left(\bar{\bar{S}}_1^{\omega},\bar{\bar{S}}^{\omega^+}\right)=\frac{1}{24}\big((|0.4562-0.1358|+|\,0.1807-0.1358|+|0.2675-0.1324\,|)$$

$$+(|0.6328-0.69|+|0.69-0.2533\,|+|0.3390-0.2426\,|)$$

$$+(|0.3960-0.4875|+|0.2066-0.1539\,|+|0.2838-0.1699\,|)$$

$$+(|0.5685-0.6476|+|\,0.3537-0.2834|+|0.4179-0.2871\,|)$$

$$+(|0.3588-0.5515|+|0.2226-0.1261\,|+|0.1703-0.1507\,|)$$

$$+(|0.5047-0.7237|+|0.3657-0.2439\,|+|\,0.3021-0.2282|)$$

$$+(|0.2092-0.4400|+|\,0.3446-0.1756|+|0.2661-0.2622|)$$

$$+(|0.326-0.6032|+|\,0.4816-0.3027|+|\,0.4375-0.4216|)\big)$$

$$\mathbb{D}^{1+}\left(\bar{\bar{S}}_1^{\omega},\bar{\bar{S}}^{\omega^+}\right)=0.1081$$

Similarly, we will find the normalized hamming distance between the alternatives and negative ideal solution using

$$\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega},\bar{\bar{A}}_{j}^{\omega^-}\right)=\frac{1}{6b}\sum_{j=1}^{b}\Bigg(\left|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|+\left|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|+\left|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|$$

$$+\left|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|+\left|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|+\left|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}(u_i)-\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_j^k}{}^{-}(u_i)\right|\Bigg)$$

We get,

$$\mathbb{D}^{1-}\left(\bar{\bar{S}}_1^{\omega},\bar{\bar{S}}^{\omega^-}\right)=0.0183$$

$$\mathbb{D}^{2-}\left(\bar{\bar{S}}_2^{\omega},\bar{\bar{S}}^{\omega^-}\right)=0.1023$$

$$\mathbb{D}^{3-}\left(\bar{\bar{S}}_3^{\omega},\bar{\bar{S}}^{\omega^-}\right)=0.0679$$

$$\mathbb{D}^{4-}\left(\bar{\bar{S}}_4^{\omega},\bar{\bar{S}}^{\omega^-}\right)=0.0968$$

One calculation is provided for the convenience of reader for $i = 1$

$$\mathbb{D}^{1-}\left(\bar{\bar{S}}_1^{\omega}, \bar{\bar{S}}^{\omega^-}\right) = \frac{1}{6(4)}\Bigg(\Big(\Big|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{T}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_1^1}\Big| + \Big|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{T}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_1^1}\Big| + \Big|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{I}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_1^1}\Big|$$

$$+ \Big|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{I}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_1^1}\Big| + \Big|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{F}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_1^1}\Big| + \Big|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_1^1}(s_1) - \mathbb{F}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_1^1}\Big|\Big)$$

$$+ \Big(\Big|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{T}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_2^3}\Big| + \Big|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{T}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_2^3}\Big| + \Big|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{I}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_2^3}\Big|$$

$$+ \Big|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{I}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_2^3}\Big| + \Big|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{F}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_2^3}\Big|\Big) + \Big|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_2^3}(s_1) - \mathbb{F}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_2^3}\Big|$$

$$+ \Big(\Big|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{T}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_3^2}\Big| + \Big|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{T}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_3^2}\Big| + \Big|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{I}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_3^2}\Big|$$

$$+ \Big|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{I}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_3^2}\Big| + \Big|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{F}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_3^2}\Big|\Big) + \Big|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_3^2}(s_1) - \mathbb{F}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_3^2}\Big|$$

$$+ \Big(\Big|\mathbb{T}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{T}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_4^2}\Big| + \Big|\mathbb{T}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{T}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_4^2}\Big| + \Big|\mathbb{I}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{I}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_4^2}\Big|$$

$$+ \Big|\mathbb{I}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{I}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_4^2}\Big| + \Big|\mathbb{F}^{\mathbb{L}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{F}^{\mathbb{L}^{\omega^-}}_{\mathbb{L}_4^2}\Big|\Big) + \Big|\mathbb{F}^{\mathbb{U}^{\omega}}_{\mathbb{L}_4^2}(s_1) - \mathbb{F}^{\mathbb{U}^{\omega^-}}_{\mathbb{L}_4^2}\Big|\Bigg)$$

$$\mathbb{D}^{1-}\left(\bar{\bar{S}}_1^{\omega}, \bar{\bar{S}}^{\omega^-}\right) = \frac{1}{24}\big((|0.4562 - 0.4347| + |\,0.1807 - 0.1807| + |0.2675 - 0.2675\,|)$$
$$+ (|0.6328 - 0.5898| + |\,0.3175 - 0.3175| + |0.3390 - 0.3390\,|)$$
$$+ (|0.3960 - 0.3960| + |0.2066 - 0.2066\,| + |0.2838 - 0.2838\,|)$$
$$+ (|0.5685 - 0.5685| + |0.3537 - 0.3537\,| + |0.4179 - 0.4179\,|)$$
$$+ (|0.3588 - 0.3588| + |0.2226 - 0.2226\,| + |0.1703 - 0.1882\,|)$$
$$+ (|0.5047 - 0.5047| + |0.3657 - 0.3657\,| + |0.3021 - 0.3195\,|)$$
$$+ (|0.2092 - 0.1569| + |\,0.3446 - 0.3446| + |0.2661 - 0.4564|)$$
$$+ (|0.326 - 0.2626| + |\,0.4816 - 0.4816| + |0.4375 - 0.621\,|)\big)$$
$$\mathbb{D}^{1-}\left(\bar{\bar{S}}_1^{\omega}, \bar{\bar{S}}^{\omega^-}\right) = 0.0183$$

Step 7: Calculate the relative closeness co-efficient

Now we will calculate relative closeness index using

$$\mathbb{RC}^{i} = \frac{\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^-}\right)}{\max\limits_{i}\left\{\mathbb{D}^{i-}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^-}\right)\right\}} - \frac{\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^+}\right)}{\min\limits_{i}\left\{\mathbb{D}^{i+}\left(\bar{\bar{A}}_{ij}^{\omega}, \bar{\bar{A}}_{j}^{\omega^+}\right)\right\}}$$

We get,

$$\mathbb{RC}^{1} = \frac{0.0183}{0.1023} - \frac{0.1081}{0.0283} = -3.64$$

$$\mathbb{RC}^{2} = \frac{0.1023}{0.1023} - \frac{0.0361}{0.0283} = -0.27$$

$$\mathbb{RC}^{3} = \frac{0.0629}{0.1023} - \frac{0.0723}{0.0283} = -1.93$$

$$\mathbb{RC}^4 = \frac{0.0968}{0.1023} - \frac{0.0283}{0.0283} = -0.05$$

Since we know that our set of four selected proposal is $\{\mathbb{R}^2, \mathbb{R}^4, \mathbb{R}^5, \mathbb{R}^6\}$ for $i = 1,2,3,4$ respectively. So, we rank the selected alternatives (shown in Figure: 3) according to the descending order of relative closeness index as;

$$\mathbb{R}^6 > \mathbb{R}^4 > \mathbb{R}^5 > \mathbb{R}^2$$

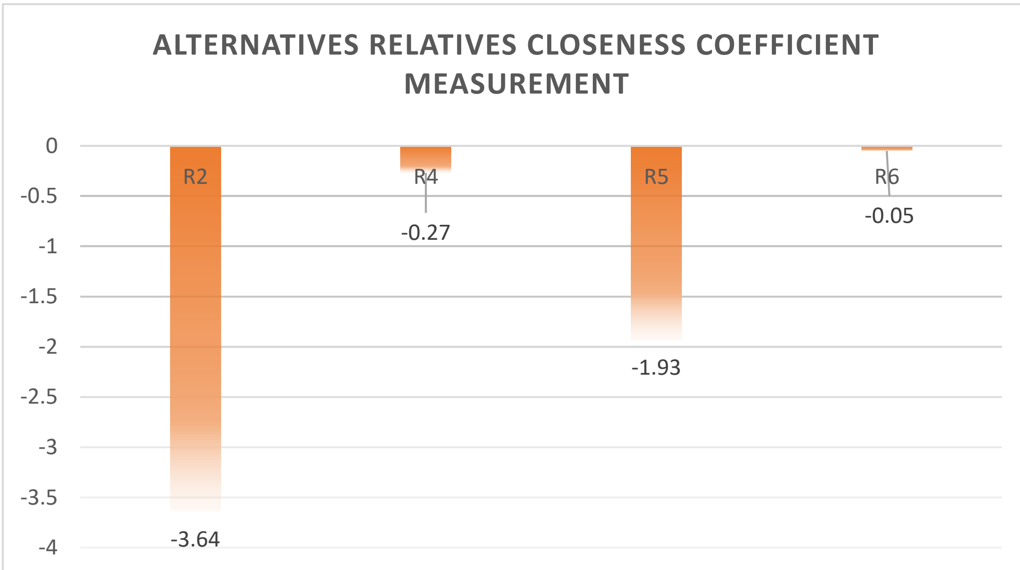

Figure: 3 Alternatives relative closeness coefficient measurement and ranking

This show that $\mathbb{R}^6$ is the best proposal.

Result Comparison

The proposed Algorithm is compared with the existing interval valued techniques which are widely used in decision-making problems and results are presented in Table: 1 listing the ranking of top four alternatives and optimal alternative.

Methods	Ranking of alternatives	Best alternative
I. Deli [39]	$\mathbb{R}^6 > \mathbb{R}^4 > \mathbb{R}^5 > \mathbb{R}^2$	$\mathbb{R}^6$
S. Alkhazaleh [40]	$\mathbb{R}^6 > \mathbb{R}^5 > \mathbb{R}^4 > \mathbb{R}^2$	$\mathbb{R}^6$
Riaz et al. (Proposed TOPSIS)	$\mathbb{R}^6 > \mathbb{R}^4 > \mathbb{R}^5 > \mathbb{R}^2$	$\mathbb{R}^6$

Table 1: Comparison analysis of final ranking with existing methods

6. Conclusion

In this chapter, TOPSIS technique has been proposed for interval valued neutrosophic hypersoft set IVNHSS by using generalized weighted similarity measure and distance measures for both IVNHSS and IVNHSM and the implementation is shown by letting a case study of life partner section. Also, chapter consists of the definitions of IVNHSM, generalized weighted similarity measures for interval valued neutrosophic hypersoft set and interval valued neutrosophic hypersoft matrix. In this process, decision makers' opinions are consolidated initially. Further, we construct ideal matrix to find the weight of decision makers using the concept of interval valued similarity measure. Afterwards, we compute a group opinion by accumulating all discrete opinions. We also gauge the weights of selected attributes to find the interval valued neutrosophic hypersoft 'positive ideal' and interval valued neutrosophic hypersoft 'negative ideal'. Interval valued having distance measure is employed to compute the distance of alternatives from positive ideal and negative ideal. Thereafter, we rank the alternatives on the basis of relative closeness index.

- This proposed TOPSIS technique with interval valued neutrosophic hypersoft set IVNHSS cannot be compared because no work is present in this direction.
- It has immense chances for multi criteria decision making issues in several fields like HR selection, supplier determination, manufacturing frameworks, and various other areas of management frameworks.
- In enlargement, the proposed TOPSIS approach can be augmented in various directions to involve wide range of decision-making issues in several interval valued neutrosophic hypersoft conditions.

In future, we propose matrix operations, theorems and propositions for interval valued neutrosophic hypersoft set along with algorithms and also the aggregate operators for IVNHSS.

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Chapter 7

The Application of the Score Function of Neutrosophic Hypersoft Set in the Selection of SiC as Gate Dielectric For MOSFET

Muhammad Umer Farooq[1], Muhammad Saqlain[1*], Zaka-ur-Rehman[2]

[1] Department of Physics, Govt. Degree College Bhera, Sargodha, Pakistan. E-mail: umerfarooq900@gmail.com
[1*] School of Mathematics, Northwest University, Xian, China. E-mail: msgondal0@gmail.com
[2] Department of Physics, University of Lahore, Sargodha Campus, Pakistan. E-mail: zakaurrehman52@gmail.com

Abstract: Neutrosophic hypersoft set are useful when attributes are more than one and are further bi-furcated. These sets are more accurate, precise and suitable for the ranking in decision making problems. Keeping in view; in this chapter, we present the study of various high k gate dielectrics for the metal oxide semiconductor filed effect transistor (MOSFET). The objective of this work is to choose the most suitable high k gate dielectric for the MOSFETs which can be used as gate dielectric. By applying the score function of neutrosophic hypersoft set for the selection of best MOSFETS since these are the most important part of all the electronic communication systems today. In the last results conclude that $D_4 = Al_2O_3$ is the best alternative that can be used as an optimal choice in digital communication devices.

Keywords: *SiC, MOSFET, Gate Dielectric, Score Function, Neutrosophic Hypersoft Set (NHSS), MCDM.*

1. Introduction

Si based semiconductor devices have been used in power electronics applications for years. But such devices have certain limitations in terms of their operation at higher voltages, temperatures and switching speeds [1]. The voltage range for Si based semiconductor devices is limited to 6.5 kV which the switching frequency is limited to several hundred hertz [2, 3]. Also, the maximum temperature limit for such devices is 200 ^{0}C [1]. Due to these problems, the wide band gap semiconductors like GaN and SiC have caught the attention of the researchers since first realization of SiC based metal oxide semiconductor field effect transistor (MOSFET). These semiconductor devices operate at higher voltages, higher temperatures and have higher switching frequencies as compared to Si based semiconductor devices [4-6]. SiC based field effect transistors (FET) have a voltage range 650 V to 1.7 kV [7]. Semiconductors are the essential part of all the communication systems today. The semiconductor devices have significantly reduced both power losses and cost of the communication systems. These devices are used for high power applications in all type of environments [8-17]. SiC is very suitable candidate for high power and high temperature applications due to their larger band gap and larger conductivity. Moreover, these materials offer larger carrier saturation velocity. Such exceptional properties of SiC makes it ideal for its applications in high power communication devices [17-27]. It is used largely in devices like Schottky Diode [10], junction field effect transistors [15, 16], bipolar junction transistors [11-14], metal oxide semiconductor field effect transistors [18-22] and insulated gate field effect transistors [23-27]. In spite of such novel properties, SiC has some drawbacks too. It has low dielectric constant and poor interface properties at SiC/SiO_2 junction in MOSFETs. This causes an increase in electric field at junction interface. So, the search for new gate dielectrics as an alternate of SiC is required. The new gate dielectric may have dielectric constant similar to SiC but smaller interface densities than SiC for its use as gate dielectric in MOSFETs. The high k gate dielectrics like HfO_2, AlN, Al_2O_3, Y_2O_3, Ta_2O_5 and La_2O_3 are the potential candidates for gate dielectric as an alternate of SiC in MOSFETs [28-45]. With the development of fuzzy sets [46] decision making becomes easier but later on this theory was extended by [47] named as Intuitionistic fuzzy number theory. To deal with more precision, accuracy and indeterminacy this idea was extended by [48] called as neutrosophy theory. To, discuss the applications of these theory number of developments were made but the most important one is the theory of soft set [49]. Later on, fuzzy, intuitionist and neutrosophy theories were extended to fuzzy softset [50], intuitionistic soft set [51] and neutrosophic soft set [52]. In different fields the applications of these theories are presented by many researchers [53-60], but with the development of TOPSIS, WSM and WPM techniques [61-66] it becomes more powerful tool to solve the MCDM problems [67-72]. Samarandache [73] came up with strategy to handle uncertainty. Soft sets were generalized to hyper soft set (HSS's) by changing the function into multi decision function. When attributes are more than one and further diverge, neutrosophic soft set environment cannot help to handle such type of issue because of soft sets as soft sets deal with single argument function. For this purpose, neutrosophic hypersoft set (NHSS) [74] was introduced. Later, aggregate operators, similarity measures and their applications are presented by [75-78].

Now the question arises why we are using these techniques in this case study? To get the answer of this question, firstly you need to know the attribute and alternatives; since dielectrics are of many types having different properties which makes it a perfect problem to apply the above-mentioned MCDM techniques. Semiconductors are the essential part of all the communication systems today. The novel characteristics of SiC make it very suitable for high power communication. But due to problem of its low dielectric constant and poor interface properties, the search for new gate dielectric is essential. These new dielectrics may have dielectric constant similar to SiC but have smaller interface densities. In this research, ten such high k-gate dielectrics are being analyzed. The purpose of this research is to find which dielectric is most suitable alternate of SiC for high power communication with the help of MOSFETs. For this purpose, we apply neutrosophic hypersoft sets. The layout of this chapter is shown in Figure:1. Introduction, Literature review, motivation and contribution is presented in section 1. Next section includes the basic definitions. Section 3, comprises of score function of NHSS. The case study is presented in section 4. Result discussion is made in section 5. Finally, in section 6, the chapter is concluded with limitation and future directions.

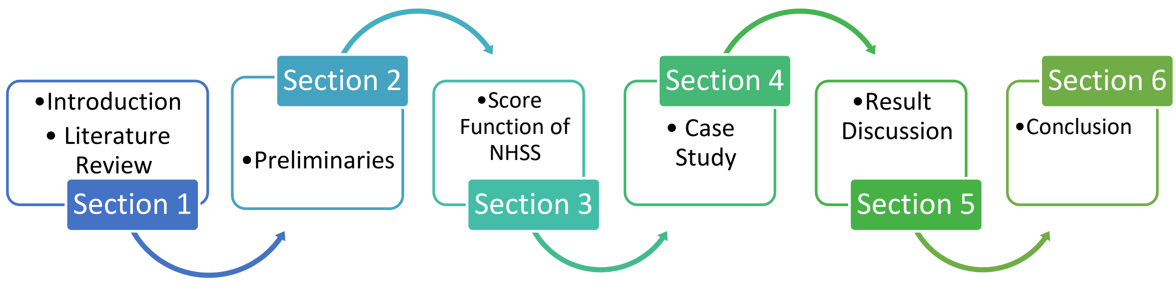

Figure 1: The layout of the chapter

2.Preliminaries

Definition 2.1: Semiconductors [79]

These are the materials which are poorer conductors than metals but better than insulators. For example, Silicon (Si), Germanium (Ge) and Gallium Arsenide.

Definition 2.2: Transistors [79]

It is an electrical device which consist of two PN junctions fabricated on a same single crystal. It has three main parts i.e. emitter, base and collector. Transistors are used to amplify and switch the electronic signals and electrical power.

Definition 2.3: Field Effect Transistors (FET) [79]

It is a type of the transistor in which flow of majority charge carrier (current) is controlled by the signal voltage applied to a reverse biased pn junction. This reverse biased pn junction is called gate. FET has three terminals i.e. source, gate and drain.

Definition 2.4: Metal Oxide Semiconductor FET (MOSFET) [79]

The MOSFET is a four terminal device i.e. source, gate, drain and substrate. The substrate is usually grounded. The charge carriers enter into conducting channel through source and exit through drain. The width of the channel is controlled by applying the voltage at the gate. The gate is present between source and drain. It is insulated from the channel near an extremely thin layer of metal oxide. The MOS capacity that exists in the device is a very important part as the entire operation occur across this. The metal contact over the insulator is known as gate electrode which is separated by a dielectric like Silicon dioxide or high k dielectric from the substrate.

Definition 2.4: Dielectrics

These are the materials which are poor conductors of electricity. It is an insulator but an effective supporter to electric filed.

Definition 2.5: Dielectrics Constant [80]

It can be defined as the ratio of the charge stored in the presence of an insulating material placed between two metallic plates to the charge that can be stored when the insulating material is replaced by vacuum or air.

Definition 2.6: High k Dielectrics Materials [81]

Dielectric materials which have high value of dielectric constant are called high k dielectric materials. Metal oxides have usually have high dielectric constant. They are usually used in MOSFETs as gate dielectric.

Definition 2.7: Band Gap [82]

The energy difference between the top of the valence band and the bottom of the conduction band is called band gap.

Definition 2.7: Wide Band Gap Materials [83]

These are the materials which have relatively larger band gap as compared to conventional semiconductors. Conventional semiconductors like silicon have a bandgap in the range of 1 - 1.5 eV, whereas wide-bandgap materials have bandgaps in the range of 2 - 4 eV.

Definition 2.8: Neutrosophic Hypersoft Set (NHSS) [74]

Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$be the power set of $\mathbb{U}$. Consider $l^1, l^2, l^3 \ldots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \ldots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$ and their relation $L^1 \times L^2 \times L^3 \ldots L^n = \$$, then the pair $(₣, \$)$ is said to be Neutrosophic Hypersoft set (NHSS) over $\mathbb{U}$ where;

$₣: L^1 \times L^2 \times L^3 \ldots L^n \to \mathbb{P}(\mathbb{U})$and

$₣(L^1 \times L^2 \times L^3 \ldots L^n) = \{< x, T(₣(\$)), I(₣(\$)), F(₣(\$)) >, x \in \mathbb{U}\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T, I, F: \mathbb{U} \to [0,1]$ also $0 \leq T(₣(\$)) + I(₣(\$)) + F(₣(\$)) \leq 3$.

3. Algorithm of Score Function of Neutrosophic Hypersoft set (NHSS)

In this section an algorithm is presented to solve MCDM problem under neutrosophic hypersoft set environment.

Suppose that there are some decision makers who wish to select from α number of objects. Each object is further characterized by β number of attributes, whose respective attributes form a relation just like Neutrosophic Hypersoft Matrix (NHSM). Each decision makes gives different Neutrosophic values to these respective attributes. Corresponding to these Neutrosophic values for the required relation we get a NHSM of order $\alpha \times \beta$. From these NHSM we calculate values matrices which helps to obtain a score matrix. And finally, we calculate the total score of each object from score matrix. Algorithm is presented in Figure 2.

Let $\mathcal{O} = [\mathcal{O}_{ij}]$ be the NHSM of order $\alpha \times \beta$, where $\mathcal{O}_{ij} = (\mathcal{T}^{o}_{ijk}, \mathcal{I}^{o}_{ijk}, \mathcal{F}^{o}_{ijk})$ then the value of matrix $\mathcal{O}$ is denoted as $\mathcal{V}(\mathcal{O})$ and it is defined as $\mathcal{V}(\mathcal{O}) = [\mathcal{V}^{O}_{ij}]$ of order $\alpha \times \beta$, where$\mathcal{V}^{O}_{ij} = \mathcal{T}^{o}_{ijk} - \mathcal{I}^{o}_{ijk}, -\mathcal{F}^{o}_{ijk}$. The Score of two NHSM $\mathcal{O} = [\mathcal{O}_{ij}]$ and $\mathcal{M} = [\mathcal{M}_{ij}]$ of order $\alpha \times \beta$ is given as $\mathcal{S}(\mathcal{O}, \mathcal{M}) = \mathcal{V}(\mathcal{O}) + \mathcal{V}(\mathcal{M})$ and $\mathcal{S}(\mathcal{O}, \mathcal{M}) = [\mathcal{S}_{ij}]$ where $\mathcal{S}_{ij} = \mathcal{V}^{O}_{ij} + \mathcal{V}^{\mathcal{M}}_{ij}$. The total score of each object in universal set is $\left|\sum_{j=1}^{n} \mathcal{S}_{ij}\right|$.

Step 1: Construct a NHSM.

Step 2: Calculate the value matrix from NHSM. Let $\mathcal{O} = [\mathcal{O}_{ij}]$ be the NHSM of order $\alpha \times \beta$, where $\mathcal{O}_{ij} = (\mathcal{T}^{o}_{ijk}, \mathcal{I}^{o}_{ijk}, \mathcal{F}^{o}_{ijk})$ then the value of matrix $\mathcal{O}$ is denoted as $\mathcal{V}(\mathcal{O})$ and it is defined as $\mathcal{V}(\mathcal{O}) = [\mathcal{V}^{O}_{ij}]$ of order $\alpha \times \beta$, where$\mathcal{V}^{O}_{ij} = \mathcal{T}^{o}_{ijk} - \mathcal{I}^{o}_{ijk}, -\mathcal{F}^{o}_{ijk}$.

Step 3: Compute score matrix with the help of value matrices. . The Score of two NHSM $\mathcal{O} = [\mathcal{O}_{ij}]$ and $\mathcal{M} = [\mathcal{M}_{ij}]$ of order $\alpha \times \beta$ is given as $\mathcal{S}(\mathcal{O}, \mathcal{M}) = \mathcal{V}(\mathcal{O}) + \mathcal{V}(\mathcal{M})$ and $\mathcal{S}(\mathcal{O}, \mathcal{M}) = [\mathcal{S}_{ij}]$ where $\mathcal{S}_{ij} = \mathcal{V}^{O}_{ij} + \mathcal{V}^{\mathcal{M}}_{ij}$.

Step 4: Compute total score from score matrix. The total score of each object in universal set is $\left|\sum_{j=1}^{n} \mathcal{S}_{ij}\right|$.

Step 5: Find optimal solution by selecting an object of maximum score from total score matrix.

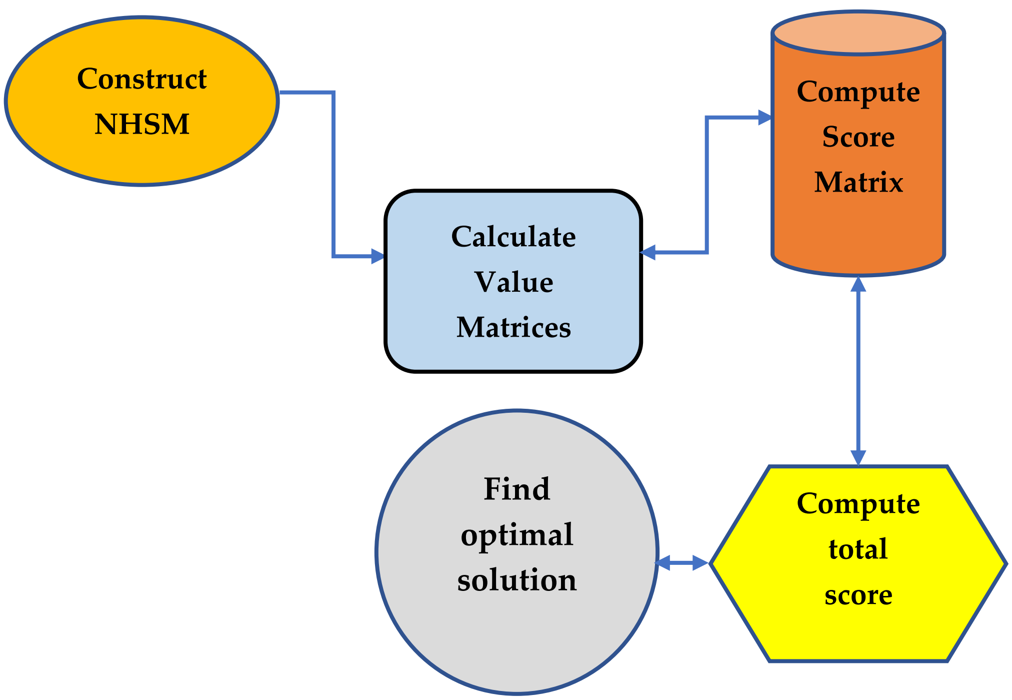

Figure 2: Graphical representation of Score Function Algorithm of NHSS

4: Case Study

In this section a case study of SiC selection as gate dielectric for high power communication is considered and the selection is made by applying all the above-mentioned algorithm.

4.1 Problem Formulation

SiC based MOSFETs are widely used in high power communication devices. But due to its poor interface properties, the search for new dielectrics which have similar dielectric constant as SiC but have good interface properties is required. So, the more efficient high-power communication devices can be constructed with the help of these high k-gate dielectrics. These devices can operate at higher temperatures and higher voltages.

4.3 Assumptions

Sr. No.	**Alternative**	**Dielectric Constant (k)**	**Band Gap $E_g\ (eV)$**	**Conduction band with respect to Si $\Delta E_c\ (eV)$**	**Conduction band with respect to 4 H-SiC $\Delta E_c\ (eV)$**
1	SiO_2	3.9	8.9	3.2	2.2 - 2.7
2	Si_3N_4	7.0	5.1	2.0	--
3	$SiON$	4.0 - 7.0	5.0 – 9.0	2.8	--
4	Al_2O_3	9.0	8.7	2.8	1.7
5	HfO_2	25	5.7	1.5 – 1.7	0.7 t0 1.6
6	ZrO_2	25	7.8	1.4	1.6
7	Ta_2O_5	26	4.5	1 – 1.5	--
8	Y_2O_3	15	5.6	2.3	--
9	La_2O_3	30	4.3	2.3	--
10	AlN	9.14	6.2	2.2	1.7

Let U be the set of all dielectric materials that can be used at junction interface with SiC,
U= $\{D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9, D_{10}\}$

Where $D_1 = SiO_2$, $D_2 = Si_3N_4$, $D_3 = SiON$, $D_4 = Al_2O_3$, $D_5 = HfO_2$, $D_6 = ZrO_2$, $D_7 = Ta_2O_5$, $D_8 = Y_2O_3$, $D_9 = La_2O_3$, $D_{10} = AlN$

Let us consider the following attributes;

$\boldsymbol{A_1} =$ Dielectric constant

$\boldsymbol{A_2} =$ Band Gap $E_g\ (eV$

$\boldsymbol{A_3} =$ Conduction band with respect to Si $\Delta E_c\ (eV)$

$\boldsymbol{A_4} =$ Conduction band with respect to 4 H-SiC $\Delta E_c\ (eV)$

So D_i = Universal set of dielectrics where i= 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

and

A_i = Set of attributes where i= 1, 2, 3, 4

$A_1^a =$ Dielectric constant $= \{3.9, 7.0, 4.0-7.0, 9.0, 25, 25, 26, 15, 30, 9.14\}$

$A_2^b =$ Band Gap $E_g\ (eV) = \{8.9, 5.1, 5-9, 8.7, 5.7, 7.8, 4.5, 5.6\ , 4.3, 6.2\}$

$$A_3^c = \text{ Conduction band with respect to Si } \Delta E_c\ (eV)$$
$$= \{3.2, 2.0, 2.8, 2.8, 1.5-1.7, 1.4, 1-1.5, 2.3, 2.3, 2.2\}$$

$A_4^d =$ Conduction band with respect to 4 H − SiC $\Delta E_c\ (eV) = \{2.2-2.7, -, -, 1.7, 0.54, 1.6, -.-, -, 1.7\}$

Let's assume the relation for the function $\mathcal{F}: A_1^a \times A_2^b \times A_3^c \times A_4^d \to P(U)$ *as;*

$$\mathcal{F}(A_1^a \times A_2^b \times A_3^c \times A_4^d \times A_5^e) = (\ 3.9, 5.1, 1.4, 1.6)$$

is the actual requirement for the selection of material for making dielectric.

Ten dielectric materials $\{D_1\ , D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9, D_{10}\}$ are selected on the basis of assumed relation i.e.(3.9,5.1,1.4,1.6).

Two decision makers $\{S^1, S^2\}$ are intended to select the most suitable material for the formation of respective dielectric. These decision makers give their opinion in the form of NHSM separately as;

Step 1: Construction of NHSM

$$[S^1]_{10\times 4} = \begin{bmatrix} (3.9,(0.8,0.1,0.05)) & (5.1,(0.2,0.2,0.6)) & (1.4,(0.5,0.3,0.1)) & (1.6,(0.6,0.05,0.2)) \\ (3.9,(0.7,0.2,0.1)) & (5.1,(0.5,0.1,0.5)) & (1.4,(0.7,0.2,0.05)) & (1.6,(0.1,0.5,0.7)) \\ (3.9,(0.5,0.05,0.2)) & (5.1,(0.7,0.05,0.1)) & (1.4,(0.7,0.1,0.05)) & (1.6,(0.5,0.2,0.3)) \\ (3.9,(0.8,0.05,0.05)) & (5.1,(0.8,0.05,0.05)) & (1.4,(0.7,0.05,0.05)) & (1.6,(0.8,0.05,0.1)) \\ (3.9,(0.2,0.2,0.6)) & (5.1,(0.8,0.1,0.05)) & (1.4,(0.5,0.05,0.2)) & (1.6,(0.6,0.05,0.2)) \\ (3.9,(0.7,0.2,0.1)) & (5.1,(0.7,0.2,0.1)) & (1.4,(0.8,0.05,0.05)) & (1.6,(0.2,0.5,0.1)) \\ (3.9,(0.7,0.1,0.05)) & (5.1,(0.7,0.05,0.1)) & (1.4,(0.8,0.05,0.05)) & (1.6,(0.7,0.2,0.05)) \\ (3.9,(0.5,0.2,0.3)) & (5.1,(0.7,0.05,0.05)) & (1.4,(0.7,0.05,0.05)) & (1.6,(0.1,0.2,0.7)) \\ (3.9,(0.8,0.05,0.1)) & (5.1,(0.7,0.2,0.1)) & (1.4,(0.6,0.05,0.2)) & (1.6,(0.5,0.2,0.3)) \\ (3.9,(0.2,0.2,0.6)) & (5.1,(0.7,0.05,0.1)) & (1.4,(0.6,0.05,0.2)) & (1.6,(0.2,0.5,0.1)) \end{bmatrix}$$

$$[S^2]_{10\times 4} = \begin{bmatrix} (3.9,(0.5,0.2,0.3)) & (5.1,(0.1,0.2,0.7)) & (1.4,(0.2,0.5,0.1)) & (1.6,(0.2,0.5,0.1)) \\ (3.9,(0.8,0.05,0.05)) & (5.1,(0.8,0.05,0.05)) & (1.4,(0.8,0.05,0.05)) & (1.6,(0.7,0.2,0.05)) \\ (3.9,(0.7,0.2,0.1)) & (5.1,(0.8,0.1,0.05)) & (1.4,(0.8,0.05,0.05)) & (1.6,(0.1,0.2,0.7)) \\ (3.9,(0.2,0.2,0.6)) & (5.1,(0.8,0.05,0.1)) & (1.4,(0.6,0.05,0.2)) & (1.6,(0.5,0.2,0.3)) \\ (3.9,(0.2,0.2,0.6)) & (5.1,(0.8,0.1,0.05)) & (1.4,(0.5,0.05,0.2)) & (1.6,(0.6,0.05,0.2)) \\ (3.9,(0.7,0.2,0.1)) & (5.1,(0.7,0.2,0.1)) & (1.4,(0.8,0.05,0.05)) & (1.6,(0.2,0.5,0.1)) \\ (3.9,(0.8,0.05,0.1)) & (5.1,(0.2,0.2,0.6)) & (1.4,(0.7,0.05,0.5)) & (1.6,(0.2,0.1,0.05)) \\ (3.9,(0.7,0.05,0.05)) & (5.1,(0.7,0.05,0.1)) & (1.4,(0.5,0.2,0.3)) & (1.6,(0.7,0.2,0.1)) \\ (3.9\ ,(0.7,0.1,0.05)) & (5.1,(0.7,0.1,0.05)) & (1.4,(0.8,0.05,0.2)) & (1.6,(0.2,0.5,0.1)) \\ (3.9,(0.8,0.05,0.05)) & (5.1,(0.8,0.05,0.1)) & (1.4,(0.7,0.5,0.2)) & (1.6,(0.8,0.05,0.05)) \end{bmatrix}$$

Step 2: Calculate the value matrices of NHSMs defined above using $\mathcal{V}_{ij}^{s} = \mathcal{T}_{ijk}^{s} - \mathcal{I}_{ijk}^{s} - \mathcal{F}_{ijk}^{s}$

$$[S^1]_{10\times 4} = \begin{bmatrix} (3.9,(0.65)) & (5.1,(-0.6)) & (1.4,(0.1)) & (1.6,(0.35)) \\ (3.9,(0.4)) & (5.1,(-0.1)) & (1.4,(0.45)) & (1.6,(-1.1)) \\ (3.9,(0.25)) & (5.1,(0.55)) & (1.4,(0.55)) & (1.6,(0.0)) \\ (3.9,(0.7)) & (5.1,(0.7)) & (1.4,(0.6)) & (1.6,(0.65)) \\ (3.9,(-0.6)) & (5.1,(0.65)) & (1.4,(0.25)) & (1.6,(0.35)) \\ (3.9,(0.4)) & (5.1,(0.4)) & (1.4,(0.7)) & (1.6,(-0.4)) \\ (3.9,(0.55)) & (5.1,(0.55)) & (1.4,(0.7)) & (1.6,(0.45)) \\ (3.9,(0.0)) & (5.1,(0.6)) & (1.4,(0.6)) & (1.6,(-0.8)) \\ (3.9,(0.65)) & (5.1,(0.4)) & (1.4,(0.35)) & (1.6,(0.0)) \\ (3.9,(-0.6)) & (5.1,(0.55)) & (1.4,(0.35)) & (1.6,(-0.4)) \end{bmatrix}$$

$$[S^2]_{10\times 4} = \begin{bmatrix} (3.9,(0.0)) & (5.1,(-0.8)) & (1.4,(-0.4)) & (1.6,(-0.4)) \\ (3.9,(0.7)) & (5.1,(0.7)) & (1.4,(0.7)) & (1.6,(0.45)) \\ (3.9,(-0.6)) & (5.1,(0.65)) & (1.4,(0.7)) & (1.6,(-0.8)) \\ (3.9,(0.6)) & (5.1,(0.65)) & (1.4,(0.35)) & (1.6,(0.0)) \\ (3.9,(0.6)) & (5.1,(0.65)) & (1.4,(0.25)) & (1.6,(0.35)) \\ (3.9,(0.4)) & (5.1,(0.4)) & (1.4,(0.7)) & (1.6,(-0.4)) \\ (3.9,(0.65)) & (5.1,(-0.6)) & (1.4,(0.15)) & (1.6,(0.05)) \\ (3.9,(0.6)) & (5.1,(0.55)) & (1.4,(0.0)) & (1.6,(0.4)) \\ (3.9,(0.55)) & (5.1,(0.55)) & (1.4,(0.55)) & (1.6,(-0.4)) \\ (3.9,(0.7)) & (5.1,(0.65)) & (1.4,(0.0)) & (1.6,(0.7)) \end{bmatrix}$$

Step 3: Now compute score matrix by adding value matrices obtained in Step 2.

$$[S(S^1S^2)]_{10\times 4} = \begin{bmatrix} (3.9,(0.65)) & (5.1,(-1.4)) & (1.4,(-0.3)) & (1.6,(-0.05)) \\ (3.9,(1.1)) & (5.1,(0.6)) & (1.4,(1.15)) & (1.6,(-0.65)) \\ (3.9,(-0.35)) & (5.1,(1.2)) & (1.4,(1.25)) & (1.6,(-0.8)) \\ (3.9,(1.3)) & (5.1,(1.35)) & (1.4,(0.95)) & (1.6,(0.65)) \\ (3.9,(0.0)) & (5.1,(1.3)) & (1.4,(0.5)) & (1.6,(0.7)) \\ (3.9,(0.8)) & (5.1,(0.8)) & (1.4,(1.4)) & (1.6,(-0.8)) \\ (3.9,(1.2)) & (5.1,(-0.05)) & (1.4,(0.85)) & (1.6,(0.5)) \\ (3.9,(0.6)) & (5.1,(1.15)) & (1.4,(0.6)) & (1.6,(-0.4)) \\ (3.9,(1.2)) & (5.1,(0.95)) & (1.4,(0.9)) & (1.6,(-0.4)) \\ (3.9,(0.1)) & (5.1,(1.2)) & (1.4,(0.35)) & (1.6,(0.3)) \end{bmatrix}$$

Step 4: Now total score of score matrix is given as;

$$Total\ score = \begin{bmatrix} \mathbf{1.1} \\ \mathbf{2.2} \\ \mathbf{1.3} \\ \mathbf{4.25} \\ \mathbf{2.5} \\ \mathbf{2.2} \\ \mathbf{2.5} \\ \mathbf{1.95} \\ \mathbf{2.65} \\ \mathbf{1.95} \end{bmatrix}$$

Step 5: The material of dielectric D^4 will be selected as the total score of D^4 is highest among the rest of the total score of dielectrics.

Alternative	Total Score Value
D1	1.1
D2	2.2
D3	1.3
D4	4.25
D5	2.5
D6	2.2
D7	2.5
D8	1.95
D9	2.65
D10	1.95

Table 1: The total score function of each dielectric taken as alternative

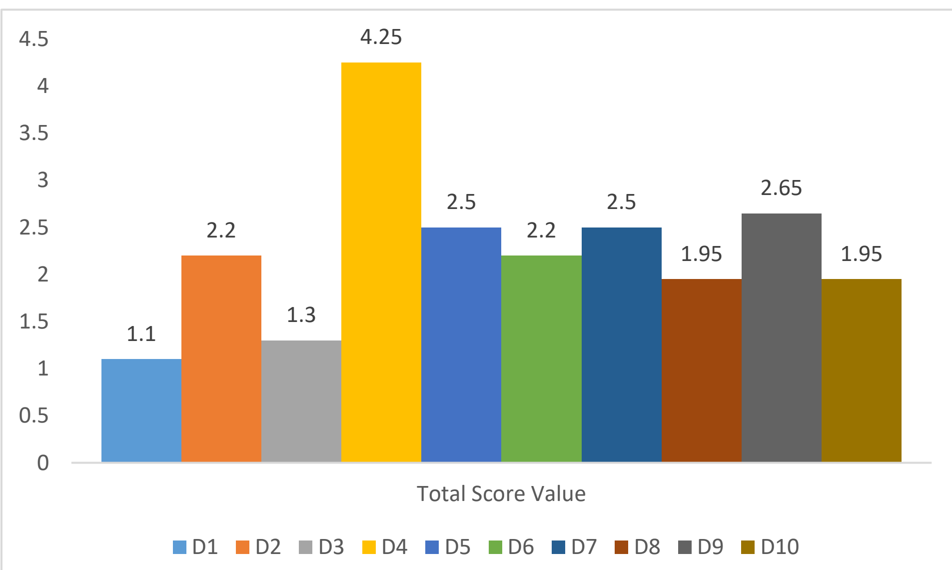

Figure 3: Total score value comparison of alternatives

6. Result Discussion

Everyday many researchers are trying to find the best operational research technique that can be useful in decision making problems. In very recent with the development of NHSS and its score function; it seems to be the success of the researchers who are working on it. Since NHSS is the tool which is most suitable in decision making problems and can be applied when attributes are more than one and are further bi-furcated. For the validity of the algorithm of score function in the applied mathematical issues and MCDM environment the case study of SiC gate dielectric for the MOSFET in communication devices is used.

In these calculations, the ranking of each dielectric with respect to each criterion is calculated which are shown in Table 1 and Figure 4. Result shows that the above-mentioned techniques can be used to find the optimal SiC dielectric in communication devises.

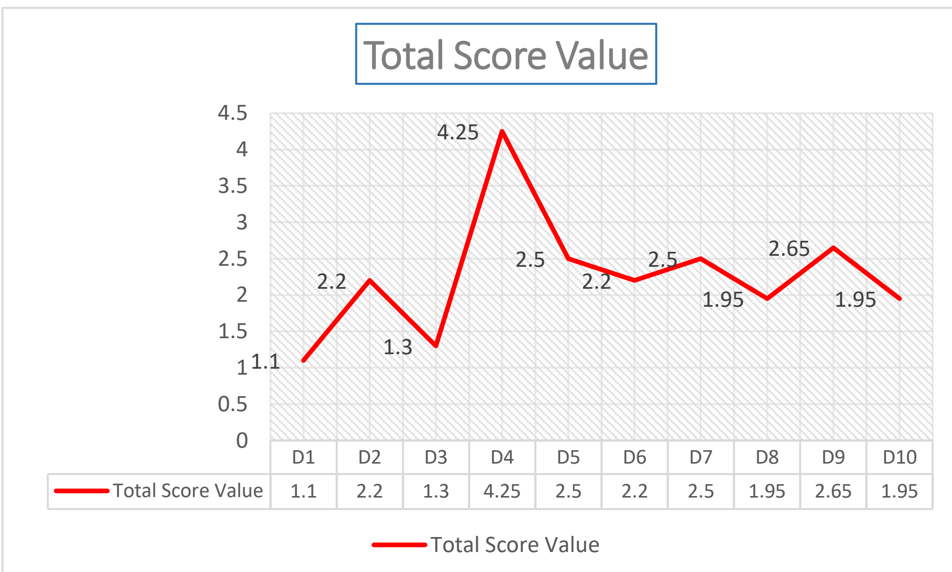

Figure 4: Ranking comparison of alternatives

The dielectric Al_2O_3 can be used as best alternative dielectric material which is most suitable in communication devices. So, the more efficient high-power communication devices can be constructed with the help of this high k-gate dielectric.

5. Conclusions

Among the ten potential alternatives for gate dielectric, the calculations show that Al_2O_3 is the most suitable high k-gate dielectric for the MOSFETs. Also Al_2O_3 has excellent lattice matching with SiC. It has good thermal stability and high conduction band off-set between Al_2O_3 and 4H-SiC. Moreover, it has high k- value and relatively larger dielectric band gap [28]. Such properties of Al_2O_3 makes it a very suitable substitute of SiC as gate dielectric. These calculations are done by applying score function of NHSS.

Since this study has not yet been studied yet, comparative study cannot be done with the existing methods.

In our forthcoming work, this method can be used to find the best alternative having same physical and structural properties in manufacturing process of different materials with low lost and good impact on environment.

Conflicts of Interest

The authors declare no conflict of interest.

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Chapter 8

Tangent, Cosine, and Ye Similarity Measures of m-Polar Neutrosophic Hypersoft Sets

Mahrukh Irfan[1], Maryam Rani[2], Muhammad Saqlain[3], Muhammad Saeed [1*]

[1] University of Management and Technology, Township, Lahore, 54000, Pakistan. Email: f2018265015@umt.edu.pk

[2] Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan. Email: Maryum.rani009@gmail.com

[3] School of Mathematics, Northwest University, Xian, China. Email: msgondal0@gmail.com

*Correspondence: Muhammad.saeed@umt.edu.pk

Abstract: In our daily lives, most of the problems are created due to wrong decisions. Similarity measures are very useful to make good decisions. In this manuscript, different similarities like Tangent, Ye, and Cosine similarity of m polar neutrosophic hypersoft sets and their comparison is presented. These techniques are very useful to deal with the problems whose attributes are numerous. In m-polar neutrosophic hypersoft set, attributes are large in number and further classified, that is why making effective and fruitful decision is cumbersome. Decision making become simple, easy and accurate through comparison of different proposed similarities. Finally, the proposed similarity measures applied to diagnose the Covid-19 will be extremely useful to prevent the spread of this virus.

Keywords: *m-Polar Neutrosophic Hypersoft Set (mPNHSS), Similarity Measure, Tangent Similarity Measure, Ye Similarity, Cosine Similarity.*

1. Introduction

A great researcher Lotfi A. Zadeh [4] proposed a revolutionary theory, known as fuzzy set theory in 1965 was based on membership values. In some problems it is difficult to allot the membership value, to overcome this problem Smarandache [10] extended this concept with the addition of indeterminacy and non-membership values along with the membership value, it is introduced as a neutrosophic set. Neutrosophic set is a scientific tool for handling all the issues which including indeterminacy, uncertain and conflicting information [5]. In this idea membership, indeterminacy, and non-membership values are independent. Theory of soft set was proposed by Molodtsov [2], it resolves the issue of uncertain conditions. He introduced the soft set by considering a universal set and the parameterized family of its subsets. Each element of the soft set is considered as an approximate element [7]. Soft sets play an important role in game prediction and basic decision-making problems [11]. From Molodtsov till now it is used in many applications and problems to connect the different fields of science and data research [2, 8, 9, 11]. Maji [15] introduced the concept of the neutrosophic soft set by combining the concept of neutrosophic set and soft set, it is used in decision-making problems [14] and for research purpose [13]. To deal with uncertainty Smarandache [6] introduced the concept of the hypersoft set by converting soft set function values into multi-decision function values. In TOPSIS and MCDM Smarandache gave brief explanations of numerous extensions of neutrosophic sets [24–29]. Saqlain [18] introduced neutrosophic hypersoft set converting it from hypersoft set to handle the problems of uncertainty. The neutrosophic hypersoft set can be used to predict games [20, 21]. Saeed and Saqlain introduced the concept of m polar neutrosophic soft set [3] and used in medical diagnosis and decision making. we convert the m-polar neutrosophic soft set into the m-polar neutrosophic hypersoft sets to handle a large number of attributes in medical diagnosis.

Many methods are used to measure the similarity between two neutrosophic hypersoft sets. The object of this paper is to use several similarity methods for m-polar neutrosophic hypersoft sets. The tangent similarity measure is introduced by Pramanik and Mondal [6], its properties and application also explained. J.Ye gave the idea of Ye similarity measure and explain its properties and applications [17]. Said Broumi and Florentin Smarandache [1] introduced cosine similarity which is based on Bhattacharya's distance [12], its properties also briefly described. Anjan Mukherjee, Abhik Mukherjee [33] introduced similarity measures of interval-valued fuzzy soft set for determining COVID-19 patients.

1.1. *Structure of this manuscript*

The proposed work is cataloged as follows, In Section 1 the related definitions of concepts for understanding soft set, neutrosophic set, neutrosophic hypersoft set, m-polar neutrosophic soft set are presented. In section 2 we proposed the basic definition of m polar neutrosophic hypersoft set and its aggregate operators like equal, null, subset, union, intersection. In section 3 comparison among the several similarity measures of m polar neutrosophic hypersoft set and graphical representation of the results have been done with the help of an illustrated example of covid-19. In section 4 finally, conclusion and future directions are presented.

1.2. *Motivation*

Saqlain et al presented the Tangent similarity measure of single-valued NHSS [23]. Many pieces of research involve multi-attributes, multi-agent, multi-object, multi-Polar, and multi-index information.To tackle these kind of situations, we theorized several similarity methods for m-Polar neutrosophic hyper soft sets. This approach will be extremely useful in MCDM, personal selection, coding theory, management problems, and medical diagnosis. In comparison with single-valued NHSS, m-Polar NHSS give more accurate results. We also applied this theory on medical diagnosis to show its importance in real life problems.

2. **Preliminaries**

In this section, soft set, hypersoft set, neutrosophic set, neutrosophic hypersoft set, m-polar neutrosophic soft set, m-polar neutrosophic hypersoft set are briefly explained.

2.1. *Soft Set*

Let Υ be a set of parameters or attributes concern the universal set K. P(K) represents the power set of the universal set K. To define soft set let B $\subseteq\Upsilon$. Here ϱ is a mapping which is defined as

ϱ: B $\rightarrow$ P(K)

Then a pair (ϱ, B) over K is called a soft set. For any a$\in$B, ϱ(a) may be written as a set of a-element of the soft set (ϱ, B). Then (ϱ, B) is defined as

(ϱ, B)=ϱ(a)$\in$ P(K) if a $\in$ B

ϱ(a)=ϕ if $a \notin B$

2.2. *HyperSoft Set*

The pair (ϱ, $k^1{\times}k^2{\times}k^3{\times}\ldots k^n$) is said to be a hypersoft set over K where ϱ is defined as

$\varrho : k^1{\times}k^2{\times}k^3{\times}\ldots k^n \rightarrow$ P(K)

Here K is the universal set and P(K) is its power set. Consider κ^1, κ^2, κ^3 ... κ^n for n$\geq$1 is the n well-defined attributes, whose corresponding attribute values are k^1, k^2, $k^3, \ldots k^n$ respectively with the condition

$k^i{\cap}k^j{=}\phi$ for i$\neq$j and i$\in${1, 2, 3, ...n}, j $\in$ {1, 2, 3, ...n}

2.3. *Neutrosophic Set*

Let K be a universal set and the neutrosophic set B is an object having the form

B={⟨x: T_B(x), I_B(x), F_B(x)⟩, x$\in$K}

Where the function T, I, F: K $\rightarrow$]-0, 1+[define respectively the truth membership function, indeterminacy function, and falsity membership function. A set is a neutrosophic set with the condition

$0^- \leq T_B(x)+I_B(x)+F_B(x)\leq 3^+$

To select the interval for the neutrosophic set from the philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]0^-, 1^+[$ it is difficult to apply in real applications for technical applications we take interval [0, 1] [1].

2.4. *Neutrosophic Hypersoft set*

Let K be the universal set and P(K) be the power set of K. Consider n well-defined attributes κ^1, κ^2, κ^3 ... κ^n for n≥1 whose corresponding attribute values are respectively $k^1,\ k^2,\ k^3 \ldots k^n$ with the condition

$k^i \cap k^j = \phi$ for $i \neq j$ and $i \in \{1, 2, 3, ...n\}$ $j \in \{1, 2, 3, ...n\}$

consider their relation

$k^1 \times\ k^2 \times\ k^3 \times \ldots k^n = \Psi$

define a mapping

λ: $\Psi \rightarrow$ P(K)

$\lambda(\Psi) = \{\langle x, T(\lambda(\Psi)), I(\lambda(\Psi)), F(\lambda(\Psi))\rangle, x \in K\}$

Then the pair (λ, Ψ) is said to be a neutrosophic hypersoft set over universal set K. Where $T(\lambda(\Psi))$ is truth membership function, $I(\lambda(\Psi))$ is the indeterminacy function and $F(\lambda(\Psi))$ is the falsity membership function such that

$T(\lambda(\Psi)), I(\lambda(\Psi)), F(\lambda(\Psi)) \rightarrow [0, 1]$

with the condition

$0 \leq T(\lambda(\Psi)) + I(\lambda(\Psi)) + F(\lambda(\Psi)) \leq 3$

2.5. *m-Polar Neutrosophic soft set*

An m-polar neutrosophic soft set is defined as if K is a universal set and P(K) is its power set Υ be the set of attributes concerning to K. B is said to be m-polar neutrosophic soft set over K if $B \subseteq \Upsilon$.

Its mapping is defined as

λ: B → P(K)

$(\lambda, B) = \{ \langle T^i\ \lambda(B), I^i\ \lambda(B), F^i\ \lambda(B)\rangle | k, k \in K\}$ $\forall$ i= 1, 2, 3, ...n

Here $T^i\lambda(B)$ denotes the degree of i^{th} truth membership function, $I^i\lambda(B)$ denotes the degree of i^{th} indeterminacy function and $F^i\lambda(B)$ denotes the degree of i^{th} falsity membership function for each element of $k \in K$ to the set B and hold the condition

$0 \leq T^i\lambda(B) + I^i\lambda(B) + F^i\lambda(B) \leq 3$

$\forall$ i=1, 2, 3, ...n

3. m-Polar Neutrosophic Hypersoft Set

Let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively $\Upsilon_1{}^1$, $\Upsilon_2{}^2$, ...$\Upsilon_m{}^n$ and their relation is $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$ then the pair(λ, $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$) is said to be m-polar neutrosophic hypersoft set over K. Let $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n=\Omega$

where λ is defined as

λ: $\Omega \to$ P(K)

$\lambda(\Omega)=\{\langle T_\Upsilon{}^i(k), I_\Upsilon{}^i(k), F_\Upsilon{}^i(k) \rangle\ k\in K, \Upsilon\in \Omega\}$ i=1, 2, ...,n

also

$0\leq\sum_{i=1}{}^n T_\Upsilon{}^i(k)\leq1$ $0\leq\sum_{i=1}{}^n I_\Upsilon{}^i(k)\leq1$ $0\leq\sum_{i=1}{}^n F_\Upsilon{}^i(k)\leq1$

where

$T_\Upsilon{}^i(k)\subseteq[0, 1]$ $I_\Upsilon{}^i(k)\subseteq[0, 1]$ $F_\Upsilon{}^i(k)\subseteq[0, 1]$

and holds the condition

$0\leq \sum_{i=1}{}^n T_\Upsilon{}^i(k)+\sum_{i=1}{}^n I_\Upsilon{}^i(k)+\sum_{i=1}{}^n F_\Upsilon{}^i(k)\leq3$

3.1. *Example*

Let K be the set of different teachers those are nominated for the best teacher of the year. consider

K{T^1, T^2, T^3, T^4, T^5}

Assumptions:

- Every teacher has the same probability to select.
- Independent attributes are considered.
- Hesitant Environment is not yet considered.

Problem formulation:

Assume the set of attributes as

$Q_1{}^a$= Academic Qualification (below masters, masters, above masters)

$Q_2{}^b$= Professional Qualification (yes, no)

$Q_3{}^c$= Professional Skills (good, average, poor)

$Q_4{}^d$= Teaching Methods (traditional, up to date)

Formulation

λ: $Q_1{}^a\times Q_2{}^b\times Q_3{}^c\times Q_4{}^d \to$ P(K)

consider

λ(masters, yes, good, up to date)={T^2, T^4}

Then the Neutrosophic hypersoft set of above assumed relation

λ(masters, yes, good, up to date)={⟨T^2,($Q_1{}^a$[0.9, 0.4, 0.1], $Q_2{}^b$[0.7, 0.3, 0.5], $Q_3{}^c$[0.4, 0.9, 0.7], $Q_4{}^d$[0.6, 0.3, 0.5]⟩, ⟨T^4 ($Q_1{}^a$[0.5, 0.3, 0.7], $Q_2{}^b$[0.5, 0.1, 0.3], $Q_3{}^c$[0.7, 0.5, 0.8], $Q_4{}^d$[0.4, 0.6, 0.3]⟩}

Then m-Polar Neutrosophic Hypersoft Set of above relation is

λ_1(masters, yes, good, up to date)={ T^2 ⟨ $Q_1{}^a$[0.01, 0.003, 0.1, 0.023, 0.07], [0.092, 0.073, 0.08, 0.2, 0.4], [0.2, 0.17, 0.06, 0.13, 0.3]⟩, ⟨$Q_2{}^b$[0.2, 0.1, 0.5, 0.019, 0.051], [0.21, 0.14, 0.27, 0.009, 0.1], [0.113, 0.35, 0.25, 0.12, 0.03]⟩, ⟨$Q_3{}^c$[0.12, 0.13, 0.14, 0.15, 0.39], [0.17, 0.20, 0.24, 0.15, 0.1], [0.2, 0.1, 0.5, 0.019, 0.051]⟩, ⟨$Q_4{}^d$[0.2, 0.17, 0.06, 0.13, 0.3], [0.12, 0.025, 0.07, 0.22, 0.074], [0.01, 0.003, 0.1, 0.023, 0.07]⟩}

λ_2(masters, yes, good, up to date)={T^4⟨ $Q_1{}^a$[0.09, 0.08, 0.7, 0.026, 0.05], [0.04, 0.03, 0.02, 0.1, 0.09], [0.32, 0.51, 0.06, 0.03, 0.02]⟩, ⟨$Q_2{}^b$[0.12, 0.13, 0.14, 0.15, 0.39], [0.17, 0.20, 0.041, 0.05, 0.1], [0.2, 0.1, 0.5, 0.019, 0.051]⟩, ⟨$Q_3{}^c$[0.09, 0.08, 0.7, 0.026, 0.05], [0.04, 0.03, 0.02, 0.1, 0.09], [0.02, 0.51, 0.06, 0.03, 0.12]⟩, ⟨$Q_4{}^d$[0.12, 0.13, 0.14, 0.15, 0.39], [0.12, 0.025, 0.07, 0.22, 0.074], [0.12, 0.025, 0.07, 0.22, 0.4]⟩}

3.2. *Aggregate Operators of m-Polar Neutrosophic Hypersoft Set*

In this manuscript, aggregate operators of m polar neutrosophic hypersoft set like the equal set, null set, subset, union, and intersection are explained. The validity and restrictions of proposed operators are briefly discussed. With the Help of aggregate operators, any set can be used to obtained desired results these operators are tools that are beneficial to use different sets in problems.

3.3. *m-Polar Neutrosophic Equal Hypersoft Set*

let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively $\Upsilon_1{}^1$, $\Upsilon_2{}^2$, ...$\Upsilon_m{}^n$ and their relation are $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$.

Let $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n=\Omega$

A(Ω)={⟨$T_{\Upsilon(A)}{}^i$(k), $I_{\Upsilon(A)}{}^i$(k), $F_{\Upsilon(A)}{}^i$(k) ⟩ k∈K, $\Upsilon\in\Omega$}

B(Ω)={⟨$T_{\Upsilon(B)}{}^i$(k), $I_{\Upsilon(B)}{}^i$(k), $F_{\Upsilon(B)}{}^i$(k) ⟩ k∈K, $\Upsilon\in\Omega$}

i=1, 2, ...,n

are two m-polar neutrosophic hypersoft sets over K. They are said to be m-polar neutrosophic equal hypersoft sets if it satisfied the following conditions

$T_{\Upsilon(A)}{}^{i}(k)=T_{\Upsilon(B)}{}^{i}(k)$ $I_{\Upsilon(A)}{}^{i}(k)=I_{\Upsilon(B)}{}^{i}(k)$ $F_{\Upsilon(A)}{}^{i}(k)=F_{\Upsilon(B)}{}^{i}(k)$

3.4. *m-Polar Neutrosophic Null Hypersoft Set*

let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively $\Upsilon_1{}^1$, $\Upsilon_2{}^2$, ...$\Upsilon_m{}^n$ and their relation are $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$.

Let $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n=\Omega$

$A(\Omega)=\{\langle T_{\Upsilon(A)}{}^{i}(k), I_{\Upsilon(A)}{}^{i}(k), F_{\Upsilon(A)}{}^{i}(k)\rangle\ k\in K,\ \Upsilon\in\Omega\}$

is said to be an m-polar neutrosophic Null hypersoft set over K.

if it satisfied the following conditions

$T_{\Upsilon(A)}{}^{i}(k)=0$ $I_{\Upsilon(A)}{}^{i}(k)=0$ $F_{\Upsilon(A)}{}^{i}(k)=0$

3.5. *m-Polar Neutrosophic Subset Hypersoft Set*

let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively $\Upsilon_1{}^1$, $\Upsilon_2{}^2$, ...$\Upsilon_m{}^n$ and their relation are $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$.

Let $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n=\Omega$

$A(\Omega)=\{\langle T_{\Upsilon(A)}{}^{i}(k), I_{\Upsilon(A)}{}^{i}(k), F_{\Upsilon(A)}{}^{i}(k)\rangle\ k\in K,\ \Upsilon\in\Omega\}$

$B(\Omega)=\{\langle T_{\Upsilon(B)}{}^{i}(k), I_{\Upsilon(B)}{}^{i}(k), F_{\Upsilon(B)}{}^{i}(k)\rangle\ k\in K,\ \Upsilon\in\Omega\}$

i=1, 2, ...,n

are two m-polar neutrosophic hypersoft sets over K. Then $A(\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n)$ is the m polar neutrosophic subset hypersoft set.

if it satisfied the following conditions

$T_{\Upsilon(A)}{}^{i}(k)\leq T_{\Upsilon(B)}{}^{i}(k)$ $I_{\Upsilon(A)}{}^{i}(k)\geq I_{\Upsilon(B)}{}^{i}(k)$ $F_{\Upsilon(A)}{}^{i}(k)\geq F_{\Upsilon(B)}{}^{i}(k)$

3.6. *Union of Two m-Polar Neutrosophic Hypersoft Sets*

let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively $\Upsilon_1{}^1$, $\Upsilon_2{}^2$, ...$\Upsilon_m{}^n$ and their relation are $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n$.

Let $\Upsilon_1{}^1\times\Upsilon_2{}^2\times...\Upsilon_m{}^n=\Omega$

$A(\Omega)=\{\langle T_{\Upsilon(A)}{}^{i}(k), I_{\Upsilon(A)}{}^{i}(k), F_{\Upsilon(A)}{}^{i}(k)\rangle\ k\in K,\ \Upsilon\in\Omega\}$

$B(\Omega)=\{\langle T_{\Upsilon(B)}{}^{i}(k), I_{\Upsilon(B)}{}^{i}(k), F_{\Upsilon(B)}{}^{i}(k)\rangle\ k\in K,\ \Upsilon\in\Omega\}$

i=1, 2, ...,n

are two m-polar neutrosophic hypersoft sets over K.

Then A(Ω) ∪ B(Ω) is given as

$$T(A(\Omega)\cup B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \max(T(A(\Omega)),T(B(\Omega))) & if x\in A\cup B)\end{cases}$$

$$I(A(\Omega)\cup B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \{\frac{(I(A(\Omega))+I(B(\Omega)))}{2}\} & if x\in A\cup B)\end{cases}$$

$$F(A(\Omega)\cup B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \min(F(A(\Omega)),T(B(\Omega))) & if x\in A\cup B)\end{cases}$$

3.7. *The Intersection of Two m-Polar Neutrosophic Hypersoft Sets*

let K={k^1, k^2, ...k^n} be the universal set and P(K) be the power set of K. Consider κ_1, κ_2, ...κ_m for m≥1 be m well-defined attributes whose corresponding attribute values are respectively ${\Upsilon_1}^1$, ${\Upsilon_2}^2$, ...${\Upsilon_m}^n$ and their relation are ${\Upsilon_1}^1\times{\Upsilon_2}^2\times...{\Upsilon_m}^n$.

Let ${\Upsilon_1}^1\times{\Upsilon_2}^2\times...{\Upsilon_m}^n=\Omega$

A(Ω)={⟨${T_{\Upsilon(A)}}^i$(k), ${I_{\Upsilon(A)}}^i$(k), ${F_{\Upsilon(A)}}^i$(k) ⟩ k∈K, ϒ∈ Ω }

B(Ω)={⟨${T_{\Upsilon(B)}}^i$(k), ${I_{\Upsilon(B)}}^i$(k), ${F_{\Upsilon(B)}}^i$(k) ⟩ k∈K, ϒ∈ Ω }

i=1, 2, ...,n

are two m-polar neutrosophic hypersoft sets over K.

Then A(Ω) ∩ B(Ω) is given as

$$T(A(\Omega)\cap B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \min(T(A(\Omega)),T(B(\Omega))) & if x\in A\cup B)\end{cases}$$

$$I(A(\Omega)\cap B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \{\frac{(I(A(\Omega))+I(B(\Omega)))}{2}\} & if x\in A\cup B)\end{cases}$$

$$F(A(\Omega)\cap B(\Omega))=\begin{cases}A(\Omega) & if x\in A\\ B(\Omega) & if x\in B\\ \max(F(A(\Omega)),T(B(\Omega))) & if x\in A\cup B)\end{cases}$$

4. Comparison among The Several Similarity Measure of m-Polar Neutrosophic Hypersoft sets

In two objects similarity measure is a numerical value of the degree according to the objects are same or nearly equal to each other. Usually, the values of similarities are positive and mostly lies between 0 and 1. Here 0 means there is no similarity and 1 means completely similar to each other, any similarity who answer near to 1 is more accurate and beneficial for research proposes. A similarity measure is very important in decision-making problems through calculating similarity measures among the ideal and given options. Let A and B be two m-polar neutrosophic hypersoft set in the universal set K=$\{ k_1, k_2, ..., k_n\}$. Explain several similarities to solve m-polar neutrosophic hypersoft set.

4.1. *Tangent Similarity Measure for m Polar neutrosophic Hypersoft Sets*

Let K=$\{k_1, k_2, k_3, ...,k_n\}$ be a universal set P(K) be its power set. Consider two m-polar neutrosophic hypersoft set

A=$\{\langle T_a{}^i(k), I_a{}^i(k), F_a{}^i(k) \rangle$ k∈K, a∈$(a_1{}^1\times a_2{}^2\times ...a_m{}^l)\}$

B=$\{\langle T_b{}^i(k), I_b{}^i(k), F_b{}^i(k) \rangle$ k∈K, b∈$(b_1{}^1\times b_2{}^2\times ...b_n{}^l)\}$

To measure the tangent similarity for these two m-polar neutrosophic hypersoft set use the given tangent similarity

$$T(A, B)=\{\tfrac{1}{n}\sum_{i=1}^{n}[1-\tan\tfrac{\pi}{12}(\mid T_a^i(k)-T_b^i(k)\mid+\mid I_a^i(k)-I_b^i(k)\mid+\mid F_a^i(k)-F_b^i(k)\mid\}$$

here

k∈K, a∈$(a_1{}^1\times a_2{}^2\times ...a_m{}^l$, b ∈ $(b_1{}^1\times b_2{}^2\times ...b_n{}^l)$

$0\le\sum_{i=1}{}^n T_{A,B}{}^i(k)\le 1$ $0\le\sum_{i=1}{}^n I_{A,B}{}^i(k)\le 1$ $0\le \sum_{i=1}{}^n F_{A,B}{}^i(k)\le 1$

where

$T_\Upsilon{}^i(k)\subseteq[0, 1]$ $I_\Upsilon{}^i(k)\subseteq[0, 1]$ $F_\Upsilon{}^i(k)\subseteq[0, 1]$

and holds the conditions

$0\le \sum_{i=1}{}^n T_{a,b}{}^i(k)+\sum_{i=1}{}^n I_{a,b}{}^i(k)+\sum_{i=1}{}^n F_{a,b}{}^i(k)\le 3$

4.2. *Proposition*

Tangent similarity measure between two m-polar neutrosophic hypersoft sets satisfies the following properties

(1) $0\le T_{mPNHSS}(A, B)\le 1$

(2) $T_{mPNHSS}(A, B)=1$ if and only if A=B

(3) $T_{mPNHSS}(A, B)=T_{mPNHSS}(B, A)$

(4) if C is a mPNHSS and A⊂B⊂C then T_{mPNHSS}(A, C) ≤ T_{mPNHSS}(A, B) and T_{mPNHSS}(A, C) ≤ T_{mPNHSS}(B, C)

4.3. *Proofs*

(1). 0≤ T_{mPNHSS}(A, B)≤1

As the truth membership function, indeterminacy function, and falsity membership function of T_{mPNHSS} lies between 0 and 1, because the values of the tangent function are within [0, 1] and the similarity measure which is based on tangent also lies within [0, 1]. Hence proved

0≤ $T_{m-PNHSS}$(A, B)≤1

(2) $T_{m-PNHSS}$(A, B)=1 if and only if A=B

For any two m-PNHSS A and B, let A and B are equal m-PNHSS as given in statement (means A=B) this implies

T_A(x)=T_B(x) I_A(x)=I_B(x) F_A(x)=F_B(x)

Hence there difference also equal to zero means

| T_A(x)-T_B(x)|=0 | I_A(x)-I_B(x)|=0 | F_A(x)-F_B(x)|=0

after putting these values in tangent similarity formula, since tan(0)=0 so we get the answer is 1.Hence if A=B then T_{mPNHSS}(A, B)=1

Conversely

If T_{mPNHSS}(A, B)=1 then it is obvious that

| T_A(x)-T_B(x)|=0 | I_A(x)-I_B(x)|=0 | F_A(x)-F_B(x)|=0

these imply that

T_A(x)=T_B(x) I_A(x)=I_B(x) F_A(x)=F_B(x)

by definition of equal mPNHss A=B

(3)T_{mPNHSS}(A, B)=T_{mPNHSS}(B, A)

To calculate the value of T_{mPNHSS}(A, B) firstly find

| T_A(x)-T_B(x)| | I_A(x)-I_B(x)| | F_A(x)-F_B(x)|

As we know the property of mod

|-x|=x

by using this property it is obvious that

| T_A(x)-T_B(x)|=| T_B(x)-T_A(x)| | I_A(x)-I_B(x)|=| I_B(x)-I_A(x)|

| F_A(x)-F_B(x)|=| F_B(x)-F_A(x)|

So the values of $T_{m-PNHSS}$(A, B)= $T_{m-PNHSS}$(A, B)

(4)if C is a m-PNHSS and A⊂B⊂C then $T_{m-PNHSS}(A, C) \leq T_{m-PNHSS}(A, B)$ and $T_{m-PNHSS}(A, C) \leq T_{m-PNHSS}(B, C)$

If A⊂B⊂C then by definition if subset m-PNHSS $T_A(x) \leq T_B(x) \leq T_C(x)$, $I_A(x) \geq I_B(x) \geq I_C(x)$, $F_A(x) \geq F_B(x) \geq F_C(x)$ for x∈X

by using definition we get the inequalities:

$| T_A(x)-T_B(x)| \leq | T_A(x)-T_C(x)|$ $| T_B(x)-T_C(x)| \leq | T_A(x)-T_C(x)|$

$| I_A(x)-I_B(x)| \leq | I_A(x)-I_C(x)|$ $| I_B(x)-I_C(x)| \leq | I_A(x)-I_C(x)|$

$| F_A(x)-F_B(x)| \leq | F_A(x)-F_C(x)|$ $| F_B(x)-F_C(x)| \leq | F_A(x)-F_C(x)|$

Thus we conclude that

$T_{m-PNHSS}(A, C) \leq T_{m-PNHSS}(A, B)$ and $T_{m-PNHSS}(A, C) \leq T_{m-PNHSS}(B, C)$,because the tangent function is increasing in the interval$[0, \frac{\pi}{4}]$

4.4. *Ye's similarity*

Ye's similarity is defined as

$S_{ye}(A, B)=1-\frac{1}{6}\sum_{i=1}{}^{n}w_i[| T_A{}^j(k_i)-T_B{}^j(k_i)| +| I_A{}^j(k_i)-I_B{}^j(k_i)|+|F_A{}^j(\frac{k}{i})+F_B{}^j(\frac{k}{i})|$

where

$T_{A,B}{}^i(k) \subseteq [0, 1]$ $I_{A,B}{}^i(k) \subseteq [0, 1]$ $F_{A,B}{}^i(k) \subseteq [0, 1]$

and holds the condition

$0 \leq \sum_{i=1}{}^{n}T_{A,B}{}^i(k)+\sum_{i=1}{}^{n}I_{A,B}{}^i(k)+\sum_{i=1}{}^{n}F_{A,B}{}^i(k) \leq 3$

4.5. *Proposition*

Ye similarity measure between two m-polar neutrosophic hypersoft set satisfies the following properties

(1) $0 \leq S_{yemPNHSS}(A, B) \leq 1$
(2) $S_{yemPNHSS}(A, B)=1$ if and only if A=B
(3) $S_{yemPNHSS}(A, B)=S_{yemPNHSS}(B, A)$

4.6. *Proofs*

(1). $0 \leq S_{yemPNHSS}(A, B) \leq 1$

As the truth membership function, indeterminacy function, and falsity membership function of $S_{yemPNHSS}$ lie between 0 and 1, the similarity measure which is based on the above functions also lies within [0, 1]. Hence proved

$0 \leq S_{yemPNHSS}(A, B) \leq 1$

(2). $S_{yemPNHSS}(A, B)=1$ if and only if A=B

Let A=B then by definition of equal m polar neutrosophic hypersoft set

$T_A(x)=T_B(x) \qquad I_A(x)=I_B(x) \qquad F_A(x)=F_B(x)$

then

$S_{yemPNHSS}(A, B)=1-\frac{1}{6}\sum_{i=1}{}^{n} w_i[| T_A{}^j(k_i)-T_B{}^j(k_i)| +| I_A{}^j(k_i)-I_B{}^j(k_i)|+|F_A{}^j(\binom{k}{i})+F_B{}^j(\binom{k}{i})|$

$S_{yemPNHSS}(A, B)=1-0$

$S_{yemPNHSS}(A, B)=1$

Conversely,

Let $S_{yemPNHSS}(A, B)=1$ then only one possibility is

$T_A(x)-T_B(x)=0 \qquad I_A(x)-I_B(x)=0 \qquad F_A(x)-F_B(x)=0$

this implies that

$T_A(x)=T_B(x) \qquad I_A(x)=I_B(x) \qquad F_A(x)=F_B(x)$

so, A=B

(3)$S_{yemPNHSS}(A, B)=S_{yemPNHSS}(B, A)$

$S_{yemPNHSS}(A, B)=1-\frac{1}{6}\sum_{i=1}{}^{n} w_i[| T_A{}^j(k_i)-T_B{}^j(k_i)| +| I_A{}^j(k_i)-I_B{}^j(k_i)|+|F_A{}^j(\binom{k}{i})+F_B{}^j(\binom{k}{i})|$

it can be write as

$S_{yemPNHSS}(A, B)=1-\frac{1}{6}\sum_{i=1}{}^{n} w_i[| T_B{}^j(k_i)-T_A{}^j(k_i)| +| I_B{}^j(k_i)-I_A{}^j(k_i)|+|F_B{}^j(\binom{k}{i})+F_A{}^j(\binom{k}{i})|$

$S_{yemPNHSS}(A, B)=S_{yemPNHSS}(B, A)$

4.7. *Cosine Similarity*

Candan and sapino [33] give us the idea of cosine similarity which is a fundamental angle based similarity. It helps to calculate the similarity between two n-dimensional vectors by using the cosine of the angle between them. Consider two vectors A, B. Their attributes are

A=((a_1, a_2, ..., a_n) and B=((b_1, b_2, ..., b_n), θ is the angle between A and B, similarity can be calculated by using dot product and magnitudes of vectors A and B

$$\cos\theta = \{\frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}}\}$$

now consider if there is an m-polar neutrosophic hypersoft sets A and B in K={ k_1, k_2, ..., k_n } then the cosine similarity between A and B can be calculated by the following formula

$$C(A, B)=\frac{1}{n}\sum_{i=1}^{n}\{\frac{\Delta T_A(k_i)\Delta T_B(k_i)+\Delta I_A(k_i)\Delta I_B(k_i)+\Delta F_A(k_i)\Delta F_B(k_i)}{\sqrt{(\Delta T_A(k_i))^2+(\Delta I_A(k_i))^2+(\Delta F_A(k_i))^2}\sqrt{(\Delta T_B(k_i))^2+(\Delta I_B(k_i))^2+(\Delta F_B(k_i))^2}}\}$$

where

$\Delta\ T_A(k_i)=\sum_{j=1}{}^{n}T\Upsilon^{j}(k)$

$\Delta\ I_A(k_i)=\sum_{j=1}{}^{n}I\Upsilon^{j}(k)$

$\Delta\ F_A(k_i)=\sum_{j=1}{}^{n}F\Upsilon^{j}(k)$

and holds the condition

$0\leq \sum_{j=1}{}^{n}T\Upsilon^{j}(k)+\sum_{j=1}{}^{n}I\Upsilon^{j}(k)+\sum_{j=1}{}^{n}F\Upsilon^{j}(k)\leq 3$

4.8. *Proposition*

Cosine similarity measure between two m-polar neutrosophic hypersoft sets satisfies the following properties

(1) $0\leq C_{m-PNHSS}(A, B)\leq 1$
(2) $C_{m-PNHSS}(A, B)=1$ if and only if A=B
(3) $C_{m-PNHSS}(A, B)=C_{m-PNHSS}(B, A)$

4.9. *Proofs*

(1). $0\leq C_{m-PNHSS}(A, B)\leq 1$

As the truth membership function, indeterminacy function, and falsity membership function of $S_{yem-PNHSS}$ lie between 0 and 1, values of cosine function also lies between 0 and 1, the similarity measure which is based on cosine also lies within [0, 1]. Hence proved

$0\leq C_{m-PNHSS}(A, B)\leq 1$

(2). $C_{m-PNHSS}(A, B)=1$ if and only if A=B

Let A=B then

$T_A(x)=T_B(x)$ $\quad$ $I_A(x)=I_B(x)$ $\quad$ $F_A(x)=F_B(x)$

so,

$$C_{m-PNHSS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\{\frac{\Delta T_A(k_i)\Delta T_B(k_i)+\Delta I_A(k_i)\Delta I_B(k_i)+\Delta F_A(k_i)\Delta F_B(k_i)}{\sqrt{(\Delta T_A(k_i))^2+(\Delta I_A(k_i))^2+(\Delta F_A(k_i))^2}\sqrt{(\Delta T_B(k_i))^2+(\Delta I_B(k_i))^2+(\Delta F_B(k_i))^2}}\}$$

after using the definition of equal m-polar neutrosophic hypersoft set it became

$C_{m-PNHSS}(A,B)=1$

Conversely,

Let $C_{m-PNHSS}(A, B)=1$ then there is the only possibility

$T_A(x)=T_B(x)$ $\quad$ $I_A(x)=I_B(x)$ $\quad$ $F_A(x)=F_B(x)$

Hence A=B

(3). $C_{m-PNHSS}(A, B)=C_{m-PNHSS}(B, A)$

$$C_{m-PNHSS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\{\frac{\Delta T_A(k_i)\Delta T_B(k_i)+\Delta I_A(k_i)\Delta I_B(k_i)+\Delta F_A(k_i)\Delta F_B(k_i)}{\sqrt{(\Delta T_A(k_i))^2+(\Delta I_A(k_i))^2+(\Delta F_A(k_i))^2}\sqrt{(\Delta T_B(k_i))^2+(\Delta I_B(k_i))^2+(\Delta F_B(k_i))^2}}\}$$

It can be written as

$$C_{m-PNHSS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\{\frac{\Delta T_B(k_i)\Delta T_A(k_i)+\Delta I_B(k_i)\Delta I_A(k_i)+\Delta F_B(k_i)\Delta F_A(k_i)}{\sqrt{(\Delta T_B(k_i))^2+(\Delta I_B(k_i))^2+(\Delta F_B(k_i))^2}\sqrt{(\Delta T_A(k_i))^2+(\Delta I_A(k_i))^2+(\Delta F_A(k_i))^2}}\}$$

$C_{m-PNHSS}(A, B)=C_{m-PNHSS}(B, A)$

4.10. *Application of Tangent Similarity Measure for m-Polr Neutrosophic Hypersoft set to Diagnosis Covid-19*

In medical diagnosis, decision making became very much complicated because medical test are too expansive and time taking. Sometime condition of patient is not stable, the doctor did not have enough time to wait for the result of a medical test, they should take urgent steps to save the patient's life. In such conditions these kind of studies help them to take effective decisions to save patient's life. In the days of covid-19 patients are large in number and the medical test is very much expensive and time-taking. Some patients had financial problems and the condition of some patients is critical. To handle that pandemic situation, we demonstrate the application of the proposed tangent similarity measure for the m-polar neutrosophic hypersoft set to medical diagnosis. we discuss the covid-19 diagnosis as follow:

For example, the patients of covid-19 reported the symptoms like

serious symptoms= SS= difficulty in breathing, chest pain, loss of speech most common symptoms= MCS= fever, dry cough, tiredness

less common symptoms= LCS= pain, sore throat, headache

we construct a model(patient) who is suffering from covid-19. Note down the symptoms of covid-19 according to the list SS, MCS, LCS and figure out their scale for diagnosis take under observation and figure out the m polar neutrosophic hypersoft set

P_M={SS⟨(0.087, 0.68, 0.073, 0.060), (0.087, 0.073, 0.79, 0.05), (0.68, 0.084, 0.079, 0.16)⟩, MCS⟨(0.53, 0.067, 0.071, 0.068), (0.43, 0.087, 0.32, 0.064), (0.411, 0.080, 0.076, 0.4)⟩, LCS⟨(0.03, 0.51, 0.063, 0.041), (0.063, 0.072, 0.54, 0.010), (0.083, 0.079, 0.076, 0.71)⟩}

Five patients are suffering from fever and also having some symptoms due to these symptoms they thought they are suffering from covid-19. To diagnosis that those patients are suffering from covid-19 or not. For this purpose construct the m polar neutrosophic hypersoft set for every patient. Like

First Patient

P_1={SS⟨(0.078, 0.77, 0.068, 0.05), (0.072, 0.086, 0.70, 0.06), (0.58, 0.085, 0.075, 0.13)⟩, MCS⟨(0.43, 0.087, 0.061, 0.059), (0.53, 0.057, 0.12, 0.034), (0.011, 0.07, 0.46, 0.3)⟩, LCS⟨(0.04, 0.61, 0.053, 0.051), (0.073, 0.52, 0.31, 0.02), (0.073, 0.069, 0.080, 0.72)⟩}

Second Patient

P_2={SS⟨(0.002, 0.01, 0.02, 0.03), (0.032, 0.056, 0.02, 0.05), (0.042, 0.051, 0.035, 0.03)⟩, MCS⟨(0.04, 0.05, 0.041, 0.029), (0.053, 0.062, 0.09, 0.11), (0.001, 0.04, 0.16, 0.05)⟩, LCS⟨(0.063, 0.049, 0.023, 0.03), (0.053, 0.42, 0.21, 0.01), (0.063, 0.054, 0.073, 0.74)⟩}

Third Patient

P_3={SS⟨(0.001, 0.003, 0.0012, 0.004), (0.021, 0.014, 0.053, 0.061), (0.012, 0.003, 0.0012, 0.021)⟩, MCS⟨(0.002, 0.013, 0.053, 0.060), (0.0017, 0.023, 0.04, 0.0029), (0.0017, 0.023, 0.04, 0.0029)⟩, LCS⟨(0.009, 0.0014, 0.0049, 0.0021), (0.007, 0.004, 0.013, 0.0029), (0.002, 0.004, 0.0012, 0.0014)⟩}

Forth Patient

P_4={SS⟨(0.0011, 0.0012, 0.0002, 0.0010), (0.0021, 0.0041, 0.0035, 0.061), (0.0012, 0.0001, 0.0021, 0.0022)⟩, MCS⟨(0.004, 001, 0.0051, 0.006), (0.0017, 0.0023, 0.004, 0.0019), (0.003, 0.00041, 0.0003, 0.005)⟩, LCS⟨(0.0009, 0.0013, 0.0009, 0.0012), (0.0005, 0.0014, 0.0002, 0.00091), (0.0003, 0.0041, 0.0012, 0.0001)⟩}

Fifth Patient

P_5={SS⟨(0.077, 0.66, 0.073, 0.060), (0.082, 0.070, 0.69, 0.05), (0.66, 0.083, 0.070, 0.11)⟩, MCS⟨(0.51, 0.077, 0.061, 0.069), (0.53, 0.077, 0.22, 0.074), (0.41, 0.81, 0.074, 0.3)⟩, LCS⟨(0.032, 0.49, 0.061, 0.042), (0.073, 0.051, 0.31, 0.05), (0.083, 0.078, 0.075, 0.70)⟩}

Our aim is to diagnosis which patient is suffering from covid-19, for this purpose use several similarities to analyze this problem.

From the tangent similarity formula, we compute the similarity between P_i (i=1, 2, 3, 4, 5) and P_M as follows:

$T_{m-PNHSS}(P_1, P_M)$=0.9148

$T_{m-PNHSS}(P_2, P_M)$=0.6182

$T_{m-PNHSS}(P_3, P_M)$=0.2499

$T_{m-PNHSS}(P_4, P_M)$=0.2030

$T_{m-PNHSS}(P_5, P_M)$=0.9077

After that calculation we diagnosis that the first patient is suffering from covid-19, and his condition is severe. The fifth patient is also suffering from covid-19. The second Patient also having some symptoms of covid-19 he needs to adopt the preventing measures. Third and Fourth Patients are suffering from seasonal temperature.

From the ye similarity formula, we compute the similarity between P_i (i=1, 2, 3, 4, 5) and P_M as follows:

$S_{yemPNHSS}(P_1, P_M)$=0.8377

$S_{yemPNHSS}(P_2, P_M)$=0.3195

$S_{yemPNHSS}(P_3, P_M)$=0

$S_{yemPNHSS}(P_4, P_M)$=0

$S_{yemPNHSS}(P_5, P_M)$=0.8248

Table 1. Several Similarity Measures

Patient	**Tangent Similarity**	**Ye Similar-ity**	**Cosine Similarity**
P_1	0.9148	0.8377	0.9961
P_2	0.6182	0.3195	0.9599
P_3	0.2489	0	0.7727
P_4	0.203	0	0,7342
P_5	0.9077	0.8248	0.9864

After that calculation, we diagnosis that First and Fifth patient is suffering from covid-19. The second Patient also having some symptoms od covid-19 he needs to adopt the prevention measures. This Similarity is not useful to the diagnosis of covid-19 for the Third and Fourth.

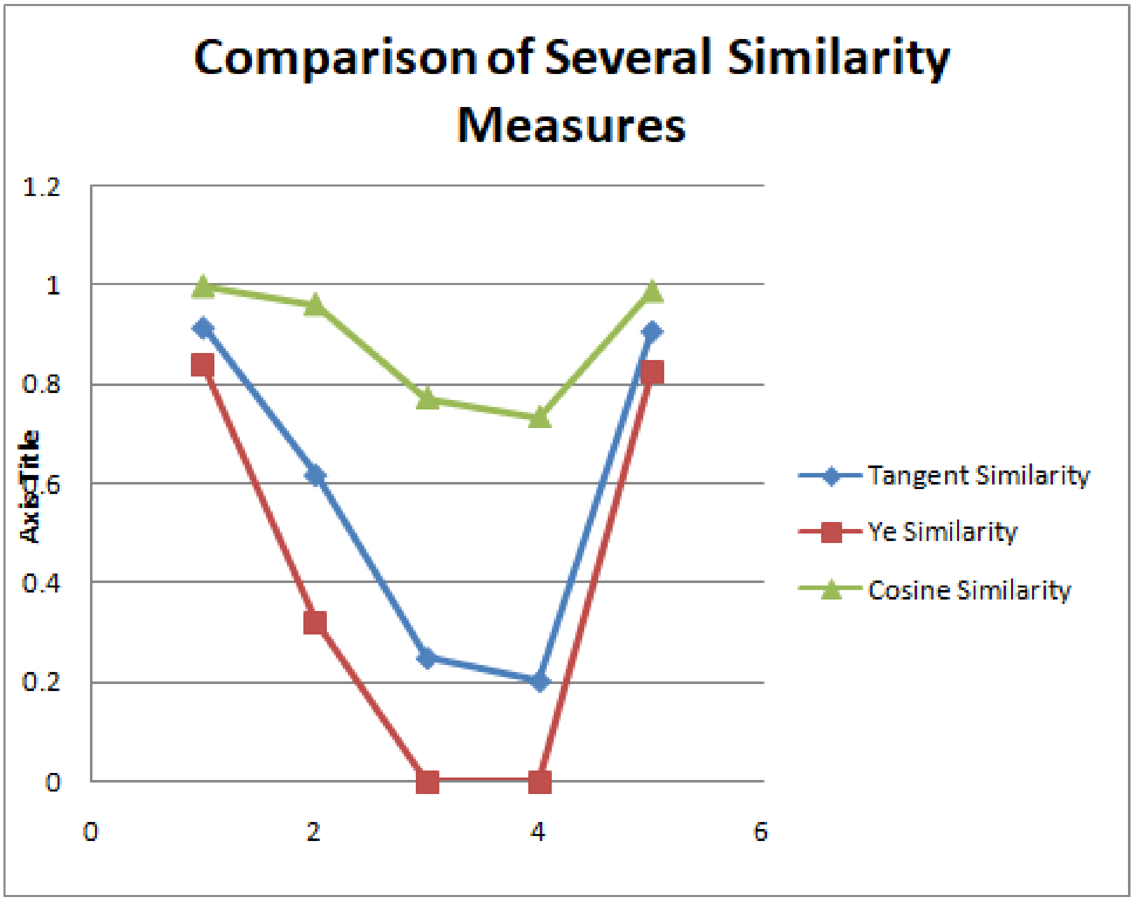
From the cosine similarity formula, we compute the similarity between P_i (i=1, 2, 3, 4, 5) and P_M as follows:

$C_{mPNHSS}(P_1, P_M)$=0.9961
$C_{mPNHSS}(P_2, P_M)$=0.9599
$C_{mPNHSS}(P_3, P_M)$=0.7727
$C_{mPNHSS}(P_4, P_M)$=0.7342
$C_{mPNHSS}(P_5, P_M)$=0.9864

After that calculation, we diagnosis that the first patient is suffering from covid-19, and his condition is severe. The fifth and second patient is also suffering from covid-19. The third and fourth patients also having some symptoms od covid-19 he needs to adopt the prevention measures.

4.11. *Tabular and Graphical Representation*

A comparison among several similarities also expressed through tabular and graphical representation for quick analysis.



5. Conclusion

Similarity concept is extremely beneficial in MCDM, management problems, personal selection, coding theory, medical diagnosis, and feature extraction,etc. The aim of this paper is to establish Tangent, Cosine, Ye similarity measures of multi-polar neutrosophic hypersoft sets. This extension can be applied immensely in decision-making problems, management problems, time series, forecasting, and supply chain, etc. Coronavirus disease 2019 (Covid-19) is an infectious disease caused of ongoing pandemic. In the end, we applied this technique in medical diagnosis of covid-19 to show the practical application of this idea. In future, we would like to generalize entropy measure, TOPSIS, VIKOR, etc of m-Polar and Bipolar neutrosophic hypersoft sets. These concepts will be extremely useful for solving the problems having large number of attributes.

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Chapter 9

A Novel Approach to Mappings on Hypersoft Classes with Application

Muhammad Saeed [1], Muhammad Ahsan [1*], Atiqe Ur Rahman [2]

[1,2] Department of Mathematics, University of Management and Technology Lahore, Pakistan.

E-mail: muhammad.saeed@umt.edu.pk , E-mail: ahsan1826@gmail.com , E-mail: aurkhb@gmail.com

Abstract: The soft set is inadequate to tackle the problems involving attribute-valued sets. Hypersoft set, an extension of the soft set, can tackle such issues efficiently. This work aims to adequate the existing concepts of mappings on fuzzy soft and soft classes for multi attribute-valued functions. First, mapping is characterized under a hypersoft set environment, then some of its essential properties like HS images, HS inverse images, etc., are developed with more generalized results. Moreover, practical application and comparative study is given to show the validity and predominance of the proposed technique.

Keywords: *Fuzzy soft classes; Soft classes, Hypersoft set, Hypersoft classes, Hypersoft images, Hypersoft inverse images.*

1. Introduction

For solving multifaceted problems in robotics, engineering, shortest-path selection, economics, and the environment, we cannot practice the conventional means successfully. Despite the variety of incomplete information, there are four theories specific to these problems Probability set theory (PST), Fuzzy set theory (FST) Zadeh [1], Rough set theory (RST) Pawlak [2] and Period mathematics (PM) can be assumed as a mathematical tool for dealing with lacking information. Every one of these apparatuses acquires the pre-determination of few parameters, to begin with, density function (DF) in PST, membership degree in FST, and a congruence relation in RST. Such a prerequisite, observed in the scrim of flawed or deficient information, escalate numerous issues. Simultaneously, fragmented information stays the most glaring attribute of humanitarian, organic, monetary, social, political, and large man-machine frameworks of different kinds. Heilpern [3], presented the idea of fuzzy mapping and demonstrated a fixed-point theory for fuzzy contraction mappings, which speculates the fixed-point hypothesis for multi-valued mappings of Nadler [4]. Estruch and Vidal give a fixed-point theory for fuzzy contraction mappings over a complete metric space, which is a generalization of the fundamental Heilpern's fixed point hypothesis [5]. In the pioneer research of Zhu and Xiao [6] they have examined the convexity, and quasiconvexity of fuzzy mappings by considering the idea of ordering due to Goetschel-Voxman [7]. Syau [8] demonstrated the idea of convex and concave fuzzy mappings. Syau [9], presented the idea of differentiability, summed up convexity e.g., pseudoconvexity and invexity for fuzzy mappings of a few factors. His methodology is equal to Goetschel- Voxman approach for fuzzy mapping of a single variable in which the arrangement of fuzzy numbers is embedded in a topological vector space.

Molodtsov [10] successfully implemented soft set (SS) theory in several directions likes the ease of function, Riemann integration (RI), Peran integration (PI), Probability theory (PT), Measurement theory (MT) and so on. Hopeful results have been found by Kokovo et al. [11]. The SS theory is used for the process of optimization in Optimization theory (OPT), Game theory (GT), and Operations research (OR). Maji et al. [12] presented S-sets applications in decision-making problems. In [13], Yang et al. highlighted the requirements of S-sets in engineering extended applications. Maji et al. [14] presented the concept of fuzzy SS and its many features. They proposed it as an attractive enlargement of S-sets, additional features to uncertainty and ambiguity on the highest level of incompleteness. Present researches have explained [15, 16] how to combine the two ideas into a more flexible, high expression structure for modeling and refined foggy data in the information system. The concept of SS is applied to solve a lot of problems in [17–23].

Karaaslan [24] presented the soft class and its relevant operations. He applied its utilizations in decision making successfully. Athar et al. [25, 26] presented the concept of mappings on fuzzy soft classes and mappings on soft classes in 2009 and 2011 respectively. They considered S-images' properties, S-inverse images, fuzzy SS, fuzzy S-images, fuzzy S-inverse images of fuzzy S-sets, and illustrated these concepts with examples and counterexamples.

Alkhazaleh [27] et al. introduced the notion of a mapping on classes where the neutrosophic soft classes are collections of the neutrosophic soft sets. Additionally, they characterized and study the properties of neutrosophic soft images and neutrosophic soft inverse images of neutrosophic soft sets. Sulaiman [28] et. al presented the idea of mappings on multi-aspect fuzzy soft classes. They explored a few properties related to the image and pre-image of multiaspect fuzzy soft sets and further illustrate with some numerical examples. Maruah [29] et. al characterized the notation of mapping on intuitionistic fuzzy soft classes with some properties of intuitionistic fuzzy soft images and inverse images. Manash et al. [30] gave the idea of composite mappings on hesitant fuzzy soft classes in 2016 and discussed some interesting properties of this idea.

Samarandache [31] introduced the concept of HS set as a generalization of soft set in 2018. At that point, he made the differentiation between the sorts of initial universes, crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic respectively. Thus, he also showed that a HS set can be crisp, fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic respectively. Saeed et al. [32] explained some basic concepts like HS subset, HS complement, not HS set, union, intersection, HS set relation, sub relation, complement relation, HS representation in matrices form and different operations on matrices. In this study, an extension is made in existing literature regarding mapping on fuzzy soft and soft classes by defining mapping for HS classes. We develop some properties of mapping on HS classes like HS images, HS inverse images. Moreover, a practical application and comparative study is given to show the validity and predominance of the proposed technique.

The ordering of remaining article is sort out as follows. In Section 2, some basic definition regarding soft set, soft class, soft image, soft inverse image, HS set, HS relation and HS function are re-imagined. In Section 3, mapping on HS classes, HS image, HS inverse image and its relevant theorems with proofs are characterized. In Section 4, a practical application is given to show the validity of the proposed approach. In the last section, some concluding remarks are described.

1.1. *Motivation*

In a diversity of real-life applications, the attributes should be further sub-partitioned into attribute values for clear understanding. Samarandache [31] fulfilled this need and developed the concept of the HSS as a generalization of the SS. Now, It will be a question that how and where it tends to be applied? HSS set is significant? How do we define mappings for HSS classes? To answers these questions and getting inspiration from the above writing, it is relevant to broaden the idea of mappings for those sets managing disjoint arrangements of attributed values, i.e., HS set. In this investigation, an extension is made in existing theories with respect to mappings on fuzzy soft and soft classes by characterizing mappings on HS classes. The striking component of mappings on HS classes is that it can mirror the inter-relationship between the multi-attribute function. Moreover, specific generalized properties of mappings on HS classes like HS images and HS inverse images, are established since it is not yet characterized. Some related results are proved with the help of illustrative examples. Moreover, a practical application and comparative study is given to show the validity and predominance of the proposed technique.

2. **Preliminaries**

In this section, some basic definition is presented over the universe $\breve{U}$.

Definition 2.1. [10] A pair $(\breve{\zeta}, \breve{A})$ is said to be soft set over $\breve{U}$, where $\breve{\zeta}$ is a mapping in such a way

$$\breve{\zeta} : \breve{A} \to \breve{P}(\breve{U}),$$

In other words, a soft set over $\breve{U}$ is a parameterized family of subsets of the universe. For $\epsilon \in A$. $\breve{\zeta}(\epsilon)$ may be considered as the set of ϵ approximate elements of the soft set $(\breve{\zeta}, \breve{A})$.

Definition 2.2. [24] Let $\breve{U}$ consider as initial universe, suppose $\breve{\eth}$ be set of parameters, and let $\breve{F} = \{\breve{\varsigma}_i : \breve{i} = 1, 2, ..., n\}$ be a set of decision makers. Indexed class of S-sets $\{\breve{\vartheta}_{\breve{\varsigma}_i} : \breve{\vartheta}_{\breve{\varsigma}_i} : \breve{\eth} \to P(\breve{U}), \breve{\varsigma}_i \in \breve{F}\}$ is said to be soft class and is denoted by $\breve{\vartheta}_{\breve{F}}$. If, for any $\breve{\varsigma}_i \in \breve{F}$, $\breve{\vartheta}_{\breve{\varsigma}_i} = \Phi$, the SS $\breve{\vartheta}_{\breve{\varsigma}_i} \notin \breve{\vartheta}_{\breve{F}}$.

Definition 2.3. [26] Let $(\breve{\Re}, \breve{\eth})$ and $(\breve{\wp}, \breve{\eth}')$ be two classes of S-sets over the universal set $\breve{\Re}$ and $\breve{\wp}$ respectively. Let $\breve{\mu} : \breve{\Re} \to \breve{\wp}$ and $\breve{\gamma} : \breve{\eth} \to \breve{\eth}'$ be mappings. Then a mapping $\breve{\vartheta} = (\breve{\mu}, \breve{\gamma}) : (\breve{\Re}, \breve{\eth}) \to (\breve{\wp}, \breve{\eth}')$ is defined as, for SS $(\breve{\hbar}, \breve{\aleph})$ in $(\breve{\Re}, \breve{\eth})$ and $\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})$ is SS in $(\breve{\wp}, \breve{\eth}')$ obtained as follows, For $\breve{\beta} \in \breve{\gamma}(\breve{\eth}) \subseteq \breve{\eth}'$ and $\breve{y} \in \breve{\wp}$, then

$$\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})(\breve{\beta})(\breve{y}) = \begin{cases} \bigcup_{\breve{x} \in \breve{\mu}^{-1}(\breve{y})} \left(\bigcup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\aleph}} \breve{\hbar}(\breve{\alpha}) \right)(\breve{x}), & \text{if } \breve{\mu}^{-1}(\breve{y}) \neq \Phi, \\ & \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\aleph} \neq \Phi \\ 0 & \text{if } otherwise \end{cases} \tag{1}$$

$\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})$ is called a soft image of SS $(\breve{\hbar}, \breve{\aleph})$.

Now, let $(\breve{\ell}, \breve{\Im})$ a SS in $(\breve{\wp}, \breve{\eth}')$, where $\breve{\Im} \subseteq \breve{\eth}'$ then $\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})$ is a SS in $(\breve{\Re}, \breve{\eth})$ given as follows,

$$\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})(\breve{\alpha})(\breve{x}) = \begin{cases} \breve{\ell}(\breve{\gamma}(\breve{\alpha})(\breve{\mu}(\breve{x}) & \text{if } \breve{\gamma}(\breve{\alpha}) \in \breve{\Im} \\ 0 & \text{if } otherwise \end{cases} \tag{2}$$

where $\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\Im}) \subset \breve{\breve{\eth}}$, then $\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})$ said to be the soft inverse image of $(\breve{\ell}, \breve{\Im})$.

Definition 2.4. [31] Let $\breve{o}_1, \breve{o}_2, \breve{o}_3, \cdots, \breve{o}_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $\breve{\eth}_1, \breve{\eth}_2, \breve{\eth}_3, \cdots, \breve{\eth}_n$ respectively, where $\breve{\eth}_i \cap \breve{\eth}_j = \breve{\Phi}$ for $\breve{i} \neq \breve{j}$. A pair $(\breve{\Upsilon}, \breve{J})$ is called a HS set over the universal set $\breve{U}$, where $\breve{\Upsilon}$ is the mapping given by $\breve{\Upsilon} : \breve{J} \longrightarrow \breve{P}(\breve{U})$, where $\breve{J} = \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times ... \times \breve{\eth}_n$. For more definition see [32–35].

Definition 2.5. [32] Let $(\breve{\hbar}, \breve{\aleph})$ and $(\breve{\ell}, \breve{\Im})$ be the two HS sets over the same universal set $\breve{U}$. Then the relation from $(\breve{\hbar}, \breve{\aleph})$ to $(\breve{\ell}, \breve{\Im})$ is called a HS set relation $(\breve{R}, \breve{C})$ or it is in simple way $\breve{R}$ is a HS subset of $(\breve{\hbar}, \breve{\aleph}) \times (\breve{\ell}, \breve{\Im})$, where $\breve{C} \subseteq \breve{\aleph} \times \breve{\Im}$ and $\forall\ (\breve{a}, \breve{b}) \in \breve{C}\ \breve{R}(\breve{a}, \breve{b}) = \breve{H}(\breve{a}, \breve{b})$, where $(\breve{H}, \breve{\aleph} \times \breve{\Im}) = (\breve{\hbar}, \breve{\aleph}) \times (\breve{\ell}, \breve{\Im})$. A HS set relation on $(\breve{\hbar}, \breve{\aleph})$ is a HS subset of $(\breve{\hbar}, \breve{\aleph}) \times (\breve{\hbar}, \breve{\aleph})$. In similar way, the parameterized form of relation R on the HS set $(\breve{\hbar}, \breve{\aleph})$ is defined as follows. If $(\breve{\hbar}, \breve{\aleph}) = \{\breve{\hbar}(\breve{a}), \breve{\hbar}(\breve{b}), ...\}$, then $\breve{\hbar}(\breve{a})\breve{R}\breve{\hbar}(\breve{b}) \Leftrightarrow \breve{\hbar}(\breve{a}) \times \breve{\hbar}(\breve{b})) \in \breve{R}$.

Definition 2.6. [32] Let $(\breve{\hbar}, \breve{\aleph})$ and $(\breve{\ell}, \breve{\Im})$ be the two HS sets over the same universal set $\breve{U}$. Then the HS relation from$(\breve{\hbar}, \breve{\aleph})$ to $(\breve{\ell}, \breve{\Im})$ can be symbolize as $\breve{\vartheta} : (\breve{\hbar}, \breve{\aleph}) \longrightarrow (\breve{\ell}, \breve{\Im})$ is called a HS set function.

(1) If every element in the domain of $\breve{\vartheta}$ has unique element in range of $\breve{\vartheta}$.

(2) If it is closed $\breve{\hbar}(\breve{a})\breve{\vartheta}\breve{\ell}(\breve{b})$ i.e $\breve{\hbar}(\breve{a})\times\breve{\ell}(\breve{b}) \in \breve{\vartheta}$ then we can represent it in the form $\breve{\vartheta}(\breve{\hbar}(\breve{a})) = \breve{\ell}(\breve{b})$

3. **Mappings on Hypersoft Classes**

In this section, mapping on HS classes, HS image, HS inverse image and its relevant theorems with proofs are characterized. Throughout this section, consider $\breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times ... \times \breve{\eth}_n = \breve{J}$, $\breve{\eth}'_1\times, \breve{\eth}'_2 \times \breve{\eth}'_3 \times ... \times \breve{\eth}'_n = \breve{K}$, $\breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times ... \times \breve{\aleph}_n = \breve{\aleph}$, $\breve{\Im}_1 \times \breve{\Im}_2 \times \breve{\Im}_3 \times ... \times \breve{\Im}_n = \breve{\Im}$, $(\breve{\alpha}_1, \breve{\alpha}_2, \breve{\alpha}_3, ..., \breve{\alpha}_n) = \breve{\alpha}$ and $(\breve{\beta}_1, \breve{\beta}_2, \breve{\beta}_3, ..., \breve{\beta}_n) = \breve{\beta}$.

Definition 3.1. Suppose $\breve{U}$ be an initial universe, let $\epsilon_1, \epsilon_2, \epsilon_3, \cdots, \epsilon_n$ be the distinct attributes whose attribute values belongs to the sets $\breve{\eth}_1, \breve{\eth}_2, \breve{\eth}_3, \cdots, \breve{\eth}_n$ respectively, where $\breve{\eth}_i \cap \breve{\eth}_j = \breve{\Phi}$ for $\breve{i} \neq \breve{j}$, let $\breve{F} = \{\breve{\varsigma}_i : \breve{i} = 1, 2, ..., n\}$ be a collection of decision makers. Indexed class of HSS $\{\breve{\vartheta}_{\breve{\varsigma}_i} : \breve{\vartheta}_{\breve{\varsigma}_i} : \breve{J} \to P(\breve{U}), \breve{\varsigma}_i \in \breve{F}\}$, where $\breve{J} = \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \cdots \times \breve{\eth}_n$ is said to be HS class and it can be symbolized in such a form $\breve{\vartheta}_{\breve{F}}$. If, for any $\breve{\varsigma}_i \in \breve{F}$, $\breve{\vartheta}_{\breve{\varsigma}_i} = \Phi$, the HSS $\breve{\vartheta}_{\breve{\varsigma}_i} \notin \breve{\vartheta}_{\breve{F}}$.

Example 3.2. Let $\breve{\Re} = \{\breve{a} =$ Line Interactive, $\breve{b} =$ Standby-Ferro, $\breve{c} =$ Delta Conversion On-Line$\}$ be types of UPS (Uninterruptible Power Supply) is considered as universe of discourse. Let $\epsilon_1 =$ efficiency, $\epsilon_2 =$ size, $\epsilon_3 =$ colour, distinct attributes whose attribute values belong to the sets $\breve{\eth}_1, \breve{\eth}_2, \breve{\eth}_3$. Let $\breve{\eth}_1 = \{\breve{\tau}_1 =$ Good, $\breve{\tau}_2 =$ Very Good $\}$, $\breve{\eth}_2 = \{\breve{\tau}_3 =$ medium, $\breve{\tau}_4 = small\}$, $\breve{\eth}_3 = \{\breve{\tau}_5 =$ brown $\}$ and let $\breve{F} = \{\breve{\varsigma}_1, \breve{\varsigma}_1, \breve{\varsigma}_1\}$ be a set of decision makers. If we consider HS sets $\breve{\vartheta\varsigma}_1, \breve{\vartheta\varsigma}_2, \breve{\vartheta\varsigma}_3$ given as

$$\begin{aligned}
\breve{\vartheta\varsigma}_1 &= \{((\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5), \{\breve{a}, \breve{b}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{b}, \breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{b}\})\},\\
\breve{\vartheta\varsigma}_2 &= \{((\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5), \{\breve{b}, \breve{c}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{a}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{a}, \breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{b}\})\},\\
\breve{\vartheta\varsigma}_3 &= \{(\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5)\{\breve{c}, \breve{a}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{b}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{b}, \breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{c}\})\},
\end{aligned}$$

then $\breve{\vartheta}_{\breve{F}} = \{\breve{\vartheta\varsigma}_1, \breve{\vartheta\varsigma}_2, \breve{\vartheta\varsigma}_3\}$ is a HS class. Now let

$$\begin{aligned}
g\breve{\varsigma}_1 &= \{((\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5), \{\breve{a}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{a}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{c}, \breve{a}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{a}\})\},\\
g\breve{\varsigma}_2 &= \{((\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5), \{\breve{c}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{b}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{b}, \breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{c}\})\},\\
g\breve{\varsigma}_3 &= \{((\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5), \{\breve{b}\}), ((\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5), \{\breve{b}\}), ((\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5), \{\breve{a}, \breve{c}\}), ((\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5), \{\breve{a}\})\},
\end{aligned}$$

is also HS class. Then HS classes can be written as $\{\breve{\vartheta\varsigma}_1, \breve{\vartheta\varsigma}_2, \breve{\vartheta\varsigma}_3\}$,$\{g\breve{\varsigma}_1, g\breve{\varsigma}_2, g\breve{\varsigma}_3\}$.

Definition 3.3. Let $(\breve{\Re}, \breve{J})$ and $(\breve{\wp}, \breve{K})$ be two classes of HS sets over the universal set $\breve{\Re}$ and $\breve{\wp}$ respectively. Let $\breve{\mu} : \breve{\Re} \to \breve{\wp}$ and $\breve{\gamma} : \breve{J} \to \breve{K}$ be mappings. Then a mapping $\breve{\vartheta} = (\breve{\mu}, \breve{\gamma}) : (\breve{\Re}, \breve{J}) \to (\breve{\wp}, \breve{K})$ is defined as for HS set $(\breve{\hbar}, \breve{\aleph})$ in $(\breve{\Re}, \breve{J})$ and $\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})$ is HS set in $(\breve{\wp}, \breve{K})$ obtained as follows, For $\breve{\beta} \in \breve{\gamma}(\breve{J}) \subseteq \breve{K}$ and $\breve{y} \in \breve{\wp}$, then

$$\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})(\breve{\beta})(\breve{y}) = \begin{cases} \cup_{\breve{x} \in \breve{\mu}^{-1}(\breve{y})} \left(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\aleph}} \breve{\hbar}(\breve{\alpha}) \right)(\breve{x}), & \text{if } \breve{\mu}^{-1}(\breve{y}) \neq \Phi, \\ & \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\aleph} \neq \Phi \\ 0 & \text{if } otherwise \end{cases} \tag{3}$$

$\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})$ is called a HS image of HS set $(\breve{\hbar}, \breve{\aleph})$.

Definition 3.4. Let $(\breve{\Re}, \breve{J})$ and $(\breve{\wp}, \breve{K})$ be two classes of HS sets over the universal set $\breve{\Re}$ and $\breve{\wp}$ respectively. Let $\breve{\mu} : \breve{\Re} \to \breve{\wp}$ and $\breve{\gamma} : \breve{J} \to \breve{K}$ be mappings. Now, let $(\breve{\ell}, \breve{\Im})$ be a HS set in $(\breve{\wp}, \breve{K})$, where $\breve{\Im} \subseteq \breve{K}$ then $\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})$ is a HS set in $(\breve{\Re}, \breve{J})$ defined as follows,

$$\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})(\breve{\alpha})(\breve{x}) = \begin{cases} \breve{\ell}(\breve{\gamma}(\breve{\alpha})(\breve{\mu}(\breve{x}) & \text{if } \breve{\gamma}(\breve{\alpha}) \in \breve{\Im} \\ 0 & \text{if } otherwise \end{cases} \tag{4}$$

where $\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\Im}) \subset \breve{J}$, then $\breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})$ said to be the HS inverse image of HS set $(\breve{\ell}, \breve{\Im})$.

Example 3.5. Let $\breve{\Re} = \{\breve{a}$ = Line Interactive, $\breve{b}$ = Standby-Ferro, $\breve{c}$ = Delta Conversion On-Line$\}$ and $\breve{\wp} = \{\breve{x}$ = Microwave cum Convection, $\breve{y}$ = Conventional oven, $\breve{z}$ = Convection oven $\}$ be types of UPS (Uninterruptible Power Supply) and Ovens respectively, considered as universes of discourses. A Marketing manger wants to know the relationship between UPS and Ovens characteristics which will be more effective regarding his marketing. Let ϵ_1 = efficiency, ϵ_2 = size, ϵ_3 = colour, ϵ_4 = price and $\breve{b}_1$ = efficiency, $\breve{b}_2$ = colour, $\breve{b}_3$ = price, $\breve{b}_4$ = size be the two types of distinct attributes whose attribute values belong to the sets $\breve{\eth}_1, \breve{\eth}_2, \breve{\eth}_3, \breve{\eth}_4$ and $\breve{\eth}'_1, \breve{\eth}'_2, \breve{\eth}'_3, \breve{\eth}'_4$ respectively. Let $\breve{\eth}_1 = \{\breve{\tau}_1$ = Good, $\breve{\tau}_2$ = Very Good$\}$, $\breve{\eth}_2 = \{\breve{\tau}_3$ = medium, $\breve{\tau}_4$ = small $\}$, $\breve{\eth}_3 = \{\breve{\tau}_5$ = brown $\}$ $\breve{\eth}_4 = \{\breve{\tau}_7$ = low price $\}$. Similarly, $\breve{\eth}'_1 = \{\breve{\tau}'_1$ = Good, $\breve{\tau}'_2$ = effective $\}$, $\breve{\eth}'_2 = \{\breve{\tau}'_3 = black, \breve{\tau}'_4$ = white $\}$, $\breve{\eth}'_3 = \{\breve{\tau}'_5$ = low price $\}$ $\breve{\eth}'_4 = \{\breve{\tau}'_7$ = medium $\}$ and $(\breve{\Re}, \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)$ and $(\breve{\wp}, \breve{\eth}'_1 \times \breve{\eth}'_2 \times \breve{\eth}'_3 \times \breve{\eth}'_4)$ be the classes of HSS. Let $\breve{\mu} : \breve{\Re} \to \breve{\wp}$, $\breve{\gamma} : \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4 \to \breve{\eth}'_1 \times \breve{\eth}'_2 \times \breve{\eth}'_3 \times \breve{\eth}'_4$ be mappings as follows $\breve{\mu}(\breve{a}) = \breve{z}$, $\breve{\mu}(\breve{b}) = \breve{y}$, $\breve{\mu}(\breve{c}) = \breve{y}$ and

$$\begin{aligned} \breve{\gamma}(\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5, \breve{\tau}_7) &= (\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \\ \breve{\gamma}(\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7) &= (\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \\ \breve{\gamma}(\breve{\tau}_2, \breve{\tau}_3, \breve{\tau}_5, \breve{\tau}_7) &= (\breve{\tau}_2{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \\ \breve{\gamma}(\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7) &= (\breve{\tau}_2{}', \breve{\tau}_4{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \end{aligned}$$

choose two HSS in $(\breve{\Re}, \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)$ and $(\breve{\wp}, \breve{\eth}'_1 \times \breve{\eth}'_2 \times \breve{\eth}'_3 \times \breve{\eth}'_4)$ respectively

$$
\begin{aligned}
(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4) = & \\
\{\{\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5, \breve{\tau}_7\} &= \Phi, \\
\{\breve{\tau}_1, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7\} &= \{\breve{a}\}, \\
\{\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7\} &= \{\breve{a}, \breve{b}, \breve{c}\}\} \\
(\breve{\ell}, \breve{\Im}_1 \times \breve{\Im}_2 \times \breve{\Im}_3 \times \breve{\Im}_4) &= \\
\{\{\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}'\} &= \{\breve{x}, \breve{y}\} \\
\{\breve{\tau}_1{}', \breve{\tau}_4{}', \breve{\tau}_5{}', \breve{\tau}_7{}'\} &= \{\breve{y}\},
\end{aligned}
$$

For a HS set $(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4)$ in $(\breve{\Re}, \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)$, then the HS image can be written as $(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4), \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)$ is a HS set in $(\breve{\wp}, \breve{\eth}'_1 \times \breve{\eth}'_2 \times \breve{\eth}'_3 \times \breve{\eth}'_4)$, where $(\breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4) = \breve{\gamma}(\breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4) = \{(\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}'), (\breve{\tau}_2{}', \breve{\tau}_4{}', \breve{\tau}_5{}', \breve{\tau}_7{}'))\}$ obtained as follows:

$$
\begin{aligned}
&= (\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4), \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)(\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \\
&= \breve{\mu}\Bigg(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\tau}_2{}', \breve{\tau}_4{}', \breve{\tau}_5{}', \breve{\tau}_7{}') \cap (\breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4)} \breve{\hbar}(\breve{\alpha})\Bigg)(\breve{s}) \\
&= \breve{\mu}\Bigg(\cup_{\breve{\alpha} \in \{(\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5, \breve{\tau}_7), (\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7)\}} \breve{\hbar}(\breve{\alpha})\Bigg)(\breve{s}) \\
&= \breve{\mu}\Bigg(\breve{\hbar}(\breve{\tau}_1, \breve{\tau}_3, \breve{\tau}_5, \breve{\tau}_7) \cup \breve{\hbar}(\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7)\Bigg)(\breve{s}) \\
&= \breve{\mu}\Bigg(\Phi \cup \{\breve{a}, \breve{b}, \breve{c}\})\Bigg)(\breve{s}) \\
&= \{\breve{y}, \breve{z}\}
\end{aligned}
$$

$$
(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4), \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)(\breve{\tau}_1{}', \breve{\tau}_3{}', \breve{\tau}_5{}', \breve{\tau}_7{}') = \{\breve{y}, \breve{z}\}
$$

Similarly,

$$
(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}_1 \times \breve{\aleph}_2 \times \breve{\aleph}_3 \times \breve{\aleph}_4), \breve{\eth}_1 \times \breve{\eth}_2 \times \breve{\eth}_3 \times \breve{\eth}_4)(\breve{\tau}_2{}', \breve{\tau}_4{}', \breve{\tau}_5{}', \breve{\tau}_7{}') = \{\breve{y}, \breve{z}\}
$$

Now for HS inverse image,

$$
\breve{\vartheta}^{-1}((\breve{\ell}, \breve{\Im}_1 \times \breve{\Im}_2 \times \breve{\Im}_3 \times \breve{\Im}_4), \breve{\Theta}_1 \times \breve{\Theta}_2 \times \breve{\Theta}_3 \times \breve{\Theta}_4)(\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7) = \{\breve{b}, \breve{c}\}
$$

where

$$
\Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4 = \breve{\gamma}^{-1}(\breve{\Im}_1 \times \breve{\Im}_2 \times \breve{\Im}_3 \times \breve{\Im}_4) = \{(\breve{\tau}_2, \breve{\tau}_4, \breve{\tau}_5, \breve{\tau}_7)\}.
$$

Definition 3.6. $\breve{\vartheta} : (\breve{\Re}, \breve{J}) \to (\breve{\wp}, \breve{K})$ be a mapping and $(\breve{\hbar}, \breve{\aleph})$ and $(\breve{\ell}, \breve{\Im})$ be the two HS sets in $(\breve{\Re}, \breve{J})$. Then for $\breve{\beta} \in \breve{K}$, HS union and intersection of HS images of $(\breve{\hbar}, \breve{\aleph})$ and $(\breve{\ell}, \breve{\Im})$ in $(\breve{\Re}, \breve{J})$

are defined as;
$(\breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \cup \breve{\vartheta}(\breve{\ell},\breve{\Im}))(\breve{\beta}) = \breve{\vartheta}(\breve{\hbar},\breve{\aleph})(\breve{\beta}) \cup \breve{\vartheta}(\breve{\ell},\breve{\Im})(\breve{\beta})$,
$(\breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \cap \breve{\vartheta}(\breve{\ell},\breve{\Im}))(\breve{\beta}) = \breve{\vartheta}(\breve{\hbar},\breve{\aleph})(\breve{\beta}) \cap \breve{\vartheta}(\breve{\ell},\breve{\Im})(\breve{\beta})$

Definition 3.7. $\breve{\vartheta} : (\breve{\Re}, \breve{J}) \rightarrow (\breve{\wp}, \breve{K})$ be a mapping and $(\breve{\hbar},\breve{\aleph})$ and $(\breve{\ell},\breve{\Im})$ be the two HS sets in $(\breve{\Re}, \breve{J})$. Then for $\breve{\alpha} \in \breve{J}$, HS union and intersection of HS inverse images of $(\breve{\hbar},\breve{\aleph})$ and $(\breve{\ell},\breve{\Im})$ in $(\breve{\Re}, \breve{J})$ are defined as;
$(\breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph}) \cup \breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im}))(\breve{\alpha}) = \breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph})(\breve{\alpha}) \cup \breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im})(\breve{\alpha})$,
$(\breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph}) \cap \breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im}))(\breve{\alpha}) = \breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph})(\breve{\alpha}) \cap \breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im})(\breve{\alpha})$

Theorem 3.8. *Let $\breve{\vartheta} : (\breve{\Re}, \breve{J}) \rightarrow (\breve{\wp}, \breve{K})$ be a HS-mapping, let $(\breve{\hbar},\breve{\aleph})$, $(\breve{\ell},\breve{\Im})$ be the two HS sets in $(\breve{\Re}, \breve{J})$ and with $(\breve{\hbar}_i,\breve{\aleph}_i)$ as the family of a HS sets in $(\breve{\Re}, \breve{J})$ we have,*

(1) $\breve{\vartheta}(\breve{\Phi}) = \breve{\Phi}$
(2) $\breve{\vartheta}(\breve{\Re}) \breve{\subset} \breve{\wp}$
(3) $\breve{\vartheta}((\breve{\hbar},\breve{\aleph}) \cup (\breve{\ell},\breve{\Im})) = \breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \cup \breve{\vartheta}(\breve{\ell},\breve{\Im})$. *Generally,* $\breve{\vartheta}(\cup_i(\breve{\hbar}_i,\breve{\aleph}_i)) = \cup_i \breve{\vartheta}(\breve{\hbar}_i,\breve{\aleph}_i)$.
(4) $\breve{\vartheta}((\breve{\hbar},\breve{\aleph}) \cap (\breve{\ell},\breve{\Im})) \breve{\supseteq} \breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \cap \breve{\vartheta}(\breve{\ell},\breve{\Im})$. *Generally,* $\breve{\vartheta}(\cap_i(\breve{\hbar}_i,\breve{\aleph}_i)) \breve{\subseteq} \cap_i \breve{\vartheta}(\breve{\hbar}_i,\breve{\aleph}_i)$.
(5) *If* $(\breve{\hbar},\breve{\aleph}) \breve{\subseteq} (\breve{\ell},\breve{\Im})$ *then* $\breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \breve{\subseteq} \breve{\vartheta}(\breve{\ell},\breve{\Im})$.

Proof. Here $(1), (2)$ are trivial case,
(3): for $\breve{\beta} \in \breve{K}$, we have to prove $\breve{\vartheta}((\breve{\hbar},\breve{\aleph}) \cup (\breve{\ell},\breve{\Im}))(\breve{\beta}) = (\breve{\vartheta}(\breve{\hbar},\breve{\aleph}) \cup \breve{\vartheta}(\breve{\ell},\breve{\Im}))(\breve{\beta})$.
Take

$$\breve{\vartheta}((\breve{\hbar},\breve{\aleph}) \cup (\breve{\ell},\breve{\Im}))(\breve{\beta}) = \breve{\vartheta}(\breve{S},\breve{\aleph} \cup \breve{\Im})(\breve{\beta}) = \begin{cases} \breve{\mu}\left(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cup \breve{\Im})} \breve{S}(\breve{\alpha})\right) & \text{if } \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cup \breve{\Im}) \neq \breve{\Phi}, \\ \breve{\Phi}, & \text{Otherwise} \end{cases} \tag{5}$$

where

$$\breve{S}(\breve{\alpha}) = \begin{cases} \breve{\hbar}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\aleph} - \breve{\Im}) \\ \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\Im} - \breve{\aleph}) \\ \breve{\hbar}(\breve{\alpha}) \cup \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\Im} \cap \breve{\aleph}) \end{cases} \tag{6}$$

L.H.S: For a non trivial case when $\breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cup \breve{\Im}) \neq \breve{\Phi}$, we have $\breve{\vartheta}((\breve{\hbar},\breve{\aleph}) \cup (\breve{\ell},\breve{\Im}))(\breve{\beta})$

$$= \breve{\mu}\left(\cup \begin{cases} \breve{\hbar}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\aleph} - \breve{\Im}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \\ \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\Im} - \breve{\aleph}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \\ \breve{\hbar}(\breve{\alpha}) \cup \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\aleph} \cap \breve{\Im}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \end{cases}\right) \tag{7}$$

R.H.S: for non trivial case, we have

$$(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}) \cup \breve{\vartheta}(\breve{\ell}, \breve{\Im}))(\breve{\beta}) = \quad u\Big(\cup_{\alpha \in p^{-1}(\beta) \cap \breve{\aleph}} \breve{\hbar}(\alpha) \Big) \cup u\Big(\cup_{\alpha \in p^{-1}(\beta) \cap \breve{H}} \breve{\ell}(\alpha) \Big) = u\Big(\cup_{\alpha \in p^{-1}(\beta) \cap \breve{\aleph}} \breve{\hbar}(\alpha) \cup_{\alpha \in p^{-1}(\beta) \cap \breve{H}} \breve{\ell}(\alpha) \Big)$$

$$= \breve{\mu} \left(\cup \begin{cases} \breve{\hbar}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\aleph} - \breve{\Im}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \\ \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\Im} - \breve{\aleph}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \\ \breve{\hbar}(\breve{\alpha}) \cup \breve{\ell}(\breve{\alpha}) & \text{if } \breve{\alpha} \in (\breve{\aleph} \cap \breve{\Im}) \cap \breve{\gamma}^{-1}(\breve{\beta}) \end{cases} \right) \tag{8}$$

From 7 and 8, we have 3.
(4): for $\breve{\beta} \in \breve{K}$ and $y \in Y$ we have to show that

$$\breve{\vartheta}((\breve{\hbar}, \breve{\aleph}) \cap (\breve{\ell}, \breve{\Im}))(\breve{\beta}) \breve{\supseteq} (\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}) \cap \breve{\vartheta}(\breve{\ell}, \breve{\Im}))(\breve{\beta})$$

$$(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}) \cap (\breve{\ell}, \breve{\Im}))(\breve{\beta}) = \breve{\vartheta}(\breve{S}, \breve{\aleph} \cap \breve{\Im})(\breve{\beta})$$

$$= \begin{cases} \breve{\mu}\left(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im})} \breve{S}(\breve{\alpha}) \right) & \text{if } \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im}) \neq \breve{\Phi}, \\ \breve{\Phi}, & \text{Otherwise,} \end{cases} \tag{9}$$

Where $\breve{S}(\breve{\alpha}) = \breve{\hbar}(\breve{\alpha}) \cap \breve{\ell}(\breve{\alpha})$.
For the non trivial case, $\breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im}) \neq \breve{\Phi}$

$$\breve{\vartheta}(\breve{S}, \breve{\aleph} \cap \breve{\Im})(\breve{\beta}) = \breve{\mu}\Big(\cup_{\alpha \in \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im})} \breve{S}(\breve{\alpha}) \Big) = \breve{\mu}\Big(\cup_{\alpha \in \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im})} (\breve{\hbar}(\breve{\alpha}) \cap \breve{\ell}(\breve{\alpha})) \Big)$$

$$(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}) \cap (\breve{\ell}, \breve{\Im}))(\breve{\beta}) = \breve{\mu}\Big(\cup_{\alpha \in \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im})} (\breve{\hbar}(\breve{\alpha}) \cap \breve{\ell}(\breve{\alpha})) \Big)$$

on the other side, by using definition 3.6 we have,

$$(\breve{\vartheta}(\breve{\hbar}, \breve{\aleph}) \cap (\breve{\ell}, \breve{\Im}))(\breve{\beta}) = (\breve{\vartheta}(\breve{\hbar}, \breve{\aleph})(\breve{\beta}) \cap \breve{\vartheta}(\breve{\ell}, \breve{\Im})(\breve{\beta}))$$

$$= \left(\begin{cases} \breve{\mu}\left(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\aleph}} \breve{\hbar}(\breve{\alpha}) \right) & \text{if } \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im}) \neq \breve{\Phi} \\ \breve{\Phi}, & \text{Otherwise,} \end{cases} \right) \cap \left(\begin{cases} \breve{\mu}\left(\cup_{\breve{\alpha} \in \breve{\gamma}^{-1}(\breve{\beta}) \cap \breve{\Im}} \breve{\ell}(\breve{\alpha}) \right) & \text{if } \breve{\gamma}^{-1}(\breve{\beta}) \cap (\breve{\aleph} \cap \breve{\Im}) \neq \breve{\Phi} \\ \breve{\Phi}, & \text{Otherwise,} \end{cases} \right) \tag{10}$$

Neglecting the trivial case, we obtain

$$(\breve{\vartheta}(\breve{\hbar},\breve{\aleph})\cap(\breve{\ell},\breve{\Im}))(\breve{\beta}) = \breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\aleph}}\breve{\hbar}(\breve{\alpha})\right)\cap\breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\Im}}\breve{\ell}(\breve{\alpha})\right)\breve{\supseteq}\breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap(\breve{\aleph}\cap\breve{\Im})}(\breve{\hbar}(\breve{\alpha})\cap\breve{\ell}(\breve{\alpha}))\right)$$
$$= \breve{\vartheta}((\breve{\hbar},\breve{\aleph})\cap(\breve{\ell},\breve{\Im}))(\breve{\beta})$$

(5): for a non-trivial case for $\breve{\beta}\in\breve{K}$

$$\breve{\vartheta}((\breve{\hbar},\breve{\aleph})(\breve{\beta}) = \begin{cases}\breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\aleph}}\breve{\hbar}(\breve{\alpha})\right) & \text{if } \breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\aleph}\neq\breve{\Phi}\\ \breve{\Phi}, & \text{Otherwise.}\end{cases} \tag{11}$$

Then

$$\breve{\vartheta}((\breve{\hbar},\breve{\aleph})(\breve{\beta}) = \breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\aleph}}\breve{\hbar}(\breve{\alpha})\right)\breve{\subseteq}\breve{\mu}\left(\cup_{\breve{\alpha}\in\breve{\gamma}^{-1}(\breve{\beta})\cap\breve{\Im}}\breve{\ell}(\breve{\alpha})\right) = \breve{\vartheta}(\breve{\ell},\breve{\Im})(\breve{\beta}).$$

This yields 5. In theorem 3.8, reversible of (2) and (4) does not hold. It can be explain in the following example.

Example 3.9. From example 3.5,

$$\breve{\wp}\not\subseteq\breve{\vartheta}(\breve{\Re}) = \{\{(\breve{\tau_1}',\breve{\tau_3}',\breve{\tau_5}',\breve{\tau_7}') = \{\breve{y},\breve{z}\},(\breve{\tau_2}',\breve{\tau_3}',\breve{\tau_5}',\breve{\tau_7}') = \{\breve{y},\breve{z}\},(\breve{\tau_2}',\breve{\tau_4}',\breve{\tau_5}',\breve{\tau_7}') = \{\breve{y},\breve{z}\}\}.$$

This indicate that reverse of (2) does not hold. Now we are going to show reverse of (4) also does not hold. For this purpose we choose two HS sets in $(\breve{\Re},\breve{\eth}_1\times\breve{\eth}_2\times\breve{\eth}_3\times\breve{\eth}_4)$ as

$$(\breve{\hbar},\breve{\aleph}_1\times\breve{\aleph}_2\times\breve{\aleph}_3\times\breve{\aleph}_4) = \{(\breve{\tau}_1,\breve{\tau}_3,\breve{\tau}_5,\breve{\tau}_7) = \Phi,(\breve{\tau}_1,\breve{\tau}_4,\breve{\tau}_5,\breve{\tau}_7) = \{\breve{a}\},$$
$$(\breve{\tau}_1,\breve{\tau}_4,\breve{\tau}_5,\breve{\tau}_7) = \{\breve{a},\breve{b},\breve{c}\},(\breve{\tau}_2,\breve{\tau}_3,\breve{\tau}_5,\breve{\tau}_7) = \{\breve{b}\}\},$$

$$(\breve{\ell},\breve{\Im}_1\times\breve{\Im}_2\times\breve{\Im}_3\times\breve{\Im}_4) = \{(\breve{\tau}_1,\breve{\tau}_3,\breve{\tau}_5,\breve{\tau}_7) = \{\breve{a}\},(\breve{\tau}_2,\breve{\tau}_4,\breve{\tau}_5,\breve{\tau}_7) = \{\breve{b},\breve{c}\}\}$$

Then calculations indicate that

$$\breve{\vartheta}(\breve{\hbar},\breve{\aleph}_1\times\breve{\aleph}_2\times\breve{\aleph}_3\times\breve{\aleph}_4)\cap\breve{\vartheta}(\breve{\ell},\breve{\Im}_1\times\breve{\Im}_2\times\breve{\Im}_3\times\breve{\Im}_4)\not\subseteq\breve{\vartheta}((\breve{\hbar},\breve{\aleph}_1\times\breve{\aleph}_2\times\breve{\aleph}_3\times\breve{\aleph}_4)\cap(\breve{\ell},\breve{\Im}_1\times\breve{\Im}_2\times\breve{\Im}_3\times\breve{\Im}_4))$$

Theorem 3.10. *Let $\breve{\vartheta}:(\breve{\Re},\breve{J})\to(\breve{\wp},\breve{K})$ be a mapping, $(\breve{\hbar},\breve{\aleph})$ and $(\breve{\ell},\breve{\Im})$ be the two HS sets in $(\breve{\Re},\breve{J})$ and the family of a HS sets $(\breve{\hbar}_i,\breve{\aleph}_i)$ in $(\breve{\Re},\breve{J})$ we have,*

(1) $\breve{\vartheta^{-1}}(\breve{\Phi}) = \breve{\Phi}$
(2) $\breve{\vartheta^{-1}}(\breve{\wp}) = \breve{\Re}$
(3) $\breve{\vartheta^{-1}}((\breve{\hbar},\breve{\aleph})\cup(\breve{\ell},\breve{\Im})) = \breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph})\cup\breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im})$. *Generally,* $\breve{\vartheta^{-1}}(\breve{\hbar}_i,\breve{\aleph}_i)) = \cup_i\breve{\vartheta^{-1}}(\breve{\hbar}_i,\breve{\aleph}_i)$.
(4) $\breve{\vartheta^{-1}}((\breve{\hbar},\breve{\aleph})\cap(\breve{\ell},\breve{\Im})) = \breve{\vartheta^{-1}}(\breve{\hbar},\breve{\aleph})\cap\breve{\vartheta^{-1}}(\breve{\ell},\breve{\Im})$. *Generally,* $\breve{\vartheta^{-1}}(\cap_i(\breve{\hbar}_i,\breve{\aleph}_i)) = \cap_i\breve{\vartheta^{-1}}(\breve{\hbar}_i,\breve{\aleph}_i)$.

(5) *If* $(\breve{\hbar},\breve{\aleph})\breve{\subseteq}(\breve{\ell},\breve{\Im})$ *then* $\vartheta^{\breve{-}1}((\breve{\hbar},\breve{\aleph})\breve{\subseteq}\vartheta^{\breve{-}1}(\breve{\ell},\breve{\Im})$.

Proof. Here $(1),(2)$ are trivial case,
(3): for $\breve{\alpha}\in\breve{J}$, we have to prove $\vartheta^{\breve{-}1}((\breve{\hbar},\breve{\aleph})\cup(\breve{\ell},\breve{\Im}))(\breve{\alpha})=(\vartheta^{\breve{-}1}(\breve{\hbar},\breve{\aleph})\cup\vartheta^{\breve{-}1}(\breve{\ell},\breve{\Im}))(\breve{\alpha})$.
Take

$$\begin{aligned}
& \vartheta^{\breve{-}1}((\breve{\hbar},\breve{\aleph})\cup(\breve{\ell},\breve{\Im}))(\breve{\alpha}) \\
= \;& \vartheta^{\breve{-}1}(\breve{S},\breve{\aleph}\cup\breve{\Im})(\breve{\alpha}) \\
= \;& u^{-1}(\breve{S}(\breve{\gamma}(\breve{\alpha})),\ \breve{\gamma}(\breve{\alpha})\in\breve{\aleph}\cup\breve{\Im} \\
= \;& u^{-1}(\breve{S}(\breve{\beta})),\ \breve{\gamma}(\breve{\alpha})=\breve{\beta}
\end{aligned}$$

where

$$=\breve{\mu}^{-1}\left(\begin{cases}\breve{\hbar}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\aleph}-\breve{\Im})\\ \breve{\ell}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\Im}-\breve{\aleph})\\ \breve{\hbar}(\breve{\beta})\cup\breve{\ell}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\Im}\cap\breve{\aleph})\end{cases}\right) \tag{12}$$

Now by using definition 3.7, we have

$$\begin{aligned}
& (\vartheta^{\breve{-}1}(\breve{\hbar},\breve{\aleph})\cup\vartheta^{\breve{-}1}(\breve{\ell},\breve{\Im}))\breve{\alpha} \\
= \;& \vartheta^{\breve{-}1}(\breve{\hbar},\breve{\aleph})(\breve{\alpha})\cup\vartheta^{\breve{-}1}(\breve{\ell},\breve{\Im})(\breve{\alpha}) \\
= \;& u^{\breve{-}1}(\breve{\hbar}(\breve{\gamma}(\breve{\alpha})))\cup u^{\breve{-}1}(\breve{\ell}(\breve{\gamma}(\breve{\alpha}))),\ \ \breve{\gamma}(\breve{\alpha})\in\breve{\aleph}\cap\breve{\Im}
\end{aligned}$$

$$=\breve{\mu}^{-1}\left(\begin{cases}\breve{\hbar}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\aleph}-\breve{\Im})\\ \breve{\ell}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\Im}-\breve{\aleph})\\ \breve{\hbar}(\breve{\beta})\cup\breve{\ell}(\breve{\beta}) & \text{if } \breve{\beta}\in(\breve{\Im}\cap\breve{\aleph})\end{cases}\right) \tag{13}$$

Where $\breve{\beta}=\breve{\gamma}(\breve{\alpha})$. From 12 and 13 we get (3).

Now for (4) we take $\breve{\alpha}\in\breve{J}$

$$\begin{aligned}
& \breve{\vartheta}^{-1}(\breve{\hbar},\breve{\aleph})\cap(\breve{\ell},\breve{\Im})(\breve{\alpha}) \\
= \;& (\breve{\vartheta}^{-1}(\breve{S},\breve{\aleph}\cap\breve{\Im})(\breve{\alpha}) \\
= \;& \breve{\mu}^{-1}(\breve{S}(p(\alpha))),p(\alpha)\in\breve{\aleph}\cap\breve{\Im}, \\
= \;& \breve{\mu}^{-1}(\breve{\hbar}(\breve{\beta})\cap\breve{\ell}(\breve{\beta})),p(\breve{\alpha})=\breve{\beta}, \\
= \;& \breve{\mu}^{-1}(\breve{\hbar}(\breve{\beta}))\cap\breve{\mu}^{-1}(\breve{\ell}(\breve{\beta})) \\
= \;& (\breve{\vartheta}^{-1}(\breve{\hbar},\breve{\aleph})\cap\breve{\vartheta}^{-1}(\breve{\ell},\breve{\Im}))(\breve{\alpha})
\end{aligned}$$

This gives (4).

(5): for $\breve{\alpha} \in \breve{J}$, consider $\breve{\vartheta}^{-1}(\breve{\hbar}, \breve{\aleph})(\breve{\alpha})$

$$\begin{aligned}
&= \breve{\mu}^{-1}(\breve{\hbar}(\breve{\gamma}(\breve{\alpha}))) \\
&= \breve{\mu}^{-1}(\breve{\hbar}(\breve{\beta})), \breve{\gamma}(\breve{\alpha}) = \breve{\beta} \\
&\subseteq \breve{\mu}^{-1}(\breve{\ell}(\breve{\beta})) \\
&= \breve{\mu}^{-1}(\breve{\ell}(\breve{\gamma}(\breve{\alpha}))) \\
&= \breve{\vartheta}^{-1}(\breve{\ell}, \breve{\Im})(\breve{\alpha})
\end{aligned}$$

This yields (5).

4. **Statement of Problem**

4.1. *Case study and Objective*

Before starting a new construction business, it is essential to understand the investment and work that is involved. Establishing a new business of any kind is never easy, and there are always things to consider that may or may not be at the forefront of one's mind, even an experienced entrepreneur. Are you ready to launch your construction startup? Here are ten things that should be at the foundation of your plans.

(1) Put Together a Solid Business Plan
(2) Find a Good Home Base for Your Startup
(3) Get Your Legal Ducks in a Row
(4) Understand Your Tax Requirements
(5) Understand Your Insurance Responsibilities
(6) Network With Suppliers, Business Associates, and Other Contractors
(7) Decide Whether to Hire Employees or Contractors
(8) Establish an Advertising and Marketing Budget
(9) Allocate Funds for Construction Software
(10) Access Small Business Loans and Financing
(11) Be organized.
(12) Stay involved.

and ten big challenges a construction company may face.

(1) Lack of Skilled Workers
(2) Lack of Communication
(3) Unreliable Subcontractors
(4) Scheduling
(5) High Insurance Costs
(6) Changing Minds of Homeowners

(7) Available Cash

(8) Document Management

(9) The Blame Game

(10) Ever-changing Regulations

A person decides to set a construction company. To tackle and plan for future he create an interrelationship between the Construction Companies of two major economic bodies in the world (China and U.S.) by considering multi-attribute function. For this, let $\breve{X} = \{\breve{a} =$ Beijing Construction Engineering Group, $\breve{b} =$ Anhui Construction Engineering Group, $\breve{c} =$ China State Construction Engineering $\}$ and $\breve{Y} = \{\breve{x} =$ Kiewit Corporation, $\breve{y} =$Skanska Construction, $\breve{z} =$ Whiting-Turner Contracting Company$\}$ be the two universal sets of construction companies China and U.S respectively. Let $\breve{x}_1 =$ Communication, $\breve{x}_2 =$ Leadership, $\breve{x}_3 =$ Standard, and $\breve{y}_1 =$ Innovative, $\breve{y}_2 =$ Planning, $\breve{y}_3 =$ Budget potency be the two distinct attributes whose attribute values belong to the sets $\breve{E}_1, \breve{E}_2, \breve{E}_3$ and $\breve{E'}_1, \breve{E'}_2, \breve{E'}_3$ respectively, where $\breve{E}_1 = \{\breve{e}_1 =$ Conciseness, $\breve{e}_2 =$ Effective Communication, $\breve{e}_3 =$Low level Communication $\}$, $\breve{E}_2 = \{\breve{e}_4 =$ Learning agility, $\breve{e}_5 =$ Influence$\}$, $\breve{E}_3 = \{\breve{e}_6 =$ High Standard, $\breve{e}_7 =$ Feasible$\}$ and $\breve{E'}_1 = \{\breve{e'}_1 =$ relative advantage, $\breve{e'}_2 =$ compatibility$\}$, $\breve{E'}_2 = \{\breve{e'}_3 =$ Corrective Action Plan, $\breve{e'}_4 =$ Traditional planning$\}$, $\breve{E'}_3 = \{\breve{e'}_5 =$ High budget$\}$.

Let $\breve{u} : \breve{X} \to \breve{Y}$, $\breve{p} : \breve{E}_1 \times \breve{E}_2 \times \breve{E}_3 \to \breve{E'}_1 \times \breve{E'}_2 \times \breve{E'}_3$ be mappings as follows $\breve{u}(\breve{a}) = \breve{x}$, $\breve{u}(\breve{b}) = \breve{y}$, $\breve{u}(\breve{c}) = \breve{x}$ and

$$\begin{aligned}
\breve{p}(\breve{e}_1, \breve{e}_4, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_1, \breve{e}_4, \breve{e}_7) &= (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) \\
\breve{p}(\breve{e}_1, \breve{e}_5, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_1, \breve{e}_5, \breve{e}_7) &= (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) \\
\breve{p}(\breve{e}_2, \breve{e}_4, \breve{e}_6) &= (\breve{e}'_2, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_2, \breve{e}_4, \breve{e}_7) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_2, \breve{e}_5, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) \\
\breve{p}(\breve{e}_2, \breve{e}_5, \breve{e}_7) &= (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) \\
\breve{p}(\breve{e}_3, \breve{e}_4, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_3, \breve{e}_4, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) \\
\breve{p}(\breve{e}_3, \breve{e}_5, \breve{e}_6) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) \\
\breve{p}(\breve{e}_3, \breve{e}_5, \breve{e}_7) &= (\breve{e}'_1, \breve{e}'_3, \breve{e}'_5)
\end{aligned}$$

respectively. This data can be encoded into two HSS from two classes $(\breve{X}, \breve{E}_1 \times \breve{E}_2 \times \breve{E}_3)$ and $(\breve{Y}, \breve{E'}_1 \times \breve{E'}_2 \times \breve{E'}_3)$ respectively,

$$(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3) = \{(\breve{e}_1, \breve{e}_4, \breve{e}_6) = \{\breve{a}, \breve{b}\}, (\breve{e}_1, \breve{e}_4, \breve{e}_7) = \{\breve{b}\}\},$$
$$(\breve{\Delta}, \breve{\Omega}_1 \times \breve{\Omega}_2 \times \breve{\Omega}_3) = \{(\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) = \{\breve{x}\}, \quad (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) = \{\breve{x}, \breve{y}\}, (\breve{e}'_2, \breve{e}'_4, \breve{e}'_5) = \{\breve{z}\}\}$$

Then the HS image of $(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3)$ under

$$\breve{f} : (\breve{X}, \breve{E}_1 \times \breve{E}_2 \times \breve{E}_3) \to (\breve{Y}, \breve{E}'_1 \times \breve{E}'_2 \times \breve{E}'_3)$$

is obtained by

$$\breve{f}(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3)(\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) = \{\breve{x}, \breve{y}\},$$
$$\breve{f}(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3)(\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) = \{\breve{y}\}$$
$$\breve{f}(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3) = \{(\breve{e}'_1, \breve{e}'_3, \breve{e}'_5) = \{\breve{x}, \breve{y}\}, (\breve{e}'_1, \breve{e}'_4, \breve{e}'_5) = \{\breve{y}\}\},$$

which means that, with respect to the attributes values (Relative advantage, corrective action plan, High budget), Kiewit Corporation and Skanska Construction is best companies. On the other hand, with respect to the attributes values (Relative advantage, Traditional planning, High budget), Skanska Construction is best. Similarly,

$$\breve{f}^{-1}(\breve{\Delta}, \breve{\Omega}_1 \times \breve{\Omega}_2 \times \breve{\Omega}_3)(\breve{e}_1, \breve{e}_4, \breve{e}_6) = \{\breve{a}, \breve{c}\}$$
$$\breve{f}^{-1}(\breve{\Delta}, \breve{\Omega}_1 \times \breve{\Omega}_2 \times \breve{\Omega}_3)(\breve{e}_1, \breve{e}_4, \breve{e}_7) = \{\breve{a}, \breve{b}, \breve{c}\}$$
$$\breve{f}^{-1}(\breve{\Lambda}, \breve{\Sigma}_1 \times \breve{\Sigma}_2 \times \breve{\Sigma}_3) = \{(\breve{e}_1, \breve{e}_4, \breve{e}_6) = \{\breve{a}, \breve{c}\}, (\breve{e}_1, \breve{e}_4, \breve{e}_7) = \{\breve{a}, \breve{b}, \breve{c}\}\},$$

which means that, with respect to the attributes values (Conciseness, Learning agility, High standard), Beijing Construction Engineering Group and Anhui Construction Engineering is best companies. On the other side, with respect to the attributes values (Conciseness, Learning agility, Feasible), Beijing Construction Engineering Group, Anhui Construction Engineering Group, China State Construction Engineering is best.

4.2. *Comparative studies*

In the following, few comparisons of the initiated techniques with shortcomings are talked about to analyze the validity and predominance of the proposed technique. Additionally, we will compare our proposed mappings based hypersoft classes with nine other existing theories, Estruch and Vidal [5] give a fixed point theory for fuzzy contraction mappings over a complete metric space, Syau [8] demonstrated the idea of convex and concave fuzzy mappings, Karaaslan [24] presented the soft class and its relevant operations, including the idea presented by Athar et al. [25, 26] the concept of mappings on fuzzy soft classes and mappings on soft classes, Alkhazaleh [27] et al. introduced the notion of a mapping on classes where the neutrosophic soft classes are collections of the neutrosophic soft sets, Sulaiman [28] et. al presented the idea of mappings on multi-aspect fuzzy soft classes, Maruah [29] et. al characterized the notation of mapping on intuitionistic fuzzy soft classes with some properties of intuitionistic fuzzy soft images and inverse images, Manash et al. [30] gave the idea of composite mappings on hesitant fuzzy soft classes in 2016 and discussed some interesting properties of this idea. However, when the attributes are further sub-divided into attribute values then all existing theories are fails to manage. This need is fulfilled in the proposed mappings based hypersoft classes, see table 1.

TABLE 1. Comparison of the proposed mappings based Hypersoft classes with existing theories

SN	References	Disadvantage	Ranking
1	[5]	lack of sub- partition values of parameter	inapplicable
2	[8]	lack of sub- partition values of parameter	inapplicable
3	[24]	lack of sub- partition values of parameter	inapplicable
4	[25]	lack of sub- partition values of parameter	inapplicable
5	[26]	lack of sub- partition values of parameter	inapplicable
6	[27]	lack of sub- partition values of parameter	inapplicable
7	[28]	lack of sub- partition values of parameter	inapplicable
8	[29]	lack of sub- partition values of parameter	inapplicable
9	[30]	lack of sub- partition values of parameter	inapplicable
10	Proposed Method in this paper	Lengthy and heavy calculations in decision-making	applicable

5. Conclusions

In this study, an extension is made in the existing mappings of fuzzy and soft classes to HS classes. The presented work is more effective than the existing one. Some important properties and results are discussed with the support of illustrated examples. The future extension can be made by developing such mappings on other hybrid structures of HS sets, topological structures, algebraic structures, graph theory, computer science, Electromagnetic analysis for detection, Rocket launch trajectory analysis, Materials science, Modeling of airflow over airplane bodies and Behavioural sciences. Moreover, a practical application and comparative study is given to show the validity and predominance of the proposed technique. This concept may help the researchers in decision-making problems by considering disjoints sets of attribute values rather than taking one set of parameters.

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Chapter 10

Development of Rough Hypersoft Set with Application in Decision Making for the Best Choice of Chemical Material

Atiqe Ur Rahman [1*], Abida Hafeez[1], Muhammad Saeed[2], Muhammad Rayees Ahmad[3], Ume-e-Farwa[3]

[1,2,3] Department of Mathematics, University of Management and Technology Lahore, Pakistan.

E-mail: aurkhb@gmail.com , E-mail: f2019109046@umt.edu.pk, E-mail: muhammad.saeed@umt.edu.pk , E-mail: rayeesmalik.ravian@gmail.com , E-mail: f2019109047@umt.edu.pk

Abstract: Hypersoft set is the generalization of soft set as it replaces single set of distinct attributes with their corresponding disjoint attribute valued sets. This makes the hypersoft set more effective and useful parameterized tool to tackle vague information. Rough soft set, the combination of rough set and soft set, is studied by many researchers to undertake imperfect knowledge through lower and upper approximations. But it is inadequate to deal with disjoint attribute valued sets. In this study, this meagerness is addressed and rough hypersoft set is conceptualized with the development of its lower and upper approximations. Moreover, some of its elementary properties and results are discussed. A new algorithm is proposed to solve decision making problems and is demonstrated with illustrative examples.

Keywords: *Soft set, Rough set, Rough soft set, Hypersoft set, Rough hypersoft set.*

1. Introduction

Zadeh's theory of fuzzy sets [1] is considered as mathematical tool to undertake many convoluted problems involving various uncertainties in different branches of mathematical sciences. But this theory is unable to solve these problems successfully due to the meagerness of the parameterization tool. This inadequacy is addressed by Molodtsov known as soft set theory [2] which is free from all such hindrances and appeared as a new parameterized family of subsets of the universe of discourse. Rough set theory [3-4] is assumed to be a novel mathematical device to tackle imperfect knowledge and vagueness through approximations. Rough soft set, a combination of soft set and rough set [5] was proposed to ensure the adequacy of rough set for parameterization purpose. This is achieved by developing lower and upper approximations for soft set. Rough soft set is studied by many researchers [6-11] through different approaches with applications in certain fields. The concept of hypersoft set as a generalization of soft set was proposed by [14]. Saeed et al. [15] extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, sub relation,complement relation, function, matrices and operations on hypersoft matrices. Mujahid et al. [16] discussed basic ingrediants for topological structures on the collection of hypersoft sets. Moreover they introduced hypersoft points in different envorinments like fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic hypersoft, plithogenic hypersoft set. Rahman et al. [17] defined complex hypersoft set and developed the hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutrosophic set respectively. They also discussed their fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. complement, union, intersection etc. Rahman et al. [18] conceptualized convexity cum concavity on hypersoft set and presented its pictorial versions with illustrative examples.

Having motivation from the work described in [5], [14], [15] and [16], we develop rough hypersoft set along with its some fundamental properties. Further we propose a new algorithm to solve decision making problems and apply it for the best choice of chemical materials. This work is the extension of existing concept [5] in the sense that it deals with attribute valued sets instead of taking just attributes.

The rest of the paper is organized as: Section 2 recalls basic definitions from literature. Section 3 presents hypersoft upper set, hypersoft lower set and rough hypersoft set. It too presents some important properties and results of rough hypersoft set. Section 4 presents application of rough hypersoft set in decision making. Section 5 finally concludes the paper.

2. **Preliminaries**

Here we recall some supporting basic definitions and concepts from existing literature.

Definition 2.1. [3] A relation $\mathbb{R}$ on $A \neq \emptyset$ is called an *equivalence relation* on A if it is reflexive, symmetric and transitive. We denote the equivalence class of an element $y \in S$ with respect to $\mathbb{R}$ as $\mathbb{R}[y]$ and $\mathbb{R}[y] = \{z \in A : z\mathbb{R}y\}$.

Definition 2.2. [3] A set $W \subseteq \mathbb{U}$ is said to be a *Rough set* w.r.t $\mathbb{R}$ on $\mathbb{U}$, if

$$\beta_{\mathbb{R}}(y) = \mathbb{R}^*(y) - \mathbb{R}_*(y)$$

is non empty, where $\mathbb{R}_*(y) = \{y : \mathbb{R}[y] \subseteq W\}$ and $\mathbb{R}^*(y) = \{y : \mathbb{R}[y] \cap W \neq \emptyset\}$ are lower and upper approximation of W respectively. For more definitions and theoretic operations on rough set, see [2].

Definition 2.3. [2]

A pair (ψ, A) is called a *soft set* over $\mathbb{U}$, if $A \subset E$ and $\psi : A \to P(\mathbb{U})$.We write ψ_A for (ψ, A) .

Definition 2.4. [12]

Let ψ_A and φ_B be soft sets over a common universe set $\mathbb{U}$ and $A, B \subset E$.Then we say that

- ψ_A is a *soft subset* of φ_B , denoted by $\psi_A \subset \varphi_B$,if
  (1) $A \subset E$ and
  (2) $\psi(e) \subset \varphi(e) \forall\ e \in A$
- $\psi_A = \varphi_B$, denoted by $\psi_A = \varphi_B$ if $\psi_A \subset \varphi_B$ and $\varphi_B \subset \psi_A$.

For more definitions and operations of soft set, see [2], [12] and [13] .

Definition 2.5. [5] Let ψ_E be a soft set and $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ then we define two soft sets $\overrightarrow{\psi} : E \to P(\mathbb{U})$ and $\overleftarrow{\psi} : E \to P(\mathbb{U})$ as follows.

$$\overrightarrow{\psi}(\,e) = \overset{*}{\mathbb{R}}(\psi(e))$$

and

$$\overleftarrow{\psi}(\,e) = \underset{*}{\mathbb{R}}(\psi(e))$$

for every $e \in E$. We call the soft sets $\underset{E}{\overrightarrow{\psi}}$ and $\underset{E}{\overleftarrow{\psi}}$, the *upper soft set* and the *lower soft set* respectively.

Definition 2.6. [5]

We say that a soft set ψ_E, is a *Rough soft set* if the difference $\underset{E}{\overrightarrow{\psi}} \setminus \underset{E}{\overleftarrow{\psi}}$ is a non-null soft set.

Definition 2.7. [5]

By a *Crisp soft set*, we mean a soft set ψ_E such that $\underset{E}{\overrightarrow{\psi}} \cong \underset{E}{\overleftarrow{\psi}}$.

Definition 2.8. [14]

Let $A_i, i = 1, 2, ..., n$, are disjoint sets having attributive values of distinct attributes $a_i, i = 1, 2, ..., n$. Then the pair (ψ, G), where $G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $\psi : G \to P(\mathbb{U})$, is called a *hypersoft Set* over $\mathbb{U}$.We simply use ψ_G for hypersoft set.

More definitions and examples can be seen from [14]- [28].

3. Rough Hypersoft Set

In this section, the lower hypersoft set ,upper hypersoft set and rough hypersoft sets are defined. Some important results are established too.

Definition 3.1. Let R be an equivalence relation on $\mathbb{U}$ and ψ_G is a hypersoft set then we define two hypersoft sets $\overrightarrow{\psi} : G \to P(\mathbb{U})$ and $\overleftarrow{\psi} : G \to P(\mathbb{U})$ as follows.

$$\overrightarrow{\psi}(\,e) = \overset{*}{R}(\,\psi(e))$$

and

$$\overleftarrow{\psi}(e) = \underset{*}{R}(\psi(e))$$

for every $e = (e_1, e_2, e_3, ..., e_n) \in G$. We call the hypersoft sets $\underset{G}{\overrightarrow{\psi}}$ and $\underset{G}{\overleftarrow{\psi}}$, the *upper hypersoft set* and the *lower hypersoft set* respectively.

Definition 3.2. We say that a hypersoft set ψ_G , a *Rough hypersoft set* if the difference $\underset{G}{\overrightarrow{\psi}} \setminus \underset{G}{\overleftarrow{\psi}}$ is a non-null hypersoft set.

Definition 3.3. By a *Crisp hypersoft set*, We mean a hypersoft set ψ_G such that $\underset{G}{\overrightarrow{\psi}} \cong \underset{G}{\overleftarrow{\psi}}$.

Theorem 3.4. *If ψ_G is a hypersoft set over $\mathbb{U}$ the universe set then $\underset{G}{\overleftarrow{\psi}} \subseteq \psi_G \subseteq \underset{G}{\overrightarrow{\psi}}$.*

Proof. Let ψ_G be a hypersoft set over the universe set $\mathbb{U}$ and R be an equivalence relation on $\mathbb{U}$.Let $x \in R_*(\psi(e))$ then $R[x] \subseteq \psi(e)$ for $e \in G$ and

$$R[x] \cap \psi(e) = R[x] \neq \emptyset$$

so $x \in R^*(\psi(e))$ which implies

$$R_*(\psi(e)) \subseteq R^*(\psi(e)) \tag{1}$$

for every $e \in G$.
If $x \in R_*(\psi(e))$ then $R[x] \subseteq \psi(e)$ so $x \in \psi(e)$
which implies

$$R_*(\psi(e)) \subseteq \psi(e) \tag{2}$$

for every $e \in G$.
Let $x \in \psi(e)$, then $x \in R(x) \cap \psi(e)$ so $R[x] \cap \psi(e) \neq \emptyset$
which implies

$$x \in R^*(\psi(e))$$

therefore,

$$\psi(e) \subseteq R^*(\psi(e)) \tag{3}$$

for every $e \in G$.
From equations (1), (2) and (3), we have

$$R_*(\psi(e)) \subseteq \psi(e) \subseteq R^*(\psi(e))$$

for every $e \in G$.
It follows that

$$\underset{G}{\overleftarrow{\psi}} \subseteq \psi_G \subseteq \underset{G}{\overrightarrow{\psi}}.$$

Hence the theorem. □

Proposition 3.5. *If Φ_G is the null hypersoft set and X_G is the absolute hypersoft set, defined by $\Phi_G = \emptyset$ and $X_G = U$ for every $e \in G$, then*

(1) $\overleftarrow{\Phi}_G \cong \overrightarrow{\Phi}_G \cong \Phi_G$

(2) $\overleftarrow{X}_G \cong \overrightarrow{X}_G \cong X_G$

Proposition 3.6. *If ψ_G and φ_G are any two hypersoft sets such that $\psi(e) \mathrel{\tilde{\subseteq}} \varphi(e)$ then*

(1) $\overleftarrow{\psi}(e) \mathrel{\tilde{\subseteq}} \overleftarrow{\varphi}(e)$

(2) $\overrightarrow{\psi}(e) \mathrel{\tilde{\subseteq}} \overrightarrow{\varphi}(e)$

Proposition 3.7. *If ψ_G and φ_G be two hypersoft sets then*

(1) $\overrightarrow{\psi_G \cup \varphi_G} \cong \overrightarrow{\psi}_G \cup \overrightarrow{\varphi}_G$

(2) $\overrightarrow{\psi_G \cap \varphi_G} \mathrel{\tilde{\subseteq}} \overrightarrow{\psi_G} \cap \overrightarrow{\varphi_G}$

(3) $\overleftarrow{\psi_G \cap \varphi_G} \cong \overleftarrow{\psi}_G \cap \overleftarrow{\varphi}_G$

(4) $\overleftarrow{\psi}_G \cup \overleftarrow{\varphi}_G \mathrel{\tilde{\subseteq}} \overleftarrow{\psi_G \cup \varphi_G}$

Proposition 3.8. *If ψ_G is any hypersoft set then*

(1) $\overrightarrow{\overrightarrow{\psi}_G} \cong \overrightarrow{\psi}_G$

(2) $\overleftarrow{\overleftarrow{\psi}_G} \cong \overleftarrow{\psi}_G$

(3) $\overrightarrow{\overrightarrow{\psi}_G} \cong \overrightarrow{\overleftarrow{\psi}_G} \cong \overrightarrow{\psi}_G$

(4) $\overleftarrow{\overleftarrow{\psi}_G} \cong \overleftarrow{\overrightarrow{\psi}_G} \cong \overleftarrow{\psi}_G$

4. **Application of Rough Hypersoft set in Decision Making**

Here a new decision making method is constructed for rough hypersoft sets to find $\psi(e)$, the appropriate material on ψ_G w.r.t R on $\mathbb{U}$ and is illustrated by two examples.
Let $\psi_G = (\psi, G)$ be an original description hypersoft set over $\mathbb{U}$. The proposed algorithm is as under:

TABLE 1. table for weighted hypersoft set ψ_G.

U	e_1	e_2	e_3	e_4
	w_1	w_2	w_3	w_4
m_1	1	1	0	0
m_2	1	0	1	0
m_3	0	1	0	1
m_4	1	1	1	0
m_5	0	0	1	0
m_6	0	0	0	1
m_7	0	0	0	1
m_8	0	1	0	1

Step 1: Take ψ_G as input.
Step 2: Compute $\underset{G}{\overrightarrow{\psi}}$ and $\underset{G}{\overleftarrow{\psi}}$ on ψ_G, respectively.
Step 3: Compute the different values of $\parallel \psi(e_i) \parallel$, where

$$\parallel \psi(e_i) \parallel = \frac{|\underset{G}{\overrightarrow{\psi}}(e_i)| - |\underset{G}{\overleftarrow{\psi}}(e_i)|}{|\psi(e_i|} \times w_i$$

where w_i are assigned weights.

Step 4: Find the minimum value $\parallel \psi(e_l) \parallel$ of $\parallel \psi(e_i) \parallel$, where

$$\parallel \psi(e_l) \parallel = mini_i \parallel \psi(e_i) \parallel .$$

Step 5: The output is $\parallel \psi(e_l) \parallel$.

Example 4.1. Let $\mathbb{U} = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$ be the materials. A drug manufacturing company intends to select the most relevant material for its specific molecule structure. The relevancy of these materials on basis of chemical properties is as under:
$m_1 \sim m_2$,
$m_3 \sim m_4 \sim m_5$, and
$m_6 \sim m_7 \sim m_8$, where $\sim$ is for close matching.
Now, there are four materials to be detected, denoted by $G = \{e_1 = (e_{11}, e_{21}), e_2 = (e_{11}, e_{22}), e_3 = (e_{12}, e_{21}), e_4 = (e_{12}, e_{22})\}$, where $G = E_1 \times E_2$ with $E_1 = \{e_{11}, e_{12}\}$ and $E_2 = \{e_{21}, e_{22}\}$. And their ingredients are represented by $\psi(e_1) = \{m_1, m_2, m_4\}, \psi(e_2) = \{m_1, m_3, m_4, m_8\}, \psi(e_3) = \{m_2, m_4, m_5\}$, and $\psi(e_4) = \{m_3, m_6, m_7, m_8\}$, respectively.
Let following weights are allocated to each element in G: $w_1 = 0.7$ is for e_1,
$w_2 = 0.5$ is for e_2,
$w_3 = 0.6$ is for e_3, and
$w_4 = 0.3$ is for e_4. Then we can find the table (4) for weighted hypersoft set ψ_G. According

Table 2. table for hypersoft set $\underset{G}{\overleftarrow{\psi}}$.

U	e_1 w_1	e_2 w_2	e_3 w_3	e_4 w_4
m_1	1	1	0	0
m_2	1	0	0	0
m_3	0	1	0	0
m_4	0	1	0	0
m_5	0	0	0	0
m_6	0	0	0	1
m_7	0	0	0	1
m_8	0	1	0	1

Table 3. table for hypersoft set $\underset{G}{\overrightarrow{\psi}}$.

U	e_1 w_1	e_2 w_2	e_3 w_3	e_4 w_4
m_1	1	1	1	0
m_2	1	1	1	0
m_3	1	1	1	1
m_4	1	1	1	1
m_5	1	1	1	1
m_6	0	1	0	1
m_7	0	1	0	1
m_8	0	1	0	1

to the proposed method and by definition (3.1), we have two hypersoft sets $\underset{G}{\overrightarrow{\psi}}$ and $\underset{G}{\overleftarrow{\psi}}$ over $\mathbb{U}$. They are presented in Tables (6) and (5).

Now by applying proposed algorithm, we have,

$\| \psi(e_1) \| = 0.7$,

$\| \psi(e_2) \| = 0.75$,

$\| \psi(e_3) \| = 1.0$, and

$\| \psi(e_4) \| = 0.215$.

Thus the minimum value for $\| \psi(e_i) \|$ is $\| \psi(e_4) \| = 0.215$.

Which implies that $\psi(e_4)$ is the most appropriate and suitable one.

Example 4.2. A company is working on cancer drug discovery. Some materials can be checked against a target on the basis of molecular structure. The materials under consideration are given is the universe $\mathbb{U} = \{M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8\}$. The Company wants to choose the best one material by observing following attributes of cancer

TABLE 4. Table for weighted hypersoft set ψ_G.

$\mathbb{U}$	e_1	e_2	e_3	e_4
	w_1	w_2	w_3	w_4
M_1	0.7	0.61	0	0
M_2	0.8	0	0.4	0
M_3	0	0.85	0	0.73
M_4	0.5	0.43	0.6	0
M_5	0	0	0.51	0
M_6	0	0	0	0.49
M_7	0	0	0	0.8
M_8	0	0.92	0	0.67

$a_1 = Tumors, a_2 = Fatigue$. The attribute values of the attributes are given in the set correspondingly as,

$E_1 = \{\text{benign}(e_{11}), \text{malignant}(e_{12})\}$,

$E_2 = \{\text{mild fatigue}(e_{21}), \text{severe fatigue}(e_{22})\}$.

In terms of their chemical properties, we regard as $M_1 \sim M_2$,

$M_3 \sim M_4 \sim M_5$,

and

$M_6 \sim M_7 \sim M_8$,

where $\sim$ represent the chemical properties of these materials are equivalent.

Now, Set of materials to be under observation, is

$$G = \{e_1 = (e_{11}, e_{21}), e_2 = (e_{11}, e_{22}), e_3 = (e_{12}, e_{21}), e_4 = (e_{12}, e_{22})\},$$

where each $G = E_1 \times E_2$ with $E_1 = \{e_{11}, e_{12}\}$ and $E_2 = \{e_{21}, e_{22}\}$.And each kind of material containing $\psi(e_1) = \{M_1, M_2, M_4\}, \psi(e_2) = \{M_1, M_3, M_4, M_8\}, \psi(e_3) = \{M_2, M_4, M_5\}$, and $\psi(e_4) = \{M_3, M_6, M_7, M_8\}$, respectively.

Let allocation of weights by the company to each element of G: $w_1 = 0.81$ is assigned to e_1,

$w_2 = 0.73$ is assigned to e_2,

$w_3 = 0.66$ is assigned to e_3, and

$w_4 = 0.45$ is assigned to e_4.

Then we have tables (4) for weighted ψ_G. And $\underset{G}{\overrightarrow{\psi}}$ and $\underset{G}{\overleftarrow{\psi}}$ over $\mathbb{U}$, are presented in Tables (6) and (5), respectively. Then we have

$\| \psi(e_1) \| = 0.2872$,

$\| \psi(e_2) \| = 0.1903$,

$\| \psi(e_3) \| = 1.0372$, and

$\| \psi(e_4) \| = 0.1422$.

TABLE 5. Table for hypersoft set $\underset{G}{\overleftarrow{\psi}}$.

$\mathbb{U}$	e_1 w_1	e_2 w_2	e_3 w_3	e_4 w_4
M_1	0.7	0.61	0	0
M_2	0.8	0	0	0
M_3	0	0.85	0	0
M_4	0	0.43	0	0
M_5	0	0	0	0
M_6	0	0	0	0.49
M_7	0	0	0	0.8
M_8	0	0.92	0	0.67

TABLE 6. Table for hypersoft set $\underset{G}{\overrightarrow{\psi}}$.

$\mathbb{U}$	e_1 w_1	e_2 w_2	e_3 w_3	e_4 w_4
M_1	0.7	0.61	0.7	0
M_2	0.8	0.5	0.4	0
M_3	0.6	0.85	0.81	0.73
M_4	0.5	0.43	0.6	0.63
M_5	0.67	0.3	0.51	0.5
M_6	0	0.7	0	0.49
M_7	0	0.65	0	0.8
M_8	0	0.92	0	0.67

Thus the minimum value for $\parallel \psi(e_i) \parallel$ is $\parallel \psi(e_4) \parallel = 0.1422$.
This means that $\psi(e_4)$ is the expected material for the drug discovery of cancer disease.

5. **Conclusion**

In this study, rough hypersoft set is conceptualized with the development of its relevant lower and upper rough approximations. Some interesting results and properties are discussed. Moreover, an application in decision making is discussed with the help of illustrative examples. This work may help the researchers to extend the work for other fuzzy-like structures i.e. Interval-valued fuzzy, Intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, neutrosophic fuzzy etc. so that imperfect information may be handled properly with attribute valued sets through lower and upper approximations. The proposed decision making algorithm may further be applied to other daily life problems.

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Chapter 11

A Novel Approach to the Rudiments of Hypersoft Graphs

Muhammad Saeed [1], Muhammad Khubab Siddique [2], Muhammad Ahsan [2], Muhammad Rayees Ahmad [3]*, Atiqe Ur Rahman [3]

[1,2,3] Department of Mathematics, University of Management and Technology Lahore, Pakistan.

E-mail: muhammad.saeed@umt.edu.pk , E-mail: khubabsiddique@hotmail.com , E-mail: ahsan1826@gmail.com , E-mail: rayeesmalik.ravian@gmail.com , E-mail: aurkhb@gmail.com

Abstract: Graph theory, soft computing and machine learning are being used in our daily life problems in the field of science involving mathematics, optimization and decision sciences. Zadeh introduced the idea of fuzzy set. This idea helped Kauffman to present the concept of fuzzy graph. Molodtsov presented the idea of soft set. Using this concept, Thumbakara et al. discovered a novel idea of soft graphs and Akram et al. discussed the fundamentals of soft graphs. Smarandache conceptualized the hypersoft set (HS) which is the generalization of soft set. Hypersoft set transforms single attribute function to multi-attribute function. In this study, the existing concept of soft graph is extended to HS-graph and some of its rudiments like HS-subgraph, not HS-graph, HS-complete graph, HS-tree, etc., are conceptualized with the help of graphical representation and illustrative examples. Moreover, some theoretic operations are discussed with generalized results on hypersoft set. This study will help the researchers in multi-dimensional fields involving artificial intelligence (AI), soft computing, graph theory and networking, data sciences, etc.

Keywords: *Soft set, Soft graph, Hypersoft set (HSS), Hypersoft graph (HS-graph), Hypersoft complete graph (HSC-graph), Hypersoft subgraph (HS-subgraph), Hypersoft tree (HS-tree).*

1. Introduction

The world is full of uncertainties, complexities of incomplete network-based information with vague data. To deal with such kind of uncertain environment, there must be a tool which handles these kinds of problems very efficiently. To overcome this problem, Zadeh [1] introduced the idea of fuzzy set in 1965. This idea was motivated Kauffman, who presented the concept of fuzzy graphs using the notion of fuzzy set. Many researchers worked in this area and implement the new concept to solve real life problems. Poulik et al. [3–7] has many researches on bipolar fuzzy graphs in different environments and presented many applications in diverse fields of sciences.

Molodtsov [8] introduced a novel notion of soft set theory in 1999. He used the idea of soft set theory in several areas of mathematics including Riemann integration, theory of probability, Perron integration, smoothness of functions effectively. Maji et al. [9] presented fundamental study on soft sets including subsets, super set, equations and operations of soft set containing union, intersection, complement and more among others in 2003. Pei et al. [10] described the relationship with information system and soft sets in 2005. Ali et al. [11] presented some new operations including restricted union, restricted intersection and their basics characteristics in 2009. Babitha et al. [12] described the concept of soft set relation, soft set function, equivalence soft set, ordering on soft sets, soft sets distribution, anti-symmetric relation and transitive closure in 2010. Nagarajan et al. [13] studied the algebraic structure of soft set theory.

There are lot of network applications in which the bulk of data is available but no tool gave the solution to the problem properly. To overcome this situation, Thumbakara et al. [14] presented the concept of soft graph as a parameterized family of graphs. Having motivation from this work, Thenge et al. [15] contributed further to soft graphs including tabular representation of soft graph, center, degree, radius and diameter of soft graph. Akram et al. [16] explained the concept of vertex and edge induced soft graph. Ratheesh [17] worked on soft graphs and their applications.

Smarandache [18], a pioneer of hypersoft set theory, generalized the soft set to hypersoft set in 2018. He introduced the hybrids of hypersoft set with other sets. Fundamentals of hypersoft set such as hypersoft subset, complement, not standard aggregation operators have been discussed in [19]. Rahman et al. [20] defined complex hypersoft set and developed the hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutro- sophic set respectively. They also discussed their fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. complement, union, intersection etc. Rahman et al. [21] conceptualized convexity cum concavity on hypersoft set and presented its pictorial versions with illustrative examples.

Having motivation from [17], [18] and [19], the novel notions of hypersoft graphs, hypersoft subgraph, not hypersoft graph, hypersoft complete graph and hypersoft tree are proposed. Moreover, some of their properties including intersection of hypersoft graphs, AND operation of hypersoft graphs are discussed with the support of graphical representation and illustrative examples.

The overview of this paper is presented as follow.

In section 2, some fundamental definitions are recalled from literature to support main results. Section 3 presents the proposed concept of hypersoft graphs, their properties and some important results with their examples. At the end, the conclusion is presented in section 4.

2. Preliminaries

Here some important and most relevant definitions are recalled from literature regarding soft sets and soft graph for clear understanding.

Definition 2.1. [8]
Let χ be the nonempty finite universe, $\mathcal{T}$ be the set of attributes and $S \subseteq \mathcal{T}$. A pair (γ, S) represents a **soft set** over χ such that $\gamma : S \to P(\chi)$.

Definition 2.2. [12]
Let (γ_1, J) and (γ_2, K) be two soft sets over the same universe χ.

A **soft relation** $R : (\gamma_1, J) \to (\gamma_2, K)$ is (R, L) with $L \subseteq J \times K$, $\forall\ (x, y) \in L$, there exist $R(x, y) = H(x, y)$ such that $(H, J \times K) = (\gamma_1, J) \times (\gamma_2, K)$.

Definition 2.3. [12]
A soft relation (R, L) is called a **soft function**, if every element of domain of R, a unique element in range of R. If $\gamma_1(x)R\gamma_2(y)$ for $x \in J$ and $y \in K$ then it can be represented $R(\gamma_1(x)) = \gamma_2(y)$.

For more detail, see [8, 9, 11, 12].

Definition 2.4. [14]
Suppose that $G = (\mathcal{V}, \mathcal{T})$ represents the graph such that $\mathcal{V}$ and $\mathcal{T}$ represents the set of vertices and edges of the graph respectively. Consider $J \subseteq \mathcal{V}$ such that $J \neq \phi$ and R is an arbitrary relation such that $R \subseteq J \times \mathcal{V}$. Let a soft set is (γ, S), then **soft graph** of G is represented by (F, J), if the subgraph induced in G by $F(x)$, implies that $\mathcal{F}(x)$ is a connected sub graph of G, $\forall\ x \in J$.

Definition 2.5. [14]
Let $G_1 = (F_1, K_1, J)$ and $G_2 = (F_2, K_2, K)$ be two soft graphs of G. Then G_2 is a **soft sub graph** of G_1 if

(1) $K \subseteq J$.
(2) $H_2(x)$ is sub graph of $H_1(x)$, $\forall\ x \in K$.

Definition 2.6. [14]
Let (F, J) be a soft graph of G, then (F, J) represents the **soft tree** $\mathcal{F}_t(x), \ \forall \ x \in J$.

For more definitions, see [14–17].

Definition 2.7. [18]
Let $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \ldots, \mathcal{T}_n$ be n-distinct attributes whose attribute values belongs to disjoint sets $J_1, J_2, J_3, \ldots, J_n$ respectively. A pair (γ, J) is called a **hypersoft set** over the universal set χ, where γ is the mapping given by $\gamma : J \to P(U)$ such that $J = J_1 \times J_2 \times J_3 \times \ldots \times J_n$.

Definition 2.8. [19]
Let (γ_1, J) and (γ_2, K) be the two hypersoft sets over the same universal sets χ, where $J = J_1 \times J_2 \times J_3 \times \ldots \times J_n$ and $K = K_1 \times K_2 \times K_3 \times \ldots \times K_n$. Then the relation from (γ_1, J) to (γ_2, K) is called a **hypersoft set relation** $(\check{R}, W)$ or it is in simple way $\check{R}$ is a hypersoft subset and it is denoted by $(\gamma_1, J) \times (\gamma_2, K)$, where $W \subseteq J \times K$ and $\forall \ (x, y) \in W$, implies that $R(x, y) = H(x, y)$, where $x = (x_1, x_2, x_3, \ldots, x_n) \in J$ and $y = (y_1, y_2, y_3, \ldots, y_n) \in K$ and $(H, J \times K) = (\gamma_1, J) \times (\gamma_2, K)$.

Definition 2.9. [19]
Let (γ_1, J) and (γ_2, K) be the two hypersoft sets over the same universal sets χ. Then the hypersoft relation from (γ_1, J) to (γ_2, K) defined as $\eta : (\gamma_1, J) \to (\gamma_2, K)$ is called a **hypersoft set function**, if every element of domain has unique element in range of γ. If it is closed $\gamma_1(x)\eta\gamma_2(y)$ i.e $\gamma_1(x) \times \gamma_2(y) \in \eta$ for $x \in J$ and $y \in K$, then we can represent it in the form $\eta(\gamma_1(x)) = \gamma_2(y)$.

For more detail regarding HS-set, see [18, 19].

3. Hypersoft Graph

In this section, we presents the HS-set, HS-graph,HS-intersection, AND operation, HSC graph, HS-subgraph, HS-tree, and their fundamentals along with their related important results.

For this, let $G = (\mathcal{V}, \mathcal{T})$ be a simple connected graph. Let $J = J_1 \times J_2 \times J_3 \times \ldots \times J_n$ and $J_i \subseteq \mathcal{V}$ with $i = 1, 2, \ldots, n$ as $J_i \cap J_j = \phi, \ i \neq j$.

Definition 3.1. Let R be any relation with J and $\mathcal{V}$ i.e. $R \subseteq J \times \mathcal{V}$. A function $F : J \to P(\mathcal{V})$ represented as

$$F(x) = \{y \in \mathcal{V} | \ x \ R \ y\}, \forall \ x \in J.$$

The pair (F, J) is hypersoft set over $\mathcal{V}$.

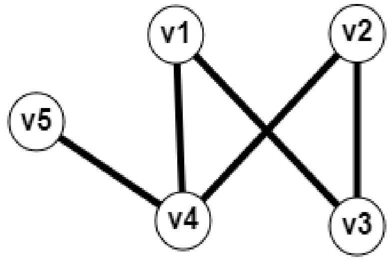

FIGURE 1. HS-Graph

Definition 3.2. A HS-set (F, J) of G over $\mathcal{V}$, represents the **HS-graph**, if $\mathcal{F}(x)$ is a connected subgraph of G, induced by $F(x)$, $\forall\ x \in J$. A set containing all the HS-graphs is denoted by $H_sG(G)$.

Example 3.3. Consider $G = (V, \mathcal{T})$ as in figure 1. We have $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$. Let $J = \{J_1, J_2\}, K = \{K_3, K_4\}$ and $L = \{L_5\}$ then

$$\begin{aligned} J \times K \times L &= \{(J_1, K_3, L_5), (J_2, K_3, L_5), (J_1, K_4, L_5), (J_2, K_4, L_5)\} \\ &= \{x_1, x_2, x_3, x_4\}. \end{aligned}$$

$$R = \{(x_1, v_2), (x_1, v_4), (x_2, v_1), (x_2, v_4), (x_3, v_2), (x_3, v_3), (x_4, v_1), (x_4, v_3)\}$$

Then

$$\check{F}(\check{x}) = \check{F}(J_i, J_j, J_k)) = \{\check{y} \in \check{V} | \check{x}R\check{y} \Leftrightarrow \check{y} \notin \{v_i, v_j, v_k\}\}$$

and

$$\begin{aligned} F(x_1) &= F((J_1, J_3, J_5)) = \{v_2, v_4\}, \\ F(x_2) &= F((J_2, J_3, J_5)) = \{v_1, v_4\}, \\ F(x_3) &= F((J_1, J_4, J_5)) = \{v_2, v_3\}, \\ F(x_4) &= F((J_2, J_4, J_5)) = \{v_1, v_3\}. \end{aligned}$$

Hence $(F, A) \in H_sG(G)$.

Example 3.4. Suppose $G = (\mathcal{V}, \mathcal{T})$, represented as figure 2. We have $V = \{v_1, v_2, v_3, v_4, v_5\}$. Let $J = \{v_1\},\ K = \{v_2, v_3\}$ and $L = \{v_4, v_5\}$ then

$$\begin{aligned} M = J \times K \times L &= \{(v_1, v_2, v_4), (v_1, v_2, v_5), (v_1, v_3, v_4), (v_1, v_3, v_5)\} = \{x_1, x_2, x_3, x_4\}, \\ R &= \{(x_1, v_2), (x_1, v_4), (x_2, v_1), (x_2, v_4), (x_3, v_2), (x_3, v_3), (x_4, v_1), (x_4, v_3)\} \subseteq J \times V \end{aligned}$$

Then

$$F(x) = F((v_i, v_j, v_k)) = \{y \in V | xRy \rightarrow y \notin \{v_i, v_j, v_k\}\}$$

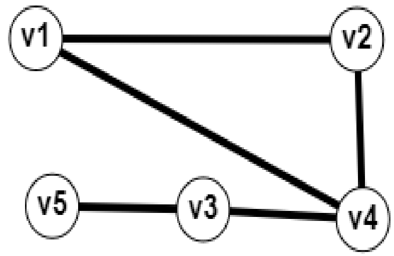

FIGURE 2. Not HS-Graph

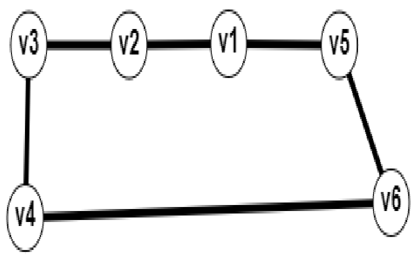

FIGURE 3. Intersection of HS-Graphs

and

$$F(x_1) = F((v_1, v_2, v_4)) = \{v_3, v_5\},$$

$$F(x_2) = F((v_1, v_2, v_5)) = \{v_3, v_4\},$$

$$F(x_3) = F((v_1, v_3, v_4)) = \{v_2, v_5\},$$

$$F(x_4) = F((v_1, v_3, v_5)) = \{v_2, v_4\}.$$

As $\mathcal{F}((v_1, v_3, v_4)) = \{v_2, v_5\}$ is not a connected subgraph of G. Hence $(F, J) \notin H_sG(G)$.

Proposition 3.5. *Every simple graph is HS-graph.*

Proof. Let $G = (V, \mathcal{T})$ be a simple connected graph and $\mathcal{T} \neq \emptyset$. Let $J = J_1 \times J_2 \times J_3 \times \ldots \times J_n$, where $J_i = \{v_i\}$ for $i = 1, 2, \ldots, n$ and a function is $F : J \longrightarrow P(V)$ as

$$F((v_1, v_2, \ldots, v_n)) = \{y \in V |\ y = \text{ end vertex of edge } \mathcal{T}\}$$

So, by definition of $\mathcal{F}(x)$, we have, for all $x \in A$, $(F, J) \in H_sG(G)$. □

Remark 3.6. The intersection of HS-graphs is not necessarily a HS-graph.

Example 3.7. Let $G = (\mathcal{V}, \mathcal{T})$ as given in figure 2. Let $J = J_1 \times J_2 \times J_3$, where $J_1 = \{v_1\}, J_2 = \{v_2\}, J_3 = \{v_3\}$.

$$\Rightarrow F((v_1, v_2, v_3)) = \{y \in V | x\ Ry \Leftrightarrow \min\{d(v_1, y), d(v_2, y), d(v_3, y)\} \leq 1\}$$

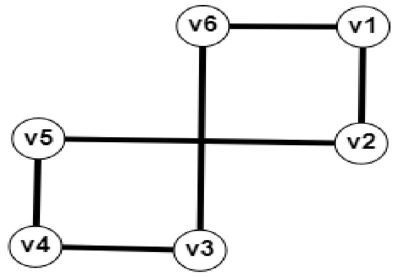

FIGURE 4. AND operation of HS-Graphs

$$\Rightarrow F((v_1, v_2, v_3)) = \{v_1, v_2, v_3, v_4, v_5\}.$$

Suppose the function H by

$$H((v_1, v_2, v_3)) = \{y \in V | xRy \Leftrightarrow \min\{d(v_1, y), d(v_2, y), d(v_3, y)\} \geq 1\}$$

$$\Rightarrow H((v_1, v_2, v_3)) = \{v_4, v_5, v_6\}.$$

$$\Rightarrow F((v_1, v_2, v_3)) \cap H((v_1, v_2, v_3)) = \{v_4, v_5\}$$

that is not induced connected subgraph of G.

Proposition 3.8. *Let (F, J) and $(H, K) \in S_hG(G)$ with $J \cap K \neq \emptyset$, then $\mathcal{F}(x) \cap \mathcal{H}(x)$ $\forall\ x \in J \cap K$ is a connected subgraph of G such that*

$$(F, J) \cap (H, K) \in H_sG(G).$$

Proof. For (F, J) and $(H, K) \in H_sG(G)$, we have

$$(F, J) \cap (H, K) = (\chi, C)\ , C = J \cap K \neq \emptyset$$

and

$$\chi(\mathcal{T}) = F(\mathcal{T}) \cap H(\mathcal{T}), \quad \forall\ \mathcal{T} \in C.$$

Let $x \in C = J \cap K$. As considered by $\chi(x) = \mathcal{F}(x) \cap \mathcal{H}(x)$ is a connected subgraph of G, so

$$(\chi, C) = (F, J) \cap (H, K) \in H_sG(G).$$

□

Example 3.9. Suppose $G = (\mathcal{V}, \mathcal{T})$ as in figure 4. Let $J = J_1 \times J_2$, with $J_1 = \{v_1\}$, $J_2 = \{v_2\}$, then $J = \{(v_1, v_2)\}$ and $K = K_1 \times K_2$, with $K_1 = \{v_3\}, K_2 = \{v_4\}$, then $K = \{(v_3, v_4)\}$. Define a function F by

$$F((x, y)) = \{z \in \mathcal{V} | (x, y)Rz \Leftrightarrow z \notin \{x, y\}\}.$$

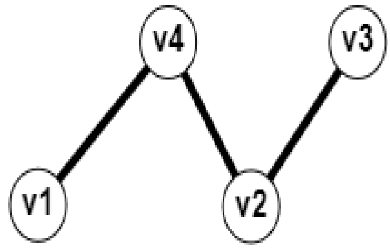

FIGURE 5. HS-Subgraphs

Then

$$F((v_1, v_2)) = \{v_3, v_4, v_5, v_6\},$$

$$F((v_3, v_4)) = \{v_1, v_2, v_5, v_6\},$$

$$F((v_1, v_2)) \wedge F((v_3, v_4)) = \{v_5, v_6\}$$

which is not an induced connected subgraph of G. Hence $(F, J) \wedge (F, K) \notin H_s G(G)$.

Definition 3.10. If (F, J) and (H, K) are two HS-graphs then (H, K) be the HS-subgraph of (F, J), if

(1) $K \subseteq J$,
(2) $\mathcal{H}(x)$ is a connected subgraph of $\mathcal{F}(x)$, $\forall\, x \in K$.

Example 3.11. Consider $G = (\mathcal{V}, \mathcal{T})$ as in figure 5. Let

$$\mathcal{V} = \{v_1, v_2, v_3, v_4\}$$

and

$$J = J_1 \times J_2 \text{ with } J_1 = \{v_1, v_2, v_3\},\ J_2 = \{v_4\},\ \text{then } J = \{(v_1, v_4), (v_2, v_4), (v_3, v_4)\}$$

and

$$K = K_1 \times K_2,\ \text{with } K_1 = \{v_1, v_2\},\ K_2 = \{v_4\},\ then\ K = \{(v_1, v_4), (v_2, v_4)\} \subseteq J.$$

Now define function F by,

$$F((x, y)) = \{z \in \mathcal{V} | (x, y)Rz \Leftrightarrow d(z, \check{t}) \leq 1 \text{ where } \check{t} \in \{x, y\}\}$$

Then

$$F((v_1, v_4)) = \{v_1, v_2, v_4\},\ F((v_2, v_4)) = \{v_1, v_2, v_3, v_4\}$$

and

$$F((v_3, v_4)) = \{v_1, v_2, v_3, v_4\}.$$

Suppose a function $H : K \longrightarrow P(\mathcal{V})$ by

$$H((x, y)) = \{z \in \mathcal{V} | (x, y)Rz \Leftrightarrow d(z, \check{t}) < 1,\ \text{where} \check{t} \in \{x, y\}\}.$$

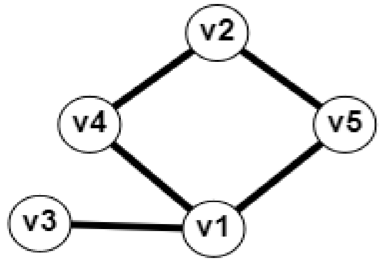

FIGURE 6. HS-Tree

Then $H((v_1, v_4)) = \{v_1, v_4\}$ and $H((v_2, v_4)) = \{v_2, v_4\}$.
Therefore (F, J), $(H, K) \in H_sG(G)$. Here $K \subseteq J$ and $\mathcal{H}(x)$ is a connected subgraph of $\mathcal{F}(x)$, $\forall\ x \in K$. Hence (H, K) is a HS-subgraph of (F, J).

Definition 3.12. Let (F, J) be a HS-graph of G, then (F, J) represents the HS-tree $\forall\ x \in J$.

Example 3.13. Consider $G = (\mathcal{V}, \mathcal{T})$ as in figure 6. Here

$$\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}.$$

Let $J_1 = \{v_1, v_2\}$, $J_2 = \{v_3\}$, $J_3 = \{v_4, v_5\}$, then

$$J = J_1 \times J_2 \times J_3 = \{(v_1, v_3, v_4), (v_1, v_3, v_5), (v_2, v_3, v_4), (v_2, v_3, v_5)\}.$$

Define a function $F : J \to P(\mathcal{V})$ by

$$F(x) = (v_i, v_j, v_k) = \{y \in \mathcal{V} | xRy \Leftrightarrow y \notin \{v_i, v_j, v_k\}\}.$$

Then

$$F((v_1, v_3, v_4)) = \{v_2, v_5\},\ F((v_1, v_3, v_5)) = \{v_2, v_4\},$$

$$F((v_2, v_3, v_4)) = \{v_1, v_5\},$$

$$F((v_2, v_3, v_5)) = \{v_1, v_4\}.$$

Here $\mathcal{F}(x)$, $\forall x \in J$ is a tree. Hence (F, J) is a HS-tree.

Theorem 3.14. *Every member in set of HS-graph of trees is also a HS-tree.*

Proof. Proof is straight forward. □

Remark 3.15. A HS-graph (F, J) is a HS-tree iff $\mathcal{F}(x)$ is acyclic.

Definition 3.16. Let (F, J) be a HS-graph, then (F, J) represents a HSC-graph, if the graph $\mathcal{F}(x)$ is complete for all $x \in J$.

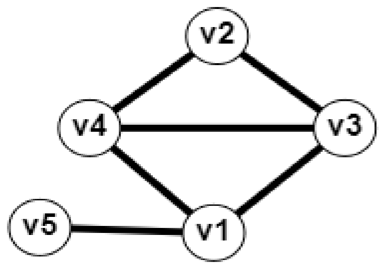

FIGURE 7. HS Complete Graph

Example 3.17. Consider $G = (\mathcal{V}, \mathcal{T})$ as in figure 7

Here $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$. Let $J_1 = \{v_1, v_2\},\ J_2 = \{v_3\},\ J_3 = \{v_4\}$, then

$$J = J_1 \times J_2 \times J_3 = \{(v_1, v_3, v_4), (v_2, v_3, v_4)\}.$$

Then we have

$$F : J_1 \times J_2 \times J_3 \to P(\mathcal{V})$$

by

$$F(x) = F(v_i, v_j, v_k) = \{y \in \check{V} | xRy \Leftrightarrow y \in \{v_i, v_j, v_k\}\}.$$

Then $F((v_1, v_3, v_4)) = \{v_1, v_3, v_4\},\ F((v_2, v_3, v_4)) = \{v_2, v_3, v_4\}$. Since $\mathcal{F}(x)$ is complete $\forall\ x \in J$. Therefore (F, J) is a HSC-graph.

Proposition 3.18. *Let $G = (\mathcal{V}, \mathcal{T})$ is a complete graph. Then every HS-graph in $H_sG(G)$ is a HSC-graph.*

Proof. Let $(F, J) \in H_sG(G)$. Since it is known that every induced subgraph of a complete graph is complete, therefore $\mathcal{F}(x)$ is a complete subgraph for all $x \in A$. Hence (F, J) is a HSC-graph. □

Remark 3.19. A HS-graph (F, J) is a HSC-graph iff $\mathcal{F}(x)$ is complete for all $x \in J$.

Example 3.20. For the same figure 7, if we take $\mathcal{V}$ and J same as example 3.17 and define $F : J \to P(\mathcal{V})$ as

$$F(x) = F(v_i, v_j, v_k) = \{y \in \mathcal{V} | xRy \Leftrightarrow y \notin \{v_i, v_j, v_k\}\} \tag{1}$$

then

$$F((v_1, v_3, v_4)) = \{v_2, v_5\} \tag{2}$$

$$F((v_2, v_3, v_4)) = \{v_1, v_5\} \tag{3}$$

Since $F((v_2, v_3, v_4))$ is not complete, so $(F, J) \notin H_SC(G)$. Hence the result is verified.

4. Conclusions

Mathematical modeling, graph theory, soft computing, machine learning involves in our daily life which contains uncertain situations in the field of science. In this study, a novel concept of hypersoft graph is developed by extending the existing concept of soft graph. Some of the fundamentals, properties and the results of hypersoft graph and hypersoft tree are discussed with the help of illustrated examples and graphs. Further extension can be sought by introducing new hybrid structures with hypersoft graph. This novel concept is very useful in developing new algorithms in the fields of artificial intelligence, machine learning, networking, soft computing, etc.

Conflicts of Interest:

The authors declare no conflict of interest.

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Chapter 12

On Neutrosophic Hypersoft Topological Spaces

Taha Yasin Ozturk [1], Adem Yolcu [2]

[1] Department of Mathematics, Kafkas University, 36100, Kars, Turkey. E-mail: taha36100@gmail.com
[2] Department of Mathematics, Kafkas University, 36100, Kars, Turkey. E-mail: yolcu.adem@gmail.com

Abstract: In this study, we redefine some operations on the neutrosophic hypersoft sets differently from the study [26]. Some basic properties of these operations have been characterized. Under the guidance of these redefined operations, we introduced the neutrosophic hypersoft topological spaces. Finally, on neutrosophic hypersoft topological spaces, we present simple concepts and theorems.

Keywords: *Hypersoft sets; neutrosophic hypersoft sets; neutrosophic hypersoft topology; neutrosophic hypersoft interior; neutrosophic hypersoft closure.*

1. Introduction

Complexity typically emerges out of confusion in the context of ambiguity in the real world. The challenges of modeling ambiguous data are discussed regularly by researchers in economics, history, medical science and many other areas. Classical approaches do not necessarily provide fruitful outcomes and there can be various forms of unknown presence in these domains. The theory of probability has become an age-old and powerful method to work with ambiguity, but it can only be extended to the random process. After that, to solve unknown problems, evidence theory, fuzzy set theory by Zadeh [32], and intuitionistic fuzzy set theory by Atanassov [4] were introduced. But each of these hypotheses, as Molodtsov [15] points out, has underlying difficulties. The underlying explanation for these problems is the inadequacy of the theory's parametrization device.

The soft set theory was introduced by Molodtsov [15] as a new mathematical technique away from the parametrization inadequacy syndrome of multiple theories dealing with instability in 1999. This makes the theory, in practice, very convenient and easily applicable. Molodtsov [15] successfully applied many criteria for soft set theory applications, such as function smoothness, game theory, operational analysis, Riemann integration, integration and probability of Perron, soft set theory and its implementations are now evolving quickly in numerous fields. Shabir and Naz [27] introduced soft topological spaces and defined some notions of soft sets. In addition, topological structure of the fuzzy, fuzzy soft, intuitionistic fuzzy and intuitionistic fuzzy soft sets have been defined by different researchers [2, 5, 7, 8, 12, 31].

The neutrosophic sets (NS) was first proposed by Smarandache [28, 29]. This is a classical set generalization, a fuzzy set, an intuitionist fuzzy set, etc. Later, a combined neutrosophic soft set (NSS) was proposed by Maji [13]. The topological structure on neutrosophic soft sets is defined by Bera [6]. Due to some structural deficiencies in the study [6], it was necessary to redefine some concepts on neutrosophic soft sets. Therefore, Ozturk et al. redefined operations on neutrosophic soft sets and studied neutrosophic soft topological spaces, neutrosophic soft separation axioms [3, 19]. Also several mathematicians have developed their research work in various mathematical systems using neutrosophic sets, neutrosophic soft sets etc. [3, 11, 14, 16-21, 24].

Smarandache [30] generalized the notion of a soft set to a hypersoft set by replacing the function with a multi-argument function specified in the cartesian product with a different set of parameters. This idea is more versatile than the soft set and more applicable in the sense of decision-making issues. Hypersoft set structure has attracted the attention of researchers because it is more suitable than soft set structure in decision making problems. Although it is a new concept, many studies have been done and the field of study continues to expand [1, 10, 22, 23, 25, 26, 33].

Due to some structural difficulties in the study [26], some problems were encountered in defining the neutrosophic hypersoft topological structure. For this reason, there was a need to redefine some concepts. In this study, firstly, basic operations on neutrosophic hypersoft sets such as complement, union, intersection, AND, OR are re-defined differently from [26] study and several properties have been characterized. After that, by using these redefined operations, the neutrosophic hypersoft topological spaces are introduced. On this topological structure, operations such as open (closed) set, interior and closure are defined and their properties are examined. The study is supported by appropriate examples.

2. Preliminaries

Definition 2.1 [15] Let Δ be an initial universe $£$ be a set of parameters and $P(\Delta)$ be the power set of Δ. A pair $(F,£)$ is called a soft set over $\Delta,$ where F is a mapping $F:£\rightarrow P(\Delta).$ In other words, the soft set is a parameterized family of subsets of the set $\Delta.$

Definition 2.2 [28,29] Let Δ be an initial universe and Λ be a neutrosophic set (NS) on Δ is defined as $\Lambda=\{<x,T_{\Lambda}(x),I_{\Lambda}(x),F_{\Lambda}(x)>:x\in\Delta\},$ where T,I,F: $\Delta\rightarrow]^{-}0,1^{+}[$ and $0\leq T_{\Lambda}(x)+I_{\Lambda}(x)+F_{\Lambda}(x)\leq 3^{+}.$

Definition 2.3 [9] Let Δ be an initial universe $£$ be a set of parameters and $NP(\Delta)$ denote the set of all neutrosophic sets of $\Delta.$ A pair $(F,£)$ is called a neutrosophic soft set over Δ and its mapping is given as $F:£\rightarrow NP(\Delta).$

Definition 2.4 [30] Let Δ be the universal set and $P(\Delta)$ be the power set of Δ. Consider $\hbar_1, \hbar_2, \hbar_3, ..., \hbar_k$ for $k \geq 1$, be k well-defined attributes, whose corresponding attribute values are respectively the sets $£_1, £_2, ..., £_k$ with $£_i \cap £_j = \varnothing$, for $i \neq j$ and $i, j \in \{1, 2, ...k\}$, then the pair $(\Upsilon, £_1 \times £_2 \times ... \times £_k)$ is said to be hypersoft set over Δ where $\Upsilon : £_1 \times £_2 \times ... \times £_k \to P(\Delta)$.

Definition 2.5 [26] Let Δ be the universal set and $NP(\Delta)$ be a family of all neutrosophic sets over Δ and $£_1, £_2, ..., £_k$ the pairwise disjoint sets of parameters. Let C_i be the nonempty subset of then the pair $£_i$ for each $i = 1, 2, ..., k$. A neutrosophic hypersoft set (NHSS) over Δ defined as the pair $(\Upsilon, C_1 \times C_2 \times ...C_k)$ where $\Upsilon : C_1 \times C_2 \times ...C_k \to NP(\Delta)$ and

$$\Upsilon(C_1 \times C_2 \times ...C_k) = \{(\alpha, < x, T_{\Upsilon(\alpha)}(x), I_{\Upsilon(\alpha)}(x), F_{\Upsilon(\alpha)}(x) >) \\ : x \in \Delta, \alpha \in C_1 \times C_2 \times ...C_k \subseteq £_1 \times £_2 \times ... \times £_k\}$$

where T is the membership value of truthiness, I is the membership value of indeterminancy and F is the membership value of falsity such that $T_{\Upsilon(\alpha)}(x), I_{\Upsilon(\alpha)}(x), F_{\Upsilon(\alpha)}(x) \in 0,1]$ also $0 \leq T_{\Upsilon(\alpha)}(x) + I_{\Upsilon(\alpha)}(x) + F_{\Upsilon(\alpha)}(x) \leq 3.$ For sake of simplicity, we write the symbols Σ for $£_1 \times £_2 \times ... \times £_k$, $\Im$ for $C_1 \times C_2 \times ...C_k$ and α for an element of the set $\Im$.

Throughout this paper, we denoted the family of all neutrosophic hypersoft sets over the universe set Δ with $NHSS(\Delta, \Sigma)$

3. Some Basic Operations on Neutrosophic Hypersoft Sets

In this section, operations of neutrosophic hypersoft subset, null neutrosophic hypersoft set, complement, union, intersection, AND, OR on neutrosophic hypersoft sets are defined differently from the study [26]. We also presented basic properties of these operations.

Definition 3.1 Let Δ be the universal set and $(\Upsilon_1, \Im_1), (\Upsilon_2, \Im_2)$ be two neutrosophic hypersoft sets over Δ. Then $(\Upsilon_1, \Im_1)$ is the neutrosophic hypersoft subset of $(\Upsilon_2, \Im_2)$ if

1. $\Im_1 \subseteq \Im_2$,
2. For $\forall \alpha \in \Im_1$ and $\forall x \in \Delta$, $T_{\Upsilon_1(\alpha)}(x) \leq T_{\Upsilon_2(\alpha)}(x), I_{\Upsilon_1(\alpha)}(x) \leq I_{\Upsilon_2(\alpha)}(x),$ $F_{\Upsilon_1(\alpha)}(x) \geq F_{\Upsilon_2(\alpha)}(x).$

It is denoted by $(\Upsilon_1, \Im_1) \subseteq (\Upsilon_2, \Im_2)$.

Definition 3.2 Let Δ be the universal set and $(\Upsilon, \Im)$ be neutrosophic hypersoft set over Δ. $(\Upsilon, \Im)^c$ is the complement of neutrosophic hypersoft set of $(\Upsilon, \Im)$ if

$$T^c_{\Upsilon(\alpha)}(x) = F_{\Upsilon(\alpha)}(x)$$
$$I^c_{\Upsilon(\alpha)}(x) = 1 - I_{\Upsilon(\alpha)}(x)$$
$$F^c_{\Upsilon(\alpha)}(x) = T_{\Upsilon(\alpha)}(x)$$

where $\forall \alpha \in \Im$ and $\forall x \in \Delta.$ It is clear that $\left((\Upsilon,\Im)^c\right)^c = (\Upsilon,\Im).$

Definition 3.3

1. A neutrosophic hypersoft set $(\Upsilon,\Im)$ over the universe set Δ is said to be null neutrosophic hypersoft set if $T_{\Upsilon(\alpha)}(x)=0,\ \ I_{\Upsilon(\alpha)}(x)=0,\ \ F_{\Upsilon(\alpha)}(x)=1$ where $\forall \alpha \in \Im$ and $\forall x \in \Delta$. It is denoted by $0_{(\Delta_{NH},\Sigma)}$.

2. A neutrosophic hypersoft set $(\Upsilon,\Im)$ over the universe set Δ is said to be absolute neutrosophic hypersoft set if $T_{\Upsilon(\alpha)}(x)=1,\ \ I_{\Upsilon(\alpha)}(x)=1,\ \ F_{\Upsilon(\alpha)}(x)=0$ where $\forall \alpha \in \Im$ and $\forall x \in \Delta$. It is denoted by $1_{(\Delta_{NH},\Sigma)}$.

Clearly, $0^c_{(\Delta_{NH},\Sigma)} = 1_{(\Delta_{NH},\Sigma)}$ and $1^c_{(\Delta_{NH},\Sigma)} = 0_{(\Delta_{NH},\Sigma)}$.

Definition 3.4 Let Δ be the universal set and $(\Upsilon_1,\Im_1)$ and $(\Upsilon_2,\Im_2)$ be neutrosophic hypersoft sets over Δ. Extended union $(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)$ is defined as

$$T((\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)) = \begin{cases} T_{\Upsilon_1(\alpha)}(x) \text{if } \alpha \in \Im_1 - \Im_2 \\ T_{\Upsilon_2(\alpha)}(x) \text{if } \alpha \in \Im_2 - \Im_1 \\ \max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha \in \Im_1 \cap \Im_2 \end{cases},$$

$$I((\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)) = \begin{cases} I_{\Upsilon_1(\alpha)}(x) \text{if } \alpha \in \Im_1 - \Im_2 \\ I_{\Upsilon_2(\alpha)}(x) \text{if } \alpha \in \Im_2 - \Im_1 \\ \max\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha \in \Im_1 \cap \Im_2 \end{cases},$$

$$F((\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)) = \begin{cases} F_{\Upsilon_1(\alpha)}(x) \text{if } \alpha \in \Im_1 - \Im_2 \\ F_{\Upsilon_2(\alpha)}(x) \text{if } \alpha \in \Im_2 - \Im_1 \\ \min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha \in \Im_1 \cap \Im_2 \end{cases}.$$

Definition 3.5 Let Δ be the universal set and $(\Upsilon_1,\Im_1)$ and $(\Upsilon_2,\Im_2)$ be neutrosophic hypersoft sets over Δ. Extended intersection $(\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)$ is defined as

$$T((\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)) = \begin{cases} T_{\Upsilon_1(\alpha)}(x) \text{if } \alpha \in \Im_1 - \Im_2 \\ T_{\Upsilon_2(\alpha)}(x) \text{if } \alpha \in \Im_2 - \Im_1 \\ \min\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha \in \Im_1 \cap \Im_2 \end{cases},$$

$$I((\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2))=\begin{cases} I_{\Upsilon_1(\alpha)}(x) \text{if } \alpha\in\Im_1-\Im_2 \\ I_{\Upsilon_2(\alpha)}(x) \text{if } \alpha\in\Im_2-\Im_1 \\ \min\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha\in\Im_1\cap\Im_2 \end{cases},$$

$$F((\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2))=\begin{cases} F_{\Upsilon_1(\alpha)}(x) \text{if} \alpha\in\Im_1-\Im_2 \\ F_{\Upsilon_2(\alpha)}(x) \text{if} \alpha\in\Im_2-\Im_1 \\ \max\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\} \text{if} \alpha\in\Im_1\cap\Im_2 \end{cases}.$$

Definition 3.6 Let $\left\{(\Upsilon_i,\Im_i)\middle| i\in I\right\}$ be a family of neutrosophic hypersoft sets over the universe set Δ. Then

$$\bigcup_{i\in I}(\Upsilon_i,\Im_i)=\left\{\left\langle x,\sup\left[T_{\Upsilon_i(\alpha)}(x)\right]_{i\in I},\sup\left[I_{\Upsilon_i(\alpha)}(x)\right]_{i\in I},\inf\left[F_{\Upsilon_i(\alpha)}(x)\right]_{i\in I}\right\rangle : x\in\Delta\right\},$$
$$\bigcap_{i\in I}(\Upsilon_i,\Im_i)=\left\{\left\langle x,\inf\left[T_{\Upsilon_i(\alpha)}(x)\right]_{i\in I},\inf\left[I_{\Upsilon_i(\alpha)}(x)\right]_{i\in I},\sup\left[F_{\Upsilon_i(\alpha)}(x)\right]_{i\in I}\right\rangle : x\in\Delta\right\}.$$

Definition 3.7 Let Δ be the universal set and $(\Upsilon_1,\Im_1)$ and $(\Upsilon_2,\Im_2)$ be neutrosophic hypersoft sets over Δ. The "AND" operator $(\Upsilon_1,\Im_1)\wedge(\Upsilon_2,\Im_2)=\Upsilon(\Im_1\times\Im_2)$ is defined as

$$T((\Upsilon_1,\Im_1)\wedge(\Upsilon_2,\Im_2))=\min\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\},$$
$$I((\Upsilon_1,\Im_1)\wedge(\Upsilon_2,\Im_2))=\min\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\},$$
$$F((\Upsilon_1,\Im_1)\wedge(\Upsilon_2,\Im_2))=\max\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\}.$$

Definition 3.8 Let Δ be the universal set and $(\Upsilon_1,\Im_1)$ and $(\Upsilon_2,\Im_2)$ be neutrosophic hypersoft sets over Δ. The "OR" operator $(\Upsilon_1,\Im_1)\vee(\Upsilon_2,\Im_2)=\Upsilon(\Im_1\times\Im_2)$ is defined as

$$T((\Upsilon_1,\Im_1)\vee(\Upsilon_2,\Im_2))=\max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\},$$
$$I((\Upsilon_1,\Im_1)\vee(\Upsilon_2,\Im_2))=\max\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\},$$
$$F((\Upsilon_1,\Im_1)\vee(\Upsilon_2,\Im_2))=\min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\}.$$

Proposition 3.9 Let $(\Upsilon_1,\Im_1)$, $(\Upsilon_2,\Im_2)$ and $(\Upsilon_3,\Im_3)$ be neutrosophic hypersoft sets over the universe set Δ. Then,

1. $(\Upsilon_1,\Im_1)\cup\left[(\Upsilon_2,\Im_2)\cup(\Upsilon_3,\Im_3)\right]=\left[(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)\right]\cup(\Upsilon_3,\Im_3)$ and
   $(\Upsilon_1,\Im_1)\cap\left[(\Upsilon_2,\Im_2)\cap(\Upsilon_3,\Im_3)\right]=\left[(\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)\right]\cap(\Upsilon_3,\Im_3)$;

2. $(\Upsilon_1,\Im_1)\cup[(\Upsilon_2,\Im_2)\cap(\Upsilon_3,\Im_3)]=[(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)]\cap[(\Upsilon_1,\Im_1)\cup(\Upsilon_3,\Im_3)]$ and $(\Upsilon_1,\Im_1)\cap[(\Upsilon_2,\Im_2)\cup(\Upsilon_3,\Im_3)]=[(\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)]\cup[(\Upsilon_1,\Im_1)\cap(\Upsilon_3,\Im_3)]$;
3. $(\Upsilon_1,\Im_1)\cup 0_{(\Delta_{NH},\Sigma)}=(\Upsilon_1,\Im_1)$ and $(\Upsilon_1,\Im_1)\cap 0_{(\Delta_{NH},\Sigma)}=0_{(\Delta_{NH},\Sigma)}$;
4. $(\Upsilon_1,\Im_1)\cup 1_{(\Delta_{NH},\Sigma)}=1_{(\Delta_{NH},\Sigma)}$ and $(\Upsilon_1,\Im_1)\cap 1_{(\Delta_{NH},\Sigma)}=(\Upsilon_1,\Im_1)$.

Proof. Straightforward.

Proposition 3.10 Let $(\Upsilon_1,\Im_1)$ and $(\Upsilon_2,\Im_2)$ be two neutrosophic hypersoft sets over the universe set Δ. Then,

1. $[(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)]^c=(\Upsilon_1,\Im_1)^c\cap(\Upsilon_2,\Im_2)^c$;
2. $[(\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)]^c=(\Upsilon_1,\Im_1)^c\cup(\Upsilon_2,\Im_2)^c$.

Proof. 1. For all $x\in\Delta$,

$$(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)=\left\{\left\langle \begin{array}{c} x,\max\left\{T_{\Upsilon_1(\alpha)}(x),T_{\Upsilon_2(\alpha)}(x)\right\},\max\left\{I_{\Upsilon_1(\alpha)}(x),I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \min\left\{F_{\Upsilon_1(\alpha)}(x),F_{\Upsilon_2(\alpha)}(x)\right\} \end{array}\right\rangle\right\}$$

$$[(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)]^c=\left\{\left\langle \begin{array}{c} x,\min\left\{F_{\Upsilon_1(\alpha)}(x),F_{\Upsilon_2(\alpha)}(x)\right\},1-\max\left\{I_{\Upsilon_1(\alpha)}(x),I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x),T_{\Upsilon_2(\alpha)}(x)\right\} \end{array}\right\rangle\right\}.$$

Now,

$$(\Upsilon_1,\Im_1)^c=\left\{\left\langle x,F_{\Upsilon_1(\alpha)}(x),1-I_{\Upsilon_1(\alpha)}(x),T_{\Upsilon_1(\alpha)}(x)\right\rangle\right\},$$

$$(\Upsilon_2,\Im_2)^c=\left\{\left\langle x,F_{\Upsilon_2(\alpha)}(x),1-I_{\Upsilon_2(\alpha)}(x),T_{\Upsilon_2(\alpha)}(x)\right\rangle\right\}.$$

Then,

$$(\Upsilon_1,\Im_1)^c\cap(\Upsilon_2,\Im_2)^c=\left\{\left\langle \begin{array}{c} x,\min\left\{F_{\Upsilon_1(\alpha)}(x),F_{\Upsilon_2(\alpha)}(x)\right\},\min\left\{\left(1-I_{\Upsilon_1(\alpha)}(x)\right),\left(1-I_{\Upsilon_2(\alpha)}(x)\right)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x),T_{\Upsilon_2(\alpha)}(x)\right\} \end{array}\right\rangle\right\}$$

$$=\left\{\left\langle \begin{array}{c} x,\min\left\{F_{\Upsilon_1(\alpha)}(x),F_{\Upsilon_2(\alpha)}(x)\right\},1-\max\left\{I_{\Upsilon_1(\alpha)}(x),I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x),T_{\Upsilon_2(\alpha)}(x)\right\} \end{array}\right\rangle\right\}.$$

Therefore, $[(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)]^c=(\Upsilon_1,\Im_1)^c\cap(\Upsilon_2,\Im_2)^c$.

2. It is obtained similarly.

Proposition 3.11 Let $(\Upsilon_1, \Im_1)$ and $(\Upsilon_2, \Im_2)$ be two neutrosophic hypersoft sets over the universe set Δ. Then,

1. $\left[(\Upsilon_1, \Im_1) \vee (\Upsilon_2, \Im_2)\right]^c = (\Upsilon_1, \Im_1)^c \wedge (\Upsilon_2, \Im_2)^c$;
2. $\left[(\Upsilon_1, \Im_1) \wedge (\Upsilon_2, \Im_2)\right]^c = (\Upsilon_1, \Im_1)^c \vee (\Upsilon_2, \Im_2)^c$.

Proof. 1. For all $x \in \Delta$

$$(\Upsilon_1, \Im_1) \vee (\Upsilon_2, \Im_2) = \left\{ \left\langle \begin{array}{c} x, \max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\}, \max\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\} \end{array} \right\rangle \right\},$$

$$\left[(\Upsilon_1, \Im_1) \vee (\Upsilon_2, \Im_2)\right]^c = \left\{ \left\langle \begin{array}{c} x, \min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\}, 1-\max\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\} \end{array} \right\rangle \right\}.$$

On the other hand,

$$(\Upsilon_1, \Im_1)^c = \left\{ \left\langle x, F_{\Upsilon_1(\alpha)}(x), 1-I_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_1(\alpha)}(x) \right\rangle \right\},$$
$$(\Upsilon_2, \Im_2)^c = \left\{ \left\langle x, F_{\Upsilon_2(\alpha)}(x), 1-I_{\Upsilon_2(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x) \right\rangle \right\}.$$

Then,

$$(\Upsilon_1, \Im_1)^c \wedge (\Upsilon_2, \Im_2)^c = \left\{ \left\langle \begin{array}{c} x, \min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\}, \min\left\{\left(1-I_{\Upsilon_1(\alpha)}(x)\right), \left(1-I_{\Upsilon_2(\alpha)}(x)\right)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\} \end{array} \right\rangle \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \min\left\{F_{\Upsilon_1(\alpha)}(x), F_{\Upsilon_2(\alpha)}(x)\right\}, 1-\max\left\{I_{\Upsilon_1(\alpha)}(x), I_{\Upsilon_2(\alpha)}(x)\right\}, \\ \max\left\{T_{\Upsilon_1(\alpha)}(x), T_{\Upsilon_2(\alpha)}(x)\right\} \end{array} \right\rangle \right\}.$$

Hence, $\left[(\Upsilon_1, \Im_1) \vee (\Upsilon_2, \Im_2)\right]^c = (\Upsilon_1, \Im_1)^c \wedge (\Upsilon_2, \Im_2)^c$.

Example 3.12 Let $\Delta = \{x_1, x_2, x_3\}$ be an initial universe and $£_1, £_2, £_3$ be sets of attributes. Attributes are given as;

$$£_1 = \{\epsilon_1, \epsilon_2, \epsilon_3\}, \quad £_2 = \{\xi_1, \xi_2\}, \quad £_3 = \{\alpha_1, \alpha_2\}$$

Suppose that

$$C_1 = \{\epsilon_1\}, C_2 = \{\xi_1, \xi_2\}, C_3 = \{\alpha_1, \alpha_2\}$$
$$D_1 = \{\epsilon_1, \epsilon_3\}, D_2 = \{\xi_1, \xi_2\}, D_3 = \{\alpha_2\}$$

are subset of $£_i$ for each $i = 1,2,3.$ Then the neutrosophic hypersoft sets $(\Upsilon_1, \Im_1)$ and $(\Upsilon_2, \Im_2)$ over the universe Δ as follows.

$$(\Upsilon_1,\Im_1)=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_1),\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.6,0.5,0.4},\frac{x_3}{0.3,0.6,0.2}\}>, \\ <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.8,0.2,0.5},\frac{x_2}{0.5,0.7,0.1},\frac{x_3}{0.5,0.5,0.4}\}>, \\ <(\epsilon_1,\xi_2,\alpha_1),\{\frac{x_1}{0.6,0.4,0.5},\frac{x_2}{0.5,0.7,0.4},\frac{x_3}{0.7,0.3,0.2}\}>, \\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.6,0.2,0.8},\frac{x_2}{0.3,0.4,0.5},\frac{x_3}{0.1,0.3,0.7}\}> \end{array}\right\},$$

$$(\Upsilon_2,\Im_2)=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.9,0.2,0.1},\frac{x_2}{0.2,0.2,0.4},\frac{x_3}{0.2,0.1,0.7}\}>, \\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.7,0.4,0.2},\frac{x_3}{0.4,0.6,0.8}\}>, \\ <(\epsilon_3,\xi_1,\alpha_2),\{\frac{x_1}{0.5,0.2,0.6},\frac{x_2}{0.6,0.5,0.3},\frac{x_3}{0.8,0.3,0.7}\}>, \\ <(\epsilon_3,\xi_2,\alpha_2),\{\frac{x_1}{0.4,0.5,0.2},\frac{x_2}{0.7,0.3,0.8},\frac{x_3}{0.6,0.1,0.5}\}> \end{array}\right\}$$

The union, intersection, "AND", "OR" operation of these sets are follows:

$$(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_1),\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.6,0.5,0.4},\frac{x_3}{0.3,0.6,0.2}\}>, \\ <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.9,0.2,0.1},\frac{x_2}{0.5,0.7,0.1},\frac{x_3}{0.5,0.5,0.4}\}>, \\ <(\epsilon_1,\xi_2,\alpha_1),\{\frac{x_1}{0.6,0.4,0.5},\frac{x_2}{0.5,0.7,0.4},\frac{x_3}{0.7,0.3,0.2}\}>, \\ <((\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.7,0.4,0.2},\frac{x_3}{0.4,0.6,0.7}\}>, \\ <(\epsilon_3,\xi_1,\alpha_2),\{\frac{x_1}{0.5,0.2,0.6},\frac{x_2}{0.6,0.5,0.3},\frac{x_3}{0.8,0.3,0.7}\}>, \\ <((\epsilon_3,\xi_2,\alpha_2),\{\frac{x_1}{0.4,0.5,0.2},\frac{x_2}{0.7,0.3,0.8},\frac{x_3}{0.6,0.1,0.5}\}> \end{array}\right\},$$

$$(\Upsilon_1,\Im_1)\cap(\Upsilon_2,\Im_2)=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.8,0.2,0.5},\frac{x_2}{0.2,0.2,0.4},\frac{x_3}{0.2,0.1,0.7}\}>, \\ <((\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.6,0.2,0.8},\frac{x_2}{0.3,0.4,0.5},\frac{x_3}{0.1,0.3,0.8}\}> \end{array}\right\},$$

Let's assume $(\epsilon_1,\xi_1,\alpha_1)=a_1$, $(\epsilon_1,\xi_1,\alpha_2)=a_2$, $(\epsilon_1,\xi_2,\alpha_1)=a_3$, $(\epsilon_1,\xi_2,\alpha_2)=a_4$ in $(\Upsilon_1,\Im_1)$ and $(\epsilon_1,\xi_1,\alpha_2)=b_1$, $(\epsilon_1,\xi_2,\alpha_2)=b_2$, $(\epsilon_3,\xi_1,\alpha_2)=b_3$, $(\epsilon_3,\xi_2,\alpha_2)=b_4$ in $(\Upsilon_2,\Im_2)$ for easier operation.

$$(\Upsilon_1,\Im_1)\wedge(\Upsilon_2,\Im_2)=\left\{\begin{array}{l}
(a_1\times b_1,\{\frac{x_1}{0.8,0.2,0.2},\frac{x_2}{0.2,0.2,0.4},\frac{x_3}{0.2,0.1,0.7}\}),\\
(a_1\times b_2,\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.6,0.4,0.4},\frac{x_3}{0.3,0.6,0.8}\}),\\
(a_1\times b_3,\{\frac{x_1}{0.5,0.2,0.6},\frac{x_2}{0.6,0.5,0.4},\frac{x_3}{0.3,0.3,0.7}\}),\\
(a_1\times b_4,\{\frac{x_1}{0.4,0.5,0.2},\frac{x_2}{0.6,0.3,0.8},\frac{x_3}{0.3,0.1,0.5}\}),\\
(a_2\times b_1,\{\frac{x_1}{0.8,0.2,0.5},\frac{x_2}{0.2,0.2,0.4},\frac{x_3}{0.2,0.1,0.7}\}),\\
(a_2\times b_2,\{\frac{x_1}{0.8,0.2,0.5},\frac{x_2}{0.5,0.4,0.2},\frac{x_3}{0.4,0.5,0.8}\})\\
(a_2\times b_3,\{\frac{x_1}{0.5,0.2,0.6},\frac{x_2}{0.5,0.5,0.3},\frac{x_3}{0.5,0.3,0.7}\}),\\
(a_2\times b_4,\{\frac{x_1}{0.4,0.2,0.5},\frac{x_2}{0.5,0.3,0.8},\frac{x_3}{0.5,0.1,0.5}\}),\\
(a_3\times b_1,\{\frac{x_1}{0.6,0.2,0.5},\frac{x_2}{0.2,0.2,0.4},\frac{x_3}{0.2,0.1,0.7}\}),\\
(a_3\times b_2,\{\frac{x_1}{0.6,0.4,0.5},\frac{x_2}{0.5,0.4,0.4},\frac{x_3}{0.4,0.3,0.8}\}),\\
(a_3\times b_3,\{\frac{x_1}{0.5,0.2,0.6},\frac{x_2}{0.5,0.5,0.4},\frac{x_3}{0.7,0.3,0.7}\}),\\
(a_3\times b_4,\{\frac{x_1}{0.4,0.4,0.5},\frac{x_2}{0.5,0.3,0.8},\frac{x_3}{0.6,0.1,0.5}\}),\\
(a_4\times b_1,\{\frac{x_1}{0.6,0.2,0.8},\frac{x_2}{0.2,0.2,0.5},\frac{x_3}{0.1,0.1,0.7}\}),\\
(a_4\times b_2,\{\frac{x_1}{0.6,0.2,0.8},\frac{x_2}{0.3,0.4,0.5},\frac{x_3}{0.1,0.3,0.8}\}),\\
(a_4\times b_3,\{\frac{x_1}{0.5,0.2,0.8},\frac{x_2}{0.3,0.4,0.5},\frac{x_3}{0.1,0.3,0.7}\}),\\
(a_4\times b_4,\{\frac{x_1}{0.4,0.2,0.8},\frac{x_2}{0.3,0.3,0.8},\frac{x_3}{0.1,0.1,0.7}\})
\end{array}\right\},$$

$$(\Upsilon_1,\Im_1)\vee(\Upsilon_2,\Im_2)=\left\{\begin{array}{l}
(a_1\times b_1,\{\frac{x_1}{0.9,0.5,0.1},\frac{x_2}{0.6,0.5,0.4},\frac{x_3}{0.3,0.6,0.2}\}),\\
(a_1\times b_2,\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.7,0.5,0.2},\frac{x_3}{0.4,0.6,0.2}\}),\\
(a_1\times b_3,\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.6,0.5,0.3},\frac{x_3}{0.8,0.6,0.2}\}),\\
(a_1\times b_4,\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.7,0.5,0.4},\frac{x_3}{0.6,0.6,0.2}\}),\\
(a_2\times b_1,\{\frac{x_1}{0.9,0.2,0.1},\frac{x_2}{0.5,0.7,0.1},\frac{x_3}{0.5,0.5,0.4}\}),\\
(a_2\times b_2,\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.7,0.7,0.1},\frac{x_3}{0.5,0.6,0.4}\})\\
(a_2\times b_3,\{\frac{x_1}{0.8,0.2,0.5},\frac{x_2}{0.6,0.7,0.1},\frac{x_3}{0.8,0.5,0.4}\}),\\
(a_2\times b_4,\{\frac{x_1}{0.8,0.5,0.2},\frac{x_2}{0.7,0.7,0.1},\frac{x_3}{0.6,0.5,0.4}\}),\\
(a_3\times b_1,\{\frac{x_1}{0.9,0.4,0.1},\frac{x_2}{0.5,0.7,0.4},\frac{x_3}{0.7,0.3,0.2}\}),\\
(a_3\times b_2,\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.7,0.7,0.2},\frac{x_3}{0.7,0.6,0.2}\}),\\
(a_3\times b_3,\{\frac{x_1}{0.6,0.4,0.5},\frac{x_2}{0.6,0.7,0.3},\frac{x_3}{0.8,0.3,0.2}\}),\\
(a_3\times b_4,\{\frac{x_1}{0.6,0.5,0.2},\frac{x_2}{0.7,0.7,0.4},\frac{x_3}{0.7,0.3,0.2}\}),\\
(a_4\times b_1,\{\frac{x_1}{0.9,0.2,0.1},\frac{x_2}{0.3,0.4,0.4},\frac{x_3}{0.2,0.3,0.7}\}),\\
(a_4\times b_2,\{\frac{x_1}{0.8,0.4,0.3},\frac{x_2}{0.7,0.4,0.2},\frac{x_3}{0.4,0.6,0.7}\}),\\
(a_4\times b_3,\{\frac{x_1}{0.6,0.2,0.6},\frac{x_2}{0.6,0.5,0.3},\frac{x_3}{0.8,0.3,0.7}\}),\\
(a_4\times b_4,\{\frac{x_1}{0.6,0.5,0.2},\frac{x_2}{0.7,0.4,0.5},\frac{x_3}{0.6,0.3,0.5}\})
\end{array}\right\}.$$

4. Neutrosophic Hypersoft Topological Spaces

Definition 4.1 Let $NHSS(\Delta,\Sigma)$ be the family of all neutrosophic hypersoft sets over the universe set Δ and $\tilde{\tau}\subseteq NHSS(\Delta,\Sigma)$. Then $\tilde{\tau}$ is said to be a neutrosophic hypersoft topology on Δ if

1. $0_{(\Delta_{NH},\Sigma)}$ and $1_{(\Delta_{NH},\Sigma)}$ belongs to $\tilde{\tau}$
2. the union of any number of neutrosophic hypersoft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$
3. the intersection of finite number of neutrosophic hypersoft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

Then $(\Delta,\Sigma,\tilde{\tau})$ is said to be a neutrosophic hypersoft topological space over Δ. Each members of $\tilde{\tau}$ is said to be neutrosophic hypersoft open set.

Definition 4.2 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon,\Im)$ be a neutrosophic hypersoft set over Δ. Then $(\Upsilon,\Im)$ is said to be neutrosophic hypersoft closed set iff its complement is a neutrosophic hypersoft open set.

Proposition 4.3 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ. Then

1. $0_{(\Delta_{NH},\Sigma)}$ and $1_{(\Delta_{NH},\Sigma)}$ are neutrosophic hypersoft closed sets over Δ

2. The intersection of any number of neutrosophic hypersoft closed sets is a neutrosophic hypersoft closed set over Δ

3. The union of finite number of neutrosophic hypersoft closed sets is a neutrosophic hypersoft closed set over Δ.

Proof. It is clear that the definition of neutrosophic hypersoft topological space.

Definition 4.4 Let $NHSS(\Delta,\Sigma)$ be the family of all neutrosophic hypersoft sets over the universe set Δ.

1. If $\tilde{\tau}=\left\{0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)}\right\}$, then $\tilde{\tau}$ is said to be the neutrosophic hypersoft indiscrete topology and $(\Delta,\Sigma,\tilde{\tau})$ is said to be a neutrosophic hypersoft indiscrete topological space over Δ.
2. If $\tilde{\tau}=NHSS(\Delta,\Sigma)$, then $\tilde{\tau}$ is said to be the neutrosophic hypersoft discrete topology and $(\Delta,\Sigma,\tilde{\tau})$ is said to be a neutrosophic hypersoft discrete topological space over Δ.

Proposition 4.5 Let $(\Delta,\Sigma,\tilde{\tau}_1)$ and $(\Delta,\Sigma,\tilde{\tau}_2)$ be two neutrosophic hypersoft topological spaces over the same universe set Δ. Then $\left(\Delta,\Sigma,\tilde{\tau}_1\cap\tilde{\tau}_2\right)$ is neutrosophic hypersoft topological space over Δ.

Proof. 1. Since $0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)}\in\tilde{\tau}_1$ and $0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)}\in\tilde{\tau}_2$, then $0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)}\in\tilde{\tau}_1\cap\tilde{\tau}_2$.

2. Suppose that $\left\{(\Upsilon_i,\Im_i)\middle|i\in I\right\}$ be a family of neutrosophic hypersoft sets in $\tilde{\tau}_1\cap\tilde{\tau}_2$. Then $(\Upsilon_i,\Im_i)\in\tilde{\tau}_1$ and $(\Upsilon_i,\Im_i)\in\tilde{\tau}_2$ for all $i\in I$, so $\underset{i\in I}{\cup}(\Upsilon_i,\Im_i)\in\tilde{\tau}_1$ and $\underset{i\in I}{\cup}(\Upsilon_i,\Im_i)\in\tilde{\tau}_2$. Thus $\underset{i\in I}{\cup}(\Upsilon_i,\Im_i)\in\tilde{\tau}_1\cap\tilde{\tau}_2$.

3. Let $\left\{(\Upsilon_i,\Im_i)\middle|i=\overline{1,k}\right\}$ be a family of the finite number of neutrosophic hypersoft sets in $\tilde{\tau}_1\cap\tilde{\tau}_2$. Then $(\Upsilon_i,\Im_i)\in\tilde{\tau}_1$ and $(\Upsilon_i,\Im_i)\in\tilde{\tau}_2$ for $i=\overline{1,k}$, so $\overset{n}{\underset{i=1}{\cap}}(\Upsilon_i,\Im_i)\in\tilde{\tau}_1$ and $\overset{n}{\underset{i=1}{\cap}}(\Upsilon_i,\Im_i)\in\tilde{\tau}_2$. Thus $\overset{n}{\underset{i=1}{\cap}}(\Upsilon_i,\Im_i)\in\tilde{\tau}_1\cap\tilde{\tau}_2$.

Remark 4.6 The union of two neutrosophic hypersoft topologies over Δ may not be a neutrosophic hypersoft topology on Δ.

Example 4.7 We consider attributes of Example-3.12.

$$\tilde{\tau}_1=\left\{0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)},(\Upsilon_1,\Im_1),(\Upsilon_2,\Im_2)\right\}$$

and

$$\tilde{\tau}_2=\left\{0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)},(\Upsilon_3,\Im_3),(\Upsilon_4,\Im_4)\right\}$$

be two neutrosophic hypersoft topologies over Δ. Here, the neutrosophic hypersoft sets $(\Upsilon_1, \Im_1)$, $(\Upsilon_2, \Im_2), (\Upsilon_3, \Im_3)$ and $(\Upsilon, \Im_4)$ over Δ are defined as following:

$$(\Upsilon_1, \Im_1) = \left\{\begin{array}{l} <(\epsilon_1, \xi_1, \alpha_1), \{\frac{x_1}{0.8, 0.5, 0.2}, \frac{x_2}{0.6, 0.5, 0.4}, \frac{x_3}{0.3, 0.6, 0.2}\}>, \\ <(\epsilon_1, \xi_1, \alpha_2), \{\frac{x_1}{0.8, 0.2, 0.5}, \frac{x_2}{0.5, 0.7, 0.1}, \frac{x_3}{0.5, 0.5, 0.4}\}>, \\ <(\epsilon_1, \xi_2, \alpha_1), \{\frac{x_1}{0.6, 0.4, 0.5}, \frac{x_2}{0.5, 0.7, 0.4}, \frac{x_3}{0.7, 0.3, 0.2}\}>, \\ <(\epsilon_1, \xi_2, \alpha_2), \{\frac{x_1}{0.6, 0.2, 0.8}, \frac{x_2}{0.3, 0.4, 0.5}, \frac{x_3}{0.1, 0.3, 0.7}\}> \end{array}\right\},$$

$$(\Upsilon_2, \Im_2) = \left\{\begin{array}{l} <(\epsilon_1, \xi_1, \alpha_1), \{\frac{x_1}{0.8, 0.6, 0.1}, \frac{x_2}{0.7, 0.6, 0.3}, \frac{x_3}{0.4, 0.8, 0.1}\}>, \\ <(\epsilon_1, \xi_1, \alpha_2), \{\frac{x_1}{0.8, 0.5, 0.3}, \frac{x_2}{0.7, 0.8, 0.1}, \frac{x_3}{0.7, 0.6, 0.2}\}>, \\ <(\epsilon_1, \xi_2, \alpha_1), \{\frac{x_1}{0.9, 0.6, 0.2}, \frac{x_2}{0.8, 0.8, 0.3}, \frac{x_3}{0.7, 0.6, 0.2}\}>, \\ <(\epsilon_1, \xi_2, \alpha_2), \{\frac{x_1}{0.9, 0.8, 0.5}, \frac{x_2}{0.6, 0.7, 0.3}, \frac{x_3}{0.4, 0.4, 0.4}\}> \end{array}\right\},$$

$$(\Upsilon_3, \Im_3) = \left\{\begin{array}{l} <(\epsilon_2, \xi_1, \alpha_2), \{\frac{x_1}{0.2, 0.6, 0.4}, \frac{x_2}{0.6, 0.6, 0.2}, \frac{x_3}{0.4, 0.5, 0.2}\}>, \\ <(\epsilon_2, \xi_2, \alpha_2), \{\frac{x_1}{0.8, 0.2, 0.5}, \frac{x_2}{0.6, 0.3, 0.1}, \frac{x_3}{0.3, 0.7, 0.8}\}>, \\ <(\epsilon_3, \xi_1, \alpha_2), \{\frac{x_1}{0.3, 0.6, 0.2}, \frac{x_2}{0.6, 0.2, 0.5}, \frac{x_3}{0.7, 0.3, 0.2}\}>, \\ <(\epsilon_3, \xi_2, \alpha_2), \{\frac{x_1}{0.5, 0.3, 0.4}, \frac{x_2}{0.5, 0.1, 0.3}, \frac{x_3}{0.4, 0.5, 0.5}\}> \end{array}\right\},$$

$$(\Upsilon_4, \Im_4) = \left\{\begin{array}{l} <(\epsilon_2, \xi_1, \alpha_2), \{\frac{x_1}{0.4, 0.7, 0.2}, \frac{x_2}{0.8, 0.7, 0.2}, \frac{x_3}{0.7, 0.6, 0.2}\}>, \\ <(\epsilon_2, \xi_2, \alpha_2), \{\frac{x_1}{0.8, 0.5, 0.2}, \frac{x_2}{0.7, 0.6, 0.1}, \frac{x_3}{0.5, 0.9, 0.4}\}>, \\ <(\epsilon_3, \xi_1, \alpha_2), \{\frac{x_1}{0.6, 0.7, 0.1}, \frac{x_2}{0.8, 0.5, 0.3}, \frac{x_3}{0.8, 0.6, 0.1}\}>, \\ <(\epsilon_3, \xi_2, \alpha_2), \{\frac{x_1}{0.8, 0.5, 0.3}, \frac{x_2}{0.4, 0.2, 0.2}, \frac{x_3}{0.6, 0.6, 0.2}\}> \end{array}\right\}$$

Since $(\Upsilon_1,\Im_1)\cup(\Upsilon_4,\Im_4)\notin \tilde{\tau}_1\cup \tilde{\tau}_2$, then $\tilde{\tau}_1\cup \tilde{\tau}_2$ is not a neutrosophic hypersoft topology over Δ.

Remark 4.8 In the $(\Upsilon,\Im)$ hypersoft set, $\Im$ consists of the cartesian product set $C_1\times C_2\times\ldots\times C_k$. If the parameters are selected from a single attribute set C_i and the empty set from the other attribute sets while creating a neutrosophic hypersoft set, it is clear that the resulting structure will be a neutrosophic soft set structure. Neutrosophic hypersoft set structure is a generalized version of neutrosophic soft structure. Therefore, the neutrosophic hypersoft topological structure also provides all the conditions of the neutrosophic soft topological structure. (see Example-4.7)

Proposition 4.9 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $\tilde{\tau}=\{(\Upsilon_i,\Im_i):(\Upsilon_i,\Im_i)\in NHSS(\Delta,\Sigma)\}=\{[\alpha,(\Upsilon_i(\alpha))]_{\alpha\in\Im}:(\Upsilon_i,\Im_i)\in NHSS(\Delta,\Sigma)\}$ where $\Upsilon_i(\alpha)=\left\{\left\langle x,T_{\Upsilon_i(\alpha)}(x),I_{\Upsilon_i(\alpha)}(x),F_{\Upsilon_i(\alpha)}(x)\right\rangle:x\in\Delta\right\}$. Then

$$\tau_1=\left\{\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}$$
$$\tau_2=\left\{\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}$$
$$\tau_3=\left\{\left[F_{\Upsilon_i(\alpha)}(x)\right]^c_{\alpha\in\Im}\right\}$$

define fuzzy soft topologies on Δ.

Proof. 1. $0_{(\Delta_{NH},\Sigma)},1_{(\Delta_{NH},\Sigma)}\in\tilde{\tau}\Rightarrow 0,1\in\tau_1$, $0,1\in\tau_2$ and $0,1\in\tau_3$

2. Suppose that $\{(\Upsilon_i,\Im_i)|i\in I\}$ be a family of neutrosophic hypersoft sets in $\tilde{\tau}$. Then $\left\{\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i\in I}$ is a family of fuzzy soft sets in τ_1, $\left\{\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i\in I}$ is a family of fuzzy soft sets in τ_2 and $\left\{\left[F_{\Upsilon_i(\alpha)}(x)\right]^c_{\alpha\in\Im}\right\}_{i\in I}$ is a family of fuzzy soft sets in τ_3. Since $\tilde{\tau}$ is a neutrosophic hypersoft topology, then $\underset{i\in I}{\cup}(\Upsilon_i,\Im_i)\in\tilde{\tau}$. That is,

$$\underset{i\in I}{\cup}(\Upsilon_i,\Im_i)=\left\{\left\langle \sup\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im},\sup\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im},\inf\left[F_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\rangle\right\}_{i\in I}\in\tilde{\tau}.$$

Therefore,

$$\left\{\sup\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i\in I}\in\tau_1$$
$$\left\{\sup\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i\in I}\in\tau_2$$
$$\left\{\sup\left[F_{\Upsilon_i(\alpha)}(x)\right]^c_{\alpha\in\Im}\right\}_{i\in I}\in\tau_3.$$

3. Suppose that $\left\{(\Upsilon_i,\Im_i)\middle| i=\overline{1,k}\right\}$ be a family of finite neutrosophic hypersoft sets in $\tilde{\tau}$. Then $\left\{\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i=\overline{1,k}}$ is a family of fuzzy soft sets in τ_1, $\left\{\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\}_{i=\overline{1,k}}$ is a family of fuzzy soft sets in τ_2 and $\left\{\left[F_{\Upsilon_i(\alpha)}(x)\right]^c_{\alpha\in\Im}\right\}_{i=\overline{1,k}}$ is a family of fuzzy soft sets in τ_3. Since $\tilde{\tau}$ is a neutrosophic hypersoft topology, then $\bigcap_{i=1}^{n}(\Upsilon_i,\Im_i)\in\tilde{\tau}$. That is,

$$\bigcap_{i=1}^{n}(\Upsilon_i,\Im_i)=\left\{\left\langle \min\left[T_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im},\min\left[I_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im},\max\left[F_{\Upsilon_i(\alpha)}(x)\right]_{\alpha\in\Im}\right\rangle\right\}_{i=\overline{1,k}}\in\tilde{\tau}.$$

Therefore,

$$\left\{\min\left[T_{\Upsilon_i(\alpha)}(x)\right]\right\}_{i\in I}\in\tau_1$$
$$\left\{\min\left[I_{\Upsilon_i(\alpha)}(x)\right]\right\}_{i\in I}\in\tau_2$$
$$\left\{\min\left[F_{\Upsilon_i(\alpha)}(x)\right]^c\right\}_{i\in I}\in\tau_3.$$

This completes the proof.

Definition 4.10 *Let* $(\Delta,\Sigma,\tilde{\tau})$ *be a neutrosophic hypersoft topological space over* Δ *and* $(\Upsilon,\Im)\in NHSS(\Delta,\Sigma)$ *be a neutrosophic hypersoft set. Then, the neutrosophic hypersoft interior of* $(\Upsilon,\Im)$, *denoted* $(\Upsilon,\Im)^\circ$, *is defined as the neutrosophic hypersoft union of all neutrosophic hypersoft open subsets of* $(\Upsilon,\Im)$.

Clearly, $(\Upsilon,\Im)^\circ$ is the biggest neutrosophic hypersoft open set that is contained by $(\Upsilon,\Im)$.

Example 4.11 Let us consider the neutrosophic hypersoft topology $\tilde{\tau}_1$ given in Example 4.7. Suppose that an any $(\Upsilon,\Im)\in NHSS(\Delta,\Sigma)$ is defined as following:

$$(\Upsilon,\Im)=\left\{\begin{array}{l}<(\epsilon_1,\xi_1,\alpha_1),\{\frac{x_1}{0.9,0.7,0.1},\frac{x_2}{0.8,0.7,0.2},\frac{x_3}{0.6,0.8,0.1}\}>,\\ <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.9,0.6,0.2},\frac{x_2}{0.8,0.9,0.1},\frac{x_3}{0.8,0.7,0.1}\}>,\\ <(\epsilon_1,\xi_2,\alpha_1),\{\frac{x_1}{0.9,0.8,0.1},\frac{x_2}{0.9,0.8,0.2},\frac{x_3}{0.8,0.7,0.2}\}>,\\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.9,0.9,0.3},\frac{x_2}{0.7,0.8,0.1},\frac{x_3}{0.6,0.7,0.3}\}>\end{array}\right\}.$$

Then $0_{(\Delta_{NH},\Sigma)}$, $(\Upsilon_1,\Im_1)$, $(\Upsilon_2,\Im_2)\subseteq(\Upsilon,\Im)$. Therefore, $(\Upsilon,\Im)^\circ=0_{(\Delta_{NH},\Sigma)}\cup(\Upsilon_1,\Im_1)\cup(\Upsilon_2,\Im_2)=(\Upsilon_2,\Im_2)$.

Theorem 4.12 Let $(\Delta, \Sigma, \tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon, \Im) \in NHSS(\Delta, \Sigma)$. $(\Upsilon, \Im)$ is a neutrosophic hypersoft open set iff $(\Upsilon, \Im) = (\Upsilon, \Im)^{\circ}$.

Proof. Let $(\Upsilon, \Im)$ be a neutrosophic hypersoft open set. Then the biggest neutrosophic hypersoft open set that is contained by $(\Upsilon, \Im)$ is equal to $(\Upsilon, \Im)$. Hence, $(\Upsilon, \Im) = (\Upsilon, \Im)^{\circ}$.
Conversely, it is known that $(\Upsilon, \Im)^{\circ}$ is a neutrosophic hypersoft open set and if $(\Upsilon, \Im) = (\Upsilon, \Im)^{\circ}$, then $(\Upsilon, \Im)$ is a neutrosophic hypersoft open set.

Theorem 4.13 Let $(\Delta, \Sigma, \tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon_1, \Im_1), (\Upsilon_2, \Im_2) \in NHSS(\Delta, \Sigma)$. Then,

1. $\left[(\Upsilon_1, \Im_1)^{\circ}\right]^{\circ} = (\Upsilon_1, \Im_1)^{\circ}$,
2. $\left(0_{(\Delta_{NH}, \Sigma)}\right)^{\circ} = 0_{(\Delta_{NH}, \Sigma)}$ and $\left(1_{(\Delta_{NH}, \Sigma)}\right)^{\circ} = 1_{(\Delta_{NH}, \Sigma)}$,
3. $(\Upsilon_1, \Im_1) \subseteq (\Upsilon_2, \Im_2) \Rightarrow (\Upsilon_1, \Im_1)^{\circ} \subseteq (\Upsilon_2, \Im_2)^{\circ}$,
4. $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} = (\Upsilon_1, \Im_1)^{\circ} \cap (\Upsilon_2, \Im_2)^{\circ}$,
5. $(\Upsilon_1, \Im_1)^{\circ} \cup (\Upsilon_2, \Im_2)^{\circ} \subseteq \left[(\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)\right]^{\circ}$.

Proof. 1. Let $(\Upsilon_1, \Im_1)^{\circ} = (\Upsilon_2, \Im_2)$. Then $(\Upsilon_2, \Im_2) \in \tilde{\tau}$ iff $(\Upsilon_2, \Im_2) = (\Upsilon_2, \Im_2)^{\circ}$. So, $\left[(\Upsilon_1, \Im_1)^{\circ}\right]^{\circ} = (\Upsilon_1, \Im_1)^{\circ}$.
2. Straighforward.
3. It is known that $(\Upsilon_1, \Im_1)^{\circ} \subseteq (\Upsilon_1, \Im_1) \subseteq (\Upsilon_2, \Im_2)$ and $(\Upsilon_2, \Im_2)^{\circ} \subseteq (\Upsilon_2, \Im_2)$. Since $(\Upsilon_2, \Im_2)^{\circ}$ is the biggest neutrosophic hypersoft open set contained in $(\Upsilon_2, \Im_2)$ and so, $(\Upsilon_1, \Im_1)^{\circ} \subseteq (\Upsilon_2, \Im_2)^{\circ}$.
4. Since $(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2) \subseteq (\Upsilon_1, \Im_1)$ and $(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2) \subseteq (\Upsilon_2, \Im_2)$, then $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} \subseteq (\Upsilon_1, \Im_1)^{\circ}$ and $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} \subseteq (\Upsilon_2, \Im_2)^{\circ}$ and so, $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} \subseteq (\Upsilon_1, \Im_1)^{\circ} \cap (\Upsilon_2, \Im_2)^{\circ}$.
On the other hand, since $(\Upsilon_1, \Im_1)^{\circ} \subseteq (\Upsilon_1, \Im_1)$ and $(\Upsilon_2, \Im_2)^{\circ} \subseteq (\Upsilon_2, \Im_2)$, then $(\Upsilon_1, \Im_1)^{\circ} \cap (\Upsilon_2, \Im_2)^{\circ} \subseteq (\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)$. Besides, $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} \subseteq (\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)$ and it is the biggest neutrosophic hypersoft open set. Therefore, $(\Upsilon_1, \Im_1)^{\circ} \cap (\Upsilon_2, \Im_2)^{\circ} \subseteq \left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ}$. Thus, $\left[(\Upsilon_1, \Im_1) \cap (\Upsilon_2, \Im_2)\right]^{\circ} = (\Upsilon_1, \Im_1)^{\circ} \cap (\Upsilon_2, \Im_2)^{\circ}$.
5. Since $(\Upsilon_1, \Im_1) \subseteq (\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)$ and $(\Upsilon_2, \Im_2) \subseteq (\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)$, then $(\Upsilon_1, \Im_1)^{\circ} \subseteq \left[(\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)\right]^{\circ}$ and $(\Upsilon_2, \Im_2)^{\circ} \subseteq \left[(\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)\right]^{\circ}$. Therefore, $(\Upsilon_1, \Im_1)^{\circ} \cup (\Upsilon_2, \Im_2)^{\circ} \subseteq \left[(\Upsilon_1, \Im_1) \cup (\Upsilon_2, \Im_2)\right]^{\circ}$.

Definition 4.14 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon,\Im)\in NHSS(\Delta,\Sigma)$ be a neutrosophic hypersoft set. Then, the neutrosophic hypersoft closure of $(\Upsilon,\Im)$, denoted $\overline{(\Upsilon,\Im)}$, is defined as the neutrosophic hypersoft intersection of all neutrosophic hypersoft closed supersets of $(\Upsilon,\Im)$.

Clearly, $\overline{(\Upsilon,\Im)}$ is the smallest neutrosophic hypersoft closed set that containing $(\Upsilon,\Im)$.

Example 4.15 Let us consider the neutrosophic hypersoft topology $\tilde{\tau}_2$ given in Example 4.7. Suppose that an any $(\Upsilon,\Im)\in NHSS(\Delta,\Sigma)$ is defined as following:

$$(\Upsilon,\Im)=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.1,0.2,0.5},\frac{x_2}{0.1,0.2,0.9},\frac{x_3}{0.2,0.3,0.8}\}>, \\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.2,0.6,0.9},\frac{x_2}{0.1,0.2,0.8},\frac{x_3}{0.3,0.1,0.6}\}>, \\ <(\epsilon_3,\xi_1,\alpha_2),\{\frac{x_1}{0.1,0.1,0.8},\frac{x_2}{0.1,0.3,0.8},\frac{x_3}{0.1,0.2,0.8}\}>, \\ <(\epsilon_3,\xi_2,\alpha_2),\{\frac{x_1}{0.2,0.4,0.9},\frac{x_2}{0.1,0.5,0.6},\frac{x_3}{0.2,0.2,0.7}\}> \end{array}\right\}.$$

Obviously, $0^c_{(\Delta_{NH},\Sigma)}$, $1^c_{(\Delta_{NH},\Sigma)}$, $(\Upsilon_3,\Im_3)^c$ and $(\Upsilon_4,\Im_4)^c$ are all neutrosophic hypersoft closed sets over $(\Delta,\Sigma,\tilde{\tau})$. They are given as following:

$$0^c_{(\Delta_{NH},\Sigma)}=1_{(\Delta_{NH},\Sigma)},1^c_{(\Delta_{NH},\Sigma)}=0_{(\Delta_{NH},\Sigma)}$$

$$(\Upsilon_3,\Im_3)^c=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.4,0.4,0.2},\frac{x_2}{0.2,0.4,0.6},\frac{x_3}{0.2,0.5,0.4}\}>, \\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.5,0.8,0.8},\frac{x_2}{0.1,0.7,0.6},\frac{x_3}{0.8,0.3,0.3}\}>, \\ <(\epsilon_3,\xi_1,\alpha_2),\{\frac{x_1}{0.2,0.4,0.3},\frac{x_2}{0.5,0.8,0.6},\frac{x_3}{0.2,0.7,0.7}\}>, \\ <(\epsilon_3,\xi_2,\alpha_2),\{\frac{x_1}{0.4,0.7,0.5},\frac{x_2}{0.3,0.9,0.5},\frac{x_3}{0.5,0.5,0.4}\}> \end{array}\right\},$$

$$(\Upsilon_4,\Im_4)^c=\left\{\begin{array}{l} <(\epsilon_1,\xi_1,\alpha_2),\{\frac{x_1}{0.2,0.3,0.4},\frac{x_2}{0.2,0.3,0.8},\frac{x_3}{0.2,0.4,0.7}\}>, \\ <(\epsilon_1,\xi_2,\alpha_2),\{\frac{x_1}{0.2,0.5,0.8},\frac{x_2}{0.1,0.4,0.7},\frac{x_3}{0.4,0.1,0.5}\}>, \\ <(\epsilon_3,\xi_1,\alpha_2),\{\frac{x_1}{0.1,0.3,0.6},\frac{x_2}{0.3,0.5,0.8},\frac{x_3}{0.1,0.4,0.8}\}>, \\ <(\epsilon_3,\xi_2,\alpha_2),\{\frac{x_1}{0.3,0.5,0.8},\frac{x_2}{0.2,0.8,0.4},\frac{x_3}{0.2,0.4,0.6}\}> \end{array}\right\}$$

Then $0^c_{(\Delta_{NH},\Sigma)}$, $(\Upsilon_3,\Im_3)^c,(\Upsilon_4,\Im_4)^c \supseteq (\Upsilon,\Im)$. Therefore, $\overline{(\Upsilon,\Im)} = 0^c_{(\Delta_{NH},\Sigma)} \cap (\Upsilon_3,\Im_3)^c \cap (\Upsilon_4,\Im_4)^c = (\Upsilon_4,\Im_4)^c$.

Theorem 4.16 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon,\Im) \in NHSS(\Delta,\Sigma)$. $(\Upsilon,\Im)$ is neutrosophic hypersoft closed set iff $(\Upsilon,\Im) = \overline{(\Upsilon,\Im)}$.

Proof. Straightforward.

Theorem 4.17 Let $(\Delta,\Sigma,\tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon_1,\Im_1),(\Upsilon_2,\Im_2) \in NHSS(\Delta,\Sigma)$. Then,

1. $\overline{\left[\overline{(\Upsilon_1,\Im_1)}\right]} = \overline{(\Upsilon_1,\Im_1)}$,
2. $\overline{\left(0_{(\Delta_{NH},\Sigma)}\right)} = 0_{(\Delta_{NH},\Sigma)}$ and $\overline{\left(1_{(\Delta_{NH},\Sigma)}\right)} = 1_{(\Delta_{NH},\Sigma)}$
3. $(\Upsilon_1,\Im_1) \subseteq (\Upsilon_2,\Im_2) \Rightarrow \overline{(\Upsilon_1,\Im_1)} \subseteq \overline{(\Upsilon_2,\Im_2)}$,
4. $\overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]} = \overline{(\Upsilon_1,\Im_1)} \cup \overline{(\Upsilon_2,\Im_2)}$,
5. $\overline{[(\Upsilon_1,\Im_1) \cap (\Upsilon_2,\Im_2)]} \subseteq \overline{(\Upsilon_1,\Im_1)} \cap \overline{(\Upsilon_2,\Im_2)}$.

Proof. 1. Let $\overline{(\Upsilon_1,\Im_1)} = (\Upsilon_2,\Im_2)$. Then, $(\Upsilon_2,\Im_2)$ is a neutrosophic hypersoft closed set. Hence, $(\Upsilon_2,\Im_2)$ and $\overline{(\Upsilon_2,\Im_2)}$ are equal. Therefore, $\overline{\left[\overline{(\Upsilon_1,\Im_1)}\right]} = \overline{(\Upsilon_1,\Im_1)}$.
2. Straightforward.
3. It is known that $(\Upsilon_1,\Im_1) \subseteq \overline{(\Upsilon_1,\Im_1)}$ and $(\Upsilon_2,\Im_2) \subseteq \overline{(\Upsilon_2,\Im_2)}$ and so, $(\Upsilon_1,\Im_1) \subseteq (\Upsilon_2,\Im_2) \subseteq \overline{(\Upsilon_2,\Im_2)}$. Since $\overline{(\Upsilon_1,\Im_1)}$ is the smallest neutrosophic hypersoft closed set containing $(\Upsilon_1,\Im_1)$, then $\overline{(\Upsilon_1,\Im_1)} \subseteq \overline{(\Upsilon_2,\Im_2)}$.
4. Since $(\Upsilon_1,\Im_1) \subseteq (\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)$ and $(\Upsilon_2,\Im_2) \subseteq (\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)$, then $\overline{(\Upsilon_1,\Im_1)} \subseteq \overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]}$ and $\overline{(\Upsilon_2,\Im_2)} \subseteq \overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]}$ and so, $\overline{(\Upsilon_1,\Im_1)} \cup \overline{(\Upsilon_2,\Im_2)} \subseteq \overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]}$.
Conversely, since $(\Upsilon_1,\Im_1) \subseteq \overline{(\Upsilon_1,\Im_1)}$ and $(\Upsilon_2,\Im_2) \subseteq \overline{(\Upsilon_2,\Im_2)}$, then $(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2) \subseteq \overline{(\Upsilon_1,\Im_1)} \cup \overline{(\Upsilon_2,\Im_2)}$. Besides, $\overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]}$ is the smallest neutrosophic hypersoft closed set that containing $(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)$. Therefore, $\overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]} \subseteq \overline{(\Upsilon_1,\Im_1)} \cup \overline{(\Upsilon_2,\Im_2)}$. Thus, $\overline{[(\Upsilon_1,\Im_1) \cup (\Upsilon_2,\Im_2)]} = \overline{(\Upsilon_1,\Im_1)} \cup \overline{(\Upsilon_2,\Im_2)}$.
5. Since $(\Upsilon_1,\Im_1) \cap (\Upsilon_2,\Im_2) \subseteq \overline{(\Upsilon_1,\Im_1)} \cap \overline{(\Upsilon_2,\Im_2)}$ and $\overline{[(\Upsilon_1,\Im_1) \cap (\Upsilon_2,\Im_2)]}$ is the smallest neutrosophic hypersoft closed set that containing $(\Upsilon_1,\Im_1) \cap (\Upsilon_2,\Im_2)$, then $\overline{[(\Upsilon_1,\Im_1) \cap (\Upsilon_2,\Im_2)]} \subseteq \overline{(\Upsilon_1,\Im_1)} \cap \overline{(\Upsilon_2,\Im_2)}$.

Theorem 4.18 Let $(\Delta, \Sigma, \tilde{\tau})$ be a neutrosophic hypersoft topological space over Δ and $(\Upsilon, \Im) \in NHSS(\Delta, \Sigma)$. Then,

1. $\left[\overline{(\Upsilon, \Im)}\right]^c = \left[(\Upsilon, \Im)^c\right]^\circ$,
2. $\left[(\Upsilon, \Im)^\circ\right]^c = \overline{\left[(\Upsilon, \Im)^c\right]}$.

Proof. 1. $\overline{(\Upsilon, \Im)} = \cap\left\{(\Upsilon_2, \Im_2) \in \tilde{\tau}^c : (\Upsilon_2, \Im_2) \supseteq (\Upsilon, \Im)\right\}$

$\Rightarrow \left[\overline{(\Upsilon, \Im)}\right]^c = \left[\cap\left\{(\Upsilon_2, \Im_2) \in \tilde{\tau}^c : (\Upsilon_2, \Im_2) \supseteq (\Upsilon, \Im)\right\}\right]^c =$

$\cup\left\{(\Upsilon_2, \Im_2)^c \in \tilde{\tau} : (\Upsilon_2, \Im_2)^c \subseteq (\Upsilon, \Im)^c\right\} = \left[(\Upsilon, \Im)^c\right]^\circ$.

2. $(\Upsilon, \Im)^\circ = \cup\left\{(G, E) \in \tilde{\tau} : (G, E) \subseteq (\Upsilon, \Im)\right\}$

$\Rightarrow \left[(\Upsilon, \Im)^\circ\right]^c = \left[\cup\left\{(\Upsilon_2, \Im_2) \in \tilde{\tau} : (\Upsilon_2, \Im_2) \subseteq (\Upsilon, \Im)\right\}\right]^c =$

$\cap\left\{(\Upsilon_2, \Im_2)^c \in \tilde{\tau}^c : (\Upsilon_2, \Im_2)^c \supseteq (\Upsilon, \Im)^c\right\} = \overline{\left[(\Upsilon, \Im)^c\right]}$.

5. Conclusions

We re-defined neutrosophic hypersoft operations such as complement, null set, absolute set, union, intersection, "AND", "OR" differently from the study [26]. Providing de-morgan laws and other properties of these operations are demonstrated with various proofs and examples. Thus, operations on the neutrosophic hypersoft set structure are well defined. After that, by using these operations, we introduced neutrosophic hypersoft topological spaces, neutrosophic hypersoft open(closed) sets, neutrosophic hypersoft interior and neutrosophic hypersoft closure. Various properties of them have been studied. These are illustrated with appropriate examples. In the future, many topics such as compactness, continuity, connectedness and separation axioms can be studied on neutrosophic hypersoft topological spaces by considering this study. We hope this article will support future studies on neutrosophic hypersoft topological structure and many other general frameworks.

Conflicts of Interest

The authors declare no conflict of interest.

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Contributors

Prof. Dr. Florentin Smarandache
Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA.
E-mail: smarand@unm.edu

Prof. Dr. Muhammad Saeed
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: muhammad.saeed@umt.edu.pk

Prof. Dr. Irfan Deli
Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey.
E-mail: irfandeli@kilis.edu.tr

Prof. Dr. Xiao Long Xin
School of Mathematics, Northwest University Xi'an, China.
School of Science, Xi'an Polytechnic University, Xi'an 710048, China.
E-mail: xlxin@nwu.edu.cn

Associate Prof. Dr. Taha Yasin Ozturk
Department of Mathematic, Kafkas University, Kars, Turkey.
E-mail: taha36100@hotmail.com

Assistant Prof. Dr. Adem Yolcu
Department of Mathematic, Kafkas University, Kars, Turkey.
E-mail: yolcu.adem@gmail.com

Muhammad Saqlain
School of Mathematics, Northwest University, Xi'an, China.
E-mail: msgondal0@gmail.com
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan.
E-mail: msaqlain@lgu.edu.pk

Atiqe Ur Rahman
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: aurkhb@gmail.com

Rana Muhammad Zulqarnain
School of Mathematics, Northwest University, Xi'an, China.
E-mail: ranazulqarnain7777@gmail.com

Muhammad Naveed Jafar
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: 15009265004@umt.edu.pk
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C,
Lahore, 54000, Pakistan.
E-mail: naveedjafar@lgu.edu.pk

Muhammad Rayees Ahmad
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: rayeesmalik.ravian@gmail.com

Muhammad Khubab Siddique
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: khubabsiddique@hotmail.com

Muhammad Ahsan
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: ahsan1826@gmail.com

Mahrukh Irfan
Department of Mathematics, University of Management and Technology
Township, 54000, Lahore, Pakistan.
E-mail: f2018265015@umt.edu.pk

Adeel Saleem
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C,
Lahore, 54000, Pakistan.
E-mail: adeelsaleem1992@hotmail.com

Sana Moin
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan.
E-mail: moinsana64@gmail.com

Maryam Rani
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan.
E-mail: Maryum.rani009@gmail.com

Abida Hafeez
Department of Mathematics, University of Management and Technology Township, 54000, Lahore, Pakistan.
E-mail: f2019109046@umt.edu.pk

Ume-e-Farwa
Department of Mathematics, University of Management and Technology Township, 54000, Lahore, Pakistan.
E-mail: f2019109047@umt.edu.pk

Muhammad Umer Farooq
Department of Physics, Govt. Degree College Bhera, Sargodha, Pakistan.
E-mail: umerfarooq900@gmail.com

Zaka-ur-Rehman
Department of Physics, University of Lahore, Sargodha Campus, Pakistan.
E-mail: zakaurrehman52@gmail.com

Muhammad Haseeb
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan.
E-mail: hasich1122@gmail.com

Ahtasham Habib
Department of Mathematic, Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan.
E-mail: ahtashamhabib1@gmail.com

Theory and Application of Hypersoft Set

The soft set was generalized in 2018 by Smarandache to the hypersoft set by transforming the function F into a multi-argument function. This extension reveals that the hypersoft set with neutrosophic, intuitionistic, and fuzzy set theory will be very helpful to construct a connection between alternatives and attributes. The Book "Theory and Application of Hypersoft Set" focuses on theories, methods, and algorithms for decision making problems. It also involves applications of neutrosophic, intuitionistic, and fuzzy information. Our goal is to develop a strong relationship with the MCDM solving techniques and to reduce the complexion in the methodologies.

Editors

Florentin Smarandache
University of New Mexico, Mathematics and Science Division, 705 Gurley Ave., Gallup, NM 87301, USA.

Muhammad Saqlain
School of Mathematics, Northwest University, Xi'an, China.
Department of Mathematics, Lahore Garrison University, Sector-C, Phase VI, DHA, Lahore, Pakistan.

Muhammad Saeed
Department of Mathematics, University of Management and Technology Township, 54000, Lahore, Pakistan.

Mohamed Abdel-Baset
Department of Computer Science, Faculty of Computers and Informatics, Zagazig University, Egypt.

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