A new approach for multi-attribute decision-making problems in bipolar neutrosophic sets

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Abstract: In this study, we give a new outranking approach for multi-attribute decision-making problems in bipolar neutrosophic environment. To do this, we firstly propose some outranking relations for bipolar neutrosophic number based on ELECTRE, and the properties in the outranking relations are further discussed in detail. Also, we developed a ranking method based on the outranking relations of for bipolar neutrosophic number. Finally, we give a real example to illustrate the practicality and effectiveness of the proposed method.

Keywords: Single valued neutrosophic sets, single valued bipolar neutrosophic sets, outranking relations, multi-attribute decision making.

1 Introduction

As a generalization of fuzzy set [100] and intuitionistic fuzzy set [1] and so on, neutrosophic set was presented by Smarandache [67, 68] to capture the incomplete, indeterminate and inconsistent information. The neutrosophic set have three completely independent parts, which are truth-membership degree, indeterminacy-membership degree and falsity-membership degree, therefore it is applied to many different areas, such as deci-sion making problems [2, 16, 24]. In additionally, since the neutrosophic sets are hard to be apply in some real problems because of the truth-membership degree, indeterminacy-membership degree and falsity-membership degree lie in $]^{-}0,\ 1^{+}[$, single valued neutrosophic set, as a example of the neutrosophic set introduced by Wang et al. [73].

Recently, Lee [27, 28] proposed notation of bipolar fuzzy set and their operations based on fuzzy sets. A bipolar fuzzy set have a $T^{+} \to [0,1]$ and $T^{-} \to [-1,0]$ is called positive membership degree and negative membership degree $T^{-}(u)$. Also the bipolar fuzzy models have been studied by many authors both theory and application in [17, 20, 30, 69, 98]. After the definition of Smarandache's neutrosophic set, neutrosophic sets and neutrosophic logic have been applied in many real applications to handle uncertainty. The neutrosophic set uses one single value in $]^{-}0, 1^{+}[$ to represent the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of a element in the universe X. Then, Deli et al. [23] introduced the concept of bipolar neutrosophic sets, as an extension of neutrosophic sets. In the bipolar neutrosophic sets, the positive membership degree $T^{+}(x)$, $I^{+}(x)$, $F^{+}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^{-}(x)$, $I^{-}(x)$, $F^{-}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Similarity measure is an important tool in constructing multi-criteria decision making methods in many areas such as medical diagnosis, pattern recognition, clustering analysis, decision making and so on. Similarity measures under all sorts of fuzzy environments including single-valued neutrosophic environments have been studied by many researchers in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 26, 31, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 79, 80, 81, 82, 83, 84, 85, 86, 85, 86, 87, 88, 89, 90, 65, 91, 92, 93, 94, 95, 96]. Also, S¸ahin et al.[72] presented a similarity measure on bipolar neutrosophic sets based on Jaccard vector similarity measure of neutrosophic set and applied to a decision making problem.

This paper is constructed as follows. In Sect. 2, some basic definitions of neutrosophic sets and bipolar neutrosophic sets are introduced. In Sect. 3, we propose the outranking relations of bipolar neutrosophic sets and investigate its several proprieties. In Sect. 4, an outranking approach for MCDM with simplified bipolar neutrosophic information is given. In Sect. 5, Illustrative examples is given. In Sect. 6, the conclusions are summarized.

2 Preliminary

In the subsection, we give some concepts related to neutrosophic sets and bipolar neutrosophic sets.

Definition 2.1. [67] Let E be a universe. A neutrosophic sets A over E is defined by

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E\}.$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by

$$T_A : E \rightarrow]^{-}0, 1^{+}[, \quad I_A : E \rightarrow]^{-}0, 1^{+}[, \quad F_A : E \rightarrow]^{-}0, 1^{+}[$$

such that $0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.2. [73] Let E be a universe. An single valued neutrosophic set (SVN-set) over E is a neutrosophic set over E, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$T_A : E \rightarrow [0, 1], \quad I_A : E \rightarrow [0, 1], \quad F_A : E \rightarrow [0, 1]$$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3. [23] A bipolar neutrosophic set A in X is defined as an object of the form

$$A = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\rangle : x \in X\}.$$

where

$$T^+, I^+, F^+ : E \to [0,1], T^-, I^-, F^- : X \to [-1,0].$$

The positive membership degree $T^+(x), I^+(x), F^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^-(x), I^-(x), F^-(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Definition 2.4. [23] Let $A_1 = \langle x, T_1^+(x), I_1^+(x), F_1^+(x), T_1^-(x), I_1^-(x), F_1^-(x)\rangle$ and $A_2 = \langle x, T_2^+(x), I_2^+(x), F_2^+(x), T_2^-(x), I_2^-(x), F_2^-(x)\rangle$ be two bipolar neutrosophic sets in a universe of discourse X, then the following operations are defined as follows:

1. $A_1 = A_2$ if and only if $T_1^+(x) = T_2^+(x), I_1^+(x) = I_2^+(x), F_1^+(x) = F_2^+(x)$ and $T_1^-(x) = T_2^-(x), I_1^-(x) = I_2^-(x), F_1^-(x) = F_2^-(x)$.

2. $$\begin{aligned} A_1 \cup A_2 = \; & \{\langle x, max(T_1^+(x), T_2^+(x)), \tfrac{I_1^+(x)+I_2^+(x)}{2}, min(F_1^+(x), F_2^+(x)), \\ & min(T_1^-(x), T_2^-(x)), \tfrac{I_1^-(x)+I_2^-(x)}{2}, max(F_1^-(x), F_2^-(x))\rangle\} \end{aligned}$$
   $\forall x \in X.$

3. $$\begin{aligned} A_1 \cap A_2 = \; & \{\langle x, min(T_1^+(x), T_2^+(x)), \tfrac{I_1^+(x)+I_2^+(x)}{2}, max(F_1^+(x), F_2^+(x)), \\ & max(T_1^-(x), T_2^-(x)), \tfrac{I_1^-(x)+I_2^-(x)}{2}, min(F_1^-(x), F_2^-(x))\rangle\} \end{aligned}$$
   $\forall x \in X.$

4. $$A^c = \; \{\langle x, 1 - T_A^+(x), 1 - I_A^+(x), 1 - F_A^+(x), 1 - T_A^-(x), 1 - I_A^-(x), 1 - F_A^-(x)\rangle\}$$

5. $A_1 \subseteq A_2$ if and only if $T_1^+(x) \le T_2^+(x), I_1^+(x) \le I_2^+(x), F_1^+(x) \ge F_2^+(x)$ and $T_1^-(x) \ge T_2^-(x), I_1^-(x) \ge I_2^-(x), F_1^-(x) \le F_2^-(x)$.

Definition 2.5. [23] Let $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-\rangle$ and $\tilde{a}_2 = \langle T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^-\rangle$ be two bipolar neutrosophic number . Then the operations for BNNs are defined as below;

$i.$ $\lambda\tilde{a}_1 = \langle 1-(1-T_1^+)^\lambda, (I_1^+)^\lambda, (F_1^+)^\lambda, -(-T_1^-)^\lambda, -(-I_1^-)^\lambda, -(1-(1-(-F_1^-))^\lambda)\rangle$

$ii.$ $\tilde{a}_1^\lambda = \langle (T_1^+)^\lambda, 1-(1-I_1^+)^\lambda, 1-(1-F_1^+)^\lambda, -(1-(1-(-T_1^-))^\lambda), -(-I_1^-)^\lambda, -(-F_1^-)^\lambda\rangle$

$iii.$ $\tilde{a}_1 + \tilde{a}_2 = \langle T_1^+ + T_2^+ - T_1^+T_2^+, I_1^+I_2^+, F_1^+F_2^+, T_1^-T_2^-, -(-I_1^- - I_2^- - I_1^-I_2^-), -(-F_1^- - F_2^- - F_1^-F_2^-)\rangle$

$iv.$ $\tilde{a}_1 + \tilde{a}_2 = \langle T_1^+T_2^+, I_1^+ + I_2^+ - I_1^+I_2^+, F_1^+ + F_2^+ - F_1^+F_2^+, -(-T_1^- - T_2^- - T_1^-T_2^-, -I_1^-I_2^-, -F_1^-F_2^-\rangle$

where $\lambda > 0$.

Definition 2.6. [32] Let $A = \langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle$ and

$B = \langle T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ be any two SVNSs, then the normalized Euclidean distance between A and B can be defined as follows:

$$d(A,B) = \sqrt{\frac{1}{3n}(|\tilde{T}_A - \tilde{T}_B|^2 + |\tilde{I}_A - \tilde{I}_B|^2 + |\tilde{F}_A - \tilde{F}_B|^2}.$$

3 The outranking relations of Bipolar Neutrosophic Sets

In this section, The binary relations between two bipolar neutrosophic sets that are based on ELECTRE are now defined.

Definition 3.1. Let $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$ and $B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ be two BNSs in the set $X = \{x_1, x_2, ..., x_n\}$. Then,then the strong dominance relation, weak dominance relation, and indifference relation of BNSs can be defined as follows:

1. If $T_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- > F_B^-$ or $T_A^+ > T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- < T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$,then A strongly dominates B (B is strongly dominated by A), denoted by $A \succ_s B$.

2. If $T_A^+ \geq T_B^+, I_A^+ \geq I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- \leq I_B^-, F_A^- > F_B^-$ or $T_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ \geq F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- \leq F_B^-$,then A weakly dominates B (B is weakly dominated by A), denoted by $A \succ_w B$.

3. If $T_A^+ = T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- = T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$,then A is indifferent to B, denoted by$A \sim_l B$.

4. If none of the relations mentioned above exist between A and B for any $x \in X$, then A and B are incomparable, denoted by $A \perp B$.

Proposition 3.2. *Let $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$ and $B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ be two BNSs in the set $X = \{x_1, x_2, ..., x_n\}$, then the following properties can be obtained:*

1. *If $B \subset A$,then $A \succ_s B$;*

2. *If $A \succ_s B$, then $B \subseteq A$;*

3. *$A \sim_l B$ if and only if $A = B$.*

Proof. 1. If $B \subset A$,then $T_B^+ < T_A^+, I_B^+ > I_A^+, F_B^+ > F_A^+, T_B^- > T_A^-, I_A^- < I_B^-, F_B^- < F_A^-$. $A \succ_s B$ is definitely validated according to the strong dominance relation in Definition 3.1.

2. $A \succ_s B$, then based on Definition 3.1,$T_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- > F_B^-$ or $T_A^+ > T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- < T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$ are realized. From Definition 2.4.

3. Necessity:$A \sim_l B \Rightarrow A = B$. According to the indifference relation in Definition 3.1 it is known that it is known that$T_A^+ = T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- = T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$. Clearly $A \subseteq B$ and $B \subseteq A$ are achieved, then $A = B$.

   Sufficiency:$A = B \Rightarrow A \sim_l B$. If $A = B$,then it is known that $A \subseteq B$ and $B \subseteq A$, which means $T_A^+ \leq T_B^+, I_A^+ \geq I_B^+, F_A^+ \geq F_B^+, T_A^- \geq T_B^-, I_A^- \leq I_B^-, F_A^- \leq F_B^-$ and $T_A^+ \geq T_B^+, I_A^+ \leq I_B^+, F_A^+ \leq F_B^+, T_A^- \leq T_B^-, I_A^- \geq I_B^-, F_A^- \geq F_B^-$; then $T_A^+ = T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- = T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$, are obtained. Due to the indifference relation in Definition 3.1, $A \sim_l B$ is definitely validated.

□

Proposition 3.3. *Let* $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$, $B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ *and* $C = \langle T_C^+(x_i), I_C^+(x_i), F_C^+(x_i), T_C^-(x_i), I_C^-(x_i), F_C^-(x_i)\rangle$ *and be three BNSs in the set* $X = \{x_1, x_2, ..., x_n\}$, *if* $A \succ_s B$ *and* $B \succ_s C$,*then* $A \succ_s C$.

Proof. According to the strong dominance relation in Definition3.1,if $A \succ_s B$, then $T_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- > F_B^-$ or $T_A^+ > T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- < T_B^-, I_A^- = I_B^-, F_A^- = F_B^-$. If $B \succ_s C$, then $T_B^+ \geq T_C^+, I_B^+ < I_C^+, F_B^+ < F_C^+, T_B^- \leq T_C^-, I_B^- > I_C^-, F_B^- > F_C^-$ or $T_B^+ > T_C^+, I_B^+ = I_C^+, F_B^+ = F_C^+, T_B^- < T_C^-, I_B^- = I_C^-, F_B^- = F_C^-$. Therefore the further derivations are:

$$IfT_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- > F_B^- \quad (1)$$

$$T_B^+ \geq T_C^+, I_B^+ < I_C^+, F_B^+ < F_C^+, T_B^- \leq T_C^-, I_B^- > I_C^-, F_B^- > F_C^- \quad (2)$$

from (1) and (2)

$$T_A^+ \geq T_C^+, I_A^+ < I_C^+, F_A^+ < F_C^+, T_A^- \leq T_C^-, I_A^- > I_C^-, F_A^- > F_C^-$$

then based on Definition 3.1$A \succ_s C$ is realized.

$$IfT_A^+ \geq T_B^+, I_A^+ < I_B^+, F_A^+ < F_B^+, T_A^- \leq T_B^-, I_A^- > I_B^-, F_A^- > F_B^- \quad (3)$$

$$T_B^+ > T_C^+, I_B^+ = I_C^+, F_B^+ = F_C^+, T_B^- < T_C^-, I_B^- = I_C^-, F_B^- = F_C^- \quad (4)$$

from (3) and (4)

$$T_A^+ > T_C^+, I_A^+ = I_C^+, F_A^+ = F_C^+, T_A^- < T_C^-, I_A^- = I_C^-, F_A^- = F_C^-$$

then based on Definition 3.1$A \succ_s C$ is achieved.

$$IfT_A^+ > T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- < T_B^-, I_A^- = I_B^-, F_A^- = F_B^- \quad (5)$$

$$T_B^+ \geq T_C^+, I_B^+ < I_C^+, F_B^+ < F_C^+, T_B^- \leq T_C^-, I_B^- > I_C^-, F_B^- > F_C^- \quad (6)$$

from (5) and (6)

$$T_A^+ > T_C^+, I_A^+ < I_C^+, F_A^+ < F_C^+, T_A^- < T_C^-, I_A^- > I_C^-, F_A^- > F_C^-$$

then based on Definition 3.1$A \succ_s C$ is obtained.

$$IfT_A^+ > T_B^+, I_A^+ = I_B^+, F_A^+ = F_B^+, T_A^- < T_B^-, I_A^- = I_B^-, F_A^- = F_B^- \quad (7)$$

$$T_B^+ \geq T_C^+, I_B^+ = I_C^+, F_B^+ = F_C^+, T_B^- \leq T_C^-, I_B^- = I_C^-, F_B^- = F_C^- \quad (8)$$

from (7) and (8)

$$T_A^+ > T_C^+, I_A^+ = I_C^+, F_A^+ = F_C^+, T_A^- < T_C^-, I_A^- = I_C^-, F_A^- = F_C^-$$

then based on Definition 3.1 $A \succ_s C$ is realized. Therefore, if $A \succ_s B$ and $B \succ_s C$,then $A \succ_s C$. □

Proposition 3.4. *Let* $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$,
$B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ *and* $C = \langle T_C^+(x_i), I_C^+(x_i), F_C^+(x_i),$ $T_C^-(x_i), I_C^-(x_i), F_C^-(x_i)\rangle$ *and be three BNSs in the set* $X = \{x_1, x_2, ..., x_n\}$*, if* $A \sim_l B$ *and* $B \sim_l C$*,then* $A \sim_l C$.

Proof. Clearly,if $A \sim_l B$ and $B \sim_l C$,then $A \sim_l C$ is surely validated. □

Proposition 3.5. *Let* $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$,
$B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ *and* $C = \langle T_C^+(x_i), I_C^+(x_i), F_C^+(x_i),$ $T_C^-(x_i), I_C^-(x_i), F_C^-(x_i)\rangle$ *and be three BNSs in the set* $X = \{x_1, x_2, ..., x_n\}$*, then the following results can be achieved.*

(1).*The strong dominance relations are categorized into:*

$$1.irreflexivity : \forall A \in BNSs, A \not\succ_s A;$$
$$2.asymmetry : \forall A, B \in BNSs, A \succ_s B \Rightarrow B \not\succ_s A;$$
$$3.transitivity : \forall A, B, C \in BNSs, A \succ_s B, B \succ_s C \Rightarrow A \succ_s C.$$

(2).*The weak dominance relations are categorized into:*

$$4.irreflexivity : \forall A \in BNSs, A \not\succ_w A;$$
$$5.asymmetry : \forall A, B \in BNSs, A \succ_w B \Rightarrow B \not\succ_w A;$$
$$6.non-transitivity \exists A, B, C \in BNSs, A \succ_w B, B \succ_w C \Rightarrow A \succ_w C.$$

(3).*The indifference relations are categorized into:*

$$7.reflexivity : \forall A \in BNSs, A \sim_l A;$$
$$8.symmetry : \forall A, B \in BNSs, A \sim_l B \Rightarrow B \sim_l A;$$
$$9.transitivity \exists A, B, C \in BNSs, A \sim_l B, B \sim_l C \Rightarrow A \sim_l C.$$

According to Definition 3.1,it is clear that $3, 7, 8$ *and* 9 *are true, and* $1, 2, 4, 5$ *and* 6 *need to be proven.*

Example 3.6. $1, 2, 4, 5$ and 6 are exemplified as follows.

1. If $A = \langle 0.5, 0.3, 0.1, -0.6, -0.4, -0.2\rangle$ is a BNSs, then $A \not\succ_s A$ can be obtained.

2. If $A = \langle 0.7, 0.4, 0.2, -0.5, -0.2, -0.1\rangle$ and $B = \langle 0.6, 0.5, 0.3, -0.4, -0.3, -0.2\rangle$ are two BNSs, then $A \succ_s B$,but $B \not\succ_s A$ is achieved.

3. If $A = \langle 0.5, 0.3, 0.1, -0.6, -0.4, -0.2\rangle$ is a BNSs, then $A \not\succ_w A$ is realized.

4. If $A = \langle 0.8, 0.5, 0.2, -0.6, -0.3, -0.3\rangle$ and $B = \langle 0.5, 0.5, 0.3, -0.4, -0.4, -0.2\rangle$ are two BNSs, then $A \succ_w B$ is obtained, however $B \not\succ_w A$.

5. If $A = \langle 0.8, 0.5, 0.4, -0.6, -0.5, -0.3\rangle$, $B = \langle 0.7, 0.2, 0.5, -0.5, -0.3, -0.2\rangle$ and $C = \langle 0.7, 0.4, 0.4, -0.3, -0.3, -0.1\rangle$ are three BNSs, then $A \succ_w B$ and $B \succ_w C$ are achieved, however $A \perp C$.

Proposition 3.7. *Let x_1 and x_2 be two actions, the performances for actions x_1 and x_2 be in the form of BNSs, and $P = s \cup w \cup l$ mean that " x_1 is at least as good as x_2",then four situations may arise:*

1. *$x_1 P x_2$ and not $x_2 P x_1$,that is $x_1 \succ_s x_2$ or $x_1 \succ_w x_2$;*
2. *$x_2 P x_1$ and not $x_1 P x_2$,that is $x_2 \succ_s x_1$ or $x_2 \succ_w x_1$;*
3. *$x_1 P x_2$ and $x_2 P x_1$,that is $x_1 \sim_l x_2$*
4. *not $x_1 P x_2$ and not $x_2 P x_1$,that is $x_1 \perp x_2$.*

Definition 3.8. Let $A = \langle T_A^+(x_i), I_A^+(x_i), F_A^+(x_i), T_A^-(x_i), I_A^-(x_i), F_A^-(x_i)\rangle$, and $B = \langle T_B^+(x_i), I_B^+(x_i), F_B^+(x_i), T_B^-(x_i), I_B^-(x_i), F_B^-(x_i)\rangle$ and and be two BNSs, then the normalized Euclidean distance between A and B can be defined as follows:

$$d(A,B) = \sqrt{\frac{1}{6n}[(|T_A^+ - T_B^+|^2 + |I_A^+ - I_B^+|^2 + |F_A^+ - F_B^+|^2) - (|T_A^- - T_B^-|^2 + |I_A^- - I_B^-|^2 + |F_A^- - F_B^-|^2)]}.$$

Proposition 3.9. *Let $d(A,B)$ be a normalized Euclidean distance between bipolar neutrosophic sets A and B. Then, we have*

1. $0 \le d(A,B) \le 1$;
2. $d(A,B) = d(B,A)$;
3. $d(A,B) = 1$ *for* $A = B$ *i.e.,*$T_A^+(x_i) = T_B^+(x_i), I_A^+(x_i) = I_B^+(x_i), F_A^+(x_i) = F_B^+(x_i), T_A^-(x_i) = T_B^-(x_i), I_A^-(x_i) = I_B^-(x_i), F_A^-(x_i) = F_B^-(x_i) (i = 1,2...,n)\ \forall x_i (i = 1,2,...,n) \in X$.

Proof. 1. It is clear from Definition 2.3.
2.

$$\begin{aligned} d(A,B) &= \sqrt{\tfrac{1}{6n}[(|T_A^+ - T_B^+|^2 + |I_A^+ - I_B^+|^2 + |F_A^+ - F_B^+|^2) - (|T_A^- - T_B^-|^2 + |I_A^- - I_B^-|^2 + |F_A^- - F_B^-|^2)]} \\ &= \sqrt{\tfrac{1}{6n}[(|T_B^+ - T_A^+|^2 + |I_B^+ - I_A^+|^2 + |F_A^+ - F_A^+|^2) - (|T_B^- - T_A^-|^2 + |I_B^- - I_A^-|^2 + |F_B^- - F_A^-|^2)]} \\ &= d(B,A) \end{aligned}$$

3. Since $T_A^+(x_i) = T_B^+(x_i), I_A^+(x_i) = I_B^+(x_i), F_A^+(x_i) = F_B^+(x_i), T_A^-(x_i) = T_B^-(x_i), I_A^-(x_i) = I_B^-(x_i), F_A^-(x_i) = F_B^-(x_i) (i = 1,2...,n)\ \forall x_i (i = 1,2,...,n) \in X$, we have $d(A,B) = 1$.
The proof is completed. $\square$

4 An outranking approach for MCDM with simplified bipolar neutrosophic information

Definition 4.1. The MCDM ranking/selection problems with simplified BNSs information consist of a group of alternatives, denoted by $U = (u_1, u_2, ..., u_n)$ be a set of alternatives, $A = (a_1, a_2, ..., a_m)$ be the set of attributes, $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of the attributes $C_j (j = 1,2,...,n)$ such that $w_j \ge 0$

and $\sum_{j=1}^{n} = 1$ and $b_{ij} = \langle T_{ij}^+, I_{ij}^+, F_{ij}^+, T_{ij}^-, I_{ij}^-, F_{ij}^- \rangle$ be the decision matrix in which the rating values of the alternatives. Then,

$$[b_{ij}]_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \end{array} & \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix} \end{array}$$

is called an NB-multi-attribute decision making matrix of the decision maker.

This method is an integration of BNSs and the outranking method to manage the MCDM problems mentioned above. In general, there are benefit criteria and cost criteria in MCDM problems and the cost-type criterion values can be transformed into benefit-type criterion values as follows:

$$\beta_{ij} = \begin{cases} b_{ij} & for\ benefit\ criterion\ a_j, \quad (i = 1,2,...,m; j = 1,2,..,n) \\ (b_{ij})^c & for\ cost\ criterion\ a_j, \quad ...(9) \end{cases}$$

here $(b_{ij})^c$ is complement of b_{ij} as defined in Definition 2.4.

The analysis given above indicates that both c_{ik} and d_{ik} include the weights of the criteria and the outranking relations among the alternatives. However, they measure different aspects of the relations, and the concordance indices and discordance indices are therefore not complementary.

To rank all alternatives, the net dominance index of b_k

$$c_k = \sum_{i=1;i\neq k}^{n} c_{ik} - \sum_{i=1;i\neq k}^{n} c_{ki}, ...(10)$$

and the net disadvantage index of b_k is

$$d_k = \sum_{i=1;i\neq k}^{n} d_{ik} - \sum_{i=1;i\neq k}^{n} d_{ki}, ...(11)$$

Here, c_k is the sum of the concordance indices between b_k and $b_k(i \neq k)$ minus the sum of the concordance indices between $b_k(i \neq k)$ and b_k, and reflects the dominance degree of the alternative b_k among the relevant alternatives. Meanwhile, d_k reflects the disadvantage degree of the alternative b_k among the relevant alternatives. Therefore, b_k obtains a greater dominance over the other alternatives that are being compared as c_k increases and d_k decreases.

Definition 4.2. The ranking rules of two alternatives are

$$\begin{array}{rl} i. & if\ c_i < c_k\ and\ d_i > d_k,\ then\ b_k\ is\ superior\ to\ b_i,\ as\ denoted\ by\ b_k \succ b_i; \\ ii. & if\ c_i = c_k\ and\ d_i = d_k,\ then\ b_k is\ indifferent\ to b_i, as\ denoted\ by\ b_k \sim b_i; \\ iii. & if\ the\ relation\ between\ b_k\ and\ b_i\ does\ not\ belong\ to\ (i)\ or\ (ii), \\ & then\ b_k\ and\ b_i\ are\ incomparable,\ as\ denoted\ by b_k \perp b_i. \end{array}$$

A ranking of alternatives obtained by the rules defined above may be only a partial ranking, and greater detail is discussed by Wu and Chen [76]

It is now feasible to develop a new approach for the MCDM problems mentioned above.

Algorithm:

Step 1. Give the decision-making matrix $[b_{ij}]_{m\times n}$; for decision; The BNSs decision matrix $R = [b_{ij}]_{m\times n}$ can be transformed into a normalized BNSs decision matrix $R = [\beta_{ij}]_{m\times n}$ based on Eq. (9).

Step 2. Determine the weighted normalized matrix. According to the weight vector for the criteria, the weighted normalized decision matrix can be constructed using the following formula:

$$\gamma_{ij} = \beta_{ij} w_j, \qquad i = 1, 2, ..., m; j = 1, 2, ..., n.$$

where w_j is the weight of the j th criterion with $\sum_{j=1}^{n} w_j = 1$.

Step 3. Determine the concordance and discordance set of subscripts. The concordance set of subscripts, which should satisfy the constraint $b_{ij}Pb_{kj}$, is represented as:

$$O_{ik} = \{j | b_{ij}Pb_{kj} \quad (i, k = 1, 2, ..., m).$$

$b_{ij}Pb_{kj}$ represents $b_{ij} >_s b_{kj}$ or $b_{ij} >_w b_{kj}$ or $b_{ij} \sim b_{kj}$.

The discordance set of subscripts for criteria is the complementary subset, therefore:

$$D_{ik} = J - O_{ik}.$$

Step 4. Determine the concordance and discordance matrix. By using the weight vector w that is associated with the criteria, the concordance index $C(b_i, b_k)$ is represented as:

$$C(b_i, b_k) = \sum_{j\in O_{ik}} w_j.$$

Thus, the concordance matrix C is:

$$C = \begin{pmatrix} - & c_{12} & \cdots & c_{1n} \\ c_{21} & - & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & - \end{pmatrix}$$

The discordance index $D(b_i, b_k)$ is represented as:

$$D_{ik} = \frac{\max_{j\in D_{ik}}\{d(b_{ij}, b_{kj})\}}{\max_{j\in J}\{d(b_{ij}, b_{kj})\}}$$

here $d(b_{ij}, b_{kj})$ denotes the normalized Euclidean distance between b_{ij} and b_{kj} as defined in Definition 3.8.

Thus, the discordance matrix D is:

$$D = \begin{pmatrix} - & d_{12} & \cdots & d_{1n} \\ d_{21} & - & \cdots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & - \end{pmatrix}$$

Step 5. Calculate the net dominance index of each alternative c_i (i=1,2,...,m) based on Formula (10), , and the net disadvantage index of each alternative d_i (i=1,2,...,m) based on Formula (11).

Step 6. Formulate the ranking of all alternatives in light of the rules given by Definition 4.2

5 Illustrative examples

In this section, an example for a MCDM problem with simplified bipolar neutrosophic information.

Example 5.1. ([37]).There is an investment company, which wants to invest a sum of money in the best option. This company has set up a panel which has to choose between four possible alternatives for investing the money:(1) b_1 is a car company; (2) b_2 is a food company; (3) b_3 is a computer company; and (4) b_4 is an arms company. The investment company must make a decision using the following three criteria: (1) a_1 is the risk; (2) a_2 is the growth; and (3) a_3 is the customer satisfaction; these are all benefit type criteria. The weight vector of the criteria is represented by $w = \{0.45, 0.15, 0.4\}$. The four possible alternatives are to be evaluated under the above three criteria in the form of BNNs for each decision-maker, as shown in the following simplified bipolar neutrosophic decision matrix R:

$$R = \begin{pmatrix} \langle 0.7, 0.5, 0.3, -0.3, -0.4, -0.5\rangle & \langle 0.8, 0.6, 0.1, -0.5, -0.3, -0.2\rangle & \langle 0.4, 0.6, 0.5, -0.2, -0.6, -0.4\rangle \\ \langle 0.6, 0.1, 0.4, -0.4, -0.3, -0.6\rangle & \langle 0.6, 0.1, 0.3, -0.4, -0.3, -0.1\rangle & \langle 0.5, 0.7, 0.3, -0.1, -0.2, -0.5\rangle \\ \langle 0.8, 0.6, 0.8, -0.3, -0.2, -0.1\rangle & \langle 0.9, 0.4, 0.5, -0.5, -0.3, -0.6\rangle & \langle 0.3, 0.4, 0.5, -0.2, -0.3, -0.4\rangle \\ \langle 0.8, 0.3, 0.1, -0.4, -0.2, -0.1\rangle & \langle 0.6, 0.1, 0.4, -0.3, -0.2, -0.3\rangle & \langle 0.6, 0.5, 0.6, -0.3, -0.4, -0.6\rangle \end{pmatrix}$$

The procedures for obtaining the best alternative are now outlined.

Step 1. Transform the decision matrix.

Since all the criteria are of the benefit type,$R^{'} = R$ can be obtained.

Step 2. Determine the weighted normalized matrix.

$$R^{'} = \begin{pmatrix} \langle 0.4128, 0.7320, 0.5817, -0.5817, -0.6621, -0.2679\rangle \\ \langle 0.3378, 0.3548, 0.6621, -0.6621, -0.5817, -0.3378\rangle \\ \langle 0.5153, 0.7946, 0.9044, -0.5817, -0.4846, -0.0463\rangle \\ \langle 0.5153, 0.5817, 0.3548, -0.6621, -0.4846, -0.0463\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.2144, 0.9262, 0.7079, -0.9012, -0.8347, -0.0329\rangle \\ \langle 0.1284, 0.7079, 0.8347, -0.8715, -0.8347, -0.0156\rangle \\ \langle 0.2920, 0.8715, 0.9012, -0.9012, -0.8347, -0.1284\rangle \\ \langle 0.1284, 0.7079, 0.8715, -0.8347, -0.7855, -0.0521\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.1848, 0.8151, 0.7578, -0.5253, -0.8151, -0.1848\rangle \\ \langle 0.2421, 0.8670, 0.6178, -0.3981, -0.5253, -0.2421\rangle \\ \langle 0.1329, 0.6931, 0.7578, -0.5253, -0.6178, -0.1848\rangle \\ \langle 0.3068, 0.7578, 0.8151, -0.6178, -0.6931, -0.3068\rangle \end{pmatrix}$$

Step 3. Determine the concordance and discordance set of subscripts.

The concordance set of subscripts is obtained as follows:

$$O_{12} = \{1,2\}; O_{21} = \{3\}; O_{31} = \{2\}; O_{41} = \{1,3\}; O_{13} = \{3\}; O_{23} = \{3\};$$

$$O_{32} = \{\}; O_{42} = \{1,2,3\}; O_{14} = \{2\}; O_{24} = \{2\}; O_{34} = \{\}; O_{43} = \{1,2,3\}.$$

The discordance set of subscripts is obtained as follows:

$$D_{12} = \{3\}; D_{21} = \{1,2\}; D_{31} = \{1,3\}; D_{41} = \{2\}; D_{13} = \{1,2\}; D_{23} = \{1,2\};$$

$$D_{32} = \{1,2,3\}; D_{42} = \{\}; D_{14} = \{1,3\}; D_{24} = \{1,3\}; D_{34} = \{1,2,3\}; D_{43} = \{\}.$$

where $\{\}$ denotes ”empty”.

Step 4. Determine the concordance and discordance matrix.

With regard to the weight vector w associated with the criteria, the concordance index is represented as follows:

$$C = \begin{pmatrix} - & 0.60 & 0.40 & 0.15 \\ 0.40 & - & 0.40 & 0.15 \\ 0.15 & 0 & - & 0 \\ 0.85 & 1 & 1 & - \end{pmatrix}$$

The discordance index can be calculated as follows. For example,

$$D_{21} = \frac{max\{d(b_{21}, b_{11}), d(b_{22}, b_{12})\}}{max\{d(b_{21}, b_{11}), d(b_{22}, b_{12}), d(b_{23}, b_{13})\}} = \frac{0.10334}{0.31501} = 0.3280$$

Here:

$$\begin{aligned} d(b_{21}, b_{11}) &= (\tfrac{1}{6}|((0.3378 - 0.4182)^2 + (0.3548 - 0.7320)^2 + (0.6621 - 0.5817)^2) \\ &-((-0.6621 - (-0.5817))^2 + (-0.5817 - (-0.6621))^2 + (-0.3378 - (-0.2679))^2)|)^{\frac{1}{2}} \\ &= 0.08973 \end{aligned}$$

$$\begin{aligned} d(b_{22}, b_{12}) &= (\tfrac{1}{6}|((0.1284 - 0.2144)^2 + (0.7079 - 0.9262)^2 + (0.8347 - 0.7079)^2) \\ &-((-0.8715 - (-0.9012))^2 + (-0.8347 - (-0.8347))^2 + (-0.0156 - (-0.0329))^2)|)^{\frac{1}{2}} \\ &= 0.10334; \end{aligned}$$

and

$$\begin{aligned} d(b_{23}, b_{13}) &= (\tfrac{1}{6}|((0.2421 - 0.1848)^2 + (0.8670 - 0.8151)^2 + (0.6178 - 0.7578)^2) \\ &-((-0.3981 - (-0.5253))^2 + (-0.5253 - (-0.8151))^2 + (-0.2421 - (-0.1848))^2)|)^{\frac{1}{2}} \\ &= 0.31501; \end{aligned}$$

Therefore, the discordance index matrix is as follows:

$$D = \begin{pmatrix} - & 0,6230 & 1 & 1 \\ 0.3280 & - & 1 & 1 \\ 1 & 1 & - & 1 \\ 1 & 0 & 0 & - \end{pmatrix}$$

Step 5. Based on Formulae (10) and (11), the net dominance index of each alternative c_i (i=1,2,3,4) and the net disadvantage index of each alternative d_i (i=1,2,3,4) can be obtained as shown below:

$$c_1 = -0.25, c_2 = -0.65, c_3 = -1.65 \; and \; c_4 = 2.55, \; \Rightarrow \; c_3 < c_2 < c_1 < c_4;$$

$$d_1 = 0.295, d_2 = 0.705, d_3 = 1 \; and \; d_4 = -3 \; \Rightarrow \; d_3 > d_2 > d_1 > d_4.$$

Step 6. According to the rules of Definition 4.2, the final ranking is $b_4 \succ b_1 \succ b_2 \succ b_3$, and the best alternative is b_4.

6 Conclusions

This paper developed a multi-criteria decision making method for bipolar neutrosophic set is developed based on these given the outranking relations. The contribution of this study is that the proposed approach is simple and convenient with regard to computing,and effective in decreasing the loss of evaluative information. More effective decision methods of this proposes a new outranking approach will be investigated in the near future and applied these concepts to engineering, game theory, multi-agent systems, decision-making and so on.

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