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ORIGINAL ARTICLE

Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information

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Abstract This paper develops a method for solving the multiple attribute decision-making problems with the single-valued neutrosophic information or interval neutrosophic information. We first propose two discrimination functions referred to as score function and accuracy function for ranking the neutrosophic numbers. An optimization model to determine the attribute weights that are partly known is established based on the maximizing deviation method. For the special situations where the information about attribute weights is completely unknown, we propose another optimization model. A practical and useful formula which can be used to determine the attribute weights is obtained by solving a proposed nonlinear optimization problem. To aggregate the neutrosophic information corresponding to each alternative, we utilize the neutrosophic weighted averaging operators which are the single-valued neutrosophic weighted averaging operator and the interval neutrosophic weighted averaging operator. Thus, we can determine the order of alternatives and choose the most desirable one(s) based on the score function and accuracy function. Finally, some illustrative examples are presented to verify the proposed approach and to present its effectiveness and practicality.

Keywords Neutrosophic sets · Single-valued neutrosophic sets · Interval neutrosophic sets · Aggregation operators · Neutrosophic multiple attribute decision making · Maximizing deviation method

1 Introduction

Zadeh [32] introduced the degree of membership/truth (t) in 1965 and proposed the concept of fuzzy set. Atanassov [1] introduced the degree of nonmembership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set (to date, the intuitionistic fuzzy sets have been widely applied in solving MCDM problems [13, 21–23]). Using the degree of indeterminacy/neutrality (i) as independent component in 1995, Smarandache initiated the neutrosophic set theory. He has coined the words "neutrosophy" and neutrosophic. In 2013, he redefined the neutrosophic set to n components: $t_1, t_2, \ldots; i_1, i_2, \ldots; f_1, f_2, \ldots$.

But, a neutrosophic set will be difficult to apply in real scientific and engineering fields. Therefore, Wang et al. [19, 20] proposed the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) which are an instance of a neutrosophic set, and provided set-theoretic operators and various properties of SVNSs and INSs. Recently, the theory of neutrosophic set has received more and more attentions [2–4, 6–8, 11, 12, 14, 15, 17, 26–31, 33, 34]. Zhang et al. [33] proposed some neutrosophic aggregation operators, such as the interval neutrosophic weighted averaging (INWA) operator and the interval neutrosophic weighted geometric (INWG) operator, and applied the operators to solve the multiple attribute group decision-making problems with interval neutrosophic information.

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For the above researches on the multiple attribute decision-making (MADM) problems with interval neutrosophic information, we can suppose the attribute weights are fully known. However, in real decision making, because of time pressure, lack of knowledge or data and the expert's limited expertise about the problem domain, the information about attribute weights is incompletely known or completely unknown. So, the existing MADMs under neutrosophic environment will be impractical for such situations. Therefore, it is necessary to study this issue. In this paper, our aim is to solve the MADM problems in which the attribute values take the form of neutrosophic information and attribute weights are incompletely known or completely unknown based on the maximizing deviation method. In Sect. 2, we summarize the some basic concepts related to a neutrosophic set and its instances, single-valued neutrosophic set and interval neutrosophic set. A score function and an accuracy function are also proposed for ranking neutrosophic numbers in this section. Section 3 introduces the neutrosophic MADM (NMADM) method under neutrosophic environment, in which the information about attribute weights is partly known and the attribute values take the form of neutrosophic numbers. An optimization model based on the maximizing deviation method is established to determine the attribute weights. For the special situations where the information about attribute weights is completely unknown, we develop another optimization model which provides a simple and exact formula. To aggregate the neutrosophic information corresponding to each alternative, we utilize the neutrosophic weighted averaging (NWA) operators which are the single-valued neutrosophic weighted averaging (SVNWA) operator and the interval neutrosophic weighted averaging (INWA) operator. Thus, we can determine the order of alternatives and choose the most desirable one(s) based on the score function and accuracy function. In Sect. 4, some illustrative examples are presented to verify the developed approach and to demonstrate its practicality and effectiveness. Section 5 concludes the paper and presents some results.

2 Preliminaries

In the subsection, we give some concepts related to neutrosophic sets, single-valued neutrosophic sets and interval neutrosophic sets.

2.1 Neutrosophic set

Definition 1 (Smarandache [16]) Let X be a universe of discourse, then a neutrosophic set is defined as:

$$A = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\},$$

which is characterized by a truth-membership function $T_A : X \to]0^-, 1^+[$, an indeterminacy-membership function $I_A : X \to]0^-, 1^+[$ and a falsity-membership function $F_A : X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

In the following, we adopt the representations $u_A(x)$, $p_A(x)$ and $v_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively.

Wang et al. [20] defined the single-valued neutrosophic set which is an instance of neutrosophic set as follows:

2.2 Single-valued neutrosophic sets

Definition 2 (Wang et al. [20]) Let X be a universe of discourse, then a single-valued neutrosophic set is defined as:

$$A = \{\langle x, u_A(x), p_A(x), v_A(x)\rangle : x \in X\}$$

where, $u_A : X \to [0, 1]$, $p_A : X \to [0, 1]$ and $v_A : X \to [0, 1]$ with $0 \leq u_A(x) + p_A(x) + v_A(x) \leq 3$ for all $x \in X$. The intervals $u_A(x), p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of x to A, respectively.

We will denote the set of all the SVNSs in X by $\mathcal{Q}$. A single-valued neutrosophic number (SVNN) is denoted by $\tilde{a} = \langle u, p, v\rangle$ for convenience.

We give a score function and an accuracy function for ranking SVNNs as follows;

Definition 3 Let $\tilde{a} = \langle u, p, v\rangle$ be a single-valued neutrosophic number. Then a score function S of the single-valued neutrosophic number can be defined by

$$S(\tilde{a}) = \frac{1 + u - 2p - v}{2} \tag{1}$$

where $S(\tilde{a}) \in [-1, 1]$.

The score function S is reduced the score function proposed by Li [5] if $p = 0$ and $u + v \leq 1$.

Example 1 Let $\tilde{a}_1 = (0.5, 0.2, 0.6)$ and $\tilde{a}_2 = (0.6, 0.4, 0.2)$ be two single-valued neutrosophic numbers for two alternatives. Then, by applying Definition 3, we can obtain

$$S(\tilde{a}_1) = \frac{1 + 0.5 - 2 \times 0.2 - 0.6}{2} = 0.25$$

$$S(\tilde{a}_2) = \frac{1 + 0.6 - 2 \times 0.4 - 0.2}{2} = 0.30.$$

In this case, we can say that alternative $\tilde{a}_2$ is better than $\tilde{a}_1$.

Definition 4 Let $\tilde{a} = \langle u, p, v\rangle$ be a single-valued neutrosophic number, an accuracy function H of the single-valued neutrosophic number can be defined by

$$H(\tilde{a}) = u - p(1-u) - v(1-p) \tag{2}$$

where $H(\tilde{a}) \in [-1, 1]$.

When the value of $H(\tilde{a})$ increases, we say that the degree of accuracy of the single-valued neutrosophic number $\tilde{a}$ increases.

Example 2 Let $\tilde{a}_1 = (0.3, 0.1, 0.4)$ and $\tilde{a}_2 = (0.5, 0.1, 0.3)$ be two single-valued neutrosophic numbers for two alternatives. Then, by applying Definition 4, we can obtain $H(\tilde{a}_1) = -0.13$ and $H(\tilde{a}_2) = 0.18$.

In this case, we can say that alternative $\tilde{a}_2$ is better than $\tilde{a}_1$.

With respect to the score function S and the accuracy function H, a method for comparing SVNNs can be defined as follows;

Definition 5 Let $\tilde{a}_1 = \langle u_1, p_1, v_1 \rangle$ and $\tilde{a}_2 = \langle u_2, p_2, v_2 \rangle$ be two single-valued neutrosophic values. Then we have

1. if $S(\tilde{a}_1) > S(\tilde{a}_2)$, then $\tilde{a}_1$. is greater than $\tilde{a}_2$, denoted by $\tilde{a}_1 \succ \tilde{a}_2$,
2. if $S(\tilde{a}_1) = S(\tilde{a}_2)$ and $H(\tilde{a}_1) > H(\tilde{a}_2)$ then $\tilde{a}_1$ is greater than $\tilde{a}_2$, denoted by $\tilde{a}_1 \succ \tilde{a}_2$.

Example 3 Let $\tilde{a}_1 = (0.6, 0.2, 0.2)$ and $\tilde{a}_2 = (0.5, 0.1, 0.3)$ be two single-valued neutrosophic numbers for two alternatives. Then, by applying Definition 5, we can obtain $S(\tilde{a}_1) = S(\tilde{a}_2) = 0.5$ and $H(\tilde{a}_1) = 0.36$, $H(\tilde{a}_2) = 0.18$. Then it implies that $\tilde{a}_1 \succ \tilde{a}_2$.

Based on the study given in Zhang et al. [33], we define two weighted aggregation operators related to SVNSs as follows;

Definition 6 Let $\tilde{a}_j = \langle u_j, p_j, v_j \rangle (j = 1, 2, \ldots, n)$ be a collection of single-valued neutrosophic numbers, and $\mathrm{SVNWA} : \mathcal{Q}_n \to \mathcal{Q}$, if

$$\begin{aligned} \mathrm{SVNWA}_{\omega}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) &= \sum_{j=1}^{n} \omega_j \tilde{a}_j \\ &= \left(1 - \prod_{j=1}^{n} \left(1 - u_j\right)^{\omega_j}, \prod_{j=1}^{n} p_j^{\omega_j}, \prod_{j=1}^{n} v_j^{\omega_j} \right) \end{aligned} \tag{3}$$

where ω_j is the weight of $\tilde{a}_j$ $(j = 1, 2, \ldots, n)$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then SVNWA is called single-valued neutrosophic weighted average operator; especially, when $\omega_j = 1/n$ $(j = 1, 2, \ldots, n)$, then the SVNWA is called an arithmetic average operator for SVNNs.

Similarly, we can define the single-valued neutrosophic weighted geometric average (SVNWG) operator.

Definition 7 Let $\tilde{a}_j = \langle u_j, p_j, v_j \rangle (j = 1, 2, \ldots, n)$ be a collection of single-valued neutrosophic numbers, and $\mathrm{SVNWG} : \mathcal{Q}_n \to \mathcal{Q}$, if

$$\begin{aligned} \mathrm{SVNWG}_{\omega}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) &= \prod_{j=1}^{n} \tilde{a}_j^{\omega_j} \\ &= \left(\prod_{j=1}^{n} u_j^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - p_j\right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - v_j\right)^{\omega_j} \right) \end{aligned} \tag{4}$$

where ω_j. is the weight of $\tilde{a}_j$ $(j = 1, 2, \ldots, n)$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then SVNWG is called single-valued neutrosophic weighted geometric average operator; especially, when $\omega_j = 1/n$ $(j = 1, 2, \ldots, n)$, then SVNWG is called a geometric average operator for SVNNs.

The aggregation results of the SVNWA and SVNWG operators are still SVNSs.

Definition 8 (Majumdar and Samanta [9]) Let $\tilde{a}_1 = \langle u_1, p_1, v_1 \rangle$ and $\tilde{a}_2 = \langle u_2, p_2, v_2 \rangle$ be two single-valued neutrosophic numbers. Then the normalized Hamming distance measure between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as:

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{3}\left(|u_1 - u_2| + |p_1 - p_2| + |v_1 - v_2|\right). \tag{5}$$

Wang et al. [19] extended the concept of single-valued neutrosophic set to interval neutrosophic set (INS) which is a further instance of the NSs. The fundamental characteristic of the INS is that the values of its truth-membership function, indeterminacy-membership function and falsity-membership function are intervals rather than exact numbers.

2.3 Interval neutrosophic sets

Definition 9 (Wang et al. [19]) Let X be a universe of discourse and Int[0,1] be the set of all closed subsets of $[0, 1]$. Then an interval neutrosophic set is defined as:

$$A = \{\langle x, u_A(x), p_A(x), v_A(x) \rangle : x \in X\}$$

where $u_A : X \to \mathrm{Int}[0, 1]$, $p_A : X \to \mathrm{Int}[0, 1]$ and $v_A : X \to \mathrm{Int}[0, 1]$ with $0 \leq \sup u_A(x) + \sup p_A(x) + \sup v_A(x) \leq 3$ for all $x \in X$. The intervals $u_A(x), p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of x to A, respectively.

For convenience, if let $u_A(x) = \left[u_A^L(x), u_A^U(x)\right]$, $p_A(x) = \left[p_A^L(x), p_A^U(x)\right]$ and $v(x) = \left[v_A^L(x), v_A^U(x)\right]$, then

$$A = \left\{\left\langle x, \left[u_A^L(x), u_A^U(x)\right], \left[p_A^L(x), p_A^U(x)\right], \left[v_A^L(x), v_A^U(x)\right]\right\rangle : x \in X\right\}$$

with the condition, $0 \leq \sup u_A^U(x) + \sup p_A^U(x) + \sup v_A^U(x) \leq 3$ for all $x \in X$. Here, we only consider the sub-unitary interval of $[0, 1]$. Therefore, an interval neutrosophic set is clearly a neutrosophic set.

We will denote the set of all the INSs in X by $\mathcal{F}$. An interval neutrosophic number (INN) is denoted by $\tilde{b} = \langle [u^-, u^+], [p^-, p^+], [v^-, v^+] \rangle$ for convenience.

We give the score function and accuracy function of an INN as follows.

Definition 10 Let $\tilde{b} = \langle [u^-, u^+], [p^-, p^+], [v^-, v^+] \rangle$ be an interval neutrosophic number, a score function S of the single-valued neutrosophic number can be defined by

$$S(\tilde{b}) = \frac{2 + u^- + u^+ - 2p^- - 2p^+ - v^- - v^+}{4} \tag{6}$$

where $S(\tilde{b}) \in [-1, 1]$.

Example 4 Let $\tilde{b}_1 = ([0.6, 0.4], [0.3, 0.1], [0.1, 0.3])$ and $\tilde{b}_2 = ([0.1, 0.6], [0.2, 0.3], [0.1, 0.4])$ be two interval neutrosophic numbers for two alternatives. Then, by Definition 10, we can obtain $S(\tilde{b}_1) = 0.65$ and $S(\tilde{b}_2) = 0.30$.

In this case, we can say that alternative $\tilde{b}_1$ is better than $\tilde{b}_2$.

Definition 11 Let $\tilde{b} = \langle [u^-, u^+], [p^-, p^+], [v^-, v^+] \rangle$ be an interval neutrosophic number, an accuracy function H of the single-valued neutrosophic number can be defined by

$$\begin{aligned} H\left(\tilde{b}\right) = \frac{1}{2}\big(&u^- + u^+ - p^+(1 - u^+) - p^-(1 - u^-) \\ &- v^+(1 - p^-) - v^-(1 - p^+)\big) \end{aligned} \tag{7}$$

where $H\left(\tilde{b}\right) \in [-1, 1]$.

The larger the value of $H\left(\tilde{b}\right)$ is, the more the degree of accuracy of the single-valued neutrosophic value $\tilde{b}$ is.

The accuracy function H is reduced the accuracy function proposed by Nayagam et al. [10] if $p^-, p^+ = 0$ and $u^+ + v^+ \leq 1$.

With respect to the score function S and the accuracy function H, we define a method for comparing INNs as follows;

Definition 12 Let $\tilde{b}_1 = \left\langle \left[u_1^-, u_1^+\right], \left[p_1^-, p_1^+\right], \left[v_1^-, v_1^+\right] \right\rangle$ and $\tilde{b}_2 = \left\langle \left[u_2^-, u_2^+\right], \left[p_2^-, p_2^+\right], \left[v_2^-, v_2^+\right] \right\rangle$ be two interval neutrosophic numbers. Then we have

1. if $S\left(\tilde{b}_1\right) > S\left(\tilde{b}_2\right)$, then $\tilde{b}_1$ is greater than $\tilde{b}_2$, denoted by $\tilde{b}_1 \succ \tilde{b}_2$,
2. if $S\left(\tilde{b}_1\right) = S\left(\tilde{b}_2\right)$ and $H\left(\tilde{b}_1\right) > H\left(\tilde{b}_2\right)$, then $\tilde{b}_1$ is greater than $\tilde{b}_2$, denoted by $\tilde{b}_1 \succ \tilde{b}_2$.

Next, we give two weighted aggregation operators related to INSs.

Definition 13 (Zhang et al. [33]) Let $\tilde{b}_j = \left\langle \left[u_j^-, u_j^+\right], \left[p_j^-, p_j^+\right], \left[v_j^-, v_j^+\right] \right\rangle (j = 1, 2, \ldots, n)$ be a collection of interval neutrosophic values, and $\mathrm{INWA} : \mathcal{F}_n \to \mathcal{F}$, if

$$\begin{aligned} &\mathrm{INWA}_\omega\left(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n\right) \\ &= \sum_{j=1}^{n} \omega_j \tilde{b}_j = \left\langle \left[1 - \prod_{j=1}^{n}\left(1 - u_j^-\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - u_j^+\right)^{\omega_j}\right], \right. \\ &\left. \left[\prod_{j=1}^{n}\left(p_j^-\right)^{\omega_j}, \prod_{j=1}^{n}\left(p_j^+\right)^{\omega_j}\right], \left[\prod_{j=1}^{n}\left(v_j^-\right)^{\omega_j}, \prod_{j=1}^{n}\left(v_j^+\right)^{\omega_j}\right] \right\rangle, \end{aligned} \tag{8}$$

where ω_j is the weight of $\tilde{b}_j$ $(j = 1, 2, \ldots, n)$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then INWA is called interval neutrosophic weighted average operator; especially, when $\omega_j = 1/n$ $(j = 1, 2, \ldots, n)$, then the INWA is called an arithmetic average operator for INNs.

Definition 14 (Zhang et al. [33]) Let $\tilde{b}_j = \left\langle \left[u_j^-, u_j^+\right], \left[p_j^-, p_j^+\right], \left[v_j^-, v_j^+\right] \right\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of interval neutrosophic numbers, and $\mathrm{INWG} : \mathcal{F}_n \to \mathcal{F}$, if

$$\begin{aligned} &\mathrm{INWG}_\omega\left(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n\right) = \prod_{j=1}^{n} \tilde{b}_j^{\omega_j} = \left\langle \left[\prod_{j=1}^{n}\left(u_j^-\right)^{\omega_j}, \prod_{j=1}^{n}\left(u_j^+\right)^{\omega_j}\right], \right. \\ &\left[1 - \prod_{j=1}^{n}\left(1 - p_j^-\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - p_j^+\right)^{\omega_j}\right], \\ &\left. \left[1 - \prod_{j=1}^{n}\left(1 - v_j^-\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - v_j^+\right)^{\omega_j}\right] \right\rangle, \end{aligned} \tag{9}$$

where ω_j is the weight of $\tilde{b}_j$ $(j = 1, 2, \ldots, n)$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then INWG is called interval neutrosophic weighted geometric average operator; especially, when $\omega_j = 1/n$ $(j = 1, 2, \ldots, n)$, then the INWG is called a geometric average operator for INNs.

The aggregation results of the INWA and INWG operators are still INSs.

Definition 15 (Ye [27]) Let $\tilde{b}_1 = \left\langle \left[u_1^-, u_1^+\right], \left[p_1^-, p_1^+\right], \left[v_1^-, v_1^+\right] \right\rangle$ and $\tilde{b}_2 = \left\langle \left[u_2^-, u_2^+\right], \left[p_2^-, p_2^+\right], \left[v_2^-, v_2^+\right] \right\rangle$ be two interval neutrosophic numbers. Then the normalized Hamming distance measure between $\tilde{b}_1$ and $\tilde{b}_2$ is defined as:

$$\begin{aligned} d\left(\tilde{b}_1, \tilde{b}_2\right) = \frac{1}{6}\Big(&\left|u_1^- - u_2^-\right| + \left|p_1^- - p_2^-\right| + \left|v_1^- - v_2^-\right| \\ &+ \left|u_1^+ - u_2^+\right| + \left|p_1^+ - p_2^+\right| + \left|v_1^+ - v_2^+\right|\Big). \end{aligned}$$

From the above analysis, we develop a method based on the maximizing deviation for the neutrosophic multiple attribute decision-making problems in which attribute values for alternatives are the single-valued neutrosophic value and the interval neutrosophic value.

3 Maximizing deviation method for neutrosophic information

Suppose that $A = \{A_1, A_2, \ldots, A_m\}$ is the set of alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is a set of criterions or attributes. The attribute weights are partly known or completely unknown. Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of attributes, such that $\sum_{j=1}^{n} \omega_j = 1$, $\omega_j \geq 0$ $(j = 1, 2, \ldots, n)$ and ω_j refers to the weight of attribute C_j. $D = \{d_1, d_2, \ldots, d_t\}$ denotes the set of decision makers (DMs), and $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_t\}$ denotes the weight vector of DMs, $\lambda_t \in [0, 1]$, $k = 1, 2, \ldots, t$, $\sum_{k=1}^{t} \lambda_t = 1$. Assume that $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n}$ is the decision matrix provided by the DM $d_k \in D$, $a_{ij}^{(k)}$ is a neutrosophic value for alternative A_i associated with the attribute C_j. If $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n} = \left(u_{ij}^{(k)}, p_{ij}^{(k)}, v_{ij}^{(k)}\right)_{m \times n}$, it is a single-valued neutrosophic decision matrix, where $u_{ij}^{(k)}$ indicates the degree that the alternative A_i satisfies the attribute C_j and $p_{ij}^{(k)}$ indicates the degree that the alternative A_i is indeterminacy on the attribute C_j, whereas $v_{ij}^{(k)}$ indicates the degree that the attribute A_i does not satisfy the attribute C_j given by the decision maker d_k. We have the conditions $u_{ij}^{(k)} \in [0, 1]$, $p_{ij}^{(k)} \in [0, 1]$, and $v_{ij}^{(k)} \in [0, 1]$, $0 \leq u_{ij}^{(k)} + p_{ij}^{(k)} + v_{ij}^{(k)} \leq 3$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Similarly, if $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n} = \left(\left(\left[\left(u_{ij}^{-}\right)^{(k)}, \left(u_{ij}^{+}\right)^{(k)}\right], \left[\left(p_{ij}^{-}\right)^{(k)}, \left(p_{ij}^{+}\right)^{(k)}\right], \left[\left(v_{ij}^{-}\right)^{(k)}, \left(v_{ij}^{+}\right)^{(k)}\right]\right)\right)_{m \times n}$, it is an interval neutrosophic decision matrix, where $\left[\left(u_{ij}^{-}\right)^{(k)}, \left(u_{ij}^{+}\right)^{(k)}\right]$ indicates the degree that the alternative A_i satisfies the attribute C_j and $\left[\left(p_{ij}^{-}\right)^{(k)}, \left(p_{ij}^{+}\right)^{(k)}\right]$ indicates the degree that the alternative A_i is indeterminacy on the attribute C_j, whereas $\left[\left(v_{ij}^{-}\right)^{(k)}, \left(v_{ij}^{+}\right)^{(k)}\right]$ indicates the degree that the attribute A_i does not satisfy the attribute C_j given by the decision maker d_k. Here, we have the condition $0 \leq \sup\left(u_{ij}^{+}\right)^{(k)} + \sup\left(p_{ij}^{+}\right)^{(k)} + \sup\left(v_{ij}^{+}\right)^{(k)} \leq 3$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Obtaining an overall preference value by synthesizing the performance values of all alternatives of each expert is an important step in decision process.

In this paper, we will utilize the SVNWA and INWA operators as the main aggregation operators for two different methods, respectively.

Definition 16 Suppose that $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n} = \left(u_{ij}^{(k)}, p_{ij}^{(k)}, v_{ij}^{(k)}\right)_{m \times n}$ is a single-valued neutrosophic decision matrix and $\tilde{r}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$ is the vector of attribute values corresponding to the alternative A_i, $i = 1, 2, \ldots, m$. Then the overall preference value of alternative A_i for DM d_k can be expressed as, $i = 1, 2, \ldots, m$,

$$\tilde{r}_i = \left(u_{ij}^{(k)}, p_{ij}^{(k)}, v_{ij}^{(k)}\right) = \mathrm{SVNWA}_{\omega}(\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in}) = \left(1 - \prod_{j=1}^{n}\left(1 - u_j^{(k)}\right)^{\omega_j}, \prod_{j=1}^{n} p_j^{(k)^{\omega_j}}, \prod_{j=1}^{n} v_j^{(k)^{\omega_j}}\right) \tag{10}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ denotes the weight vector of attributes.

Definition 17 Suppose that $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n} = \left(\left(\left[\left(u_{ij}^{-}\right)^{(k)}, \left(u_{ij}^{+}\right)^{(k)}\right], \left[\left(p_{ij}^{-}\right)^{(k)}, \left(p_{ij}^{+}\right)^{(k)}\right], \left[\left(v_{ij}^{-}\right)^{(k)}, \left(v_{ij}^{+}\right)^{(k)}\right]\right)\right)_{m \times n}$ is an interval neutrosophic decision matrix and $\tilde{z}_i = (\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{in})$ is the vector of attribute values corresponding to the alternative A_i, $i = 1, 2, \ldots, m$. Then the overall preference value of alternative A_i for DM d_k can be expressed as, $i = 1, 2, \ldots, m$,

$$\begin{aligned}\tilde{z}_i &= \left(\left[u_{ij}^{-(k)}, u_{ij}^{+(k)}\right], \left[p_{ij}^{-(k)}, p_{ij}^{+(k)}\right], \left[v_{ij}^{-(k)}, v_{ij}^{+(k)}\right]\right) \\ &= \mathrm{INWA}_{\omega}(\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{in}) \\ &= \left(\left[1 - \prod_{j=1}^{n}\left(1 - u_{ij}^{-(k)}\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1 - u_{ij}^{+(k)}\right)^{\omega_j}\right],\right. \\ &\quad \left.\left[\prod_{j=1}^{n}\left(p_{ij}^{-(k)}\right)^{\omega_j}, \prod_{j=1}^{n}\left(p_{ij}^{+(k)}\right)^{\omega_j}\right], \left[\prod_{j=1}^{n}\left(v_{ij}^{-(k)}\right)^{\omega_j}, \prod_{j=1}^{n}\left(v_{ij}^{+(k)}\right)^{\omega_j}\right]\right),\end{aligned} \tag{11}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of attributes.

Because many practical group decision-making problems are complex and uncertain, and human thinking is inherently subjective, the information about attribute weights is usually incomplete. Generally speaking, the incomplete attribute weight information can be expressed as the following relationships among the weights, for $i \neq j$:

Form 1: A weak ranking: $\omega_i \geq \omega_j$;
Form 2: A strict ranking: $\omega_i - \omega_j \geq \alpha_i (\alpha_i > 0)$;
Form 3: A ranking of differences: $\omega_i - \omega_j \geq \omega_k - \omega_l$ $(j \neq k \neq l)$
Form 4: A ranking with multiples: $\omega_i \geq \alpha_i \omega_j$ $(0 \leq \alpha_i \leq 1)$
Form 5: An interval form: $\alpha_i \leq \omega_i \leq \alpha_i + \varepsilon_i$, $(0 \leq \alpha_i \leq \alpha_i + \varepsilon_i \leq 1)$.

Wang [18] developed the maximizing deviation method for handling the multiple attribute decision-making problems characterized by numerical information. In decision-making problem, it is essential to rank them by comparing alternatives. The larger the ranking value $\tilde{r}_i$ (or $\tilde{z}_i$) is, the better corresponding alternative A_i is. If an attribute is creating little differences on all alternatives, it implies that such an attribute has a small important in decision process. Contrary, if an attribute has very clear differences in terms of the performance values of each alternative, we say that such an attribute should be in the foreground in selecting the best alternative. That is, if one attribute has a similar effect among alternatives, it should be assigned with a small weight; otherwise, the attribute which makes larger deviations should be assigned a bigger weight; especially, if all alternatives have a very similar performance value in term of a given attribute, then such an attribute will not have much effect on ranking the alternatives. In other words, such an attribute should be assigned with a very small weight. Also, Wang [18] put forward that zero should be assigned with the corresponding to attribute.

To determine the differences among the performance values of all alternatives, we adopt the deviation method. For the DM d_k and the attribute C_j, the deviation of alternative A_i to all the other alternatives can be expressed as follows:

$$H_{ij}^{(k)}(\omega) = \sum_{s=1}^{n} d\left(a_{ij}^{(k)}, a_{sj}^{(k)}\right)\omega_j, \quad \begin{array}{l} i = 1,2,\ldots,m \\ j = 1,2,\ldots,n \end{array}.$$

Let

$$H_j^{(k)}(\omega) = \sum_{i=1}^{n} H_{ij}^{(k)}(\omega) = \sum_{i=1}^{n}\sum_{s=1}^{n} d\left(a_{ij}^{(k)}, a_{sj}^{(k)}\right)\omega_j, \quad j = 1,2,\ldots,n.$$

Then $H_j^{(k)}(\omega)$ gives the deviation value of all alternatives to other alternatives for the attribute A_i and the DM d_k.

Using the single-valued neutrosophic sets and the interval neutrosophic sets, we can select a weight vector ω for maximize operator of all deviation values with respect to all the attributes and all the DMs.

3.1 Maximizing deviation method for single-valued neutrosophic sets

In the subsection, we construct a nonlinear programming model with single-valued neutrosophic information, as follows:

$$(\text{M-1}) \begin{cases} \max H(\omega) = \sum_{k=1}^{t} \lambda_k \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{n} d\left(a_{ij}^{(k)}, a_{sj}^{(k)}\right)\omega_j \\ \text{subject to } \omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1, \quad j = 1,2,\ldots,n \end{cases} \tag{12}$$

where λ_k is the weight of DM d_k, and

$$d\left(a_{ij}^{(k)}, a_{sj}^{(k)}\right) = \frac{1}{3}\left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right|\right).$$

By solving the model (M-1), we get the optimal solution $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{T}$, which can be used as the weight vector of attributes.

If the attribute weights are completely unknown, we can establish another programming model:

$$(\text{M-2}) \begin{cases} \max H(\omega) = \sum_{k=1}^{t} \lambda_k \frac{1}{3} \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{n} \omega_j \left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right|\right) \\ \text{subject to } \omega_j \geq 0, \sum_{j=1}^{n} \omega_j^2 = 1, \ j = 1,2,\ldots,n \end{cases}$$

To solve this model, we construct the Lagrange function:

$$L(\omega, \xi) = \sum_{k=1}^{t} \lambda_k \frac{1}{3} \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{n} \omega_j \left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right|\right) + \frac{\xi}{6}\left(\sum_{j=1}^{n} \omega_j^2 - 1\right) \tag{13}$$

where ξ is the Lagrange multiplier.

Then we compute the partial derivatives of L as follows:

$$\begin{cases} \frac{\partial L}{\partial \omega_j} = \sum_{k=1}^{t} \lambda_k \sum_{i=1}^{n}\sum_{s=1}^{n} \times \left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right| + \xi\omega_j = 0\right) \\ \frac{\partial L}{\partial \xi} = \sum_{j=1}^{n} \omega_j^2 - 1 = 0 \end{cases}$$

From Eq. (13), we get a simple and exact formula for determining the attribute weights as follows:

$$\omega_j^* = \frac{\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}\left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right|\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}\left(\left|u_{ij} - u_{sj}\right| + \left|p_{ij} - p_{sj}\right| + \left|v_{ij} - v_{sj}\right|\right)\right)^2}} \tag{14}$$

By normalizing ω_j^* $(j = 1, 2, \ldots, n)$ be a unit, we have

$$\omega_j = \frac{\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\left| u_{ij} - u_{sj} \right| + \left| p_{ij} - p_{sj} \right| + \left| v_{ij} - v_{sj} \right| \right)}{\sum_{j=1}^{n} \sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\left| u_{ij} - u_{sj} \right| + \left| p_{ij} - p_{sj} \right| + \left| v_{ij} - v_{sj} \right| \right)}. \tag{15}$$

3.2 Maximizing deviation method for interval neutrosophic sets

Similar to the previous method, we also construct a non-linear programming model with interval neutrosophic information, as follows:

$$(\text{M-3}) \begin{cases} \max H(\omega) = \sum_{k=1}^{t} \lambda_k \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{s=1}^{m} d\left(a_{ij}^{(k)}, a_{sj}^{(k)} \right) \omega_j \\ \text{subject to } \omega_j \geq 0, \quad \sum_{j=1}^{n} \omega_j = 1, \quad j = 1, 2, \ldots, n \end{cases}$$

where λ_k is the weight of DM d_k, and

$$d\left(a_{ij}^{(k)}, a_{sj}^{(k)} \right) = \frac{1}{6} \left(\left| u_{ij}^- - u_{sj}^- \right| + \left| p_{ij}^- - p_{sj}^- \right| + \left| v_{ij}^- - v_{sj}^- \right| + \left| u_{ij}^+ - u_{sj}^+ \right| + \left| p_{ij}^+ - p_{sj}^+ \right| + \left| v_{ij}^+ - v_{sj}^+ \right| \right).$$

Solving the model (M-3), we get the optimal solution $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{\mathrm{T}}$, which can be used as the weight vector of attributes.

If the attribute weights are completely unknown, we can establish another programming model:

$$(\text{M-4}) \begin{cases} \max H(\omega) = \sum_{k=1}^{t} \lambda_k \frac{1}{6} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{s=1}^{m} \omega_j \left(\Delta_{upv} \right) \\ \text{subject to } \omega_j \geq 0, \quad \sum_{j=1}^{n} \omega_j^2 = 1, j = 1, 2, \ldots, n \end{cases}$$

where λ_k is the weight of DM d_k and $\Delta_{upv} = \left| u_{ij}^- - u_{sj}^- \right| + \left| p_{ij}^- - p_{sj}^- \right| + \left| v_{ij}^- - v_{sj}^- \right| + \left| u_{ij}^+ - u_{sj}^+ \right| + \left| p_{ij}^+ - p_{sj}^+ \right| + \left| v_{ij}^+ - v_{sj}^+ \right|$.

To solve this model, we construct the Lagrange function:

$$L(\omega, \xi) = \sum_{k=1}^{t} \lambda_k \frac{1}{6} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{s=1}^{m} \omega_j \left(\Delta_{upv} \right) + \frac{\xi}{12} \left(\sum_{j=1}^{n} \omega_j^2 - 1 \right), \tag{16}$$

where ξ is the Lagrange multiplier and $\Delta_{upv} = \left| u_{ij}^- - u_{sj}^- \right| + \left| p_{ij}^- - p_{sj}^- \right| + \left| v_{ij}^- - v_{sj}^- \right| + \left| u_{ij}^+ - u_{sj}^+ \right| + \left| p_{ij}^+ - p_{sj}^+ \right| + \left| v_{ij}^+ - v_{sj}^+ \right|$.

Then we compute the partial derivatives of L as follows:

$$\begin{cases} \frac{\partial L}{\partial \omega_j} = \sum_{k=1}^{t} \lambda_k \sum_{i=1}^{n} \sum_{s=1}^{n} \left(\Delta_{upv} \right) + \xi \omega_j = 0 \\ \frac{\partial L}{\partial \xi} = \sum_{j=1}^{n} \omega_j^2 - 1 = 0 \end{cases} \tag{17}$$

where, $\Delta_{upv} = \left| u_{ij}^- - u_{sj}^- \right| + \left| p_{ij}^- - p_{sj}^- \right| + \left| v_{ij}^- - v_{sj}^- \right| + \left| u_{ij}^+ - u_{sj}^+ \right| + \left| p_{ij}^+ - p_{sj}^+ \right| + \left| v_{ij}^+ - v_{sj}^+ \right|$.

From Eq. (17), we get a simple and exact formula for determining the attribute weights as follows:

$$\omega_j^* = \frac{\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\Delta_{upv} \right)}{\sqrt{\sum_{j=1}^{n} \left(\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\Delta_{upv} \right) \right)^2}} \tag{18}$$

By normalizing ω_j^* $(j = 1, 2, \ldots, n)$ be a unit, we have

$$\omega_j = \frac{\sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\Delta_{upv} \right)}{\sum_{j=1}^{n} \sum_{k=1}^{t} \lambda_k \sum_{i=1}^{m} \sum_{s=1}^{m} \left(\Delta_{upv} \right)}, \tag{19}$$

where $\Delta_{upv} = \left| u_{ij}^- - u_{sj}^- \right| + \left| p_{ij}^- - p_{sj}^- \right| + \left| v_{ij}^- - v_{sj}^- \right| + \left| u_{ij}^+ - u_{sj}^+ \right| + \left| p_{ij}^+ - p_{sj}^+ \right| + \left| v_{ij}^+ - v_{sj}^+ \right|$.

Using by MATLAB software with optimization toolbox or Lindo/Lingo software package, the solution of aforementioned maximization problem could be easily solved by a few simple calculations.

With respect to the aforementioned models, we establish a practical and suitable method for solving the NMADM problems. In our methods, the attribute weights are partly known or completely unknown, and the attribute values are the single-valued neutrosophic information or interval neutrosophic information. The methods are described by the following steps:

Method (1): maximizing deviation method for single-valued neutrosophic sets

Step 1 Let $A^{(k)} = \left(a_{ij}^{(k)} \right)_{m \times n}$ be a single-valued neutrosophic decision matrix, where $a_{ij}^{(k)} = \left(u_{ij}^{(k)}, p_{ij}^{(k)}, v_{ij}^{(k)} \right)$ is an attribute value, given by the decision maker d_k, for the alternative A_i with respect to the attribute C_j and $\tilde{r}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$ be the vector of attribute values corresponding to the alternative A_i.

Step 2 If the attribute weights are partly known, then we solve the model (M-1) to obtain the attribute weights. If the information about the attribute weights is completely unknown, then we use the model (M-2).

Step 3 (i) Utilize the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{\mathrm{T}}$ of attributes and by Eq. (10), and obtain the matrix of overall single-valued neutrosophic preference values $\tilde{r}_i$ corresponding to the alternative $A_i(i = 1, 2, \ldots, m)$

(ii) By using the SVNWA operator again and weights of decision makers, compute the collective overall single-valued neutrosophic preference values $\tilde{r}_i$ of alternative $A_i(i = 1, 2, \ldots, m)$

Step 4 Calculate the scores $S(\tilde{r}_i)$ of the collective overall single-valued neutrosophic preference values $\tilde{r}_i(i = 1, 2, \ldots, m)$ to rank all the alternatives $A_i(i = 1, 2, \ldots, m)$ and then to select the best one(s).

If there is no difference between two scores $S(\tilde{r}_i)$ and $S(\tilde{r}_j)$, then we need to calculate the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$ of the collective overall single-valued neutrosophic preference values $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j corresponding to the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$ $(i, j = 1, 2, \ldots, m)$.

Step 5 Rank all the alternatives $A_i(i = 1, 2, \ldots, m)$ and select the best one(s) in accordance with $S(\tilde{r}_i)$ and $H(\tilde{r}_i)$.

Step 6 End.

Method (2): maximizing deviation method for interval neutrosophic sets

Step 1 Let $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m\times n}$ be an interval neutrosophic decision matrix, where $a_{ij}^{(k)} = \left(\left[u_{ij}^{-(k)}, u_{ij}^{+(k)}\right], \left[p_{ij}^{-(k)}, p_{ij}^{+(k)}\right], \left[v_{ij}^{-(k)}, v_{ij}^{+(k)}\right]\right)$ is an attribute value, given by the decision maker d_k, for the alternative A_i with respect to the attribute C_j and $\tilde{z}_i = (\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{in})$ be the vector of attribute values corresponding to the alternative A_i.

Step 2 If the attribute weights are partly known, then we solve the model (M-3) to obtain the attribute weights. If the information about the attribute weights is completely unknown, then we solve the model (M-4).

Step 3 (i) Utilize the weight vector $\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n)^T$ of attributes and by Eq. (11), and obtain the matrix of overall interval neutrosophic preference values $\tilde{z}_i$ corresponding to the alternative $A_i(i = 1, 2, \ldots, m)$

(ii) By using the INWA operator again and weights of decision makers, compute the collective overall interval neutrosophic preference values $\tilde{z}_i$ of alternative $A_i(i = 1, 2, \ldots, m)$.

Step 4 Calculate the scores $S(\tilde{z}_i)$ of the overall interval neutrosophic preference value $\tilde{z}_i(i = 1, 2, \ldots, m)$ to rank all the alternatives $A_i(i = 1, 2, \ldots, m)$ and then to select the best one(s).

If there is no difference between two scores $S(\tilde{z}_i)$ and $S(\tilde{z}_j)$, then we need to calculate the accuracy degrees $H(\tilde{z}_i)$ and $H(\tilde{z}_j)$ of the collective overall interval neutrosophic preference values $\tilde{z}_i$ and $\tilde{z}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H(\tilde{z}_i)$ and $H(\tilde{z}_j)$ $(i, j = 1, 2, \ldots, m)$.

Step 5 Rank all the alternatives $A_i(i = 1, 2, \ldots, m)$ and select the best one(s) in accordance with $S(\tilde{z}_i)$ and $H(\tilde{z}_i)$.

Step 6 End.

4 Numerical examples

Example 5 Let us consider decision-making problem adapted from Xu and Xia [25]. An automotive company is desired to select the most appropriate supplier for one of the key elements in its manufacturing process. After pre-evaluation, five suppliers have remained as alternatives for further evaluation. In order to evaluate alternative suppliers, a committee composed of three decision makers has been formed. The committee selects four attributes to evaluate the alternatives: (1) product quality C_1, (2) relationship closeness C_2, (3) delivery performance C_3 and (4) price C_4. Decision makers (without loss of generality), whose weight vector is $\mathrm{k} = (\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3, \mathrm{k}_4) = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$, use the single-valued neutrosophic values to evaluate the four possible alternatives $A_i(i = 1, 2, 3, 4)$ under the above four attributes and construct the single-valued neutrosophic decision matrices $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m\times n}$ $k = (1, 2, 3, 4)$, as listed in Tables 1, 2, 3 and 4.

Table 1 Decision matrices $A^{(1)}$ given by DM-1

	C_1	C_2	C_3	C_4
A_1	(0.4, 0.2, 0.3)	(0.4, 0.2, 0.3)	(0.2, 0.2, 0.5)	(0.7, 0.2, 0.3)
A_2	(0.6, 0.1, 0.2)	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.5, 0.1, 0.2)
A_3	(0.3, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.1, 0.5, 0.2)	(0.1, 0.4, 0.5)
A_4	(0.7, 0.2, 0.1)	(0.6, 0.1, 0.2)	(0.4, 0.3, 0.2)	(0.4, 0.5, 0.1)

Table 2 Decision matrices $A^{(2)}$ given by DM-2

	C_1	C_2	C_3	C_4
A_1	(0.1, 0.3, 0.5)	(0.5, 0.1, 0.5)	(0.3, 0.1, 0.6)	(0.4, 0.1, 0.4)
A_2	(0.2, 0.5, 0.4)	(0.3, 0.4, 0.3)	(0.2, 0.3, 0.1)	(0.2, 0.3, 0.5)
A_3	(0.5, 0.2, 0.6)	(0.2, 0.4, 0.3)	(0.5, 0.2, 0.5)	(0.1, 0.5, 0.3)
A_4	(0.2, 0.4, 0.2)	(0.1, 0.1, 0.3)	(0.1, 0.5, 0.4)	(0.5, 0.3, 0.1)

Table 3 Decision matrices $A^{(3)}$ given by DM-3

	C_1	C_2	C_3	C_4
A_1	$(0.3, 0.2, 0.1)$	$(0.3, 0.1, 0.3)$	$(0.1, 0.4, 0.5)$	$(0.2, 0.3, 0.5)$
A_2	$(0.6, 0.1, 0.4)$	$(0.6, 0.4, 0.2)$	$(0.5, 0.4, 0.1)$	$(0.5, 0.2, 0.4)$
A_3	$(0.3, 0.3, 0.6)$	$(0.4, 0.2, 0.4)$	$(0.2, 0.3, 0.2)$	$(0.3, 0.5, 0.1)$
A_4	$(0.3, 0.6, 0.1)$	$(0.5, 0.3, 0.2)$	$(0.3, 0.3, 0.6)$	$(0.4, 0.3, 0.2)$

Table 4 Decision matrices $A^{(4)}$ given by DM-4

	C_1	C_2	C_3	C_4
A_1	$(0.2, 0.2, 0.3)$	$(0.3, 0.2, 0.3)$	$(0.2, 0.3, 0.5)$	$(0.4, 0.2, 0.5)$
A_2	$(0.4, 0.1, 0.2)$	$(0.6, 0.3, 0.5)$	$(0.1, 0.2, 0.2)$	$(0.5, 0.1, 0.2)$
A_3	$(0.3, 0.5, 0.1)$	$(0.2, 0.2, 0.3)$	$(0.5, 0.4, 0.3)$	$(0.5, 0.3, 0.2)$
A_4	$(0.3, 0.1, 0.1)$	$(0.2, 0.1, 0.4)$	$(0.2, 0.3, 0.2)$	$(0.3, 0.1, 0.6)$

Then, we use the approach developed to obtain the most desirable alternative(s).

Case 1 Assume that the attribute weights are partly known and the weight information is given as follows:

$$\left(\begin{array}{l} 0.18 \le x_1 \le 0.20, 0.15 \le x_2 \le 0.25, 0.30 \le x_3 \le 0.35, 0.30 \le x_4 \le 0.40, \\ x_j \ge 0, \sum_{j=1}^{4} x_j = 1, \quad j = 1, 2, 3, 4 \end{array} \right)$$

Step 1 Obtain the decision matrix $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m\times n}$ given by the DM d_k and all the components $a_{ij}^{(k)}$ are single-valued neutrosophic values (See Tables 1, 2, 3 and 4).

Step 2 Utilize the model (M-1) to establish the following nonlinear programming model:

$$\begin{cases} \max H(x) = 1.06x_1 + 0.83x_2 + 1.63x_3 + 1.23x_4 \\ \text{subject to } x_j \ge 0, \sum_{j=1}^{n} x_j = 1, \quad j = 1, 2, \ldots, n \end{cases}$$

Solving this model, we obtain the weight vector of attributes $x = (0.18, 0.15, 0.35, 0.32)$.

Step 3 By the weight vector $x = (0.18, 0.15, 0.35, 0.32)$ and by Eq. (10), we obtain the overall single-valued neutrosophic preference values $\tilde{r}_i$ of the alternatives $A_i (i = 1, 2, 3, 4)$, as shown in Table 5.

By using the SVNWA operator again (here, take $k = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ as the DM's weight vector), we get the collective overall single-valued neutrosophic preference values $\tilde{r}_i$ of alternatives A_i,
$\tilde{r}_1 = (0.3630, 0.1957, 0.4018)$, $\tilde{r}_2 = (0.4511, 0.2013, 0.2335)$, $\tilde{r}_3 = (0.2931, 0.3285, 0.2763)$, $\tilde{r}_4 = (0.3868, 0.2583, 0.2169)$.

Step 4 Calculate the scores $S(\tilde{r}_i)$ the collective overall single-valued neutrosophic preference values $\tilde{r}_i (i = 1, 2, 3, 4)$.
$S(\tilde{r}_1) = 0.2848$, $S(\tilde{r}_2) = 0.4074$, $S(\tilde{r}_3) = 0.1798$, $S(\tilde{r}_4) = 0.3265$.

Step 5 Rank all the alternatives $A_i (i = 1, 2, 3, 4)$ in accordance with the scores $S(\tilde{r}_i)(i = 1, 2, 3, 4)$ of the collective overall single-valued neutrosophic preference values $\tilde{r}_i (i = 1, 2, 3, 4)$: $A_2 \succ A_4 \succ A_1 \succ A_3$ and thus A_2 is the most desirable alternative

Case 2 If the attribute weights are completely unknown, we propose another approach to determine the most desirable alternative(s).

Step 1 See (Step 1).

Step 2 Utilize the Eq. (15) to obtain the weight vector of attributes: $x = (0.2238, 0.1748, 0.3427, 0.2587)$.

Step 3 Utilize the weight vector $x = (0.2238, 0.1748, 0.3427, 0.2587)$ and by Eq. (10), we obtain the overall single-valued neutrosophic preference values $\tilde{r}_i$ of the alternatives $A_i (i = 1, 2, 3, 4)$, as shown in Table 6.

By using the SVNWA operator again (here, take $k = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ as the DM's weight vector), we get the collective overall single-valued neutrosophic preference values $\tilde{r}_i$ of alternatives A_i,
$\tilde{r}_1 = (0.3614, 0.1956, 0.3909)$, $\tilde{r}_2 = (0.4541, 0.2027, 0.2336)$,
$\tilde{r}_3 = (0.2961, 0.3188, 0.2821)$, $\tilde{r}_4 = (0.3902, 0.2537, 0.2137)$.

Step 4 Calculate the scores $S(\tilde{r}_i)$ of the collective overall single-valued neutrosophic preference values $\tilde{r}_i (i = 1, 2, 3, 4)$.
$S(\tilde{r}_1) = 0.2895$, $S(\tilde{r}_2) = 0.4075$, $S(\tilde{r}_3) = 0.1881$, $S(\tilde{r}_4) = 0.3345$

Step 5 Rank all the alternatives $A_i (i = 1, 2, 3, 4)$ in accordance with the scores $S(\tilde{r}_i)(i = 1, 2, 3, 4)$ the collective overall single-valued neutrosophic preference values $\tilde{r}_i (i = 1, 2, 3, 4)$: $A_2 \succ A_4 \succ A_1 \succ A_3$, and thus, the most desirable alternative is A_2

Table 5 Matrix of the overall preference values with respect to party known attributes weights

	d_1	d_2	d_3	d_4
$\tilde{r}_1$	$(0.4684, 0.2000, 0.3587)$	$(0.3371, 0.1218, 0.4962)$	$(0.2023, 0.2616, 0.3466)$	$(0.4142, 0.2305, 0.4224)$
$\tilde{r}_2$	$(0.5355, 0.1274, 0.2305)$	$(0.2158, 0.3434, 0.2532)$	$(0.5355, 0.2497, 0.2219)$	$(0.4634, 0.1502, 0.2294)$
$\tilde{r}_3$	$(0.2124, 0.3440, 0.3065)$	$(0.3524, 0.2975, 0.4064)$	$(0.2833, 0.3324, 0.2166)$	$(0.3171, 0.3422, 0.2162)$
$\tilde{r}_4$	$(0.5016, 0.2785, 0.1414)$	$(0.2699, 0.3204, 0.2170)$	$(0.3665, 0.3399, 0.2593)$	$(0.3866, 0.1468, 0.2784)$

Table 6 Matrix of the overall preference values with respect to completely unknown attributes weights

	d_1	d_2	d_3	d_4
$\tilde{r}_1$	$(0.4465, 0.2000, 0.3573)$	$(0.3291, 0.1278, 0.5023)$	$(0.2201, 0.2495, 0.3189)$	$(0.4329, 0.2298, 0.4078)$
$\tilde{r}_2$	$(0.5425, 0.1268, 0.2298)$	$(0.2184, 0.3536, 0.506)$	$(0.5425, 0.2451, 0.2203)$	$(0.4571, 0.1536, 0.2347)$
$\tilde{r}_3$	$(0.2323, 0.3275, 0.2979)$	$(0.3680, 0.2861, 0.4173)$	$(0.2867, 0.3189, 0.2412)$	$(0.2907, 0.3457, 0.2112)$
$\tilde{r}_4$	$(0.5213, 0.2580, 0.1431)$	$(0.2470, 0.3145, 0.2275)$	$(0.3658, 0.3503, 0.2495)$	$(0.3951, 0.1457, 0.2568)$

Table 7 Decision matrices $A^{(1)}$ given by DM-1

	C_1	C_2	C_3	C_4
A_1	$([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])$	$([0.3, 0.4], [0.3, 0.6], [0.2, 0.4])$	$([0.2, 0.5], [0.2, 0.6], [0.3, 0.5])$	$([0.5, 0.6], [0.3, 0.5], [0.2, 0.5])$
A_2	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.1, 0.3], [0.1, 0.4], [0.2, 0.5])$	$([0.4, 0.5], [0.2, 0.5], [0.3, 0.7])$	$([0.2, 0.4], [0.1, 0.4], [0.3, 0.3])$
A_3	$([0.3, 0.4], [0.2, 0.3], [0.3, 0.4])$	$([0.3, 0.6], [0.2, 0.3], [0.2, 0.5])$	$([0.2, 0.7], [0.2, 0.4], [0.3, 0.6])$	$([0.2, 0.6], [0.4, 0.7], [0.2, 0.7])$
A_4	$([0.2, 0.6], [0.1, 0.1], [0.1, 0.2])$	$([0.2, 0.5], [0.4, 0.5], [0.1, 0.6])$	$([0.3, 0.5], [0.1, 0.3], [0.2, 0.2])$	$([0.4, 0.4], [0.1, 0.6], [0.1, 0.5])$

Example 6 Let us consider decision-making problem adapted from Wei et al. [24]. Suppose an organization plans to implement ERP system. The first step is to form a project team that consists of CIO and two senior representatives from user departments. By collecting all possible information about ERP vendors and systems, project team chooses four potential ERP systems A_i $(i = 1, 2, 3, 4)$ as candidates. The company employs some external professional organizations (or experts) to aid this decision making. The project team selects four attributes to evaluate the alternatives: (1) function and technology C_1, (2) strategic fitness C_2, (3) vendor's ability C_3 and (4) vendor's reputation C_4. Decision makers (without loss of generality), take weight vector $k = (k_1, k_2, k_3) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and use the interval neutrosophic values to evaluate the four possible alternatives $A_i (i = 1, 2, 3, 4)$ under the above four attributes and construct the interval neutrosophic decision matrices $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n}$ $k = (1, 2, 3)$, as listed in Tables 7, 8 and 9.

Then, we use the approach developed to obtain the most desirable alternative(s).

Case 1 Assume that the attribute weights are partly known and the weight information is given as follows:

$$\begin{cases} 0.16 \le x_1 \le 0.18, 0.20 \le x_2 \le 0.25, 0.20 \le x_3 \le 0.30, 0.35 \le x_4 \le 0.40 \\ x_j \ge 0, \quad \sum_{j=1}^{4} x_j = 1, \quad j = 1, 2, 3, 4 \end{cases}$$

Step 1 Obtain the decision matrix $A^{(k)} = \left(a_{ij}^{(k)}\right)_{m \times n}$ given by the DM d_k and all the components $a_{ij}^{(k)}$ are interval neutrosophic values (See Tables 7, 8 and 9).

Step 2 Utilize the model (M-3) to establish the following nonlinear programming model:

$$\begin{cases} \max H(x) = 0.71x_1 + 0.69x_2 + 0.81x_3 + 1.01x_4 \\ \text{subject to } x_j \ge 0, \quad \sum_{j=1}^{n} x_j = 1, \quad j = 1, 2, \ldots, n \end{cases}$$

Solving this model, we obtain the weight vector of attributes $x = (0.16, 0.20, 0.24, 0.40)$.

Step 3 By the weight vector $x = (0.16, 0.20, 0.24, 0.40)$ and by Eq. (11), we obtain the overall interval neutrosophic preference values $\tilde{z}_i$ of the alternatives $A_i (i = 1, 2, 3, 4)$, as shown in Table 10.

By using the INWA operator again (here, take

Table 8 Decision matrices $A^{(2)}$ given by DM-2

	C_1	C_2	C_3	C_4
A_1	$([0.4,0.6],[0.1,0.3],[0.2,0.4])$	$([0.3,0.5],[0.1,0.4],[0.3,0.4])$	$([0.4,0.5],[0.2,0.4],[0.1,0.3])$	$([0.3,0.6],[0.3,0.6],[0.3,0.6])$
A_2	$([0.3,0.5],[0.1,0.2],[0.2,0.3])$	$([0.3,0.4],[0.2,0.2],[0.1,0.3])$	$([0.2,0.7],[0.3,0.5],[0.3,0.6])$	$([0.2,0.5],[0.2,0.7],[0.1,0.2])$
A_3	$([0.5,0.6],[0.2,0.3],[0.3,0.4])$	$([0.1,0.4],[0.1,0.3],[0.3,0.5])$	$([0.5,0.5],[0.4,0.6],[0.3,0.4])$	$([0.1,0.2],[0.1,0.4],[0.5,0.6])$
A_4	$([0.3,0.4],[0.1,0.2],[0.1,0.3])$	$([0.3,0.3],[0.1,0.5],[0.2,0.4])$	$([0.2,0.3],[0.4,0.5],[0.5,0.6])$	$([0.3,0.3],[0.2,0.3],[0.1,0.4])$

Table 9 Decision matrices $A^{(3)}$ given by DM-3

	C_1	C_2	C_3	C_4
A_1	$([0.1,0.3],[0.2,0.3],[0.4,0.5])$	$([0.3,0.3],[0.1,0.3],[0.3,0.4])$	$([0.2,0.6],[0.3,0.5],[0.3,0.5])$	$([0.4,0.6],[0.3,0.4],[0.2,0.3])$
A_2	$([0.3,0.6],[0.3,0.5],[0.3,0.5])$	$([0.3,0.4],[0.3,0.4],[0.3,0.5])$	$([0.3,0.5],[0.2,0.4],[0.1,0.5])$	$([0.1,0.2],[0.3,0.5],[0.3,0.4])$
A_3	$([0.4,0.5],[0.2,0.4],[0.2,0.4])$	$([0.2,0.3],[0.1,0.1],[0.3,0.4])$	$([0.1,0.4],[0.2,0.6],[0.3,0.6])$	$([0.4,0.5],[0.2,0.6],[0.1,0.3])$
A_4	$([0.2,0.4],[0.3,0.4],[0.1,0.3])$	$([0.1,0.4],[0.2,0.5],[0.1,0.5])$	$([0.3,0.6],[0.2,0.4],[0.2,0.2])$	$([0.2,0.4],[0.3,0.3],[0.2,0.6])$

Table 10 Matrix of the overall preference values with respect to party known attributes weights

	d_1	d_2	d_3
$\tilde{z}_1$	$([0.383,0.525],[0.255,0.499],[0.235,0.487])$	$([0.341,0.558],[0.183,0.449],[0.215,0.439])$	$([0.292,0.510],[0.225,0.380],[0.267,0.389])$
$\tilde{z}_2$	$([0.315,0.469],[0.118,0.377],[0.259,0.407])$	$([0.237,0.541],[0.197,0.411],[0.145,0.301])$	$([0.225,0.296],[0.272,0.478],[0.230,0.457])$
$\tilde{z}_3$	$([0.237,0.601],[0.263,0.451],[0.235,0.576])$	$([0.288,0.396],[0.155,0.397],[0.368,0.491])$	$([0.299,0.441],[0.174,0.392],[0.181,0.392])$
$\tilde{z}_4$	$([0.309,0.481],[0.131,0.367],[0.118,0.359])$	$([0.277,0.317],[0.184,0.352],[0.169,0.421])$	$([0.209,0.455],[0.250,0.372],[0.155,0.397])$

k $= (\frac{1}{3},\frac{1}{3},\frac{1}{3})$ as the DM's weight vector), we get the collective overall interval neutrosophic preference values $\tilde{z}_i$ of alternatives A_i,

$\tilde{z}_1 = ([0.3401, 0.5318], [0.2196, 0.4406], [0.2388, 0.4345])$,
$\tilde{z}_2 = ([0.2606, 0.4720], [0.1854, 0.4207], [0.2059, 0.3831])$,
$\tilde{z}_3 = ([0.2754, 0.4874], [0.1930, 0.4134], [0.2506, 0.4816])$,
$\tilde{z}_4 = ([0.2654, 0.4218], [0.1829, 0.4469], [0.1462, 0.3922])$.

Step 4 Compute the scores $S(\tilde{z}_i)$ of the collective overall interval neutrosophic preference values $\tilde{z}_i (i = 1, 2, 3, 4)$.
$S(\tilde{z}_1) = 0.2194$, $S(\tilde{z}_2) = 0.2328$, $S(\tilde{z}_3) = 0.2044$, $S(\tilde{z}_4) = 0.2222$.

Step 5 Rank all the alternatives $A_i (i = 1, 2, 3, 4)$ in accordance with the scores $S(\tilde{z}_i)(i = 1, 2, 3, 4)$ of the collective overall interval valued preference values $\tilde{z}_i (i = 1, 2, 3, 4)$: $A_2 \succ A_4 \succ A_1 \succ A_3$, and thus, A_2 is the most desirable alternative

Case 2 If the attribute weights are completely unknown, we proposed another approach to determine the most desirable alternative(s).

Step 1 See Step (1).

Step 2 Utilize the Eq. (19) to obtain the weight vector of attributes: x $= 0.2224, 0.2155, 0.2518, 0.3103)$.

Step 3 Utilize the weight vector x $= 0.2224, 0.2155, 0.2518, 0.3103)$ and by Eq. (11), we obtain the overall interval neutrosophic preference values $\tilde{z}_i$ of the alternatives $A_i (i = 1, 2, 3, 4)$ (see Table 11).
By using the INWA operator again (here, take k $= (\frac{1}{3},\frac{1}{3},\frac{1}{3})$ as the DM's weight vector), we get the collective overall interval neutrosophic preference values $\tilde{z}_i$ of alternatives A_i,

$\tilde{z}_1 = ([0.3308, 0.5203], [0.2080, 0.4260], [0.2425, 0.4344])$,
$\tilde{z}_2 = ([0.2795, 0.4892], [0.1832, 0.4008], [0.2067, 0.3938])$,
$\tilde{z}_3 = ([0.2861, 0.4915], [0.1922, 0.3941], [0.2555, 0.4745])$,
$\tilde{z}_4 = ([0.2590, 0.4303], [0.1808, 0.4389], [0.1454, 0.3747])$.

Step 4 Compute the scores $S(\tilde{z}_i)$ of the overall interval neutrosophic preference values $\tilde{z}_i (i = 1, 2, 3, 4)$.
$S(\tilde{z}_1) = 0.2265, S(\tilde{z}_2) = 0.2503, S(\tilde{z}_3) = 0.2187, S(\tilde{z}_4) = 0.2324$.

Step 5 Rank all the alternatives $A_i (i = 1, 2, 3, 4)$ in accordance with the scores $S(\tilde{z}_i)(i = 1, 2, 3, 4)$

Table 11 Matrices of the overall preference values with respect to completely unknown attributes weights

	d_1	d_2	d_3
$\tilde{z}_1$	$([0.369, 0.514], [0.247, 0.485], [0.242, 0.476])$	$([0.349, 0.556], [0.167, 0.425], [0.207, 0.421])$	$([0.270, 0.488], [0.216, 0.373], [0.282, 0.406])$
$\tilde{z}_2$	$([0.345, 0.492], [0.119, 0.362], [0.251, 0.414])$	$([0.245, 0.542], [0.189, 0.371], [0.153, 0.314])$	$([0.243, 0.427], [0.270, 0.476], [0.227, 0.466])$
$\tilde{z}_3$	$([0.245, 0.592], [0.248, 0.419], [0.242, 0.552])$	$([0.318, 0.427], [0.165, 0.390], [0.351, 0.475])$	$([0.293, 0.437], [0.172, 0.372], [0.194, 0.405])$
$\tilde{z}_4$	$([0.292, 0.496], [0.134, 0.325], [0.119, 0.336])$	$([0.276, 0.323], [0.175, 0.348], [0.174, 0.415])$	$([0.206, 0.458], [0.248, 0.383], [0.147, 0.375])$

of the overall interval neutrosophic preference values $\tilde{z}_i(i = 1, 2, 3, 4)$: $A_2 \succ A_4 \succ A_1 \succ A_3$, and thus, the most desirable alternative is A_2.

From the examples, we can see that the proposed neutrosophic decision-making methods are more suitable for real scientific and engineering applications because they can handle not only incomplete information but also the indeterminate information and inconsistent information existing in real situations. Therefore, the technique proposed in this paper extends the existing decision-making methods and provides a new way for decision makers.

By a comparative study with existing methods, we can represent the useable and feasibility of the developed group decision-making method. Here, we discuss some methods used to determine the final ranking order of all the alternatives with the single-valued neutrosophic information, which are based on the cosine similarity measure and the correlation coefficient [26], the weighted cross-entropy [28], the aggregation operators [31] and the outranking approach [11, 34]. In these methods, the weights of decision makers and attribute weights are completely known and the decision process is carried out in the opinion of only a decision maker. In fact, in many MAGDM with neutrosophic information, because of time pressure, lack of knowledge or data and the decision makers' limited expertise about the problem domain, the information about the weights of decision makers and attributes are incompletely known or completely unknown. Our method has a group decision-making approach and utilizes the maximizing deviation method to determine the weight values that are incompletely known or completely unknown of decision makers and attributes, respectively, which is more flexible and reasonable, while the Ye [26, 28, 31] 's method, Peng et al. [11] and Zhang et al.'s [34] methods ask the decision makers to provide the weight values of decision makers and attributes in advance, which is subjective and sometime cannot yield the persuasive results.

With respect to above analyses, a single-valued neutrosophic set and an interval neutrosophic set is a special case of a neutrosophic set, and a neutrosophic set is a set where each element of the universe has the degrees of truth, indeterminacy and falsity, which lie within $]0^-, 1^+[$, the non-standard unit interval. In particular, the uncertainty presented here, i.e., the indeterminacy factor, is independent of truth and falsity values, whereas the incorporated uncertainty is dependent on the degree of belongingness and non-belongingness of intuitionistic fuzzy sets. Therefore, this leads to the theory that intuitionistic fuzzy sets are a special case of single-valued neutrosophic sets. Moreover, SNSs can solve some problems that are beyond the scope of fuzzy sets and intuitionistic fuzzy sets. Therefore, the proposed MAGDM approach under single-valued neutrosophic environment can be used also to solve MADM problems with fuzzy information and intuitionistic information. Thus, the comparison shows that our method has its great superiority in handling the ambiguity and uncertainty inherent in MAGDM problems with neutrosophic information.

5 Conclusions

DMs have a major role to provide the information about alternatives in decision-making process. Because of time pressure, lack of knowledge or data and the expert's limited expertise about the problem domain, the information about attribute weights given by DMs is partly known or completely unknown. Recently, some authors proposed many of methods to overcome the limitations. In this paper, we first defined two discrimination functions such that score function and accuracy function used to rank the neutrosophic numbers. Considering by the idea that the attribute with a larger deviation value among alternatives should be assigned with a larger weight, we then established a method called the maximizing deviation method to compute the optimal weights of attributes under neutrosophic environment, in which the attribute values are characterized in terms of neutrosophic values. When aggregating the neutrosophic information corresponding to each alternative, we utilize the neutrosophic weighted averaging (NWA) operators, the single-valued neutrosophic weighted averaging (SVNWA) operator and the interval neutrosophic weighted averaging (INWA) operator. Thus, one can easily determines the order of alternatives and can chooses the most desirable one(s) based on the proposed score function and accuracy function. Finally, an application of developed approach is given to explain its effectiveness and practicality. Our method is straightforward and has no loss of information. In the future, we shall continue working in application of the neutrosophic multiple attribute decision making.

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