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Multi-objective optimization problem of system reliability under intuitionistic fuzzy set environment using Cuckoo Search algorithm

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Abstract. In designing phase of systems, design parameters such as component reliabilities and cost are normally under uncertainties. Although there have been tremendous advances in the art and science of system evaluation, yet it is very difficult to assess these parameters with a very high accuracy or precision. Therefore, to handle this issue, this paper presents an alternative approach for solving the multi-objective reliability optimization problem by utilizing the uncertain, vague and imprecise data. For this a conflicting nature between the objectives is resolved with the help of intuitionistic fuzzy programming technique by considering the nonlinear degree of membership and non-membership functions. The resultant fuzzy multi-objective optimization problem is converted into single-objective optimization problem using the satisfaction functions with exponential weights. The optimal solution of the corresponding problem has been obtained with the cuckoo search algorithm. Finally, a numerical instance is presented to show the performance of the proposed approach.

Keywords: Intuitionistic fuzzy optimization, cuckoo search, reliability optimization, membership functions

1. Introduction

Decision making involves the use of a rational process for selecting the best of several alternatives. In real life, decisions are often made on the basis of multiple, conflicting and non-commensurable criteria/objectives in uncertain/imprecise environments. Bellman and Zadeh [6] first introduced the fuzzy set theory in decision-making processes. Later, Zimmermann [29] showed that the classical algorithms could be used to solve a fuzzy linear programming problem. After their work, a great number of articles dealing with the fuzzy optimization problems have come out. In most of the existing models, it is assumed that the parameters, objective goals and constraints goals are deterministic and fixed. However, in the real world problems, the available (historical) data are often inaccurate, imprecise, vague and collected under different operating and environmental conditions. Thus, in this environment, built-in uncertainties in the data are inadequate to handle the problem in probabilistic approach. For this reason, the concept of fuzzy reliability has been introduced and formulated either in the context of the possibility measures or as a transition from a fuzzy success state of fuzzy failure state. Other important contributions to fuzzy programming with fuzzy parameters have been made by the researchers [1, 2, 15–17, 19–21, 24].

All of the above researchers have used the fuzzy optimization technique for solving the resulting reliability optimization problem. In fuzzy optimization, we search for the best possible solution which can be achieved in the presence of incomplete or imprecise or vagueness in information. But in fuzzy optimization only

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the degree of acceptance of objectives and constraints is considered. Nowadays, researchers are engaged on its modifications and generalized forms and out of that intuitionistic fuzzy set (IFS) theory, introduced by Attanassov in 1986 [4, 5], is the most successful extension during the last decades. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in the case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. They consider not only the degree of membership, as in fuzzy set theory to a given set, but also the degree of rejection such that the sum of both values is less than one so that there is a degree of interminancy between the membership functions. Applying this concept, it is possible to reformulate the optimization problem by using degree of rejection of the constraints and the value of the objective which are non-admissible. Angelov [3] implemented the optimization in an intuitionistic fuzzy environment. Pramanik and Roy [22] solved a vector optimization problem using an intuitionistic fuzzy goal programming. Garg [12] proposed a technique for analyzing the behavior of industrial systems in terms of various reliability parameters using vague set theory. Chakrabortty et al. [7] proposed a method to solve the multi-objective EPQ inventory model with fuzzy inventory costs and fuzzy demand rate and the problem is solved with IFO technique. Garg and Rani [13] presented an efficient technique for computing the membership functions of various reliability parameters using PSO and IFS theory. Apart from that a lot of work has been done to develop and enrich the IFS theory given in [8, 9, 14] and their corresponding references in terms of reliability evaluation of series-parallel system.

In general, reliability optimization problem is solved with the assumption that the coefficients or cost of components is specified in a precise way. As in the early stages, due to non-availability of the distribution function of the product design, the reliability of a component is taken as a precise number between zero and one and hence it is difficult to determine the reliability specifically. But, in today, most of the real-world decision-making problems in economic, technical and environmental ones are multidimensional and multi-objective. So, it is significant to realize that multiple objectives are often non-commensurable and conflict with each other in the optimization problem. In the single objective optimization, one attempts to obtain the global solution/decision, but in multi-objective optimization, there exists a set of solutions which are superior to the rest of the solution in the search space when all the objectives are considered, but are inferior to other solutions in the space in one or more objectives (not all). For handling such types of situations, one usually tries to search for a solution which is as close to the decision makers (DMs) expectations as possible. For this, the problem is solved interactive manner in which DM is initially asked to specify his or her preferences towards the objectives. Based on these preferences, the problem is solved and the DM is provided with a possible solution. If the DM is satisfied with this solution the problem ends there, otherwise he or she is asked to modify his or her preferences in the light of the earlier obtained results. This iterative procedure is continued till a satisfactory solution is achieved, which is closed to DM's expectations. So a multi-objective model with fuzzy objectives is more realistic than the one with deterministic [14]. However, it seems that so far there has been little research on multi-objective optimization using IFS, which is indeed one of the most important areas in decision analysis as most real world decision problems involve multi-objective optimization problem. Also, there is very rare research carried out on IFO in reliability optimization model. Therefore, the most appropriate procedure is to cautiously find a set of solutions that satisfy the decision makers' expectations to the highest possible degree. Clearly, this makes for an interactive fuzzy multi-objective optimization approach which incorporates the preferences and expectations of the decision maker, allowing for human (expert) judgment. Iteratively, it becomes possible to obtain the most satisfactory solution in a fuzzy environment. In view of the above issues, the purpose of this paper is to address the problem of system reliability optimization in an intuitionistic fuzzy environment characterized with multiple conflicting objectives. Therefore, our specific objectives are as follows:

- To develop a fuzzy multiple-objective nonlinear programming model for the reliability optimization problem;
- To use an aggregation method to transform the fuzzy model to a single-objective optimization problem; and,
- To use a global meta-heuristic optimization method to obtain a set of acceptable solutions.

Thus motivated by this idea and to handle the inaccurate parameter specified to the reliability of each component of the system, an efficient technique is proposed in this study. The major extension of the present work as compared to existing work is to express the impreciseness of the objective goals by intuitionistic fuzzy non-linear membership functions in terms of their degree of acceptance and rejection functions. For this, an exponential membership and quadratic non-membership functions has been used here. A variable weight method, instead of constant weight, has been used for converting the multi-objective optimization problem into its equivalent single-objective optimization problem and the resultant Pareto optimal solution is obtained, for different values of weights corresponding to different objective functions, by using Cuckoo search algorithms and the results are compared with the PSO. The proposed approach is explained through examples of reliability-cost benchmark optimization problems.

The rest of the manuscript is described as follows: Section 2 describe the basic concept of the intuitionistic fuzzy set theory (IFS). A formulation of the multi-objective reliability optimization problem has been discussed in Section 3. The intuitionistic fuzzy optimization (IFO) based methodology for solving the multi-objective optimization problem has been presented in Section 4. An illustrative examples have been taken in Section 6 and their corresponding results are presented in Section 7. Finally, some concrete conclusions have been presented in Section 8.

2. Basic concepts of Intuitionistic fuzzy set theory

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. Then the IFS $\tilde{A}$ in X [4, 5] is the set of ordered triplets $< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >$, i.e.

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) > \mid x \in X\} \tag{1}$$

where $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are functions from X into [0, 1], i.e. $\mu_{\tilde{A}}, \nu_{\tilde{A}} : X \to [0, 1]$. For each $x_i \in X$, $\mu_{\tilde{A}}(x_i)$ and $\nu_{\tilde{A}}(x_i)$ represent respectively the degrees of membership and non-membership functions of the element x_i to the subset $\tilde{A}$ of X such that $\mu_{\tilde{A}}(x_i) + \nu_{\tilde{A}}(x_i) \leq 1$. The function $\pi_{\tilde{A}}(x_i) = 1 - \mu_{\tilde{A}}(x_i) - \nu_{\tilde{A}}(x_i)$ is called the degree of hesitation or uncertainty level of the element x_i in the set $\tilde{A}$. Especially, if $\pi_{\tilde{A}}(x) = 0$ for all $x \in X$, then the IFS is reduced to a fuzzy set.

An IFS $\tilde{A}$ is said to be normalized [4, 5] if there exist at least two points $x_1, x_2 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_1) = 1$ and $\nu_{\tilde{A}}(x_2) = 1$ otherwise it is said to subnormal IFS. An IFS in universe X is said to convex if and only if membership functions of $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ of $\tilde{A}$ are fuzzy - convex and fuzzy - concave respectively i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \leq \lambda \leq 1$$

and

$$\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \leq \lambda \leq 1$$

An Intuitionistic fuzzy number (IFN) of the set $\tilde{A}$ is a normal, convex membership function on the real line $\mathbb{R}$ with bounded support i.e. $\{x \in X \mid \nu_{\tilde{A}}(x) < 1\}$ is bounded and $\mu_{\tilde{A}}$ is upper semi-continuous and $\nu_{\tilde{A}}$ is lower semi-continuous. Let $\tilde{A}$ be IFS denoted by $\tilde{A} =< [(a, b, c); \mu, \nu] >$, where $a, b, c \in \mathbb{R}$ then the set $\tilde{A}$ is said to be intuitionistic fuzzy number if its membership function and non-membership functions are defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} f_A(x) & ; \quad a \leq x \leq b \\ 1 & ; \quad x = b \\ g_A(x) & ; \quad b \leq x \leq c \\ 0 & ; \quad \text{otherwise} \end{cases} \quad \text{and}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} F_A(x) & ; \quad a \leq x \leq b \\ 0 & ; \quad x = b \\ G_A(x) & ; \quad b \leq x \leq c \\ 1 & ; \quad \text{otherwise} \end{cases}$$

where the functions $f_A, g_A, F_A, G_A : \mathbb{R} \to [0, 1]$ are called the sides of fuzzy number. The function f_A, G_A are nondecreasing continuous functions and the function g_A, F_A are nonincreasing continuous functions.

3. Multi-objective reliability optimization problem

Reliability is one of the vital attributes of performance in arriving at the optimal design of a system because it directly and significantly influences the system's performance. In practice, the problem of system reliability may be formed as a typical nonlinear programming problem with nonlinear cost functions. In reliability optimization problem, it is often required to minimize the system cost together with maximizing the system reliability. Therefore, multi-objective functions become an important aspect in the reliable design of the engineering systems. Hence the suitable form of the multi-objective reliability optimization problem by considering systems reliability and cost as an objective is

$$\text{Maximize}: R_s(r_1, r_2, \cdots, r_m) = \begin{cases} \prod_{i=1}^{m} r_i \text{ for series system} \\ 1 - \prod_{i=1}^{m}(1 - r_i) \text{ for parallel system} \\ \text{or combination of series and parallel system} \end{cases} \tag{2}$$

$$\text{Minimize} \; : C_s(r_1, r_2, \cdots, r_m) = \sum_{i=1}^{m} C_i(r_i)$$

$$\text{subject to} \; r_{i,\min} \leq r_i \leq r_{i,\max},$$

$$R_{s,\min} \leq R_s \leq 1 \;\; \text{for } i = 1, 2, \ldots, m$$

There are many factors which are involved during the decision related to manufacturing system to the reliability optimization problem. In most cases, the objective and constraints of these problems are not precisely known and hence in order to handle these conditions, fuzzy logic optimization method has been introduced. After successful application of their fuzzy set theory, intuitionistic fuzzy set theory is one of the successful extension of fuzzy set theory by considering degree of hesitation between the membership functions. Therefore, the reliability allocation model (2) can be represented by fuzzy nonlinear programming to make the model more flexible and adaptable to the human decision process. Thus, in a fuzzy environment, the corresponding optimization problem becomes

$$\text{Minimize} \quad \tilde{f}(r) = \{\tilde{Q}_s(r), \tilde{C}_s(r)\} \tag{3}$$

$$\text{subject to}: g(r) \leq 0$$

$$r_{i,\min} \leq r_i \leq r_{i,\max} \quad ; \quad i = 1, 2, \ldots, m$$

$$R_{s,\min} \leq R_s \leq 1 \quad ; \quad r_i \in [0, 1] \subset \mathbb{R}$$

$$\text{where} \quad Q_s = 1 - R_s$$

where $\tilde{f}$ represents that the function f in IFS environment with membership function μ_f and non-membership function ν_f. Corresponding to each objective function, the degree of satisfaction is given by $\eta_{f_j} = \mu_{f_j} - \nu_{f_j}$ and hence the dissatisfaction of each objective is defined as $\xi_{f_j} = 1 - \eta_{f_j}$. After obtaining the satisfaction functions of each objective, the overall satisfaction function of the objective $\eta(f)$ is expressed as a function of $b-$ sub-objectives as follows:

$$\eta(f) = \prod_{j=1}^{b} \omega_j (\eta_{f_j})^{\gamma_j}$$

where $\omega_j = \frac{\xi_{f_j}}{\sum_{i=1}^{b} \xi_{f_j}}$ is to represent the relative importance of the j^{th} sub-objective with the requirement of $\omega_j \geq 0$ and $\sum_{j=1}^{b} \omega_j = 1$ while γ_j represents the exponential weight which control the pace at which the j^{th} sub-objective changes between being satisfactory and non-satisfactory.

4. Intuitionistic fuzzy programming technique for solution

In order to solve the above formulated problem, an intuitionistic fuzzy optimization has been used in this paper by formulating membership and non-membership functions corresponding to each objective function. Let μ_{f_t} and ν_{f_t} be the degree of the membership and non-membership function, to be decided on the basis of interaction with DM, corresponding to the objective function f_t, $t = 1, 2, \ldots, b$. Computations of these membership functions are based on their ideal and anti-ideal values which are formulated by solving each of the objectives separately.

Step 1: **Calculation of ideal and anti ideal values of objective functions:** Solve the multi-objective optimization problem as a single objective cost function using one objective at a time and ignoring all the others. The solution of the problem so obtained is the ideal solution x_t^* for each objective function, f_t, and the corresponding objective function at the ideal solution may be given by

$$f_t^* = f_t(x_t^*), \quad t = 1, 2, \ldots, b$$

Let the minimum and maximum of feasible values of each objective function f_t at different ideal values as obtained by considering one objective at a time and ignoring the others, be m_t and M_t respectively i.e.

$$m_t = \min_{1 \leq q \leq b} f_t(x_q^*) \quad \text{and}$$

$$M_t = \max_{1 \leq q \leq b} f_t(x_q^*).$$

Step 2: **Formulation of the membership functions (μ_{f_t}) and non-membership function (ν_{f_t}) for each objective function:**
A linear membership function does not provide biasness towards objectives, however if the DM has some preference (i.e. biasness) towards one or more objectives, a nonlinear membership function may fulfill the purpose. For this, we make use of the exponential functions for the membership functions while quadratic functions for the non-membership functions and is defined as follows.

$$\mu_{f_j}(x) = \begin{cases} 1, & f_j(x) \le m_j \\ \dfrac{e^{-\omega\left(\frac{f_j(x)-m_j}{M_j-m_j}\right)} - e^{-\omega}}{1 - e^{-\omega}}, & m_j \le f_j(x) \le M_j \\ 0, & f_j(x) \ge M_j \end{cases} \tag{4}$$

and

$$\nu_{f_j}(x) = \begin{cases} 0, & f_j(x) \le m_j \\ \left(\dfrac{f_j(x) - m_j}{M_j - m_j}\right)^2, & m_j \le f_j(x) \le M_j \\ 1, & f_j(x) \ge M_j \end{cases} \tag{5}$$

Step 3: **Equivalent crisp(non-fuzzy) optimization problem:** After determining the membership and non-membership functions defined in (4)-(5) for each of the objective functions, the original fuzzy problem can be formulated as an equivalent crisp (non-fuzzy) model in the form:

$$\begin{aligned} &\text{Maximize } \eta(F) = \sum_{j=1}^{b} \omega_j (\eta_{f_j})^{\gamma_j} \\ &\text{subject to } \mu_{f_j}(x) \ge \alpha \\ &\nu_{f_j}(x) \le \beta \\ &\alpha \ge \beta \;;\; \alpha + \beta \le 1 \quad \alpha, \beta \ge 0 \end{aligned} \tag{6}$$

where α denotes the minimal degree of acceptable and β denotes the maximal degree of rejection of objective(s) and constraints which can be written in the form

$$\begin{aligned} &\text{Maximize } \eta(F) = \sum_{j=1}^{b} \omega_j (\eta_{f_j})^{\gamma_j} \\ &\text{subject to} \\ &\frac{e^{-\omega\left(\frac{f_j(x)-m_j}{M_j-m_j}\right)} - e^{-\omega}}{1 - e^{-\omega}} \ge \alpha \\ &\beta \ge \left(\frac{f_j - m_j}{M_j - m_j}\right)^2 \\ &\alpha \ge \beta \;;\; \alpha + \beta \le 1 \;; \\ &\alpha, \beta \ge 0 \;;\; \omega > 0 \end{aligned} \tag{7}$$

The obtained optimization problem is solved by using one of the meta-heuristic technique namely cuckoo search algorithm which is described in the Section 5.

Step 4: **Adjustment of the preference parameters:** If the DM is satisfied with the solution obtained in Step 3, then the approach stops successfully. Otherwise, the key preference parameters, that is, decision maker's desirability functions (DF's), in terms of their ideal values, preferences of each objective function can be altered to meet the DM's choice, and the method again goes back to Step 3. The process is repeated until the DM is satisfied. We are just showing one run of the approach here as we assume that in this problem DM is satisfied by the results obtained in Step 3.

5. Cuckoo Search(CS)

CS is a meta-heuristic search algorithm which has been proposed recently by Yang and Deb [28] getting inspired from the reproduction strategy of cuckoos. At the most basic level, cuckoos lay their eggs in the nests of other host birds, which may be of different species. The host bird may discover that the eggs are not its own so it either destroys the eggs or abandons the nest all together. This has resulted in the evolution of cuckoo eggs which mimic the eggs of local host birds. CS is based on three idealized rules:

(i) Each cuckoo lays one egg at a time, and dumps it in a randomly chosen nest.

(ii) The best nests with high quality of eggs (solutions) will carry over to the next generations.

(iii) The number of available host nests is fixed, and a host can discover an alien egg with a probability $p_a \in [0, 1]$. In this case, the host bird can either throw the egg away or abandon the nest so as to build a completely new nest in a new location.

To make the things even simpler, the last assumption can be approximated by the fraction of p_a of n nests that are replaced by new nests with new random solutions. The fitness function of the solution is defined in a similar way as in other evolutionary techniques. In this technique, egg presented in the nest will represent the solution while the cuckoo's egg represents the new solution. The aim is to use the new and potentially better solutions (cuckoos) to replace worse solutions that are in the nests. Based on these three rules, the basic steps of the cuckoo search are described in Algorithm 1.

Algorithm 1 Pseudo code of Cuckoo Search (CS)

```
Objective function: f(x), x = (x_1, x_2, ..., x_D);
Generate an initial population of n host nests x_i ; i = 1, 2, ..., n;
While (t < MaxGeneration) or (stop criterion)
    Get a cuckoo randomly (say, i)
    Generate a new solution by performing Lévy flights;
    Evaluate its fitness f_i
    Choose a nest among n (say, j) randomly;
    if (f_i > f_j)
        Replace j by new solution
    end if
    A fraction(p_a) of the worse nests are abandoned and new ones are built;
    Keep the best solutions/nests;
    Rank the solutions/nests and find the current best;
    Pass the current best solutions to the next generation;
end while
```

This algorithm uses a balanced combination of a local random walk and the global explorative random walk, controlled by a switching parameter p_a. The local random walk can be written as

$$x_i^{t+1} = x_i^t + \alpha s \otimes H(p_a - \in) \otimes (x_j^t - x_k^t) \qquad (8)$$

where x_j^t and x_k^t are two different solutions selected randomly by random permutation, $H(u)$ is a Heaviside function, $\in$ is a random number drawn from a uniform distribution, $\otimes$ represents entry-wise multiplications and s is a step size. On the other hand, the global random walk is carried out by using Lévy flights

$$x_i^{(t+1)} = x_i^{(t)} + \alpha L(s, \lambda) \qquad (9)$$

where

$$L(s, \lambda) = \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}}, \quad s > 0 \qquad (10)$$

Here $\alpha > 0$ is the step size scaling factor, which should be related to the scales of the problem of interest. In most cases, we can use $\alpha = O(L/10)$ where L is the characteristic scale of the problem of interest, t is the current iteration number.

The Lévy flight essentially provides a random walk whose random step length drawn from a Lévy distribution which has an infinite variance with an infinite mean. Here the steps essentially form a random walk process with a power-law step length distribution with a heavy tail.

5.1. *Special cases of Cuckoo search*

Recent studies show that CS is potentially far more efficient than PSO and GA [23, 27]. Moreover the number of parameters in CS to be tuned is less than GA and PSO, and thus it is potentially more generic to adapt to a wider class of optimization problems. In addition to that, if we look on the Equation. (8) such that the factor $P = \alpha s \otimes H(p_a - \in)$ is greater than zero and hence the Equation. (8) becomes the major updating equation of the differential equation (DE). On the other hand, if we replace p_j^t by the current best solution (gbest) and $k = i$ then we have $x_i^{t+1} = x_i^t + P(gbest - x_i^t)$ which is the essential a variant of the particle swarm optimization without individual historical best. Also, from Equation. (9), the simulated annealing (SA) with a stochastic cooling scheduling controlled with a parameter p_a. Therefore, DE, PSO, SA etc., can be considered as a special case of cuckoo search [27]. Therefore, Cuckoo search is a good and efficient combination of DE, PSO and SA in one algorithm. Hence, cuckoo search is very efficient than the other meta-heuristic.

5.2. *Why Cuckoo Search is so efficient*

In addition to the analysis of the previous section, it has also been analyzed by the various researchers that PSO can converge quickly to the current best solution, but not necessarily the global best solutions [10, 18, 26, 27]. Moreover, some analyses suggest that PSO updating equations do not satisfy the global convergence conditions, and thus there is no guarantee for global convergence. On the other hand, it has proved that cuckoo search satisfy the global convergence requirements and thus has guaranteed global convergence properties [26, 27]. This implies that for multimodal optimization, PSO may converge prematurely to a local optimum, while cuckoo search can usually converge to the global optimality. Furthermore, cuckoo search has two search capabilities: local search and global search, controlled by a switching/discovery probability. As the local search is very intensive with about p_a of the search time, while global search takes about $1 - p_a$ of the total search time. This allows that the search space can be explored more efficiently on the global scale, and consequently the global optimality can be found with a higher probability. A further advantage of cuckoo search is that its global search uses Lévy flights or process, rather than standard random walks. As Lévy flights have infinite mean and variance, CS can explore the search space more efficiently than algorithms using standard Gaussian processes. This advantage, combined with both local and search capabilities and guaranteed global convergence, makes cuckoo search very efficient.

6. Illustrative example

To demonstrate the proposed approach, the following four reliability optimization problem has been considered here. The first three problems are taken as reliability-cost optimization in which aim is to optimize the reliability and cost of the 5 unit series system, life support system in a space capsule and complex bridge system respectively. The last problem is of mixed series-parallel system which aim is to maximize reliability and minimize the cost and weight simultaneously under the given set of constraints so as to find their component reliability in each subsystem of the system. The detail of these problems is explained as below.

6.1. Example 1: Series system

A series system having five components, shown in Fig. 1(a) is considered [14, 17], each having component reliability $r_i, i = 1, 2, \ldots, 5$. The system reliability R_s, unreliability Q_s and system cost C_s are given by

$$R_s = \prod_{i=1}^{5} r_i \quad \text{or} \quad Q_s = (1 - R_s) = 1 - \prod_{i=1}^{5} r_i$$

$$C_s = \sum_{i=1}^{5} \left[a_i \log\left(\frac{1}{1 - r_i} \right) + b_i \right]$$

The objective of this problem is to find the decision variables r_i which minimize both Q_s and C_s, subject to $0.5 \le r_i \le 0.99; i = 1, 2, \ldots, 5$.

In other words, the problem can be posed as a MOOP given by

$$\text{Minimize } \{Q_s, C_s\}$$
$$\text{subject to } 0.5 \le r_i \le 0.99 \quad ; \quad i = 1, 2, \ldots, 5$$

where vectors of the coefficients a_i and b_i are $a = \{24, 8, 8.75, 7.14, 3.33\}$ and $b = \{120, 80, 70, 50, 30\}$ respectively [14, 17].

6.2. Example 2: Life support system in a space capsule

This problem concerns the reliability design of a life-support system in a space capsule [14, 24] whose system configuration is presented in Fig. 1(b). The system, which requires a single path for its success, has two redundant subsystems each comprising component 1 and 4. Each of the redundant subsystems is in series with component 2 and the resultant pair of series-parallel arrangement forms two equal paths. Component 3 is inserted as a third path and backup for the pair. This problem is a continuous nonlinear optimization problem and consists of four components, each having component reliability $r_i, i = 1, 2, 3, 4$ such that their system reliability R_s, unreliability Q_s and system cost C_s are given by

$$\text{Maximize } R_s = 1 - r_3[(1 - r_1)(1 - r_4)]^2 - (1 - r_3)[1 - r_2\{1 - (1 - r_1)(1 - r_4)\}]^2$$

or

$$\text{Minimize } Q_s = 1 - R_s$$

$$\text{Minimize } C_s = 2K_1 r_1^{\alpha_1} + 2K_2 r_2^{\alpha_2} + K_3 r_3^{\alpha_3} + 2K_4 r_4^{\alpha_4}$$

$$\text{subject to } \quad 0.5 \le r_i \le 1.0, \quad i = 1, 2, 3, 4$$

In other words, the problem can be posed as a MOOP given by

$$\text{Minimize } \{Q_s, C_s\}$$
$$\text{subject to } 0.5 \le r_i \le 1.0 \quad ; \quad i = 1, 2, \ldots, 4$$

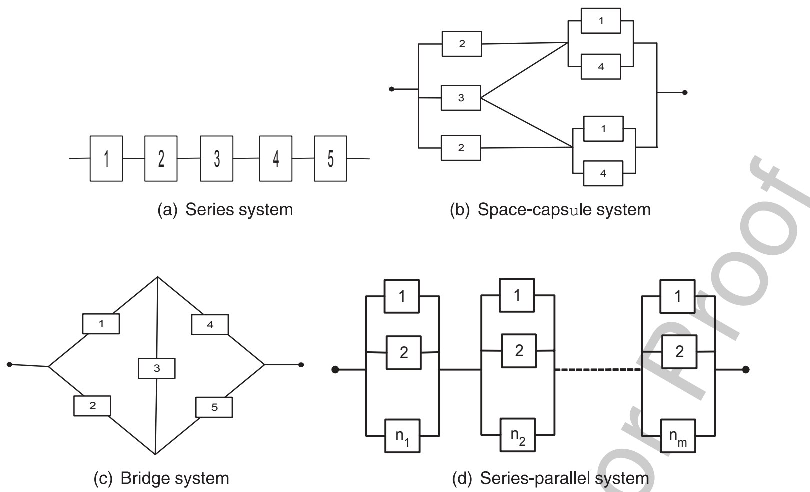

Fig. 1. Layout of the series, space capsule, bridge and series-parallel systems.

where vectors of the coefficients K_i and α_i are $K = \{100, 100, 200, 150\}$ and $\alpha = \{0.6, 0.6, 0.6, 0.6\}$ respectively [14, 24].

6.3. Example 3: Complex system

The bridge network is considered as a system of the five components [14, 24], each having component reliability $r_i, i = 1, 2, \ldots, 5$, to find out the system reliability as shown in Fig. 1(c). The objective is to minimize the cost and reliability of the system at the same time. The algebraic expression for system reliability R_s and the cost C_s of the bridge system are given as follows:

$$R_s = r_1r_4 + r_2r_5 + r_2r_3r_4 + r_1r_3r_5 + 2r_1r_2r_3r_4r_5$$
$$-r_2r_3r_4r_5 - r_1r_3r_4r_5 - r_1r_2r_3r_5 - r_1r_2r_3r_4$$

$$C_s = \sum_{i=1}^{5} a_i \exp\left(\frac{b_i}{1-r_i}\right)$$

The problem is to find the decision variables $r_i, i = 1, 2, \ldots, 5$ which minimize both Q_s and C_s subject to $0 \le r_i \le 1$. Hence, mathematically, the MOOP for the bridge network can be formulated as

$$\text{Minimize } \{Q_s, C_s\}$$
$$\text{subject to } 0 \le r_i \le 1 \quad ; \quad i = 1, 2, \ldots, 4$$

where $a_i = 1$ and $b_i = 0.0003, \forall\, i, i = 1, 2, \ldots, 5$

6.4. Example 4: Mixed series-parallel system

This multi-objective reliability optimization problem is taken from Garg et al. [14], Ravi et al. [24] and Sakawa [25] whose block diagram is shown in Fig. 1(d). The aim of this problem is to allocate the optimal reliabilities $r_i, i = 1, 2, 3, 4$ of four components whose redundancies are specified in order to achieve the following three goals

$$\text{Maximize } R_s, \quad \text{Minimize } C_s, \quad \text{Minimize } W_s$$
$$\text{or}$$
$$\text{Minimize } Q_s, \quad \text{Minimize } C_s, \quad \text{Minimize } W_s$$

$$\text{subject to } V_s = \sum_{i=1}^{4} V_i n_i \le 65 \quad ; \quad P_s \le 12000$$
$$0.5 \le r_i \le 1, \quad i = 1, 2, 3, 4$$

In other words, the problem can be posed of three objectives given in

$$\text{Minimize } \{Q_s, C_s, W_s\}$$
$$\text{subject to } V_s = \sum_{i=1}^{4} V_i n_i \le 65 \quad ; \quad P_s \le 12000$$
$$0.5 \le r_i \le 1, \quad i = 1, 2, 3, 4$$

where R_s, Q_s, C_s, W_s, V_s are the reliability, unreliability, cost, weight and volume of the system respectively. Here r_i represents the reliability of the i_{th} component of the system. In addition, we have

$$P_s = W_s \times V_s \quad ; \quad C_s = \sum_{i=1}^{4} C_i n_i$$

$$R_s = \prod_{i=1}^{4} \left[1 - (1 - r_i)^{n_i}\right] \quad ; \quad W_s = \sum_{i=1}^{4} W_i n_i$$

$$C_i = \alpha_i^c \left[\log_{10}\left(\frac{\beta_i^c}{1 - r_i}\right)\right]^{\gamma_i^c}$$

$$V_i = \alpha_i^v \left[\log_{10}\left(\frac{\beta_i^v}{1 - r_i}\right)\right]^{\gamma_i^v} ;$$

$$W_i = \alpha_i^w \left[\log_{10}\left(\frac{\beta_i^w}{1 - r_i}\right)\right]^{\gamma_i^w}$$

$$\alpha_i^c = 8.0, \quad \alpha_i^w = 6.0, \quad \alpha_i^v = 2.0; \quad \forall\, i$$

$$\gamma_i^c = 2.0, \quad \gamma_i^w = 0.5, \quad \gamma_i^v = 0.5; \quad \forall\, i$$

$$\beta_i^c = \{2, 10, 3, 18\}; \quad \beta_i^w = \{3, 2, 10, 8\},$$

$$\beta_i^v = \{2, 2, 6, 8\}$$

$$n_i = \{7, 8, 7, 8\}$$

7. Computational results

In this section, we have described and analyzed the results as obtained by the above stated approach for optimization. In order to eliminate stochastic discrepancy, in each example, 25 independent runs are made which involves 25 different initial trial solutions with a randomly generated population of size $20 \times D$ for the optimization, where D is the dimension of the problem. The termination criterion has been set either limited to a maximum number of generations (1000) or to the order of relative error equal to 10^{-6}, whichever is achieved first. The other specific parameters of CS algorithms, p_a are set to be 0.25 represent the probability that host can discover an alien egg. The parameter ω is set to be 0.1 for defining the exponential membership function of the objective function. On the other hand, the parameters for the PSO algorithm, namely Cognitive and social components which are constants that can be used to change the weighting between personal and population experience, respectively. In our experiments, both were both set to 1.5. Inertia weight, which determines how the previous velocity of the particle influences the velocity in the next iteration, is linearly decreased with the iteration from the initial weight 0.9 to final weight 0.4.

7.1. Results & Discussion

In order to solve these four problems, firstly the fuzzy multi-objective reliability optimization problem is formulated for corresponding to individual problem as given in equation (3). For this the desirability function's priori preference parameters in the form of ideal and anti-ideal values corresponding to each problem are calculated and based on that degree of satisfaction and dissatisfaction are computed from their degree of membership and non-membership functions. Using these constructed membership functions, an intuitionistic fuzzy optimization model has been reformulated into its equivalent crisp optimization model as given in equation (7). For obtaining their corresponding Pareto optimal solution, CS has been used and compared their results with PSO.

7.1.1. Results corresponding to Example 1:

The ideal and anti-ideal values of the system reliability are 0.9 and 1 while for the system cost these values are 500 and 600 respectively. The main aim of the problem is to determine the sub-systems' reliabilities $R = [R_1, R_2, \ldots, R_5]^T$ so to maximize the system reliability R_s and minimize the system cost C_s. Using these ideal values, the membership and non-membership functions for the system reliability objective are defined as

$$\mu_{R_s}(x) = \begin{cases} 0, & R_s(x) \leq 0.9 \\ \frac{e^{-\omega\left(\frac{1-R_s(x)}{0.1}\right)} - e^{-\omega}}{1-e^{-\omega}}, & 0.9 \leq R_s(x) \leq 1 \\ 1, & R_s(x) \geq 1 \end{cases} \tag{11}$$

and

$$\nu_{R_s}(x) = \begin{cases} 1, & R_s(x) \leq 0.9 \\ \left(\frac{1-R_s(x)}{0.1}\right)^2, & 0.9 \leq R_s(x) \leq 1 \\ 0, & R_s(x) \geq 1 \end{cases} \tag{12}$$

while for the system cost are defined as

$$\mu_{C_s}(x) = \begin{cases} 1, & C_s(x) \leq 500 \\ \frac{e^{-\omega\left(\frac{C_s(x)-500}{100}\right)} - e^{-\omega}}{1-e^{-\omega}}, & 500 \leq C_s(x) \leq 600 \\ 0, & C_s(x) \geq 600 \end{cases} \tag{13}$$

and

$$\nu_{C_s}(x) = \begin{cases} 0, & C_s(x) \leq 500 \\ (\frac{C_s(x)-500}{100})^2, & 500 \leq C_s(x) \leq 600 \\ 1, & C_s(x) \geq 600 \end{cases} \quad (14)$$

Using these membership functions, the overall satisfactory function of the system is given by

$$\eta(f(R, C)) = \omega_1(\eta_{R_s})^{\gamma_R} + \omega_2(\eta_{C_s})^{\gamma_C} \quad (15)$$

where

$$\eta_{R_s} = \mu_{R_s} - \nu_{R_s} \quad ; \quad \eta_{C_s} = \mu_{C_s} - \nu_{C_s},$$

$$\omega_1 = \frac{1 - \eta_{R_s}}{2 - \eta_{R_s} - \eta_{C_s}} \quad \text{and} \quad \omega_2 = \frac{1 - \eta_{C_s}}{2 - \eta_{R_s} - \eta_{C_s}}$$

The exponential weights γ_R and γ_C corresponding to reliability and cost of the system respectively can be set at different values. The results obtained corresponding to these different values are summarized in Table 1. It is observed from the table that by increasing the value of exponential weight γ_R then the systems' reliability and cost become increasing while the overall satisfaction function $\eta(f)$ will decrease. On the other hand, when the exponential weight γ_C of the system cost increases, the system reliability R_s and cost C_s are decreasing. The last column of the Table 1 indicates that the overall satisfaction level of the system will decrease with smaller the weight γ_R. When γ_R and γ_C are equal then the satisfaction level of the objective will be the lowest and hence the unsatisfactory degree in each objective is the highest and the comprehensive ability of the decision maker is the weakest. Also it has been seen that the satisfactory degree of each objective becomes larger with the gradual increase in their difference of their exponential weights and hence the total degree of dissatisfaction becomes smaller. It shows that the comprehensive coordination ability of decision maker becomes stronger, until the single-objective optimization is reached. Finally it has been observed from the analysis that when the exponential weights are greater than one then the overall degree of satisfaction of the objective is very small and hence the ideal satisfactory solution will probably not be obtained. Therefore exponential weight is suggested to be less than one.

7.1.2. Results corresponding to Example 2:

The ideal and anti-ideal values of the systems' reliability corresponding to space capsule system problem are 0.9 and 1 while for cost functions are 641 and 700 respectively. Based on these ideal values, a region of satisfaction is constructed corresponding to each of the objectives. Using these constructed membership functions, an intuitionistic fuzzy optimization model has been reformulated into its equivalent crisp optimization model as given in equation (7). Initially, the iterative process is started with the exponential weights $\gamma_R = \gamma_C = 1$ for a satisfaction region of the system reliability and cost. The outcomes of this iteration are given as $R_s = 0.9278073$ and $C_s = 658.53122$, given in the Table 2. However, the solution reported by Ravi et al. [24] is given as $R_s = 0.94743$ and $C_s = 668.28$. Thus in terms of cost, the reported solution is better than the solution reported in Ravi et al. [24], however, its reliability is less. Keeping this in view, put a more weighting on the system reliability objective as compared to the cost. For this exponential weight γ_C are decreasing from one and keeping reliability weight γ_R as fixed, then the corresponding results shows that the system reliability are increasing with the decrease of their weight γ_C and correspondingly the overall satisfaction function $\eta(f)$ will also increase. Other iterations are also performed, by varying exponential weight γ_R of the system reliability and fixing the exponential weight γ_C to be 1, to achieve the other solutions. The corresponding results for different values of exponential weights γ_R and γ_C after solving by using the CS algorithm are summarized in Table 2 along with their PSO results. Decision maker (DM) may achieve more trade-off solutions by changing the reservation value of objectives according to desire.

7.1.3. Results corresponding to Example 3:

The third example taken is the complex (bridge) system in which aim is to find the reliability of components so as to maximize reliability and minimize the cost simultaneously. For this, membership and non-membership function corresponding to objective functions are constructed based on its desirability functions, 0.99 and 1 corresponding to reliability while 4.8 and 5.5 for cost. Based on these functions, the optimization problem (7) is formulated for the considered system and hence solved with a CS approach for different values of γ_R and γ_C. The results corresponding to it are summarized in Table 3 and compared their results with PSO results. It has been examined from the table that the proposed results are better than the results obtained by PSO. Moreover, it has been also observed from the analysis that exponential weights should be less than one in order to obtain their ideal satisfactory solutions.

Table 1
Results corresponding to series system (Example 1)

Method	γ_R	γ_C	r_1	r_2	r_3	r_4	r_5	R_s	C_s	$(\eta_{R_s})^{\gamma_R}$	$(\eta_{C_s})^{\gamma_C}$	$\eta(f)$
PSO	0.01	1	0.9598808	0.9805495	0.9754804	0.9874390	0.9936032	0.9008008	5.3922423	0.952317	0.441943	0.768623
CS			0.9548417	0.9781811	0.9827367	0.9884755	0.9932975	0.9012247	5.3899495	0.956330	0.446054	0.773072
PSO	0.1	1	0.9561542	0.9845308	0.9800691	0.9889341	0.9950746	0.9078980	5.4251292	0.765566	0.381888	0.612459
CS			0.9579439	0.9845810	0.9842047	0.9826722	0.9950926	0.9077143	5.4238425	0.763920	0.384276	0.612919
PSO	0.5	1	0.9625607	0.9834595	0.9870238	0.9851548	0.9928630	0.9139156	5.4619041	0.337453	0.312299	0.326462
CS			0.9586241	0.9849617	0.9838115	0.9898147	0.9941160	0.9140513	5.4594998	0.338827	0.316927	0.329288
PSO	0.8	1	0.9624038	0.9878738	0.9844165	0.9884538	0.9932577	0.9188742	5.4895542	0.213990	0.258286	0.234573
CS			0.9614191	0.9854850	0.9870623	0.9867380	0.9939359	0.9172076	5.4788830	0.201999	0.279304	0.237143
PSO	1	1	0.9642275	0.9836446	0.9822799	0.9879260	0.9922604	0.9132783	5.4585247	0.109465	0.318801	0.200192
CS			0.9597791	0.9848955	0.9852240	0.9861095	0.9941996	0.9130514	5.4522561	0.107876	0.330805	0.203425
PSO	1	0.8	0.9605856	0.9887070	0.9874474	0.9869466	0.9916588	0.9178541	5.4869927	0.139410	0.343896	0.233720
CS			0.9548512	0.9895269	0.9848852	0.9899517	0.9936496	0.9153690	5.4719308	0.123641	0.374414	0.235627
PSO	1	0.5	0.9644641	0.9835188	0.9852261	0.9898850	0.9900836	0.9159279	5.4798021	0.127291	0.526784	0.308228
CS			0.9656811	0.9863976	0.9848967	0.9838797	0.9950794	0.9184936	5.4916518	0.143278	0.504112	0.311216
PSO	1	0.1	0.9678786	0.9885362	0.9886324	0.9899154	0.9888517	0.9259288	5.5523382	0.182533	0.815598	0.508875
CS			0.9686636	0.9910018	0.9872736	0.9864465	0.9929053	0.9282531	5.5617121	0.192647	0.802288	0.512227
PSO	1	0.01	0.9651833	0.9894481	0.9907855	0.9903399	0.9954922	0.9328347	5.5912297	0.209573	0.969922	0.625164
CS			0.9689616	0.9860122	0.9880108	0.9921780	0.9969620	0.9337246	5.6014259	0.212398	0.963751	0.628032
PSO	1.5	1	0.9564797	0.9860164	0.9847168	0.9866863	0.9910490	0.9081248	5.4251311	0.018905	0.381884	0.163922
CS			0.9600336	0.9865125	0.9824746	0.9839299	0.9931475	0.9092606	5.4319491	0.022579	0.369177	0.163553
PSO	1	1.5	0.9559413	0.9834325	0.9837203	0.9916965	0.9939694	0.9115895	5.4499493	0.097406	0.194071	0.138406
CS			0.9599507	0.9875026	0.9791179	0.9877323	0.9939681	0.9112424	5.4457422	0.094861	0.201056	0.139516

Table 2
Results corresponding to nonlinear space capsule system (Example 2)

Method	γ_R	γ_C	r_1	r_2	r_3	r_4	R_s	C_s	$(\eta_{R_s})^{\gamma_R}$	$(\eta_{C_s})^{\gamma_C}$	$\eta(f)$
PSO	0.01	1	0.5463795	0.7936154	0.5197462	0.5073802	0.9033303	647.98995	0.965736	0.862202	0.952850
CS			0.5099959	0.8384279	0.5130517	0.5038219	0.9046501	646.29902	0.968832	0.897976	0.962011
PSO	0.1	1	0.5087991	0.8560035	0.5064657	0.5400102	0.9179577	655.77969	0.821534	0.677280	0.782171
CS			0.5104167	0.8731123	0.5003880	0.5002821	0.9121247	647.96109	0.795344	0.862826	0.804281
PSO	0.5	1	0.5233016	0.9049671	0.5036342	0.5128216	0.9266019	657.45727	0.430776	0.633128	0.493619
CS			0.5938911	0.8579064	0.5030364	0.5009450	0.9297234	659.31388	0.445549	0.582475	0.492448
PSO	0.8	1	0.5133542	0.9359586	0.5211990	0.5027874	0.9290755	660.13473	0.271499	0.559481	0.373433
CS			0.5059642	0.9511007	0.5097616	0.5087910	0.9316251	660.46363	0.281992	0.550165	0.378938
PSO	1	1	0.5103233	0.9124469	0.5007069	0.5240649	0.9278073	658.53122	0.190787	0.604057	0.326563
CS			0.5266560	0.9387869	0.5095765	0.5058709	0.9334570	661.46819	0.211564	0.521344	0.328587
PSO	1	0.8	0.5350007	0.9181756	0.5143666	0.5024137	0.9304100	660.14327	0.201113	0.628172	0.352955
CS			0.5771052	0.8934357	0.5028270	0.5010461	0.9343215	661.30572	0.214208	0.598159	0.358663
PSO	1	0.5	0.5490615	0.9306721	0.5039249	0.5119007	0.9382973	664.44553	0.224537	0.657794	0.407584
CS			0.5167162	0.9783334	0.5109278	0.5076252	0.9394002	665.37428	0.226870	0.635649	0.404809
PSO	1	0.1	0.5869689	0.9306811	0.5131157	0.5097173	0.9466040	671.08232	0.236421	0.858579	0.551272
CS			0.5293290	0.5027187	0.9934818	0.5202093	0.9469042	670.82739	0.236605	0.861952	0.551359
PSO	1	0.01	0.6190601	0.8729139	0.5023595	0.5512285	0.9473234	676.50921	0.236833	0.963384	0.644567
CS			0.5170686	0.9744657	0.5025875	0.5626034	0.9509467	676.35856	0.237413	0.965405	0.645062
PSO	1.5	1	0.5539270	0.8846019	0.5061649	0.5073954	0.9281703	658.73040	0.084330	0.598596	0.255059
CS			0.5287961	0.9158787	0.5009568	0.5108942	0.9299024	658.79261	0.088906	0.596886	0.258996
PSO	1	1.5	0.5323546	0.8745094	0.5038797	0.5184860	0.9228797	656.39778	0.167707	0.537639	0.274731
CS			0.5184258	0.9283047	0.5068567	0.5100979	0.9301131	659.46462	0.200000	0.439752	0.282758

Table 3
Results corresponding to complex bridge system (Example 3)

	γ_R	γ_C	r_1	r_2	r_3	r_4	r_5	R_s	C_s	$(\eta_{R_s})^{\gamma_R}$	$(\eta_{C_s})^{\gamma_C}$	$\eta(f\ )$
PSO	0.01	1	0.9455041	0.9515035	0.9033055	0.9588207	0.9502702	0.9949021	5.0281954	0.985723	0.556687	0.354786
CS			0.9444638	0.9519407	0.9163091	0.9599051	0.9504936	0.9949962	5.0288579	0.985727	0.555106	0.354531
PSO	0.1	1	0.9434656	0.9634259	0.9041537	0.9484755	0.9507169	0.9949922	5.0286371	0.866097	0.555633	0.354638
CS			0.9624824	0.9423956	0.9014956	0.9449147	0.9552632	0.9949452	5.0284894	0.866088	0.555985	0.354690
PSO	0.5	1	0.9384203	0.9529402	0.9073707	0.9530634	0.9605881	0.9948689	5.0285758	0.487180	0.555779	0.354552
CS			0.9524374	0.9637767	0.9026942	0.9456245	0.9448926	0.9948808	5.0287228	0.487209	0.555428	0.354502
PSO	0.8	1	0.9509569	0.9409911	0.9228819	0.9550774	0.9563509	0.9948165	5.0287276	0.316284	0.555417	0.354367
CS			0.9516132	0.9471844	0.9007612	0.9526228	0.9529726	0.9947931	5.0276951	0.316192	0.557879	0.354797
PSO	1	1	0.9529831	0.9354290	0.9110737	0.9588891	0.9597933	0.9949494	5.0292506	0.237480	0.554168	0.354325
CS			0.9483112	0.9536796	0.9106658	0.9430209	0.9591303	0.9948984	5.0283287	0.237408	0.556369	0.354717
PSO	1	0.8	0.9671780	0.9213701	0.9119818	0.9504851	0.9558933	0.9948125	5.0293210	0.237176	0.623459	0.354069
CS			0.9585802	0.9367381	0.9112590	0.9417061	0.9647802	0.9949151	5.0291229	0.237437	0.623885	0.354356
PSO	1	0.5	0.9505880	0.9545951	0.9275672	0.9433014	0.9572848	0.9950326	5.0292224	0.237491	0.744470	0.354346
CS			0.9482660	0.9431433	0.9124931	0.9655812	0.9471308	0.9948724	5.0289850	0.237353	0.744850	0.354361
PSO	1	0.1	0.9337506	0.9558072	0.9070479	0.9562047	0.9592173	0.9949007	5.0288396	0.237413	0.942846	0.354475
CS			0.9513013	0.9556598	0.9020340	0.9272357	0.9647767	0.9948442	5.0287200	0.237278	0.942895	0.354434
PSO	1	0.01	0.9579344	0.9474640	0.9044894	0.9624505	0.9335544	0.9948886	5.0285765	0.237389	0.994143	0.354584
CS			0.9429274	0.9626901	0.9018209	0.9483415	0.9488921	0.9948053	5.0281152	0.237150	0.994163	0.354631
PSO	1.5	1	0.9383515	0.9613511	0.9086598	0.9355960	0.9642653	0.9949346	5.0290599	0.115717	0.554624	0.354406
CS			0.9488194	0.9565861	0.9191431	0.9613535	0.9366029	0.9949716	5.0290663	0.115740	0.554608	0.354426
PSO	1	1.5	0.9425407	0.9608263	0.9029580	0.9358527	0.9662291	0.9951962	5.0296292	0.237128	0.411527	0.353884
CS			0.9529616	0.9552172	0.9020054	0.9419079	0.9587698	0.9950994	5.0286660	0.237404	0.414096	0.354552

7.1.4. Results corresponding to Example 4:

The fourth example is of RRAP in which aim is to maximize the system reliability and minimizes the systems cost and weight simultaneously subject to various resource constraints. The degree of satisfaction corresponding to objective functions are formulated and then IFO model has been reformulated into its equivalent crisp optimization model. The constraints are handled with the help of parameter free penalty method [11] in which constrained optimization is converted into unconstrained optimization problems. The resultant problem has been solved with the help of CS for different values of exponential weight γ_R and γ_C and their corresponding results are summarized in Table 4. The results corresponding to $\gamma_R = \gamma_C = \gamma_W = 1$ show that $(R_s, C_s, W_s) =$ (0.9963176, 369.17902, 184.53879) with degree of satisfaction are 0.8174, 0.7695 and 0.9556 respectively for the system reliability, cost and weight. The overall satisfaction degree for it is 0.806699. Clearly this solution dominates the solution (R_s=0. 99671, C_s=413. 664, W_s=193. 432) reported by Ravi et al. [24]. In order to increase the reliability of the system, system analysts may prefer more weighing on the reliability objective by decreasing their preference value parameter γ_C and γ_W from 1 to 0. Not satisfied with these outcomes, the decision maker may have other preferences towards the objective functions and their corresponding results are summarized in Table 4. Finally it has been observed from the analysis that when the exponential weights are greater than one then the overall degree of satisfaction of the objective is very small and hence the ideal satisfactory solution will probably not be obtained.

7.2. Statistical analysis

In order to analyze whether the results as obtained in the above tables are statistically significantly with each other or not. For this we compute their corresponding the best, the average, the worst and the standard deviation of objective function values after 25 independent runs and are reported in Table 5 to each example. From this analysis it has been concluded that the proposed approach is of superior searching quality and robustness for these problems. Moreover, the standard deviation of the results is very small. In addition to that we perform a *t*-test on the pair of the algorithm for each problem. For this, an analysis has been conducted with the assumption that the populations have equal variances at the significance level of $\alpha = 0.05$ in the case of proposed results with PSO results. The test has been performed against the null hypothesis that there is no difference in their population means. Under this null hypothesis, the pooled *t*-test has been performed corresponding to $\gamma_R = \gamma_C = \gamma_W = 1$ for each problem and hence *t*-statistics values corresponding to example 1, example 2, example 3, example 4 are 1.8184, 3.3205, 4.2636 and 1.7479 respectively. It is indicated from these values that the values of *t*-stat are greater than the *t*-critical values (1.6715). Also the *p* values obtained corresponding to each case is less than the significance level $\alpha = 0.05$. Hence it is observed that it is highly significant and null significant, i.e., the mean of the algorithms is identical is rejected. Thus, the two types of means differ significantly. Similar observations have been computed for different parameters of γ_R, γ_C and γ_W. Further, since mean of cost functions with proposed approach is better than the mean of another algorithm, and hence we conclude that the proposed approach is better than others and this difference is statistically significant.

8. Conclusion

The problem of optimizing the reliability of complex systems has been modeled as an intuitionistic fuzzy multi-objective optimization problem, where the reliability, cost, weight and volume of the system are considered as fuzzy objectives. Four optimization problems involving different kinds of complex systems and multistage mixed systems have been successfully solved using the model. In the formulation, the membership and non-membership functions of their objective functions are formulated by using exponential and quadratic membership functions respectively. A variable weight method has been used, instead of constant ones, for reformulated the problem in a single objective optimization model using exponential weights γ_R and γ_C corresponding for reliability and cost respectively. Cuckoo search, a global metaheuristic optimization technique has been used for solving the nonconvex problems and compared their results with PSO algorithm results. Finally it has been observed from the analysis that when the exponential weights are greater than one, then the overall degree of satisfaction of the objective is very small and hence the ideal satisfactory solution will probably not be obtained. Therefore exponential weight is suggested to be less than one. The results are encouraging and they indicate that proposed intuitionistic fuzzy optimization techniques can be employed as viable alternatives to the traditional goal programming approaches to the kind of problems

Table 4
Results corresponding to mixed series-parallel system RRAP (Example 4)

	γ_R	γ_C	γ_W	r_1	r_2	r_3	r_4	R_s	C_s	W_s	$(\eta_{R_s})^{\gamma_R}$	$(\eta_{C_s})^{\gamma_C}$	$(\eta_{W_s})^{\gamma_W}$	$\eta(f)$
PSO	0.01	1	0.01	0.5915328	0.5331878	0.5984125	0.5184053	0.9912978	340.99474	179.89441	0.989746	0.932435	0.999851	0.984614
CS				0.6134969	0.5408969	0.5745892	0.5105508	0.9909525	340.29286	180.05045	0.988652	0.936103	0.999842	0.984465
PSO	0.1	1	0.1	0.6516017	0.5435637	0.6209380	0.5115890	0.9931462	349.52978	182.02784	0.941010	0.886314	0.997146	0.933249
CS				0.6156442	0.5626669	0.6389091	0.5288322	0.9942053	354.50692	182.35206	0.956721	0.858133	0.996934	0.932609
PSO	0.5	1	0.5	0.6218299	0.5801845	0.6035794	0.5758689	0.9953514	363.48416	183.29485	0.860690	0.804904	0.981680	0.847459
CS				0.6688789	0.5492630	0.6561570	0.5357159	0.9951403	361.17740	183.88545	0.850466	0.818876	0.979729	0.849393
PSO	0.8	1	0.8	0.6746666	0.5524090	0.6341508	0.5704046	0.9959718	366.50702	184.39336	0.828674	0.786286	0.965096	0.821958
CS				0.6616704	0.5622983	0.6706581	0.5701219	0.9965631	371.43226	185.10333	0.866446	0.755202	0.961338	0.816837
PSO	1	1	1	0.6417986	0.5581655	0.6554089	0.5787711	0.9962280	368.36234	184.29183	0.810535	0.774686	0.957224	0.806595
CS				0.6337711	0.5747330	0.6675002	0.5648951	0.9963176	369.17902	184.53879	0.817400	0.769538	0.955611	0.806699
PSO	1	0.8	0.8	0.6732973	0.5583721	0.6510410	0.5710502	0.9963841	369.61432	184.89961	0.822456	0.808609	0.962420	0.829700
CS				0.6613086	0.5992464	0.6475747	0.5557804	0.9966358	372.08873	185.39456	0.841373	0.795256	0.959789	0.829220
PSO	1	0.5	0.5	0.6837239	0.5685661	0.6764391	0.5668366	0.9968756	375.24641	185.97663	0.859030	0.854688	0.972698	0.869694
CS				0.6559008	0.5891153	0.6379764	0.5766484	0.9967734	372.68580	185.17522	0.851547	0.864374	0.975416	0.872128
PSO	1	0.1	0.1	0.6922808	0.5959790	0.6732593	0.6101997	0.9980994	390.52283	187.95368	0.943628	0.954230	0.993079	0.958268
CS				0.6843544	0.6173414	0.6706265	0.5959016	0.9980978	390.07548	188.00464	0.943520	0.954715	0.993042	0.958661
PSO	1	0.01	0.01	0.7363932	0.6251687	0.7028506	0.6001071	0.9986639	404.95519	190.74881	0.979546	0.993461	0.999098	0.993804
CS				0.7472045	0.6203673	0.6940327	0.6131087	0.9987501	407.39103	191.04791	0.984855	0.993094	0.999075	0.993763
PSO	1.5	1	1	0.6322412	0.6029990	0.6695628	0.5626320	0.9967078	374.15362	185.36230	0.779117	0.737629	0.950204	0.774034
CS				0.6884177	0.5752611	0.6189404	0.5603138	0.9960975	368.26658	184.93796	0.716152	0.775288	0.952996	0.767973
PSO	1	1.5	1	0.6444216	0.5639517	0.6514573	0.5667497	0.9961135	366.56357	184.18478	0.801693	0.696755	0.957922	0.766728
CS				0.6350051	0.5743333	0.6206336	0.5758728	0.9958875	365.58922	183.79647	0.784009	0.704804	0.960448	0.763520
PSO	1	1.5	1.5	0.6436523	0.5676392	0.6159764	0.5775735	0.9958101	365.10297	183.76916	0.777871	0.708814	0.941524	0.761335
CS				0.6367300	0.5968497	0.6158179	0.5696782	0.9960619	368.07825	184.29610	0.797686	0.684208	0.936487	0.756238

Table 5
Statistical Results corresponding to each case study

	γ_R	γ_C	γ_W	Problem 1		Problem 2		Problem 3		Problem 4	
				PSO	CS	PSO	CS	PSO	CS	PSO	CS
Mean				0.749834	0.762525	0.925206	0.939571	0.352927	0.353741	0.977864	0.973770
Best	0.01	1	0.01	0.768623	0.773072	0.952850	0.962011	0.354786	0.354531	0.984614	0.984465
Worst				0.729964	0.744149	0.882384	0.911402	0.352289	0.353158	0.954262	0.962145
Std dev				0.009361	0.007338	0.017732	0.011964	0.000628	0.000366	0.007231	0.006651
Mean				0.314729	0.318658	0.461431	0.473926	0.353438	0.353818	0.820029	0.833310
Best	0.5	1	0.5	0.326462	0.329288	0.493619	0.492448	0.354542	0.354502	0.847459	0.849393
Worst				0.306148	0.308425	0.424580	0.450261	0.352629	0.352426	0.794657	0.817011
Std dev				0.005562	0.005123	0.017038	0.010013	0.000531	0.000460	0.014644	0.010450
Mean				0.192236	0.194653	0.301463	0.310362	0.353440	0.353985	0.780353	0.786353
Best	1	1	1	0.200192	0.203425	0.326563	0.328587	0.354325	0.354717	0.806595	0.806699
worst				0.185130	0.183990	0.294715	0.293536	0.352298	0.353282	0.752611	0.755052
Std dev				0.004718	0.004488	0.010036	0.008465	0.000475	0.000408	0.013877	0.009499
Mean				0.303157	0.305413	0.390879	0.395827	0.353638	0.354023	0.843816	0.856798
Best	1	0.5	0.5	0.308228	0.311216	0.407584	0.404809	0.354346	0.354361	0.869694	0.872128
worst				0.294052	0.297864	0.367065	0.375117	0.352503	0.353400	0.828781	0.842150
Std dev				0.004855	0.004229	0.009987	0.007870	0.000564	0.000378	0.012112	0.007864
Mean				0.613572	0.617163	0.641106	0.642511	0.353709	0.354003	0.992228	0.993090
Best	1	0.01	0.01	0.625164	0.628032	0.644567	0.645062	0.354584	0.354631	0.993804	0.993753
worst				0.600992	0.603245	0.637047	0.634662	0.352673	0.353216	0.989061	0.992036
Std dev				0.007302	0.006954	0.003138	0.002549	0.000517	0.000402	0.001150	0.000557
Mean				0.131018	0.133768	0.258088	0.266813	0.353730	0.353837	0.722791	0.734314
Best	1	1.5	1.5	0.138406	0.139516	0.274731	0.282758	0.353884	0.354552	0.761335	0.756238
worst				0.127469	0.126203	0.229188	0.236391	0.351949	0.352709	0.690778	0.710830
Std dev				0.004198	0.003766	0.013146	0.011861	0.000640	0.000484	0.017865	0.014327

solved in this paper. The obtained result by CS algorithm is shown to be statistically significant as compared to PSO in terms of means of pooled *t*-test. The optimal design parameters help the decision-maker help the decision maker basically in following two ways:

- In deciding the related characteristics of each component
- In formulating optimal design policies and repair policies for the entire system to ensure highly reliable and efficient system.

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