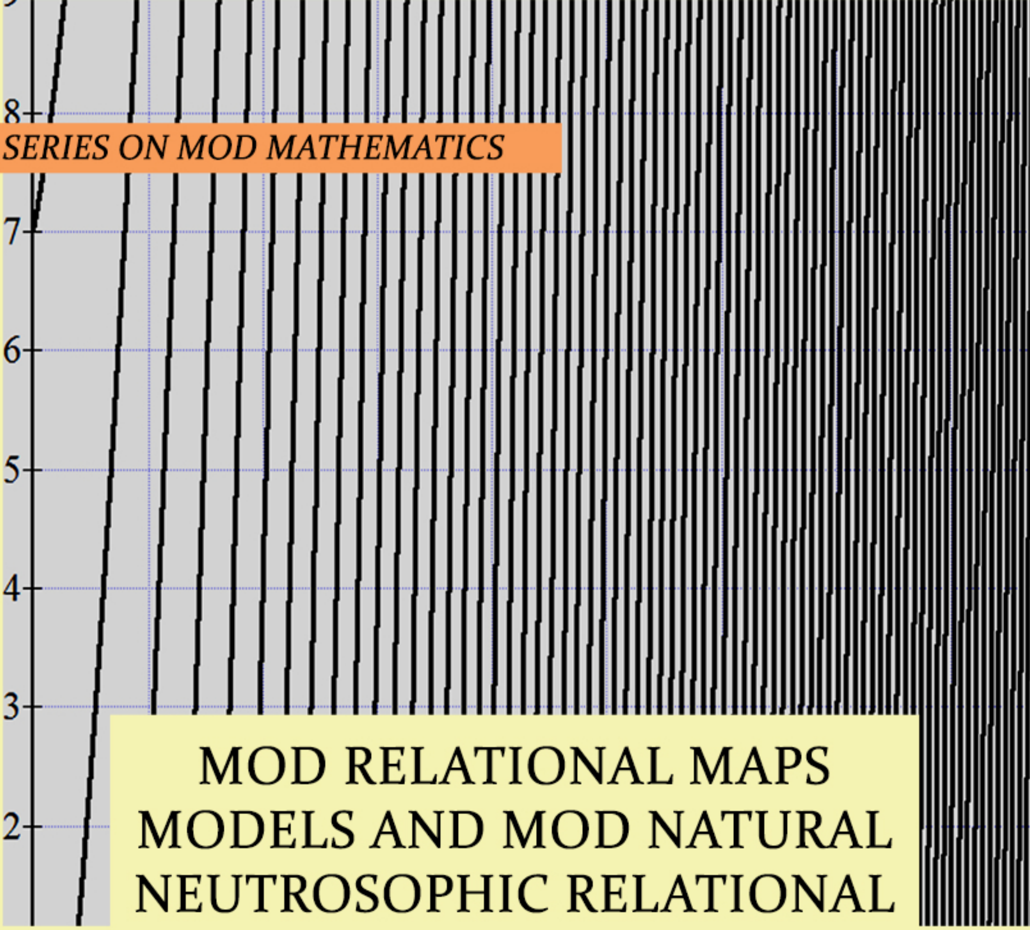
SERIES ON MOD MATHEMATICS



MOD RELATIONAL MAPS MODELS AND MOD NATURAL NEUTROSOPHIC RELATIONAL MAPS MODELS

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CONTENTS

Preface 4

Chapter One
INTRODUCTION 9

Chapter Two
MOD RELATIONAL MAPS MODELS 13

Chapter Three
MOD-NATURAL NEUTROSOPHIC RELATIONAL MAPS MODEL AND THEIR PROPERTIES 99

Chapter Four
MOD INTERVAL RELATIONAL MAPS 175

Chapter Five
SUGGESTED PROBLEMS 247

FURTHER READING 265

INDEX 273

ABOUT THE AUTHORS 278

PREFACE

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set. Atanassov introduced the degree of non membership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set. Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set on three components (t,i,f) = (truth, indeterminacy, falsehood).

The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book. Etymologically, "neutro-sophy" (noun) [French *neutre* <Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] means knowledge of neutral thought. While "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$ with not necessarily any connection between them. For software engineering proposals the classical unit interval [0, 1] may be used. T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval [0, 1] is used. For single valued neutrosophic logic, the sum of the components is:

$0 \leq t+i+f \leq 3$ when all three components are independent;
$0 \leq t+i+f \leq 2$ when two components are dependent, while the third one is independent from them;
$0 \leq t+i+f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1). If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

In 2013 Smarandache refined the neutrosophic set to n components: T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ... See http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf .

Neutrosophy <*philosophy*> (From Latin "neuter" - neutral, Greek "sophia" - skill/wisdom) A branch of philosophy, introduced by Florentin Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Neutrosophy considers a proposition, theory, event, concept, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A".

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics. {From: *The Free Online Dictionary of Computing*, is edited by Denis Howe from England. Neutrosophy is an extension of the Dialectics.}

The most important books and papers in the development of neutrosophics

1995-1998 - Introduction of neutrosophic set/ logic/ probability/ statistics; generalization of dialectics to neutrosophy; http://fs.gallup.unm.edu/eBook-neutrosophics4.pdf (4th edition)

2003 – Introduction of neutrosophic numbers (a+bI, where I = indeterminacy).
2003 – Introduction of I-neutrosophic algebraic structures.
2003 – *i*ntroduction to neutrosophic cognitive maps.
http://fs.gallup.unm.edu/NCMs.pdf

2005 - Introduction of interval neutrosophic set/logic.
http://fs.gallup.unm.edu/INSL.pdf

2009 – Introduction of N-norm and N-conorm
http://fs.gallup.unm.edu/N-normN-conorm.pdf

2013 - Development of neutrosophic probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 - Refinement of components (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)
http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic.pdf

2014 – Introduction of the law of included multiple middle
(<A>; <neut1A>, <neut2A>, …; <antiA>)
http://fs.gallup.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - Development of neutrosophic statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)
http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf

2015 - Introduction of neutrosophic precalculus and neutrosophic calculus
http://fs.gallup.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 – Refined neutrosophic numbers ($a+ b_1I_1 + b_2I_2 + \dots + b_nI_n$), where $I_1, I_2, \dots, I_n$ are subindeterminacies of indeterminacy I;
2015 – Neutrosophic graphs;
2015 - Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, *(t, i, f)*-Neutrosophic Structures, *I*-Neutrosophic Structures, Refined Literal Indeterminacy, Multiplication Law of Subindeterminacies:
http://fs.gallup.unm.edu/SymbolicNeutrosophicTheory.pdf

2015 – Introduction of the subindeterminacies of the form

$I_0^n = \frac{k}{0}$, for $k \in \{0, 1, 2, \dots, n-1\}$, into the ring of modulo integers Z_n, are called *natural neutrosophic zeros*
http://fs.gallup.unm.edu/MODNeutrosophicNumbers.pdf

In this book authors for the first time construct a MOD Relational Maps model analogous to Fuzzy Relational Maps (FRMs) model or Neutrosophic Relational Maps (NRMs) model using the MOD rectangular or relational matrices with entries from Z_n or Z_n^I $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup g\rangle_I$ or $C(Z_n)$ or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup I\rangle_I$ and so on.

The advantage of using these models is that we are sure to get the MOD fixed point pair or a MOD limit cycle pair after a finite number of iterations. However we as in case of FRMs or NRMs need not at each stage threshold the resultant state vectors. We do updating when the on state which we started becomes zero. This is yet another advantage of using these newly constructed models.

Finally the resultant state vector pair can be real MOD values or finite complex or neutrosophic or a dual number or a special quasi dual number or special dual like number.

However we can use Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ and so on. We can have various types of natural neutrosophic numbers or indeterminacies as resultant pairs.

This will address all practical problems as we cannot always get real values as resultant it can be anything or in some cases it may be unpredictable.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

In this chapter we just indicate the references of the concepts which we have used in this book.

This book basically builds the notion of MOD Relational Maps (MOD RMs) models using

$$Z_n \text{ or } \langle Z_n \cup I\rangle \text{ or } \langle Z_n \cup g\rangle \text{ or } C(Z_n) \text{ or } \langle Z_n \cup h\rangle \text{ or } \langle Z_n \cup k\rangle.$$

MOD Relational Maps model also built using Z_n^I or $R^I(n)$ or $C^I(Z_n)$ and so on.

These two models are different and distinct as the former uses only modulo integers but the later uses the MOD natural neutrosophic numbers generated by division in Z_n, $C(Z_n)$ and so on.

For more about the concept of MOD natural neutrosophic elements in particular and for properties of MOD structures in general please refer [57-66].

Further these new models are obtained analogous to Fuzzy Relational Maps (FRMs) model and Neutrosophic Relational Maps (NRMs) model [25].

In the case of FRMs or NRMs we can get the nodes as 0 or 1 or 0 or 1 or I respectively.

But in case of MODRMs we can get the nodes from

$$Z_n \text{ or } \langle Z_n \cup g\rangle \text{ or } C(Z_n)$$

$$\text{or } \langle Z_n \cup I\rangle \text{ or } \langle Z_n \cup k\rangle \text{ or } \langle Z_n \cup k\rangle.$$

Such type of study is both new and innovative.

These models need not undertake the operation of thresholding for that is taken care of by modulo integer n.

The MOD rectangular matrices as operators have been elaborately dealt with in [66]. In fact in that book these MOD rectangular matrix operator contribute to special type of fixed point pair.

This study is also carried out in [66]. The special features enjoyed by this new MOD Relational Maps model is carried out.

Next we study analogously the MOD Interval Relational Maps model using

$$[0, n), \langle [0, n) \cup I\rangle$$

$$\text{or } C([0, n)$$

$$\text{or } \langle [0, n) \cup g\rangle;$$

$$g^2 = 0 \text{ or } \langle [0,n \cup h\rangle,$$

$$h^2 = h \text{ or } \langle [0,n \cup k \rangle;$$

$$k^2 = (n-1)k.$$

Each of these MOD intervals behave in a distinct way when used as MOD Interval Relational Maps Models.

The limitations of this model is we can have only one type of operation that is thresholding and updating at each stage for us to arrive at a MOD resultant after a finite number iterations.

Next we study the MOD natural neutrosophic numbers Relational Maps model using

$$^{I}[0,n) \text{ or } \langle [0,n \cup I \rangle_I$$

$$\text{or } C^I(0, n)) \text{ or } ([0, n))$$

$$\text{or } \langle [0,n \cup g \rangle_I \text{ or } \langle [0,n \cup g \rangle_I$$

$$\text{or } \langle [0,n \cup h \rangle_I \text{ or } \langle [0,n \cup k \rangle_I.$$

We see in case of using MOD Interval Relational Maps model the initial state vectors nodes can only take

$$\{0, 1\} \text{ or } \{0, 1, g\} \text{ or } \{0, 1, h\} \text{ or } \{0, 1, I\} \text{ or } \{0, 1, i_F\}$$

and so on.

But in case of MOD natural neutrosophic Interval Relational Maps model the MOD resultant nodes can take values as

$$I_t^n \text{ or } I_t^I \text{ or } I_t^g \text{ or } I_t^h \text{ or } I_t^c \text{ or } I_t^k$$

where t varies over a finite index and all such elements form a semigroup under $\times$.

Unless this is guaranteed we will not be in a position to arrive at a MOD resultant after a finite number of iterations.

This is also discussed in the forth chapter of this book.

We have suggest several problems in this book some of which can be considered as open conjectures. On the whole the MOD Relational Maps Models is an innovative piece of work which will certainly in due course of time find lots of applications.

Chapter Two

MOD RELATIONAL MAPS MODELS

In this chapter for the first time we define the new notion of MOD Relational Maps models analogous to FRMs models.

FRMs model can be realized as the special type of generalization of the FCMs when the nodes / concepts associated with it can be divided into two classes as domain space and range space. The MOD Relational Maps (MODRMs) model can be realized as a generalization of MOD Cognitive Maps model [69]. To build them we need the concept of MOD directed relational graphs and MOD relational matrices which we will describe with examples.

Throughout this chapter $Z_n = \{0, 1, 2, \ldots, n - 1\}$ denote MOD integers $\langle Z_n \cup I\rangle = \{a + bI / a, b \in Z_n, I^2 = I\}$ will be redefined as MOD neutrosophic integers,

$\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$ is described as MOD dual number, $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ is defined as MOD special dual like numbers,

$\langle Z_n \cup k\rangle = \{a + bk / a, b \in Z_n, k^2 = (n - 1)k\}$ is defined as the MOD special quasi dual numbers and

$C(Z_n) = \{a + bi_F \,/\, a, b \in Z_n,\ i_F^2 = n - 1\}$ is the MOD finite complex modulo integers.

Example 2.1: Let G be the MOD relational directed graph. G is a bipartite graph will also be known as MOD bipartite relational directed graph with edge weights from Z_n.

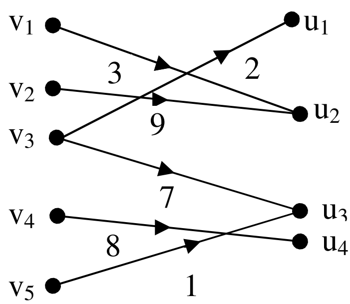


Figure 2.1

Example 2.2: Let G be MOD bipartite directed graph with edge weights from Z_{11}.

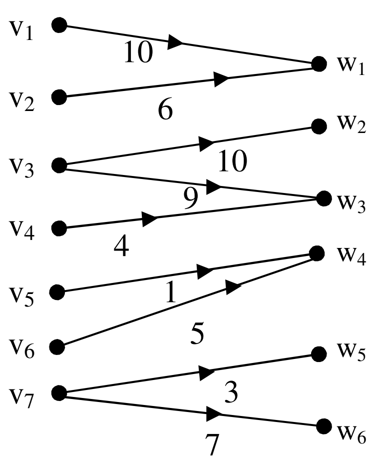


Figure 2.2

Now we proceed onto give examples of MOD rectangular matrices.

Example 2.3: Let

$$M = \begin{bmatrix} 0 & 2 & 4 \\ 5 & 6 & 8 \\ 11 & 12 & 16 \\ 17 & 18 & 0 \\ 1 & 3 & 7 \\ 6 & 10 & 15 \end{bmatrix}$$

with entries from Z_{19}.

We call M as the MOD rectangular matrix.

Example 2.4: Let

$$W = \begin{bmatrix} 0 & 3 & 4 & 5 & 7 & 1 \\ 5 & 0 & 0 & 9 & 0 & 4 \\ 0 & 1 & 2 & 0 & 3 & 0 \\ 4 & 2 & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the MOD rectangular matrix with entries from Z_{10}.

Now we define MOD directed bipartite graph and MOD rectangular matrices in the following.

Let G be a directed bipartite graph with edge weights from Z_n, then we define G to be a MOD directed bipartite graph.

Let $M = (m_{ij})_{m \times n}$ be a $m \times n$ matrix ($m \neq n$) with entries from Z_s then we define M to be a MOD rectangular matrix.

Now having seen examples we show how MOD adjacency matrix can be got related with the MOD bipartite directed graph.

Example 2.5: Let G be a MOD directed bipartite graph with edge weight from Z_6 given by the following figure.

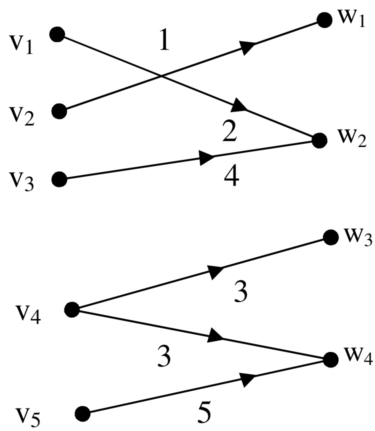


Figure 2.3

The adjacency matrix M associated with the graph G is as follows;

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix} \end{array}.$$

Example 2.6: Let H be the MOD directed bipartite graph with edge weights from Z_{10} given by the following Figure 2.4.

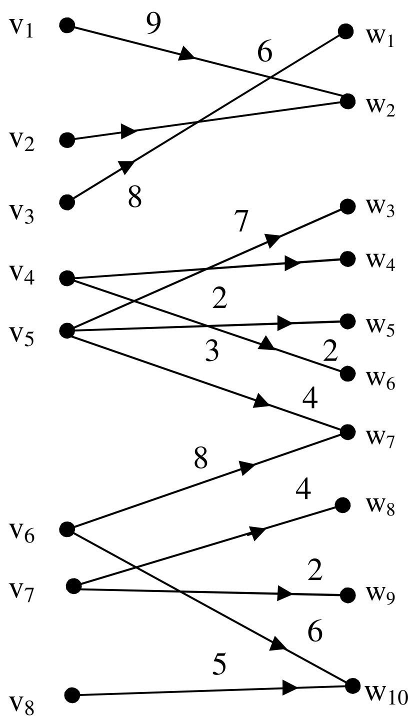


Figure 2.4

Let S be the MOD adjacency matrix associated with the MOD bipartite graph H.

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 & w_9 & w_{10} \end{array} \\ \begin{bmatrix} 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 3 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix} \end{array}.$$

Now we need to define only a special type of operations on these MOD rectangular matrices.

Already in book [66] MOD realized fixed points and MOD realized limit cycles are defined and described.

However as the idea is new and as we have no books only two [65, 66] so we just give a few examples of them.

Example 2.7: Let B be a MOD rectangular matrix with entries from Z_{12}.

$$B = \begin{bmatrix} 0 & 2 & 4 & 1 & 0 & 0 & 0 & 3 & 1 & 2 \\ 1 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 6 & 0 & 2 & 0 & 0 & 0 & 3 & 0 & 0 \end{bmatrix}.$$

$X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1\}; 1 \leq i \leq 5\}$ is defined as the MOD domain space of initial state vectors associated with B.

$Y = \{(y_1, y_2, \ldots, y_{10}) / y_i \in \{0, 1\}; 1 \leq i \leq 10\}$ is defined as the MOD range space of initial state vectors associated with B.

We now define special type of operations using elements X and Y and the MOD matrix B.

Let $x = (1, 0, 0, 0, 0) \in X$ to find the effect of x on B.

$xB = (0, 2, 4, 1, 0, 0, 0, 0, 3\ 1\ 2) = y_1$;
$y_1B^t = (11, 0, 3, 10, 11) = x_1$;
$x_1B = (4, 4, 11, 9, 0, 0, 6, 6, 3, 1) = y_2$;
$y_2B^t = (0, 4, 0, 10, 0) = x_2$;
$x_2B = (8, 0, 0, 0, 8, 4, 0, 0, 4, 0) = y_3$;
$y_3B^t = (4, 4, 0, 0, 0) = x_3$;
$x_3B = (4, 8, 4, 4, 8, 4, 0, 0, 4, 8) = y_4$;
$y_4B^t = (8, 4, 0, 8, 8) = x_4$;
$x_4B = (0, 4, 8, 0, 8, 4, 0, 0, 4, 4) = y_5$;
$y_5B^t = (4, 8, 0, 4, 0)$ and so on.

We are sure after a finite number of iterations to arrive at a MOD realized fixed point pair or a MOD realized limit cycle pair.

Let $x_2 = (0, 1, 0, 0, 0) \in X$; to find the effect of x_2 on B.

$x_2B = (1, 0, 0, 0, 2, 1, 0, 0, 0, 0) = y_1$;
$y_1B^t = (0, 6, 0, 4, 0) = x_3$;
$x_3B = (10, 0, 0, 0, 0, 6, 0, 0, 4, 0) = y_2$;
$y_2B^t = (4, 4, 0, 8, 0) = x_4$;
$x_4B = (0, 8, 4, 4, 8, 4, 0, 0, 0, 8) = y_3$;
$y_3B^t = (4, 8, 0, 0, 8) = x_5$;
$x_5B = (8, 8, 4, 8, 4, 8, 0, 0, 4, 8) = y_4$;
$y_4B^t = (0, 0, 0, 0, 4) = x_6$;
$x_6B = (0, 0, 0, 8, 0, 0, 0, 0, 0, 0) = y_5$;
$y_5B^t = (8, 0, 0, 0, 4) = x_7$;
$x_7B = (0, 4, 8, 0, 0, 0, 0, 0, 8, 4) = y_6$;
$y_6B^t = (1, 0, 0, 0, 8, 0) = x_8$;
$x_8B = (8, 8, 4, 10, 0, 0, 0, 6, 6, 8) = y_7$;
$y_7B^t = (4, 8, 0, 8, 6) = x_9$;
$x_9B = (4, 8, 4, 4, 4, 8, 0, 0, 0, 8) = y_8$;
$y_8B^t = (4, 8, 0, 4, 8) = x_{10}$;
$x_{10}B = (0, 8, 4, 8, 4, 8, 0, 0, 8) = y_9$;
$y_9B^t = (4, 4, 4, 8, 4) = x_{11}$;

$x_{11}B = (0, 8, 8, 0, 8, 4, 8, 0, 0, 0) = y_{10}$;
$y_{10}B^t = (0, 8, 0, 0, 0) = x_{12}$;
$x_{12}B = (8, 0, 0, 0, 4, 8, 0, 0, 0, 0) = y_{11}$;
$y_{11}B^t = (0, 0, 0, 8, 0) = x_{13}$;
$x_{13}B = (8, 0, 0, 0, 0, 0, 0, 0, 8, 0) = y_{12}$;
$y_{12}B^t = (8, 8, 0, 4, 0) = x_{14}$;
$x_{14}B = (0, 4, 8, 8, 4, 8, 0, 0, 0, 4) = y_{13}$;
$y_{13}B^t = (8, 4, 0, 0, 4) = x_{15}$;
$x_{15}B = (4, 4, 8, 4, 8, 4, 0, 0, 8, 4) = y_{14}$;
$y_{14}B^t = (0, 0, 0, 0, 8) = x_{16}$;
$x_{16}B = (0, 0, 0, 4, 0, 0, 0, 0, 0, 0) = y_{15}$;
$y_{15}B^t = (4, 0, 0, 0, 8) = x_{17}$;
$x_{17}B = (0, 8, 4, 8, 0, 0, 0, 0, 4, 8) = y_{16}$;
$y_{16}B^t = (0, 0, 0, 4, 4) = x_{18}$;
$x_{18}B = (4, 0, 0, 8, 0, 0, 0, 0, 4, 0) = y_{17}$;
$y_{17}B^t = (0, 4, 0, 8, 0) = x_{19}$;
$x_{19}B = (0, 0, 0, 0, 8, 4, 0, 0, 8, 0) = y_{18}$;
$y_{18}B^t = (8, 8, 0, 8, 0) = x_{20}$;
and so on.

We are sure to arrive at a MOD realized fixed point pair or a MOD realized limit cycle pair.

Let $y = (0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \in Y$.

To find the effect of y on B;

$yB^t = (2, 0, 1, 0, 0) = x_1$;
$x_1B = (0, 4, 9, 2, 0, 0, 2, 6, 2, 5) = y_1$;
$y_1B^t = (4, 0, 6, 8, 6) = x_2$;
$x_2B = (8, 8, 10, 4, 0, 0, 0, 6, 0, 2) = y_2$;
$y_2B^t = (10, 8, 0, 8, 2) = x_3$;
$x_3B = (4, 8, 6, 2, 4, 8, 0, 0, 6, 6) = y_3$ and so on.

We are sure after a finite number of iterations to get at the MOD resultant to be a MOD realized fixed point pair or a MOD realized limit cycle pair.

Here in this type of operations we do not update or threshold the resultants at each stage or in the final state.

We will illustrate this situation by some examples.

Example 2.8: Let

$$M = \begin{bmatrix} 3 & 0 & 1 & 2 \\ 0 & 0 & 0 & 4 \\ 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$$

be the MOD rectangular matrix, which we may also address as a MOD rectangular matrix operator with entries from Z_6.

Let $X= \{(a_1, a_2, \ldots, a_7) / a_i \in Z_6; 1 \leq i \leq 7\}$ and

$Y = \{(a_1, a_2, a_3, a_4) / a_i \in Z_6; 1 \leq i \leq 4\}$ be the MOD state vectors of the domain and range space respectively associated with the MOD matrix operator M.

This is yet another type of operation using the MOD matrix operator M.

Let $x = (1, 0, 0, 0, \ldots, 0) \in X$.

To find the effect of x on the MOD matrix operator M.

$xM = (3, 0, 1, 2) = y_1;$
$y_1M^t = (2, 2, 3, 0, 2, 0, 0) = x_1;$
$x_1M = (3, 0, 0, 0) = y_2;$
$y_2M^t = (3, 0, 3, 0, 0, 0, 0) = x_2;$
$x_2M = (0, 0, 3, 0) = y_3;$

$y_3M^t = (3, 0, 0, 0, 0, 0, 0) = x_3$;
$x_3M = (3, 0, 3, 0) = y_4$;
$y_4M^t = (0, 0, 3, 0, 0, 0, 0) = x_4$;
$x_4M = (3, 0, 0, 0) = y_5$ $(=y_2)$.

Thus the MOD resultant is MOD realized limit cycle pair given by {(3, 0, 3, 0, 0, 0, 0), (3, 0, 0, 0)} --- I

Let x = (2, 0, 0, 0, 0, 0, 0) $\in$ X to find the effect of x on the MOD matrix operator M.

$xM = (0, 0, 2, 4) = y_1$;
$y_1M^t = (4, 4, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 4, 0) = y_2$;
$y_2M^t = (4, 0, 0, 0, 2, 0, 0) = x_2$;
$x_2M = (0, 0, 2, 2) = y_3$;
$y_3M^t = (0, 2, 0, 0, 4, 0, 0) = x_3$;
$x_3M = (0, 0, 2, 2) = y_4$ $(=y_3)$.

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 2, 0, 0, 4, 0, 0), (0, 0, 2, 2)} II

We see when x = (1, 0, 0, …, 0) the MOD resultant is a MOD realized limit cycle pair where as when x = (2, 0, 0, …, 0) the MOD resultant is a MOD realized fixed point pair.

Let x = (3, 0, …, 0) $\in$ X to find the effect of x on the MOD operator M.

$xM = (3, 0, 3, 0) = y_1$;
$y_1M^t = (0, 0, 3, 0, 0, 0, 0) = x_1$;
$x_1M = (3, 0, 0, 0) = y_2$;
$y_2M^t = (3, 0, 3, 0, 0, 0, 0) = x_2$;
$x_2M = (0, 0, 3, 0) = y_3$;
$y_3M^t = (3, 0, 0, 0, 0, 0, 0) = x_3$ $(=x)$.

Thus the MOD resultant is a MOD realized limit cycle pair given by {(3, 0, …, 0), (3, 0, 3, 0)} --- III

So this MOD resultant is different from the earlier ones.

Consider x = (4, 0, …, 0) ∈ X; to find the effect of x on the MOD matrix operator M.

$xM = (0, 0, 4, 2) = y_1;$
$y_1M^t = (2, 2, 0, 0, 2, 0, 0) = x_1;$
$x_1M = (0, 0, 0, 0) = y_2;$
$y_2M^t = (0, 0, ..., 0) = x_2.$

This the MOD resultant is a MOD realized fixed point pair given by {(0, 0 ,…, 0), (0, 0, 0, 0)} --- IV

Let x = (5, 0, 0, …, 0) ∈ X, to find the effect of x on MOD operator M.

$xM = (3, 0, 5, 4) = y_1;$
$y_1M^t = (4, 4, 3, 0, 4, 0, 0) = x_1;$
$x_1M = (3, 0, 0, 0) = y_2;$
$y_2M^t = (3, 0, 3, 0, 0, 0, 0) = x_2;$
$x_2M = (0, 0, 3, 0) = y_3;$
$y_3M^t = (3, 0, 0, 0, \ldots ,0) = x_3;$
$x_3M = (3, 0, 3, 0) = y_4;$
$y_4M^t = (0, 0, 3, 0, 0, 0, 0) = x_4;$
$x_4M = (3, 0, 0, 0) = y_5\ (=y_2).$

Thus the MOD resultant is a MOD realized limit cycle given by {(3, 0, 3, 0, 0, 0, 0), (3, 0, 0, 0)}. --- V

Now the resultant (I) to (V) are compared and we see only the MOD state vector (1, 0, 0, 0, 0, 0, 0) and (5, 0, 0, 0, 0, 0, 0) give same resultant all the other initial state vectors give different values.

Thus this method has a significance of its own.

Further $5^2 = 1$ (mod n) may be the cause for same resultant for 1 and 5. One has to ponder over this.

Let x = (0, 1, 0, ..., 0) ∈ X to find the effect of x on M.

$xM = (0, 0, 0, 4) = y_1;$
$y_1M^t = (2, 4, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 0, 2, 2) = y_2;$
$y_2M^t = (0, 2, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (0, 0, 2, 2) = y_3.$

Thus the MOD resultant is MOD realized fixed point pair given by {(0, 2, 0, 0, 4, 0, 0), (0, 0, 2, 2)} --- I

Let x = (0, 2, 0, 0, …, 0) ∈ X, to find the effect of x on M

$xM = (0, 0, 0, 2) = y_1;$
$y_1M^t = (4, 2, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 0, 4, 4) = y_2;$
$y_2M^t = (0, 4, 0, 0, 2, 0, 0) = x_2;$
$x_2M = (0, 0, 4, 4) = y_3 (= y_2).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 4, 0, 0, 2, 0, 0), (0, 0, 4, 4)} --- II

Let x = (0, 3, 0, 0, … ,0) ∈ X to find the effect of x on the MOD rectangular matrix operator M.

$xM = (0, 0, 0, 0) = y_1;$
$y_1M^t = (0, 0, \ldots , 0).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 . . . 0), (0, 0, 0, 0)} --- III

Let x = (0, 4, 0, 0, 0, ..., 0) ∈ X to find the effect of x on M.

$xM = (0, 0, 0, 4) = y_1;$
$y_1M^t = (2, 4, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 0, 2, 4) = y_2;$
$y_2M^t = (0, 4, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (0, 0, 2, 4) = y_3 (= y_2).$

Thus the MOD resultant is a MOD fixed point pair given by {(0, 4, 0, 0, 4, 0, 0), (0, 0, 2, 4)} --- IV

Finally let x = (0, 5, 0, …, 0) ∈ X to find the effect of x on M.

$xM = (0, 0, 0, 2) = y_1$;
$y_1M^t = (4, 2, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 4, 4) = y_2$;
$y_2M^t = (0, 4, 0, 0, 2, 0, 0) = x_2$;
$x_2M = (0, 0, 4, 4) = y_3$ ($=y_2$).

Thus the MOD resultant is a MOD fixed point pair given by {(4, 2, 0, 0, 0, 0, 0), (0, 0, 4, 4)} --- V

From equations (I), (II) … (V) we see all the five are distinct but it is observed the resultant is I and II are related that is II = 2I.

Let us now consider the initial state vector

$x_1 = (0, 0, 3, 0, 0, 0, 0) \in X$; to find the effect of x_1 is as follows.

$x_1M = (3, 0, 0, 0) = y_1$;
$y_1M^t = (3, 0, 3, 0, 0, 0, 0) = x_2$;
$x_2M = (0, 0, 3, 0) = y_2$;
$y_2M^t = (3, 0, 0, 0, 0, 0, 0) = x_3$;
$x_3M = (3, 0, 3, 0) = y_3$;
$y_3M^t = (0, 0, 3, 0, 0, 0, 0) = x_4$ ($=x_1$).

Let $x_2 = (0, 0, 1, 0, 0, 0, 0) \in X$ to find the effect of x_2 on M.

$x_2M = (5, 0, 0, 0) = y_1$;
$y_1M^t = (3, 0, 1, 0, 0, 0, 0) = x_3$;
$x_3M = (2, 0, 3, 0) = y_2$;
$y_2M^t = (3, 0, 4, 0, 0, 0, 0) = x_4$;
$x_4M = (5, 0, 3, 0) = y_3$;
$y_3M^t = (0, 0, 1, 0, 0, 0, 0) = x_5$ ($=x_2$).

Thus the MOD resultant is a MOD realized limit cycle given by {(0, 0, 1, 0, 0, 0, 0), (5, 0, 0, 0)} --- I

Let x = (0, 0, 2, 0, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (4, 0, 0, 0) = y_1$;
$y_1M^t = (0, 0, 2, 0, 0, 0, 0) = x_1$;
$x_1M = (4, 0, 0, 0) = y_2$ $(=y_1)$.

Thus the MOD resultant is a MOD realized fixed point given by {(0, 0, 2, 0, 0, 0, 0), (4, 0, 0, 0)} --- II

Let x = (0, 0, 3, 0, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (3, 0, 0, 0) = y_1$;
$y_1M^t = (3, 0, 3, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 3, 0) = y_2$;
$y_2M^t = (3, 0, 0, 0, 0, 0, 0) = x_2$;
$x_2M = (3, 0, 3, 0) = y_3$;
$y_3M^t = (0, 0, 3, 0, 0, 0, 0) = x_3$ $(=x)$.

Thus the MOD resultant is MOD realized limit cycle pair given by {(0, 0, 3, 0, 0, 0, 0), (3, 0, 0, 0)} --- III

Let x = (0, 0, 4, 0, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (2, 0, 0, 0) = y_1$;
$y_1M^t = (0, 0, 4, 0, 0, 0, 0) = x_1$;
$x_1M = (2, 0, 0, 0) = y_2$ $(=y_1)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 4, 0, 0, 0, 0), (2, 0, 0, 0) --- IV

Let x = (0, 0, 5, 0, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (1, 0, 0, 0) = y_1$;
$y_1M^t = (3, 0, 5, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 3, 0) = y_2$;
$y_2M^t = (3, 0, 0, 0, 0, 0, 0) = x_2$;

$x_2M = (3, 0, 3, 0) = y_3$;
$y_3M^t = (0, 0, 3, 0, 0, 0, 0) = x_3$;
$x_3M = (3, 0, 0, 0) = y_4$;
$y_4M^t = (3, 0, 3, 0, 0, 0, 0) = x_4$;
$x_4M = (0, 0, 3, 0) = y_5$ $(=y_2)$.

Thus the MOD resultant is a MOD realized limit cycle pair given by {(3, 0, 0, 0, 0, 0, 0), (0, 0, 3, 0)} --- V

Now compare (I) to (V) MOD resultants and see all are distinct.

Let x = (0, 0, 0, 1, 0, 0, 0) ∈ X, to find the effect of x on M.

$xM = (0, 1, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 1, 0, 0, 3) = x_1$;
$x_1M = (0, 4, 0, 0) = y_2$;
$y_2M^t = (0, 0, 0, 4, 0, 0, 0) = x_2$;
$x_2M = (0, 4, 0, 0) = y_3$ $(= y_2)$.

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 4, 0, 0, 0), (0, 4, 0, 0)} --- I

Let x = (0, 0, 0, 2, 0, 0, 0) ∈ X, to find the effect of x on M.

$xM = (0, 2, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 2, 0, 0, 0) = x_1$ $(=x)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 0, 2, 0, 0, 0), (0, 2, 0, 0)} --- II

Consider x = (0, 0, 0, 3, 0, 0, 0) ∈ X, to find the effect of x on M.

$xM = (0, 3, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 3, 0, 0, 3) = x_1$;
$x_1M = (0, 0, 0, 0) = y_2$;
$y_2M^t = (0, 0, 0, 0, 0, 0, 0) = x_2$.

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- III

Let x = (0, 0, 0, 4, 0, 0, 0) ∈ X, to find the effect of x on M.

$xM = (0, 4, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 4, 0, 0, 0) = x_1 (=x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- IV

Consider x = (0, 0, 0, 5, 0, 0, 0) ∈ X

To find the effect of x on M.

$xM = (0, 5, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 5, 0, 0, 3) = x_1;$
$x_1M = (0, 2, 0, 0) = y_2;$
$y_2M^t = (0, 0, 0, 2, 0, 0, 0) = x_2;$
$x_2M = (0, 2, 0, 0) = y_3 (=y_2).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 2, 0, 0, 0), (0, 2, 0, 0)} --- V

From the 5 equation I to V we see two and 5 are same. III and IV are equal to zero pairs.

Next we find for x = (0, 0, 0, 0, 1, 0, 0) ∈ X the effect on M.

$xM = (0, 0, 2, 0) = y_1;$
$y_1M^t = (2, 0, 0, 0, 4, 0, 0) = x_1;$
$x_1M = (0, 0, 4, 4) = y_2;$
$y_2M^t = (0, 4, 0, 0, 2, 0, 0) = x_2;$
$x_2M = (0, 0, 4, 4) = y_3 (=y_2).$

Thus the MOD resultant is a MOD fixed point pair given by {(0, 4, 0, 0, 2, 0, 0), (0, 0, 4, 4)} --- a

Let x = (0, 0, 0, 0, 2, 0, 0) ∈ X; to find the effect of x on M.

$xM = (0, 0, 4, 0) = y_1;$
$y_1M^t = (4, 0, 0, 0, 2, 0, 0) = x_1;$
$x_1M = (0, 0, 0, 2) = y_2;$
$y_2M^t = (4, 2, 0, 0, 0, 0, 0) = x_2;$
$x_2M = (0, 0, 4, 4) = y_3;$
$y_3M^t = (0, 4, 0, 0, 2, 0, 0) = x_3;$
$x_3M = (0, 0, 4, 4) = y_4 (=y_3).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 4, 0, 0, 2, 0, 0), (0, 0, 4, 4)} --- b

Let x = (0, 0, 0, 0, 3, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (0, 0, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 0, 0, 0, 0) = x_1.$

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- c

Let x = (0, 0, 0, 0, 4, 0, 0) $\in$ X to find the effect of x on M.

$xM = (0, 0, 2, 0) = y_1;$
$y_1M^t = (2, 0, 0, 0, 4, 0, 0) = x_1;$
$x_1M = (0, 0, 4, 4) = y_2;$
$y_2M^t = (0, 4, 0, 0, 2, 0, 0) = x_2;$
$x_2M = (0, 0, 4, 4) = y_3 (=y_2).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 4, 0, 0, 2, 0, 0), (0, 0, 4, 4)} --- d

Let x = (0, 0, 0, 0, 5, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (0, 0, 4, 0) = y_1;$
$y_1M^t = (4, 0, 0, 0, 2, 0, 0) = x_1;$
$x_1M = (0, 0, 2, 2) = y_2;$
$y_2M^t = (0, 2, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (0, 0, 2, 2) = y_3 (=y_2).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 2, 0, 0, 4, 0, 0), (0, 0, 2, 2)} --- e

Clearly of the 5 equations a, b, c, d and e we see a b and d are the same.

Let x = (0, 0, 0, 0, 0, 1, 0) $\in$ X, the effect of x on M is

$$xM = (0, 0, 0, 0) = y_1;$$
$$y_1M^t = (0, 0, 0, 0, 0, 0, 0).$$

For every x = (0, 0, 0, 0, 0, t, 0); t = 1, 2, 3, 4 and 5 we see the MOD resultant is a MOD fixed point pair given by

{(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)}.

Let x = (0, 0, 0, 0, 0, 0, 1) $\in$ X, to find the effect of x on M .

$$xM = (0, 3, 0, 0) = y_1;$$
$$y_1M^t = (0, 0, 0, 3, 0, 0, 3) = x_1;$$
$$x_1M^t = (0, 0, 0, 0) = y_2;$$
$$y_2M^t = (0, 0, 0, 0, 0, 0, 0) = x_2.$$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- 1

Let x = (0, 0, 0, 0, 0, 0, 2) $\in$ X; to find the effect of x on M.

$$xM = (0, 0, 0, 0) = y_1;$$
$$y_1M^t = (0, 0, 0, 0, 0, 0, 0) = x_1.$$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- 2

Clearly (1) and (2) are the same.

Consider x = (0, 0, 0, 0, 0, 0, 3) $\in$ X to find the effect of x on M.

$$xM = (0, 3, 0, 0) = y_2;$$

$y_1M^t = (0, 0, 0, 3, 0, 0, 3) = x_1$;
$x_1M = (0, 0, 0, 0) = y_2$;
$y_2M^t = (0, 0, 0, 0, 0, 0, 0) = x_2$.

Thus in this case also the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} ---3

Let x = (0, 0, 0, 0, 0, 0, 4) ∈ X, to find the effect of x on M.

$xM = (0, 0, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 0, 0, 0, 0) = x_1$.

The MOD resultant is a MOD realized fixed pair
{(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- 4

All the resultants (1), (2), (3) and (4) are the same.

Let x = (0, 0, 0, 0, 0, 0, 5) ∈ X, to find the effect of x on M.

$xM = (0, 3, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 3, 0, 0, 3) = x_1$;
$x_1M = (0, 0, 0, 0) = y_2$;
$y_2M^t = (0, 0, 0, 0, 0, 0, 0) = x_2$.

Thus the MOD resultant is a MOD realized fixed point given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- 5

All the 5 resultants are the same.

Thus it is clearly observed since we have taken the coordinates to be all values in Z_6 as initial state vectors in most cases we have distinct resultants.

So this is one of the advantages and the main difference between usual models.

Secondly we are not following the theory of updating at the appropriate states.

Now we consider y = (1, 0, 0, 0) $\in$ Y and find its effect on M

$yM^t = (3, 0, 5, 0, 0, 0, 0) = x_1$;
$x_1M = (4, 0, 3, 0) = y_1$;
$y_1M^t = (3, 0, 2, 0, 0, 0, 0) = x_2$;
$x_2M = (1, 0, 3, 0) = y_2$;
$y_2M^t = (0, 0, 3, 0, 0, 0, 0) = x_3$;
$x_3M = (3, 0, 0, 0) = y_3$;
$y_3M^t = (3, 0, 3, 0, 0, 0, 0) = x_4$;
$x_4M = (0, 0, 3, 0) = y_4$;
$y_4M^t = (3, 0, 0, 0, 0, 0, 0) = x_5$;
$x_5M = (3, 0, 3, 0) = y_5$;
$y_5M^t = (0, 0, 3, 0, 0, 0, 0) = x_6\ (= x_3)$.

Thus the M is a MOD realized limit cycle pair
{(0, 0, 3, 0, 0, 0, 0), (3, 0, 0, 0)} --- 1

Let y = (2, 0, 0, 0) $\in$ Y to find the effect of y on M.

$yM^t = (0, 0, 4, 0, 0, 0, 0) = x_1$;
$x_1M = (2, 0, 0, 0) = y$.

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 4, 0, 0, 0, 0), (2, 0, 0, 0)} --- 2

Let y = (3, 0, 0, 0) $\in$ Y to find the effect of y on M.

$yM^t = (3, 0, 3, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 3, 0) = y_1$;
$y_1M^t = (3, 0, 0, 0, 0, 0, 0) = x_2$;
$x_2M = (3, 0, 3, 0) = y_2$;
$y_2M^t = (0, 0, 3, 0, 0, 0, 0) = x_3$;
$x_3M = (3, 0, 0, 0) = y_3\ (= y)$.

Thus the MOD resultant is a MOD realized limit cycle given by {(3, 0, 3, 0, 0, 0, 0), (3, 0, 0, 0)} --- 3

Let y = (4, 0, 0, 0) $\in$ Y to find the effect of y on M.

$yM^t = (0, 0, 2, 0, 0, 0, 0) = x_1;$
$x_1M = (4, 0, 0, 0) = y_1 \ (= y_5).$

Thus the MOD resultant is MOD special classical fixed point pair given by {(4, 0, 0, 0), (0, 0, 2, 0, 0, 0, 0)} --- 4

Let y = (5, 0, 0, 0) $\in$ Y to find the effect of y on M.

$yM^t = (3, 0, 1, 0, 0, 0, 0) = x_1;$
$x_1M = (2, 0, 3, 0) = y_1;$
$y_1M^t = (3, 0, 4, 0, 0, 0, 0) = x_2;$
$x_2M = (5, 0, 3, 0) = y_2;$
$y_2M^t = (0, 0, 1, 0, 0, 0, 0) = x_3;$
$x_3M = (5, 0, 0, 0) = y_3 \ (= y_1).$

Thus the MOD resultant is a MOD limit cycle given by {(3, 0, 1, 0, 0, 0, 0), (5, 0, 0, 0)} --- 5

All the 5 MOD resultants are distinct.

Let y = (0, 1, 0, 0) $\in$ Y

$yM^t = (0, 0, 0, 1, 0, 0, 3) = x_1;$
$x_1M = (0, 4, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 4, 0, 0, 0) = x_2;$
$x_2M = (0, 4, 0, 0) = y_2 \ (= y_1)$.

Thus the MOD resultant is a MOD realized fixed point given by {(0, 0, 0, 4, 0, 0, 0), (0, 4, 0, 0)} --- 1

Let y = (0, 2, 0, 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0, 0, 0, 2, 0, 0, 0) = x_1;$
$x_1M = (0, 2, 0, 0) = y_2 \ (= y).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 2, 0, 0), (0, 0, 0, 2, 0, 0, 0)} --- 2

Let y = (0, 3, 0, 0) ∈ Y, to find the effect of y on M.

$yM^t = (0, 0, 0, 3, 0, 0, 3) = x_1$;
$x_1M = (0, 0, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 0, 0, 0, 0) = x_2$.

The MOD resultant is a MOD realized fixed point given by {(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)} --- 3

(1), (2) and (3) are all distinct pair of vectors.

Let y = (0, 4, 0, 0) ∈ Y, to find the effect of y on M.

$yM^t = (0, 0, 0, 4, 0, 0, 0) = x_1$;
$x_1M = (0, 4, 0, 0) = y_1$ (= y).

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 0, 4, 0, 0, 0), (0, 4, 0, 0)} --- 4

Let y = (0, 5, 0, 0) ∈ Y, to find the effect of y on M.

$yM^t = (0, 0, 0, 5, 0, 0, 3) = x_1$;
$x_1M = (0, 2, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 2, 0, 0, 0) = x_2$;
$x_2M = (0, 2, 0, 0) = y_2$ (= y_1).

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 2, 0, 0, 0), (0, 2, 0, 0)} --- 5

This is same as equation (2)

Let y = (0, 0, 1, 0) ∈ Y to find the effect of y on M.

$yM^t = (1, 0, 0, 0, 2, 0, 0) = x_1$;
$x_1M = (3, 0, 5, 2) = y_1$;
$y_1M^t = (0, 2, 3, 0, 4, 0, 0) = x_2$;
$x_2M = (3, 0, 2, 2) = y_2$;
$y_2M^t = (3, 2, 3, 0, 4, 0, 0) = x_3$;
$x_3M = (0, 0, 5, 2) = y_3$;

$y_3M^t = (3, 2, 0, 0, 4, 0, 0) = x_4;$
$x_4M = (3, 0, 5, 2) = y_4\ (= y_1).$

Thus the MOD resultant is a MOD realized limit cycle pair given by {(0, 2, 3, 0, 4, 0, 0), (3, 0, 5, 2)} --- 1

Let y = (0, 0, 2, 0) ∈ Y to find the effect of y on M.

$yM^t = (2, 0, 0, 0, 4, 0, 0) = x_1;$
$x_1M = (0, 0, 4, 4) = y_1;$
$y_1M^t = (0, 4, 0, 0, 2, 0, 0) = x_2;$
$x_2M = (0, 0, 4, 4) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD realized limit cycle pair given by{(0, 4, 0, 0, 2, 0, 0), (0, 0, 4, 4)} --- 2

(1) and (2) are distinct

Let y = (0, 0, 3, 0) ∈ Y, to find the effect of y on M.

$yM^t = (3, 0, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (3, 0, 3, 0) = y_1;$
$y_1M^t = (0, 0, 3, 0, 0, 0, 0) = x_2;$
$x_2M = (3, 0, 0, 0) = y_2;$
$y_2M^t = (3, 0, 3, 0, 0, 0, 0) = x_3;$
$x_3M = (0, 0, 3, 0) = y_3\ (= y).$

Thus the MOD resultant is a MOD limit cycle pair given by {(0, 0, 3, 0), (3, 0, 0, 0, 0, 0, 0)} --- 3

(1), (2) and (3) are distinct.

Let y = (0, 0, 4, 0) ∈ Y, to find the effect of y on M.

$yM^t = (4, 0, 0, 0, 2, 0, 0) = x_1;$
$x_1M = (0, 0, 2, 2) = y_1;$
$y_1M^t = (0, 2, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (0, 0, 2, 2) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 2, 2), (0, 2, 0, 0, 4, 0, 0)} --- 4

(1), (2), (3) and (4) are all different.

Let y = (0, 0, 5, 0) $\in$ Y, to find the effect of y on M.

$yM^t = (5, 0, 0, 0, 4, 0, 0) = x_1$;
$x_1M = (3, 0, 1, 4) = y_1$;
$y_1M^t = (0, 4, 3, 0, 2, 0, 0) = x_2$;
$x_2M = (3, 0, 4, 4) = y_2$;
$y_2M^t = (3, 4, 3, 0, 2, 0, 0) = x_3$;
$x_3M = (0, 0, 1, 4) = y_3$;
$y_3M^t = (3, 4, 0, 0, 2, 0, 0) = x_4$;
$x_4M = (3, 0, 1, 4) = y_4$ (= y_1).

Thus the MOD resultant is a MOD realized limit cycle pair given by {(3, 0, 1, 4), (0, 4, 3, 0, 2, 0, 0)} --- 5

(5) is different from (1), (2), (3) and (4).

Thus all the 5 vector give 5 different MOD resultants.

Let y = (0, 0, 0, 1) $\in$ Y, to find the effect of y on M.

$yM^t = (2, 4, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 2, 2) = y_1$;
$y_1M^t = (0, 2, 0, 0, 4, 0, 0) = x_1$;
$x_1M = (0, 0, 2, 2) = y_2$ (=y_1).

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 2, 2), (0, 2, 0, 0, 4, 0, 0)} --- I

Let y = (0, 0, 0, 2) $\in$ Y, to find the effect of y on M

$yM^t = (4, 2, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 4, 4) = y_1$;
$y_1M^t = (0, 4, 0, 0, 2, 0, 0) = x_2$;
$x_2M = (0, 0, 4, 4) = y_2$ (= y_1).

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 0, 4, 4), (0, 4, 0, 0, 2, 0, 0)\}$ --- II

Let $y = (0, 0, 0, 3) \in Y$ to find the effect of y on M

$$yM^t = (0, 0, 0, 0, 0, 0, 0) = x_1;$$
$$x_1M = (0, 0, 0, 0) = y_1.$$

Thus the MOD resultant is the MOD fixed point pair given by $\{(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)\}$ --- III

I, II and III are distinct.

Let $y = (0, 0, 0, 4) \in Y$.

$$yM^t = (2, 4, 0, 0, 0, 0, 0) = x_1;$$
$$x_1M = (0, 0, 2, 4) = y_1;$$
$$y_1M^t = (4, 4, 0, 0, 4, 0, 0) = x_2;$$
$$x_2M = (0, 0, 0, 0) = y_2;$$
$$y_2M^t = (0, 0, 0, 0, 0, 0, 0) = x_3.$$

The MOD resultant is a MOD fixed point pair given by $\{(0, 0, 0, 0, 0, 0, 0), (0, 0, 0, 0)\}$ --- IV

Clearly III and IV are identical MOD resultants.

Let $y = (0, 0, 0, 5) \in Y$, to find the effect of y on M .

$$yM^t = (4, 2, 0, 0, 0, 0, 0) = x_1;$$
$$x_1M = (0, 0, 4, 4) = y_1;$$
$$y_1M^t = (0, 4, 0, 0, 2, 0, 0) = x_2;$$
$$x_2M = (0, 0, 4, 4) = y_2 (= y_1).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 0, 4, 4), (0, 4, 0, 0, 2, 0, 0)\}$ --- V

Clearly V and II are identical.

Now we will show for the same MOD matrix operator if the initial vector which was in the on state was updated at each stage we show certainly the MOD resultant will be different.

We will prove this for a few MOD initial state vectors.

Let x = (1, 0, 0, 0, 0, 0, 0) $\in$ X to find the effect of x on M.

$xM = (3, 0, 1, 2) = y_1;$
$y_1M^t = (1, 2, 3, 0, 2, 0, 0) = x_1;$
$x_1M = (0, 0, 5, 4) = y_2;$
$y_2M^t = (1, 4, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (3, 0, 3, 0) = y_3;$
$y_3M^t = (1, 0, 3, 0, 0, 0, 0) = x_3;$
$x_3M = (0, 0, 1, 2) = y_4;$
$y_4M^t = (1, 2, 0, 0, 2, 0, 0) = x_4;$
$x_4M = (3, 0, 5, 4) = y_5;$
$y_5M^t = (1, 4, 3, 0, 4, 0, 0) = x_5;$
$x_5M = (0, 0, 3, 0) = y_6;$
$y_6M^t = (1, 0, 0, 0, 0, 0, 0) = x_6$ (= x).

Thus the MOD resultant is a MOD realized limit cycle pair given by {(1, 0, 0, 0, 0, 0, 0), (3, 0, 1, 2)} which is different from all MOD resultants using this M.

Let x = (0, 1, 0, 0, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (0, 0, 0, 4) = y_1;$
$y_1M^t \rightarrow (2, 1, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 0, 2, 2) = y_2;$
$y_2M^t \rightarrow (0, 1, 0, 0, 4, 0, 0) = x_2;$
$x_2M = (0, 0, 2, 4) = y_3;$
$y_3M^t \rightarrow (4, 1, 0, 0, 4, 0, 0) = x_3;$
$x_3M = (0, 0, 0, 0) = y_4;$
$y_4M^t \rightarrow (0, 1, 0, 0, 0, 0, 0) = x_4$ (= x).

Thus the MOD resultant is a MOD fixed point pair given by {(0, 1, 0, 0, 0, 0, 0), (0, 0, 0, 0)}.

This is also different from all other MOD resultants associated with this M.

Let x = (0, 0, 1, 0, 0, 0, 0) $\in$ X, to find the MOD resultant of x on M.

$xM = (5, 0, 0, 0) = y_1;$
$y_1M^t \to (3, 0, 1, 0, 0, 0, 0) = x_1;$
$x_1M = (2, 0, 3, 0) = y_2;$
$y_2M^t = (3, 0, 1, 0, 0, 0, 0) = x_2\ (=x_1).$

Thus the MOD resultant is a MOD fixed point pair given by {(3, 0, 1, 0, 0, 0, 0), (2, 0, 3, 0)} which is different from all other MOD resultants.

Let x = (0, 0, 0, 1, 0, 0, 0) $\in$ X, to find the effect of x on M.

$xM = (0, 1, 0, 0) = y_1;$
$y_1M^t \to (0, 0, 0, 1, 0, 0, 3) = x_1;$
$x_1M = (0, 4, 0, 0) = y_2;$
$y_2M^t \to (0, 0, 0, 1, 0, 0, 0) = x_2\ (= x).$

Thus the MOD resultant is MOD realized limit cycle pair given by {(0, 0, 0, 1, 0, 0, 0), (0, 1, 0, 0)}.

Now we find the MOD resultant of y = (0, 1, 0, 0) $\in$ Y.

$yM^t = (0, 0, 0, 1, 0, 0, 3) = x_1;$
$x_1M \to (0, 1, 0, 0) = y_1\ (= y_1).$

Thus the MOD resultant is the MOD special classical fixed point pair given by {(0, 1, 0, 0), (0, 0, 0, 1, 0, 0, 0, 3)}.

Let y = (0, 0, 1, 0) $\in$ Y to find the effect of y on M.

$yM^t = (1, 0, 0, 0, 2, 0, 0) = x_1;$
$x_1M \to (3, 0, 1, 2) = y_1;$
$y_1M^t = (2, 2, 3, 0, 2, 0, 0) = x_2\ ;$
$x_2M \to (3, 0, 1, 0) = y_2;$

$y_2M^t = (4, 0, 3, 0, 2, 0, 0) = x_3$;
$x_3M \to (3, 0, 1, 2) = y_3$.

Thus the MOD resultant is a MOD realized limit cycle pair
{(3, 0, 1, 2), (2, 2, 3, 0, 2, 0, 0)}.

This MOD resultant is distinctly different from the existing MOD resultants so far calculated.

Let x = (0, 2, 0, 0, 0, 0, 0) $\in$ X to find the effect of x on M.

$xM = (0, 0, 0, 2) = y_1$;
$y_1M^t \to (4, 2, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 4, 4) = y_2$;
$y_2M^t \to (0, 2, 0, 0, 2, 0, 0) = x_2$;
$x_2M = (0, 0, 4, 2) = y_3$;
$y_3M^t \to (2, 2, 0, 0, 2, 0, 0) = x_3$;
$x_3M = (0, 0, 0, 0) = y_4$;
$y_4M^t \to (0, 2, 0, 0, 0, 0, 0) = x$.

Thus the MOD resultant is a MOD limit cycle pair given by
{(0, 0, 0, 0), (0, 2, 0, 0, 0, 0, 0)}.

This is also very much different from the other MOD resultants.

Let x = (0, 0, 0, 0, 4, 0, 0) $\in$ X to find the effect of x on M.

$xM = (0, 0, 2, 0) = y_1$;
$y_1M^t = (2, 0, 0, 0, 4, 0, 0) = x_1$;
$x_1M = (0, 0, 4, 4) = y_1$;
$y_1M^t \to (0, 4, 0, 0, 4, 0, 0) = x_2$;
$x_2M = (0, 0, 0, 4) = y_2$;
$y_2M^t \to (2, 4, 0, 0, 4, 0, 0) = x_3$;
$x_3M = (0, 0, 4, 4) = y_2$ $(=y_1)$.

Thus the MOD resultant is a MOD realized limit point pair given by {(0, 0, 4, 4), (0, 4, 0, 0, 4, 0, 0)}.

This MOD resultants is also different from other MOD resultants so far calculated.

Let x = (0, 0, 0, 0, 0, 5, 0) $\in$ X.

To find the effect of x on M.

$xM = (0, 0, 0, 0) = y_1;$
$y_1M^t \to (0, 0, 0, 0, 0, 5, 0).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 0, 0, 0, 5, 0), (0, 0, 0, 0)}.

Let x = (0, 0, 0, 0, 0, 0, 3) $\in$ X, to find the effect of x on M;

$xM = (0, 3, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 3, 0, 0, 3) = x_1;$
$x_1M = (0, 0, 0, 0) = y_2;$
$y_2M^t \to (0, 0, 0, 0, 0, 0, 3).$

Thus the MOD resultant is a MOD realized fixed point pair given by {(0, 0, 0, 0, 0, 0, 3), (0, 0, 0, 0)}.

Let y = (0, 0, 0, 5) $\in$ Y; to find the effect of y on M.

$yM^t = (4, 2, 0, 0, 0, 0, 0) = x_1;$
$x_1M \to (0, 0, 4, 5) = y_1;$
$y_1M^t = (2, 2, 0, 0, 2, 0, 0) = x_2;$
$x_2M \to (0, 0, 0, 5) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD realized limit cycle pair given by {(4, 2, 0, 0, 0, 0, 0), (0, 0, 0, 5)}.

This MOD resultant is also different from other MOD resultants.

Thus we see depending on the type of operations on the MOD matrix operator we get the MOD resultants.

Now we proceed onto define the new notion of MOD Relational Maps model MOD Cognitive Maps models of various types have been defined, developed and described in [68].

Here we define MOD Relational Maps model analogous to FRMs when the very casual associations can be divided into two disjoint units. We have a MOD domain space and MOD range space associated with MOD Relational Maps model.

We denote by $R_1 \ldots R_m$ the range space
$R = \{(a_1, \ldots, a_m) / a_i \in 0 \text{ or } 1;\ 1 \leq i \leq m\}$.

If $a_i = 1$ then the node R_i is in on state if $a_i = 0$ the node R_i is in the off state. Similarly D denotes the nodes $D_1, \ldots, D_n$ of the domain space where

$$D = \{(b_1, \ldots, b_n) / b_j = 0 \text{ or } 1\} \text{ for } i = 1, 2, \ldots, n.$$

If $b_i = 1$ it means the node D_i is on state and if $b_i = 0$ the node D_i is in the off state.

Further in case MOD Relational Maps we still have $R_S = \{(a_1, \ldots, a_m) / a_i \in Z_s$ where Z_s is the modulo integer $1 \leq i \leq m\}$.

If $a_i = t \in Z_s$ then we say the node has the value t associated with it.

Similarly $D_s = \{(b_1, \ldots, b_n) / b_i \in Z_s$ where Z_s is the modulo integer, $1 \leq i \leq n\}$.

DEFINITION 2.1: *A MOD Relational Map (MODRM) is a directed bipartite graph or a map from D to R (D_s to R_s) with concepts like policies or events etc, as nodes and causalities as edges. It represents causal relations between spaces D and R (D_S and R_S). Let D_i and R_j denote that two nodes of the MOD RM.*

The directed edge from D_i to R_j denotes the causality of D_i to R_j called MOD relations. Every edge in the MODRM is weighted with a number in the set Z_s. If e_{ij} is weight of the edge

$D_i R_j$, $e_{ij} \in Z_s$. The other relations are defined analogous to FRMs.

Let $D_1, \ldots, D_n$ be the nodes of the MOD domain space D of the MODRM and $R_1, \ldots, R_m$ be the nodes of the MOD range space of the MODRM.

E = (e_{ij}) where e_{ij} is the MOD weight of the directed edge $D_i R_j$ (or $R_j D_i$). E is defined as the MOD relational matrix of the MODRMs.

Let D, R, D_S and R_S (as said above) denote the MOD nodes of the MODRM.

Let A = $\{(a_1, \ldots, a_n) / a_i \in \{0, 1\}$ $(a_i \in Z_s)$ $1 \leq i \leq n\}$. A is called the MOD instantaneous state vector of the MOD domain space and it denotes the on-off position of nodes at any instant. Similarly let B = $\{(b_1, b_2, \ldots, b_m) / b_i \in \{0, 1\}$ $(b_i \in Z_s)$; $1 \leq i \leq m\}$. B is called the MOD instantaneous state vector of the MOD range space and it denotes the on-off (the value of number associated with the node) position of the nodes at an instant. Such theme is new.

Thus with $D_1, \ldots, D_n$ and $R_1, \ldots, R_n$ the nodes of an MODRM.

Let D_iR_j (or $R_j D_i$) be the edges of the MODRM $1 \leq i \leq n$ and $1 \leq j \leq m$ be the edges form a MOD directed cycle.

An MODRM is said to be MOD cycle if it posses a MOD directed cycle.

A MODRM is a said to be MOD acyclic if it does not possess any MOD directed cycle.

A MODRM with MOD cycles is said to be a MODRM with MOD feed back. A MODRM with a MOD feedback is one which MOD causal relations flow through a MOD cycle in a revolutionary manner, the MODRM is defined as the MOD dynamical system.

Let D_i R_j (or R_j D_i); $1 \leq j \leq m$, $1 \leq i \leq n$ when R_i (or D_j) is switched on if the causality flows through edges of the MOD cycle and again it causes R_i (or D_j) we say the MOD dynamical system goes rounded and round. This is true for any MOD node R_j (or D_i) for $1 \leq i \leq n$ ($1 \leq j \leq m$).

The MOD equilibrium state of this MOD dynamical system is called the MOD hidden pattern.

If the MOD equilibrium state of a MOD dynamical system is a unique MOD state vectors then it is defined as the MOD fixed point.

Consider a MODRM with $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ as MOD nodes. For instance let us start the MOD dynamical system by switching on R_1 (or D_1). Let us assume that MODRM settles down with R_1 and R_m (or D_1 and D_n) on i.e. the MOD state vector remains as (1 0 0 0 .., 0 1) in R ((t 0 0 0 … u) in R_S) (or 1 0 … 1) in D or (t, …, u) in D_S). This MOD state is called the MOD fixed point.

If the MODRM settles down with a MOD state vector repeating in the form

$$A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$$
$$(\text{or } B_1 \rightarrow B_2 \rightarrow \ldots \rightarrow B_i \rightarrow B_i).$$

Then this MOD equilibrium is called the MOD limit cycle.

We will be describing the methods of obtaining the MOD hidden patterns by some examples.

Example 2.9: Let P be a problem in hand. Here we wish to study the teacher-student relationship. Let $D_1, D_2, \ldots, D_6$ be the nodes associated with the MOD domain space.

MOD domain space

D_1 – Devoted to profession
D_2 – Good
D_3 – Poor in teaching skills
D_4 – Mediocre
D_5 – Kind
D_6 – Harsh.

Let the MOD range space represent the MOD nodes associated with the students MOD range space.

R_1 - Good student
R_2 - Bad student
R_3 - Average student
R_4 - No motivation to study.

The MOD relational directed graph G with edge weights from Z_6 of the teacher-student MODRM model is given in the following figure.

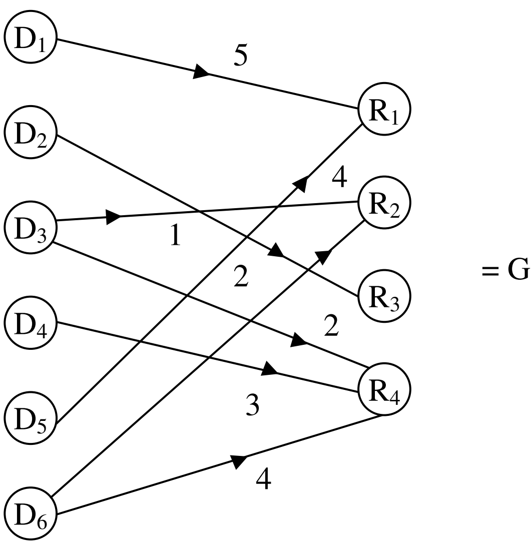


Figure 2.5

The MOD connection matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix} \end{array}.$$

M serves as the MOD dynamical system of the MODRM.

Let $X = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$,

$Y = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$,

$X_S = \{(a_1, a_2, \ldots, a_6) / a_i \in Z_6\ 1 \leq i \leq 6\}$ and

$Y_S = \{(a_1, a_2, a_3, a_4) / a_i \in Z_6; 1 \leq i \leq 4\}$ are the MOD instantaneous state vectors and MOD special state vectors.

Let $x = (1, 0, 0, 0, 0, 0) \in X$ to find the effect of x on M

$xM = (5, 0, 0, 0) = y_1$;
$y_1M^t = (1, 0, 0, 0, 4, 0) = x_1$;
$x_1M = (1, 0, 0, 0) = y_2$;
$y_2M^t = (5, 0, 0, 0, 2, 0) = x_2$;
$x_2M = (5, 0, 0, 0) = y_3\ (= y_1)$.

Thus the MOD resultant is a MOD realized limit cycle pair $\{(1, 0, 0, 0, 4, 0), (5, 0, 0, 0)\}$.

Let $x_1 = (3, 0, 0, 0, 0, 0) \in X$, to find the effect of x_1 on M.

$x_1M = (3, 0, 0, 0) = y$;
$y_1M^t = (3, 0, 0, 0, 0, 0) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD special classical fixed point given by {(3, 0, 0, 0, 0, 0), (3, 0, 0, 0)}.

Clearly the resultants are not equal

$x = (2, 0, 0, 0, 0, 0) \in X$, to find the effect of x on M.

$xM = (4, 0, 0, 0) = y_1$;
$y_1M^t = (2, 0, 0, 0, 2, 0) = x_1$;
$x_1M = (2, 0, 0, 0) = y_2$;
$y_2M^t = (4, 0, 0, 0, 4, 0) = x_2$;
$x_2M = (4, 0, 0, 0) = y_3 (=y_1)$.

Thus the MOD resultant is a MOD realized limit cycle given by {(4, 0, 0, 0), (2, 0, 0, 0, 2, 0)}. This is also different from the other two resultants.

Let $x = (0, 1, 0, 0, 0, 0) \in X$, to find the effect of x on M.

$xM = (0, 0, 2, 0) = y_1$;
$y_1M^t = (0, 4, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 2, 0) = y_2 (= y_1)$.

Thus the MOD resultant is a MOD fixed point given by {(0, 4, 0, 0, 0, 0), (0, 0, 2, 0)}.

Thus if teacher is good we are certain to get average students.

Let $x = (0, 0, 0, 0, 1, 0) \in X$, to find the effect of x on M

$xM = (2, 0, 0, 0) = y_1$;
$y_1M^t = (0, 4, 0, 0, 0, 0) = x_1$;
$x_1M = (0, 0, 2, 0) = y_2 (= y_1)$.

Thus the MOD resultant is a MOD fixed point given by {(0, 4, 0, 0, 0, 0), (0, 0, 2, 0)}.

Thus if teacher is good we are certain to get average students.

Let x = (0, 0, 0, 0, 1, 0) ∈ X, to find the effect of x on M.

$xM = (2, 0, 0, 0) = y_1;$
$y_1M^t = (0, 4, 0, 0, 4, 0) = x_2;$
$x_2M = (4, 0, 0, 0) = y_2\ ;$
$y_2M^t = (2, 0, 0, 0, 2, 0) = x_3;$
$x_3M = (2, 0, 0, 0) = y_3\ (= y_1).$

Thus the MOD resultant is a MOD limit cycle given by the pair {(4, 0, 0, 0, 4, 0), (2, 0, 0, 0)}.

Let x = (0, 0, 0, 0, 0, 2) ∈ x_S

$xM = (0, 4, 0, 2) = y_1;$
$y_1M^t = (0, 0, 2, 0, 0, 4) = x_1;$
$x_1M = (0, 4, 0, 4) = y_2\ ;$
$y_2M^t = (0, 0, 4, 0, 0, 4) = x_2;$
$x_2M = (0, 0, 0, 0) = y_3;$
$y_3M^t = (0, 0, 0, 0, 0, 0) = x_3.$

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 0, 0, 0), (0, 0, 0, 0)}.

Let x = (0, 0, 0, 0, 0, 1) ∈ X, to find the effect of x on M.

$xM = (0, 2, 0, 4) = y_1;$
$y_1M^t = (0, 0, 4, 0, 0, 4) = x_1;$
$x_1M = (0, 0, 0, 0) = y_2;$
$y_2M^t = (0, 0, 0, 0, 0, 0) = x_2.$

Thus the MOD resultant is a MOD fixed point given by {(0, 0, 0, 0, 0, 0), (0, 0, 0, 0)}.

Let y = (4, 0, 0, 0) ∈ Y to find the effect of y on M.

$yM^t = (4, 0, 0, 0, 2, 0) = x_1;$
$x_1M = (0, 0, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 0, 0, 0) = x_2.$

Thus the MOD resultant is a MOD fixed point given by {(0, 0, 0, 0, 0, 0), (0, 0, 0, 0)}.

Let y = (0, 2, 0, 0) $\in Y_S$, to find the effect of y on M.

$yM^t = (0, 0, 2, 0, 0, 4) = x_1;$
$x_1M = (0, 2, 0, 4) = y_1;$
$y_1M^t = (0, 0, 4, 0, 0, 4) = x_2;$
$x_2M = (0, 0, 0, 0) = y_2 ;$
$y_2M = (0, 0, 0, 0, 0, 0) = x_3.$

Once again the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 0), (0, 0, 0, 0, 0, 0)}.

Let y = (0, 0, 0, 1) $\in$ Y, to find the effect of y on M.

$yM^t = (0, 0, 2, 3, 0, 4) = x_1;$
$x_1M = (0, 4, 0, 5) = y_1;$
$y_1M^t = (0, 0, 2, 3, 0, 4) = x_2 \ (=x_1).$

Thus the MOD resultant is a MOD fixed pair given by {(0, 0, 2, 3, 0, 4), (0, 4, 0, 5)}.

Let y = (0, 0, 0, 3) $\in Y_S$, to find the effect of y on M.

$yM^t = (0, 0, 0, 3, 0, 0) = x_1;$
$x_1M = (0, 0, 0, 3) = y_1 \ (= y_1).$

Thus the MOD resultant is a MOD special classical fixed point given by {(0, 0, 0, 3, 0, 0), (0, 0, 0, 3)}.

This is the way operations without updating the values at each stage is worked out.

Next we see the effect of x = (1, 0, 0, 0, 0, 0) ∈ X, to find the effect of x on M.

$xM = (5, 0, 0, 0) = y_1;$
$y_1M^t = (1, 0, 0, 0, 4, 0) = x_1;$
$x_1M = (1, 0, 0, 0) = y_2;$
$y_2M^t = (1, 0, 0, 0, 2, 0) = x_2;$
$x_2M = (1, 0, 0, 0) = y_3 (= y_2).$

Thus the MOD resultant of x using updating gives a MOD fixed point pair {(1, 0, 0, 0, 2, 0), (1, 0, 0, 0)}.

Clearly this resultant is different from the other resultants which is not updated at each stage.

Let y = (0, 1, 0, 0) ∈ Y, to find the effect of y on M using the notion of updating stage by stage.

$yM^t = (0, 0, 1, 0, 0, 2) = x_1;$
$x_1M = (0, 1, 0, 4) = y_1;$
$y_1M^t = (0, 0, 3, 0, 0, 0) = x_2;$
$x_2M = (0, 1, 0, 0) = y_2 (= y).$

Thus the MOD resultant is a MOD limit cycle given by {(0, 1, 0, 0), (0, 0, 1, 0, 0, 2)}.

Let x = (0 0 0 4 0 0) ∈ X_S to find the effect of x on M using the method of updating at each stage.

$xM = (0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 4\ 0\ 0) = x_1 (= x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0 0 0 4 0 0), (0 0 0 0)}.

Let x = (0 0 0 0 1 0) ∈ X to find the MOD resultant using the method of updating

$xM = (2\ 0\ 0\ 0) = y_1$;
$y_1M^t = (4\ 0\ 0\ 0\ 1\ 0) = x_1$;
$x_1M = (4\ 0\ 0\ 0) = y_2$;
$y_2M^t = (2\ 0\ 0\ 0\ 1\ 0) = x_2$;
$x_2M = (0\ 0\ 0\ 0) = y_3$;
$y_3M^t = (0\ 0\ 0\ 0\ 1\ 0) = x_3\ (= x_1)$.

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0\ 0\ 0\ 0\ 1\ 0), (2\ 0\ 0\ 0)\}$.

Thus we see in general we have different values of the MOD resultant depending on the method used in finding the MOD resultant.

In view of all this study we have the following theorem.

THEOREM 2.1: *Let M be the MOD dyamical system associated with MODRMs model.*

$D = \{(a_1, \ldots, a_n) / a_i \in \{0, 1\}, 1 \le i \le n\}$ and

$R = \{(b_1, \ldots, b_m) / b_j \in \{0, 1\}; 1 \le j \le m\}$ be the MOD initial instantaneous state vectors of the domain and range spaces respectively.

Let $D_S = \{(a_1, \ldots, a_n) / a_i \in Z_t; 1 \le i \le n\}$ and

$R_S = \{(b_1, \ldots, b_m) / b_j \in Z_t; 1 \le j \le m\}$ be the MOD special initial state vectors of the MOD domain and range space respectively.

i) $R \subseteq R_S$ and $D \subseteq D_S$.

ii) The MOD resultant of $x \in D$ ($y \in R$) in general is different from MOD resultant of $x \in D_S \setminus D$ ($y \in R_S \setminus R$).

iii) The MOD resultant any $x \in D_S$ ($y \in R_S$) in general is distinctly different from the MOD resultant *calculated using updating at each stage from the MOD resultant calculated not using the updating process.*

Proof is direct and hence left as an exercise to the reader.

One of the advantages of using this new MODRMs model is that it can give more information about the resultant.

The resultant need not always be on or off state.

The second advantage is the nodes can take any value in Z_t their by making the weightage of it relative to other on state nodes.

This MODRMs are handy for we can always arrive at a MOD resultant after a finite number of iterations.

It is easy to find the MOD resultant once a programming using C++ is made.

Next we proceed onto describe.

Now we build the MOD Complex Relational Maps (MODCRMs) model using $C(Z_n)$.

To do this we need the notion of MOD complex directed graph and MOD complex rectangular or relational matrix.

We only give examples of them.

Example 2.10: Let G be a bipartite directed graph with edge weights from $C(Z_{10})$.

Then we define G to be a MOD complex bipartite directed graph.

The graph G is as follows.

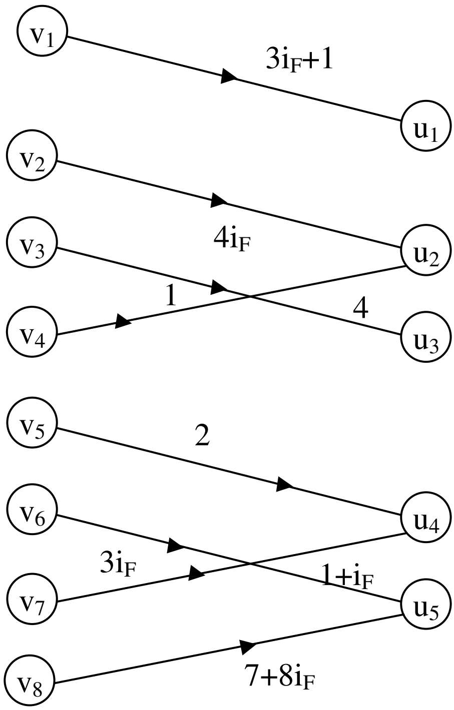


Figure 2.6

We give another example of a MOD complex bipartite graph.

Example 2.11: Let G be the MOD complex bipartite graph with edge weights from $C(Z_7)$ which is as follows.

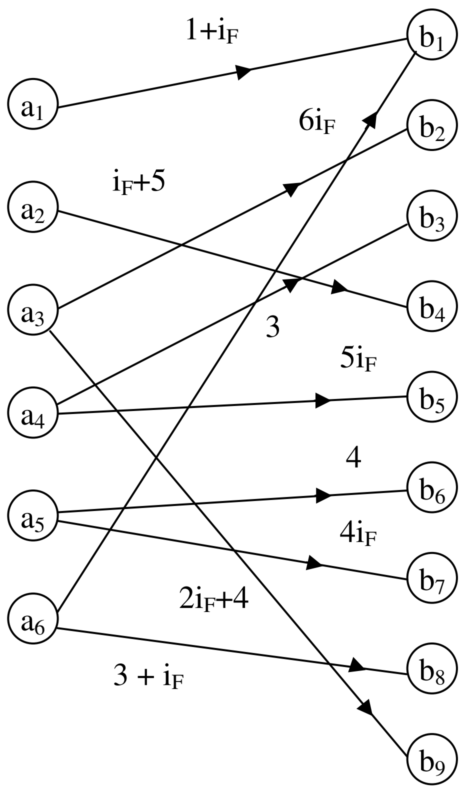


Figure 2.7

Now we just describe the MOD complex directed bipartite graph.

Let G be a MOD directed bipartite graph. If G takes edge weights from $C(Z_n)$ then we define G to be a MOD directed bipartite graph.

We have already given examples of them.

Next we proceed onto describe the MOD complex rectangular matrix or MOD complex relational matrix by some examples.

Example 2.12: Let

$$M = \begin{bmatrix} 3+4i_F & 0 & i_F+2 & 1+i_F & 3 & 4i_F \\ 0 & 3 & 2 & 0 & 0 & 0 \\ 1+i_F & 0 & 0 & 2 & 0 & 1+i_F \\ 1 & 4i_F & 2i_F+1 & 0 & 4i_F & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 4i_F & 2 & i_F & 0 & i_F & 0 \\ 2 & 0 & 2 & 1 & 0 & 1 \\ i_F+1 & 3i_F & 0 & 3+i_F & 2 & 0 \end{bmatrix}$$

be a MOD complex 8×6 rectangular matrix with entries from $C(Z_5)$.

Example 2.13: Let B be the MOD complex rectangular matrix with entries from $C(Z_{12})$.

$$B = \begin{bmatrix} 3i_F & 0 & 2 & 4i_F+3 \\ 0 & 1 & 0 & 0 \\ 5 & 2+i_F & 3i_F & 7 \\ 10+i_F & 0 & 0 & 11+10i_F \\ 3i_F & 4 & 1+i_F & 0 \end{bmatrix}.$$

Thus if M is a $m \times n$ rectangular matrix with entries from $C(Z_t)$, $2 \leq t < \infty$; then we define M to be a MOD complex rectangular matrix.

Now we proceed onto describe the MOD complex relational matrix associated with a MOD complex bipartite directed graph with entries from $C(Z_n)$.

Example 2.14: Let G be the MOD complex bipartite directed graph G with entries from $C(Z_{12})$ given by the following figure.

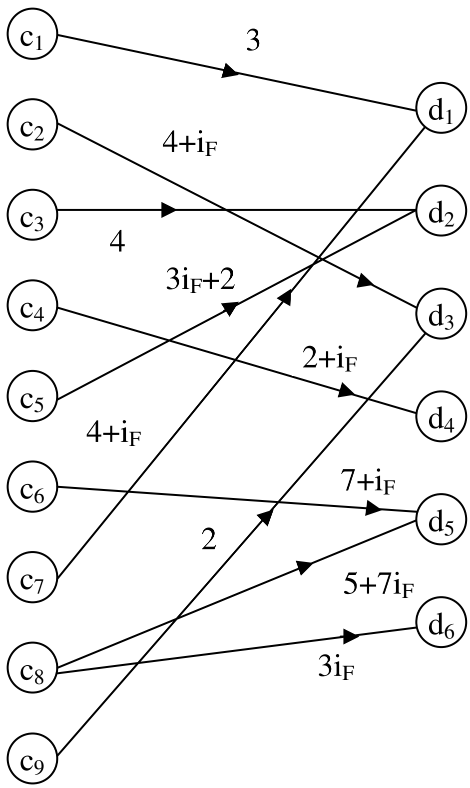


Figure 2.8

Let S be the related connection matrix which we choose to call as MOD relational matrix.

$$S = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{cccccc} d_1 & d_2 & d_3 & d_4 & d_5 & d_6 \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4+i_F & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+i_F & 0 & 0 \\ 0 & 3i_F+2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7+i_F & 0 \\ 4+i_F & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5+7i_F & 3i_F \\ 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

This is the way the matrix S is obtained using the graph G.

Example 2.15: Let G_1 be the MOD complex bipartite directed graph with edge weights from $C(Z_9)$ which is in the following figure.

$G_1 =$

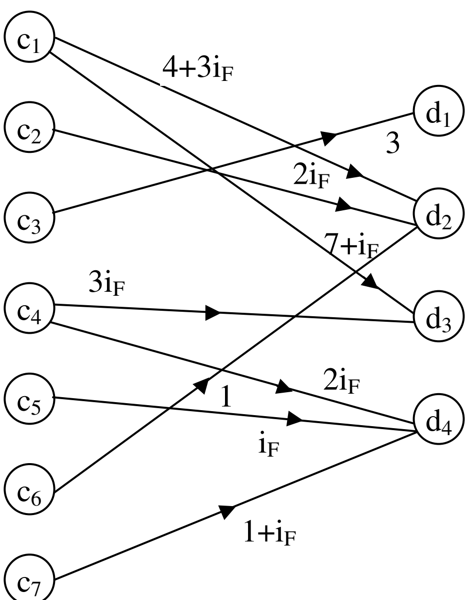


Figure 2.9

The MOD connection or relational matrix associated with G_1 is as follows.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array}\begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \left[\begin{array}{cccc} 0 & 4+3i_F & 7+i_F & 0 \\ 0 & 2i_F & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 3i_F & 2i_F \\ 0 & 0 & 0 & i_F \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1+i_F \end{array}\right] \end{array}.$$

This is the way one get the MOD relational matrix associated with the MOD directed complex bipartite graph G_1

Next we proceed onto described the MOD operation on MOD complex relational matrices by some examples.

Example 2.16: Let

$$M = \begin{bmatrix} i_F & 3 & 0 & 0 \\ 0 & 0 & 1+i_F & 2 \\ 0 & 0 & 1 & i_F+3 \\ 1+i_F & 0 & 0 & 0 \\ 0 & i_F & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

be the MOD relational complex matrix which will also be known as MOD relational complex matrix operator [66] with entries from $C(Z_4)$.

Let $X = \{(a_1\ a_2\ a_3\ \ldots\ a_6) / a_i \in \{0, 1\}, 1 \leq i \leq 6\}$ be the MOD initial state vectors of domain space and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_i \in \{0, 1\}$ be the range space of MOD initial state vectors associated with the matrix operator M.

Let $x = (1\ 0\ 0\ 0\ 0\ 0) \in X$ to find the effect of x on M.

$xM = (i_F\ 3\ 0\ 0) = y_1$
$y_1M^t = (0\ 0\ 0\ 3 + i_F\ 3i_F\ i_F) = x_1$
$x_1M = (2 + i_F\ 1\ 0\ i_F) = y_2$
$y_2M^t = (2 + 2i_F\ 2i_F\ 3i_F + 3\ 1 + 3i_F\ i_F\ 2 + 2i_F) = x_2$

and so on.

However we are sure after a finite number of iterations we will arrive at a MOD realized fixed point pair or a MOD realized limit cycle pair.

$x = (0\ 1\ 0\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$xM = (0\ \ 0\ \ 1{+}i_F\ \ 2) = y_1$;
$y_1M^t = (0\ \ 2i_F\ \ 3{+}3i_F\ \ 0\ \ 0\ \ 2) = x_1$;
$x_1M = (2\ \ 0\ \ 1{+}i_F\ \ 2) = y_2$;
$y_2M^t = (2i_F\ \ 2i_F\ \ 3{+}3i_F\ \ 2{+}2i_F\ \ 0\ \ 2) = x_2$;
$x_2M = (0\ \ 2i_F\ \ 1{+}i_F\ \ 2) = y_3$;
$y_3M^t = (2i_F\ \ 2i_F\ \ 3{+}3i_F\ \ 0\ \ 2\ \ 2) = x_3$;
$x_3M = (0\ \ 0\ \ 1{+}i_F\ \ 2) = y_4$;
$y_4M^t = (0\ \ 2i_F\ \ 3{+}3i_F\ \ 0\ \ 0\ \ 2) = x_4$;
$x_4M = (2\ \ 0\ \ 2i_F{+}2\ \ 0) = y_5$;
$y_5M^t = (2i_F\ \ 0\ \ 2{+}2i_F\ \ 2{+}2i_F\ \ 0\ \ 2) = x_5$;
$x_5M = (0\ \ 2i_F\ \ 2{+}2i_F\ \ 2) = y_6$;
$y_6M^t = (2i_F\ \ 0\ \ 2{+}2i_F\ \ 0\ \ 2\ \ 2) = x_6$;
$x_6M = (0\ \ 0\ \ 2{+}2i_F\ \ 2) = y_7$;
$y_7M^t = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2) = x_7$;
$x_7M = (2\ \ 0\ \ 0\ \ 2) = y_8$;
$y_8M^t = (2i_F\ \ 0\ \ 2{+}2i_F\ \ 2{+}2i_F\ \ 0\ \ 0) = x_8$;
$x_8M = (2\ \ 2i_F\ \ 2{+}2i_F\ \ 0) = y_9$;
$y_9M^t = (0\ \ 0\ \ 2{+}2i_F\ \ 2{+}2i_F\ \ 2\ \ 2) = x_9$;
$x_9M = (2\ \ 2i_F\ \ 0\ \ 2) = y_{10}$;
$y_{10}M^t = (0\ \ 0\ \ 0\ \ 2{+}2i_F\ \ 2i_F{+}2\ \ 0) = x_{10}$;

$x_{10}M = (0 \;\; 2+2i_F \;\; 0 \;\; 0) = y_{11};$
$y_{11}M^t = (2+2i_F \;\; 0 \;\; 0 \;\; 0 \;\; 2i_F+2 \;\; 0) = x_{11}$

and so on we are sure to arrive a MOD fixed point pair or a MOD limit cycle pair.

Let x = (0 0 0 0 0 1) ∈ X, to find the effect of x on M.

$xM = (1 \; 0 \; 0 \; 1) = y_1$
$y_1M^t = (i_F \;\; 2 \; i_F + 3 \;\; 1 + i_F \; 0 \; 2) = x_1$
$x_1M = (1 + 2i_F \; 3i_F \; 1 + 3i_F \; 2i_F + 2) = y_2$
$y_2M^t = (2i_F + 2i_F \; 2 \; 1 + 3i_F \; 3 + 3i_F \; 1 \; 3) = x_2$
$x_2M = (1 \; 2 + 3i_F \; 3 + 2i_F) = y_3$
$y_3M^t = (2 + 2i_F \;\; 0 \; 1 + i_F \; 1 + i_F \; 3i_F + 3 \; 2i_F) = x_3$

and so on.

We will arrive at a MOD fixed point pair or a MOD limit cycle pair.

Let y = (1 0 0 0) ∈ Y to find the effect of y on M.

$yM^t = (i_F \;\; 0 \; 0 \; 1 + i_F \; 0 \; 1) = x_1;$
$x_1M = (2i_F \; 3i_F \; 0 \; 1) = y_1;$
$y_1M^t = (2 + i_F \;\; 2 \; i_F + 3 \;\; 2i_F + 2 \; 1 + 2i_F) = x_2;$
$x_2M = (0 \; 2 \; 0 \; 3i_F + 1 \; 1) = y_2;$
$y_2M^t = (2 \; 0 \; 0 \; 0 \;\; 2i_F \; 1) = x_3;$
$x_3M = (2i_F + 1 \; 0 \; 0 \; 1) = y_3;$
$y_3M^t = (2 + i_F \;\; 2 \; 3 + i_F \;\; 3 + 3i_F \; 0 \; 2 + 2i_F) = x_4;$
$x_4M = (1 \; 2+3i_F \; 1 + 3i_F \; 2) = y_4;$
$y_4M^t = (2 + 2i_F \;\; 2 \; 3 + i_F \; 1 + i_F \; 2i_F \; 3) = x_5;$
$x_5M = (2i_F + 3 \; 1 + 3i_F \; 1 + i_F \; 2i_F + 3) = y_5;$

and so on.

We are sure to get after a finite number of iterations either a MOD fixed point pair or a MOD limit cycle pair.

Let y = (0 0 0 1) ∈ Y to find the effect of y on M.

$yM^t = (0\ 2\ i_F + 3\ 0\ 0\ 1) = x_1;$
$x_1M = (1\ 0\ 3i_F + 1\ \ 2i_F + 1) = y_1;$
$y_1M^t = (i_F\ \ 0\ 2 + 2i_F\ 1 + i_F\ 0\ 2 + 2i_F) = x_2;$
$x_2M = (i_F + 2\ 3i_F\ \ 2 + 2i_F\ 2 + 2i_F) = y_2;$
$y_2M^t = (3 + 3i_F\ 0\ 2 + 2i_F\ \ 1 + 3i_F\ 1\ 3i_F) = x_3$ and so on.

We are sure to arrive at a MOD resultant.

We give get another example.

Example 2.17: Let

$$M = \begin{bmatrix} 0 & i_F & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1+i_F \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

be a MOD relational complex matrix using elements from $C(Z_3)$.

$X = \{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \{0, 1\}; 1 \leq i \leq 5\}$ and

$X_S = \{(a_1 \ldots a_5) / a_i \in C(Z_3); 1 \leq i \leq 5\}$ be the MOD domain space of initial state vectors or MOD complex state vectors respectively.

Similarly let $Y = \{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$Y_S = \{(a_1\ a_2\ a_3\ a_4) / a_i \in C(Z_3); 1 \leq i \leq 4\}$ be the MOD range space of initial state vector and special state vectors respectively associated with M.

Here we will use two types of MOD operations using M and will illustrate how the MOD resultant varies.

Let $x = (1\ 0\ 0\ 0\ 0) \in X$, the effect of x on M.

$xM = (0\ i_F\ 0\ 0) = y_1;$

$yM^t = (2\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 2i_F\ 0\ 0) = y_2;$
$y_2M^t = (1\ 0\ 0\ 0\ 0) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(1\ 0\ 0\ 0\ 0), (0\ i_F\ 0\ 0)\}$.

Let $x = (0\ 1\ 0\ 0\ 0) \in X$, to find the effect of x on M

$xM = (1\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 1\ 0\ 0\ 2) = x_1;$
$x_1M = (2\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 2\ 0\ 0\ 1) = x_2;$
$x_2M^t = (1\ 0\ 0\ 0) = y_3\ (= y_1).$

Thus the MOD resultant is a MOD limit cycle pair given by

$$\{(0\ 1\ 0\ 0\ 2), (1\ 0\ 0\ 0)\}.$$

Let $x = (0\ 0\ 1\ 0\ 0) \in X$, to find the effect of x on M.

$xM = (0\ 0\ 2\ 0) = y_1;$
$y_1M^t = (0\ 0\ 1\ 0\ 0) = x_1\ (= x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by $\{(0\ 0\ 1\ 0\ 0), (0\ 0\ 2\ 0)\}$.

Let $x = (0\ 0\ 0\ 1\ 0) \in X$ to find the effect of x on M.

$xM = (0\ 0\ 0\ 1 + i_F) = y_1;$
$y_1M^t = (0\ 0\ 0\ 2i_F, 0) = x_1;$
$x_1M = (0\ 0\ 0\ 2i_F + 1) = y_2;$
$y_2M^t = (0\ 0\ 0\ 2\ 0) = x_2;$
$x_2M = (0\ 0\ 0\ 2 + i_F) = y_3;$
$y_3M^t = (0\ 0\ 0\ i_F\ 0) = x_3;$
$x_3M = (0\ 0\ 0\ i_F + 2) = y_4;$
$y_4M^t = (0\ 0\ 0\ 1\ 0) = x_4\ (= x).$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0\ 0\ 0\ 1\ 0), (0\ 0\ 0\ 1 + i_F)\}$.

Let $x = (0\ 0\ 0\ 0\ 1) \in X$ to find the effect of x on M.

$xM = (2\ 0\ 0\ 0) = y_1$;
$y_1M^t = (0\ 2\ 0\ 0\ 1) = x_1$;
$x_1M = (1\ 0\ 0\ 0) = y_2$;
$y_2M^t = (0\ 1\ 0\ 0\ 2) = x_2$;
$x_2M = (2\ 0\ 0\ 0) = y_3$;
$y_3M^t = (0\ 2\ 0\ 0\ 1) = x_3\ (=x_1)$.

The MOD resultant is a MOD limit cycle pair given by $\{(0\ 2\ 0\ 0\ 1), (2\ 0\ 0\ 0)\}$.

Let $y = (1\ 0\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t = (0\ 1\ 0\ 0\ 2) = x_1$;
$x_1M = (2\ 0\ 0\ 0) = y_1$;
$y_1M^t = (0\ 2\ 0\ 0\ 1) = x_2$;
$x_2M = (1\ 0\ 0\ 0) = y_2\ (=y)$.

Thus the MOD resultant is a MOD limit cycle given by $\{(0\ 2\ 0\ 0\ 1), (1\ 0\ 0\ 0)\}$.

Let $y = (0\ 1\ 0\ 0) \in X$ to find the effect of y on M.

$yM^t = (i_F\ 0\ 0\ 0\ 0) = x_1$;
$x_1M = (0\ 2\ 0\ 0) = y_1$;
$y_1M^t = (2i_F\ \ 0\ 0\ 0\ 0) = x_2$;
$x_2M = (0\ 1\ 0\ 0) = y_2\ (= y)$.

Thus the MOD resultant is MOD limit cycle pair given by $\{(i_F\ 0\ 0\ 0\ 0), (0\ 1\ 0\ 0)\}$.

Let $y = (0\ 0\ 1\ 0) \in Y$, to find the effect of y on M.

$yM^t = (0\ 0\ 2\ 0\ 0) = x_1$;
$x_1M = (0\ 0\ 1\ 0) = y_1\ (= y_1)$.

Thus the MOD resultant is a MOD fixed pair given by $\{(0\ 0\ 2\ 0\ 0), (0\ 0\ 1\ 0)\}$.

Let y = (0 0 0 1) $\in$ Y to find the effect of y on M.

$yM^t = (0\ 0\ 0\ 1 + i_F\ 0) = x_1$;
$x_1M = (0\ 0\ 0\ 2i_F) = y_1$;
$y_1M^t = (0\ 0\ 0\ 2i_F\ + 1\ 0) = x_2$;
$x_2M = (0\ 0\ 0\ 2) = y_2$;
$y_2M^t = (0\ 0\ 0\ 2 + 2i_F\ \ 0) = x_3$;
$x_3M = (0\ 0\ 0\ i_F) = y_3$;
$y_3M^t = (0\ 0\ 0\ i_F\ + 2\ 0) = x_4$;
$x_4M = (0\ 0\ 0\ 1) = y_4$ (= y).

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0\ 0\ 0\ i_F + 2\ 0), (0\ 0\ 0\ 1)\}$.

Now having seen how the MOD resultant for state vectors in X and Y behave we proceed onto work with yet another example.

Example 2.18: Let

$$M = \begin{bmatrix} 0 & 0 & 1 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & i_F & 0 & 0 & 0 \end{bmatrix}$$

be MOD relational matrix with entries from $C(Z_6)$.

Let X = $\{(a_1\ a_2\ a_3) / a_i \in \{0, 1\}, 1 \leq i \leq 3\}$ and

$X_S = \{(a_1\ a_2\ a_3) / a_i \in C(Z_6), 1 \leq i \leq 3\}$ be the MOD domain initial state vectors and MOD domain special initial state vectors respectively.

Let Y = $\{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \{0, 1\}; 1 \leq i \leq 5\}$ and

$Y_S = \{(a_1\ a_2\ \ldots\ a_5) / a_i \in C(Z_6); 1 \leq i \leq 5\}$ be the MOD range initial state vectors and MOD range special initial state vectors respectively.

Let x = (1 0 0) ∈ X to find the effect of x on M.

$xM = (0\ 0\ 1\ 4\ 0) = y_1;$
$y_1M^t = (5\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 5\ 2\ 0) = y_2;$
$y_2M^t = (1\ 0\ 0) = x_2\ (= x).$

Thus the MOD resultant is a MOD limit cycle pair given by {(1 0 0), (0 0 5 2 0)}.

Let x = (0 1 0) ∈ X to find the effect of x on M.

$xM = (1\ 0\ 0\ 0\ 0) = y;$
$yM^t = (0\ 1\ 2) = x_1;$
$x_1M = (5\ 2i_F\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 5\ 2) = x_2;$
$x_2M = (3\ 2i_F\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 3\ 4) = x_3;$
$x_3M = (5\ 4i_F\ 0\ 0\ 0) = y_3;$
$y_3M^t = (0\ 5\ 0) = x_4;$
$x_4M = (5\ 0\ 0\ 0\ 0) = y_4;$
$y_4M^t = (0\ 5\ 4) = x_5;$
$x_5M = (1\ 4i_F\ 0\ 0\ 0) = y_5;$
$y_5M^t = (0\ 1\ 4) = x_6;$
$x_6M = (3\ 4i_F\ 0\ 0\ 0) = y_6;$
$y_6M^t = (0\ 3\ 2) = x_7;$
$x_7M = (1\ 2i_F\ 0\ 0\ 0) = y_7;$
$y_7M^t = (0\ 1\ 0) = x_8\ (= x).$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0\ 1\ 0), (1\ 2i_F\ 0\ 0\ 0)\}$.

Let x = (0 0 1) ∈ X to find the effect of x on M.

$xM = (2,\ i_F\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 2\ 3) = x_1;$
$x_1M = (2\ 3i_F\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 2\ 1) = x_2;$
$x_2M = (4\ i_F\ 0\ 0\ 0) = y_3;$
$y_3M^t = (0\ 4\ 1) = x_3;$
$x_3M = (0\ i_F\ 0\ 0\ 0) = y_4;$
$y_4M^t = (0\ 0\ 5) = x_4;$
$x_4M = (4\ 5i_F\ 0\ 0\ 0) = y_5;$
$y_5M^t = (0\ 4\ 3) = x_5;$
$x_5M = (4\ 3i_F\ 0\ 0\ 0) = y_6;$
$y_6M^t = (0\ 4\ 5) = x_6\ ;$
$x_6M = (2\ 5i_F\ 0\ 0\ 0) = y_7;$
$y_7M^t = (0\ 2\ 5) = x_7;$
$x_7M = (0\ 5i_F\ 0\ 0\ 0) = y_8;$
$y_8M^t = (0\ 0\ 1) = x_8\ (= x).$

Thus the MOD resultant is again a MOD limit cycle pair given by $\{(0\ 0\ 1), (0\ 5i_F\ 0\ 0\ 0)\}$.

Let $x = (2\ 0\ 4) \in X_S$ to find the effect of x on M

$xM = (3\ 4i_F\ 2\ 2\ 0) = y_1;$
$y_1M^t = (4\ 3\ 2) = x_1;$
$x_1M = (1\ 2i_F\ 4\ 4\ 0) = y_2;$
$y_2M^t = (2\ 1\ 0) = x_2;$
$x_2M = (1\ 0\ 2\ 2\ 0) = y_3;$
$y_3M^t = (4\ 2\ 2) = x_3;$
$x_3M = (0\ 2i_F\ 4\ 4\ 0) = y_4;$
$y_4M^t = (2\ 0\ 4) = x_4\ (= x).$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(2\ 0\ 4), (0\ 2i_F\ 4\ 4\ 0)\}$.

Let $y = (4\ 0\ 0\ 2\ 0) \in Y_S$ to find the effect of y on M.

$yM^t = (2\ 4\ 2) = x_1;$
$x_1M = (2\ 2i_F\ 2\ 2\ 0) = y_1;$
$y_1M^t = (4\ 2\ 2) = x_2;$
$x_2M = (0\ 2i_F\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 0\ 4) = x_3;$
$x_3M = (2\ 4i_F\ 0\ 0\ 0) = y_3;$
$y_3M^t = (0\ 2\ 0) = x_4;$
$x_4M = (2\ 0\ 0\ 0\ 0) = y_4;$
$y_4M^t = (0\ 2\ 4) = x_5;$
$x_5M = (4\ 4i_F\ 0\ 0\ 0) = y_5;$
$y_5M^t = (0\ 4\ 4) = x_6;$
$x_6M = (0\ 4i_F\ 0\ 0\ 0) = y_6;$
$y_6M^t = (0\ 0\ 4) = x_7\ (= x_3).$

Thus the MOD resultant is a MOD limit cycle pair given by
$\{(0\ 0\ 4), (2\ 4i_F\ 0\ 0\ 0)\}.$

In this way one can find the MOD resultants in case of MOD Complex Relational Maps model.

Next we proceed onto describe the MOD Neutrosophic Relational Maps model.

We first describe the MOD neutrosophic directed bipartite graph and then MOD neutrosophic relational (rectangular matrix) [66].

Example 2.19: Let G be a bipartite directed graph.

The edge weight of G are taken from $\langle Z_{11} \cup I\rangle$.

We call G as the MOD directed neutrosophic bipartite graph or MOD neutrosophic bipartite directed graph which is described in the following figure;

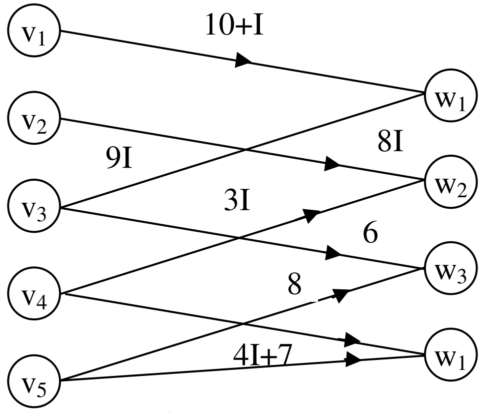


Figure 2.10

Example 2.20: Let H be a MOD neutrosophic bipartite graph with edge weights from $\langle Z_6 \cup I\rangle$, which is given in the following figure.

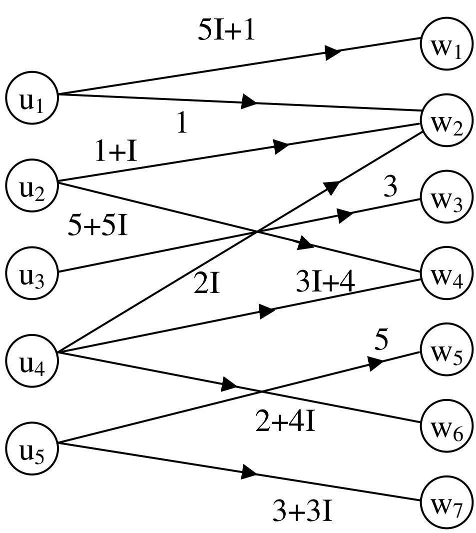


Figure 2.11

Example 2.21: Let

$$M = \begin{bmatrix} 0 & 3I+4 & 0 & 4I & 6I \\ 4 & 0 & 3I & 2 & 0 \\ 4+I & 2 & 0 & 0 & 2+5I \\ 0 & 0 & 1 & 2+3I & 0 \\ 1 & 0 & 0 & 0 & 00 \\ 0 & 2 & 1 & I & 0 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is a MOD neutrosophic relational matrix with entries from $\langle Z_7 \cup I\rangle$.

Now we describe some operations by an example or two for if in the place of complex modulo integers $C(Z_n)$ we replace $\langle Z_n \cup I\rangle$ we get MOD Neutrosophic Relational Maps model.

Hence the reader is left with the task of defining, developing and describing them.

Example 2.22: Let

$$B = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

be the MOD neutrosophic relational matrix with entries from $\langle Z_6 \cup I\rangle$.

Let $X = \{(x_1\ x_2\ \ldots\ x_7) / x_i \in \{0, 1\}; 1 \leq i \leq 7\}$ and

$Y = \{(y_1\ y_2\ y_3\ y_4) \ /\ y_i \in \{0, 1\};\ 1 \leq i \leq 4\}$ be the MOD instantaneous state vectors associated with B.

Let x = (1 0 0 0 0 0 0) ∈ X to find the effect of x on B.

$xB = (0\ 3\ 2\ 0) = y_1;$
$y_1B^t = (1\ 0\ 0\ 0\ 0\ 0\ 0) = (x_1).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(1 0 0 0 0 0 0), (0 3 2 0)}.

Let x = (0 1 0 0 0 0 0) ∈ X to find the effect of x on B.

$xB = (1\ 0\ 0\ 0) = y_1;$
$y_1B^t = (0\ 1\ 0\ 0\ 4\ 0\ 0) = x_1;$
$x_1B = (5\ 0\ 0\ 0) = y_2;$
$y_2B^t = (0\ 5\ 0\ 0\ 2\ 0\ 0) = x_2;$
$x_2B = (3\ 0\ 0\ 0) = y_3;$
$y_3B^t = (0\ 3\ 0\ 0\ 0\ 0\ 0) = x_3;$
$x_3B = (3\ 0\ 0\ 0) = y_4\ (= y_3);$
$y_4B^t = (0\ 3\ 0\ 0\ 0\ 0\ 0) = x_4\ (= x_3).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 3 0 0 0 0 0), (3 0 0 0)}.

Let x = (0 0 1 0 0 0 0) ∈ X.

$xB = (0\ 0\ 0\ I) = y_1;$
$y_1B^t = (0\ 0\ I\ 0\ 0\ 0\ 0) = x_1;$
$x_1B = (0\ 0\ 0\ I) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD fixed point pair given by
{(0 0 0 I), (0 0 I 0 0 0 0)}.

Let x = (0 0 0 1 0 0 0) ∈ X to find the effect of x on B.

$xB = (0\ 0\ 0\ 0) = y_1;$
$y_1B^t = (0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1.$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 0) (0 0 0 0 0 0 0)}.

Let x = (0 0 0 0 1 0 0) $\in$ X, to find the effect of x on B .

$xB = (4\ 0\ 0\ 0) = y_1$;
$y_1B^t = (0\ 4\ 0\ 0\ 4\ 0\ 0) = x_1$;
$x_1B = (2\ 0\ 0\ 0) = y_2$;
$y_2B^t = (0\ 2\ 0\ 0\ 2\ 0\ 0) = x_2$;
$x_2B = (4\ 0\ 0\ 0) = y_3\ (= y_1)$.

Thus the MOD resultant is a MOD limit cycle pair given by {(0 4 0 0 4 0 0), (4 0 0 0)}.

Let x = (0 0 0 0 0 1 0) $\in$ X, to find the effect of x on B.

$xB = (0\ 2\ 0\ 0) = y_1$;
$yB^t = (4\ 0\ 0\ 0\ 0\ 4\ 0) = x_1$;
$x_1B = (0\ 2\ 2\ 0) = y_2$;
$y_2B^t = (4\ 0\ 0\ 0\ 0\ 4\ 0) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by {(4 0 0 0 0 4 0), (0 2 2 0)}.

Let x = (0 0 0 0 0 0 1) $\in$ X, to find the effect of x on B.

$xB = (0\ 0\ 3\ 0) = y_1$;
$y_1B^t = (0\ 0\ 0\ 0\ \ 0\ 0\ 3) = x_1$;
$x_1B = (0\ 0\ 3\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 0 0 0 3), (0 0 3 0)}.

Let y = (1 0 0 0) $\in$ Y, to find the effect of y on B.

$yB^t = (0\ 1\ 0\ 0\ 4\ 0\ 0) = x_1$;
$x_1B = (5\ 0\ 0\ 0) = y_1$;
$y_1B^t = (0\ 5\ 0\ 0\ 2\ 0\ 0) = x_2$;
$x_2B = (3\ 0\ 0\ 0) = y_2$;

$y_2B^t = (0\ 3\ 0\ 0\ 0\ 0\ 0) = x_3$;
$x_3B = (3\ 0\ 0\ 0) = y_3\ (= y_2)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0, 3, 0, 0, 0, 0, 0), (3, 0, 0, 0)}.

Let y = (0, 1, 0, 0) ∈ Y; to find the effect of y on B

$yB^t = (3, 0, 0, 0, 0, 2, 0) = x_1$;
$x_1B = (0, 1, 0, 0) = y_1\ (= y)$.

Thus the MOD resultant is a MOD classical special fixed point pair given by {(3, 0, 0, 0, 0, 2, 0), (0, 1, 0, 0)}.

Let y = (0, 0, 1, 0) ∈ Y to find the effect of y on B.

$yBt = (2, 0, 0, 0, 0, 0, 3) = x_1$;
$x_1B = (0, 0, 1, 0) = y_1\ (= y)$.

Thus the MOD resultant in this case also is a MOD special classical fixed point pair give by
{(2, 0, 0, 0, 0, 0, 3), (0, 0, 1, 0)}.

Y = (0 0 0 1) ∈ Y, to find the effect of y on B

$yB^t = (0\ 0\ I\ 0\ 0\ 0\ 0) = x_1$;
$x_1B = (0\ 0\ 0\ I) = y_1$;
$y_1B^t = (0\ 0\ I\ 0\ 0\ 0\ 0) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 I 0 0 0 0), (0 0 0 I)}.

Let x = (1 0 0 0 0 1 0) ∈ X to find the effect of x on B.

$xB = (0\ 5\ 2\ 0) = y_1$;
$y_1B^t = (1\ 0\ 0\ 0\ 0\ 4\ 0) = x_1$;
$x_1B = (0\ 5\ 2\ 0) = y_2\ (= y_1)$.

Hence the MOD resultant is a MOD fixed point pair given by {(1 0 0 0 0 4 0), (0 5 2 0)}.

Let x = (0 1 0 0 1 0 0) ∈ X, to find the effect of x on B.

$xB = (5\ 0\ 0\ 0) = y_1;$
$y_1B^t = (0\ 5\ 0\ 0\ 2\ 0\ 0) = x_1;$
$x_1B = (1\ 0\ 0\ 0) = y_2;$
$y_2B^t = (0\ 1\ 0\ 0\ 4\ 0\ 0) = x_2;$
$x_2B = (5\ 0\ 0\ 0) = y_3\ (= y_1).$

Thus the MOD resultant is a MOD limit cycle pair given by {(5 0 0 0), (0 5 0 0 2 0 0)}.

Let y = (0 1 0 1) ∈ Y, to find the effect of y on B.

$yB^t = (3\ 0\ I\ 0\ 0\ 2\ 0) = x_1;$
$x_1B = (0\ 1\ 0\ I) = y_1;$
$y_1B^t = (3\ 0\ I\ 0\ 0\ 2\ 0) = x_2\ (= x_1).$

Thus the MOD resultant is MOD fixed point pair given by {(3 0 I 0 0 2 0), (0 1 0 I)}.

This is the way one can work with MOD Neutrosophic Relational Maps model.

Example 2.23: Let

$$V = \begin{bmatrix} 3 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}$$

be the MOD neutrosophic relational matrix with entries from $\langle Z_4 \cup I\rangle$.

$X = \{(a_1\ a_2\ a_3) / a_i \in \{0, 1, I\}; 1 \leq i \leq 3\}$ and

$X_S = \{(a_1\ a_2\ a_3) / a_i \in \langle Z_4 \cup I\rangle;\ 1 \leq i \leq 3\}$ be the MOD domain space and MOD special domain space of initial state vector respectively associated with V.

Let $Y = \{(a_1\ a_2\ \ldots\ a_6) / a_i \in \{0, 1, I\};\ 1 \leq i \leq 6\}$ and $Y_S = \{(a_1\ a_2\ \ldots\ a_6) / a_i \in \langle Z_4 \cup I\rangle,\ 1 \leq i \leq 6\}$ be the MOD range space of initial state vectors and special initial state vectors respectively associated with V.

Let $x = (1\ 0\ 0) \in X$ to find the effect of x on V.

$$xV = (3\ 0\ 1\ 0\ 0\ 1) = y_1;$$
$$y_1V^t = (3\ 0\ 0) = x_1;$$
$$x_1V = (1\ 0\ 3\ 0\ 0\ 3) = y_2;$$
$$y_2V^t = (1\ 0\ 0) = x_2\ (= x).$$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(1\ 0\ 0), (1\ 0\ 3\ 0\ 0\ 3)\}$.

Let $y = (0\ I\ 0\ 0\ 0\ 0) \in Y$, to find the effect of y on V.

$$yV^t = (0\ I\ 0) = x_1;$$
$$x_1V = (0\ I\ 0\ 0\ I\ 0) = y_1;$$
$$y_1V^t = (0\ 2I\ 0) = x_2;$$
$$x_2V = (0\ 2I\ 0\ 0\ 2I\ 0) = y_2;$$
$$y_2V^t = (0\ 0\ 0) = x_3;$$
$$x_3V = (0\ 0\ 0\ 0\ 0\ 0) = y_3.$$

The MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0), (0\ 0\ 0\ 0\ 0\ 0)\}$.

Let $x = (0\ 0\ I) \in X$ to find the effect of x on V.

$$xV = (0\ 0\ 0\ I\ 0\ 0) = y_1;$$
$$y_1V^t = (0\ 0\ I) = x_1\ (=x).$$

Thus the MOD resultant is a MOD classical special fixed point pair given by $\{(0\ 0\ I), (0\ 0\ 0\ I\ 0\ 0)\}$.

Let x = (3 + 2I 0 0) ∈ X_S to find the effect of x on V.

$xV = (1 + 2I\ 0\ 3 + 2I\ 0\ 0\ 3 + 2I) = y_1;$
$y_1V^t = (1 + 2I, 0\ 0) = x_1;$
$x_1V = (3 + 2I, 0\ 1 + 2I\ 0\ 0\ 1 + 2I) = y_2;$
$y_2V^t = (2 + 2I\ 0\ 0) = x_2;$
$x_2V = (2 + 2I, 0\ 2 + 2I\ 0\ 0\ 2 + 2I) = y_3;$
$y_3V = (2 + 2I\ 0\ \ 0) = x_3\ (=x_3).$

Thus the MOD resultant is a MOD fixed point pair given by {(2 + 2I 0 0), (2 + 2I 0 2 + 2I 0 0 2 + 2I)}.

Let y = (0 0 0 3 + I 0 0) ∈ Y_S to find the effect of y on V.

$yV^t = (0\ 0\ 0) = x_1;$
$x_1V = (0\ 0\ 0\ 0\ 0\ 0).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0), (0 0 0 0 0 0)}.

Hence we can find the MOD resultant for any initial state vector in X or X_S or Y or $Y_{S.}$

This paves way for the description of the MOD Neutrosophic Relational Maps (MODNRMs) model. Let P be a problem in hand. The expert wishes to work with a domain space and range space of nodes. The edge weight of the MOD directed bipartite graph G takes its edge weight values from $\langle Z_n \cup I\rangle$; that G is the MOD neutrosophic directed bipartite graph.

Further if M is the connection matrix associated with G, then M is the MOD Neutrosophic Relational matrix and M is also known as the dynamical system of the MOD Neutrosophic Relational Maps model.

This model also functions in a similar way as that of the MOD Relational Maps model or MOD Complex Relational Maps model.

Next we proceed onto describe and develop the notion of MOD Dual Number Relational Maps (MODDNRM) model.

To this end one needs the notion of MOD dual number directed bipartite graph which takes edge weights from $\langle Z_n \cup I\rangle$; $g^2 = 0$. This will be illustrated by examples.

Example 2.24: Let G be a MOD directed bipartite graph with edge weights from $\langle Z_{10} \cup g\rangle = \{a + bg / a, b \in Z_{10}, g^2 = 0\}$.

We call G as the MOD dual number directed bipartite graph which is as follows.

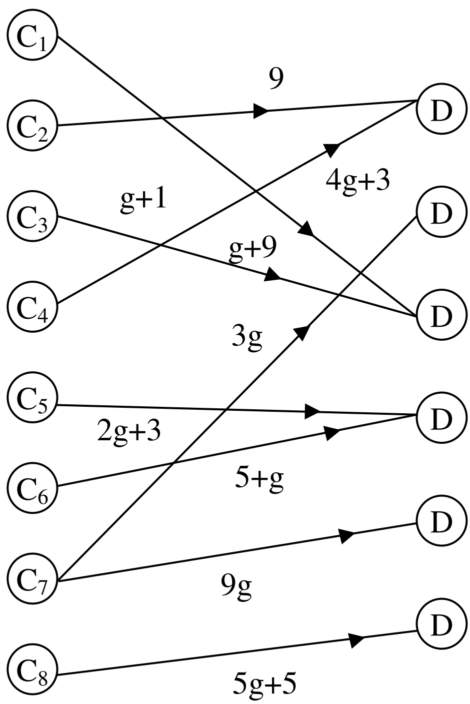


Figure 2.12

Example 2.25: Let G_1 be the MOD dual number bipartite directed graph with edge weights from $\langle Z_5 \cup g\rangle$ given by the following figure.

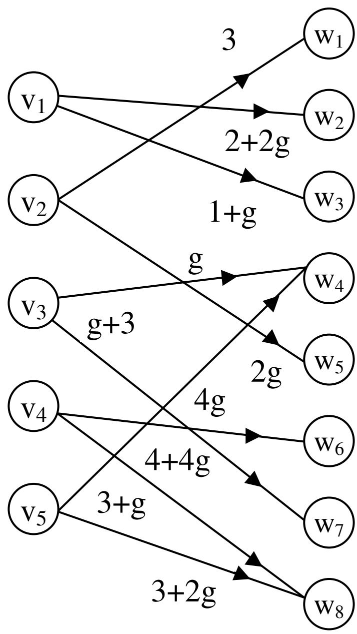


Figure 2.13

Next we give examples of MOD dual number rectangular matrix or MOD dual number relational matrix operator [66].

Example 2.26: Let

$$M = \begin{bmatrix} 2+g & g & 0 & 3 & 4g & 0 \\ 0 & 2+g & 5g & 0 & 0 & 1+g \\ 1 & 2 & 0 & 2g & 0 & 1 \\ 5g+1 & 0 & 2 & 0 & 3 & 0 \\ 4g & 3+g & 0 & g+5 & 0 & 0 \\ 0 & 0 & 5+4g & 0 & 2+3g & 3g \\ 1 & 2 & 3g & g & 0 & 0 \\ 5+g & 0 & 0 & 1+g & 2+g & 0 \end{bmatrix}$$

be the MOD dual number matrix operator or MOD dual number relational matrix operator with entries from $\langle Z_6 \cup g\rangle$.

Example 2.27: Let P =

$$\begin{bmatrix} 2g+5 & 0 & 8g & g+11g & 0 & 0 & 2 \\ 0 & 4+4g & 0 & 0 & 2g & 1+g & 0 \\ g & 0 & 5g+10 & 12 & 0 & 0 & 11g+1 \\ 4g & 2g & 0 & 0 & 5+10g & 4 & 0 \end{bmatrix}$$

be the MOD dual number relational matrix operator with entries from $\langle Z_{12} \cup g\rangle$.

Now having seen examples of MOD dual number directed bipartite graphs and MOD dual number relational (rectangular) matrix operators.

We now proceed onto describe the means of getting MOD dual number relational matrix (operators) from the MOD dual number directed bipartite graph which is nothing but the adjacency matrix associated with the graph.

Example 2.28: Let G be the MOD dual number directed bipartite graph with entries from $\langle Z_8 \cup g\rangle$.

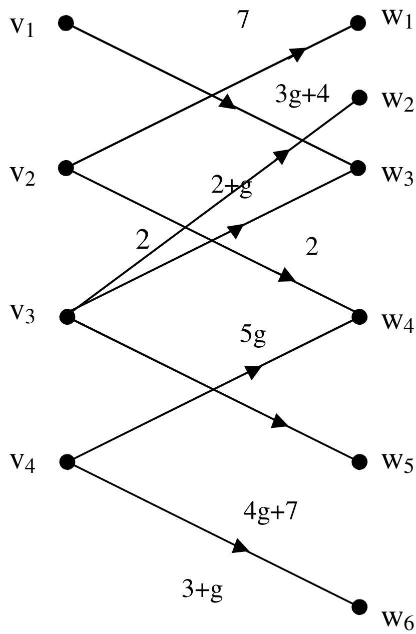
G = 

Figure 2.14

The MOD dual number relational matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 3g+4 & 0 & 0 & 0 \\ 7 & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2+g & 0 & 4g+7 & 0 \\ 0 & 0 & 0 & 5g & 0 & 3+g \end{bmatrix} \end{array}.$$

Example 2.29: Let H be the MOD dual number directed bipartite graph with edge weights from $\langle Z_{12} \cup g \rangle$.

The figure of H is as follows.

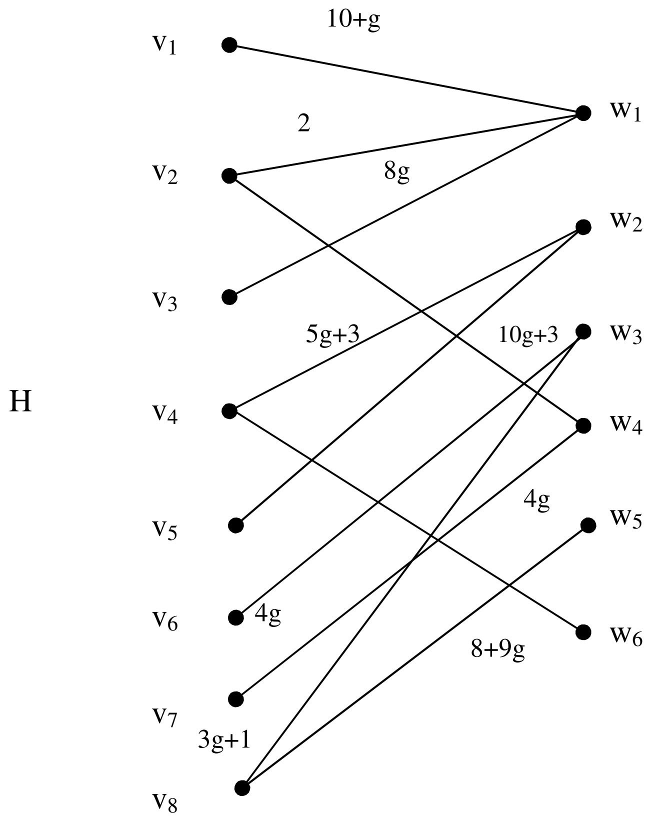


Figure 2.15

Let N be the MOD dual number relational matrix associated with the MOD dual number bipartite directed graph H.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array} \\ \begin{bmatrix} 10+g & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 10g+3 & 0 & 0 \\ 8g & 0 & 0 & 0 & 0 & 0 \\ 0 & 5g+3 & 0 & 0 & 0 & 8+9g \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4g & 0 & 0 & 0 \\ 0 & 0 & 0 & 4g & 0 & 0 \\ 0 & 0 & 3g+1 & 0 & 8+9g & 0 \end{bmatrix} \end{array}.$$

Now we can describe the MOD Dual Number Relational Maps (MOD DNRMs) model.

Let P be a problem in hand. Suppose expert wishes to work with the problem using the MOD Dual Number Relational Maps model with entries from

$\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$.

Let G the MOD dual number bipartite directed graph with edge weights from $\langle Z_n \cup g\rangle$ where the expert has $D_1 \ldots D_t$ to be the MOD domain space and $R_1, \ldots, R_S$ the MOD range space.

G is built using $D_1, \ldots, D_t$ and $R_1, \ldots, R_S$.

Let M be the MOD dual number relational (connection) matrix associated with G.

Then M is called the MOD Dual Number dynamical system of the MOD Dual Number Relational Maps model.

This will be represented by the following example.

Example 2.30: Let G be the MOD Dual Number Relational Maps model given by the following figure with edge weights from $\langle Z_{10} \cup g\rangle$.

Let $C_1, C_2, \ldots, C_5$ and $P_1, P_2, \ldots, P_6$ be the concepts associated with the domain and range spaces respectively.

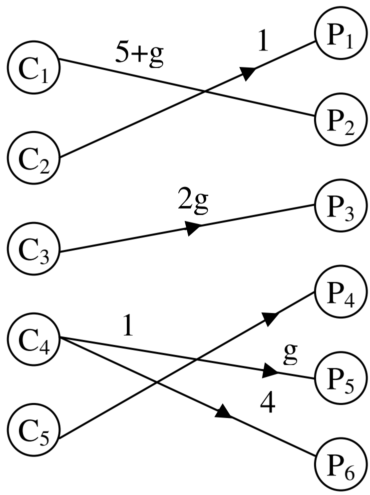
G = 

Figure 2.16

Let M be the MOD dual number relational matrix of the graph G which is as follows;

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{cccccc} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \end{array} \\ \begin{bmatrix} 0 & 5+g & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Let $X = \{(x_1, x_2, x_3, x_4, x_5) / x_i \in \{0, 1, g\}; 1 \leq i \leq 5\}$ and

$X_S = \{(x_1, x_2, \ldots, x_5) / x_i \in \langle Z_{10} \cup g\rangle, 1 \leq i \leq 5\}$ be the MOD domain space of initial and special state vectors respectively.

Let $R = \{(y_1, y_2, \ldots, y_6) / y_i \in \{0, 1, g\}, 1 \leq i \leq 6\}$ and

$R_S = \{(y_1\ y_2\ \ldots\ y_6) / y_i \in \langle Z_{10} \cup g\rangle; 1 \leq i \leq 6\}$ be the MOD range space of initial and special state vectors respectively.

Let $x = (1, 0, 0, 0, 0) \in X$, to find the effect of x on M.

$xM = (0, 5 + g, 0, 0, 0, 0) = y_1;$
$y_1M^t = (5, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 5g + 5, 0, 0, 0, 0) = y_2;$
$y_2M^t = (5, 0, 0, 0, 0) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(5, 0, 0, 0, 0), (0, 5+5g, 0, 0, 0, 0)\}$.

Let $x = (0, 0, 1, 0, 0) \in X$, to find the effect of x on M.

$xM = (0, 0, 2g, 0, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 0, 0) = x_1;$
$x_1M = (0, 0, 0, 0, 0, 0).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 0, 0, 0, 0), (0, 0, 0, 0, 0, 0)\}$.

Let $x = (0, 0, 0, 0, 1) \in X$, to find the effect of x on M.

$xM = (0, 0, 0, 1, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 1, 0) = x_1\ (= x).$

Thus the MOD resultant is MOD special classical fixed point pair $\{(0, 0, 0, 1, 0), (0, 0, 0, 1, 0, 0)\}$.

Let $y = (1, 0, 0, 0, 0, 0) \in Y;$

$yM^t = (0, 1, 0, 0, 0) = x_1;$
$x_1M = (1, 0, 0, 0, 0, 0) = y_1\ (= y).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 1, 0, 0, 0), (1, 0, 0, 0, 0, 0)\}$.

Let $y = (0, 0, 0, 1, 0, 0) \in Y$, the effect of y on M.

$yM^t = (0, 0, 0, 0, 1) = x_1$;
$x_1M = (0, 0, 0, 1, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 0, 1) = x_2 (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 0, 0, 0, 1), (0, 0, 0, 1, 0, 0)\}$.

Let $y = (0, 0, 0, g, 0, 0) \in M$.

$yM^t = (0, 0, 0, 0, g) = x_1$;
$x_1M = (0, 0, 0, g, 0, 0) = y_1 (= y)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 0, 0, 0, g), (0, 0, 0, g, 0, 0)\}$.

This is the way one can work with MOD Dual Number Relational Maps model.

The reader is left with task of giving examples of this model.

Next we proceed onto describe develop and define the notion of MOD Special Dual Like Number Relational Maps model (MOD SDLNRM).

We give examples of the new notion of MOD special dual like number directed graph and matrices.

Example 2.31: Let G be the MOD directed bipartite graph with edge weights from $\langle Z_{12} \cup h\rangle = \{a + bh / a, b \in Z_{12}, h^2 = h\}$ then we call G to be the MOD special dual like number directed bipartite graph given by the following figure.

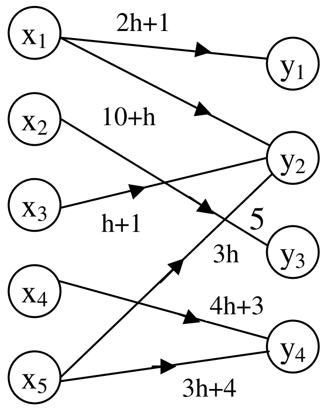


Figure 2.17

Example 2.32: Let H be the MOD special dual like number directed bipartite graph given by the following figure with edge weights from $\langle Z_4 \cup h \rangle$

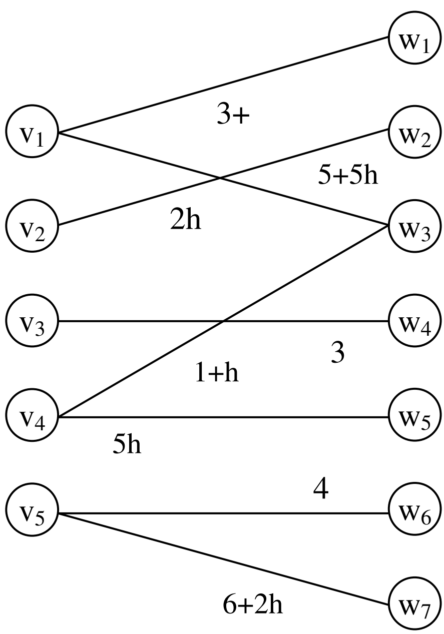


Figure 2.18

We just give examples of MOD special dual like number relational matrices (rectangular matrices). [66].

Example 2.33: Let

$$M = \begin{bmatrix} 3h & 0 & 4h+3 \\ 0 & 3+h & 0 \\ 2 & 0 & 4h \\ 0 & 1+h & 0 \\ 4+h & 0 & 3 \\ 0 & 4h & 0 \\ 6h & 0 & 2 \end{bmatrix}$$

be the MOD special dual like number relational matrix with entries from $\langle Z_8 \cup h\rangle$.

Example 2.34: Let

$$P = \begin{bmatrix} 0 & 4+h & 2h+1 & 0 & 3h & 6 & 9 \\ 3h+1 & 0 & 3 & 2h & 0 & 1+h & 3 \\ 0 & 4h+8 & 0 & 0 & 3h+1 & 0 & 0 \end{bmatrix}$$

be the MOD special dual like number relational matrix with entries from $\langle Z_{10} \cup h\rangle$.

We now describe the MOD special dual like number Relational Maps model.

Let P be the problem at hand. Suppose an expert feels that the nodes associated with P forms two disjoint classes and wishes to work with MOD special dual like number Relational Maps model with entries of the MOD dynamical system from $\langle Z_n \cup h\rangle$.

Let M be the associated MOD special dual like number matrix with entries from $\langle Z_n \cup h\rangle$.

Then this model function analogous to MOD Dual Number Relational Maps model or MOD Relational Maps model or MOD Complex Relational Maps model and so on.

Such is illustrated by an example.

Example 2.35: Let P be a problem G be the MOD special dual like number bipartite directed graph G with entries from $\langle Z_4 \cup h \rangle$ which is as follows.

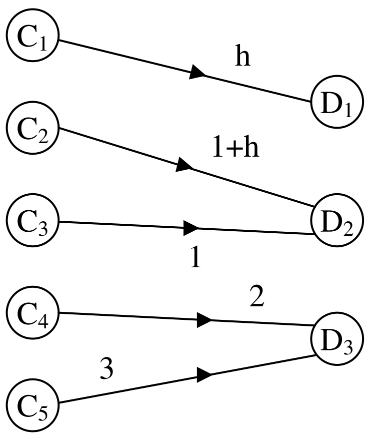


Figure 2.19

Let M be the MOD special dual like number matrix associated with G;

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} h & 0 & 0 \\ 0 & 1+h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix} \end{array}.$$

Let X = $\{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \{0, 1, h\}; 1 \leq i \leq 5\}$ and Y = $\{(b_1\ b_2\ b_3) / b_i \in \{0, 1, h\}; 1 \leq i \leq 3\}$ be the MOD initial state vectors of domain and range space respectively associated with M

$X_s = \{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \langle Z_4 \cup h\rangle; 1 \leq i \leq 5\}$ and

$Y_S = \{(b_1\ b_2\ b_3) / b_i \in \langle Z_4 \cup h\rangle; 1 \leq i \leq 3\}$ be the MOD special instantaneous state vectors of the domain and range space respectively associated with M.

Let x = (1 0 0 0 0) $\in$ X to find the effect of x on M.

$$xM = (h\ 0\ 0) = y_1;$$
$$y_1M^t = (h\ 0\ 0\ 0\ 0) = x_1;$$
$$x_1M = (h\ 0\ 0) = y_2\ (= y_1).$$

Thus the MOD resultant is a MOD fixed point pair given by {(h 0 0 0 0), (h 0 0)}.

Let x = (0 0 1 0 0) $\in$ X, to find the effect of x on M.

$$xM = (0\ 1\ 0) = y_1;$$
$$y_1M^t = (0\ 1 + h\ 1\ 0\ 0) = x_1;$$
$$x_1M = (0\ 2 + h\ 0) = y_2;$$
$$y_2M^t = (0\ 2\ 2 + h\ 0\ 0) = x_2;$$
$$x_2M = (0\ 3h\ 0) = y_3;$$
$$y_3M^t = (0\ 2h\ 3h\ 0\ 0) = x_3;$$
$$x_3M = (0\ 3h\ 0) = y_4\ (=y_3).$$

Thus the MOD resultant is a MOD fixed point pair given by {(0 2h 3h 0 0), (0 3h 0)}.

Let x = (0 0 0 0) $\in$ X;

$$xM = (0\ 0\ 3) = y_1;$$
$$y_1M^t = (0\ 0\ 0\ 2\ 1) = x_1;$$
$$x_1M = (0\ 0\ 3) = y_2\ (= y_1).$$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 2 1), (0 0 3)}.

Let $y = (0, 1, 0) \in Y$, to find the effect y on M

$yM^t = (0, 1 + h, 1, 0, 0) = x_1$;
$x_1M = (0, 2 + 3h, 0) = y_1$;
$x_2M = (0, h, 0) = y_2$;
$y_2M^t = (0, 2h, h, 0, 0) = x_3$;
$x_3M = (0, h, 0) = y_3$ $(=y_2)$.

Thus the MOD resultant can be MOD fixed point pair or a MOD limit cycle pair.

We now consider

$x = (3 + h, 0, 0, 0, 0) \in X_S$ to find the effect of x on M.

$xM = (0, 0, 0) = y_1$;
$y_1M^t = (0, 0, 0, 0, 0) = x_1$.

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 0, 0), (0, 0, 0)}.

Let $x = (0, 0, 0, 2 + 2h, 0) \in X_S$ to find the x on M.

$xM = (0, 0, 0) = y_1$;
$y_1M^t = (9, 0, 0, 0, 0)$.

Thus the MOD resultant is a MOD fixed point pair {(0, 0, 0, 0, 0), (0, 0, 0)}.

Let $y_1 = (0, 0, 2 + h) \in Y_S$, the effect of y_1 on M

$y_1M^t = (0, 0, 0, 2h, 2 + 3h) = x_1$;
$x_1M = (0, 0, 2 + h) = y_2$ $(=y_1)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0, 0, 0, 2h, 2 + 3h), (0, 0, 2 + h)}.

Interested reader can work with MOD Special Dual like number Relational Maps models.

Let us now describe the MOD special quasi dual number Relational Maps model built using
$\langle Z_n \cup k\rangle = \{a + bk/b, \in Z_n, k^2 = (n - 1)k\}$.

We just first give some examples of MOD special quasi dual number directed bipartite graphs.

Example 2.36: Let G be the MOD bipartite directed graph with edge weights from $\langle Z_{10} \cup k\rangle$ given in the following figure.

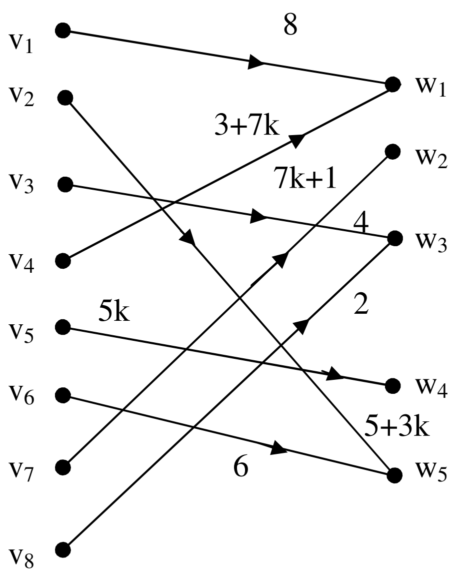


Figure 2.20

Example 2.37: Let H be a MOD special quasi dual number directed bipartite graph with entries from
$\langle Z_5 \cup k\rangle = \{a+bk / a, b \in Z_5, k^2 = 4k\}$ given by the following figure.

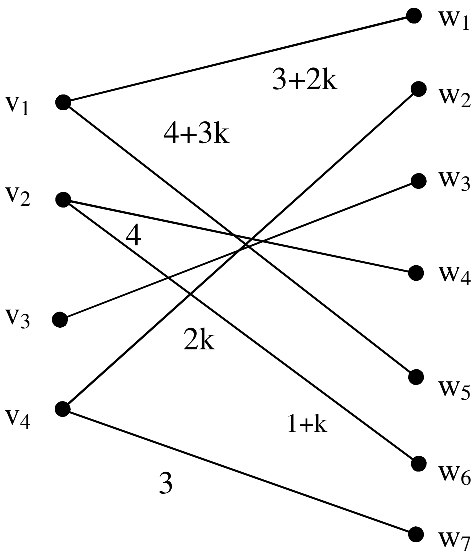


Figure 2.21

Next we proceed onto give examples of MOD special quasi dual number relational (rectangular) matrices [66].

Example 2.38: Let

$$M = \begin{bmatrix} 8+3k & 0 & 4k & 2k+9 & 3 \\ 0 & 5k & 0 & 0 & 1+k \\ 2 & 1+4k & 2 & k+5 & 0 \\ 0 & 0 & 1+5k & 0 & k+4 \\ 1 & 2 & 3 & 4 & 5 \\ 5k & 0 & 0 & k & 2k \\ 0 & 5k+7 & 1+k & 0 & 10 \\ 4k+2 & 10k+1 & 0 & 3k+1 & 0 \end{bmatrix}$$

be the MOD special quasi dual number relational matrix with entries from $\langle Z_{11} \cup k\rangle$.

Example 2.39: Let

$$P = \begin{bmatrix} 3+2k & 0 & 2 & 7k & 11k+4 & 1 & 0 \\ 0 & 2k & 0 & 1+k & 0 & 0 & 5k+1 \\ 4+k & 0 & 2k+1 & 0 & 2k & 4+k & 0 \\ 0 & 4+k & 0 & 5 & 0 & 0 & 6 \end{bmatrix}$$

be the MOD special quasi dual number rectangular matrix with entries from $\langle Z_{12} \cup k\rangle$.

These matrices serve as MOD relational operators [66]. These also are useful in building the MOD special quasi dual number Relational Maps model.

We first just indicate how these MOD relational matrices are got from the MOD special quasi dual number directed bipartite graph with edge weights from MOD special quasi dual numbers $\langle Z_n \cup k\rangle$.

Example 2.40: Let G be the MOD special quasi dual number directed bipartite graph given by the following figure.

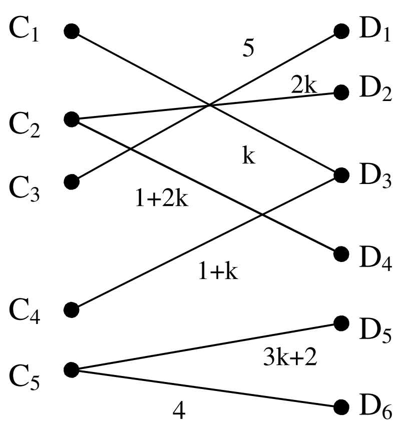


Figure 2.22

The MOD special quasi dual number connection matrix or MOD relational special quasi dual number matrix associated with G is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{cccccc} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 \end{array} \\ \begin{bmatrix} 0 & 0 & k & 0 & 0 & 0 \\ 0 & 2k & 0 & 1+2k & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3k+2 & 4 \end{bmatrix} \end{array}.$$

M the MOD special quasi dual number relational matrix will serve as the MOD special quasi dual number Relational Maps model dynamical system.

Example 2.41: Let P be a problem in hand and the expert wishes to work with the problem using MOD Relational Maps model.

Infact the edge weight of the MOD directed bipartite graph is taken from

$$\langle Z_6 \cup k\rangle = \{a + bk / a, b \in Z_6, k^2 = 5k\}.$$

Let G be MOD special quasi dual number bipartite directed graph given by the following figure with edge weights from $\langle Z_6 \cup k\rangle$.

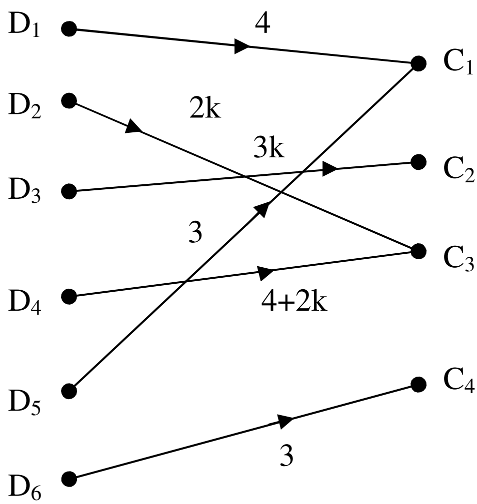


Figure 2.23

The MOD special quasi dual number relational matrix M associated with the bipartite directed graph G is as follows.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2k & 0 \\ 0 & 3k & 0 & 0 \\ 0 & 0 & 4+2k & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

Let $X = \{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6) / a_i \in \{0, 1, k\}; 1 \leq i \leq 6\}$ and

$Y = \{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1, k\}; 1 \leq i \leq 4\}$ be the MOD initial state domain and range space of vectors associated with respectively.

Let $X_S = \{(a_1, \ldots, a_6) / a_i \in \langle Z_6 \cup k\rangle; 1 \leq i \leq 6\}$ and

$Y_S = \{(d_1, d_2, d_3, d_4) / d_i \in \langle Z_6 \cup k\rangle; 1 \leq i \leq 4\}$ be the MOD special instantaneous state vectors of domain and range space respectively of M.

Let $x = (1, 0, 0, 0, 0, 0) \in X$, to find the effect of x on M.

$xM = (4, 0, 0, 0) = y_1;$
$y_1M^t = (4, 0, 0, 0, 0, 0) = x_1;$
$x_1M = (4, 0, 0, 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(4, 0, 0, 0, 0, 0), (4, 0, 0, 0)\}$.

Let $x = (0, 1, 0, 0, 0, 0) \in X$ to find the effect of x on M.

$xM = (0, 0, 2k, 0) = y_1;$
$y_1M^t = (0, 2k, 0, 4k, 0, 0) = x_1;$
$xM = (0, 0, 4k, 0) = y_2;$
$y_2M^t = (0, 4k, 0, 2k, 0, 0) = x_2;$
$x_2M = (0, 0, 2k, 0) = y_3\ (= y_1).$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0, 0, 2k, 0), (0, 2k, 0, 4k, 0, 0)\}$ or $\{(0, 0, 4k, 0), (0, 4k, 0, 2k, 0, 0)\}$.

Let $x = (0, 0, 1, 0, 0, 0) \in X$ to find the effect of x on M.

$xM = (0, 3k, 0, 0) = y_1;$
$y_1M^t = (0, 0, 3k, 0, 0, 0) = x_1$
$x_1M^t = (0, 3k, 0, 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 3k\ 0\ 0\ 0), (0\ 3k\ 0\ 0)\}$.

Let x = (0, 0, 0, 1, 0, 0) ∈ X to find the effect of x on M.

$xM = (0, 0, 4+2k, 0) = y_1;$
$y_1M^t = (0, 0, 4+2k, 0) = x_1;$
$x_1M = (0, 0, 4, 0) = y_2;$
$y_2M^t = (0, 2k, 0, 4+2k, 0, 0) = x_2;$
$x_2M = (0, 0, 4k+4, 0) = y_3;$
$y_3M^t = (0, 0, 0, 4+4k, 0, 0) = x_3;$
$x_3M = (0, 0, 4+4k, 0) = y_4$ $(= y_3)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 4 + 4k, 0, 0), (0, 0, 4 + 4k, 0)}.

Let x = (0, 0, 0, 0, 1, 0) ∈ X to find the effect of x on M.

$xM = (3, 0, 0, 0) = y_1;$
$y_1M^t = (0, 0, 0, 0, 3, 0) = x_1;$
$x_1M = (3, 0, 0, 0) = y_2$ $(= y_1)$.

Thus the MOD resultant in this case is also a MOD fixed point pair given by {(0, 0, 0, 0, 3, 0), (3, 0, 0, 0)}.

Let x = (0, 0, 0, 0, 0, 1) ∈ X, to find the effect of x on M.

$xM = (0, 0, 0, 3) = y_1;$
$y_1M^t = (0, 0, 0, 0, 0, 3) = x_1;$
$x_1M = (0, 0, 0, 3) = y_2$ $(= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0, 0, 0, 0, 0, 3), (0, 0, 0, 3)}.

Let y = (1, 0, 0, 0) ∈ Y, to find the effect of y on M.

$yM^t = (4, 0, 0, 0, 3, 0) = x_1$
$x_1M = (1, 0, 0, 0) = y$ $(= y_1)$.

Thus the MOD resultant is a MOD classical special fixed point pair given by {(1, 0, 0, 0), (4, 0, 0, 0, 3, 0)}.

Let $y = (0\ 1\ 0\ 0) \in Y$ to find the effect of y on M

$yM^t = (0\ 0\ 3k\ 0\ 0\ 0) = x_1$;
$x_1M = (0\ 3k\ 0\ 0) = y_1$;
$y_1M^t = (0\ 0\ 3k\ 0\ 0\ 0) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 3k\ 0\ 0\ 0\ 0), (0\ 3\ k\ 0\ 0)\}$.

Let $y = (0, 0, 1, 0) \in Y$ to find the effect of y on M.

$yM^t = (0, 2k, 0, 4+2k, 0, 0) = x_1$;
$x_1M = (0, 0, 4+2k, 0) = y_1$;
$y_1M^t = (0, 4k, 0, 4, 0, 0) = x_2$;
$x_2M = (0, 0, 4, 0) = y_2$;
$y_2M^t = (0, 2k, 0, 4+2k, 0, 0) = x_3\ (= x_1)$.

Thus the MOD resultant is a MOD limit cycle pair.

Let $x = (0, 2k, 0, 0, 0, k) \in X_S$ to find the effect of x on M.

$xM = (0, 0, 4k, 3k) = y_1$;
$y_1M^t = (0, 4k, 0, 0, 0, 3k) = x_1$;
$x_1M = (0, 0, 4k, 3k) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0, 4k, 0, 0, 0, 3k), (0, 0, 4k, 3k)\}$

Let $y = (k, 0, 3k, 0) \in Y_S$ to find the effect of y on M

$yM^t = (4k, 0, 0, 0, 0, 0) = x_1$;
$x_1M = (4k, 0, 0, 0) = y_1$;
$y_1M^t = (4k, 0, 0, 0, 0, 0) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(4k, 0, 0, 0, 0, 0), (4k, 0, 0, 0)\}$.

Thus the interested reader can work with any other MOD initial state vectors from X or X_S or Y or Y_S.

Here we give this task as it is a matter of routine and leave it to the reader. We have seen we can have six such models each has a special feature of its own.

Chapter Three

MOD NATURAL NEUTROSOPHIC RELATIONAL MAPS MODEL AND THEIR PROPERTIES

In this chapter we for the first time introduce the new notion of MOD natural neutrosophic Relational Maps model. The concept of MOD natural neutrosophic numbers have been studied and analysed in [60].

Several interesting features have been carried on them. The MOD natural neutrosophic Cognitive Maps model have been introduced in [68].

Here we construct 6 distinct such MOD natural neutrosophic Relational Maps model using Z_n^I, $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ $(g^2 = 0)$ or $\langle Z_n \cup h\rangle_I$, $(h^2 = h)$ or $\langle Z_n \cup k\rangle_I$, $(k^2 = (n - 1)k)$.

All of them are different and distinct in their own way. Such study will lead to lots of applications in medical, engineering and on field of social scientific problems.

For the definition and properties of Z_n^I refer [60].

However we give examples of them.

Example 3.1: Let $Z_2^I = \{0, 1, I_0^2, 1+ I_0^2\}$ be the MOD natural neutrosophic numbers.

Example 3.2: Let $Z_3^I = \{0, 1, 2, I_0^3, 1 + I_0^3, 2 + I_0^3\}$ be the MOD natural neutrosophic numbers.

Example 3.3: Let $Z_4^I = \{0, 1, 2, 3, I_0^4, I_2^4, I_0^4 + 1, 2 + I_0^4, 3 + I_0^4, 1 + I_2^4, 2 + I_2^4, 3 + I_2^4, I_0^4 + I_2^4, 1 + I_0^4 + I_2^4, 2 + I_0^4 + I_2^4, 3 + I_0^4 + I_2^4\}$ be the MOD natural neutrosophic numbers.

For more refer [60].

It is important to keep on record that Z_n^I has more natural neutrosophic numbers only when n is a composite number when n is a prome we have only one natural neutrosophic number I_0^n, and other number generated with $t \in Z_n$ are $t + I_0^n$.

We have defined several properties associated with them in [60].

Now we will give example of the notion of MOD natural neutrosophic bipartite directed graphs with edge weights from Z_n^I.

Example 3.4: Let G be a bipartite directed graph with edge weights from Z_6^I given in the following figure.

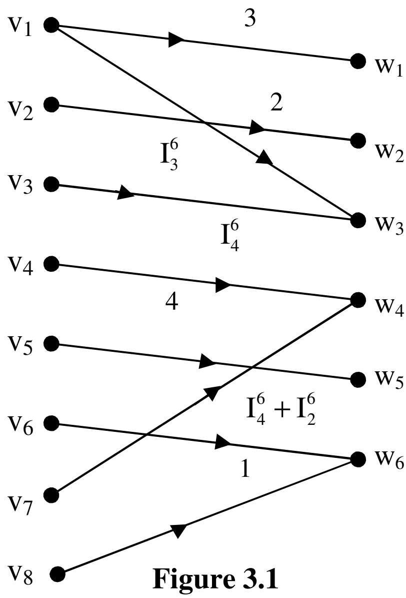


Figure 3.1

G will be known as the MOD natural neutrosophic directed bipartite graph.

Example 3.5: Let G_1 be the MOD natural neutrosophic directed bipartite graph with entries from Z_{12}^{I} given by the following.

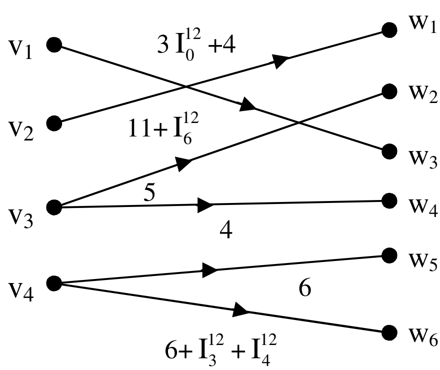


Figure 3.2

Next we give examples of MOD natural neutrosophic relational matrix (or rectangular matrix) with entries from Z_n^I.

Example 3.6: Let

$$M = \begin{bmatrix} 3 & 0 & 2+I_4^8 & 2 \\ 0 & I_6^8 & 0 & 1 \\ I_2^8+I_4^8 & 0 & 3 & 0 \\ 0 & 2+I_0^8 & 0 & I_2^8 \\ 4 & 0 & 6 & 0 \\ I_4^8 & I_2^8+1 & 0 & 4+I_2^8 \\ 0 & 6 & I_2^8 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic relational matrix.

This matrix M can work like matrix operator yielding MOD fixed point pair or MOD limit cycle pair [ ].

Example 3.7: Let

$$P = \begin{bmatrix} 1+I_5^{10} & 0 & 2 & 4 & I_2^{10} & 0 & I_0^{10}+I_6^{10} \\ 0 & I_0^{10} & 0 & 0 & 1 & I_8^{10} & 0 \\ 5 & 6 & 8 & I_2^{10}+I_5^{10} & 0 & 0 & I_8^{10}+4 \\ 4+I_0^{10} & 0 & I_0^{10} & 0 & I_2^{10}+1 & 4+I_0^{10} & 0 \end{bmatrix}$$

be the MOD natural neutrosophic relational matrix with entries from Z_{10}^I.

Now we describe some special type of operations on them by examples.

For this $X = \{(x_1\ x_2 \ldots x_n) / x_i \in \{0, 1\};\ 1 \leq i \leq n\}$ and

$Y = \{(y_1\ y_2\ \ldots\ y_m) / y_i \in \{0, 1\}, 1 \leq i \leq n\}$ be the MOD initial state vectors associated with $M = (m_{ij})_{n\times m}$ MOD natural neutrosophic relational matrix with entries from Z_s^I.

Example 3.8: Let

$$B = \begin{bmatrix} 0 & 4+I_0^6 & 0 & 2 \\ 1 & 0 & I_2^6 & 0 \\ 0 & 4 & 0 & I_4^6 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & I_0^6 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic Relational Maps model connection matrix (operator) with entries from Z_6^I.

$X = \{(a_1\ a_2\ \ldots\ a_7) / a_i \in \{0, 1\}; 1 \leq i \leq 7\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_i \in \{0, 1\}; 1 \leq i \leq 4\}$ be the MOD natural neutrosophic initial state vectors.

Let $x = (1\ 0\ 0\ 0\ 0\ 0\ 0) \in X$, to find the effect of x on B.

$xB = (0\ 4 + I_0^6\ 0\ 2) = y_1;$

$y_1B^t = (2 + I_0^6\ 0\ 4 + I_4^6\ 0\ I_0^6\ 0\ 0) = x_1;$

$x_1B = (0\ I_4^6 + I_0^6\ 0\ I_0^6 + I_4^6 + 4) = y_2;$

$y_2B^t = (2 + I_0^6 + I_4^6\ 0\ I_0^6 + I_4^6\ 0\ I_0^6\ 0\ 0) = x_2;$

$x_2B = (0\ 2 + I_0^6 + I_4^6\ 0\ 4 + I_4^6 + I_0^6) = y_3;$

$y_3B^t = (4 + I_0^6 + I_4^6\ 0\ 2 + I_0^6 + I_4^6\ 0\ I_0^6\ 0\ 0) = x_3;$

$x_3B = (0\ I_0^6 + I_4^6\ 0\ 2 + I_0^6 + I_4^6)$ and so on we are sure to arrive at a MOD fixed point pair or a MOD limit cycle pair.

Let $x = (0\ 0\ 1\ 0\ 0\ 0\ 0) \in X$, to find the effect of x on B.

$xB = (0\ 4\ 0\ I_4^6) = y_1;$
$y_1B^t = (4 + I_4^6 + I_0^6\ 0\ 4 + I_4^6\ 0\ I_0^6\ 0\ 0) = x_1;$
$x_1B = (0\ 2 + I_4^6 + I_0^6\ 0\ 2 + I_4^6 + I_2^6) = y_2;$
$y_2B^t = (I_0^6 + I_4^6\ 0\ 2 + I_4^6 + I_0^6\ 0\ I_0^6\ I_0^6 + I_4^6\ 0)$

and so on.

After a finite number of iterations we are sure to arrive at a MOD fixed point pair or a MOD limit cycle pair.

Let y = (1 0 0 0) $\in$ Y, to find the effect of y on B.

$yB^t = (0\ 1\ 0\ 3\ 0\ 1\ 0) = x_1;$
$x_1B = (5\ 0\ 3 + I_2^6\ 0) = y_1;$
$y_1B^t = (0\ 5 + I_4^6 + I_2^6\ 0\ I_2^6\ 0\ 5\ I_2^6);$
$x_2B = (5 + I_4^6 + I_2^6\ 0\ I_4^6 + I_2^6\ I_4^6 + I_2^6 + I_0^6) = y_2;$
$y_2B^t = (I_4^6 + I_2^6 + I_0^6\ 5 + I_4^6 + I_2^6\ I_4^6 + I_2^6 + I_0^6\ 3 + I_4^6 + I_2^6\ I_0^6\ 5 + I_4^6 + I_2^6\ I_4^6 + I_2^6) = x_3;$
$x_3B = (1 + I_4^6 + I_2^6\ I_4^6 + I_2^6 + I_0^6\ 3 + I_2^6 + I_4^6\ I_4^6 + I_2^6 + I_0^6) = y_3;$
$y_3B^t = (I_4^6 + I_2^6 + I_0^6\ 1 + I_4^6 + I_2^6\ I_4^6 + I_0^6 + I_2^6\ I_4^6 + I_2^6\ I_0^6\ 1 + I_4^6 + I_2^6\ I_2^6 + I_4^6)$ and so on.

Certainly after a finite number of iterations we will arrive at a MOD resultant which may be a MOD limit cycle pair or a MOD fixed point pair.

This is the way the MOD natural neutrosophic matrix operator functions yield a MOD limit cycle pair or a MOD fixed point pair.

Now we proceed onto describe and develop the MOD natural neutrosophic number Relational Maps model (MOD NRMs model).

Let P be a problem in hand D_1 D_2 … D_t and R_1, R_2, …, R_s be the MOD domain and MOD range space of nodes associated with the problem P.

Let G be the MOD directed bipartite graph given by an expert with edge weights from Z_n^I.

G will also be known as the MOD natural neutrosophic directed bipartite graph.

This given in the following figure.

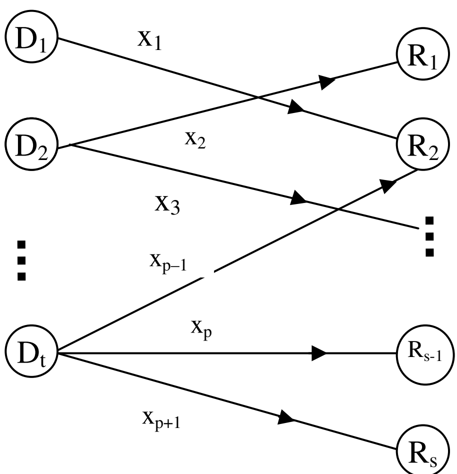


Figure 3.3

where $x_i \in Z_n^I$; $1 \leq i \leq p + 1$.

Let M denote the MOD natural neutrosophic relational matrix associated with G . M = (m_{ij}) is a t × s MOD natural neutrosophic relational matrix of G with entries $m_{ij} \in Z_n^I$.

Let X = $\{(a_1 \ldots a_t) / a_i \in \{0, 1\}; 1 \leq i \leq t\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_s) / b_j \in \{0, 1\}; 1 \leq j \leq p\}$ be the MOD natural neutrosophic initial / instantaneous state vectors associated with M.

$M = (m_{ij})$ will be defined as the MOD natural neutrosophic Relational Maps model dynamical system.

All properties associated with this new model can be developed and defined as that of the MOD Relational Maps model defined in chapter II of this book.

We will describe this MOD NRMs by some simple examples.

Example 3.9: Let P be a problem. Let $D_1 \ldots D_6$ and R_1, R_2, R_3, R_4 be the MOD domain and MOD range nodes respectively associated with the problem P.

Let G_1 be the MOD natural neutrosophic number directed bipartite graph given by the expert using the above mentioned nodes of the problem P with edge weights from Z_4^I.

Let G_1 be as follows.

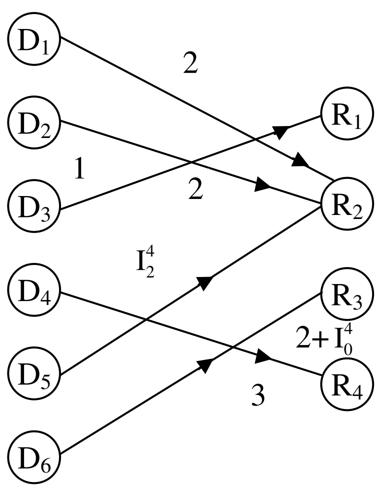


Figure 3.4

Let M be the MOD natural neutrosophic relational matrix associated with G_1.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \left[\begin{array}{cccc} 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & I_2^4 & 0 & 0 \\ 0 & 0 & 2+I_0^4 & 0 \end{array}\right] \end{array}.$$

Now we show how the MOD NRMs model function.

Let x= (1 0 0 0 0 0) $\in$ X, to find the effect of x on M

$xM = (0\ 2\ 0\ 0) = y_1;$
$y_1M^t \to (1\ 0\ 0\ 0\ I_2^4\ 0) = x_1;$
$x_1M = (0\ 2 + I_2^4\ 0\ I_2^4) = y_2;$
$y_2M^t = (I_2^4\ I_2^4\ 0\ I_2^4\ I_2^4 + I_0^4\ 0) = x_2;$
$x_2M = (0\ I_2^4 + I_0^4\ 0\ I_2^4) = y_3;$
$y_3M^t = (I_2^4 + I_0^4\ I_2^4 + I_0^4\ 0\ I_2^4\ I_0^4\ 0) = x_3;$
$x_3M = (0\ I_2^4 + I_0^4\ 0\ I_2^4) = y_4\ (=y_3).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_2^4 + I_0^4\ I_2^4 + I_0^4\ 0\ I_2^4\ I_0^4\ 0), (0\ I_2^4 + I_0^4\ 0\ I_2^4)\}$.

Let x = (0 1 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM = (0\ 2\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 1\ 0\ 0\ I_2^4 0) = x_1;$
$x_1M = (0\ 2 + I_0^4\ 0\ 0) = y_2;$
$y_2M^t = (I_0^4\ I_0^4\ 0\ 0\ I_2^4 + I_0^4\ 0) = x_2;$

$x_2M = (0\ I_0^4\ 0\ 0\ ) = y_3;$
$y_3M^t = (\ I_0^4\ I_0^4\ 0\ 0\ I_0^4\ 0) = x_3;$
$x_3M = (0\ I_0^4\ 0\ 0) = y_4\ (=y_3).$

Thus the MOD resultant is MOD fixed point pair given by $\{(\ I_0^4\ \ I_0^4\ 0\ 0\ I_0^4\ 0), (0\ I_0^4\ 0\ 0)\}$.

Let x = (0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM = (1\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 1\ 0\ 0\ 0) = x_1\ (=x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by{(0 0 1 0 0 0), (1 0 0 0)}.

Let x = (0 0 0 1 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 3) = y_1;$
$y_1M^t = (0\ 0\ 0\ 1\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 3) = y_2;$
$y_2M^t = (0\ 0\ 0\ 1\ 0\ 0) = x_2;$
$x_2M = (0\ 0\ 0\ 3) = y_3\ (=y_2).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0 0 0 1 0 0), (0 0 0 3)}.

Let x = (0 0 0 0 1 0) ∈ X.

$xM = (0\ I_2^4\ 0\ 0) = y_1;$
$y_1M^t = (\ I_2^4\ I_2^4\ 0\ 0\ I_2^4\ 0\ ) = x_1;$
$x_1M = (0\ I_2^4+I_0^4\ 0\ 0) = y_2;$
$y_2M^t = (\ I_0^4+I_2^4\ \ I_0^4\ +\ I_2^4\ 0\ 0\ I_0^4\ 0) = x_2;$
$x_2M = (0\ I_0^4\ +\ I_2^4\ 0\ 0) = y_3\ (=y_2).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(\ I_2^4\ +\ I_0^4\ \ I_2^4+I_0^4\ 0\ \ 0\ \ I_0^4\ 0), (0\ I_0^4\ +\ I_2^4\ \ 0\ \ 0)\}$.

Let x = (0 0 0 0 0 1) $\in$ X, to find the effect of x on M.

$$xM = (0\ 0\ 2 + I_0^4\ \ 0) = y_1;$$
$$y_1M^t = (0\ 0\ 0\ 0\ 0\ \ I_0^4) = x_1;$$
$$x_1M = (0\ 0\ \ I_0^4\ \ 0) = y_2;$$
$$y_2M^t = (0\ 0\ 0\ 0\ 0\ \ I_0^4) = x_2\ (=x_1).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0\ 0\ 0\ \ I_0^4), (0\ 0\ \ I_0^4\ \ 0)\}$.

Let y = (1 0 0 0) $\in$ Y, to find the effect of y on M.

$$yM^t = (0\ 0\ 1\ 0\ 0\ 0) = x_1;$$
$$x_1M = (1\ 0\ 0\ 0) = y_1\ (= y).$$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0 0 1 0 0 0), (1 0 0 0)}.

Let y = (0 1 0 0) $\in$ Y, to find the effect of y on M.

$$yM^t = (2\ 2\ 0\ 0\ \ I_2^4\ \ 0) = x_1;$$
$$x_1M = (0\ \ I_0^4\ \ 0\ 0) = y_1;$$
$$y_1M^t = (I_0^4\ I_0^4\ \ 0\ 0\ \ I_0^4\ \ 0) = x_2;$$
$$x_2M = (0\ \ I_0^4\ \ 0\ 0) = y_2\ (= y_1).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^4\ \ I_0^4\ \ 0\ 0\ \ I_0^4\ \ 0), (0\ \ I_0^4 0\ 0)\}$.

Let y = (0 0 1 0) $\in$ Y, to find the effect of y on M.

$$yM^t = (0\ 0\ 0\ 0\ 0\ \ I_0^4\ + 2) = x_1;$$
$$x_1M = (0\ 0\ \ I_0^4\ \ 0) = y_1;$$
$$y_1M^t = (0\ 0\ 0\ 0\ 0\ \ I_0^4) = x_2;$$

$x_2M = (0\ 0\ I_0^4\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0\ 0\ 0\ I_0^4), (0\ 0\ I_0^4\ 0)\}$.

Let $y = (0\ 0\ 0\ 1) \in Y$, to find the effect of y on M.

$yM^t = (0\ 0\ 0\ 3\ 0\ 0) = x_1$;
$x_1M = (0\ 0\ 0\ 1) = y_1\ (=y)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by $\{(0\ 0\ 0\ 3\ 0\ 0), (0\ 0\ 0\ 1)\}$.

Let $x = (0\ 1\ 0\ 0\ 1\ 0) \in X$, to find the effect of x on M.

$xM = (0\ 2 + I_0^4\ 0\ 0) = y_1$;
$y_1M^t = (0\ I_2^4\ I_2^4\ 0\ I_2^4 + I_0^4\ 0) = x_1$;
$x_1M = (0\ I_2^4 + I_0^4\ 0\ 0) = y_2$;
$y_2M^t = (0\ I_2^4 + I_0^4\ I_2^4 + I_0^4\ 0\ I_0^4\ 0) = x_2$;
$x_2M = (0\ I_2^4 + I_0^4\ 0\ 0) = y_3\ (= y_2)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ I_2^4 + I_0^4\ I_0^4 + I_2^4\ 0\ I_0^4\ 0), (0\ I_2^4 + I_0^4\ 0\ 0)\}$.

This is the way operations are performed using M.

Now we proceed onto find the effect of M on D_S and R_S.

Suppose $D_S = \{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6) / a_i \in Z_4^I;\ 1 \leq i \leq 6\}$ and

$R_S = \{(b_1\ b_2\ b_3\ b_4) / b_i \in Z_4^I;\ 1 \leq i \leq 4\}$ be the MOD special initial state vectors.

To find the effect of x in D_S (or $y \in R_S$) on M.

Let $x = (2 + I_0^4\ 0\ 0\ 0\ 0\ 0) \in D_S$, to find the effect of x on M.

$xM = (0\ I_0^4\ 0\ 0) = y_1;$
$y_1M^t = (I_0^4\ I_0^4\ 0\ 0\ I_0^4\ 0) = x_1;$
$x_1M = (0\ I_0^4\ 0\ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^4\ I_0^4\ 0\ 0\ I_0^4\ 0), (0\ I_0^4\ 0\ 0)\}$.

Let $x = (0\ 0\ 3 + I_2^4\ 0\ 0\ I_0^4) \in D_S$, to find the effect of x on M.

$xM = (3 + I_2^4\ 0\ I_0^4 0) = y_1;$
$y_1M^t = (0\ 0\ 3 + I_2^4\ 0\ I_2^4\ I_0^4) = x_1;$
$x_1M = (3 + I_2^4\ I_0^4\ I_0^4\ 0) = y_2;$
$y_2M^t = (I_0^4\ I_0^4\ 3 + I_2^4\ 0\ I_0^4\ I_0^4) = x_2;$
$x_2M = (3 + I_2^4\ I_0^4\ I_0^4\ 0) = y_3\ (=y_2).$

Thus the MOD resultant is a MOD fixed point pair givenby $\{(I_0^4\ I_0^4\ 3 + I_2^4\ 0\ I_0^4\ I_0^4), (3 + I_2^4\ I_0^4\ I_0^4\ 0)\}$.

Thus the nodes are natural neutrosophic zero or natural neutrosophic zero divisor.

So far we studied only MOD NRMs model we now proceed onto describe MOD natural neutrosophic Relational Maps model $\langle Z_n \cup I\rangle_I$, denotes the MOD natural neutrosophic- neutrosophic numbers.

We describe the MOD directed bipartite graphs with edge weights from $\langle Z_n \cup I\rangle_I$, which will also be known as the MOD natural neutrosophic-neutrosophic bipartite directed graph.

This will be represented by some examples.

Example 3.10: Let G be a MOD directed bipartite graph with edge weights from $\langle Z_6 \cup I\rangle_I$ given by the following figure.

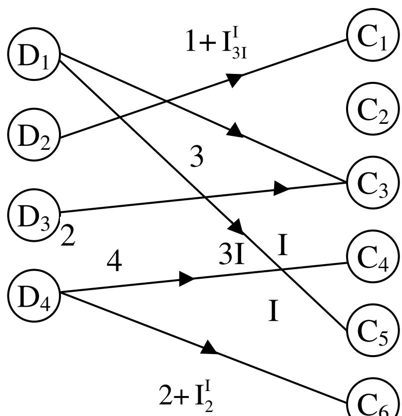


Figure 3.5

Example 3.11: Let H be the MOD directed bipartite natural neutrosophic-neutrosophic graph with edge weights from $\langle Z_{11} \cup I\rangle_I$, given by the following figure.

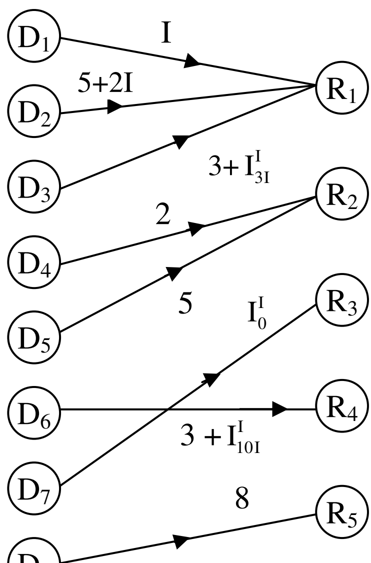


Figure 3.6

Let us now give examples of MOD rectangular natural neutrosophic-neutrosophic matrix or the MOD rectangular (relational) neutrosophic-neutrosophic matrix if its entries are from $\langle Z_n \cup I\rangle_I$.

Example 3.12: Let

$$M = \begin{bmatrix} 3+I_{3I}^{I} & 2 & I_{7I}^{I} & 0 & 5 & 1 \\ 0 & 1+I_{4I}^{I} & 0 & 2+I_{6}^{I} & 0 & I_{3}^{I} \\ 4 & 0 & 5+I_{0}^{I} & 0 & 3I & 0 \\ I_{3}^{I}+3 & I_{I}^{I} & 0 & I_{8I}^{I} & 0 & 7 \\ 0 & 0 & 3 & 0 & 1 & 2+I_{2I}^{I} \end{bmatrix}$$

be the MOD rectangular natural neutrosophic-neutrosophic matrix with entries from $\langle Z_9 \cup I\rangle_I$.

Example 3.13: Let

$$M = \begin{bmatrix} 3+I_{0}^{I} & 0 & 1 \\ 0 & 2+I_{4I}^{I} & 0 \\ 1+I_{I}^{I} & 4 & 4+I_{2I}^{I} \\ 0 & 0 & 2+I_{3I}^{I} \\ I_{3}^{I} & 0 & 0 \\ 2 & 0 & 2+I_{3}^{I} \\ 0 & 4+I_{4}^{I} & 0 \end{bmatrix}$$

be the MOD natural neutrosophic-neutrosophic relational (rectangular) matrix with entries from $\langle Z_6 \cup I\rangle_I$.

Now we show how the relation between the MOD directed bipartite natural neutrosophic-neutrosophic graphs and the MOD natural neutrosophic-neutrosophic relational matrix by examples.

Example 3.14: Let G be the MOD natural neutrosophic-neutrosophic bipartite graph with edge weights from $\langle Z_{12} \cup I\rangle_I$, given by the following figure.

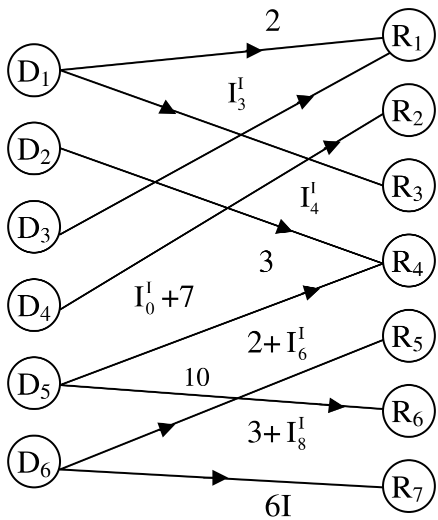


Figure 3.7

Let M be the MOD natural neutrosophic-neutrosophic connection matrix of the MOD bipartite directed graph G.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 \end{array} \\ \begin{bmatrix} 2 & 0 & I_4^I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ I_3^I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_0^I + 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 + I_6^I & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 3 + I_8^I & 0 & 6I \end{bmatrix} \end{array}.$$

We see M is a 6 × 7 MOD natural neutrosophic-neutrosophic rectangular or relational matrix associated with the graph G.

Now we proceed onto describe the MOD natural neutrosophic-neutrosophic Relational Maps model in a line or two. Just before that show the special operations are carried out using these MOD natural neutrosophic-neutrosophic relational matrix operators [66] by some examples.

Example 3.15: Let

$$M = \begin{bmatrix} I_{2I}^{I} & 0 & 2 \\ 0 & 1+I_0^I & 0 \\ 3I & 0 & I_{3I}^{I} \\ 0 & 1 & 0 \\ 2 & 0 & I \\ 0 & 3I & 0 \\ 2+I_0^I & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic-neutrosophic rectangular (relational) matrix operator with entries from $\langle Z_4 \cup I \rangle_I$.

Let X ={$(a_1\ a_2\ \ldots\ a_7)$ / $a_i \in \{0, 1\}$, $1 \leq i \leq 7$} be the MOD domain space of initial state vectors and

Y = {$(b_1\ b_2\ b_3)$ / $b_i \in \{0, 1\}$; $1 \leq i \leq 3$} be the MOD range space of initial state vectors.

Let X_S = {$(a_1 \ldots a_7)$ / $a_i \in \langle Z_4 \cup I \rangle_I$; $1 \leq i \leq 7$} and

Y_s = {$(b_1\ b_2\ b_3)$ / $b_i \in \langle Z_4 \cup I \rangle_I$; $1 \leq i \leq 3$} be the MOD special initial state vectors of MOD domain space and MOD range space respectively.

We find the MOD resultant of MOD state vectors from X or Y or X_S or Y_S on M.

Let x = (1 0 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$$xM = (I_{2I}^{I}\ 0\ 2) = y_1;$$
$$y_1M^t = (I_0^I\ \ 0\ \ I_{2I}^I + I_{3I}^I\ 0\ \ I_{2I}^I + 2I\ 0\ \ I_{2I}^2 + I_0^I) = x_1;$$
$$x_1M = (I_0^I + I_{2I}^2\ \ 0\ \ I_0^I + I_{2I}^2 + 2I) = y_2;$$
$$y_2M^t = (I_0^I + I_{2I}^2\ 0\ \ I_0^I + I_{2I}^2 + I_{3I}^2\ \ 0\ \ I_0^I + I_{2I}^2 + 2I\ 0\ \ I_0^I + I_{2I}^2) = x_2;$$

and so on.

However we are sure that after a finite number of iterations we will arrive at a MOD resultant which will be MOD fixed point pair or a MOD limit cycle pair.

Let x = (0 1 0 0 0 0 0) ∈ X;

To find the effect of x on M.

$$xM = (0\ \ 1 + I_0^I\ \ 0) = y_1;$$
$$y_1M^t = (0\ \ 1 + I_0^I\ \ 0\ 1 + I_0^I\ \ 0\ \ 3I + I_0^I\ 0) = x_1;$$
$$x_1M = (0\ \ 2 + I_0^I + I\ 0) = y_2;$$
$$y_2M^t = (0\ \ 2 + I_0^I + I\ 0\ \ 2 + I + I_0^I\ \ 0\ \ I + I_0^I\ \ 0) = x_2;$$
$$x_2M = (0\ \ I_0^I + I\ 0) = y_3;$$
$$y_3M^t = (0\ \ I + I_0^I\ \ 0\ \ I + I_0^I\ \ 0\ \ 3I + I_0^I\ \ 0) = x_3;$$
$$x_3M = (0\ \ 3I + I_0^I\ \ 0) = y_4;$$
$$y_4M^t = (0\ \ 3I + I_0^I\ \ 0\ \ 3I + I_0^I\ \ 0\ \ I + I_0^I\ \ 0) = x_4;$$
$$x_4M = (0\ \ I + I_0^I\ 0) = y_5\ (=y_3).$$

Thus the MOD resultant is a MOD limit cycle pair given by $\{(0\ I + I_0^I\ \ 0\ I + I_0^I\ \ 0\ 3 + I_0^I\ \ 0), (0\ I + I_0^I\ \ 0)\}$.

Let x = (0 0 1 0 0 0 0) ∈ X, to find the effect of x on M.

$$xM = (3I\ \ 0\ \ I_{3I}^I) = y_1;$$
$$y_1M^t = (I_{2I}^I + I_{3I}^I\ \ \ 0\ I + I_{3I}^I\ \ 0\ 2I + I_{3I}^I\ \ \ 0\ 2I + I_0^I) = x_1;$$

$$x_1M = (3I + I_{2I}^{I} + I_{3I}^{I} + I_0^{I}\ 0\ I_{2I}^{I} + I_{3I}^{I} + 2I) = y_2;$$

$$y_2M^t = (I_{2I}^{I} + I_{3I}^{I} + I_0^{I}\ 0\ I + I_{2I}^{I} + I_{3I}^{I} + I_0^{I}\ 0\ I_{2I}^{I} + I_{3I}^{I} + I_0^{I}\ 0\ 2I + I_0^{I} + I_{2I}^{I} + I_{3I}^{I}) = x_2;$$

and so on.

However we are sure after on finite number of iterations will arrive at a MOD resultant which will be a MOD fixed point pair or a MOD limit cycle pair.

So if we take y = (0 0 1) ∈ Y, we find the effect of y on M.

$$yM^t = (2\ 0\ I_{3I}^{I}\ 0\ I\ 0\ 0) = x_1;$$

$$x_1M = (I_{2I}^{I} + I_{3I}^{I} + 2I\ 0\ I_{3I}^{I} + I) = y_1;$$

$$y_1M^t = (I_0^{I} + I_{2I}^{I} + 2I + I_{3I}^{I}\ 0\ 2I + I_{2I}^{I} + I_{3I}^{I}\ 0\ I_{2I}^{I} + I_{3I}^{I} + I\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I}) = x_2;$$

$$x_2M = (I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I} + I) = y_2;$$

$$y_2M^t = (2I + I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ I + I_{2I}^{I} + I_0^{I} + I_{3I}^{I}\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I}) = x_3;$$

$$x_3M = (I_{2I}^{I} + I_0^{I} + I_{3I}^{I} + 2I\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I} + I) = y_3;$$

$$y_3M^t = (I_0^{I} + I_{2I}^{I} + I_{3I}^{I} + 2I\ 0\ 2I + I_{2I}^{I} + I_0^{I} + I_{3I}^{I}\ 0\ I_{2I}^{I} + I + I_{3I}^{I} + I_0^{I}\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I}) = x_4;$$

$$x_4M = (I_0^{I} + I_{3I}^{I} + I_{2I}^{I}\ 0\ I + I_0^{I} + I_{3I}^{I} + I_{2I}^{I}) = y_4\ (=y_3)\ .$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(2I + I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ 2I + I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ I + I_0^{I} + I_{2I}^{I} + I_{3I}^{I}\ 0\ I_0^{I} + I_{2I}^{I} + I_{3I}^{I}), (I_0^{I} + I_{3I}^{I} + I_{2I}^{I}\ 0\ I + I_0^{I} + I_{3I}^{I} + I_{2I}^{I})\}$.

This is the way this model functions.

It is interesting and important to note that the MOD resultants need not be a pairs whose nodes are just 0 or 1 or I but it can be any value a sum of natural neutrosophic element or just real or 0 or just pure neutrosophic.

The main advantage being that we are sure to arrive at a MOD fixed point pair or a MOD limit cycle pair with entries from $\langle Z_n \cup I \rangle_I$.

At an instant the nodes can take any value from $\langle Z_n \cup I \rangle_I$.

This it is not a biased result where one at each stage thresholds to get a 1 or 0 or I.

So no personal bias is possible and the operation of thresholding has no role to play.

Certainly this new innovative model will be a boon to scientists, socio scientists, technologists and engineers.

Next we proceed onto describe and develop the new notion of MOD natural neutrosophic finite complex number Relational Maps model.

We will first define and develop the essential tools needed for this yet a new model.

$C^I(Z_n) = \{(C(Z_n), \langle I_0^C, I_t^C;$ t in $C(Z_n)$ is a zero divisor, nilpotents or idempotents$\rangle\}$.

We will first describe by examples the notion of MOD natural neutrosophic finite complex number bipartite directed graph.

Example 3.16: Let G be the MOD natural neutrosophic finite complex bipartite directed graph given in the following with edge weights from $C^I(Z_6)$.

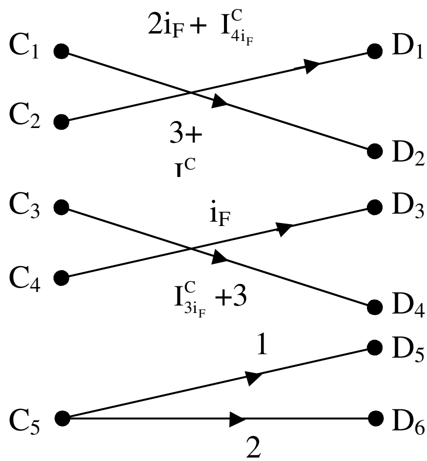


Figure 3.8

Example 3.17: Let H be the MOD finite complex number natural neutrosophic bipartite directed graph with edge weights from $C^I(Z_9)$ given by the following figure.

H =

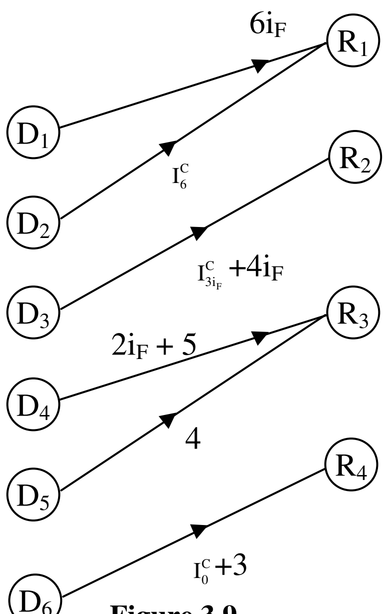


Figure 3.9

Thus G is a MOD finite complex number natural neutrosophic directed bipartite graph if edge weights are from $C^I(Z_n)$; $2 \leq n < \infty$.

We have seen examples of them. Now we proceed onto describe MOD finite complex number natural neutrosophic rectangular or relational matrices, these matrices also serve as operation [66].

We will give a few examples of them.

Example 3.18: Let M be the MOD complex number natural neutrosophic relational matrix with entries from $C^I(Z_{12})$ which is as follows.

$$M = \begin{bmatrix} I_2^C + I_{4i_F}^C + 3 & 0 & 0 & 0 & 0 & I_2^C \\ 0 & 4 & 1+3i_F & 1+i_F & 3i_F & 0 \\ 4+2i_F & 0 & 6 & 0 & 1 & 1+2i_F \\ 0 & 1+i_F & 0 & 7i_F & 0 & 0 \\ 11 & 0 & I_{3i_F}^C + 2 & 0 & I_{4i_F}^C & I_0^C \\ 10+I_0^C & 5i_F & 0 & I_{8i_F}^C & 0 & 0 \\ 0 & 0 & I_{4i_F}^C + i_F & 0 & 0 & 3+I_6^C \\ I_{6i_F+7}^0 & 6+2i_F & 0 & I_0^C & 1+2i_F & 0 \end{bmatrix}.$$

Example 3.19: Let N be the MOD finite complex number natural neutrosophic rectangular matrix with entries from $C^I(Z_{16})$.

$$N = \begin{bmatrix} I_{2i_F}^C + I_{4i_F+6}^C & 0 & 6+2i_F & 0 & 4 & i_F & 0 \\ 0 & I_0^C + I_8^C & 0 & I_{8i_F+4}^C & 0 & 0 & 1 \\ 0 & 0 & 3i_F & 0 & I_4^C & 5 & I_2^C \\ I_4^C + 7 + 6i_F & 4 + I_2^C + i_F & 0 & 3 & i_F & 0 & 3 \end{bmatrix}.$$

Next we proceed onto describe only important operations that is need for us to use in the MOD finite complex number natural neutrosophic Relational Maps model.

Let $D = \{(a_1 \ldots a_n) / a_i \in \{0, 1\}\ 1 \leq i \leq n\}$ and

$R = \{(b_1 \ldots b_m) / b_i \in \{0, 1\}; 1 \leq i \leq m\}$ be the MOD initial state vectors associated with the $n \times m$ MOD finite complex number natural neutrosophic rectangular matrix M.

Let $D_S = \{(a_1 \ldots a_n) / a_i \in C^I(Z_t); 1 \leq i \leq n\}$ $(2 \leq t < \infty)$ and $R_S = \{(b_1\ b_2 \ldots, b_m) / b_j \in C^I(Z_t); 1 \leq j \leq m\}$ be the MOD finite complex special initial state of vectors associated with the MOD finite complex numbers matrix M.

Clearly $R \subseteq R_S$ and $D \subseteq D_S$.

We will now proceed onto describe how the operations are performed on M using elements from R or D or R_S or D_S.

Example 3.20: Let

$$M = \begin{bmatrix} 3 & i_F & 0 & 0 \\ 0 & 0 & 1 & I_0^C \\ 1 & I_0^C & 0 & 0 \\ 0 & 0 & I_2^C & 0 \\ 0 & I_{2i_F}^C & 0 & 0 \\ 1+I_2^C & 0 & 0 & I_{2+2i_F}^C \end{bmatrix}$$

be the MOD finite complex number natural neutrosophic rectangular (relational) operator with entries from $C^I(Z_4)$.

Let $X = \{(a_1\ a_2 \ldots a_6) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_i \in \{0, 1\}; 1 \leq i \leq 4\}$,

$X_S = \{(a_1\ a_2 \ldots a_6) / a_i \in C^I(Z_4); 1 \leq i \leq 6\}$ and

$Y_S = \{(b_1\ b_2\ b_3\ b_4) / b_i \in C^I(Z_4);\ 1 \leq i \leq 4\}$ be the MOD initial state domain and range space of vectors and MOD special initial complete state of domain and range space of vectors respectively related with M.

We now describe how operations are performed using elements of X or Y or X_S or Y_S and M.

Let x = (1 0 0 0 0 0) ∈ X, to find the effect of x on M.

$$xM = (3\ i_F\ 0\ 0) = y_1;$$
$$y_1M^t = (0\ 0\ 3 + I_0^C\ 0\ I_{2i_F}^C\ 3 + I_2^C) = x_1;$$
$$x_1M = (2 + I_0^C + I_2^C\ I_0^C\ 0\ I_{2+2i_F}^C + I_0^C) = y_2;$$
$$y_2M^t = (2 + I_0^C + I_2^C\ I_0^C\ 2 + I_0^C + I_2^C\ 0\ I_0^C\ 2 + I_2^C + I_0^C + I_{2+2i_F}^C) = x_2;$$
$$x_2M = (I_0^C + I_2^C + I_{2+2i_F}^C + 2,\ 2i_F + I_0^C + I_2^C\ I_0^C\ I_0^C + I_{2+2i_F}^C) = y_3;$$
$$y_3M^t = (I_0^C + I_2^C + I_{2+2i_F}^C\ I_0^C\ I_0^C + 2 + I_2^C + I_{2+2i_F}^C\ I_0^C\ I_{2i_F}^C + I_0^C\ 2 + I_0^C + I_2^C + I_{2+2i_F}^C) = x_3;$$

and so on.

Thus we see after a finite number of iterations we will arrive at a MOD resultant which may be a MOD fixed point pair or a MOD limit cycle pair.

Let x = (0 1 0 0 0 0) ∈ X, to find the effect of x on M.

$$xM = (0\ 0\ 1\ I_0^C) = y_1;$$
$$y_1M^t = (0\ 1 + I_0^C\ 0\ I_2^C\ 0\ I_0^C) = x_1;$$
$$x_1M = (I_0^C + I_2^C\ 0\ 1 + I_0^C\ I_0^C) = y_2;$$
$$y_2M^t = (I_0^C + I_2^C\ 1 + I_0^C\ I_0^C + I_2^C\ 0\ 0\ I_0^C + I_2^C) = x_2;$$
$$x_2M = (I_0^C + I_2^C\ I_0^C + I_2^C\ 1 + I_0^C\ I_0^C) = y_3;$$

$$y_3M^t = (I_0^C + I_2^C \quad I_0^C + I_2^C + I_{2i_F}^C \quad I_0^C + I_2^C \quad I_0^C + I_2^C \quad I_0^C + I_{2i_F}^C \quad I_0^C + I_2^C) = x_3;$$

$$x_3M = (I_0^C + I_2^C \quad I_0^C + I_2^C \quad I_0^C + I_2^C + I_{2i_F}^C \quad I_0^C) = y_4;$$

and so on.

We are sure after a finite number of iterations we will arrive at a MOD resultant which may be a MOD fixed point pair or a MOD limit cycle pair.

Let $x = (0\ 0\ 0\ 0\ I_0^C\ 0) \in X_S$, to find the effect of x on M.

$$xM = (0 \quad I_0^C \quad 0 \quad 0) = y_1;$$
$$y_1M^t = (I_0^C \quad 0 \quad I_0^C \quad 0 \quad I_0^C \quad 0) = x_1;$$
$$x_1M = (I_0^C \quad I_0^C \quad 0 \quad 0) = y_2;$$
$$y_2M^t = (I_0^C \quad 0 \quad I_0^C \quad 0 \quad I_0^C \quad I_0^C) = x_2;$$
$$x_2M = (I_0^C \quad I_0^C \quad 0 \quad I_0^C) = y_3;$$
$$y_3M^t = (I_0^C \quad I_0^C \quad I_0^C \quad I_0^C \quad I_0^C \quad I_0^C) = x_3;$$
$$x_3M = (I_0^C \quad I_0^C \quad I_0^C \quad I_0^C) = y_4;$$
$$y_4M^t = (I_0^C \quad I_0^C \quad I_0^C \quad I_0^C \quad I_0^C \quad I_0^C) = x_4\ (=x_3).$$

MOD realized resultant is a MOD realized fixed point pair given by $\{(I_0^C\ I_0^C\ I_0^C\ I_0^C\ I_0^C\ I_0^C), (I_0^C\ I_0^C\ I_0^C\ I_0^C)\}$.

Let $y = (0\ 0\ 0\ 1) \in Y$, to find the effect of y on M.

$$yM^t = (0 \quad I_0^C\ 0\ 0\ 0 \quad I_{2+2i_F}^C) = x_1;$$
$$x_1M = (I_{2+2i_F}^C + I_0^C \quad 0 \quad I_0^C \quad I_0^C) = y_1;$$
$$y_1M^t = (I_{2+2i_F}^C + I_0^C \quad I_0^C \quad I_{2+2i_F}^C + I_0^C \quad I_0^C \quad 0 \quad I_0^C + I_{2+2i_F}^C) = x_2;$$
$$x_2M = (I_{2+2i_F}^C + I_0^C \quad I_{2+2i_F}^C + I_0^C \quad I_0^C \quad I_0^C) = y_2;$$
$$y_2M^t = (I_{2+2i_F}^C + I_0^C \quad I_0^C \quad I_{2+2i_F}^C + I_0^C \quad I_0^C \quad I_0^C \quad I_{2+2i_F}^C + I_0^C) = x_3\ (= x_2).$$

Thus the MOD realized resultant is a MOD realized fixed point pair given by

$$\{(I^C_{2+2i_F} + I^C_0 \;\; I^C_0 \;\; I^C_{2+2i_F} + I^C_0 \;\; I^C_0 \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F}),$$
$$(I^C_0 + I^C_{2+2i_F} \;\; I^C_{2+2i_F} + I^C_0 \;\; I^C_0 \;\; I^C_0)\}.$$

Now suppose we perform the same operation but at each stage do only the process of updating then we study the effect of y = (0 0 0 1) on M.

$$yM^t = (0 \;\; I^C_0 \;\; 0 \; 0 \; 0 \;\; I^C_{2+2i_F}) = x_1;$$
$$x_1M \to (I^C_{2+2i_F} + I^C_0 \;\; 0 \;\; I^C_0 \;\; 1) = y_1;$$
$$y_1M^t = (I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; 0 \;\; I^C_{2+2i_F} + I^C_0) = x_2;$$
$$x_2M \to (I^C_0 + I^C_{2+2i_F} \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; 1) = y_2;$$
$$y_2M^t = (I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F}) = x_3;$$
$$x_3M \to (I^C_0 + I^C_{2+2i_F} \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; 1) = y_3 \; (=y_2).$$

Thus the MOD resultant is a MOD realized fixed point pair given by $\{(I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F}), (I^C_0 + I^C_{2+2i_F} \;\; I^C_0 + I^C_{2+2i_F} \;\; I^C_0 \;\; 1)\}$.

Clearly the mode of updating gives a MOD realized resultant which is also a MOD realized fixed point pair but different from the one for which the resultants are not updated at each stage.

Let x = (0 0 0 0 0 1) $\in$ X to find the effect of x on M.

$$xM = (1 + I^C_2 \;\; 0 \; 0 \;\; I^C_{2+2i_F}) = y_1;$$
$$y_1M^t = (3 + I^C_2 \;\; I^C_0 \;\; 1 + I^C_2 \;\; 0 \; 0 \;\; 1 + I^C_2 + I^C_2) = x_1;$$
$$x_1M = (3 + I^C_2 + I^C_0 \;\; 3i_F + I^C_2 + I^C_0 \;\; I^C_0 \;\; I^C_0 + I^C_{2+2i_F}) = y_1;$$

$$y_1M^t = (2 + I_2^C + I_0^C \;\; I_0^C \;\; 3 + I_2^C + I_0^C \;\; I_0^C \; I_{2i_F}^C + I_0^C \;\; 3 + I_0^C + I_2^C) = x_2;$$

$$x_2M = (1 + I_0^C + I_2^C \;\; 2i_F + I_2^C + I_0^C \;\; I_0^C \;\; I_0^C + I_{2+2i_F}^C) = y_2;$$

$$y_2M^t = (1 + I_0^C + I_2^C \;\; I_0^C \; 1 + I_0^C + I_2^C \;\; I_0^C \;\; I_0^C \;\; I_0^C + I_{2+2i_F}^C + 1)\;;$$

and so on.

We will after a finite number of iterations arrive at a MOD realized fixed point pair or a MOD realized limit cycle pair.

Example 3.21: Let

$$S = \begin{bmatrix} 2 & 0 & 0 & 4+i_F & I_{3i_F}^C & 0 \\ 0 & 0 & 1+I_2^C & 0 & 0 & 1 \\ 3 & I_0^C & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the MOD finite complex number natural neutrosophic rectangular matrix operator with entries from $C^I(Z_9)$.

Let $X = \{(a_1\ a_2\ a_3) / a_i \in \{0, 1\}; 1 \leq i \leq 3\}$ and

$Y = \{(a_1\ a_2 \ldots a_6) / a_i \in \{0, 1\}, 1 \leq i \leq 6\}$ be the MOD finite complex domain and range space of initial state vectors respectively associated with S.

Let $x = (1\ 0\ 0) \in X$ to find the effect of x on S.

$$xS = (2\ 0\ 0\ 4 + i_F \;\; I_{3i_F}^C \;\; 0) = y_1;$$

$$y_1S^t = (1 + 8i_F + I_0^C \;\; 0 \;\; 1 + i_F) = x_1;$$

$$x_1S = (5 + i_F + I_0^C \;\; I_0^C \;\; 0 \;\; 3 + I_0^C \;\; I_{3i_F}^C + I_0^C \;\; 0) = y_2;$$

$$y_2M^t = (4 + 5i_F + I_0^C \;\; 3 + I_0^C + I_2^C \;\; I_0^C + 3i_F) = x_2;$$

$$x_2M = (8 + i_F + I_0^C \; I_0^C \; 3 + I_2^C + I_0^C \; 2 + I_0^C \; I_{3i_F}^C + I_0^C \; 3 + I_0^C + I_2^C) = x_3;$$

$$x_3M = (6 + 4i_F + I_0^C + I_2^C \; 6 + I_2^C + I_0^C \; 7 + I_0^C + 3i_F) = y_3;$$

and so on.

We are sure after a finite number of iterations we will arrive at a MOD realized resultant which can be a MOD realized fixed point pair or a MOD realized limit cycle pair.

We propose the following theorem.

THEOREM 3.1: *Let $M = (m_{ij})$ be a $s \times t$ MOD finite complex number natural neutrosophic rectangular matrix with entries from $C^I(Z_n)$.*

Let $X = \{(a_1 \ldots a_s) / a_i \in \{0, 1\}; 1 \le i \le s\}$,

$Y = \{(b_1\, b_2 \ldots b_t) / b_j \in \{0, 1\}; 1 \le j \le t\}$,

$X_S = \{(a_1 \ldots a_s) / a_i \in C^I(Z_n); 1 \le i \le s\}$ and

$Y_S = \{(b_1 \ldots b_t) / b_j \in C^I(Z_n); 1 \le j \le t\}$ be the MOD initial state vectors of domain and range spaces and MOD special instantaneous state vectors of domain and range space respectively associated with M.

If $x \in X$ (or Y or X_S or Y_S) then the MOD realized resultant pair in general is different for x when the MOD realized resultant is updated at each stage of the iterations.

Proof is direct and hence left as an exercise to the reader.

Let G be the MOD finite complex natural neutrosophic number bipartite directed graph with edge weights from $C^I(Z_{10})$ given by the following figure.

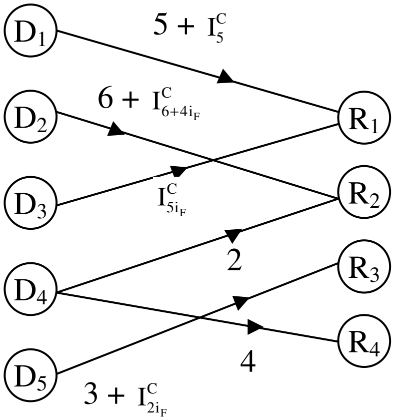


Figure 3.10

The MOD relational or connection matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 5+I_5^C & 0 & 0 & 0 \\ 0 & 6+I_{6+4i_F}^C & 0 & 0 \\ I_{5i_F}^C & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 3+I_{2i_F}^C & 0 \end{bmatrix} \end{array}.$$

Example 3.22: Let H be the MOD finite natural neutrosophic number complex directed bipartite graph with entries from $C^I(Z_n)$ given in the figure.

Let M_1 be the MOD relational connection matrix associated with H.

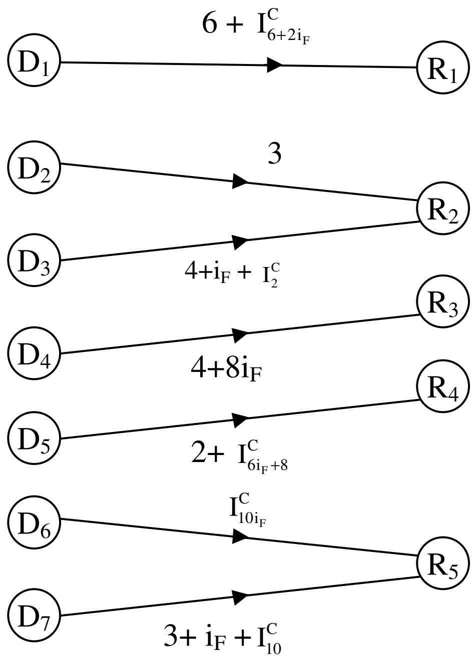


Figure 3.11

$$M_1 = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \left[\begin{array}{ccccc} 6+I^C_{6+2i_F} & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 4+i_F+I^C_2 & 0 & 0 & 0 \\ 0 & 0 & 4+8i_F & 0 & 0 \\ 0 & 0 & 0 & 2+I^C_{6i_F+8} & 0 \\ 0 & 0 & 0 & 0 & I^C_{10i_F} \\ 0 & 0 & 0 & 0 & 3+i_F+I^C_{10} \end{array}\right] \end{array}.$$

Now if P is a problem in hand and the expert wishes to work MOD finite complex number natural neutrosophic Relational Maps model using edge weights from $C^I(Z_6)$.

Let G be the MOD directed bipartite graph associated with the problem P given by the following figure.

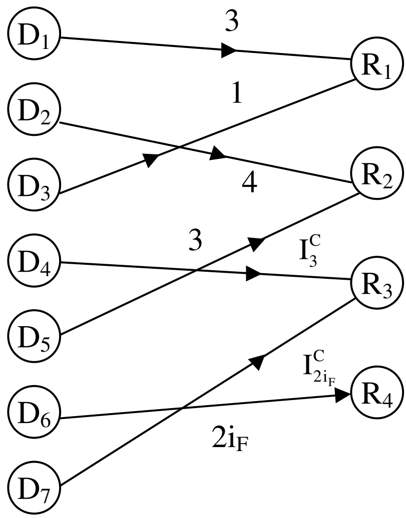


Figure 3.12

The MOD finite complex natural neutrosophic relational matrix associated with G is as follows:

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{array}\begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & I_3^C & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2i_F \\ 0 & 0 & I_{2i_F}^c & 0 \end{bmatrix} \end{array}.$$

Let X = $\{(a_1\ a_2\ \ldots\ a_7)\ /\ a_i \in \{0, 1\};\ 1 \leq i \leq 7\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_i \in \{0, 1\}; 1 \leq i \leq 4\}$ be the domain and range MOD initial state vector respectively associated with M.

We will find the effect of x = (1 0 0 0 0 0 0) ∈ X on M is as follows.

$xM = (3\ 0\ 0\ 0) = y_1;$
$y_1M^t = (3\ 0\ 3\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0) = x_2.$

Thus the MOD realized resultant is a MOD realized fixed point given by {(0 0 0 0 0 0 0), (0 0 0 0)}.

Let x = (0 1 0 0 0 0 0) ∈ X; to find the effect of x on M.

$xM = (0\ 4\ 0\ 0) = y_1;$
$y_1M^t = (0\ 4\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 4\ 0\ 0) = y_2\ (=y_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by {(0 4 0 0 0 0 0), (0 4 0 0)}.

Let x = (0 0 1 0 0 0 0) ∈ X, to find the effect of x on M.

$xM = (1\ 0\ 0\ 0) = y_1;$
$y_1M^t = (3\ 0\ 1\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (4\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 0\ 4\ 0\ 0\ 0\ 0) = x_2;$
$x_2M = (4\ 0\ 0\ 0) = y_3\ (= y_2).$

Thus the MOD realized resultant is a MOD realized fixed pair given by {(0 0 4 0 0 0 0), (4 0 0 0)}.

Let x = (0 0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ I_3^C\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ I_3^C\ 0\ 0\ I_0^C) = x_1;$

$x_1M = (0\ 0\ I_3^C + I_0^C\ 0) = y_2;$

$y_2M^t = (0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_0^C) = x_2;$

$x_2M = (0\ 0\ I_3^C + I_0^C\ 0) = y_3\ (= y_2).$

Thus the MOD realized resultant is MOD realized fixed point pair given by $\{(0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_0^C), (0\ 0\ I_3^C + I_0^C\ 0)\}$.

Let x = (0 0 0 0 1 0 0) $\in$ X, to find the effect of x on M.

$xM = (0\ 3\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 3\ 0\ 0) = x_1;$
$x_1M = (0\ 3\ 0\ 0) = y_2;$
$y_2M^t = (0\ 0\ 0\ 0\ 3\ 0\ 0) = x_2\ (= x_1).$

The MOD realized resultant is a MOD realized fixed point pair given by {(0 0 0 0 3 0 0), (0 3 0 0)}.

Let x = (0 0 0 0 0 1 0) $\in$ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 2i_F) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 2\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 4i_F) = y_2;$
$y_2M^t = (0\ 0\ 0\ 0\ 0\ 4\ 0) = x_2;$
$x_2M = (0\ 0\ 0\ 4) = y_3;$
$y_3M = (0\ 0\ 0\ 0\ 0\ 2i_F\ 0) = x_3;$
$x_3M = (0\ 0\ 0\ 2) = y_4;$
$y_4M^t = (0\ 0\ 0\ 0\ 0\ 4i_F\ 0) = x_4;$
$x_4M = (0\ 0\ 0\ 4) = y_5\ (=y_3).$

Thus the MOD realized resultant is a MOD realized limit cycle pair given by $\{(0\ 0\ 0\ 0\ 0\ 2i_F\ 0), (0\ 0\ 0\ 4)\}$.

Let x = (0 0 0 0 0 0 1) $\in$ X, to find the effect of x on M.

$xM = (0\ 0\ I_{2i_F}^C\ 0) = y_1;$

$y_1M^t = (0\ 0\ 0\ I_0^C\ 0\ 0\ I_2^C) = x_1;$

$x_1M = (0\ 0\ I_0^C + I_{4i_F}^C\ \ 0) = y_2;$

$y_2M^t = (0\ 0\ 0\ I_0^C\ 0\ 0\ I_0^C + I_2^C) = x_2;$

$x_2M = (0\ 0\ I_0^C + I_{4i_F}^C\ \ 0) = y_3\ (= y_2).$

Thus the MOD realized resultant is a MOD realized fixed point pair given by $\{(0\ 0\ 0\ I_0^C\ 0\ 0\ I_0^C + I_2^C), (0\ 0\ I_0^C + I_{4i_F}^C\ 0)\}$.

Let y = (1 0 0 0) ∈ Y, to find the effect of y on M.

$yM^t = (3\ 0\ 1\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (4\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 4\ 0\ 0\ 0\ 0) = x_2;$
$x_2M = (4\ 0\ 0\ 0) = y_2\ (= y_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by {(0 0 4 0 0 0 0), (4 0 0 0)}.

Y = (0 1 0 0) ∈ Y, to find the effect of y on M.

$yM^t = (0\ 4\ 0\ 0\ 3\ 0\ 0) = x_1;$
$x_1M = (0\ 1\ 0\ 0) = y_1\ (=y).$

Thus the MOD realized resultant is a MOD special classical fixed point pair given by {(0 4 0 0 3 0 0), (0 1 0 0)}.

Let y = (0 0 1 0) ∈ Y to find the effect of y on M.

$yM^t = (0\ 0\ 0\ I_3^C\ 0\ 0\ I_{2i_F}^C) = x_1;$

$x_1M = (0\ 0\ I_3^C + I_2^C\ \ 0) = y_1;$

$y_1M^t = (0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_{4i_F}^C + I_0^C) = x_2;$

$x_2M = (0\ 0\ I_3^C + I_0^C + I_4^C\ \ 0) = y_2;$

$y_2M^t = (0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_0^C + I_{2i_F}^C) = x_3;$

$x_3M = (0\ 0\ I_3^C + I_0^C + I_2^C\ \ 0) = y_3;$

$y_3M^t = (0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_0^C + I_{4i_F}^C) = x_4\ (= x_2).$

Thus the MOD realized resultant is MOD limit cycle pair given by

$\{(0\ 0\ 0\ I_3^C + I_0^C\ 0\ 0\ I_0^C + I_{4i_F}^C),\ (0\ 0\ I_3^C + I_2^C\ 0)$ or

$(0\ 0\ I_3^C + I_0^C + I_4^C\ 0)$ or $(0\ 0\ I_3^C + I_0^C + I_2^C\ 0)\}$.

Let y = (0 0 0 1) ∈ Y, to find the effect of y on M.

$yM^t = (0\ 0\ 0\ 0\ 0\ 2i_F\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 2) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 4i_F\ 0) = x_2;$
$x_2M = (0\ 0\ 0\ 4) = y_2;$
$y_2M^t = (0\ 0\ 0\ 0\ 0\ 2i_F\ 0) = x_3\ (= x_1).$

Thus the MOD resultant is a MOD limit cycle pair.

We see we can get various types of MOD resultants and we are not in a position to predict anything related with it.

However we leave this study for the reader as it is considered to be a matter of routine.

Thus the new MOD finite complex number natural neutrosophic Relational Maps model built using $C^I(Z_n)$ has several advantageous for in the first place it can give any state to the node from $C^I(Z_n)$ depending only on the initial state vector.

Secondly one need not use the method of thresholding at each stage for mod n takes the role of it.

Certainly this not only saves time but also it has the power to stop difference of opinion while fixing the thresholding values for that job of thresholding never comes to play any role.

So the nodes can be complex real or natural neutrosophic complex or real or zero.

Next we proceed onto briefly describe the MOD natural neutrosophic dual number bipartite directed graph by examples.

Example 3.23: Let G be the MOD natural neutrosophic dual number bipartite directed graph with edge weights from $\langle Z_{10} \cup g\rangle_I$ G is the MOD natural neutrosophic dual number bipartite directed graph.

The graph G is as follows.

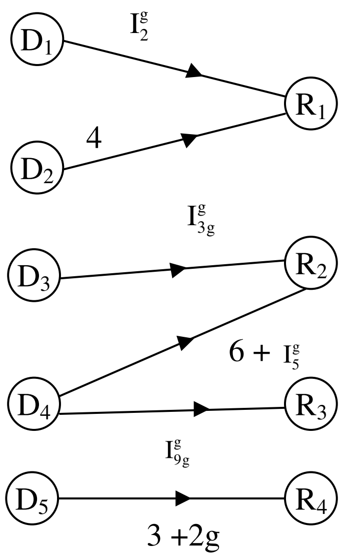


Figure 3.13

Example 3.24: Let G be a MOD natural neutrosophic dual number bipartite directed graph with edge weights from $\langle Z_{11} \cup g\rangle_I$ given by the following figure.

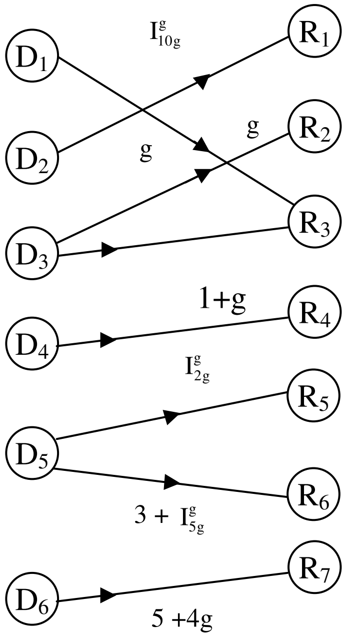


Figure 3.14

Next we proceed onto describe the notion of MOD natural neutrosophic dual number rectangular or relational matrix operator [66] by some examples.

Example 3.25: Let

$$M = \begin{bmatrix} 2+g & I_{2g}^g & I_{3g}^g + 4 \\ 0 & 1 & 0 \\ 5 & 2+g & 6+I_{5g}^g \\ I_0^g & 0 & 0 \\ 1 & 4 & 5g \\ 6g+I_g^g & 0 & g \end{bmatrix}$$

be the MOD natural neutrosophic dual number relational matrix with entries from $\langle Z_7 \cup I\rangle_I$.

Example 3.26: Let

$$S = \begin{bmatrix} g + I^g_{4g} & 0 & 2g+4 & 0 & I^g_g + 2 & 0 & I^g_{5g} + 2 \\ I^g_8 & I^g_5 & I^g_{2+4g} & 1 & 0 & 5g+1 & 0 \\ 9+2g & 0 & 0 & 1+4g & I^g_0 & 0 & I^g_{6g} \end{bmatrix}$$

be the MOD natural neutrosophic dual number relational matrix with entries from $\langle Z_{10} \cup g\rangle_I$.

Now we proceed onto define special type of operations using these MOD natural neutrosophic dual number relation matrix operator.

Further for some properties about $\langle Z_n \cup g\rangle_I$ please refer [60].

$X = \{(a_1 \ldots a_n) / a_i \in \{0, 1\}; 1 \leq i \leq n\}$ will be known as the MOD initial state vectors of MOD domain space associated with the $n \times m$ MOD natural neutrosophic dual number relational matrix M.

$Y = \{(b_1\, b_2 \ldots b_n) / b_j \in \{0, 1\}; 1 \leq j \leq m\}$ is the MOD initial state vectors of the MOD range space associated with $n \times m$ MOD natural neutrosophic dual number $n \times n$ relational matrix M.

We give a few of the related operations using X, Y and M by some examples.

Example 3.27: Let

$$S = \begin{bmatrix} 3+g & 0 & I_{2g}^{g} \\ 0 & 1 & 0 \\ g & 0 & 0 \\ 1 & 2+I_{g}^{g} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

be the MOD natural neutrosophic dual number matrix operator with entries from $\langle Z_4 \cup g\rangle_I$.

Let X = $\{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \{0, 1\}; 1 \leq i \leq 5\}$ and

Y = $\{(b_1\ b_2\ b_3) / b_j \in \{0, 1\}, 1 \leq i \leq 3\}$ be the MOD domain and range space of initial state vectors associated with S.

Let x = (1 0 0 0 0) $\in$ X, to find the effect of x on S.

$$xS = (3 + g\ \ 0\ \ I_{2g}^{g}) = y_1;$$
$$y_1S^t = (1 + 2g + I_0^g\ \ 0\ \ 3g\ \ 3 + g\ \ I_{2g}^{g}) = x_1;$$
$$x_1S = (2 + I_0^g\ \ 2 + 2g + I_g^g\ \ I_{2g}^{g}) = y_2;$$
$$y_2S^t = (2 + 2g + I_0^g\ \ 2 + 2g + I_g^g\ \ 2g + I_0^g\ \ 2 + I_g^g + I_0^g\ \ I_{2g}^{g}) = x_2;$$
$$x_2S = (I_g^g + I_0^g\ \ 2 + 2g + I_g^g + I_0^g\ \ I_{2g}^{g} + I_0^g) = y_3;$$
$$y_3S^t = (I_g^g + I_0^g\ \ 2 + 2g + I_g^g + I_0^g\ \ I_g^g + I_0^g\ \ I_g^g + I_0^g\ \ I_{2g}^{g} + I_0^g) = x_3;$$

and so on.

This is the way the MOD realized resultant is obtained after a finite number of iterations.

Let x = (0 0 0 1 0) $\in$ X. To find the effect of x on S.

$xS = (1\ 2 + I_g^g\ 0) = y_1;$

$y_1S^t = (3 + g\ \ 2 + I_g^g\ \ g\ \ 1 + I_0^g + I_g^g\ \ 0) = x_1;$

$x_1S = (2 + 2g + I_0^g + I_g^g\ \ I_g^g + I_0^g\ \ I_{2g}^g) = y_2;$

$y_2S^t = (2 + I_0^g + I_g^g\ \ I_g^g + I_0^g\ 2g + I_0^g + I_g^g\ \ 2 + 2g + I_0^g + I_g^g\ \ I_{2g}^g) = x_2;$

$x_2S = (I_0^g + I_g^g\ \ I_g^g + I_0^g\ \ I_{2g}^g + I_0^g) = y_3;$

$y_3S^t = (I_0^g + I_g^g\ \ I_0^g + I_g^g\ I_0^g + I_g^g\ \ I_0^g + I_g^g\ I_{2g}^g + I_0^g) = y_3;$

$x_3S = (I_0^g + I_g^g\ \ I_0^g + I_g^g\ I_{2g}^g + I_0^g) = y_4\ (=y_3).$

Thus the MOD realized resultant is a MOD fixed point pair given by $\{(I_0^g + I_g^g\ I_0^g + I_g^g\, I_g^g + I_0^g\ I_0^g + I_g^g\ \ I_{2g}^g + I_0^g),$
$(I_0^g + I_g^g\ I_0^g + I_g^g\ I_{2g}^g + I_0^g)\}.$

Let y = (1 0 0) $\in$ Y, to find the effect of y on S.

$yS^t = (3 + g\ 0\ g\ 1\ 0) = x_1;$
$x_1S = (2 + 2g\ 2 + I_g^g\ \ I_{2g}^g) = y_1;$

$y_1S^t = (2 + 2g + I_0^g\ \ 2 + I_g^g\ \ 2g\ \ 2g + I_g^g\ \ I_{2g}^g) = x_2;$

$x_2S = (2 + 2g + I_0^g + I_g^g\ \ 2 + I_g^g + I_0^g\ \ I_{2g}^g + I_0^g) = y_2;$

$y_2S^t = (2 + I_g^g + I_0^g\ \ 2 + I_g^g + I_0^g\ 2g + I_0^g + I_g^g\ 2g + I_0^g + I_g^g\ \ I_0^g + I_{2g}^g) = x_3;$

and so on.

Certainly after a finite number of iterations we arrive at a MOD realized resultant.

Let y = (0 0 1) $\in$ Y, to find the effect of y on S.

$yS^t = (I_{2g}^g\ \ 0\ 0\ \ 0\ 2) = x_1;$

$x_1S = (I^g_{2g}\ 0\ \ I^g_0) = y_1;$

$y_1S^t = (I^g_{2g}\ \ 0\ \ I^g_{2g}\ \ I^g_{2g}\ \ I^g_0) = x_2;$

$x_2S = (I^g_{2g}\ \ I^g_{2g} + I^g_0\ \ I^g_0) = y_2;$

$y_2S^t = (I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g}\ \ I^g_{2g} + I^g_0\ \ I^g_0) = x_3;$

$x_3S = (I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0) = y_3;$

$y_3S^t = (I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0)$
$= x_4;$

$x_4S = (I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_{2g} + I^g_0) = y_4\ (= y_3).$

Thus the MOD realized resultant is a MOD realized fixed point pair given by $\{(I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g}), (I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g})\}$.

Let x = (1 0 1 0 1) $\in$ X to find the effect of x on M.

$xS = (3 + 2g\ 0\ \ I^g_{2g} + 2) = y_1;$

$y_1S^t = (1 + g + I^g_{2g} + I^g_0\ \ 0\ \ 3g\ \ 3 + 2g\ \ I^g_{2g}) = x_1;$

$x_1S = (I^g_0 + I^g_{2g} + 2 + 2g\ \ 2 + I^g_g\ \ I^g_{2g} + I^g_0) = y_2;$

$y_2S^t = (2 + I^g_0 + I^g_{2g}\ \ 2 + I^g_g\ \ 2g + I^g_0\ \ 2 + 2g + I^g_0$
$+ I^g_{2g}\ \ I^g_0 + I^g_{2g}) = x_2;$

$x_2S = (I^g_0 + I^g_{2g}\ \ 2 + I^g_g + I^g_0\ \ I^g_0 + I^g_{2g}) = y_3;$

$y_3S^t = (I^g_0 + I^g_{2g}\ \ 2 + I^g_g + I^g_0\ \ I^g_0 + I^g_{2g}\ \ I^g_0 + I^g_{2g} + I^g_{2g}$
$I^g_0 + I^g_{2g}) = x_3;$

$x_3S = (I^g_0 + I^g_{2g} + I^g_g\ \ 2 + I^g_g + I^g_0\ \ I^g_0 + I^g_{2g}) = y_4;$

$y_4S^t = (I^g_0 + I^g_{2g} + I^g_g\ \ I^g_0 + I^g_g + 2\ \ I^g_0 + I^g_g + I^g_{2g}\ \ I^g_0 +$
$I^g_g + I^g_{2g}\ \ I^g_0 + I^g_{2g}) = x_4;$

and so on.

We are sure after a finite number of iterations we will arrive at a MOD fixed point pair or MOD limit cycle pair.

Next for the MOD initial state vector from the MOD range space Y.

We take y = (0 0 1) but follow the operation which updates at each stage.

$$yS^t = (I_{2g}^g\ 0\ 0\ 0\ 2) = x_1;$$
$$x_1S \to (I_{2g}^g\ \ 0\ 1) = y_1;$$
$$y_1S^t = (I_{2g}^g\ \ 0\ \ I_{2g}^g\ \ I_{2g}^g\ \ 2) = x_2;$$
$$x_2S \to (I_{2g}^g\ \ I_{2g}^g + I_0^g\ \ 1) = y_2;$$
$$y_2S^t = (I_{2g}^g\ \ I_{2g}^g + I_0^g\ \ I_{2g}^g\ \ I_{2g}^g + I_0^g\ \ 2) = x_3;$$
$$x_3S = (I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ 1) = y_3;$$
$$y_3S^t = (I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ 2) = x_4;$$
$$x_4S \to (I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ 1) = y_4\ (=y_3).$$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ 2), (I_{2g}^g + I_0^g\ \ I_{2g}^g + I_0^g\ \ 1)\}.$$

Let us illustrate this by one more example.

Example 3.28: Let

$$B = \begin{bmatrix} g+2 & 0 & 1 & I_{2g}^2 & 0 & 2g & I_{4g}^g \\ 0 & 2g & 0 & 0 & I_g^g & 0 & I_0^g \\ 4 & 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number relational matrix operator with entries from $\langle Z_5 \cup g\rangle_I$.

Let $X = \{(a_1\ a_2\ a_3) / a_i \in \{0, 1\}, 1 \leq i \leq 3\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_7) / b_i \in \{0,1\};\ 1 \leq i \leq 7\}$ be the MOD special initial state vectors domain or range space respectively associated with B.

Let $X_S = \{(a_1\ a_2\ a_3) / a_i \in \langle Z_5 \cup g\rangle_I;\ 1 \leq i \leq 3\}$ and

$Y_S = \{(b_1\ b_2\ \ldots\ b_7) / b_i \in \langle Z_5 \cup g\rangle_I;\ 1 \leq i \leq 7\}$ be the MOD special initial state vectors of the domain and range space respectively associated with the MOD natural neutrosophic dual number matrix B.

Let $x = (1\ 0\ 0) \in X$,

$xB = (g + 2\ \ 0\ \ 1\ \ I^g_{2g}\ \ 0\ \ 2g\ \ I^g_{4g}) = y_1$;

$y_1B^t = (I^g_0 + 4g\ \ I^g_0\ \ 4g) = x_1$;

$x_1B = (I^g_0 + 4g\ \ I^g_0\ \ I^g_0 + 2g\ \ I^g_0 + I^g_{2g}\ \ I^g_0\ \ I^g_0\ \ I^g_0) = y_2$;

$y_2B^t = (I^g_0 + I^g_{2g} + 3g\ \ I^g_0\ \ I^g_0) = x_2$;

$x_2B = (I^g_0 + I^g_{2g} + g\ \ I^g_0\ \ I^g_0 + I^g_{2g}\ \ I^g_{2g} + I^g_0\ \ I^g_0$

$g + I^g_0 + I^g_{2g}\ \ I^g_{4g} + I^g_0) = y_3$.

Let $y = (1\ 0\ 0\ 0\ 0\ 0\ 0) \in Y$, to find the effect of y on B.

$yB^t = (g + 2\ \ 0\ \ 4) = x_1$;

$x_1B = (4g\ \ 0\ \ g\ \ I^g_{2g}\ \ 0\ 4g\ \ I^g_{4g}) = y_1$;

$y_1B^t = (4g + I^g_0\ \ \ I^g_0\ \ 0) = x_2$;

$x_2B = (3g + I^g_0\ \ I^g_0\ \ 4g + I^g_0\ \ I^g_{2g} + I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0) = y_2$;

$y_2B^t = (I^g_0\ \ I^g_0\ \ I^g_0) = x_3$;

$x_3B = (I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0) = y_3$;

$y_3B^t = (I^g_0\ \ I^g_0\ \ I^g_0) = x_4\ (= x_3)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(I^g_0\ \ I^g_0\ \ I^g_0), (I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0\ \ I^g_0)\}$.

Let us now use the same y = (1 0 0 0 0 0) but use the updating procedure and find the MOD resultant on B.

$yB^t = (g + 2 \;\; 0 \;\; 4) = x_1;$
$x_1B \to (1 \; 0 \; g \;\; I^g_{2g} \;\; 0 \;\; 4g \;\; I^g_{4g}) = y_1;$
$y_1B^t = (I^g_0 + 2g + 2 \;\; I^g_0 \;\; 4 + 2g) = x_2;$
$x_2B \to (1 \;\; I^g_0 \;\; g + I^g_0 \;\; I^g_0 + I^g_{2g} \;\; I^g_0 \;\; 4g + I^g_0 \;\; I^g_{4g} + I^g_0) = y_2;$
$y_2B^t = (2g + 2 + I^g_0 \;\; I^g_0 + I^g_0 \;\; 4 + 2g + I^g_0) = x_3;$
$x_3B \to (1 \;\; I^g_0 + I^g_g \;\; I^g_0 + g \;\; I^g_{2g} + I^g_0 \;\; I^g_0 \;\; 2g \;\; I^g_{4g} + I^g_0) = y_3$
and so on.

We are sure after a finite number of iterations we will arrive at a MOD realized fixed point pair or a MOD realized limit cycle pair.

Since this work is realized as a matter of routine we leave it as an exercise to the reader the task of finding MOD realized fixed points by usual method and the method of updating.

Next we study the effect of y = (0 0 0 0 0 1 0) ∈ Y.

$yB^t = (2g \;\; 0 \;\; 0) = x_1;$
$x_1B \to (4g \;\; 0 \;\; 2g \;\; I^g_{2g} \;\; 0 \; 1 \;\; I^g_{4g}) = y_1;$
$y_1B^t = (I^g_0 + 2g \;\; I^g_0 \;\; 0) = x_2;$
$x_2B \to (I^g_0 + 4g \;\; I^g_0 \;\; I^g_0 + 2g \;\; I^g_{2g} + I^g_0 \;\; I^g_0 \;\; 1 \;\; I^g_{4g} + I^g_0) = y_2;$
$y_2B^t = (I^g_0 + 2g \;\; I^g_0 \;\; I^g_0) = x_3;$
$x_3B = (4g + I^g_0 \;\; I^g_0 \;\; I^g_0 + 2g \;\; I^g_0 + I^g_0 \;\; I^g_0 \;\; 1 \;\; I^g_{4g} + I^g_0) = y_3;$
$y_3B^t = (I^g_0 + 2g \;\; I^g_0 \;\; I^g_0) = x_4 \; (=x_3).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(I^g_0 + 2g \, I^g_0 \;\; I^g_0), (4g + I^g_0 \;\; I^g_0 \;\; I^g_0 + 2g \;\; I^g_0 + I^g_{2g} \;\; I^g_0 \;\; 1 \;\; I^g_0 + I^g_{4g})\}.$$

Let x = (I_0^g 0 0) ∈ X_S, to find the effect of x on B,

$xB = (I_0^g\ 0\ I_0^g\ I_0^g\ 0\ I_0^g\ I_0^g) = y_1;$
$y_1B^t = (I_0^g\ I_0^g\ I_0^g) = x_1;$
$x_1B = (I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g) = y_2;$
$y_2B^t = (I_0^g\ I_0^g\ I_0^g) = x_2\ (=x_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g), (I_0^g\ I_0^g\ I_0^g)\}$.

We make the following observation even if x is updated we will get the same MOD resultant.

Let y = (0 0 0 0 0 0 I_g^g) ∈ Y_S, to find the effect of y on B

$yB^t = (I_0^g\ I_0^g\ 0) = x_1;$
$x_1B \rightarrow (I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g) = y_1;$
$y_1B^t = (I_0^g\ I_0^g\ I_0^g) = x_2;$
$x_2B \rightarrow (I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g).$

Thus the MOD realized resultant is a MOD fixed point pair given by $\{(I_0^g\ I_0^g\ I_0^g), (I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g\ I_0^g)\}$.

We can work with any of the MOD initial state vectors from X or Y or X_S or Y_S.

This is realized as a matter of routine so left as an exercise to the reader.

In view of all these the following observation is vital.

THEOREM 3.2: *Let S be the MOD natural neutrosophic dual number relational matrix operator M with entries from $\langle Z_n \cup g\rangle_I$ M is a s × t matrix.*

Let $X = \{(a_1 \ldots a_s) / a_i \in \{0, 1\}; 1 \leq i \leq s\}$ *and*

$Y = \{(b_1\, b_2 \ldots b_t) / b_t \in \{0, 1\}, 1 \leq i \leq t\}$ *be the MOD initial state vectors of the domain and range space respectively associated with M.*

Let $X_S = \{(a_1 \ldots a_s) / a_i \in \langle Z_n \cup g\rangle_I; 1 \leq i \leq s\}$ *and*

$Y_S = \{(b_1\, b_2 \ldots b_t) / b_j \in \langle Z_n \cup g\rangle_I; 1 \leq j \leq t\}$ *be the MOD special initial domain and range space of vectors respectively associated with M.*

In general if $x \in X$ *(or* Y *or* Y_S *or* X_S*) we see the MOD resultant got using M without updating in general is distinctly different from the MOD resultant got using M by updating at each stage.*

The proof is direct and hence left as an exercise to the reader.

Now we make the formal definition of MOD natural neutrosophic dual number Relational Maps model.

Let $M = (m_{ij})_{s \times t}$ be the MOD natural neutrosophic dual number connection matrix associated MOD natural neutrosophic dual number directed bipartite graph.

We see as in case of usual MOD Relational Maps model this M will serve as the MOD dynamical system associated with the MOD natural neutrosophic dual number Relational Maps model.

This functions analogous to other MOD natural neutrosophic Relational Maps models.

The reader is left with the task of constructing such models using $\langle Z_n \cup g\rangle_I$ and work with them.

Next we proceed onto describe and develop the notion of MOD natural neutrosophic special dual like number Relational Maps models.

To this end we need first the notion of MOD natural neutrosophic special dual like number bipartite directed graphs.

Let G be a MOD directed bipartite graph with entries from $\langle Z_n \cup h\rangle_I$. the MOD natural neutrosophic special dual like number set then the directed bipartite graph G as defined as the MOD natural neutrosophic special dual like number bipartite directed graph. First we will supply some examples.

Example 3.29: Let G be the MOD natural neutrosophic bipartite directed graph with edge weights from $\langle Z_6 \cup h\rangle_I$. G will known as the MOD natural neutrosophic special dual like number bipartite directed graph with edge weights from $\langle Z_6 \cup h\rangle_I$. The figure of G is as follows.

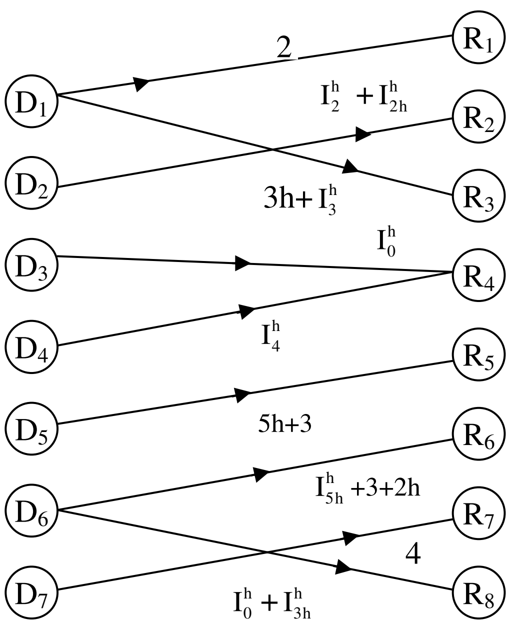


Figure 3.15

Example 3.30: Let H be the MOD natural neutrosophic bipartite graph with edge weights from $\langle Z_{11} \cup I\rangle_I$. Thus H is the MOD natural neutrosophic special dual like number directed graph given by the following figure.

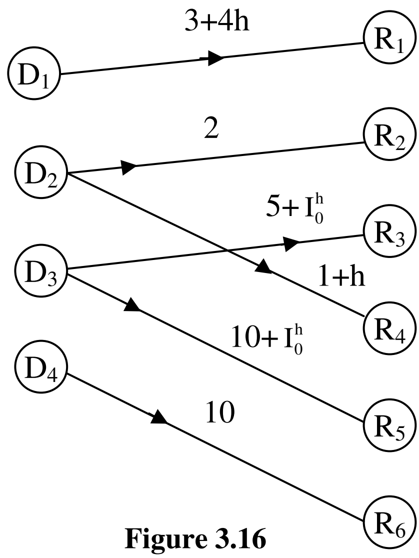


Figure 3.16

Now we proceed onto describe and develop notion of MOD natural neutrosophic special dual like number rectangular (relational) matrix by some examples.

Example 3.31: Let

$$M = \begin{bmatrix} 3+h+I_{2h}^h & I_h^h+4 & 0 \\ 0 & 0 & 4h+I_{8h}^h \\ 4+I_{8h}^h & I_0^h & 0 \\ 0 & 0 & I_2^h \\ 1 & 5 & 3 \\ 0 & 2+I_{4h}^h & 0 \\ 4h+3 & 9h+1 & I_8^h \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number relational (rectangular) matrix with entries from $\langle Z_{10} \cup h\rangle_I$.

This will also act as the operator.

Example 3.32: Let

$$S = \begin{bmatrix} 4h+3 & I_{3h}^{h} & 0 & 6 & 0 & 8h & 7 \\ 0 & 0 & 2+I_{h}^{h} & 0 & 5h+3 & 0 & 0 \\ 1+I_{0}^{h} & 3h+5 & 0 & I_{6}^{h} & 0 & I_{0}^{h} & 2 \\ 0 & 0 & 7+6h & 0 & 4h & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number rectangular matrix operator.

Now we proceed onto describe special type of operations using these MOD natural neutrosophic special dual like number rectangular matrix operator.

To this end we need the notion of MOD domain and MOD range initial state of vectors.

Thus if $M = (m_{ij})$ is a $s \times t$ $(s \neq t)$ MOD natural neutrosophic special dual like number rectangular matrix operator with entries from $\langle Z_n \cup h\rangle_I$; then $X = \{(a_1\ a_2 \ldots a_s) / a_i \in \{0, 1\}; 1 \leq i \leq s\}$ and $Y = \{(b_1\ b_2 \ldots b_t) / b_j \in \{0, 1\}, 1 \leq j \leq t\}$ are the MOD natural neutrosophic special domain and range spaces initial state vectors respectively associated with M.

Let $X_S = \{(a_1\ a_2 \ldots a_s) / a_i \in \langle Z_n \cup h\rangle_I; 1 \leq i \leq s\}$ and

$Y_S = \{(b_1\ b_2 \ldots b_t) / b_j \in \langle Z_n \cup h\rangle_I\ 1 \leq j \leq t\}$ be the MOD domain and range space of special state vectors respectively associated with M.

We will show by examples how operations are carried out.

Example 3.33: Let

$$M = \begin{bmatrix} 3 & 0 & 0 & 2h \\ 0 & h & 0 & 0 \\ 0 & 0 & I_2^h & 1 \\ 0 & 0 & 0 & 2+I_4^h \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number rectangular matrix operator with entries from $\langle Z_6 \cup h\rangle_I$.

Let $X = \{(a_1\ a_2 \ldots a_6) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_j \in \{0, 1\}; 1 \leq j \leq 4\}$ be the MOD natural neutrosophic dual like number domain and range space respectively of initial state vectors associated with M.

Let $x = (1\ 0\ 0\ 0\ 0\ \ 0) \in X$, the effect of x on M is as follows.

$xM = (3\ 0\ 0\ \ 2h) = y_1;$
$y_1M^t = (3 + 4h\ \ 0\ \ 0\ \ 4h + I_4^h\ \ 3\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 4h + I_4^h) = y_2;$
$y_2M^t = (2h + I_4^h\ \ 0\ 4h + I_4^h\ \ 2h + I_4^h\ \ 0\ 0) = x_2;$
$x_2M = (I_4^h\ 0\ I_2^h\ \ I_4^h) = y_3;$
$y_3M^t = (I_4^h\ \ 0\ I_4^h\ \ I_4^h\ \ I_4^h\ \ I_2^h) = x_4;$
$x_4M = (I_4^h\ \ I_4^h\ \ I_2^h\ I_4^h) = y_4;$
$y_4M^t = (I_4^h\ I_4^h\ I_4^h\ I_4^h\ I_4^h\ \ I_2^h) = x_5;$
$x_5M = (I_4^h\ I_4^h\ \ I_2^h\ \ I_4^h) = y_5\ (=y_4).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_4^h\ \ I_4^h\ \ I_4^h\ \ I_4^h\ \ I_4^h\ \ I_2^h), (I_4^h\ I_4^h\ I_4^h\ I_2^h)\}$.

Let $x = (0\ 1\ 0\ 0\ 0\ 0\ 0) \in X$ to find the effect of x on M.

$xM = (0\ \ h\ \ 0\ \ 0) = y_1;$
$y_1M^t = (0\ \ h\ \ 0\ \ 0\ \ 2h\ \ 0) = x_1;$
$x_1M = (2h\ \ 3h\ \ 0\ \ 0) = y_2;$
$y_2M^t = (0\ \ 3h\ \ 0\ \ 0\ \ 2h\ \ 0) = x_2;$
$x_2M = (2h\ \ h\ \ 0\ \ 0) = y_3;$
$y_3M^t = (0\ \ h\ \ 0\ \ 0\ \ 4h\ \ 0) = x_3;$
$x_3M = (4h\ \ 3h\ \ 0\ \ 0) = y_4;$
$y_4M^t = (0\ \ 3h\ \ 0\ \ 0\ \ 4h\ \ 0) = x_4;$
$x_4M = (4h\ \ 5h\ \ 0\ \ 0) = y_5;$
$y_5M^t = (0\ \ 5h\ \ 0\ \ 0\ \ 2h\ \ 0) = x_5;$
$x_5M = (2h\ \ 3h\ \ 0\ \ 0) = y_6;$
$y_6M^t = (0\ \ 3h\ \ 0\ \ 0\ \ 2h\ \ 0) = x_6;$
$x_6M = (2h\ \ h\ \ 0\ \ 0) = y_7;$
$y_7M^t = (0\ \ h\ \ 0\ \ 0\ \ 4h\ \ 0) = x_7\ (= x_3).$

Thus the MOD realized resultant is a MOD limit cycle pair given by {(0 h 0 0 4 h 0), (4h 3h 0 0)}.

Let x = (0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ I_2^h\ \ 1) = y_1;$
$y_1M^t = (0\ 0\ I_4^h + 1\ 2 + I_4^h\ \ 0\ \ I_2^h) = x_1;$
$x_1M = (0\ 0\ I_2^h\ \ I_4^h + 5) = y_2;$
$y_2M^t = (4h + I_4^h\ \ 0\ \ I_4^h + 5\ 4 + I_4^h\ \ 0\ \ I_2^h) = y_2;$
$y_2M^t = (I_4^h + 2h\ 0\ \ I_2^h + 4 + I_4^h\ \ 2 + I_4^h\ \ 4h + I_4^h\ \ I_4^h + 5)$
$= x_2;$
$x_2M = (I_4^h + 4h\ 2h + I_4^h\ \ 5 + I_4^h + I_2^h\ \ I_4^h + 4 + I_2^h) = y_3;$
and so on.

We are sure that certainly after a finite number of iterations we will arrive at a MOD fixed point pair or a MOD limit cycle pair.

Let x = (0 0 0 1 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 2 + I_4^h) = y_1;$
$y_1M^t = (4h + I_4^h\ \ 0\ \ 2 + I_4^h\ \ 4 + I_4^h\ \ 0\ 0) = x_1;$

$x_1M = (I_4^h \; 0 \; I_2^h \;\; 2 + 2h + \; I_4^h \;) = y_2;$
$y_2M^t = (I_4^h + 2h \; 0 \; I_4^h + 2 + 2h \; 4 + 4h + I_4^h \;\; I_4^h \;\; I_2^h \;) = x_2;$
$x_2M = (I_4^h \; I_4^h \; I_2^h \; I_4^h + 2 + 2h) = y_3;$
$y_3M^t = (I_4^h \; + 2h \; I_4^h \;\; I_4^h + 2 + 2h \;\; 4 + 4h + \; I_4^h \;\; I_4^h \;\; I_2^h \;)$
$= x_3;$
$x_3M = (I_4^h \; I_4^h \;\; I_2^h \;\; 4 + 2h + \; I_4^h \;) = y_4;$
$y_4M^t = (I_4^h \; I_4^h \; I_4^h \;\; + 4 + 2h \;\; 2 + 4h + \; I_4^h \;\; I_4^h \; I_2^h \;) = x_4;$
$x_4M = (I_4^h \; I_4^h \;\; I_2^h \;\; I_4^h \; + 2 + 4h) = y_5;$
$y_5M^t = (I_4^h \; I_4^h \; I_4^h + 2 + 4h \; I_4^h + 4 + 2h \; I_4^h \;\; I_2^h \;) = x_5 \; (=x_4).$

Thus the MOD resultant is a MOD fixed point pair or a MOD limit cycle pair which we are sure to get after a finite number of iterations.

Let $y = (1 \;\; 0 \;\; 0 \;\; 0) \in \; Y$ to find the effect of y on M.

$yM^t = (3 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0) = x_1;$
$x_1M = (4 \;\; 2 \;\; 0 \;\; 0) = y_1;$
$y_1M^t = (0 \;\; 2h \;\; 0 \;\; 0 \;\; 2 \;\; 0) = x_2;$
$x_2M = (2 \;\; 2h + 4 \;\; 0 \;\; 0) = y_2;$
$y_2M^t = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 4 + 4h \;\; 0) = x_3;$
$x_3M = (4 + 4h \;\; 2 + 2h \;\; 0 \;\; 0) = y_3;$
$y_3M^t = (0 \;\; 4h \;\; 0 \;\; 0 \;\; 2 + 2h \;\; 0) = x_4;$
$x_4M = (2 + 2h \;\; 4 + 2h \;\; 0 \;\; 0) = y_4;$
$y_4M^t = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 4 \;\; 0) = x_5;$
$x_5M = (4 \;\; 2 \;\; 0 \;\; 0) = y_5;$
$y_5M^t = (0 \;\; 2h \;\; 0 \;\; 0 \;\; 2 \;\; 0) = x_6;$
$x_6M = (2 \;\; 0 \;\; 0 \;\; 0) = y_6;$
$y_6M^t = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 0) = x_7;$
$x_7M = (2 \;\; 4 \;\; 0 \;\; 0) = y_7;$
$y_7M^t = (0 \;\; 4h \;\; 0 \;\; 0 \;\; 4h \;\; 0) = x_8;$
$x_8M = (4h \;\; 0 \;\; 0 \;\; 0) = y_8;$
$y_8M^t = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 4h \;\; 0) = x_9;$
$x_9M = (4h \;\; 2h \;\; 0 \;\; 0) = y_9;$
$y_9M^t = (0 \;\; 2h \;\; 0 \;\; 0 \;\; 2h \;\; 0) = x_{10};$
$x_{10}M = (2h \;\; 0 \;\; 0 \;\; 0) = y_{10};$
$y_{10}M^t = (0 \;\; 0 \;\; 0 \;\; 0 \;\; 2h \;\; 0) = x_{11};$

$x_{11}M = (2h\ \ 4h\ \ 0\ \ 0) = y_{11};$
$y_{11}M^t = (0\ \ 4h\ \ 0\ \ 0\ \ 4h\ \ 0) = x_9\ (=x_8).$

Thus the MOD resultant is a MOD limit cycle pair given by {(0 4h 0 0 4h 0), (4h 0 0 0)}.

Let y = (0 1 0 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0\ \ h\ \ 0\ \ 0\ \ 2\ \ 0) = x_1;$
$x_1M = (2\ \ 4 + h\ \ 0\ \ 0) = y_1;$
$y_1M^t = (0\ \ 5h\ \ 0\ \ 0\ \ 4 + 2h\ \ 0) = x_2;$
$x_2M = (4 + 2h\ \ 2 + 3h\ \ 0\ \ 0) = y_2;$
$y_2M^t = (0\ \ 5h\ \ 0\ \ 0\ \ 2\ \ 0) = x_3;$
$x_3M = (2\ \ 5h + 4\ \ 0\ \ 0) = y_3;$
$y_3M^t = (0\ \ 3h\ \ 0\ \ 0\ \ 4 + 4h\ \ 0) = x_4;$
$x_4M = (4 + 4h\ \ 5h + 2\ \ 0\ \ 0) = y_4;$
$y_4M^t = (0\ \ h\ \ 0\ \ 0\ \ 2h + 2\ \ 0) = x_5;$
$x_5M = (2h + 2\ \ 5h + 4\ \ 0\ \ 0) = y_5;$
$y_5M^t = (0\ \ 3h\ \ 0\ \ 0\ \ 4\ \ 0) = x_6;$
$x_6M = (4\ \ 3h + 2\ \ 0\ \ 0) = y_6;$
$y_6M^t = (0\ \ 5h\ \ 0\ \ 0\ \ 2\ \ 0) = x_7\ (= x_3).$

Thus the MOD resultant is a MOD limit cycle pair given by
{(0 5h 0 0 2 0), (2 5h + 4 0 0)}.

Let y = (0 0 1 0) $\in$ Y to find the effect of y on M.

$yM^t = (0\ 0\ I_2^h\ 0\ 0\ 1) = x_1;$
$x_1M = (0\ 0\ I_4^h + 1\ \ I_2^h) = y_1;$
$y_1M^t = (I_2^h\ \ 0\ I_2^h\ \ I_2^h\ \ 0\ 1 + I_4^h) = x_2;$
$x_2M = (I_2^h\ \ 0\ I_4^h + 1\ \ I_2^h) = y_2;$
$y_2M^t = (I_2^h\ \ 0\ I_2^h\ \ I_2^h\ \ I_2^h\ \ 1 + I_4^h) = x_3;$
$x_3M = (I_2^h\ \ I_2^h\ \ 1 + I_4^h\ \ I_2^h) = y_3;$
$y_3M^t = (I_2^h\ \ I_2^h\ \ I_2^h\ \ I_2^h\ \ I_2^h\ \ 1 + I_4^h) = x_4;$
$x_4M = (I_2^h\ \ I_2^h\ \ 1 + I_4^h + I_2^h\ \ I_2^h) = y_4;$
$y_4M^t = (I_2^h\ \ I_2^h\ \ I_2^h\ \ I_2^h\ \ I_2^h\ \ I_2^h + I_4^h + 1) = x_5;$

$$x_5M = (I_2^h\ I_2^h\ 1 + I_2^h + I_4^h\ I_2^h) = y_5\ (= y_4).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_2^h\ I_2^h\ I_2^h\ I_2^h\ I_2^h\ I_2^h + I_4^h + 1), (I_2^h\ I_2^h\ 1 + I_2^h + I_4^h\ I_2^h)\}$.

Let y = (0 0 0 1) ∈ Y to find the effect of y on M.

$$yM^t = (2h\ 0\ 1\ 2 + I_4^h\ 0\ 0) = x_1;$$
$$x_1M = (0\ 0\ I_2^h\ 5 + I_4^h + 4h) = y_1;$$
$$y_1M^t = (I_4^h\ 0\ I_2^h + 4h + 5\ I_4^h) = x_2;$$
$$x_2M = (I_0^h\ \ 0\ I_2^h + I_4^h\ \ I_4^h\ \ I_4^h + I_2^h + 4h + 5) = y_2$$ and so on.

We are sure to arrive at a MOD fixed point pair or a MOD limit cycle pair after a finite number of iterations.

Example 3.34: Let

$$S = \begin{bmatrix} 0 & 3 & I_2^h & 0 & 6 & I_5^h & 1 \\ h & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_0^h \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number relational matrix operator with entries from $\langle Z_{10} \cup h\rangle_I$.

Let X = {(a b c) / a, b, c ∈ {0, 1}} and

Y = ({a_1 a_2, … a_7) / a_i ∈ {0, 1}; 1 ≤ i ≤ 7} be the MOD domain and MOD range space of initial state vectors associated with S.

Let x = (1 0 0) ∈ X to find the effect of x on S.

$$xS = (0\ 3\ I_2^h\ 0\ 6\ I_5^h\ 1) = y_1;$$
$$y_1S^t = (6 + I_4^h + I_5^h\ 0\ I_0^h) = x_1;$$

$$x_1S = (0\ 8 + I_4^h + I_5^h\quad I_2^h + I_0^h + I_8^h\ \ 0\ \ 8 + I_4^h + I_5^h\ \ I_5^h +$$
$$I_0^h\ \ 6 + I_4^h + I_5^h\,) = y_1;$$
$$y_1S^t = (8 + I_4^h + I_5^h + I_0^h + I_6^h\ \ 0\ \ I_0^h\,) = x_2;$$
$$x_2S = (0\ 4 + I_4^h + I_5^h + I_0^h + I_6^h\ \ 0\ \ I_0^h\,) = x_2;$$
$$x_2S = (0\ 4 + I_4^h + I_5^h + I_6^h + I_0^h\quad I_2^h + I_8^h + I_0^h\ 0\ 8 + I_4^h +$$
$$I_5^h + I_0^h + I_6^h\ \ I_5^h + I_0^h\ \ 8 + I_4^h + I_5^h + I_0^h + I_6^h\,) = y_2;$$

and so on.

However we are sure after a finite number of iterations we are sure to arrive at a MOD fixed point pair or MOD limit cycle pair.

Let $x = (0\ 1\ 0) \in X$ to find the effect of x on S.

$$xS = (1\ \ 0\ \ 0\ \ 4\ \ 0\ \ 0\ \ 0) = y_1;$$
$$y_1S^t = (0\ \ h + 6\ \ 0) = x_1;$$
$$x_1S = (7h\ \ 0\ \ 0\ \ 4 + 4h\ \ 0\ \ 0\ \ 0) = y_2;$$
$$y_2S^t = (0\ \ 6 + 3h\ \ 0) = x_3;$$
$$x_3S = (5h\ \ 0\ \ 0\ \ 6 + 4h\ \ 0\ \ 0\ \ 0) = y_3;$$
$$y_3S^t = (0\ \ 4 + h\ \ 0) = x_4;$$
$$x_4S = (5h\ \ 0\ \ 0\ \ 6 + 5h\ \ 0\ \ 0\ \ 0) = y_4\ (= y_3).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 4 + h\ 0), (5h\ \ 0\ \ 0\ \ 6 + 4h\ \ 0\ \ 0\ \ 0)\}$.

Let $x = (0\ 0\ 1) \in X$, to find the effect of x on S.

$$xS = (0\ 0\ 0\ 0\ 0\ 0\ I_0^h\,) = y_1;$$
$$y_1S^t = (I_0^h\ \ 0\ \ I_0^h\,) = x_1;$$
$$x_1S = (0\ \ I_0^h\ \ I_0^h\ \ 0\ \ I_0^h\ \ I_0^h\ \ I_0^h\,) = y_2;$$
$$y_2S^t = (I_0^h\ \ 0\ \ I_0^h\,) = x_2\ (= x_1).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^h\ \ 0\ \ I_0^h), (0\ \ I_0^h\ \ I_0^h\ \ 0\ \ I_0^h\ \ I_0^h\ \ I_0^h)\}$.

Let y = (0 0 0 0 0 1 0) ∈ Y to find the effect of y on S.

$$yS^t = (I_5^h\ 0\ 0) = x_1;$$
$$x_1S = (0\ I_5^h\ I_0^h\ 0\ I_5^h\ I_5^h\ I_5^h) = y_1;$$
$$y_1S^t = (I_5^h + I_0^h\ 0\ I_0^h) = x_2;$$
$$x_2S = (0\ I_5^h + I_0^h\ I_0^h\ 0\ I_5^h + I_0^h\ I_5^h + I_0^h\ I_5^h + I_0^h) = y_2;$$
$$y_2S^t = (I_5^h + I_5^h\ 0\ I_0^h) = x_3\ (=x_2).$$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^h + I_5^h\ 0\ I_0^h), (0\ I_5^h + I_0^h\ I_0^h\ 0\ I_5^h + I_0^h\ I_5^h + I_0^h\ I_5^h + I_0^h)\}$.

Let y = (0 0 0 1 0 0 0) ∈ Y, to find the effect of y on S.

$$yS^t = (0\ 4\ 0) = x_1;$$
$$x_1S = (4h\ 0\ 0\ 6\ 0\ 0\ 0) = y_1;$$
$$y_1S^t = (0\ 4 + 4h\ 0) = x_2;$$
$$x_2S = (8h\ 0\ 0\ 6 + 6h\ 0\ 0\ 0) = y_2;$$
$$y_2S^t = (0\ 4 + 2h\ 0) = x_3;$$
$$x_3S = (6h\ 0\ 0\ 6 + 8h\ 0\ 0\ 0) = y_3;$$
$$y_3S^t = (0\ 8h + 4\ 0) = x_4;$$
$$x_4S = (2h\ 0\ 0\ 6 + 2h\ 0\ 0\ 0) = y_4;$$
$$y_4S^t = (0\ 4\ 0) = x_5\ (x_1).$$

Thus the MOD realized resultant is a MOD limit cycle pair given by {(0 4 0), (4h 0 0 6 0 0 0)}.

Thus we may arrive at a MOD limit cycle pair or MOD fixed point pair, but we are sure to arrive at a MOD resultant after a finite number of iterations.

Now we proceed onto briefly describe the MOD natural neutrosophic special dual like number Relational Maps model.

Let P be a problem in hand and suppose the expert wishes to work using MOD relational map model. He / She has feeling that the problem is very vague so it is full of indeterminates of very different types like zero indeterminate (or zero neutrosophic number), zero divisor indeterminate, idempotents indeterminate and nilpotent indeterminate. Here the term indeterminate is used analogous to neutrosophic so uses entries from $\langle Z_n \cup h\rangle_I$. He / She gives the MOD natural neutrosophic special quasi dual number directed bipartite graph G with $D_1, D_2, \ldots, D_m$ as domain nodes of the problem and nodes of the problem and $R_1, R_2 \ldots, R_s$ as the range nodes of the problem with edge weight from $\langle Z_n \cup h\rangle_I$.

Let $M = (e_{ij})$ be the $m \times s$ connection (relational) matrix associated with G. That is M is the MOD natural neutrosophic special dual like number relational matrix. M is defined as the MOD dynamical system of the MOD natural neutrosophic special dual like number Relational Maps model.

We will illustrate this by a simple example.

Example 3.35: Let P be the problem at hand $D_1, D_2, D_3, D_4, D_5, D_6$ are the domain nodes associated with the problem and R_1, R_2, R_3 and the range nodes of the problem P. The expert wishes to work with the problem with edge weights from $\langle Z_4 \cup h\rangle_I$.

Let G be the MOD natural neutrosophic special dual like number bipartite directed graph given in the following Figure.

G =

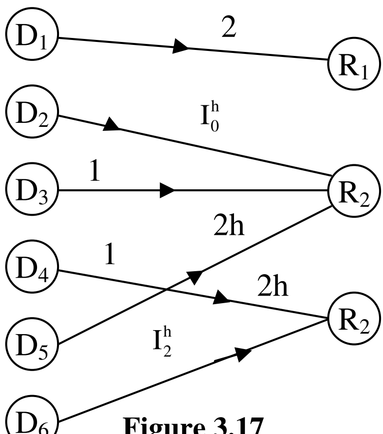


Figure 3.17

Let M be the MOD natural neutrosophic special dual like number relational matrix associated with G.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 \\ 0 & I_0^h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2h \\ 0 & 2h & 0 \\ 0 & 0 & I_2^h \end{bmatrix} \end{array}.$$

Let $X = \{(x_1\ x_2 \ldots x_6) / x_i \in \{0, 1\}; 1 \leq i \leq 6\}$ and

$Y = \{(y_1\ y_2\ y_3) / y_i \in \{0, 1\}; 1 \leq i \leq 3)\}$ be the MOD initial state vectors of the domain and range space respectively associated with M.

Let $x = (1\ 0\ 0\ 0\ 0\ 0) \in X$. The effect of x on M;

$xM = (2\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ \ 0\ \ 0).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0\ 0\ 0\ 0), (0\ 0\ 0)\}$

Let $x = (0\ 1\ 0\ 0\ 0\ 0) \in X$ to find the effect of x on M.

$xM = (0\ \ I_h^h\ \ 0) = y_1;$
$y_1M^t = (0\ \ I_h^h\ \ I_h^h\ \ 0\ \ I_h^h\ \ 0) = x_1;$
$x_1M = (0\ \ I_h^h\ \ 0) = x_2\ (= x_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by $\{(0\ \ I_h^h\ \ 0), (0\ \ I_h^h\ \ I_h^h\ \ 0\ \ I_h^h\ \ 0)\}$.

Let x = (0 0 1) $\in$ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 2h\ 0\ 3) = y_1;$
$y_1M^t = (0\ 0\ 1) = x_2\ (=x_1).$

Thus the MOD realized resultant is a MOD special classical fixed point pair given by {(0 0 1), (0 0 0 2h 0 3)}.

Let y = (1 0 0 0 0 0) $\in$ Y; to find the effect of y on M.

$yM^t = (2\ \ 0\ \ 0) = x_1;$
$x_1M = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0) = y_1;$
$y_1M^t = (0\ \ 0\ \ 0) = x_2;$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0), (0 0 0 0 0 0)}.

Let y = (0 1 0 0 0 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0\ \ I_h^h\ \ 0) = x_1;$
$x_1M = (0\ \ I_h^h\ \ I_h^h\ \ 0\ \ I_h^h\ 0) = y_1;$
$y_1M^t = (0\ \ I_h^h\ \ 0) = x_2\ (=x_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by $\{(0\ \ I_h^h\ \ 0), (0\ \ I_h^h\ \ I_h^h\ \ 0\ \ I_h^h\ \ 0)\}$.

Let y = (0 0 1 0 0 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0\ 1\ 0) = x_1;$
$x_1M = (0\ \ I_h^h\ \ \ 1\ \ 0\ \ 2\,h\ \ 0) = y_1;$
$y_1M^t = (0\ \ I_h^h + 1\ 0) = x_2;$
$x_2M = (0\ \ I_h^h\ \ 1 +\ I_h^h\ \ 0\ 2h +\ I_h^h\ \ 0) = y_2;$
$y_2M^t = (0\ \ I_h^h + 1\ 0) = x_3\ (= x_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 1 + I_h^h\ \ 0), (0\ \ I_h^h\ \ 1 + I_h^h\ \ 0\ 2h + I_h^h\ \ 0)\}.$$

Let y = (0 0 0 1 0 0) ∈ Y to find the effect of y on M.

$yM^t = (0\ 0\ 2h) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ 0\ 2h) = y_1;$
$y_1M^t = (0\ 0\ 2h) = x_2\ (=x_1).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 2h), (0 0 0 0 0 2h)}.

Let y = (0 0 0 0 1 0) ∈ Y, to find the effect of y on M.

$yM^t = (0\ 2\ h\ 0) = x_1;$
$x_1M = (0\ \ I_h^h\ \ 2h\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ \ I_h^h\ \ 0) = x_2;$
$x_2M = (0\ \ I_h^h\ \ I_h^h\ 0\ \ I_h^h\ \ 0) = y_2;$
$y_2M^t = (0\ \ I_h^h\ \ 0) = x_3\ (=x_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ \ I_h^h\ \ 0), (0\ \ I_h^h\ \ I_h^h\ 0\ \ I_h^h\ \ 0)\}.$$

Let y = (0 0 0 0 0 1) ∈ Y to find the effect of y on M.

$yM^t = (0\ 0\ 3) = x_1;$
$x_1M = (0\ 0\ 0\ 2h\ 0\ 1) = y_1;$
$y_1M^t = (0\ \ 0\ \ 3) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 3), (0 0 0 2h 0 1)}.

From the MOD resultant we make the following observations.

i) This new model will give the MOD resultant after a finite number of iterations.

ii) The MOD resultant can give values like reals from Z_n or special dual like numbers of the form a + bh or natural neutrosophic numbers of the form I_t^h; $t \in \langle Z_n \cup h\rangle$ is an idempotent or a nilpotent or a zero divisor in $\langle Z_n \cup h\rangle$.

So this sort of getting different values for the nodes depending on the MOD initial state vector from X or Y is very different from the usual FRMs and NRMs.

Hence this is the special feature enjoyed by the MOD natural neutrosophic special dual like number Relational Maps model.

Deriving other properties of these models is a matter of routine so is left as an exercise to the reader.

Next we proceed onto describe the MOD natural neutrosophic special quasi dual number Relational Maps model using $\langle Z_n \cup k\rangle_I$.

For more about $\langle Z_n \cup k\rangle_I$ please refer [60].

To this end one needs the notion of MOD natural neutrosophic special quasi dual number bipartite directed graphs and rectangular or relational matrices which will be described by examples.

Example 3.36: Let G be a MOD directed bipartite graph with edge weights from $\langle Z_5 \cup k\rangle_I$ that is G is the MOD natural neutrosophic special quasi dual number directed bipartite graph with edge weights from $\langle Z_5 \cup h\rangle_I$ given by the following figure.

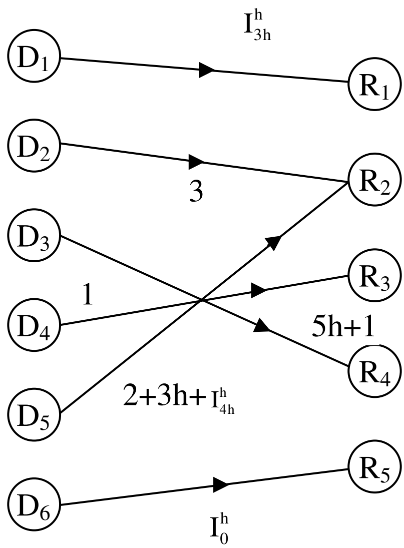


Figure 3.18

Example 3.37: Let H be the MOD natural neutrosophic special quasi dual number directed bipartite graph with edge weights from $\langle Z_{10} \cup k\rangle_I$ given by the following figure.

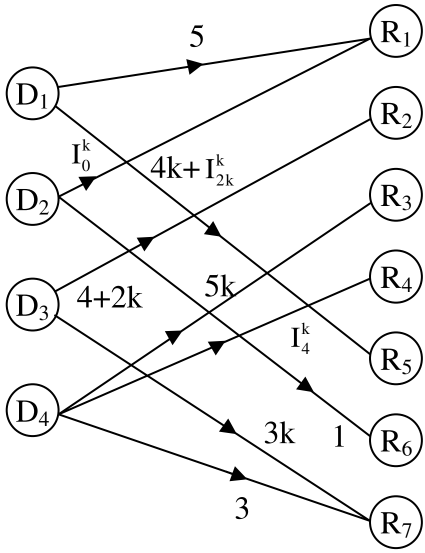


Figure 3.19

Next we proceed onto describe the MOD natural neutrosophic special quasi dual number rectangular (relational) matrix.

These will also be defined or called as MOD natural neutrosophic special quasi dual number relational (rectangular matrix operator) [66].

Example 3.38: Let

$$M = \begin{bmatrix} 3 & 0 & 4k+2 & 0 & I_{2k}^k & 0 & 1 \\ 0 & I_0^k+5 & 0 & 5 & 0 & 6k+2 & 0 \\ 1 & 0 & 0 & I_{2+5k}^k & 1 & 0 & 0 \\ 0 & 1 & 6 & 0 & 2 & 0 & 4 \end{bmatrix}$$

be the MOD natural neutrosophic special quasi dual number relational (rectangular) matrix with entries from $\langle Z_8 \cup k\rangle_I$.

Example 3.39: Let

$$P = \begin{bmatrix} 0 & 4k + I_3^k & 0 \\ 5k & 0 & 7k + I_0^k \\ 0 & I_k^k & 0 \\ 1 & 2 & 3 \\ 0 & 0 & 4k \\ I_{3k}^k & 2 & 0 \\ 0 & 0 & I_{5k+1}^k \end{bmatrix}$$

be the MOD natural neutrosophic special quasi dual number rectangular (or relational) matrix with entries from $\langle Z_9 \cup k\rangle_I$.

Hence if $M = (m_{ij})_{s\times t}$ be a MOD rectangular matrix ($s \neq t$) with entries from $\langle Z_n \cup k\rangle_I$ that is $m_{ij} \in \langle Z_n \cup k\rangle_I$ then M will be known as the MOD natural neutrosophic special quasi dual number relational (rectangular) matrix operator [66].

We will be defining only some special type of operations on M or using M.

To this end let $D = \{(x_1 \ldots x_s) / x_i \in \{0, 1\}; 1 \leq i \leq s\}$ and

$R = \{(y_1, y_2 \ldots y_t) / y_j \in \{0, 1\}; 1 \leq i \leq t\}$ be the MOD natural neutrosophic special quasi dual number initial domain and range space of state vectors associated with M.

So with each MOD natural neutrosophic quasi dual number matrix rectangular operator.

We will illustrate the special type of operations on these matrices by examples [66].

Example 3.40: Let

$$M = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & I_0^k \\ 1+k & 0 & 0 \\ 0 & I_2^k & 0 \\ 0 & 0 & 1+I_2^k \end{bmatrix}$$

be the MOD natural neutrosophic quasi dual number relational matrix with entries from $\langle Z_4 \cup k\rangle_I$.

Let X = $\{(a_1\ a_2\ \ldots\ a_6) / a_i \in \{0, 1\}, 1 \leq i \leq 6\}$ and

Y = $\{(a\ \ b\ \ c) / a, b, c \in \{0, 1\}\}$ be the MOD domain and MOD range space of initial state vectors associated with M.

Let x = (1 0 0 0 0 0) $\in$ X to find the effect of x on M.

$xM = (0\ \ 2\ \ 0) = y_1;$
$y_1M^t = (0\ \ 0\ \ 0\ \ 0\ \ I_2^k\ \ 0) = x_1;$
$x_1M = (0\ \ I_0^k\ \ 0) = y_2;$
$y_2M^t = (I_0^k\ \ 0\ \ 0\ \ 0\ \ I_0^k\ \ 0) = x_2;$
$x_2M = (0\ \ I_0^k\ \ 0) = y_3\ (= y_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(I_0^k\ \ 0\ \ 0\ \ 0\ \ I_0^k\ \ 0), (0\ \ I_0^k\ \ 0)\}.$$

Let x = (0 1 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM = (1\ \ 0\ \ 0) = y_1$;
$y_1M^t = (0\ \ 1\ \ 0\ \ 1 + k\ \ 0\ \ 0) = x_1$;
$x_1M = (2 + k\ \ 0\ \ 0) = y_2$;
$y_2M^t = (0\ \ 2 + k\ \ 0\ \ 2 + 2k\ \ 0\ \ 0) = x_2$;
$x_2M = (3k\ \ 0\ \ 0) = y_3$;
$y_3M^t = (0\ \ 3k\ \ 0\ \ 0\ \ 0\ \ 0) = x_3$;
$x_3M = (3k\ \ 0\ \ 0) = y_4\ (= y_3)$.

Thus the MOD resultant is a MOD realized fixed point pair give by {(0 3k 0 0 0 0), (3k 0 0)}.

We see the on state of the 2nd node leads to the value of the 2nd node to be 3k and that of first node in the range of nodes to be also 3k.

The earlier resultant gives the nodes value as natural neutrosophic numbers.

Let $y = (1\ \ 0\ \ 0) \in Y$ to find the effect of y on M.

$yM^t = (0\ \ 1\ \ 0\ \ 1 + k\ \ 0\ \ 0) = x_1$;
$x_1M = (2 +k\ \ 0\ \ 0) = y_1$;
$y_1M^t = (0\ \ 2 + k\ \ 0\ \ 2 + 2k\ \ 0\ \ 0) = x_2$;
$x_2M = (3k\ \ 0\ \ 0) = y_2$;
$y_2M^t = (0\ \ 3k\ \ 0\ \ 0\ \ 0\ \ 0) = x_3$;
$x_3M = (3k\ \ 0\ \ 0) = y_3\ (= y_2)$.

Thus the MOD resultant is a fixed point pair given by {(0 3k 0 0 0 0), (3k 0 0)}.

Let y = (0 1 0) $\in$ Y, to find the effect of y on M.

$yM^t = (2\ \ 0\ \ 0\ \ 0\ \ I_2^k\ \ 0) = x_1;$
$x_1M = (0\ \ I_0^k\ \ 0) = y_1;$
$y_1M^t = (I_0^k\ \ 0\ \ 0\ \ 0\ \ I_0^k\ \ 0) = x_2;$
$x_2M = (0\ \ I_0^k\ \ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^k\ 0\ 0\ 0\ I_0^k\ 0), (0\ I_0^k\ 0)\}$.

Let y = (0 0 1) $\in$ Y, to find the effect of y on M.

$yM^t = (0\ 0\ I_0^k\ 0\ 0\ 1 + I_2^k) = x_1;$
$x_1M = (0\ 0\ I_0^k + 1 + I_2^k) = y_1;$
$y_1M^t = (0\ 0\ I_0^k\ \ 0\ 0\ 1 + I_0^k + I_2^k) = x_2;$
$x_2M = (0\ 0\ 1 + I_0^k + I_2^k) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ 0\ I_0^k\ 0\ 0\ 1 + I_0^k + I_2^k), (0\ 0\ 1 + I_0^k + I_2^k)\}$.

Let x = (0 0 0 0 1 0) $\in$ X to find the effect of x on M.

$xM = (0\ \ I_2^k\ \ 0) = y_1;$
$y_1M^t = (I_2^k\ \ 0\ 0\ 0\ I_0^k\ \ 0) = x_1;$
$x_1M = (0\ \ I_2^k + I_0^k\ \ 0) = y_2;$
$y_2M^t = (I_2^k + I_0^k\ \ 0\ 0\ 0\ I_0^k\ 0) = x_2;$
$x_2M = (0\ \ I_2^k + I_0^k\ \ 0) = y_3\ (= y_2).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_2^k + I_0^k\ \ 0\ 0\ 0\ I_0^k\ \ 0), (0\ I_2^k + I_0^k\ \ 0)\}$.

Hence the interested reader can work with any of the initial state vectors and arrive at a MOD resultant.

Based on this working we proceed onto describe and develop the MOD natural neutrosophic special quasi dual number Relational Maps model.

Let P be a problem in hand. Let the expert work with the problem using MOD natural neutrosophic special quasi dual number Relational Maps model.

Let G be the MOD natural neutrosophic special quasi dual number directed bipartite graph with edge weights from $\langle Z_9 \cup k\rangle_I$ which is given in the following figure.

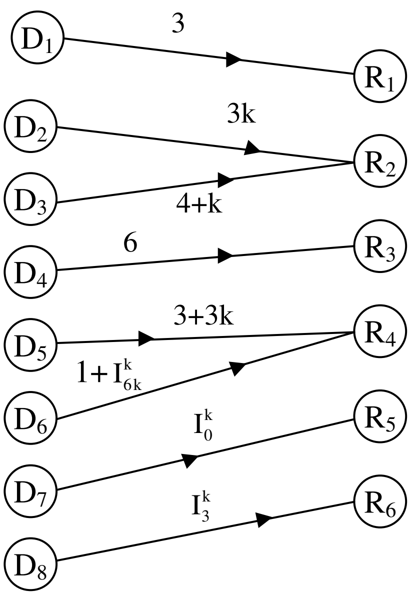


Figure 3.20

Let M be the MOD natural neutrosophic special quasi dual number relational matrix associated with G.

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array} \begin{array}{c} \begin{array}{cccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3k & 0 & 0 & 0 & 0 \\ 0 & 4+k & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3+3k & 0 & 0 \\ 0 & 0 & 0 & 1+I_{6k}^{k} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^k & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3^k \end{bmatrix} \end{array}.$$

Let $X = \{(a_1\ a_2\ \ldots\ a_8) / a_i \in \{0, 1\};\ 1 \leq i \leq 8\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_6) / b_j \in \{0, 1\};\ 1 \leq i \leq 6\}$ be the MOD initial state vectors of the MOD domain space and MOD range space respectively.

M serves as the MOD natural neutrosophic special quasi dual number Relational Maps models dynamical system.

Let $x = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \in X$; to find the effect of x on M.

$xM = (3\ 0\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1$;
$x_1M = (0\ 0\ 0\ 0\ 0\ 0)$.

Thus the MOD resultant is MOD realized fixed point pair given by $\{(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0), (0\ 0\ 0\ 0\ 0\ 0)\}$.

Let us now define the operation of updating for the same x.
The effect of x on M is

$xM = (3\ 0\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1\ (=x_1)$.

Thus the MOD resultant is MOD special classical fixed point pair given by {(1 0 0 0 0 0 0 0), (3 0 0 0 0 0)}.

Let x = (0 1 0 0 0 0 0 0) $\in$ X, to the effect of x on M.

$xM = (0\ 3k\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1$;
$x_1M = (0\ 0\ 0\ 0\ 0\ 0)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}.

If for the same x we perform the updating operation then we see

$xM = (0\ 3k\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0) = x_1\ (= x)$.

Thus the MOD resultant in this case is also a MOD special classical fixed point pair given by

{(0 1 0 0 0 0 0 0), (0 3k 0 0 0 0)}.

Let x = (0 0 1 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM = (0\ 4 + k\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t = (0\ 0\ 7 + 7k\ 0\ 0\ 0\ 0\ 0) = x_1$;
$x_1M = (0\ 1 + k\ 0\ 0\ 0\ 0) = y_2$;
$y_2M^t = (0\ 0\ 4 + 4k\ 0\ 0\ 0\ 0\ 0) = x_2$;
$x_2M = (0\ 7 + 7k\ 0\ 0\ 0\ 0) = y_3$;
$y_3M^t = (0\ 0\ 1\ + k\ 0\ 0\ 0\ 0\ 0) = x_3$;
$x_3M = (0\ 4 + 4k\ 0\ 0\ 0\ 0) = y_4$;
$y_4M^t = (0\ 0\ 7 + 7k\ 0\ 0\ 0\ 0\ 0) = x_4\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 7 + 7k 0 0 0 0 0), (0 4 + 4k 0 0 0 0 0)}.

We study the effect of x = (0 0 0 1 0 0 0 0) ∈ X on M.

$xM = (0\ 0\ 6\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ 0\ 0) = y_2.$

Thus the MOD resultant is a MOD fixed point pair given by
{(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}.

Let x = (0 0 0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 6\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ 0\ 0) = y_2.$

Thus the MOD resultant is a MOD fixed point pair given by
{(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}

Let x = (0 0 0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 3 + 3k\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1;$
$xM = (0\ 0\ 0\ 0\ 0\ 0) = y_2.$

Thus the MOD resultant is a MOD fixed point pair given by
{(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}.

Now we study the effect of x = (0 0 0 0 1 0 0 0) ∈ X on the M using the operation of updating.

$xM = (0\ 0\ 0\ 3 + 3k\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) = x_1\ (= x_1).$

Thus the MOD resultant is a MOD special classical fixed point pair given by {(0 0 0 0 1 0 0 0), (0 0 0 3 + 3k 0 0)}.

Let x = (0 0 0 0 0 1 0 0) ∈ X, the effect of x on M is

$xM = (0\ 0\ 0\ 1 + I^k_{6k}\ 0\ 0) = y_1;$

$y_1M^t = (0\ 0\ 0\ 0\ 3k + 3 + I^k_{6k},\ 1 + I^k_0 + I^k_{6k}\ 0\ 0) = x_1;$

$x_1M = (0\ 0\ 0\ I^k_{6k} + 1 + I^k_0,\ 0\ 0) = y_2;$

$y_2M^t = (0\ 0\ 0\ 0\ 3 + 3k + I^k_{6k} + I^k_0,\ 1 + I^k_{6k} + I^k_0\ 0\ 0) = x_2;$

$x_2M = (0\ 0\ 0\ I^k_{6k} + I^k_0 + 1\ 0\ 0);$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0\ 0\ 3 + 3k + I^k_{6k} + I^k_0\ 1 + I^k_{6k} + I^k_0\ 0\ 0), (0\ 0\ 0\ I^k_{6k} + I^k_0 + 1\ 0\ 0)\}$.

Even if updating is done at each stage we would arrive only at this MOD resultant which is not different from the other operations.

Let x = (0 0 0 0 0 0 1 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 0\ I^k_0\ 0) = y_1;$

$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ I^k_0\ 0) = x_1;$

$x_1M = (0\ 0\ 0\ 0\ I^k_0\ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ I^k_0\ 0), (0\ 0\ 0\ 0\ I^k_0\ 0)\}.$$

Even with updating operation we get the same MOD resultant.

Let y = (1 0 0 0 0 0) ∈ Y, to find the effect of y on M.

$yM^t = (3\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1;$

$x_1M = (0\ 0\ 0\ 0\ 0\ 0) = y_1;$

$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_2;$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}.

Suppose for the same y = (1 0 0 0 0 0) $\in$ Y we do get the resultant by performing the operation of updating.

Let y = (1 0 0 0 0 0) $\in$ Y the effect of y on M.

$yM^t = (3\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (1\ 0\ 0\ 0\ 0\ 0) = y_1\ (= y_1).$

Thus the MOD resultant is a MOD classical special fixed point pair given by {(3 0 0 0 0 0 0 0), (1 0 0 0 0 0)}.

Let y = (0 1 0 0 0 0) $\in$ Y to find the effect of y on M.

$yM^t = (0\ 3k\ 4 + k\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 7 + 7k\ 0\ 0\ 0\ 0) = y_1;$
$y_2M^t = (0\ 0\ 1 + k\ 0\ 0\ 0\ 0\ 0) = x_2;$
$x_2M = (0\ 4 + 4k\ 0\ 0\ 0\ 0) = y_2;$
$y_2M^t = (0\ 0\ 7 + 7k\ 0\ 0\ 0\ 0\ 0) = x_3;$
$x_3M = (0\ 1 + k\ 0\ 0\ 0\ 0) = y_3;$
$y_3M^t = (0\ 0\ 4 + 4k\ 0\ 0\ 0\ 0\ 0) = x_4;$
$x_4M = (0\ 7 + 7k\ 0\ 0\ 0\ 0) = y_4\ (= y_1).$

Thus the MOD resultant is a MOD limit cycle pair given by
{(0 0 7 + 7k 0 0 0 0 0), (0 7 + 7k 0 0 0 0)}.

Hence even with the updating operation at each stage we will arrive at the same MOD resultant.

Let y = (0 0 1 0 0 0) $\in$ Y to find the effect of y on M.

$yM^t = (0\ 0\ 0\ 6\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = x_2;$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 0 0 0 0 0 0), (0 0 0 0 0 0)}.

Let us now do the updating operation on the same

$$y = (0\ 0\ 1\ 0\ 0\ 0) \in Y$$

The effect of y on M is

$yM^t = (0\ 0\ 0\ 6\ 0\ 0\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 1\ 0\ 0\ 0) = y_1\ (= y_1).$

Thus the MOD resultant is MOD classical special fixed point pair given by {(0 0 0 6 0 0 0 0), (0 0 1 0 0 0)}.

This is different from the other resultant.

Let y = (0 0 0 1 0 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0\ 0\ 0\ 0\ 3 + 3k\ 1 + I_{6k}^k\ 0\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 1 + I_{6k}^k + I_0^k\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 3 + 3k + I_{6k}^k + I_0^k\ 1 + I_{6k}^k + I_0^k\ 0\ 0) = x_2;$
$x_2M = (0\ 0\ 0\ I_{6k}^k + I_0^k\ 1\ 0\ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ 0\ 0\ 3 + 3k + I_{6k}^k + I_0^k\ I_{6k}^k + I_0^k + 1\ 0\ 0), (0\ 0\ 0\ I_{6k}^k + I_0^k + 1\ 0\ 0)\}$.

Let y_1 = (0 0 0 0 1 0) $\in$ Y to found the effect of y_1 on M.

$yM^t = (0\ 0\ 0\ 0\ 0\ 0\ I_0^k\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ I_0^k\ 0) = y_1;$
$y_1M^t\ (0\ 0\ 0\ 0\ 0\ 0\ I_0^k\ 0) = x_2\ (= x_1).$

The MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ 0\ I_0^k\ 0), (0\ 0\ 0\ 0\ I_0^k\ 0)\}.$$

The MOD resultant does not vary as it is not different from the updating operator.

Let y = (0 0 0 0 0 1) $\in$ Y the effect of y on M,

$yM^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ I_3^k) = x_1;$
$x_1M = (0\ 0\ 0\ 0\ 0\ I_0^k) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0\ I_0^k) = x_2;$
$x_2M = (0\ 0\ 0\ 0\ 0\ I_0^k) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ 0\ I_0^k), (0\ 0\ 0\ 0\ 0\ I_0^k)\}.$$

The same MOD resultant would be obtained even after applying or using the updating operator.

Thus we see we can have such new models and the nodes of these new models can be real, mixed real and special quasi dual number or natural neutrosophic depending on the initial state vectors and so on .

As this study is new the reader is expected to analyse the problem.

The advantageous of using this new model is

i) When edge weights are used from $\langle Z_n \cup k\rangle_I$ certainly after finite number of iterations we are sure to get a MOD fixed point pair or a MOD limit cycle pair.

ii) The advantage of using the new model is we see the nodes can get any value in $\langle Z_n \cup k\rangle_I$ depending on the on state of the node at the time of working.

iii) Further this method or this new model does not need the thresholding of the state vectors at each of this stage. This is an advantage as we see the thresholding factor in general is not uniform but depends on the expert; this bias is not in this model.

Chapter Four

MOD INTERVAL RELATIONAL MAPS

In this chapter we proceed onto define the new models viz. MOD interval Relational Maps model built using the interval [0, n) and MOD natural neutrosophic interval Relational Maps model using the MOD natural neutrosophic interval $I_{[0,n)}$; $2 \leq n < \infty$.

We will illustrate this by examples.

Thus we will give examples of MOD interval directed bipartite graphs.

Example 4.1: Let G be the MOD interval bipartite directed graph with edge weights from [0, 8) given by the following example in the following figure.

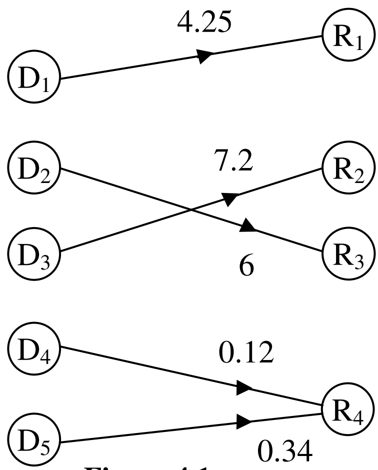


Figure 4.1

Example 4.2: Let H be the MOD interval directed bipartite graph with edge weights from [0, 19) is given by the following figure.

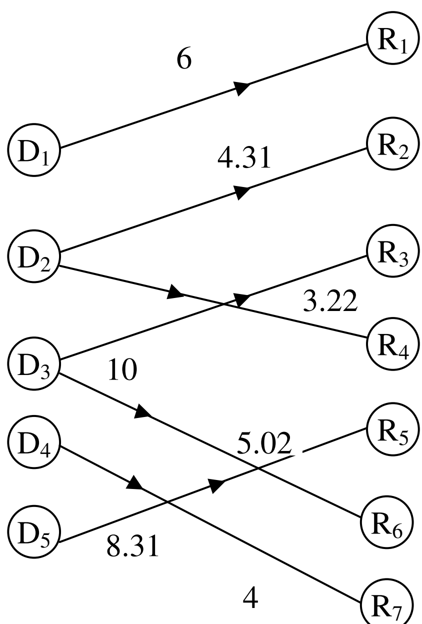


Figure 4.2

Thus if G is any MOD interval directed bipartite graph with edge weights from [0, n) ($2 \leq n < \infty$) is defined as the MOD interval directed bipartite graph.

We have seen examples of them.

We now proceed onto describe by examples the MOD interval rectangular (relational) matrix.

Example 4.3: Let

$$M = \begin{bmatrix} 3 & 1.231 & 0 & 4.21 & 0 & 1.75 \\ 0 & 0 & 4.52 & 0 & 3.72 & 0 \\ 4.32 & 3 & 0 & 9.32 & 0 & 4.3 \\ 8.01 & 0 & 3.2 & 0.7 & 1 & 0.311 \end{bmatrix}$$

be the MOD interval rectangular (relational) matrix.

Example 4.4: Let

$$A = \begin{bmatrix} 0 & 3.35 & 2 \\ 1.77 & 0 & 0 \\ 0 & 1.02 & 4.01 \\ 1.5 & 0 & 0 \\ 2.7 & 0.115 & 0 \\ 0 & 0 & 1.117 \\ 0.33 & 7 & 0 \\ 6 & 0 & 0 \end{bmatrix}$$

be the MOD interval relational matrix with entries from MOD interval [0, 10).

In view of this we can define and describe the MOD interval relational matrix.

Let $M = (m_{ij})$ be a $m \times n$ matrix with entries from $[0, t)$; that is $m_{ij} \in [0, t)$. $M = (m_{ij})$ is defined as the MOD interval relational matrix with entries from the MOD interval $[0, t)$.

Now we describe two types of operations using these matrices as MOD operators [66].

To this end we need the following MOD initial state vectors.

$X = \{(a_1 \ldots a_m) / a_i \in \{0, 1\}; 1 \leq i \leq m\}$ and

$Y = \{(b_1\ b_2 \ldots b_n) / b_i \in \{0, 1\}; 1 \leq j \leq n\}$ be the MOD domain space of initial state vectors and MOD range space of initial state vectors respectively associated with the MOD interval rectangular matrix operator M.

We will illustrate this situation by some examples.

Example 4.5: Let

$$M = \begin{bmatrix} 0.3 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0.2 & 0 & 0 & 0 \end{bmatrix}$$

be MOD interval rectangular matrix with entries from [0, 6).

Let $X= (\{a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4\ b_5) / b_j \in \{0, 1\}; 1 \leq j \leq 5\}$ be the MOD domain and MOD range spaces of initial state vectors respectively associated with M.

$x = (1\ 0\ 0\ 0) \in X$, to find the effect of x on M

$xM = (0.3\ 0\ 1\ 0\ 0) = y_1;$

$y_1M^t = (1.09\ 0.3\ 0\ 0) = x_1$;
$x_1M = (0.627\ \ 0.6\ \ 1.09\ \ 0\ 0.3) = y_2$;
$y_2M^t = (1.5781\ \ 1.827\ \ 0\ \ 0.12) = x_2$;
$x_2M = (2.40043\ \ 3.678\ \ 1.5781\ \ 0\ \ 1.827)$;

and so on.

Certainly we may not get the MOD resultant after a finite number of iterations so this type of operations will yield nothing to us.

Hence we will use the thresholding technique to arrive at a resultant after a finite number of iterations.

Let x = (1 0 0 0) ∈ X to find the effect using thresholding techniques.

$xM = (0.3\ 0\ 1\ 0\ 0) \to (0\ 0\ 1\ 0\ 0) = y_1$;

(Here if an entry in the resultant is greater than or equal to 1 replace by 1 if less than 1 replace by 0)

$y_1M^t = (1\ 0\ 0\ 0) = x_1\ (= x)$.

This the MOD resultant is a MOD classical special fixed point pair given by {(1 0 0 0), (0 0 1 0 0)}.

Let x = (0 1 0 0) ∈ X, to find the effect of x on M.

$xM = (1\ 2\ 0\ 0\ 1) \to (1\ 1\ 0\ 0\ 1) = y$;
$yM^t = (0.3\ 4\ 0\ 0.2) \to (0\ 1\ 0\ 0) = x_1$;

Thus the MOD resultant is a MOD classical special fixed point pair given by {(0 1 0 0), (1 1 0 0 1)}.

Let x = (0 0 1 0) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ 4\ 0) \to (0\ 0\ 0\ 1\ 0) = y_1;$
$y_1M^t = (0\ 0\ 4\ 0) \to (0\ 0\ 1\ 0).$

The MOD interval resultant is a MOD special classical realized fixed point pair given by {(0 0 1 0), (0 0 0 1 0)}.

Let x = (0 0 0 1) ∈ X, to find the effect of x on M.

$xM = (0\ 0\ 2\ 0\ 0\ 0) \to (0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M_t = (0\ 0\ 0\ 0) \to (0\ 0\ 0\ 1).$

The MOD resultant is a MOD classical fixed point pair given by

{(0 0 0 1), (0 0 0 0 0)}.

Let y = (1 0 0 0 0) ∈ Y to find the effect of y on M.

$yM^t = (0.3\ 1\ 0\ 0) \to (0\ 1\ 0\ 0) = x_1;$
$x_1M = (1\ 2\ 0\ 0\ 1) \to (1\ 1\ 0\ 0\ 1) = y_1;$
$y_1M^t = (0.3\ \ 4\ \ 0\ 0.2) \to (0\ 1\ 0\ 0) = x_2\ (= x_1).$

The MOD interval resultant is a MOD fixed point pair given by {(0 1 0 0), (1 1 0 0 1)}.

Let y = (0 1 0 0 0) ∈ Y.

To find the effect of y on M.

$yM^t = (0\ 2\ 0\ 0\ 2) \to (0\ 1\ 0\ 0) = x_1;$
$x_1M = (1\ 2\ 0\ 0\ 1) \to (1\ 1\ 0\ 0\ 1) = y_1;$
$y_1M^t = (1.3\ 4\ 0\ 0.2) \to (1\ 1\ 0\ 0) = x_2;$
$x_2M = (1.3\ 2\ 1\ 0\ 1) \to (1\ 1\ 1\ 0\ 1) = y_2;$
$y_2M^t = (1.3\ \ 4\ 0\ 0.2) \to (1\ 1\ 0\ 0).$

Thus the MOD interval resultant is a MOD interval fixed point pair given by {(1 1 0 0), (1 1 1 0 1)}.

Let y = (0 0 1 0 0) ∈ Y, to find the effect of y on M.

$yM^t = (1\ 0\ 0\ 0) = x_1;$
$x_1M = (0.3\ 0\ 1\ 0\ 0) \to (0\ 0\ 1\ 0\ 0) = y_1\ (=y).$

Thus the MOD interval resultant is a MOD classical special fixed point pair given by

$$\{(1\ 0\ 0\ 0), (0\ 0\ 1\ 0\ 0)\}.$$

Let $y = (0\ 0\ 0\ 1\ 0) \in Y$, to find the effect of y on M.

$yM^t = (0\ 0\ 4\ 0) \to (0\ 0\ 1\ 0) = x_1;$
$x_1M = (0\ 0\ 0\ 1\ 0) = y_1\ (= y_1).$

Thus the MOD interval resultant is a MOD special classical fixed point pair given

$$\{(0\ 0\ 1\ 0), (0\ 0\ 0\ 1\ 0)\}.$$

Let $y = (0\ 0\ 0\ 0\ 1) \in Y$, to find the effect of y on M.

$yM^t = (0\ 1\ 0\ 0) = x_1;$
$x_1M = (1\ 2\ 0\ 0\ 1) \to (1\ 1\ 0\ 0\ 1) = y_1;$
$y_1M^t = (1.3\ \ 4\ 0\ 0.2) \to (1\ 1\ 0\ 0) = x_2;$
$x_2M = (1.3\ 2\ 1\ 0\ 1) \to (1\ 1\ 1\ 0\ 1) = y_2;$
$y_2M^t = (1.3\ \ 4\ \ 0\ \ 0.2) \to (1\ 1\ 0\ 0) = x_3\ (=x_2).$

Thus the MOD interval resultant is a MOD interval fixed pair point given by

$$\{(1\ 1\ 0\ 0), (1\ 1\ 1\ 0\ 1)\}.$$

Only the operation of thresholding at each stage can give the MOD interval resultant after a finite number of iterations we give yet another example of this situation.

Example 4.6: Let

$$S = \begin{bmatrix} 0 & 0.7 & 1.2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 2.4 \\ 0.3 & 0 & 0.2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.2 & 0 & 0 \end{bmatrix}$$

be the MOD interval relational matrix with entries from [0, 4).

Let $X = \{(a_1\ a_2, \ldots\ a_7) / a_i \in \{0, 1\}, 1 \leq i \leq 7\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4)$ where by $\in \{0, 1\}; 1 \leq j \leq 4\}$ be the MOD interval domain and range space of initial state vectors respectively associated with S.

Let x = (1 0 0 0 0 0 0) $\in$ X, to find the effect of x on S.

$xS = (0\ 0.7\ 1.2\ 0) \rightarrow (0\ 0\ 1\ 0) = y_1;$
$y_1S^t = (1.2\ 0\ 0\ 0\ 2\ 1\ 0) \rightarrow (1\ 0\ 0\ 0\ 1\ 1\ 0) = x_4;$
$x_1S = (0.3\ 0.7\ 2.4\ 0) \rightarrow (0\ 0\ 1\ 0) = y_2\ (=y_1).$

Thus the MOD interval resultant is a MOD fixed point pair given by {(1 0 0 0 1 1 0), (0 0 1 0)}.

Let x = (0 1 0 0 0 0 0) $\in$ X, to find the effect of x on S.

$xS = (1\ 0\ 0\ 0) = y_1;$
$y_1S^t = (0\ 1\ 0\ 0\ 0.3\ 0\ 0) \rightarrow (0\ 1\ 0\ 0\ 0\ 0\ 0) = x_1\ (=x).$

Thus the MOD interval resultant is a MOD interval resultant is a MOD special classical fixed point pair given by {(0 1 0 0 0 0 0), (1 0 0 0)}.

Let x = (0 0 1 0 0 0 0) ∈ X to find the effect of x on S.

$xS = (0\ 2.5\ 0\ 0) \to (0\ 1\ 0\ 0) = y_1;$
$y_1S^t = (0.7\ 0\ 2.5\ 0\ 0\ 0\ 0) \to (0\ 0\ 1\ 0\ 0\ 0\ 0) = x_1 = (x).$

Thus in this case also the MOD interval resultant is a MOD special classical fixed point give by

{(0 0 1 0 0 0 0), (0 1 0 0)}.

Let x = (0 0 0 1 0 0 0) ∈ X to find the effect of x on s.

$xS = (0\ 0\ 0\ 2.4) \to (0\ 0\ 0\ 1) = y_1;$
$y_1S^t = (0\ 0\ 0\ 1\ 0\ 0\ 0) = x_1\ (= x).$

Thus the MOD interval resultant is MOD special classical fixed point pair given by

{(0 0 0 1 0 0 0), (0 0 0 1)}.

Let x = (0 0 0 0 1 0 0) ∈ X to find the effect of x on S.

$xS = (0.3\ 0\ 0\ 20) = y_1;$
$y_1S \to (0\ 0\ 0\ 0\ 1\ 0\ 0) = x_1\ (x).$

Thus in this case also the MOD interval resultant is a MOD special classical fixed point pair given by

{(0 0 0 0 1 0 0), (0 0 0 0)}

Let x = (0 0 0 0 0 1 0) ∈ X.

$xS = (0\ 0\ 1\ 0) = y_1;$
$y_1S^t = (1\ 2\ 0\ 0\ 0\ 0.2\ 1\ 0) \to (1\ 0\ 0\ 0\ 0\ 1\ 0) = x_1;$
$x_1S = (0\ 0.7\ 2.2\ 0) \to (0\ 0\ 1\ 0) = y_2\ (= y_1).$

Thus the MOD interval resultant is a MOD fixed point pair given by {(1 0 0 0 0 1 0), (0 0 1 0)}.

Let x = (0 0 0 0 0 0 1) $\in$ X to find the effect of x on S.

$xS = (0\ 0\ 2\ 0\ 0) \rightarrow (0\ 0\ 0\ 0) = y_1;$
$y_1S^t \rightarrow (0\ 0\ 0\ 0\ 0\ 0\ 1)$.

Thus the MOD interval resultant is a MOD special classical fixed point pair given by {(0 0 0 0 0 0 1), (0 0 0 0)}.

Let y = (1 0 0 0) $\in$ Y, to find the effect of y on S.

$yS^t = (0\ 1\ 0\ 0\ 0\ 0.3\ 0\ 0) \rightarrow (0\ 1\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1S = (1\ 0\ 0\ 0) = y_1\ (= y).$

Thus the MOD integral resultant is a MOD classical special fixed point pair given by {(0 1 0 0 0 0 0), (1 0 0 0)}.

Let Y = (0 1 0 0) $\in$ Y the effect of y on S is as follows.

$yS^t = (0.7\ 0\ 2.5\ 0\ 0\ 0\ 0) \rightarrow (0\ 0\ 1\ 0\ 0\ 0\ 0) = x_1;$
$x_1S = (0\ 2.5\ 0\ 0) \rightarrow (0\ 1\ 0\ 0) = y_1\ (=y).$

Thus the MOD interval resultant is a MOD special classical fixed point pair given by

{(0 0 1 0 0 0 0), (0 1 0 0)}.

Let y = (0 0 1 0) $\in$ Y, to find the effect of y on S is as follows.

$yS^t = (1.2\ 0\ 0\ 0\ 0.2\ 1\ 0) \rightarrow (1\ 0\ 0\ 0\ 0\ 1\ 0) = x_1;$
$x_1S = (0\ 0\ 7\ 2.2\ 0) \rightarrow (0\ 0\ 1\ 0) = y_1\ (= y).$

Thus once again the MOD interval resultant is MOD classical special fixed point pair given by

{(1 0 0 0 0 1 0), (0 0 1 0)}.

Let y = (0 0 0 1) $\in$ Y, to find the effect of y on S.

$yS^t \to (0\ 0\ 0\ 1) = y_1\ (= y)$.

Hence the MOD resultant in this case is also a MOD classical special fixed point pair given by

$$\{(0\ 0\ 0\ 1\ 0\ 0\ 0), (0\ 0\ 0\ 1)\}.$$

Thus we can built the MOD interval Relational Maps model using entries from $[0, n)$; $2 \leq n < \infty$.

Unless the operation of thresholding is done at each stage it is impossible to arrive at a MOD interval resultant by finite number of iterations.

Hence updating and thresholding operations have become mandatory.

We now proceed onto describe the MOD interval Relational Maps model in the following.

Let P be the problem in hand. Suppose the expert uses the edge weights of the graph G for the model from the interval $[0, n)$.

Further the expert uses the Relational Maps model.

The graph G will be known as the MOD interval bipartite directed graph with edge weights from $[0, n)$; $2 \leq n < \infty$.

Let M be the MOD interval connection matrix of the MOD interval directed graph G. M with the graph G will be known as the MOD interval Relational Maps model.

We have already explains all the properties related with this new model.

It is noted that only we use the operations of thresholding and updating at each stage.

Next we proceed onto describe the MOD interval finite complex number Relational Maps model or MOD finite complex number Relational Maps model.

To this end we need to define MOD finite complex bipartite directed graph and the related connection matrices.

All these concepts will be described by examples.

Example 4.7: Let G be the MOD finite complex directed bipartite graph with edge weights from

$C([0, 6)) = \{a + bi_F / a, b \in [0, 6), i_F^2 = 5\}$ given by the following figure.

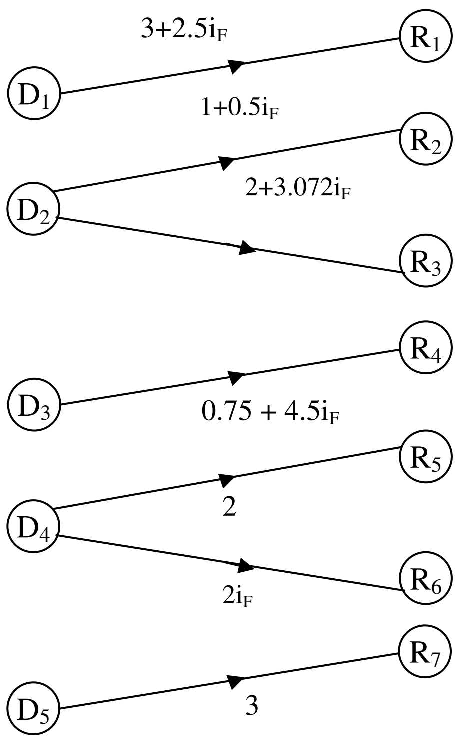


Figure 4.3

Example 4.8: Let H be the MOD complex interval directed bipartite graph with entries from

C([0, 4)) = $\{a + bi_F / a, b \in [0, 4), i_F^2 = 3\}$ given in the following;

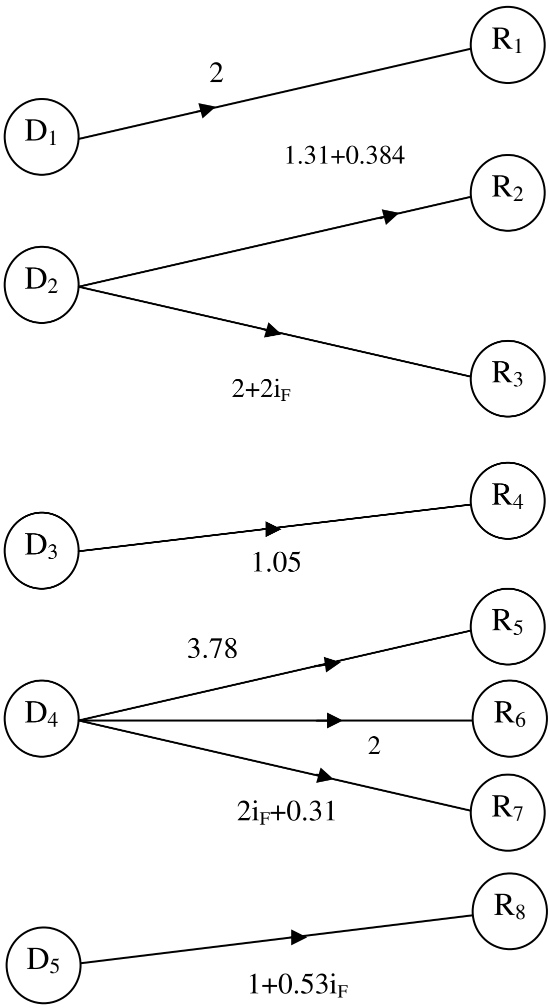


Figure 4.4

Next we proceed onto describe the MOD complex relational (rectangular) matrix.

Example 4.9: Let

$$M = \begin{bmatrix} 3.03+2.9i_F & 0 & 0 & 0 \\ 0 & 4.35i_F & 0 & 5.2 \\ 0.115 & 0 & 1.34i_F & 0 \\ 0 & 2 & 0 & 1+2.5i_F \\ 1.95i_F & 0 & 1.3i_F+7 & 0 \\ 0 & 3.72 & 0 & 1+2i_F \end{bmatrix}$$

be the MOD complex relational (rectangular matrix) with entries from C([0, 5)).

Example 4.10: Let P =

$$\left[\begin{matrix} 3+i_F & 0 & 0.1353 & 0 \\ 0 & 1.332 & 0 & 1 \\ 1+0.35i_F & 0 & 2.0017i_F & 0 \\ 0 & 4.335i_F & 0 & 2i_F+4 \end{matrix}\right.$$

$$\left.\begin{matrix} 3.2i_F & 0 & 1.324+0.57i_F \\ 0 & 0.4i_F+8.32 & 0 \\ 1.55i_F+4 & 0 & 6.325i_F \\ 0 & 1.532+0.3i_F & 0 \end{matrix}\right]$$

be the MOD complex rectangular matrix with entries from

$$C([0, 7)) = \{a + bi_F / a, b \in [0, 7);\ i_F^2 = 6\}.$$

Now we will proceed onto define operations on these types of matrices.

To this end X = {(a_1 a_2 …, a_t) / $a_i \in$ {0, 1, i_F}, $1 \leq i \leq t$} will be known as the MOD domain space of initial state vectors.

Y = {(b_1 … b_s) / $b_j \in$ {0, 1, i_F}; $1 \leq j \leq s$} is the known as the MOD range space of initial state vectors.

These will be associated with a MOD complex rectangular $t \times s$ matrix.

The operation is taken as thresholding and updating at each other for otherwise we will not in general be in a position to arrive at a MOD resultant after a finite number of iterations.

We will illustrations by some examples.

$$M = \begin{bmatrix} 0 & 3+i_F & 0 & 0.32 & 0 & 1.5i_F \\ 0.4 & 0 & 0.2i_F & 0 & i_F & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the MOD complex 3×6 matrix with entries from

C([0, 5) = {a + bi_F / a, b $\in$ [0, 5), $i_F^2 = 4$}.

X = {(a_1 a_2 a_3) / $a_i \in$ {0, 1, i_F}; $1 \leq i \leq 3$} and

Y = {(b_1 b_2 b_3 b_4 b_5 b_6) $b_{ij} \in$ {0, 1, i_F}; $1 \leq j \leq 6$} be the MOD domain space and MOD range space of initial state vector associated with A.

x = (1 0 0) $\in$ X, to find the effect of x on M.

xM = (0 3 + i_F 0 0.32 0 1+ 5i_F) $\to$ (0 1 0 0 0 i_F) = y_1;
y_1M^t = (4 + i_F 0 1) $\to$ (1 0 1) = x_1;

$$x_1M = (0\ 4 + i_F\ 0\ 0.32\ 0\ \ 1.5i_F) \to (0\ 1\ 0\ 0\ 0\ i_F) = y_2$$
$$(= y_1).$$

The MOD resultant is a MOD fixed point pair given by $\{(1\ 0\ 1), (0\ 1\ 0\ 0\ 0\ i_F)\}$.

We see the node is also complex.

Let $x = (0\ 1\ 0) \in X$, to find the effect of x on M.

$$xM = (0.4\ 0\ 0.2i_F\ 0\ i_F\ 0) \to (0\ 0\ 0\ 0\ i_F\ 0) = y_1;$$
$$y_1M^t = (0\ \ 4\ \ 0) \to (0\ \ 1\ \ 0)\ ;$$
$$x_1M = (0.4\ \ 0\ 0.2i_F\ \ 0\ \ i_F\ 0) \to (0\ 0\ 0\ 0\ i_F\ 0) = y_2\ (=y_1).$$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 1\ 0), (0\ 0\ 0\ 0\ i_F\ 0)\}.$$

Let $x = (0\ 0\ 1) \in X$ to find the effect of x on M.

$$xM = (0\ 1\ 0\ 0\ 0\ 0) = y_1;$$
$$y_1M^t = (3 + i_F\ 0\ 1) \to (1\ \ 0\ \ 1) = x_1;$$
$$x_1M = (\ 0\ 4 + i_F\ 0\ 0.32\ 0\ 1.5i_F) \to (0\ \ 1\ \ 0\ 0\ 0\ \ i_F) = y_2;$$
$$y_2M^t = (1\ \ 0\ \ 1) = x_2\ (=x_1).$$

The MOD resultant is a MOD fixed point pair given by $\{(1\ 0\ 1), (0\ 1\ 0\ 0\ 0\ i_F)\}$.

Let $y = (1\ 0\ 0\ 0\ 0\ 0) \in Y$, to find the effect of y on M

$$yM^t = (0, 0.4\ 0) \to (0\ 0\ 0) = x_1;$$
$$x_1M \to (1\ \ 0\ 0\ 0\ 0\ 0) = y_1\ (= y).$$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0), (1\ 0\ 0\ 0\ 0\ 0)\}.$$

Let $y = (0\ 1\ 0\ 0\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t = (3 + i_F\ 0\ 1) \to (1\ 0\ 1) = x_1;$
$x_1M = (0\ 4 + i_F\ 0\ 0.32\ 0\ 1.5i_F) \to (0\ 1\ 0\ 0\ 0\ i_F) = y_1;$
$y_1M^t = (4 + i_F\ 0\ 1) \to (1\ 0\ 1) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(1\ 0\ 1), (0\ 1\ 0\ 0\ i_F)\}.$$

Let $y = (0\ 0\ 1\ 0\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t = (\ 0\ 0.2i_F\ 0) \to (0\ 0\ 0) = x_1;$
$xM \to (0\ 0\ 1\ 0\ 0\ 0) = y_1\ (= y).$

Thus the MOD resultant is a MOD special classical realized fixed point pair given by

$$\{(0\ 0\ 0), (0\ 0\ 1\ 0\ 0\ 0)\}.$$

Let $y = (0\ 0\ 0\ 1\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t = (0.32\ 0\ 0) \to (0\ 0\ 0) = x_1;$
$x_1M \to (0\ 0\ 0\ 1\ 0\ 0) = y_1\ (= y).$

The MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0), (0\ 0\ 0\ 1\ 0\ 0)\}$$

Let $y = (0\ 0\ 0\ 0\ 1\ 0) \in Y$, to find the effect of y on M.

$yM^t = (0\ i_F\ 0) = x_1;$
$x_1M = (0.4i_F\ 0\ 0.8\ 0\ 4\ 0) \to (0\ 0\ 0\ 0\ 1\ 0) = y_1\ (y).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ i_F\ 0), (0\ 0\ 0\ 0\ 1\ 0)\}\ .$$

Let $y = (0\ 0\ 0\ 0\ 0\ 1) \in Y$, to find the effect of y on M,

$yM^t = (1.5i_F\ 0\ 0) \to (i_F\ 0\ 0) = x_1;$
$x_1M =(0.4i_F\ \ 0\ 0.8\ \ 0.4\ \ 0) \to (0\ 0\ 0\ 0\ 1\ 0) = y_1\ (y).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ i_F\ 0), (0\ 0\ 0\ 0\ 1\ 0)\}.$$

Let y = (0 0 0 0 0 1) ∈ Y, to find the effect of y on M.

$yM^t = (1.5i_F\ \ 0\ 0) \to (i_F\ 0\ 0) = x_1;$
$x_1M = (0\ 3i_F + 4\ 0\ \ 0.32i_F\ 0\ 1) \to (0\ 1\ 0\ 0\ 0\ 1) = y_1;$
$y_1M^t = (3 + 2.5\ i_F\ 0\ 1) \to (1\ 0\ 1) = x_2;$
$x_2M = (0\ 4 + i_F\ 0\ 0.32\ 0\ 1.5\ i_F) \to (0\ 1\ 0\ 0\ 0\ i_F) = y_2;$
$y_2M^t = (4 + i_F\ 0\ 1) \to (1\ 0\ 1) = x_3\ (= x_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(1\ 0\ 1), (0\ 1\ 0\ 0\ 0\ i_F)\}.$$

We observe this is different from MOD relational matrix model for the nodes in [0, n) can take only values 0 or 1 but in case of MOD complex rectangular matrix with entries from C([0, n)) = $\{a + bi_F$ / a, b ∈ [(0, n) $i_F^2 = n - 1\}$; the nodes can take values 0 or 1 or i_F.

Now we proceed onto describe one more example.

Example 4.11: Let

$$M = \begin{bmatrix} 0 & 0.3i_F & 1 & 0 & 0.32 \\ 0 & 0 & 0 & i_F & 0 \\ 0.1 & 0 & 0 & 0 & 2 \\ 0 & 0.01 & 0.03 & 0.2 & 0 \\ 2.2 & 0 & 0 & 0 & 0 \\ 0 & 2i_F & 0 & 0 & 0 \\ 0.03+0.i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0.3 & 0 \end{bmatrix}$$

be the MOD complex rectangular matrix with entries from $C([0.3)) = a + bi_F$ / a, b $\in$ [0, 3); $i_F^2 = 2\}$.

Let $X = \{(a_1\ a_2 \ldots a_8) / a_i \in \{0, 1, i_F\}; 1 \leq i \leq 8\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4\ b_5) / b_j \in \{0, 1, i_F\}; 1 \leq j \leq 5\}$

be the MOD domain and MOD range space of initial state vectors associated with M we perform only the operation of updating and thresholding for otherwise it is impossible to arrive at the MOD resultant after a finite number of iterations.

Let x = (1 0 0 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM = (0\ \ 0.3i_F\ 1\ 0\ 0.32) \to (0\ 0\ 1\ 0\ 0) = y_1;$
$y_1M^t = (1\ 0\ 0\ 0.03\ 0\ 0\ 0\ 0.02) \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$
$= x_1\ (= x).$

Thus the MOD realized resultant of x is a MOD special classical fixed point pair given by {(1 0 0 0 0 0 0 0) (0 0 1 0 0)}.

Let x = (0 1 0 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM = (0\ 0\ 0\ i_F\ 0) = y_1;$
$y_1M_t = (0\ 2\ 0\ 0.2i_F\ 0\ 0\ 0\ 0.3i_F) \to (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0)$
$= x_1\ (=x).$

Thus the MOD realized resultant is once again a MOD special classical fixed point pair given by
$\{(0\ 1\ 0\ 0\ 0\ 0\ 0\ 0), (0\ 0\ 0\ i_F\ 0)\}.$

Let x = (0 0 1 0 0 0 0 0) $\in$ X to find the effect of x on M.

$xM = (0.1\ 0\ 0\ 0\ 2) \to (0\ 0\ 0\ 0\ 1) = y_1;$
$y_1M^t = (0.32\ 0\ 2\ 0\ 0\ 0\ 0\ 0) = x_1 \to (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)$
$= x_1\ (= x).$

Thus once again the MOD realized resultant is a MOD classical special fixed point pair given by

$$\{(0\ 0\ 1\ 0\ 0\ 0\ 0\ 0),\ (0\ 0\ 0\ 0\ 1)\}.$$

Let $x = (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$$xM = (0\ 0.01\ 0.03\ 0.2\ 0) \to (0\ 0\ 0\ 0\ 0) = y_1;$$
$$y_1M^t = (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0) = x_1\ (= x).$$

Thus the MOD realized resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 1\ 0\ 0\ 0\ 0),\ (0\ 0\ 0\ 0\ 0)\}.$$

Let $x = (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$$xM = (2.2\ 0\ 0\ 0\ 0) \to (1\ 0\ 0\ 0\ 0) = y_1;$$
$$y_1M^t = (0\ 0\ 0.1\ 0\ 2.2\ 0\ 0.03 + 0\ i_F\ 0) \to$$
$$(0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) = x_1\ (= x).$$

Thus once again the MOD realized resultant is the MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 1\ 0\ 0\ 0),\ (1\ 0\ 0\ 0\ 0)\}.$$

Let $x = (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0) \in X$, to find the effect of x on M,

$$xM = (0\ 2i_F\ 0\ 0\ 0) \to (0\ i_F\ 0\ 0\ 0) = y_1$$
$$y_1M^t = (0\ 0.6\ 0\ 0\ 0.0\ i_F\ 4\ 0\ 0) \to (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0)$$
$$= x_1\ (= x).$$

Thus the MOD resultant once again is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 1\ 0\ 0),\ (0\ i_F\ 0\ 0\ 0)\}.$$

It is important and interesting to note that one of the nodes take the imaginary value i_F, which is the marked difference between the MOD interval matrix operator.

Let $x = (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) \in X$ to find the effect of x on M.

$xM = (0.03 + 0\ i_F\ 0\ 0\ 0\ 0) \to (0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) = x_1\ (= x).$

Thus the MOD realized resultant is the MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ 1\ 0), (0\ 0\ 0\ 0\ 0)\}.$$

Let $x = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 1) \in X$, to find the effect of x on M.

$xM = (0\ 0\ 0.2\ 0.3\ 0) \to (0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0\ 0\ 0\ 0\ 0\ 0\ 0) = x_1\ (= x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ 0\ 1), (0\ 0\ 0\ 0\ 0)\}.$$

Consider $y = (1\ 0\ 0\ 0\ 0) \in X$, to find the effect of y on M.

$yM^t = (0\ 0\ 0.1\ 0\ 2.2\ 0\ 0.03 + 0.i_F\ 0) \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) = x_1;$
$x_1M \to (1\ 0\ 0\ 0\ 0) = y_1\ (= y).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 1\ 0\ 0\ 0), (1\ 0\ 0\ 0\ 0)\}.$$

Let $y = (0\ 1\ 0\ 0\ 0\ 0) \in Y$ to find the effect of y on M.

$yM^t = (0.3i_F\ 0\ 0\ 0.01\ 0\ 2i_F\ 0\ 0) \to (0\ 0\ 0\ 0\ 0\ i_F\ 0\ 0) = x_1;$
$x_1M = (0.2i_F\ 0\ 0\ 0) \to (0\ i_F\ 0\ 0\ 0) = y_1;$
$y_1M^t = (0.6\ 0\ 0\ 0.0\ i_F\ 0\ 1\ \ 0\ 0) \to (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0) = x_2;$
$x_2M = (\ 0\ 2i_F\ 0\ 0\ 0) \to (0\ i_F\ 0\ 0\ 0) = y_2\ (=y_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 1\ 0\ 0), (0\ i_F\ 0\ 0\ 0)\}.$$

Let y = (0 0 1 0 0) $\in$ Y to find the effect of y on M.

yM^t = (1 0 0 0.03 0 0 0 0.02) $\to$ (1 0 0 0 0 0 0 0) = x_1;
x_1M = (0 0.3i_F 1 0 0.32) $\to$ (0 0 1 0 0)= y_1 (= y).

Thus the MOD realized resultant is a MOD special classical fixed point pair given by

{(1 0 0 0 0 0 0 0), (0 0 1 0 0)}.

Let y = (0 0 0 1 0) $\in$ Y, to find the effect of y on M.

yM^t = (0 i_F 0 0.2 0 0 0 0.3) $\to$ (0 i_F 0 0 0 0 0 0) = x_1;
x_1M = (0 0 0 2 0) $\to$ (0 0 0 1 0) = y_1 (= y).

Thus once again the MOD realized resultant is a MOD special classical fixed point pair given by

{(0 i_F 0 0 0 0 0 0), (0 0 0 1 0)}.

Let y = (0 0 0 0 1) $\in$ Y, to find the effect of y on M.

yM^t = (0.32 0 2 0 0 0 0 0) $\to$ (0 0 1 0 0 0 0 0) = x_1;
x_1M = (0.1 0 0 0 2) $\to$ (0 0 0 0 1) = y_1 (= y).

Thus the MOD realized resultant is a MOD special classical fixed point pair given by

{(0 0 1 0 0 0 0 0), (0 0 0 0 1)}.

Now we proceed onto mention some of the special features about this matrix operator M.

i) We need the stage by state updating and thresholding of the resultant for us to arrive at a MOD resultant after a finite number of iterations.

ii) This MOD resultant can give the node values for any initial state vector the value 0 or 1 or i_F.

This is yet another major difference between the usual FRMs or NRMs and MODRMs.

Now we leave the task of building the MOD complex Relational Maps model using

$C([0, n)) = \{a + bi_F / a, b \in [0, n), i_F^2 = (n - 1)\}$; for it is considered as a matter of routine so left as an exercise to the reader.

Next we proceed onto describe the MOD neutrosophic Relational Maps model using

$\langle [0, n \cup I \rangle = \{a + bI / a, b \in [0, n); I^2 = I\}$ where I is an indeterminate for more refer [6,7].

We first describe by examples the MOD neutrosophic bipartite directed graphs.

Example 4.12: Let G be the MOD bipartite directed graph with edge weights from $\langle [0, 8) \cup I \rangle$. G is given in the following figure.

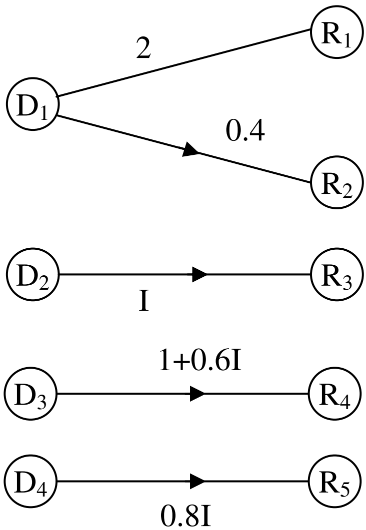


Figure 4.3

Example 4.13: Let H be MOD directed bipartite graph with entries from $\langle[0,90)\, n \cup I\rangle$ given by the following figure.

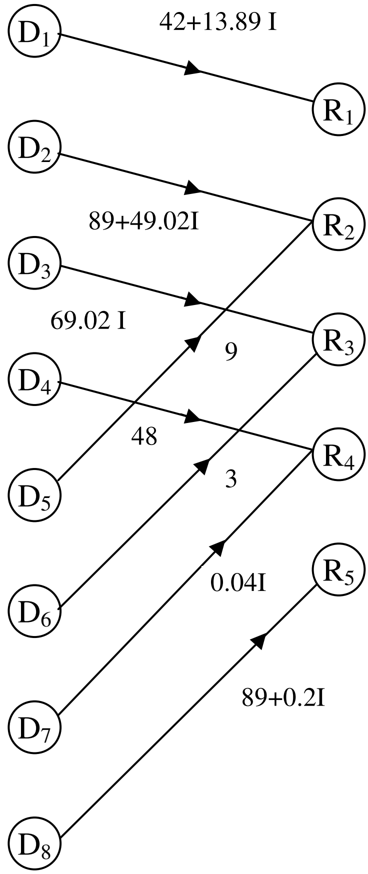


Figure 4.6

Thus if we have a MOD bipartite directed graph with its edge weights from $\langle [0,n) \cup I\rangle = \{a + bI \,/\, a, b \in [0, n); I^2 = I\}$ then we define G to be a MOD neutrosophic directed bipartite graph.

This is well illustrated by examples 4.12 and 4.13.

Next we proceed onto describe the MOD neutrosophic rectangular or relational matrices by some examples.

Example 4.14: Let

$$M = \begin{bmatrix} 0 & 2.3I & 0 & 1+I & 0 & 5 \\ 0.35 & 0 & 1 & 0 & 1.5I & 0 \\ 0 & 1+0.3I & 0 & 6 & 0 & 0.3 \\ I & 0 & 0.7I+2 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD neutrosophic rectangular or relational matrix with entries from $\langle[0, 9) \cup I\rangle = \{a + bI / a,b \in [0,9), I^2 = I\}$.

Example 4.15: Let

$$S = \begin{bmatrix} 0 & 3+0.2I & 0 & 0 & 0.004 & 0 \\ 1+I & 0 & 0.2I & 0.3I & 0 & 0.12 \\ 0 & 0 & 4 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0.333 & 0 & 0.221 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3I & 0 \\ 0.31 & 0 & 0.04I & 0 & 0 & 0.3 \\ 0 & 0.01 & 0 & 0.011 & 0.1 & 0 \\ 0.02 & 0 & 0 & 0 & 0 & 0.5I \\ 0 & 0 & 0.101 & 0.21I & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2I \end{bmatrix}$$

be the MOD neutrosophic relational 11×6 matrix with entries $\langle[0, 5) \cup I\rangle = \{a + bI / a, b \in [0,5); I^2 = I$.

Now we will describe only one special type of operation using these matrices.

For we see all types of operations will certainly not yield a MOD fixed point pair or a MOD limit cycle pair after a finite number of iterations.

This type of operation in particular and this study in general is important for it alone can give the applications and help in the construction of MOD neutrosophic Relational Maps model.

To this end we define the MOD domain and MOD range of initial state vectors associated with a MOD neutrosophic relational or rectangular $s \times t$ matrix M with entries from

$$\langle [0, n) \cup I \rangle = \{a + bI / a, b \in [0, n)' \; I^2 = I\}.$$

Example 4.16: Let

$$M = \begin{bmatrix} 0 & 0.1 & 0.01 & 1 & 0 & 0 & 0 & 2.4I & 0 \\ 1 & 0 & 0 & 0 & 0.2 & 0.02I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0.06 & 0.07 & 0.08 \\ 2I & 0 & 2 & 0 & I & 0.1 & 6 & 0 & 6.2I \end{bmatrix}$$

be the MOD neutrosophic relational or rectangular matrix M with entries from $\langle [0, 9) \cup I \rangle = \{a + bI / a, b \in [0, 9); I^2 = I\}$.

Let $X = \{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, I, I\}, 1 \leq i \leq 4\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_9) / b_j \in \{0, 1, I\}; 1 \leq j \leq 9\}$ be the MOD domain space and MOD range space of initial state vectors respectively associated with M.

Let $x = (1\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$$xM = (0\ 0.1\ 0.01\ 1\ 0\ 0\ 0\ 2.4I\ 0) \to (0\ 0\ 0\ 1\ 0\ 0\ 0\ I\ 0) = y_1$$

$$y_1M^t = (1 + 2.4I\ \ 0\ \ 0.4 + 0.07I\ \ 0) \to (I\ 0\ 0\ 0) = x_1$$

$$x_1M = (0\ 0.I\ 0.0I\ I\ 0\ 0\ 0\ 2.4I\ 0) \to (0\ 0\ 0\ I\ 0\ 0\ 0\ I\ 0) = y_2$$

$$y_2M^t = (34I\ 0\ 0.47I\ 0) \to (I\ 0\ 0\ 0) = x_2\ (=x_1).$$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(I\ 0\ 0\ 0), (0\ 0\ 0\ I\ 0\ 0\ 0\ I\ 0)\}.$$

Thus the nodes in this case has become indeterminates.

Let $x = (0\ 1\ 0\ 0) \in X$, to find the effect of x on M.

$xM = (1\ 0\ 0\ 0\ 0.2\ 0.02I\ 0\ 0\ 0) \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$
$= y_1;$
$y_1M^t = (0\ 1\ 0\ 2I) \to (0\ 1\ 0\ I) = x_2;$
$x_1M = (1 + 2I\ 0\ 2I\ 0\ 0.2 + I\ 0.02I + 0.I,\ 6I\ 0\ 6.2I) \to$
$(I\ 0\ I\ 0\ 0\ 0\ I\ 0\ I) = y_2;$
$y_2M^t = (I\ I\ 0\ I) = x_2;$
$x_2M \to (I\ 0\ I\ I\ I\ 0\ I\ I\ I) = y_3;$
$y_3M^t \to (I\ I\ 0\ I) = x_3\ (=x_2).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(I\ I\ 0\ I), (I\ 0\ I\ I\ I\ 0\ I\ I\ I)\}.$$

Let $x = (0\ 0\ 1\ 0) \in X$ to find the effect of x on M.

$xM = (0\ 0\ 0\ 0.4\ 0\ 0\ 0.6\ 0.07\ 0.08) \to (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y_1$
$y_1M^t \to (0\ 0\ 1\ 0) = x_1\ (= x).$

Thus MOD realized resultant is a MOD special classical fixed point pair

$$\{(0\ 0\ 1\ 0), (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)\}.$$

Let $x = (0\ 0\ 0\ 1) \in X$, to find the effect of x on M.

$xM = (2I\ 0\ 2\ 0\ I\ 0.1\ 6\ 0\ 6.2I) \to (I\ 0\ 1\ 0\ I\ 0\ 1\ 0\ I) = y_1;$
$y_1M^t \to (0\ I\ 0\ I) = x_1;$
$x_1M \to (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_2;$
$y_2M^t \to (0\ I\ 0\ I) = x_3\ (=x_1).$

Thus the MOD realized resultant is the MOD fixed point

$$\{(0\ I\ 0\ I), (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I)\}.$$

Let y = (1 0 0 0 0 0 0 0 0) $\in$ Y, to find the effect of y on M.

$yM^t = (0.01\ 1\ 0\ 2I) \to (0\ 1\ 0\ I) = x_1;$
$x_1M = (1 + 2I\ 0\ 2I\ 0\ I\ 0.12\ I\ 6I\ 0\ 6.2I) \to$
$(I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_1;$
$y_1M^t \to (0\ I\ 0\ I) = x_2;$
$x_2M \to (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_2\ (= y_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(0\ I\ 0\ I), (I\ 0\ I\ 0\ I\ 0\ I\ 0I)\}.$$

Thus all the nodes in both the spaces for this particular state of vector are indeterminates or zero only.

Let y = (0 1 0 0 0 0 0 0 0) $\in$ Y to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 0) = x_1;$
$xM \to (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y_1\ (=y).$

Thus the MOD resultant is the MOD classical special fixed point pair given by

$$\{(0\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)\}.$$

Let y = (0 0 1 0 0 0 0 0 0) $\in$ Y to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 1) = x_1;$
$x_1M \to (I\ 0\ 1\ 0\ I\ 0\ 1\ 0\ I) = y_1;$
$y_1M^t \to (0\ I\ 0\ I) = x_2;$
$x_2M^t \to (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_2;$
$y_2M \to (0\ I\ 0\ I) = x_3\ (= x_2).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(0\ I\ 0\ I), (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I)\}.$$

Let y = (0 0 0 1 0 0 0 0 0) ∈ Y, to find the effect of y on M.

$yM^t \to (1\ 0\ 0\ 0) = x_1;$
$x_1M \to (0\ 0\ 0\ 1\ 0\ 0\ 0\ I\ 0) = y;$
$y_1M^t \to (I\ 0\ 0\ 0) = x_2;$
$x_2M \to (0\ 0\ 0\ I\ 0\ 0\ 0\ I\ 0) = y_2;$
$y_2M^t \to (I\ 0\ 0\ 0) = x_3\ (=x_2).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(I\ 0\ 0\ 0), (0\ 0\ 0\ I\ 0\ 0\ 0\ I\ 0)\}.$$

Let y = (0 0 0 0 0 0 1 0 0) ∈ Y, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 1) = x_1;$
$x_1M \to (I\ 0\ 1\ 0\ I\ 0\ 1\ 0\ I) = y_1;$
$y_1M^t \to (0\ I\ 0\ I) = x_2;$
$x_2M \to (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_2;$
$y_2M^t = (0\ I\ 0\ I) = x_3\ (=x_2).$

Thus the MOD realized resultant is a MOD fixed point pair given by

$$\{(0\ I\ 0\ I), (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I)\}.$$

All nodes which have become on are indeterminates.

Let y = (0 0 0 0 0 0 0 0 1) ∈ Y, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ I) = x_1;$
$x_1M \to (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_1;$
$y_1M^t \to (0\ I\ 0\ I) = x_2;$
$x_2M^t = (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ I\ 0\ I), (I\ 0\ I\ 0\ I\ 0\ I\ 0\ I)\}.$$

This is the way MOD realized resultant are obtained.

We will give one more example.

Example 4.17: Let

$$S = \begin{bmatrix} 0 & 3I & 0 & 0.1 \\ 1 & 0 & 0 & 0.2I \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

be the MOD neutrosophic relational matrix with entries from $\langle[0, 4) \cup I\rangle = \{a + bI / a, b \in [0,4), I^2 = I\}$.

Let $X = \{(a_1\ a_2 \ldots a_6) / a_i \in \{0, 1, I\}; 1 \leq i \leq 6\}$ and

$Y = \{(b_1\ b_2\ b_3\ \ b_4) / b_j \in \{0, 1, I\}; 1 \leq j \leq 4\}$ be the MOD domain and MOD range space respectively associated with S.

Let x = (1 0 0 0 0 0) ∈ X, to find the effect of x on S.

$xS = (0\ 3I\ 0\ 0\ 1) \to (0\ I\ 0\ 0) = y_1$;
$y_1S^t = (3I\ 0\ 0\ 0\ 0\ 2I\ 0) \to (I\ 0\ 0\ 0\ 0\ 0) = x_1$;
$x_1S \to (0\ I\ 0\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is MOD fixed point pair given by

$$\{(I\ 0\ 0\ 0\ 0\ 0), (0\ I\ 0\ 0)\}.$$

Let x = (0 1 0 0 0 0) ∈ X, to find the effect of x on S.

$xS \to (1\ 0\ 0\ 0) = y_1$;
$y_1S^t \to (0\ 1\ 0\ 0\ 0\ 0) = x_1\ (= x)$.

Thus the MOD resultant is a MOD classical special fixed point pair given by

$$\{(0\ 1\ 0\ 0\ 0\ 0), (1\ 0\ 0\ 0)\}.$$

Let x = (0 0 1 0 0 0) $\in$ X, to find the effect of x on S.

xS $\rightarrow$ (0 0 1 0) = y_1;
$y_1S^t \rightarrow$ (0 0 1 0 0 0) = x_1 (= x).

Once again the MOD realized resultant is a MOD classical special fixed point pair given by {(0 0 1 0 0 0), (0 0 1 0)}

Let x = (0 0 0 1 0 0) $\in$ X, to find the effect of x on S.

xS $\rightarrow$ (0 0 0 0) = y_1;
$y_1S^t \rightarrow$ (0 0 0 1 0 0) = x_1 (= x).

Thus the MOD realized resultant is a MOD classical special fixed point pair given by

{(0 0 0 1 0 0), (0 0 0 0)}.

Let x = (0 0 0 0 0 1) $\in$ X to find the effect of x on S.

xS $\rightarrow$ (0 0 0 1) = y_1
$y_1S^t \rightarrow$ (0 0 0 0 0 1) = x_1 (= x).

Thus once again the MOD resultant is the MOD special classical fixed point pair given by

{(0 0 0 0 0 1), (0 0 0 1)}.

Let y = (1 0 0 0) $\in$ Y, to find the effect of y on S.

$yS^t \rightarrow$ (0 1 0 0 0) = x_1;
$x_1S \rightarrow$ (1 0 0 0) = y_1 (= y).

This MOD realized resultant is also a MOD special classical fixed point pair given by

{(0 1 0 0 0 0), (1 0 0 0)}.

Let y = (0 1 0 0) $\in$ Y, to find the effect of y on S.

$yS^t \to (I\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1S \to (0\ I\ 0\ 0) = y_1;$
$y_1S^t \to (I\ 0\ 0\ 0\ 0\ 0) = x_2\ (= x_1).$

Thus the MOD realized resultant is a MOD fixed point pair given by {(I 0 0 0 0 0), (0 I 0 0)}, both the nodes which are on are indeterminates.

Let y = (0 0 1 0) ∈ Y, to find the effect of y on S,

$yS^t \to (0\ 0\ 1\ 0\ 0\ 0) = x_1;$
$x_1S \to (0\ 0\ 1\ 0) = y_1\ (= y_1).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

{(0 0 1 0 0 0), (0 0 1 0)}.

Let y = (0 0 0 1) ∈ Y to find the effect of y on S.

$yS^t \to (0\ 0\ 0\ 0\ 0\ 1) = x_1;$
$x_1S \to (0\ 0\ 0\ 1) = y_1\ (= y).$

Thus the MOD resultant is a MOD classical special fixed point pair given by

{(0 0 0 0 0 1), (0 0 0 1)}.

Interested reader can construct any such number of MOD neutrosophic matrices and work for the MOD resultants [66].

Now we just describe briefly the MOD neutrosophic Relational Maps model built using

$$\langle [0, n) \cup I \rangle = \{a + bI / a, b \in [0, n), I^2 = I\}.$$

Suppose there is a problem in hand and the expert wishes to work with MOD Interval Relational Maps model with indeterminates involved then this model which has its entries from $\langle [0, n) \cup I \rangle$ will be known as the MOD neutrosophic interval Relational Maps model or just MOD neutrosophic Relational Maps model.

The functioning of this model and their associated properties are left as an exercise to the reader as it is considered as a matter of routine.

Next we proceed on to describe MOD dual number Relational Maps model.

We first describe MOD dual number bipartite directed graph G with entries from

$\langle [0, n) \cup g \rangle = \{a + bg / g^2 = 0, a, b \in [0, n)\}$ by some examples.

Example 4.18: Let G be a MOD directed bipartite graph with edge weights from $\langle [0, n) \cup g \rangle = \{a + bg / g^2 = 0, a, b \in [0, 9)\}$ given by the following figure.

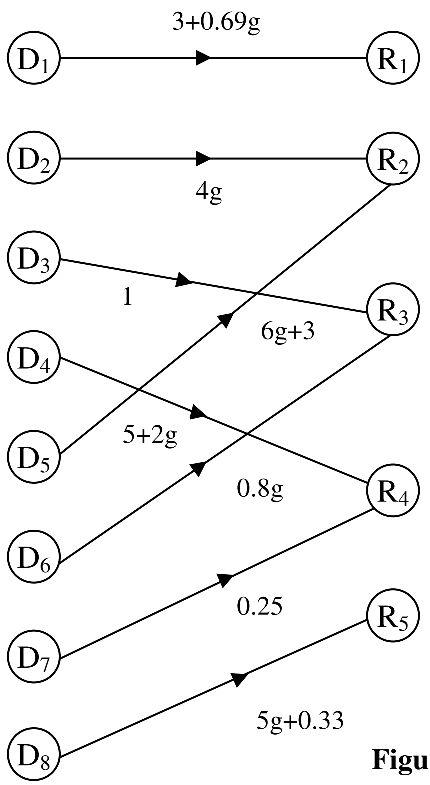


Figure 4.7

This MOD dual number directed bipartite graph.

Example 4.19: Let G be the MOD dual number directed bipartite graph with edge weights from

$\langle [0, 1) \cup g \rangle = \{a + bg / a, b \in [0, 11), g^2 = 0\}$ given by the following figure.

We now describe the MOD dual number relational matrix operator.

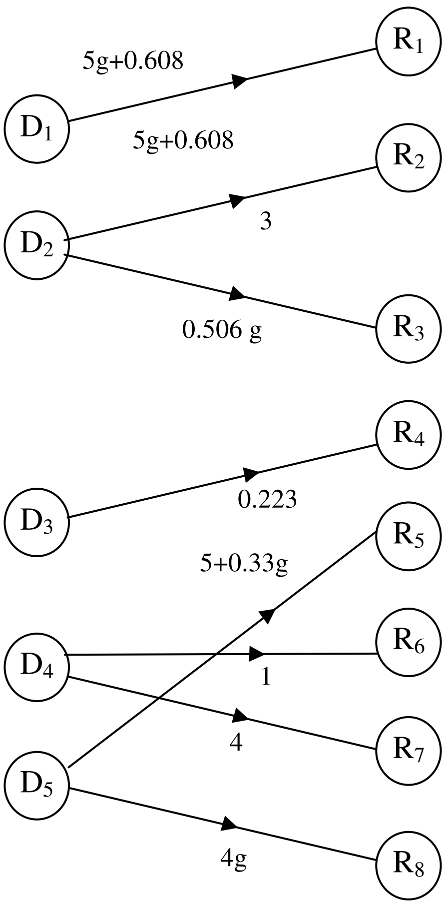


Figure 4.8

Example 4.20: Let

$$M = \begin{bmatrix} 0 & 0.3g & 0 & 0.2 & 0 & g \\ 1 & 0 & 0.03 & 0 & 0.01 & 0 \\ 0.2g & 0.4 & 0 & 2g & 0 & 0.1 \\ 0.6 & 0 & 0.2g & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0.2g & 0 \\ 0.32g & 0 & 2g & 0.42 & 0 & 1 \\ 0 & 0.21 & 0 & 0.01 & 1 & 0 \end{bmatrix}$$

be the MOD dual number rectangular or relational matrix with entries from

$$\langle [0, 14) \cup g \rangle = \{a + bg / a, b \in [0, 4), g^2 = 0\}.$$

Example 4.21: Let

$$S = \begin{bmatrix} 0 & 3.2g & 0.01 & 0 & 0.02g & 0 & 0.g & 0.3 \\ 0.2 & 0 & 0 & 1 & 0 & 0.2g & 0 & 0 \\ 0 & 0 & 2g & 0 & 0.3 & 0 & 0.1 & 0 \\ 1.2 & 0 & 0 & 0 & 0 & 0 & 1.02 & 0.4g \\ 0 & 0.32 & 0 & 0.52g & 0 & 0 & 0 & g \end{bmatrix}$$

be the MOD dual number relational matrix operator with entries from

$$\langle [0, 4) \cup g \rangle = \{a + bg/a, b \in [0, 4); g^2 = 0\}.$$

In this case also we cannot perform usual operations we can opt only for the thresholding and updating operations.

Thus if G be a MOD dual number directed bipartite graph with edge weights from

$$\langle [0, n) \cup g \rangle = \{a + bg / a, b \in [0, n)\ g^2 = 0\}.$$

Let $M = (a_{ij})_{t \times s}$ be the MOD dual number relational matrix of the graph G.

Clearly $a_{ij} \in \langle[0, n) \cup g\rangle$; $1 \leq i \leq t$, and $1 \leq j \leq s$.

Let $X = \{(a_1\ a_2\ \ldots\ a_t) / a_i \in \{0, 1, g\};\ 1 \leq i \leq t\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_s) / b_j \in \{0, 1, g\};\ 1 \leq j \leq s\}$ be the MOD domain space and MOD range space of initial state vectors respectively associated with $M = (a_{ij})_{t \times s}$.

We will M the dynamical system of the MOD dual number Relational Maps model.

We will find for any $x \in X$ (or $y \in Y$) MOD resultants using M by the method of updating and thresholding at each stage for otherwise we will not be in a position to arrive the MOD resultant after a finite number of iterations.

We will illustrate by an example or two.

Example 4.22: Let M be the MOD dual number relational matrix which is the MOD dual number dynamical system given in the following with entries from

$$\langle[0, 9) \cup g\rangle = \{a + bg / a, b \in [0, 9), g^2 = 0\}.$$

$$M = \begin{bmatrix} 0 & 2.1g & 0 & 0 & 0.2g & 0 & 0 & 0.12 \\ 0.g & 0 & 0.2 & 1 & 0 & 0.21 & 0.12g & 0 \\ 0 & 0 & 0 & 0.2g & 0 & 1 & 0 & 0 \\ 0.14g & 0 & 0 & 0 & 4 & 0 & 0 & 0.2g \\ 0 & 0.4g & 6 & 0 & 0 & 0 & 0.25 & 0 \end{bmatrix}$$

Let $X = \{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in \{0, 1, g\};\ 1 \leq i \leq 5\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_8) / b_j \in \{0, 1, g\};\ 1 \leq j \leq 8\}$ be the MOD domain and MOD range space of initial state vectors respectively associated with the MOD dynamical system M.

Let x = (1 0 0 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ g\ 0\ 0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (1\ 0\ 0\ 0\ 0) = x_1\ (= x).$

Thus the MOD resultant is a MOD classical special fixed point pair given by

$$\{(1\ 0\ 0\ 0\ 0\ 0), (0\ g\ 0\ 0\ 0\ 0\ 0\ 0)\}.$$

Let x = (0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 1\ 0\ 0\ 0) = x_1\ (= x).$

Thus once again the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 1\ 0\ 0\ 0), (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)\}.$$

Let x = (0 0 1 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 1\ 0\ 0) = x_1\ (= x);$
$x_1M \to (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0)$.

Thus the MOD resultant again is a MOD classical special fixed point pair given by

$$\{(0\ 0\ 1\ 0\ 0), (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0)\}$$

Let x = (0 0 0 1 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 1\ 0) = x_1\ (= x).$

Thus in this case also the MOD resultant is a MOD classical special fixed point pair given by

$$\{(0\ 0\ 0\ 1\ 0), (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0)\}.$$

Let x = (0 0 0 0 1) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 0\ 1) = x_1$ (= x).

Thus in this case also the MOD resultant is a MOD special classical fixed point pair given by

$\{(0\ 0\ 0\ 0\ 1), (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)\}.$

Let y = $(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \in$ Y, to find the effect of y on M .

$yM^t \to (0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y$ (= y_1).

Thus the MOD resultant in this case is also a MOD classical special fixed point pair given by

$\{(0\ 0\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 0\ 0\ 0\ 0)\}.$

Let y = $(0\ 0\ 1\ 0\ 0\ 0\ 0\ 0) \in$ Y, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 0\ 1) = x_1;$
$x_1M \to (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0) = y_1$ (= y).

Thus the MOD resultant is a MOD special classical fixed point pair given by

$\{(0\ 0\ 0\ 0\ 1), (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)\}.$

Let y = $(0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) \in$ Y to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 1\ 0) = x_1;$
$x_1M \to (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0) = y_1$ (= y).

Thus in this case also the MOD resultant is a MOD special classical fixed point pair given by

$\{(0\ 0\ 0\ 1\ 0), (0\ 0\ 0\ 0\ 1\ 0\ 0\ 0)\}.$

Let y = $(0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) \in$ Y, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 0\ 0) = x_1;$

$x_1M \to (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) = y_1$ (= y).

Thus the MOD resultant is the MOD classical special fixed point pair, given by

$$\{(0\ 0\ 0\ 0\ 0), (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0)\}.$$

Thus we can find MOD resultants.

We will give yet another example of this MOD dual number plane Relational Maps model.

Example 4.23: Let M be the MOD dual number relational matrix with entries from $\langle[0, 12) \cup g\rangle = \{a + bg / a, b \in [0, 12)\ g^2 = 0\}$ which is given in the following.

$$M = \begin{bmatrix} 0 & 3.2 & 0 & 0.2g+0.1 & 0 \\ 1 & 0 & 0 & 0 & 0.01 \\ 0 & 0.1 & 0.04 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0.32g & 0 & 0 & 0.5g & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0.17g & 0.21g & 0 & 4.1 & 0 \end{bmatrix}.$$

Let $X = \{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7) / a_i \in \{0, 1, g\}; 1 \leq i \leq 7\}$ and

$Y = (\{b_1\ b_2\ b_3\ b_4\ b_5) / b_i \in \{0, 1, g\}; 1 \leq i \leq 5\}$ be the MOD domain and range space of initial state vectors respectively associated with M.

Let $x = (1\ 0\ 0\ 0\ 0\ 0\ 0) \in X$ to find the effect of x on M.

$xM \to (0\ 1\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (1\ 0\ 0\ 0\ 0\ 0\ 0) = x_1$ (= x).

Thus the MOD resultant is a MOD special classical fixed point pair given by $\{(1\ 0\ 0\ 0\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 0)\}$.

Let x = (0 1 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM \to (1\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (0\ 1\ 0\ 0\ 0\ 0\ 0) = x_1$ (= x).

Thus the MOD resultant in this case is also a MOD special classical fixed point pair given by
{(0 1 0 0 0 0 0), (1 0 0 0 0)}.

Let x = (0 0 0 0 1 0 0) $\in$ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (0\ 0\ 0\ 0\ 1\ 0\ 0) = x_1$ (= x).

Thus the MOD resultant is a MOD special classical fixed point pair given by
{(0 0 0 0 1 0 0), (0 0 0 0 0)}.

Let y = (0 1 0 0 0) $\in$ Y, to find the effect of y on M.

$yM^t \to (1\ 0\ 0\ 0\ 0\ 0\ 0) = x_1$;
$x_1M \to (0\ 1\ 0\ 0\ 0) = y_1$ (= y).

Thus the MOD realized resultant is a MOD classical special fixed point pair given by
{(1 0 0 0 0 0 0), (0 1 0 0 0)}.

Let y = (0 0 0 0 1) $\in$ Y, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 1\ 0\ 0\ 0) = x_1$;
$x_1M \to (0\ 0\ 0\ 0\ 1) = y_1$ (= y).

Thus the MOD resultant is a MOD special classical fixed point pair given by
{(0 0 0 1 0 0 0), (0 0 0 0 1)}.

Let x = (0 0 0 0 0 0 g) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ g\ 0) = y_1$;
$y_1M \to (0\ 0\ 0\ 0\ 0\ 0\ g) = x_1\ (= x_1)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0\ 0\ 0\ g), (0\ 0\ 0\ g\ 0)\}.$$

Thus the nodes state are the dual element g.

Interested reader can work with any of the MOD dual number plane Relational Maps model.

This is considered as a matter of routine exercise so left for the reader.

Next we proceed onto describe the MOD special dual like number plane Relational Maps model.

To this end we first describe the basic tools needed for the construction for this model by some examples.

Example 4.24: Let G be a MOD directed bipartite graph with edge weights from

$$\langle [0, 3) \cup h \rangle = \{a + bh \mid a, b \in [0, 3), h^2 = h\}.$$

Then G will be known as the MOD special dual like number directed bipartite graph.

G is given by the following figure.

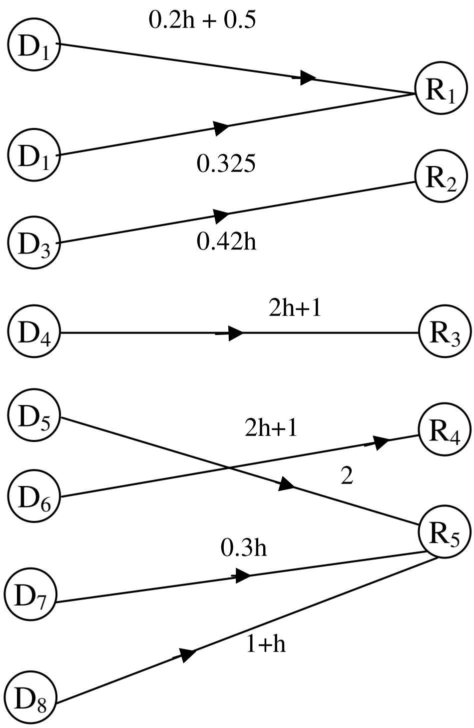


Figure 4.9

Interested reader can have more such MOD special dual like number plane directed bipartite graph.

Next we give one example of the MOD special dual like number relational or rectangular matrices.

Example 4.25: Let

$$M = \begin{bmatrix} 0 & 0.35h & 0 & h \\ 1 & 0 & 0.3+0.5h & 0 \\ 0.2h & 0 & 0 & 0.2 \\ 0 & 2h & 1 & 0 \\ 0.3 & 0 & 0.4 & 0 \\ 0.1+0.2h & 0 & 0 & 1+h \\ 0 & 1 & h & 0 \end{bmatrix}$$

special dual like number relational or rectangular matrix with entries from $\langle[0,4) \cup h\rangle = \{a + bh / a, b \in [0,4); h^2 = h\}$.

It is left as an exercise for the reader the task of giving examples and getting the MOD special dual like number connection matrices given the MOD special dual like number directed bipartite graphs.

Now we proceed onto describe how the MOD resultants can be obtained using MOD special dual like number relational matrix operator.

At first it is important to mention that only special type of operations which involves both updating and thresholding at each stage alone can give the MOD resultant after finite number of iterations, otherwise finding a resultant is a NP hard problems.

To this end we first describe the tools needed for this operator to function.

Let $M = (m_{ij})_{s\times t}$ $(s\neq t)$ be a MOD special dual like number rectangular or relational matrix with entries from $\langle[0,n) \cup h\rangle = \{a + bh / a, b \in [0,n); h^2 = h\}$.

Let $X = \{(a_1 \ldots a_s) / a_i \in \{0, 1, h\}; 1 \leq i \leq s\}$ and

Y = {(b_1 … b) / $b_j \in$ {0, 1, h}; $1 \leq j \leq t$} be the MOD special dual like number MOD domain and MOD range space of initial state of vector respectively associated with M.

We will describe this situation by some examples.

Example 4.26: Let

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 & h \\ 0.2 & 0 & 1 & 0 & 0 \\ 0 & 0.3h & 0 & 0.2 & 0 \\ 0 & h & 0 & 0 & 0.32 \\ 0.5 & 0 & 0 & 1+2h & 0 \\ 0 & 0.1 & 0.3 & 0 & 0 \\ h & 0 & 0 & 0 & 2 \end{bmatrix}$$

be the MOD special dual like number relational matrix operator with entries from

$\langle [0,4) \cup h \rangle$ = {a + bh / a, b $\in$ [0,4); $h^2 = h$}.

Let X = {(a_1 a_2 … a_7) / $a_i \in$ {0, 1, h}; $1 \leq i \leq 7$} and

Y = {(b_1 b_2 b_3 b_4 b_5) / $b_j \in$ {0, 1, h}, $1 \leq j \leq 5$} be the MOD domain and MOD range of initial state vectors respectively associated with the MOD relational matrix M.

Let x = (1 0 0 0 0 0) $\in$ X to find the effect of x on M.

xM $\to$ (1 0 0 0 h) = y_1;
$y_1M^t \to$ (1 0 0 0 0 0 h) = x_1;
$x_1M \to$ (1 0 0 0 h) = y_2 (= y_1)

Thus the MOD resultant is a MOD fixed point pair given by
{(1 0 0 0 0 0 h), (1 0 0 0 h)}.

Let x = (0 1 0 0 0 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 1\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 1\ 0\ 0\ 0\ 0\ 0) = x_1\ (= x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$\{(0\ 1\ 0\ 0\ 0\ 0\ 0), (0\ 0\ 1\ 0\ 0)\}.$

Let x = (0 0 1 0 0 0 0) ∈ ×, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M \to (0\ 0\ 1\ 0\ 0\ 0\ 0) = x_1\ (=x).$

Thus the MOD resultant is a MOD classical special fixed point pair given by

$\{(0\ 0\ 1\ 0\ 0\ 0\ 0), (0\ 0\ 0\ 0\ 0)\}.$

Let x = (0 0 0 1 0 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ h\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ h\ 0\ 0\ 0) = x_1;$
$x_1M \to (0\ h\ 0\ 0\ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$\{(0\ 0\ 0\ h\ 0\ 0\ 0), (0\ h\ 0\ 0\ 0)\}$

Let x = (0 0 0 0 0 1 0) ∈ X to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 0\ 0\ 1\ 0) = x_1\ (= x).$

Thus the MOD resultant is a MOD special classical fixed point pair given by

$\{(0\ 0\ 0\ 0\ 0\ 1\ 0),\ (0\ 0\ 0\ 0\ 0)\}.$

Let x = (0 0 0 0 0 0 1) ∈ X, to find the effect of x on M.

$xM \to (h\ 0\ 0\ 0\ 1) = y_1;$
$y_1M^t \to (h\ 0\ 0\ 0\ 0\ 0\ h);$
$x_1M \to (h\ 0\ 0\ 0\ h) = y_1;$
$y_1M^t \to (h\ 0\ 0\ 0\ 0\ 0\ h).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(h\ 0\ 0\ 0\ 0\ 0\ h), (h\ 0\ 0\ 0\ h\}.$

Let $y = (0\ 0\ 0\ 0\ 1) \in Y$, to find the effect of y on M.

$yM^t \to (h\ 0\ 0\ 0\ 0\ 0\ 1) = x_1;$
$x_1M \to (h\ 0\ 0\ 0\ 1) = y_1;$
$y_1M^t \to (h\ 0\ 0\ 0\ 0\ 0\ 1) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(h\ 0\ 0\ 0\ 0\ 0\ 1), (h\ 0\ 0\ 0\ 1)\}.$

Let $y = (0\ 0\ h\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t \to (0\ h\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (0\ 0h\ 0\ 0) = y_1\ (= y).$

Thus the MOD resultant is a MOD classical special fixed point pair given by
$\{(0\ h\ 0\ 0\ 0\ 0\ 0), (0\ 0\ h\ 0\ 0)\}.$

Hence the study of MOD special dual like number plane Relational Maps model is left as an exercise to the reader.

Now we proceed onto describe the MOD special quasi dual number model maps using from
$\langle [0,n) \cup h\rangle = \{a + bk / a, b \in [0,n); k^2 = h\}.$

We first describe the MOD special dual number directed bipartite graphs by example.

Example 4.27: Let G be the MOD special quasi dual number plane directed bipartite graph G with edge weights from from $\langle [0,6) \cup k \rangle = \{a + bk / k^2 = 5k \text{ where } a, b \in [0, 6)\}$.

G is given by the following figure.

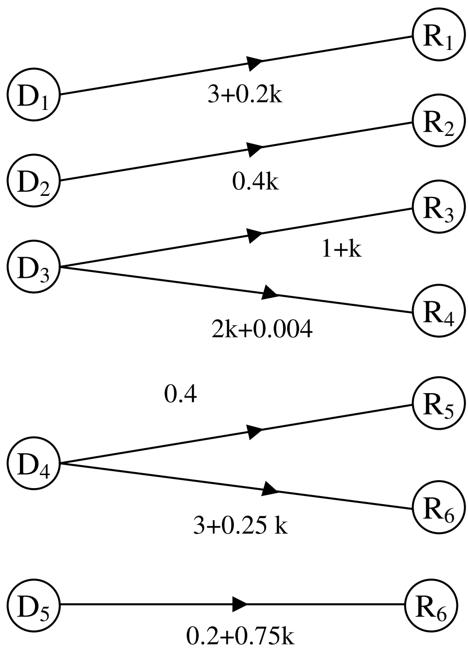


Figure 4.10

The interested reader is expected to give more examples of MOD special quasi dual number directed bipartite graph with edge weights from $\langle [0,n) \cup k \rangle = \{a + bk / a, b \in [0,n); k^2 = k\}$; $2 \leq n < \infty$.

Next we proceed on to give one example of a MOD special quasi dual number rectangular matrix with entries from

$$\langle [0,n) \cup k \rangle = \{a + bk / a, b \in [0,n); k^2 = (n-1)k\}.$$

Example 4.28: Let

$$M = \begin{bmatrix} 0.3k & 0 & 1 & 0.4 & 0 & 0 & 0.2+0.5k \\ 0 & 2 & 0 & 0 & 0 & 0.2k & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.62k \\ 0.1 & 0 & 0.5k & 0 & 0 & 0.25 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0.125 \\ 0 & 0 & 0 & 2k & 0 & 0 & 0 \\ 0 & 0.2k & 0 & 0 & 0 & 4+2k & 0 \\ k & 0 & 0.3k & 4 & 0.2k & 0 & 2+5k \end{bmatrix}$$

be the MOD special quasi dual number relational matrix with entries from

$$\langle [0,5) \cup k \rangle = \{a + bk / a, b \in [0,5); k^2 = 4k\}.$$

The reader is expected to give more examples of them.

Now we proceed onto work with special operations using MOD special quasi dual number relational or $t \times s$ rectangular matrices $M = (m_{ij})$ with entries from

$\langle [0,n) \cup k \rangle = \{a + bk / a, b \in [0,n); k^2 = (n-1) k\}$; that is

$m_{ij} \in \langle [0,n) \cup k \rangle$; $1 \leq i \leq 5$, $1 \leq j \leq s$.

Let $X = \{(a_1\ a_2 \ldots a_t) / a_i \in \{0, 1, k\}; 1 \leq i \leq t\}$ and

$Y = \{(b_1\ b_2 \ldots b_s) / b_j \in \{0, 1, k\}; 1 \leq j \leq t\}$ be the MOD domain and range space of initial state vectors associated with M.

We can do special type of operation only on M to get the final resultant after a finite number of iterations.

This will be described by an example.

Example 4.29: Let

$$M = \begin{bmatrix} 0 & 3.2k & 0 & 0.12 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0.1 & 0 & 0 & 0.2k & 0 & 0 & 0 \\ 2 & 0.2k & 0 & 0 & 1.2k & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & k & 2 \end{bmatrix}$$

be the MOD special quasi dual number relational matrix operator with entries from $\langle [0,4) \cup k \rangle$.

Let $X = \{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1, k\}; 1 \leq i \leq 4\}$ and

$Y = \{(b_1\ b_2\ \ldots\ b_9) / b_j \in \{0, 1, k\}\ 1 \leq j \leq 9\}$ be the MOD domain and MOD range of special state vectors respectively associated with M.

Let $x = (0\ 1\ 0\ 0) \in X$, to find the effect of x on M.

$xM \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (0\ k\ 0\ 0\ 0\ 0\ 1\ 0\ 0) = x_1$;
$y_1M^t \to (0\ 1\ 1\ 0) = x_1$
$x_1M \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)\}$.

Let $x = (0\ 0\ 0\ 1) \in X$, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ 0\ 0\ 0\ k\ 1) = y_1$;
$y_1M^t \to (0\ 0\ 0\ k) = x_1$;
$x_1M \to (0\ 0\ 0\ 0\ 0\ 0\ 0\ k\ k) = y_2$;
$y_2M^t \to (0\ 0\ 0\ k) = x_2\ (= x_1)$.

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ 0\ 0\ k), (0\ 0\ 0\ 0\ 0\ 0\ 0\ k\ k)\}$.

Let $y = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \in Y$ to find the effect of y on M.

$yM^t \to (0\ 1\ 1\ 0) = x_1$;
$x_1M \to (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) = y_1\ (= y)$.

Thus the MOD resultant in this case is a MOD special classical fixed point pair given by

$$\{(0\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)\}.$$

Let $y = (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 0) = x_1$;
$x_1M \to (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0) = y_1\ (= y)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 0), (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0)\}.$$

Let $y = (0\ 0\ 0\ 0\ 0\ k\ 0\ 0\ 0) \in Y$, to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ 0) = x_1$;
$x_1M \to (0\ 0\ 0\ 0\ 0\ k\ 0\ 0\ 0) = y_1\ (= y)$.

Thus the MOD resultant in this case is also a MOD special classical fixed point pair

$$\{(0\ 0\ 0\ 0), (0\ 0\ 0\ 0\ 0\ k\ 0\ 0\ 0)\}.$$

Let $y = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0) \in Y$ to find the effect of y on M.

$yM^t \to (0\ 0\ 0\ k) = x_1$;
$x_1M \to (0\ 0\ 0\ 0\ 0\ 0\ 0\ k\ k) = y_1$;
$y_1M^t \to (0\ 0\ 0\ k) = x_2\ (= x)$.

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ k), (0\ 0\ 0\ 0\ 0\ 0\ 0\ k\ k)\}.$$

Thus the interested reader can construct such models and study the special features enjoyed by them.

Hence these on new models only one type of operation can be made and also they can give node values as 0 or 1 or I or i_F

or g or h or k depending of the plane used for that relational maps model.

Next we proceed onto describe and develop the MOD natural neutrosophic interval Relational Maps model built using ${}^{I}[0,n)$, $C^{I}([0,n))$ $\langle [0, n) \cup I\rangle_I$, $\langle [0, n) \cup g\rangle_I$ $\langle [0, n) \cup h\rangle_I$ and $\langle [0, n) \cup k\rangle_I$.

We will be sparing in our description and give only an example or so for each case.

Let ${}^{I}[0,n) = \{[0, n), I_t^n$ where t is zero divisor or an idempotent or a pseudo zero divisor or pseudo unit or pseudo idempotent [60].

The notions of MOD natural neutrosophic numbers was introduced in [60].

Here we give examples of MOD natural neutrosophic interval directed bipartite graph built using ${}^{I}[0, n)$.

Example 4.30: Let ${}^{I}[0\ 3) = \{[0, 3),\ I_0^3,\ I_{1.5}^3,\ I_2^3$ and so on$\}$.

However these compatibility with respect to product is not always guaranteed.

Further $I_2^3 \times I_2^3 = I_1^3$ but I_1^3 is not in the set as it is an impossibility 1 can create a natural neutrosophic zero divisor in this set up.

For more refer [60].

Example 4.31: Let G be a MOD bipartite directed graph with edge weights from ${}^{I}[0,6)$.

We define G to be a MOD natural neutrosophic interval directed bipartite graph which is given by the following figure.

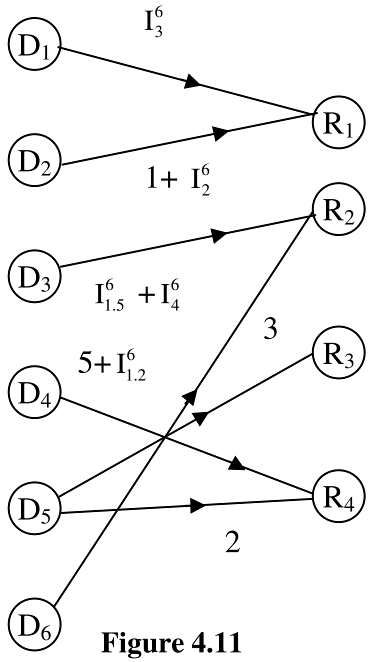


Figure 4.11

Interested reader can get more examples of MOD natural neutrosophic interval bipartite directed graphs with edge weights from $^I[0, n)$; $2 \leq n < \infty$.

Next we give an example of MOD natural neutrosophic interval relational (or rectangular) matrix operator with entries from $^I[0, n)$.

Example 4.32: Let

$$M = \begin{bmatrix} I_0^5 & 0 & 2 & 3.0011 & 0 & 1+I_{2.5}^5 \\ 0 & I_2^5+3 & 0 & 0 & 1+I_{1.25}^5 0 & 0 \\ 0 & 0 & I_{0.625}^5 & 1 & 0 & 0 \\ 2 & 0.25 & 0 & 1.001 & 0 & 1.521 \end{bmatrix}$$

be the MOD real natural neutrosophic interval relational or rectangular matrix with entries from ${}^{I}[0,5)$.

We can perform only one type of operation using the MOD natural neutrosophic real interval relational matrix operators.

Let ${}^{I}[0, n)$; $2 \leq n < \infty$ be the natural neutrosophic real interval. Let G be the MOD directed bipartite graph with edge weighs from ${}^{I}[0, n)$.

Then we define G to be the MOD natural neutrosophic real interval directed bipartite graph.

Likewise if $M = (m_{ij})_{t \times s}$ is a $t \times s$ MOD relational or rectangular matrix with entries $e_{ij} \in {}^{I}[0, n)$; $2 \leq n < \infty$ then we define M to be a MOD natural neutrosophic real interval relational or rectangular matrix operator [66].

We have given an example of it.

Interested reader can construct more such examples.

We now proceed onto describe some special types of operations using them for which we need the notion of MOD domain and MOD range initial state of vectors.

Let M be as given above.

Let $X = \{(a_1, a_2, a_3, \ldots, a_t) / a_i \in \{0, 1, I_0^n, I\ I_t^n$ / t runs over only finite such selected MOD natural neutrosophic elements as per need}; $1 \leq i \leq t\}$ be the MOD natural neutrosophic domain space of special initial state vectors associated with this M.

Let $Y = \{(b_1, b_2, \ldots b_s) / b_j \in \{0, 1, I_0^n, I, I_t^n$ / t runs only over finite MOD such natural neutrosophic elements as given in ×}; $1 \leq j \leq s\}$ be the MOD interval natural neutrosophic special initial state vectors of range space associated with M.

Thus for us to do operations on M we need elements of both X and Y further the operation at each stage is updated and thresholded.

We will describe this situation by an example.

Example 4.33: Let

$$M = \begin{bmatrix} 0 & I_0^6 & 0.21 & 0.32 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & I_{1.5}^6 & 0 \\ 0 & 0 & 1 & 0 & 4.5 & 0 & 0 \\ 0 & 0 & 0 & I_4^6 & 0 & 0 & 2 \end{bmatrix}$$

be the MOD natural neutrosophic interval relational matrix.

$X = \{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1, I\ I_{1.5}^6, I_4^6, I_2^6, I_3^6\ I_0^6\};$
$1 \leq i \leq 4\}$ and

$Y = \{(b_1\ b_2 \ldots b_7) / b_j \in \{0, 1, I_{1.5}^6, I_0^6, I_4^6, I_2^6, I_3^6\};$
$1 \leq j \leq 7\}$

be the MOD natural neutrosophic initial state vectors of MOD domain and MOD range space respectively.

Let $x = (1\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$xM \to (0\ I_0^6\ 0\ 0\ 0\ 0\ 0)$;
$y_1M^t \to (I_0^6\ 0\ 0\ 0) = x_1$;
$xM \to (0\ I_0^6\ 0\ 0\ 0\ 0\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by $\{(I_0^6\ 0\ 0\ 0), (0\ I_0^6\ 0\ 0\ 0\ 0\ 0)\}$ both the nodes which has come to on state has become natural neutrosophic zero for the initial state vector $x = (1\ 0\ 0\ 0) \in X$.

Thus this property is unique and related only to this model.

Let x = (0 1 0 0) ∈ X, to find the effect of x on M.

$xM \to (1\ 0\ 0\ 0\ 0\ I^6_{1.5}\ \ 0) = y_1;$
$y_1M^t \to (0\ \ I^6_{1.5} + 1\ 0\ 0) = x_1;$
$x_1M \to (\ I^6_{1.5}\ + 1\ 0\ 0\ 0\ 0\ I^6_{2.25}\ \ 0) = y_2;$
$y_2M^t \to (0\ 1 +\ I^6_{1.9} +\ I^6_{3.975}\ \ 0 \ldots 0).$

This example is mainly provided to the reader to show that an expert or a researcher cannot select any form of natural neutrosophic numbers from ${}^I[0,6)$ he/she should judicially select a natural neutrosophic element so that product among themselves can we well define here in this case products are not defined so the matrix M is not a properly constructed MOD natural neutrosophic real interval relational matrix operator.

So we have illustrated by an example how the MOD natural neutrosophic values should be taken from ${}^I[0,6)$ in particular and ${}^I[0, n)$ in general.

We will give an example which works out well.

Example 4.34: Let

$$M = \begin{bmatrix} 0 & 3.2 & I^8_2 & 0 \\ 1.2 & 0 & 0 & 1 \\ 0 & 0 & 0.31 & 0 \\ 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & I^8_0 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic real interval relational matrix with entries from ${}^I[0, 8)$.

Let $X = \{(a_1\ a_2 \ldots a_6) / a_i \in \{0, 1, I\ I^8_2,\ I^8_4,\ I^8_0\}; 1 \leq i \leq 6\}$ and

$Y = \{(b_1\ b_2\ b_3\ b_4) / b_{ij} \in \{0, 1, I_0^8, I_4^8\ I_2^8\}; 1 \leq j \leq 4\}$ be the MOD natural neutrosophic domain and range space of initial state of vectors.

Let $x = (1\ 0\ 0\ 0\ 0\ 0) \in X$, to find the effect of x on M.

$xM \to (0\ 1\ I_2^8\ 0) = y_1$;
$y_1M^t \to (1 + I_4^8\ 0\ 0\ 0\ 0\ I_0^8) = x_1$;
$x_1M \to (0\ 1 + I_4^8 + I_0^8\ \ I_2^8 + I_0^8\ 0) = y_2$;
$y_2M^t \to (1 + I_0^8 + I_4^8 + I_2^8\ 0\ 0\ 0\ 0\ I_0^8) = x_2$;
$x_2M \to (0, 1 + I_0^8 + I_4^8 + I_2^8\ \ I_0^8 + I_4^8 + I_2^8\ 0) = y_3$;
$y_3M^t \to (1 + I_0^8 + I_4^8 + I_2^8\ \ 0\ 0\ 0\ 0\ I_0^8) = x_3\ (= x_2)$.

Thus the MOD natural neutrosophic models resultant is a MOD fixed point pair given by

$$\{(0\ 1 + I_0^8 + I_4^8 + I_2^8\ \ I_0^8 + I_4^8 + I_2^8\ 0), (1 + I_0^8 + I_4^8 + I_2^8\ 0\ 0\ 0\ 0\ I_0^8)\}$$

It is clearly observed that the nodes take sum of the MOD natural neutrosophic values.

The reader is expected to work with more such initial state vectors using M.

Next we proceed onto describe and develop MOD natural neutrosophic-neutrosophic interval Relational Maps model using $\langle[0, n) \cup I\rangle_I = \{[0, n), I_0^I, I_t^I$, t is a zero divisor or idempotent or nilpotent or pseudo zero divisor and so on $t \in [0, n)\}$.

To this first we need the notion of MOD natural neutrosophic-neutrosophic interval directed bipartite graph G, which we described by an example.

Example 4.35: Let G be a MOD interval directed bipartite graph with edge weights from $\langle [0, 12) \cup I \rangle_I$.

G will be known as the MOD natural neutrosophic-neutrosophic interval directed bipartite graph. G is given by the following figure.

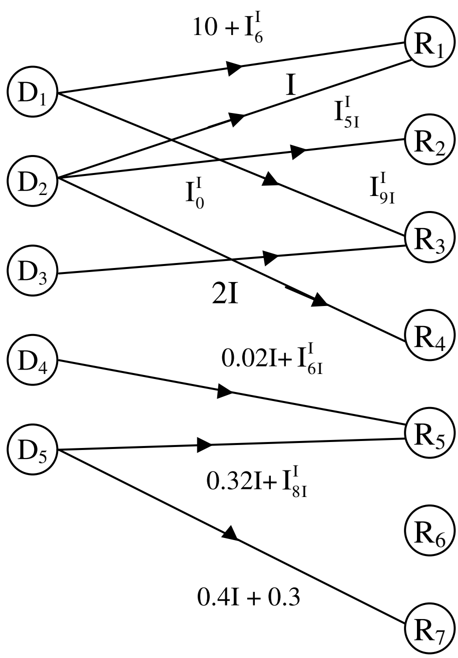


Figure 4.12

Interested reader can construct any number of such MOD natural neutrosophic-neutrosophic interval bipartite directed graphs using $\langle [0, n) \cup I \rangle_I$.

We give one example of the MOD natural neutrosophic-neutrosophic interval relational (rectangular) matrix operator [66].

Example 4.36: Let

$$M = \begin{bmatrix} 0 & 0.31 & I_{2I}^{I} & 0 & I_0^I + 0.2 & 0.72I \\ 0.21 & 0 & 0 & I_{4I}^{I} & 0 & 0 \\ 0.112I & I & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 + I_4^I & 1 & 2I & 1.32 \\ 1 & 0 & 0 & 0 & 0 & 0.14I \end{bmatrix}$$

be the MOD natural neutrosophic-neutrosophic interval rectangular or relational matrix with entries from $\langle [0, 5) \cup I \rangle_I$.

One can obtain all special features associated with these matrices as it is realized as a matter of routine.

We will describe by one example how the MOD natural neutrosophic-neutrosophic interval matrix operations.

Example 4.37: Let

$$M = \begin{bmatrix} I_{3I}^{I} & 0 & 0 & 0.2 \\ 0 & I & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0.1 & 0.4I & 0 & 0.7I \\ 0 & 0 & I_0^I & I_2^I \\ 0 & 0.2 & 0 & 4 \end{bmatrix}$$

be the MOD natural neutrosophic-neutrosophic interval relational matrix operator with entries from $\langle [0, 5) \cup I \rangle_I$.

Let X = $\{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6) / a_i \in \{0, 1, I_0^I\ I, I_{2I}^I\ I_I^I, I_{3I}^I\ I_{4I}^I\}$ $1 \leq i \leq 6\}$ and

Y = $\{(b_1\ b_2\ b_3\ b_4) / b_j \in \{0, 1\ I_0^I, I\ I_I^I\ I_{2I}^I\ I_{3I}^I\ I_{4I}^I\}, 1 \leq j \leq 4\}$ be the MOD domain range space of initial state vectors associated with M.

Let x = (1 0 0 0 0 0) ∈ X to find the effect of x on M.

$xM \to (I^I_{3I}\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (I^I_{4I}\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (I^I_{2I}\ 0\ 0\ 0) = y_2;$
$y_2M^t \to (I^I_I\ 0\ 0\ 0\ 0\ 0) = x_2;$
$x_2M \to (I^I_{3I}\ 0\ 0\ 0) = y_3\ (= y_1).$

Thus the MOD resultant is a MOD realized limit cycle pair given by $\{(I^I_{4I}\ 0\ 0\ 0\ 0\ 0), (I^I_{3I}\ 0\ 0\ 0)\}$ that is both the on nodes which are in the on state are MOD natural neutrosophic-neutrosophic elements.

Let x = (0 1 0 0 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ I\ 0\ 0) = y_1;$
$y_1M^t \to (0\ I\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (0\ I\ 0\ 0) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ I\ 0\ 0\ 0\ 0), (0\ I\ 0\ 0)\}.$

Clearly both the nodes are just indeterminate I.

Let x = (0 0 1 0 0 0) ∈ X, to find the effect of x on M,

$xM \to (0\ 0\ 1\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 1\ 0\ I^I_0\ 0) = x_1;$
$x_1M \to (0\ 0\ 1 + I^I_0\ \ I^I_0) = y_2;$
$y_2M^t \to (0\ 0\ 1 + I^I_0\ 0\ I^I_0\ I^I_0) = x_2;$
$x_2M \to (0\ 0\ I^I_0 + 1\ \ I^I_0) = y_3\ (= y_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 1 + I_0^I\ \ 0\ \ I_0^I\ \ I_0^I), (0\ 0\ 1 + I_0^I\ \ I_0^I)\}.$$

Let x = (0 0 0 1 0 0) ∈ X to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0) = y_1;$
$y_1M^t \to (0\ 0\ 0\ 1\ 0\ 0) = x_1\ (= x).$

The MOD resultant is a MOD special classical fixed point pair given by

$$\{(0\ 0\ 0\ 1\ 0\ 0), (0\ 0\ 0\ 0)\}.$$

Let x = (0 0 0 0 1 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ I_0^I\ \ I_2^I) = y_1;$
$y_1M^t \to (0\ 0\ I_0^I\ \ 0\ \ I_0^I + I_4^I\ \ I_2^I) = x_1;$
$x_1M \to (0\ 0\ I_0^I\ \ I_2^I + I_0^I + I_3^4) = y_2;$
$y_2M^t \to (0\ 0\ I_0^I\ \ 0\ \ I_0^I + I_1^I).$

Thus we see even though we have taken X and Y properly still the MOD resultant does not exist.

The main reason attributed to it is that even one has to properly take entries of the MOD natural neutrosophic-neutrosophic interval matrix.

Whenever the MOD natural neutrosophic-neutrosophic values are taken those elements should generate a finite semigroup under product otherwise we will face a problem as in case o the present example for $I_2^I \times I_2^I = I_4^I$ and $I_4^I \times I_4^I = I_1^I$ which is never a MOD natural neutrosophic neutrosophic number so the entry I_2^I is not correct entry of M according to our analysis.

Thus we make one more stipulation in this direction the MOD natural neutrosophic neutrosophic directed bigraph should have edge weights if are from natural neutrosophic neutrosophic numbers then those numbers must be closed with respect to product.

Thus we can briefly describe the MOD natural neutrosophic neutrosophic relational maps model.

Let $\langle [0, n) \cup I \rangle_I$ be the MOD natural neutrosophic neutrosophic interval collection.

Let G be a MOD directed graph with entries from $\langle [0, n) \cup I \rangle_I$.

For if I_t^I are edge weights of G then for the resultant to exist in the MOD natural neutrosophic neutrosophic interval Relational Maps model, we must have the collection of I_t^I,s are such that they generate a finite semigroup under $\times$.

Consequent of this we will get the entries of the MOD natural neutrosophic neutrosophic relational matrix M to have the I_t^I's to be such that they generate a semigroup under $\times$; and M forms the MOD dynamical system of the model.

Unless this is achieved it is impossible to work with the model as finding resultants would be a NP hard problem.

The construction of this new model and related studies is a matter or routine hence left as an exercise to the reader.

Next we proceed onto develop the MOD natural neutrosophic dual number interval Relational Maps model with entries from $\langle [0, n) \cup g \rangle_I = \{[0, n), I_t^g$; t is a zero divisor or an idempotent or a pseudo zero divisor and so on in $[0, n)\}$

We will first describe the notion of MOD natural neutrosophic dual number interval directed bipartite graph G with edge weights from $\langle [0, n) \cup g \rangle_I$. $g^2 = 0$; $2 \leq n < \infty$ by an example.

Example 4.38: Let G be a MOD natural neutrosophic dual number interval directed bigraph with edge weights from $\langle [0, 7) \cup g \rangle_I$ given by the following figure.

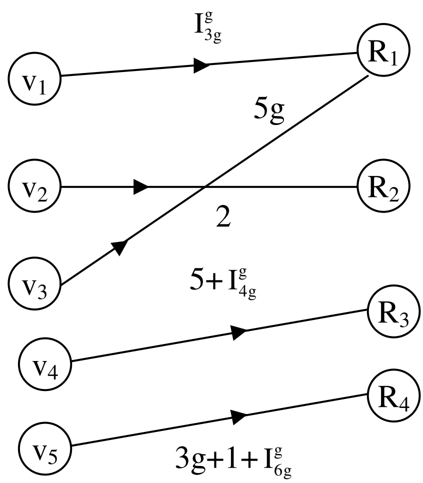


Figure 4.13

Interested reader can get more number of such graph.

Thus if G a MOD natural neutrosophic interval dual number directed bipartite graph then the edge weights are essential from $\langle [0, n) \cup g \rangle_I$; $g^2 = 0$, $2 \leq n < \infty$.

Next we give one example of a MOD natural neutrosophic interval dual number relational or rectangular matrix.

Example 4.39: Let

$$M = \begin{bmatrix} 0 & 1+I_{4g}^{g} & I_{2}^{g} & 0 & 0 & 0 & 5 \\ 0.72g & 0 & 0 & 0.15g & 1 & 0 & 0.331 \\ 0 & 0 & 1+I_{5g}^{g} & 0 & 0 & 2 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic interval dual number relational or rectangular matrix with entries from $\langle[0, 6) \cup I\rangle_I$

Thus $M = (m_{ij})_{t\times s}$ is a $t \times s$ MOD natural neutrosophic interval dual number relational matrix if $m_{ij} \in \{\langle[0, n) \cup g\rangle_I$, $g^2 = 0$; $1 \leq i \leq t$ and $1 \leq j \leq s\}$.

We as in case of MOD natural neutrosophic neutrosophic interval numbers demand the set of I_t^g in M must generate a finite semigroup under product.

Then only the MOD natural neutrosophic interval dual number Relational Maps model will exist.

$X = \{(x_1\ x_2 \ldots x_t) / x_i \in \{0, 1, g\ I_t^g / I_t^g$ for all t such that t running only over a finite index must generate a finite semigroup under x}; $1 \leq i \leq t\}$ and

$Y = \{(y_1\ y_2 \ldots y_s) / y_i \in \{0, 1, g, I_t^g$; t as said in $\times\}$ $1 \leq i \leq s\}$ are the MOD domain and MOD range spaces of initial vectors, respectively associated with M.

We give an example to show how this new model or the MOD natural neutrosophic dual number relational matrix operator functions.

Example 4.40: Let

$$P = \begin{bmatrix} 0 & 3 & I_2^g & 0 & 1 & 0 & 0 & 0.2g & 0 \\ I_0^g & 0 & 0 & 0.3g & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 2+0.3g & 0 & 0 & 0 & 0 & 3g \\ 0 & 0.22g & 0.01 & 0 & 0.2g & 3 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number interval relational matrix operator with entries from $\langle [0, 4) \cup I \rangle_I$; $g^2 = 0$.

Let X = $\{(a_1\ a_2\ a_3\ a_4) / a_i \in \{0, 1, I_2^g\ I_2^g\ I_g^g\ I_{3g}^g\ I_{2+2g}^g\ I_0^g\};$ $1 \leq i \leq 4\}$ and

Y = $\{(b_1\ b_2\ b_3 \ldots b_9) / b_j \in \{1, 0, g, I_2^g, I_{2g}^g\ I_0^g\ I_g^g\ I_{3g}^g\ I_{2+2g}^g\};$ $1 \leq j \leq 9\}$ be the MOD domain and MOD range of special initial state vectors respectively associated with P.

Let x = (1 0 0 0) ∈ X, to find the effect of x on P.

$$xP \to (0\ 1\ I_2^g\ 0\ 1\ 0\ 0\ 0\ 0) = y_1;$$
$$y_1P^t \to (1 + I_0^g\ 0\ 0\ 0) = x_1;$$
$$x_1P \to (0\ I_0^g + 1\ I_2^g\ 0\ 1 + I_2^g\ 0\ 0\ 0\ 0) = y_2;$$
$$y_2P^t \to (1 + I_0^g + I_2^g\ 0\ 0\ 0) = x_2;$$
$$x_2P \to (0\ I_0^g + I_2^g+1\ \ I_2^g\ 0\ 1 + I_0^g + I_2^g\ 0\ 0\ 0\ 0) = y_3;$$
$$y_3P^t \to (1 + I_0^g + I_2^g, 0\ 0\ 0\ 0) = x_3\ (= x_2)\ .$$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(1 + I_0^g + I_2^g\ 0\ 0\ 0), (0\ 1 + I_0^g\ \ I_2^g\ 0\ 1 + I_2^g\ \ 0\ 0\ 0\ 0)\}.$$

Thus the on nodes are natural neutrosophic elements combination with real value.

Let y = (0 0 0 1 0 0 0 0 0) ∈ Y, to find the effect of y on P.

$yP^t \to (0\ 0\ 1\ 0) = x_1;$
$x_1P \to (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ g) = y_1;$
$y_1P^t \to (0\ 0\ 1\ 0) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by {(0 0 1 0), (0 0 0 1 0 0 0 0 g)}.

The on state nodes are 1 or g only.

Interested reader can work with any number of such initial vectors as it is considered as a matter of routine.

Now at this juncture we wish to mention the MOD natural neutrosophic interval special quasi dual number bipartite directed $\langle[0, n) \cup k\rangle_I$; $k^2 = (n - 1) k$ graph and MOD natural neutrosophic interval special dual like number directed bipartite graph using edge weights $\langle[0, n) \cup h\rangle_I$, $h^2 = h$ and $2 \leq n < \infty$ can be defined and developed analogous to MOD natural neutrosophic interval dual number bipartite directed graph using edge weights from $\langle[0, n) \cup g\rangle_I$; $g^2 = 0$.

This work is considered as a matter of routine and left as an exercise to the reader.

Likewise we can define the MOD natural neutrosophic special dual like number interval rectangular or Relational Maps model matrix with entries from

$\langle[0, n) \cup h\rangle_I$; $h^2 = h$, $2 \leq n < \infty$ is considered as a matter of routine so is left as an exercise to the reader.

Further the MOD natural neutrosophic special quasi dual number interval rectangular or matrix and its related Relational Maps models with entries from

$\langle[0, n) \cup k\rangle_I$, $k^2 = (n - 1)k$, $2 \leq n < \infty$ is also left as an exercise to the reader.

However all these three models function in a similar way with making appropriate changes while product operation is done on them.

We will illustrate by some examples.

Example 4.41: Let M be the MOD special dual like number Relational Maps models dynamical system with entries from $\langle[0, 9) \cup h\rangle_I$; $h^2 = h$. M is as follows.

$$M = \begin{bmatrix} 0 & 8+0.32h & 0 & 0 & 0.14h+0.5 \\ 0.2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3h & 0 \\ 0 & 0 & 0 & 0 & I_3^h \\ I_{6h}^h & 0.32 & 0 & 0 & 0 \\ 0.3h & 0 & 0.2h & 1 & 0 \end{bmatrix}$$

$X = \{(x_1\ x_2\ \dots\ x_6) / x_i \in \{0, 1, h, I_3^h, I_{6h}^h, I_0^h, I_{3h}^h\},$
$1 \leq i \leq 6\}$ and

$Y = \{(y_1\ y_2\ \dots\ y_5) / y_j \in \{0, 1, h, I_3^h, I_{6h}^h, I_0^h, I_{3h}^h; 1 \leq j \leq 5\}$ be the MOD natural neutrosophic special dual like number Relational Maps models, dynamical systems associated MOD domain and MOD range spaces respectively.

Let x = (1 0 0 0 0 0) $\in$ X, to find the effect of x on M.

$xM \to (0\ 1\ 0\ 0\ 0) = y_1$;
$y_1M^t \to (1\ 0\ 0\ 0\ 0\ 0) = x_1$ (= x).

Thus the MOD resultant is a MOD special classical fixed point pair given by

{(1 0 0 0 0 0), (0 1 0 0 0)}.

Let x = (0 1 0 0 0 0) $\in$ X to find the effect of x on M.

$xM \to (1\ 0\ 0\ 0\ 0) = y_1$;

$y_1M^t \to (0\ 1\ 0\ 0\ I_{6h}^h\ 0) = x_1;$
$x_1M \to (I_0^h\ 0\ 1\ 0\ 0) = y_2;$
$y_2M^t \to (0\ 1\ 0\ 0\ I_0^h\ 0) = x_2;$
$x_2M \to (I_0^h\ 0\ 1\ 0\ 0) = y_3\ (= y_2).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(I_0^h\ 0\ 1\ 0\ 0), (0\ 1\ 0\ 0\ I_0^h\ 0)\}.$$

Let x = (0 0 0 1 0 0) ∈ X, to find the effect of x on M.

$xM \to (0\ 0\ 0\ 0\ I_3^h) = y_1;$
$y_1M^t \to (0\ 0\ 0\ I_3^h\ 0\ 0) = x_1;$
$x_1M \to (0\ 0\ 0\ 0\ I_3^h) = y_2\ (= y_1).$

Thus the MOD resultant is a MOD fixed point pair given by

$$\{(0\ 0\ 0\ I_3^h\ 0\ 0), (0\ 0\ 0\ 0\ I_3^h)\}.$$

Let y = (0 1 0 0 0) ∈ Y, to find the effect of y on M.

$yM^t \to (1\ 0\ 0\ 0\ 0\ 0) = x_1;$
$x_1M \to (0\ 1\ 0\ 0\ 0) = y_1\ (= y).$

Thus the MOD resultant is the MOD classical special fixed point pair given by

$$\{(1\ 0\ 0\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 0)\}.$$

The reader is expected to work with other initial state vectors.

Example 4.42: Let

$$S = \begin{bmatrix} 0 & 2k+0.33 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.3k & 0 & 0.2 & 0 \\ 0 & 0.335 & 0 & I_{2k}^k & 0 & 0 \\ 0 & 0 & 0 & 0 & 2k & 3 \end{bmatrix}$$

be the MOD special quasi dual number relational matrix operator with entries from $\langle[0, 4) \cup k\rangle_I$; $k^2 = 3k$.

Let $X = \{(x_1\ x_2\ x_3\ x_4) / x_i \in \{0, 1\ k\ I_{2k}^k\ \ I_0^I\ I_{2k}^k\}; 1 \leq i \leq 4\}$ and

$Y = \{(y_1\ y_2\ \ldots\ y_6) / y_j \in \{0, 1, k, I_{2k}^k, I_0^I\ I_2^k\}; 1 \leq j \leq 6\}$

be the MOD domain and MOD range space of special initial state vectors associated with S.

Let x = (1 0 0 0) ∈ X, to find the effect of x on S.

$xS \to (0\ k\ 0\ 0\ 0\ 0) = y_1$;
$y_1S^t \to (k\ 0\ 0\ 0) = x_2$;
$x_2S \to (0\ k\ 0\ 0\ 0\ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point pair given by
{(k 0 0 0), (0 k 0 0 0 0)}.

Let y = (0 0 1 0 0 0) ∈ Y, to find the effect of y on S.

$yS^t \to (0\ 0\ 0\ 0) = x_1$;
$x_1S \to (0\ 0\ 1\ 0\ 0\ 0) = y_1\ (= y)$.

Thus the MOD resultant is a MOD special classical fixed point pair given by
{(0 0 1 0 0 0), (0 0 0 0)}

Let x = (0 0 k 0) ∈ X, to find the effect of x on S.

$xS \to (0\ 0\ 0\ I^{*}_{2k}\ 0\ 0) = y_1;$
$y_1S^t \to (0\ 0\ I^{*}_{2k}\ 0) = x_1;$
$x_1S \to (0\ 0\ 0\ I^{k}_{0}\ 0\ 0) = y_2;$
$y_2S^t \to (0\ 0\ I^{k}_{0}\ 0) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ 0\ I^{k}_{0}\ 0), (0\ 0\ 0\ I^{k}_{0}\ 0\ 0)\}.$

Let y = (0 0 0 0 0 1) $\in$ Y to find the effect of y on S.

$yS^t \to (0\ 0\ 0\ 1) = x_1;$
$x_1S \to (0\ 0\ 0\ 0\ k\ 1) = y_1;$
$y_1S^t \to (0\ 0\ 0\ 1) = x_2\ (= x_1).$

Thus the MOD resultant is a MOD fixed point pair given by
$\{(0\ 0\ 0\ 1), (0\ 0\ 0\ 0\ k\ 1)\}.$

Let y = (0 0 0 1 0 k) $\in$ Y, to find the effect of y on S.0

$yS^t \to (0\ 0\ I^{k}_{2k}\ k) = x_1;$
$x_1S \to (0\ 0\ 0\ I^{k}_{0}\ k\ k) = y_1;$
$y_1S^t \to (0\ 0\ I^{k}_{0}\ k) = x_2;$
$x_2S \to (0\ 0\ 0\ I^{k}_{0}\ k\ k) = y_2\ (=y_1).$

Thus the MOD resultant is a MOD fixed point pair given by $\{(0\ 0\ I^{k}_{0}\ k), (0\ 0\ 0\ I^{k}_{0}\ k\ k)\}$.

Thus the on state of the nodes are special quasi dual number k or the MOD natural neutrosophic zero.

Thus one can work with any initial state vector from X or Y and find the MOD resultants.

The MOD natural neutrosophic number Relational Maps model with entries from

$^{I}[0, n)$ or $C^{I}([0, n))$ or $\langle [0, n) \cup I\rangle_I$ or $\langle [0, n) \cup g\rangle_I$, $g^2 = 0$ or $\langle [0, n) \cup h\rangle_I$ $h^2 = h$ or $\langle [0, n) \cup k\rangle_I$, $k^2 = (n - 1)k$

can be functional that they can have MOD resultants after a finite number of iterations if and only if the MOD natural neutrosophic numbers in the MOD dynamical system associated with the model has the collection of MOD natural neutrosophic elements is such that it generates a finite semigroup under X; for otherwise the problem of finding the MOD resultant would be a NP hard problem.

This can be equivalently stated as follows.

Let G be the MOD natural neutrosophic number interval directed bipartite graph with edge weights from

$^{I}[0, n)$ or C^{I} $([0, n))$ or $\langle [0, n) \cup I\rangle_I$ or $\langle [0, 5) \cup g\rangle_I$, $g^2 = 0$ or $\langle [0, 5) \cup h\rangle_I$, $h^2 = h$ or $\langle [0, n) \cup k\rangle_I$; $k^2 = (n - 1)$ k; $2 \leq n < \infty$

then the collection of all edge weights of the I_t^n or I_t^g or I_t^h or I_t^I or I_t^c or I_t^k for $0 < t < \infty$ should generate only a finite semigroup under product else the MOD resultant cannot be got in general after a finite number of iterations.

Secondly it is mandatory that the MOD operation using the dynamical system is done by updating and thresholding at each stage of the operation else we will not be in a position to arrive at a MOD resultant after a finite number of iterations it would once again become a NP hard problem.

Finally by this method we are able to get as nodes in the MOD resultant which are MOD natural neutrosophic numbers or I or i_F or g or h or k or 1.

Interested reader can obtain all other special features associated with them.

This task is left as an exercise to the reader.

Chapter Five

SUGGESTED PROBLEMS

In this chapter we suggest some problems some of which can also be realized as open conjectures.

1. Characterize the special features associated with MOD directed bipartite graphs with entries from Z_n.

2. Bring out the difference between MOD directed bipartite graphs and the usual bipartite graphs.

3. Show by illustrations that MOD bipartite directed graphs has applications in MOD Relational Maps models.

4. Enumerate all the special features enjoyed by the MOD rectangular matrices.

5. Let G be the MOD directed bipartite graph G given by the following figure with edge weights from Z_{10}

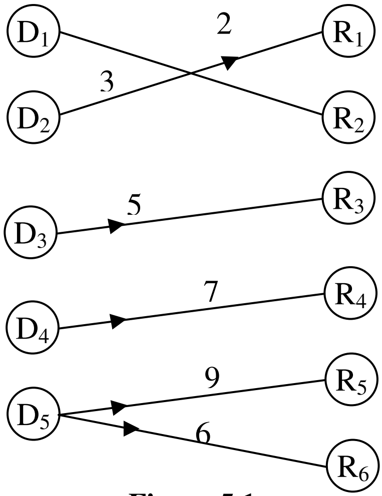
G = 

Figure 5.1

i) Find the MOD relational matrix M associated with G.

ii) If x = (1 0 0 0 0) $\in$ X = $\{(a_1\ a_2\ a_3\ a_4\ a_{5)} / a_i \in \{0, 1\}, 1 \leq i \leq 5\}$ be the initial state vectors associated with M.

(a) Find the MOD resultant using usual operation.
(b) Find the MOD resultant when updating is carried out at each stage.

iii) Find all x in X for which MOD resultant is the same for both the types of operations.

iv) Let Y = $\{(b_1\ b_2\ \ldots.\ b_6)/b_i \in \{0, 1\};\ 1 \leq i \leq 6\}$ be the MOD range initial state vector associated with M.

a) If y = (0 0 0 0 1 0) $\in$ Y find the MOD results under usual operations and MOD resultant using updating operations.

v) Find all y $\in$ Y which has different MOD resultants under usual operations and updating operations.

6. Enumerate all the special features associated with MOD Relational Maps models.

7. Let

$$M = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

be the MOD dynamical system of the MOD Relational Maps model with entries from Z_6.

i) Find all the MOD classical special fixed point pairs of M.
ii) Find all MOD limit cycle pairs of M which are obtained after 5 iterations.
iii) What is maximum number of iterations needed for any MOD domain state vector or a MOD range state vector to reach a MOD fixed point pair?
iv) What is the maximum number of iterations needed for any MOD domain state vector or MOD range state vector to arrive at a MOD limit cycle pair?
v) In case of questions (iii) and (iv) obtain the result if maximum is replaced by minimum.
vi) How many MOD resultants are arrived exactly after 6 iterations?
vii) Obtain any other special feature enjoyed by this M.

8. Let $S = \begin{bmatrix} 3 & 0 & 10 & 0 & 2 & 0 & 11 \\ 0 & 14 & 0 & 0 & 0 & 5 & 0 \\ 1 & 0 & 2 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15 & 2 & 1 & 13 \end{bmatrix}$

   be the MOD relational maps model associated MOD dynamical system built using Z_{17}.

   Study questions (i) to (vii) of problem (7) for this S.

9. Let $P = \begin{bmatrix} 2+I & 0 & 2 & 1 \\ 0 & 6I & 0 & 0 \\ 2I & 0 & I & 0 \\ 1+I & 0 & 0 & 2+5I \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 2I & 6 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

   be the MOD neutrosophic relational matrix with entries from $\langle Z_7 \cup I \rangle$.

   P is the dynamical system of a MOD neutrosophic relational maps model.

   Study questions (i) to (vii) of problem (7) for this P.

10. Obtain all special features associated with MOD neutrosophic relational maps model built using $\langle Z_{n7} \cup I \rangle$. Compare this model with MOD relational maps model built using Z_n.

11. Let

$$W = \begin{bmatrix} 3+i_F & 0 & 2i_F & 1 & 0 & 0 & 2 \\ 0 & 7+5i_F & 0 & 0 & 2+i_F & 1 & 0 \\ 0 & 0 & 2+4i_F & 9 & 0 & 0 & 3+5i_F \\ 2 & 0 & 0 & 0 & 5 & 6i_F & 0 \\ 7 & 9+i_F & 0 & 0 & 1 & 0 & 9 \end{bmatrix}$$

be the MOD finite complex number relational matrix of the MOD finite complex number Relational Maps model dynamical system with entries from $C(Z_{10})$.

i) Study questions (i) to (vii) of problem 7 for this W.
ii) Enumerate all special features enjoyed by this W.
iii) Compare this model with any MOD Relational Maps model built using Z_{10}.
iv) Compare this model with any MOD Relational Maps model built using $\langle Z_{10} \cup I \rangle$.

12. Let $V = \begin{bmatrix} 0 & 2g & 0 & 5+4g & 1 \\ 2 & 0 & 1+4g & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 4+g & 1 & 0 & 3g+9 \\ 10 & 0 & 11+g & 2g & 0 \\ 11g & 1 & 0 & 4+9g & 1 \\ 0 & 0 & 0 & 0 & 2+11g \end{bmatrix}$

be the MOD dual number Relational Maps model with entries from $\langle Z_{12} \cup I \rangle = \{a + bg / a, b \in Z_{12}; g^2 = 0\}$.

i) Study questions (i) to (vii) of problem 7 for this V.
ii) Enumerate all the special features enjoy by this model.

iii) Compare this model with other models with entries from $C(Z_{12})$, $\langle Z_{12}\cup I\rangle$ and Z_{12}.

13. Let B = $$\begin{bmatrix} 0 & 2h & 0 & 5 & 0 & 4h+2 \\ 4 & 0 & 2 & 0 & h & 0 \\ 0 & 4h & 0 & 1+4h & 0 & 0 \\ 0 & 0 & 4+5h & 0 & 1 & 2 \\ 1 & 0 & 0 & 2 & 0 & 4h \\ 0 & 1+h & 0 & h & 0 & 0 \\ 4+3h & 0 & 3 & 0 & 5 & 0 \\ 1+5h & 0 & 0 & 0 & 5+5h & 1 \\ 0 & 2 & 0 & h & 0 & 0 \end{bmatrix}$$

be the MOD special dual like number relational matrix of the MOD special dual like number Relational Maps model with entries from $\langle Z_6 \cup h\rangle = \{a + bh \,/\, a, b \in Z_6, h^2 = h\}$.

i) Study questions (i) to (vii) of problem 7 for this B.
ii) Derive all the special features associated with B.
iii) Compare B with MOD relational models built using Z_6 $\langle Z_6\cup g\rangle$, $\langle Z_6\cup I\rangle$ and $C(Z_6)$.

14. Let M = $$\begin{bmatrix} 0 & 2+k & 0 & 1 & 2k \\ k & 0 & 1+k & 0 & 2 \\ 1+k & k & 0 & 2k & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

be the MOD special quasi dual number Relational Maps model matrix with entries from $\langle Z_3 \cup I\rangle$.

Study questions (i) to (vii) of problem 7 for this M.

15. Enumerate all special features enjoyed by MOD special quasi dual number Relational Maps model built using $\langle Z_n \cup k\rangle$; $k^2 = (n - 1)k$.

16. Let G be a MOD natural neutrosophic modulo integer bipartite directed graph with edge weights from $\langle Z_9 \cup I\rangle_O$ given by the following figure.

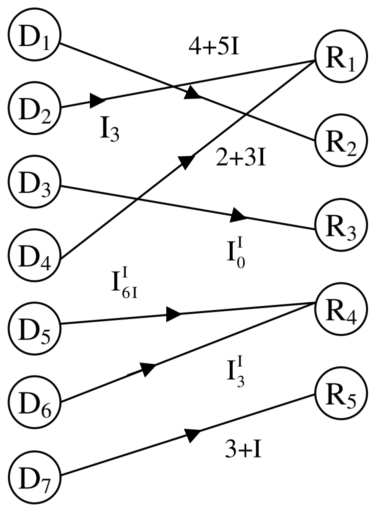


Figure 5.2

i) Find the MOD relational matrix M associated with the graph G.

ii) Using M find all MOD resultants which are MOD classical special fixed point pairs.

iii) Study questions (i) to (vii) of problem (7) for this M.

17. Obtain all special features associated with the MOD natural neutrosophic integer bipartite directed graph G with edge weights.

18. Let S = $\begin{bmatrix} 3+I_2^8 & 0 & 0 & 1 \\ 0 & I_4^8 & 2 & 0 \\ 0 & 0 & 1+I_6^8 & 0 \\ I_0^8 & 6 & 0 & 3 \\ 0 & I_6^8 & 0 & I_0^8 \end{bmatrix}$

be the MOD natural neutrosophic relational matrix with entries from Z_8^I.

i) Study questions (i) to (vii) of problem (7) for this S.
ii) Obtain any other special feature enjoyed by S.

19. Let V = $\begin{bmatrix} 0 & 2+4I & I_8^I & I_{10I}^I \\ 10+I_{6I}^I & 0 & 0 & 5+6I \\ 0 & 4+I_{2I}^I & 1 & 7 \\ 5 & 0 & 2 & I_0^I \\ 0 & I_0^I+I_2^I & 0 & 0 \end{bmatrix}$

be the MOD natural neutrosophic neutrosophic relational matrix with entries from $\langle Z_{12} \cup I\rangle_I$.

i) Study questions (i) to (vii) of problem 7 for this V.
ii) Obtain all special features enjoy by this V.
iii) Hence or otherwise study MOD natural neutrosophic neutrosophic relational matrix $M = (m_{ij})_{t\times s}$ with $m_{ij} \in \langle Z_n \cup I\rangle_I$.

20. Let M = $(m_{ij})_{t\times s}$; $m_{ij} \in Z_n^I$ be the MOD natural neutrosophic relational matrix operator.

    Study questions (i) to (vii) of problem 7 for this M.

21. Let W = $\begin{bmatrix} 2+I_{2g}^g & 0 & 1 & I_g^g \\ 0 & I_0^g & 0 & 0 \\ 4 & 2 & 5 & I_{5g}^g \\ 3g & 0 & g & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

    be the MOD natural neutrosophic dual number relational maps model.

    Study questions (i) to (vii) of problem 7 for this W.

22. Let M = $(m_{ij})_{s\times t}$ MOD natural neutrosophic dual number matrix model with entries from neutrosophic dual number matrix model with entries $\langle Z_n \cup g\rangle_I$; $2 \leq n < \infty$.

    i) Find all special features enjoyed by this MOD operator.
    ii) Compare this model with other models.

23. Let T = $\begin{bmatrix} 0 & I_{3h}^h & 0 & 1+2h & 0 \\ 1 & 0 & 2h & 0 & I_0^h \\ 0 & 3+5h & 0 & 6+6h & 0 \\ I_{6h}^h & 0 & I_0^h+1 & 0 & 3h \\ 0 & 1 & 0 & 2 & 0 \\ 8+7h & 0 & I_{6h+3}^h & 0 & 7 \end{bmatrix}$

    be the MOD natural neutrosophic special dual like number relational matrix operator.

Study questions (i) to (vii) of problem 7 for this T.

24. Let M = $(m_{ij})_{s\times t}$ be the MOD natural neutrosophic special dual like number matrix operator with
$m_{ij} \in \langle Z_n \cup h\rangle_I$; $1 \leq i \leq s$ and $1 \leq j \leq t$, $h^2 = h$.

    i) Find all special features enjoyed by M.
    ii) How for this model is different from other models built using $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup I\rangle_I$ and $C^I(Z_n)$?

25. Let M be a MOD natural neutrosophic special quasi dual like number rectangular matrix M operator with entries from $\langle Z_n \cup k\rangle_I$.

    Study all special features enjoyed by this new model.

26. Let M = $$\begin{bmatrix} 0 & 2+I_2^k & 0 & 5 & I_0^k \\ I_4^k & 0 & 2 & 0 & 2k \\ 0 & k & 0 & I_{4k}^k & 0 \\ 1 & 0 & 4k+2 & 0 & 7 \\ I_{6k}^k & 1 & 0 & 2 & 0 \\ 0 & 0 & I_{6k+4}^k & 0 & 1 \end{bmatrix}$$

    be the MOD natural neutrosophic special quasi dual number matrix operator.

    Study questions (i) to (vii) of problem 7 for this M.

27. Compare all the six models on fixed problem P.

    i) Which MOD integers is best suited for the problem Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup I\rangle_I$, or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$?

ii) Does the MOD model relates to the problem P?
iii) Compare MOD cognitive maps model for P with FCMs and NCMs for the same P by bringing out the advantages and disadvantages.

28. Let G be any directed graph with edge weights from [0,5).

Enumerate all the special properties enjoyed by G.

29. Let M be any square matrix with entries from [0, 12).

Study all the special features enjoyed by M.

30. Using $M = \begin{bmatrix} 0 & 0.312 & 3.2 & 0 & 1 \\ 0 & 0 & 0 & 2.5 & 0 \\ 4.5 & 0.0801 & 0 & 0 & 0 \\ 0.001 & 0.005 & 0.02 & 0 & 0.01 \\ 0.005 & 0 & 0 & 0.015 & 0 \end{bmatrix}$

be the MOD interval matrix which is the dynamical system of a MOD interval cognitive map model with entries from [0, 6).

i) Find all MOD special classical fixed points of M.

ii) What is the maximum number of iterations needed to arrive at a MOD resultant?

iii) Give the minimum number iterations that is used on M to get the MOD resultant.

iv) How many MOD resultants given by M are MOD fixed points?

v) How many MOD resultants given by M are MOD limit cycle?

vi) Show if x and y are MOD initial state vectors them in general the sum of the MOD resultants of x and y on M is not the same as the MOD resultant of x + y on M.

vii) Obtain any other special feature enjoyed by this MOD interval cognitive maps model.

viii) Show this model can give face values of nodes in the resultant provided {0, 1} is replaced by Z_n in the MOD initial state vectors.

31. Let $P = \begin{bmatrix} 0 & 0.32 & 1.5 & 0 \\ 2.3 & 0 & 0 & 0.002 \\ 0 & 2 & 0 & 2.4 \\ 0.001 & 0 & 0.001 & 0 \end{bmatrix}$

be the MOD interval cognitive maps model dynamical system using [0, 3).

Let X = $\{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, 2\}; 1 \leq i \leq 4\}$ be the MOD initial state vector.

Study questions (i) to (viii) of problem 30 for this P.

32. Let $M = \begin{bmatrix} 0 & 2I+0.001 & 0 \\ 0.001 & 0 & 4+2I \\ 3 & 0.001 & 0 \end{bmatrix}$

be MOD interval neutrosophic cognitive maps model with entries from $\langle [0,5) \cup I \rangle$.

i) Study questions (i) to (viii) of problem 30 for this M.

ii) If $X_S = \{(x_1\ x_2\ x_3) / x_i \in \langle Z_5 \cup I\rangle;\ 1 \leq i \leq 3\}$ as the MOD initial state vector.

Study questions (i) to (viii) of problem 30 for M.

33. Give a real world model for which MOD interval neutrosophic cognitive maps model is appropriate.

34. Let W = $\begin{bmatrix} 0 & 2g+0.001 & 0 & 0.21 \\ 0 & 0 & 3+0.12g & 6g \\ 0.002 & 0.001 & 0 & 0.02g \\ 9.02 & 0 & 0 & 0 \end{bmatrix}$

be the MOD interval dual number cognitive maps model.

Study questions (i) to (viii) of problem 30 for this W.

35. Give a real world problem model for which MOD interval dual n umber cognitive maps model is appropriate.

Compare it with MOD interval cognitive maps model and MOD interval neutrosophic cognitive maps model.

36. Let V = $\begin{bmatrix} 0 & 2i_F & 0.002 & 0.05i_F & 0 \\ 4+i_F & 0 & 0 & 0 & 0.023 \\ 0.111 & 0 & 0 & 6.2i_F & 0 \\ 0 & 0.11+0.2i_F & 0.3 & 0 & 3.2 \\ 0 & 0 & 2.38 & 0 & 0 \end{bmatrix}$

be the MOD interval complex cognitive maps model dynamical system with entries from C([0, 7)).

Study questions (i) to (viii) of problem 30 for this V.

37. Compare for the same problem P the four models MOD interval cognitive maps model, MOD interval neutrosophic cognitive maps model, MOD interval complex number cognitive maps model and MOD interval dual number cognitive maps model for same set of modes $C_1, C_2, \ldots, C_t$ using $[0,n)$, $\langle [0,n) \cup I \rangle$, $C([0, n))$ and $\langle [0, n) \cup g \rangle$ respectively.

38. Let S = $$\begin{bmatrix} 0 & 0.2+5k & 0.1113 & 0 \\ 0.13k & 0 & 6.2+0.3k & 0.011 \\ 5.3+0.2k & 0.12 & 0 & 6k \\ 0.302 & 0 & 0.225k & 0 \end{bmatrix}$$

be the MOD interval special quasi dual number Cognitive Maps model with entries from $\langle [0,9) \cup k \rangle$.

Study questions (i) to (viii) of problem 30 for this S.

39. Show by an appropriate example that use of MOD interval special quasi dual number cognitive maps model is more suited to some problems than the other MOD interval Cognitive Maps models.

40. Let B = $$\begin{bmatrix} 0 & 0.231 & 0 & 0.22h & 0 \\ 6.2h & 0 & 0.331 & 0 & 0.102 \\ 0 & 9.32 & 0 & 0.112 & 0 \\ 0.335 & 0 & 4.72 & 0 & 6.3h \\ 0 & 0.112 & 0.32+0.5h & 8h & 0 \end{bmatrix}$$

be the MOD interval special dual like number cognitive maps model.

Study questions (i) to (viii) of problem 30 for this B.

41. Show by an illustrative example MOD interval special dual MOD interval special dual like number cognitive maps model is more suited than other model.

42. For the same problem for the same set of nodes compare the six MOD interval cognitive maps models.

43. Let $M = \begin{bmatrix} 0 & I_0^6 & I_3^6 & 0 & 0 \\ 0.32 & 0 & 2+I_2^6 & 0 & 0.335 \\ 0.5+I_3^6 & 0 & 0 & 0.2+I_4^6 & 0 \\ 0 & 0.335 & 0 & 0 & 2+I_4^6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic cognitive maps model with entries from ${}^{I}[0,6)$.

i) Prove if T = { I_t^6 / t $\in$ [0,6)} $\subseteq$ ${}^{I}[0,6)$ entries in M does not generate a semigroup under X then M will not have a MOD resultant.

ii) Study questions (i) to (viii) of problem 30 for this M.

iii) List out all the special features enjoyed by this MOD interval natural neutrosophic cognitive maps model.

iv) If $I_{1.5}^6$ is taken as an entry in M instead of I_3^6 can we say the collection T will generate a semigroup under x.

44. Let $V = \begin{bmatrix} 0 & I_{2.5}^{5} & 0 & 0 \\ 1.27 & 0 & I_{1.25}^{5} & 0.31 \\ 0 & 0.352 & 0 & I_{0.625}^{5} \\ 0.315 & 0 & 1.1115 & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic cognitive maps model.

i) Will V have MOD resultants if $x = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, 2, 3, 4, I_{2.5}^{5}, I_{1.25}^{5}, I_{0.625}^{5}\}; 1 \leq i \leq 4\}$?

ii) Does $T = \langle I_{2.5}^{5}, I_{1.25}^{5}, I_{0.625}^{5} \rangle$ generate a finite semigroup under x?

iii) Can we say finding MOD resultant is a NP hard problem?

45. Let $P = \begin{bmatrix} 0 & I_{2g}^{g} + g & 2 \\ 2g & 0 & I_{g}^{g} \\ 0 & 0.331 & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic dual number cognitive maps model.

i) Study questions (i) to (viii) of problem 30 for this P.
ii) Prove $\langle I_{2g}^{5}, I_{g}^{g} \rangle$ generates a finite order MOD natural neutrosophic number semigroup under +.
iii) If $I_{2g}^{g} + g$ is replaced by $I_{1.5}^{g}$ in P can this have MOD resultants for all initial state vectors.

46. Let W be the MOD dynamical system of the MOD interval natural neutrosophic complex number Cognitive Maps model with related matrix entries in $C^{I}([0,12))$.

$$W = \begin{bmatrix} 0 & 8+0.3i_F & 0 & I_4^0 & 0.25 \\ 0.7i_F & 0 & 0.1+5i_F & 0 & I_{2+6i_F}^C \\ 0 & 0.332i_F & 0 & 0.25i_F & 0.9+3i_F \\ 0.2+0.3i_F & 0 & 0.7+0.2i_F & 0 & 0 \\ I_{4i_F}^C + 0.12 & 0.113 & 0 & 4+0.2i_F & 0 \end{bmatrix}$$

i) Study questions (i) to (viii) of problem 30 for this W.

ii) Prove { I_4^C, $I_{4i_F}^C$, $I_{2+6i_F}^C$ } will generate a finite semigroup under product.

iii) If I_4^C is replaced by $I_{1.2}^C$ then { $I_{1.2}^C$ $I_{4i_F}^C$, $I_{2+6i_F}^C$ } will not generate a finite semigroup under $\times$.

47. Prove the criteria of generating a finite semigroup under I_t^C is essential for one to have MOD resultants in $C^I([0, n))$; $2 \leq n < \infty$.

48. Let $S = \begin{bmatrix} 0 & 3+I_4^I & 0 & 0.332I \\ 0.42 & 0 & 2.5+I_{2I}^I & 0 \\ 0 & 0.352I & 0 & I_0^I \\ 0.53I & 0 & 0.72I & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic-neutrosophic cognitive maps model dynamical system with entries from $\langle [0,6) \cup I \rangle$.

i) Study questions (i) to (viii) of problem 30 for this S.

ii) If I_0^0 is replaced by $I_{1.5I}^I$ prove the MOD resultant will not exist.

iii) Find the largest semigroup under product by element I_t^I in $\langle [0,6) \cup I \rangle_I$.

49. Let B = $\begin{bmatrix} 0 & I_{6h}^{h}+0.32 & 0 & 0.33h & 0 \\ I_{3}^{h} & 0 & 1+I_{0}^{h} & 0 & 0.225 \\ 0 & 0.333h & 0 & I_{3+3h}^{h} & 0 \\ 0.21 & 0 & 0.801 & 0 & I_{6h}^{h} \\ 0.45 & 0.112+0.32h & 0 & 0.405 & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic special dual like number cognitive maps model dynamical system with entries from $\langle[0,9) \cup h\rangle_I$.

i) Study questions (i) to (viii) of problem 30 for this B.
ii) Prove $\{I_{6h}, I_{6h}^{h}, I_{0}^{h}, I_{3+3h}^{h}, I_{3}^{h}\}$ generates a finite semigroup under product.
iii) If I_{3}^{h} is replaced by $I_{4.5}^{h}$ in the matrix B prove MOD resultant in general will not exist.

50. Let W = $\begin{bmatrix} 0 & I_{2k+3}^{k} & 0.331 & 0 \\ 0.53 & 0 & I_{3k}^{k} & 0.21 \\ 0 & 0.73+0.4k & 0 & I_{4k}^{k} \\ 4+I_{2+4k}^{k} & 0 & 0.221 & 0 \end{bmatrix}$

be the MOD interval natural neutrosophic special quasi dual number cognitive maps model connection matrix with entries from $\langle[0,6) \cup k\rangle_I$.

i) Study questions (i) to (viii) of problem 30 for this W.
ii) If I_{2k+3}^{k} is replaced by $I_{1.5+0.75k}^{l}$ then in general we will not be in a position to find the MOD resultants.
iii) Find the largest MOD natural neutrosophic semigroup of $\langle[0,6) \cup k\rangle_I$.
iv) Can we say $\langle[0,6) \cup k\rangle_I$ is a semigroup under $\times$?
v) Obtain any other special feature associated with W.

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INDEX

F

Fuzzy Relational Maps (FRMs) model, 7

M

MOD interval dual number bipartite directed graph, 206-210
MOD acyclic, 43
MOD complex bipartite directed graph, 53-6
MOD complex rectangular or relational matrix, 53-7
MOD Complex Relational Maps (MOD CRMs) model, 52-5
MOD directed cycle, 43
MOD directed neutrosophic bipartite graph, 67-9
MOD domain space of initial state vectors, 17-9
MOD dual number directed bipartite graphs, 76-9
MOD dual number rectangular matrix, 77-9
MOD dual number relational maps model, 81-9
MOD dual number, 13
MOD dynamical system, 43
MOD equilibrium state, 44
MOD finite complex number natural neutrosophic rectangular or relational matrix, 117-129
MOD finite complex number natural neutrosophic Relational Maps model, 117-129
MOD hidden pattern, 44

MOD interval dual number dynamical system, 209-214
MOD interval dual number Relational Maps model, 206-210
MOD interval dual number relational or rectangular matrix operator, 207-215
MOD interval finite complex number directed bipartite graph, 184-9
MOD interval finite complex number Relational Maps model, 184-8
MOD interval finite complex number relational or rectangular matrix operator, 186-9
MOD interval neutrosophic bipartite directed graph, 195-9
MOD interval neutrosophic interval bipartite directed graph, 224-9
MOD interval neutrosophic rectangular or relational matrix operator, 198-205
MOD interval neutrosophic Relational Maps model, 195-205
MOD interval rectangular or relational matrix, 175-9
MOD interval Relational Maps model, 173-9
MOD interval special dual like number (plane) Relational Maps model, 214-220
MOD interval special dual like number directed bipartite graph, 214-18
MOD interval special dual like number plane rectangular or relational matrices, 215-9
MOD interval special quasi dual number plane directed bipartite graphs, 219-225
MOD interval special quasi dual number plane rectangular matrix, 220-9
MOD interval special quasi dual number plane Relational Maps model, 219-225
MOD natural neutrosophic interval special dual like number directed bipartite graphs, 239-243
MOD natural neutrosophic bipartite directed graphs, 98-103
MOD natural neutrosophic Cognitive Maps model, 97-9
MOD natural neutrosophic dual number directed bipartite graph, 131-6
MOD natural neutrosophic dual number interval bipartite directed graph, 235-9

MOD natural neutrosophic dual number interval Relational Maps model, 235-9
MOD natural neutrosophic dual number rectangular or relational matrix operator, 133-9
MOD natural neutrosophic dual number Relational Maps model, 142-9
MOD natural neutrosophic elements, 7-10
MOD natural neutrosophic finite complex number bipartite directed graph, 116-129
MOD natural neutrosophic finite complex number Relational Maps model, 116-9
MOD natural neutrosophic interval dual number relational or rectangular matrix, 235-9
MOD natural neutrosophic interval Relational Maps model, 173-9
MOD natural neutrosophic interval Relational Maps model, 225-9
MOD natural neutrosophic interval relational or rectangular matrix operator, 225-9
MOD natural neutrosophic interval special dual like number Relational Maps model, 239-243
MOD natural neutrosophic interval special dual like number relational matrix operator, 240-244
MOD natural neutrosophic interval special quasi dual number rectangular matrix operator, 238-240
MOD natural neutrosophic interval special quasi number bipartite directed graph, 239-243
MOD natural neutrosophic numbers Relational Maps model, 8
MOD natural neutrosophic numbers, 98-102
MOD natural neutrosophic Relational Maps model, 97-9
MOD natural neutrosophic relational or rectangular matrix, 99-105
MOD natural neutrosophic special dual like number bipartite directed graph, 143-9
MOD natural neutrosophic special dual like number rectangular or relational matrix operator, 144-9
MOD natural neutrosophic special dual like number Relational Maps model, 143-9

MOD natural neutrosophic special quasi dual interval Relational Maps model, 238-242
MOD natural neutrosophic special quasi dual number bipartite directed graph, 157-170
MOD natural neutrosophic special quasi dual number rectangular or relational matrix operator, 159-163
MOD natural neutrosophic special quasi dual number Relational Maps model, 164-9
MOD natural neutrosophic-neutrosophic directed bipartite graph, 110-7
MOD natural neutrosophic-neutrosophic interval Relational Maps model, 229-235
MOD natural neutrosophic-neutrosophic interval relational or rectangular matrix operator, 230-5
MOD natural neutrosophic-neutrosophic plane directed bipartite graph, 229-35
MOD natural neutrosophic-neutrosophic Relations Maps model, 113-9
MOD neutrosophic integers, 13
MOD neutrosophic Relational Maps model, 67-9
MOD neutrosophic relational or rectangular matrix, 67-9
MOD range space of initial state vectors, 17-9
MOD realized fixed point pair, 17
MOD realized limit cycles pair, 17
MOD rectangular matrices as operators, 7
MOD rectangular matrices, 15
MOD rectangular natural neutrosophic-neutrosophic matrix, 111-9
MOD relational bipartite directed graph, 14
MOD Relational Maps (MOD RMs) model, 7, 42
MOD relations, 42
MOD special classical point pair, 44-9
MOD special dual like number directed bipartite graph, 84-9
MOD special dual like number relational maps model, 85-9
MOD special dual like number relational or rectangular matrix, 86-92
MOD special dual like numbers, 13
MOD special quasi dual number directed bipartite graph, 90-6
MOD special quasi dual number Relational Maps model, 90-6

MOD special quasi dual number relational matrix, 91-7
MOD special quasi dual numbers, 13

N

Neutrosophic Relational Maps (NRMs) model, 7

S

Special type of MOD fixed point pair, 8-9

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In this book authors for the first time construct MOD Relational Maps model analogous to Fuzzy Relational Maps (FRMs) model or Neutrosophic Relational Maps (NRMs) model using the MOD rectangular or relational matrix.
The advantage of using these models is that the MOD fixed point pair or a MOD limit cycle pair is obtained after a finite number of iterations.

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