Lower and Upper Soft Interval Valued Neutrosophic Rough Approximations of An IVNSS-Relation

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Abstract: In this paper, we extend the lower and upper soft interval valued intuitionstic fuzzy rough approximations of IVIFS –relations proposed by Anjan et al. to the case of interval valued neutrosophic soft set relation(IVNSS-relation for short)

Keywords: Interval valued neutrosophic soft , Interval valued neutrosophic soft set relation

0. Introduction

This paper is an attempt to extend the concept of interval valued intuitionistic fuzzy soft relation (IVIFSS-relations) introduced by A. Mukherjee et al [45]to IVNSS relation .

The organization of this paper is as follow: In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, relation interval neutrosophic soft relation is presented. In section 4 various type of interval valued neutrosophic soft relations. In section 5, we concludes the paper

1. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set, interval neutrosophic set, soft set, neutrosophic soft set and interval neutrosophic soft set.

Definition 2.1.

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= {< x: $\mu_{A(x)}$, $\nu_{A(x)}$, $\omega_{A(x)}$ >,x ∈ U}, where the functions μ, ν, ω : U→]$^{-}$0,1^{+}[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+}.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$.so instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications, because $]^{-}0,1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2. A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in U$, $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$, $\omega_A(x) \geq \omega_B(x)$.

Definition 2.3. Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\boldsymbol{\mu_A(x)}$, indeteminacy-membership function $\boldsymbol{\nu_A(x)}$ and falsity-membership function $\boldsymbol{\omega_A(x)}$. For each point **x in X, we have that $\boldsymbol{\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]}$.**
For two IVNS , A_{IVNS} ={ <x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | $x \in X$ }
And B_{IVNS} ={ <x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | $x \in X$ } the two relations are defined as follows:
(1) $A_{\text{IVNS}} \subseteq B_{\text{IVNS}}$ if and only if $\mu_A^L(x) \leq \mu_B^L(x)$,$\mu_A^U(x) \leq \mu_B^U(x)$,$\nu_A^L(x) \geq \nu_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$, $\omega_A^L(x) \geq \omega_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$
(2) $A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$
As an illustration ,let us consider the following example.
Example 2.4. Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x2 characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness,
the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,
A = {< x_1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x_2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x_3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 2.5.
Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (K, A) is called a soft set over U, where K is a mapping given by $K : A \rightarrow P(U)$.
As an illustration, let us consider the following example.
Example 2.6 .
Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K,A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:
A={e_1,e_2,e_3,e_4,e_5};
$K(e_1)$ = {h_2, h_3, h_5}, $K(e_2)$ = {h_2, h_4}, $K(e_3)$ = {h_1}, $K(e_4)$ = U, $K(e_5)$ = {h_3, h_5}.

Definition 2.7 .
Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IVNS(U) denotes the

set of all interval neutrosophic subsets of U. The collection (K,A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K : A → IVNS(U).
The interval neutrosophic soft set defined over an universe is denoted by INSS.
To illustrate let us consider the following example:
Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is a interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, in the green surroundings, moderate, expensive }. In this case, to define a interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1,h_2,h_3,h_4,h_5\}$ and the set of parameters A = $\{e_1,e_2,e_3,e_4\}$, where each e_i is a specific criterion for houses:
e_1 stands for 'beautiful',
e_2 stands for 'costly',
e_3 stands for 'in the green surroundings',
e_4 stands for 'moderate',
Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(costly)={< b_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.
K(in the green surroundings)= {< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< b_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(moderate)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

Definition 2.8.
Let U be an initial universe and (F,A) and (G,B) be two interval valued neutrosophic soft set . Then a relation between them is defined as a pair (H, AxB), where H is mapping given by H: AxB→IVNS(U). This is called an interval valued neutrosophic soft sets relation (IVNSS-relation for short).the collection of relations on interval valued neutrosophic soft sets on Ax Bover U is denoted by $\sigma_U(A\text{x } B)$.

Defintion 2.9. Let P, Q ∈ $\sigma_U(Ax\ B)$ and the ordre of their relational matrices are same. Then P ⊆ Q if H $(e_{j,}e_j)$ ⊆ J $(e_{j,}e_j)$ for $(e_{j,}e_j)$ ∈ A x B where P=(H, A x B) and Q = (J, A x B)
Example:
P

U	$(e_1\ ,e_2)$	$(e_1\ ,e_4)$	$(e_3\ ,e_2)$	$(e_3\ ,e_4)$
h_1	([0.2, 0.3],[0.2, 0.3],[0.4, 0.5])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Q

U	$(e_1\ ,e_2)$	$(e_1\ ,e_4)$	$(e_3\ ,e_2)$	$(e_3\ ,e_4)$
h_1	([0.3, 0.4],[0, 0],[0, 0])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])

h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Definition 2.10**.**
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then a null relation between them is denoted by 0_U and is defined as 0_U =(H_O , A xB) where H_O (e_i,e_j)={<h_k , [0, 0],[1, 1],[1, 1]>; h_k ∈ U} for (e_i,e_j) ∈ A xB.
Example. Consider the interval valued neutrosophic soft sets (F, A) and (G, B). Then a null relation between them is given by

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_2	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_3	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_4	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])

Remark. It can be easily seen that P ∪ 0_U =P and P ∩ 0_U =0_U for any P ∈ $\sigma_U(Ax\ B)$
Definition 2.11.
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then an absolute relation between them is denoted by I_U and is defined as I_U =(H_I , A xB) where H_I (e_i,e_j)={<h_k , [1, 1],[0, 0],[0, 0]>; h_k ∈ U} for (e_i,e_j) ∈ A xB.

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_2	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_3	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_4	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])

Definition.2.12 Let P ∈ $\sigma_U(Ax\ B)$, P= (H, AxB) ,Q = (J, AxB) and the order of their relational matrices are same.Then we define

(i) P ∪ Q= (H ∘J, AxB) where H∘ J :AxB →IVNS(U) is defined as (H ∘J)(e_i,e_j)= H(e_i,e_j) ∨ J(e_j,e_j) for (e_i,e_j) ∈ A x B, where ∨ denotes the interval valued neutrosophic union.

(ii) P ∩ Q= (H ■J, AxB) where H■J :AxB →IVNS(U) is defined as (H■J)(e_i,e_j)= H(e_i,e_j) ∧ J(e_i,e_j) for (e_j,e_j) ∈ A x B, where ∧ denotes the interval valued neutrosophic intersection

(iii) P^c= (∼H, AxB) , where ∼H :AxB →IVNS(U) is defined as ∼H(e_i,e_j)=[H(e_i,e_j)] ^{c} for (e_i,e_j) ∈ A x B, where c denotes the interval valued neutrosophic complement.

Defintion.2.13.
Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R_*X and R^* X and are defined by

$R_*X=\{x\in U: [x]_R \subseteq X\}$,
$R^*X=\{ x\in U: [x]_R \cap X \neq \}$.
Now if $R_*X = R^* X$, then X is called definable; otherwise X is called a rough set.

3-Lower and upper soft interval valued neutrosophic rough approximations of an IVNSS-relation

Defntion 3.1 .Let R $\in \sigma_U(A\text{x } A)$ and R=(H, Ax A). Let Θ=(f,B) be an interval valued neutrosophic soft set over U and S= (U, Θ) be the soft interval valued neutrosophic approximation space. Then the lower and upper soft interval valued neutrosophic rough approximations of R with respect to S are denoted by $\text{Lwr}_S(R)$and $\text{Upr}_S(R)$ respectively, which are IVNSS- relations over AxB in U given by:
$\text{Lwr}_S(R)$= (J, A xB) and $\text{Upr}_S(R)$ =(K, A xB)
J(e_i ,e_k**) ={<x, [**$\wedge_{e_j\in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))$],

[$\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))$],
[$\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))$] :x ∈ U}.

K(e_i ,e_k**) ={<x, [**$\wedge_{e_i\in A}(\inf \mu_{H(e_i,e_j)}(x) \vee \inf \mu_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \vee \sup \mu_{f(e_k)}(x))$],

[$\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \wedge \inf \nu_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \wedge \sup \nu_{f(e_k)}(x))$],
[$\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \wedge \inf \omega_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \wedge \sup \omega_{f(e_k)}(x))$] :x ∈ U}.

For $e_i \in A$, $e_K \in B$

Theorem 3.2. Let be an interval valued neutrosophic soft over U and S = (U,Θ) be the soft approximation space. Let R_1 , $R_2 \in \sigma_U(A\text{x } A)$ and R_1=(G,Ax A) and R_2=(H,Ax A).Then

(i) $\text{Lwr}_S(0_U)= 0_U$
(ii) $\text{Lwr}_S(1_U)= 1_U$
(iii) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \text{Lwr}_S(\boldsymbol{R_1}) \subseteq \text{Lwr}_S(\boldsymbol{R_2})$
(iv) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \text{Upr}_S(\boldsymbol{R_1}) \subseteq \text{Upr}_S(\boldsymbol{R_2}$
(v) $\text{Lwr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \text{Lwr}_S(\boldsymbol{R_1}) \cap \text{Lwr}_S(\boldsymbol{R_2})$
(vi) $\text{Upr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \text{Upr}_S(\boldsymbol{R_1}) \cap \text{Upr}_S(\boldsymbol{R_2})$
(vii) $\text{Lwr}_S(\boldsymbol{R_1}) \cup \text{Lwr}_S(\boldsymbol{R_2}) \subseteq \text{Lwr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$
(viii) $\text{Upr}_S(\boldsymbol{R_1}) \cup \text{Upr}_S(\boldsymbol{R_2}) \subseteq \text{Upr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$

Proof. (i) –(iv) are straight forward.
Let Lwrs($R_1 \cap R_2$) =(S, Ax B).Then for $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB , we have

S($\boldsymbol{e_i}, \boldsymbol{e_k}$) ={<x, [$\wedge_{e_j\in A}(\inf \mu_{G\circ H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x))$, $\wedge_{e_j\in A}(\sup \mu_{G\circ H(e_i,e_j)}(x) \wedge$
$\sup \mu_{f(e_k)}(x))$],
[$\wedge_{e_j\in A}(\inf \nu_{G\circ H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x))$, $\wedge_{e_j\in A}(\sup \nu_{G\circ H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))$],

[$\wedge_{e_j\in A}(\inf \omega_{G\circ H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)) ,\wedge_{e_j\in A}(\sup \omega_{G\circ H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))$] :x ∈ U}

$=\{<x, [\wedge_{e_j\in A}(\min(\inf \mu_{G(e_i,e_j)}(x), \inf \mu_{H(e_i,e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x))$
$, \wedge_{e_j\in A}(\min(\sup \mu_{G(e_i,e_j)}(x), \sup \mu_{H(e_i,e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x))],$

$[\wedge_{e_j\in A}(\max(\inf \nu_{G(e_i,e_j)}(x), \inf \nu_{H(e_i,e_j)}(x)) \vee \inf \nu_{f(e_k)}(x))$
$, \wedge_{e_j\in A}(\max(\sup \nu_{G(e_i,e_j)}(x), \sup \nu_{H(e_i,e_j)}(x)) \vee \sup \nu_{f(e_k)}(x))],$

$[\wedge_{e_j\in A}(\max(\inf \omega_{G(e_i,e_j)}(x), \inf \omega_{H(e_i,e_j)}(x)) \vee \inf \omega_{f(e_k)}(x))$
$, \wedge_{e_j\in A}(\max(\sup \omega_{G(e_i,e_j)}(x), \sup \omega_{H(e_i,e_j)}(x)) \vee \sup \omega_{f(e_k)}(x))] : x \in U\}$

Also for $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cap \mathrm{Lwr}_S(\boldsymbol{R_2})$ =(Z,A x B) and $(\boldsymbol{e_i}, \boldsymbol{e_K}) \in A \times B$,we have ,
$Z(\boldsymbol{e_i}, \boldsymbol{e_K}) = \{<x, [\,\mathrm{Min}\,(\wedge_{e_j\in A}(\inf \mu_{G(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge$
$\inf \mu_{f(e_k)}(x))\,),\ \mathrm{Min}(\wedge_{e_j\in A}(\sup \mu_{G(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge$
$\sup \mu_{f(e_k)}(x))\,)],$
$[\mathrm{Max}\,(\wedge_{e_j\in A}(\inf \nu_{G(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x))\,),$
$\mathrm{Max}(\wedge_{e_j\in A}(\sup \nu_{G(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))\,)],$

$[\mathrm{Max}\,(\wedge_{e_j\in A}(\inf \omega_{G(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x))\,),$
$\mathrm{Max}(\wedge_{e_j\in A}(\sup \omega_{G(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))\,)] : x \in$
$U\}$

Now since $\min(\inf \mu_{G(e_i,e_j)}, \inf \mu_{H(e_i,e_j)}(x)) \leq \inf \mu_{G(e_i,e_j)}(x)$ and
$\min(\inf \mu_{G(e_i,e_j)}, \inf \mu_{H(e_i,e_j)}(x)) \leq \inf \mu_{H(e_i,e_j)}(x)$ we have
$\wedge_{e_j\in A}(\min(\inf \mu_{G(e_i,e_j)}(x), \inf \mu_{H(e_i,e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j\in A}(\inf \mu_{G(e_i,e_j)}(x) \wedge$
$\inf \mu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x))\,).$
Similarly we can get
$\wedge_{e_j\in A}(\min(\sup \mu_{G(e_i,e_j)}(x), \sup \mu_{H(e_i,e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j\in A}(\sup \mu_{G(e_i,e_j)}(x) \wedge$
$\sup \mu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))\,).$

Again as $\max(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \geq \inf \nu_{G(e_i,e_j)}(x)$,and
$\max(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \geq \inf \nu_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j\in A}(\max(\inf \nu_{G(e_i,e_j)}(x), \inf \nu_{H(e_i,e_j)}(x)) \vee \inf \nu_{f(e_k)}(x)) \geq \mathrm{Max}\,(\wedge_{e_j\in A}(\inf \nu_{G(e_i,e_j)}(x) \vee$
$\inf \nu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x))\,).$
Similarly we can get

$\wedge_{e_j\in A}(\max(\sup \nu_{G(e_i,e_j)}(x), \sup \nu_{H(e_i,e_j)}(x)) \vee \sup \nu_{f(e_k)}(x)) \geq \mathrm{Max}\,(\wedge_{e_j\in A}(\sup \nu_{G(e_i,e_j)}(x) \vee$
$\sup \nu_{f(e_k)}(x)),\ \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))\,).$
Again as $\max(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \geq \inf \omega_{G(e_i,e_j)}(x)$,and
$\max(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \geq \inf \omega_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\max(\inf \omega_{\mathbf{G}(e_i, e_j)}(x), \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)) \geq \operatorname{Max}(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \omega_{\mathbf{G}(e_i, e_j)}(x), \sup \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x)) \geq \operatorname{Max}(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))$.

Consequently, $\mathrm{Lwr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \mathrm{Lwr}_S(\boldsymbol{R_1}) \cap \mathrm{Lwr}_S(\boldsymbol{R_2})$

(vi) Proof is similar to (v)

(vii) Let $\mathrm{Lwr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$ **=(S, A xB).Then for** $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB , we have

$S(\boldsymbol{e_i}, \boldsymbol{e_k})=\{<x, [\wedge_{e_j \in A}(\inf \mu_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\inf \nu_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{G}\blacksquare\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x))] : x \in U\}$

$=\{<x, [\wedge_{e_j \in A}(\max(\inf \mu_{\mathbf{G}(e_i, e_j)}(x), \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\max(\sup \mu_{\mathbf{G}(e_i, e_j)}(x), \sup \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \sup \mu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\min(\inf \nu_{\mathbf{G}(e_i, e_j)}(x), \inf \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\min(\sup \nu_{\mathbf{G}(e_i, e_j)}(x), \sup \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \nu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\min(\inf \omega_{\mathbf{G}(e_i, e_j)}(x), \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\min(\sup \omega_{\mathbf{G}(e_i, e_j)}(x), \sup \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x))] : x \in U\}$

Also for $\mathrm{Lwrs}(\boldsymbol{R_1}) \cup \mathrm{Lwrs}(\boldsymbol{R_2})$ **= (Z, AxB) and** $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB ,we have ,

$Z(\boldsymbol{e_i}, \boldsymbol{e_K})= \{<x, [\operatorname{Max}(\wedge_{e_j \in A}(\inf \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x))), \operatorname{Max}(\wedge_{e_j \in A}(\sup \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)))]$,

$[\operatorname{Min}(\wedge_{e_j \in A}(\inf \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x))), \operatorname{Min}(\wedge_{e_j \in A}(\sup \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)))]$,

$[\operatorname{Min}(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x))), \operatorname{Min}(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))] : x \in U\}$

Now since $\max(\inf \mu_{\mathbf{G}(e_i, e_j)}, \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \geq \inf \mu_{\mathbf{G}(e_i, e_j)}(x)$ and $\max(\inf \mu_{\mathbf{G}(e_i, e_j)}, \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \geq \inf \mu_{\mathbf{H}(e_i, e_j)}(x)$ we have

$\wedge_{e_j \in A}(\max(\inf \mu_{\mathbf{G}(e_i, e_j)}(x), \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)) \geq \max(\wedge_{e_j \in A}(\inf \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \mu_{\mathbf{G}(e_i, e_j)}(x), \sup \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)) \geq \max(\wedge_{e_j \in A}(\sup \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)))$.

Again as $\min(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \leq \inf \nu_{G(e_i,e_j)}(x)$,and
$\min(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \leq \inf \nu_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \nu_{G(e_i,e_j)}(x), \inf \nu_{H(e_i,e_j)}(x)) \vee \inf \nu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \nu_{G(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \nu_{G(e_i,e_j)}(x), \sup \nu_{H(e_i,e_j)}(x)) \vee \sup \nu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \nu_{G(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)))$.

Again as $\min(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \leq \inf \omega_{G(e_i,e_j)}(x)$,and
$\min(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \leq \inf \omega_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \omega_{G(e_i,e_j)}(x), \inf \omega_{H(e_i,e_j)}(x)) \vee \inf \omega_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \omega_{G(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \omega_{G(e_i,e_j)}(x), \sup \omega_{H(e_i,e_j)}(x)) \vee \sup \omega_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \omega_{G(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x)))$.

Consequently $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cup \mathrm{Lwr}_S(\boldsymbol{R_2}) \subseteq \mathrm{Lwrs}(\boldsymbol{R_1} \cap \boldsymbol{R_2})$
(vii) Proof is similar to (vii).

Conclusion

In the present paper we extend the concept of Lower and upper soft interval valued intuitionstic fuzzy rough approximations of an IVIFSS-relation to the case IVNSS and investigated some of their properties.

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