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An Evidence Fusion Method with Importance Discounting Factors based on Neutrosophic Probability Analysis in DSmT Framework

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Abstract:
To obtain effective fusion results of multi source evidences with different importance, an evidence fusion method with importance discounting factors based on neutrosopic probability analysis in DSmT framework is proposed. First, the reasonable evidence sources are selected out based on the statistical analysis of the pignistic probability functions of single focal elements. Secondly, the neutrosophic probability analysis is conducted based on the similarities of the pignistic probability functions from the prior evidence knowledge of the reasonable evidence sources. Thirdly, the importance discounting factors of the reasonable evidence sources are obtained based on the neutrosophic probability analysis and the reliability discounting factors of the real-time evidences are calculated based on probabilistic-based distances. Fourthly, the real-time evidences are discounted by the importance discounting factors and then the evidences with the mass assignments of neutrosophic empty sets are discounted by the reliability discounting factors. Finally, DSmT+PCR5 of importance discounted evidences is applied. Experimental examples show that the decision results based on the proposed fusion method are different from the results based on the existed fusion methods. Simulation experiments of recognition fusion are performed and the superiority of proposed method is testified well by the simulation results.

Keywords: Information fusion; Belief function; Dezert-Smarandache Theory; Neutrosophic probability; Importance discounting factors.

1. Introduction

As a high-level and commonly applicable key technology, information fusion can integrate partial information from multisource, and decrease potential redundant and incompatible information between different sources, thus reducing uncertainties and improving the quick and correct decision ability of high intelligence systems. It has drawn wide attention attention by scholars and has found many successful applications in the military and economy fields in recent years [1-9]. With the increment of information environmental complexity, effective highly conflict evidence reasoning has huge demands on information fusion. Belief function also called evidence theory which includes Dempster- Shafer theory (DST) and Dezert-Smarandache theory (DSmT) has made great efforts and contributions to solve this problem. Dempster-Shafer theory (DST) [10,11] has been commonly applied in information fusion field since it can represent uncertainty and full ignorance effectively and includes Bayesian theory as a special case. Although very attractive, DST has some limitations, especially in dealing with highly conflict evidences fusion [9]. DSmT, jointly proposed by Dezert and Smarandache, can be considered as an extension of DST. DSmT can solve the complex fusion problems beyond the exclusive limit of the DST discernment framework and it can get more reasonable fusion results when multisource evidences are highly conflicting and the refinement of the discernment framework is unavailable. Recently, DSmT has many successful applications in many areas, such as, Map Reconstruction of Robot [12,13], Clustering [14,15], Target Type Tracking [16,17], Image Processing [18], Data Classification [19-21], Decision Making Support [22], Sonar Imagery [23], and so on. Recently the research on the discounting factors based on DST or DSmT have been done by many scholars [24,25]. Smarandache and et al [24] put forward that discounting factors in the procedure of evidence fusion should conclude importance discounting factors and reliability discounting factors, and they also proved that effective fusion could not be carried out by Dempster combination rules when the importance discounting factors were considered. However, the method for calculating the importance discounting factors was not mentioned. A method for calculating importance or reliability discounting factors was proposed in article [25]. However, the importance and reliability discounting factors could not be distinguished and the focal element of empty set or full ignorance was processed based on DST. As the exhaustive limit of DST, it could not process empty set effectively. So, the fusion results based on importance and reliability discounting factors are the same in [25], which is not consist with real situation. In this paper, an evidence fusion method with importance discounting factors based on neutrosophic probability analysis in DSmT framework is proposed. In Section 2, basic theories including DST, DSmT and the dissimilarity measure of evidences are introduced briefly. In Section 3, the contents and procedure of the proposed fusion method are given. In Section 4, simulation experiments in the application background of recognition fusion are also performed for testifying the superiority of proposed method. In Section 5, the conclusions are given.

2. Basic Theories

2.1. DST

Let $\Theta = \{\theta_1, \theta_2, L, \theta_n\}$ be the discernment frame having n exhaustive and exclusive hypotheses $\theta_i, i = 1, 2, L, n$. The exhaustive and exclusive limits of DST assume that the refinement of the fusion problem is accessible and the hypotheses are precisely defined. The set of all subsets of Θ, denoted by 2^Θ, is defined as the power set of Θ. 2^Θ is under closed-world assumption. If the discernment frame Θ is defined as above, the power set can be obtained as follows [10,11]:

$$2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, L, \{\theta_n\}, \{\theta_1, \theta_2\}, L, \{\theta_1, \theta_2, L, \theta_n\}\}. \quad (1)$$

In Shafer's model, a basic belief assignment (bba) $m(.): 2^\Theta \to [0,1]$ which consists evidences is defined by

$$m_k(\emptyset) = 0 \text{ and } \sum_{A \in 2^\Theta} m(a) = 1. \quad (2)$$

The DST rule of combination (also called the Dempster combination rule) can be considered as a conjunctive normalized rule on the power set 2^Θ. The fusion results based on the Dempster combination rule are obtained by the bba's products of the focal elements from different evidences which intersect to get the focal elements of the results. DST also assumes that the evidences are independent. The i^{th} evidence source's bba is denoted m_i. The Dempster combination rule is given by [10,11]:

$$(m_1 \oplus m_2)(C) = \frac{1}{1-K} \sum_{AI\, B=C} m_1(A) m_2(B), \forall C \subseteq \Theta \quad (3)$$

$$K = \sum_{\substack{A,B \subseteq \Theta \\ AI\, B=\emptyset}} m_1(A) m_2(B) \quad (4)$$

In some applications of multisource evidences fusion, some evidences influenced by the noise or some other conditions are highly conflicting with the other evidences. The reliability of an evidence can represent its accuracy degree of describing the given problem. The reliability discounting factor α in [0, 1] is considered as the quantization of the reliability of an evidence. The reliability discounting method of DST (also called the Shafer's discounting method) is widely accepted and applied. The method consists of two steps. First, the mass assignments of focal elements are multiplied by the reliability discounting factor α. Second, all discounted mass assignments of the evidence are transferred to the focal element of full ignorance Θ. The Shafer's discounting method can be mathematically defined as follows [10,11]

$$\begin{cases} m_\alpha(X) = \alpha \cdot m(X), \text{for } X \neq \Theta \\ m_\alpha(X) = \alpha \cdot m(\Theta) + (1-\alpha) \end{cases} \quad (5)$$

where the reliability discounting factor is denoted by α and $0 \le \alpha \le 1, X$ denotes the focal element which is not the empty set, $m(.)$ denotes the original bba of evidence, $m_\alpha(.)$ denotes the bba after importance discounting.

2.2. DSmT

For many complex fusion problems, the elements can not be separated precisely and the refinement of discernment frame is inaccessible. For dealing with this situation, DSmT [9] which overcomes the exclusive limit of DST, is jointly proposed by Dezert and Smarandache. The hyper-power set in DSmT framework denoted by D^Θ consists of the unions and intersections elements in

Θ. Assume that $\Theta = \{\theta_1, \theta_2\}$, the hyper-power set of Θ can be defined as $D^{\Theta} = \{\emptyset, \theta_1, \theta_2, \theta_3, \theta_1 \cup \theta_2, \theta_1 \cap \theta_2\}$. The bba which consists the body of the evidence in DSmT framework is defined on the hyper-power set as $m(.): D^{\Theta} \to [0,1]$.

Dezert Smarandache Hybrid (DSmH) combination rule transfers partial conflicting beliefs to the union of the corresponding elements in conflicts which can be considered as partial ignorance or uncertainty. However, the way of transferring the conflicts in DSmH increases the uncertainty of fusion results and it is not convenient for decision-making based on the fusion results. The Proportional Conflict Redistribution (PCR) 1-6 rules overcome the weakness of DSmH and gives a better way of transferring the conflicts in multisource evidence fusion. PCR 1-6 rules proportionally transfer conflicting mass beliefs to the involved elements in the conflicts [9,26,27]. Each PCR rule has its own and different way of proportional redistribution of conflicts and PCR5 rule is considered as the most accurate rule among these PCR rules [9,26,27]. The combination of two independent evidences by PCR5 rule is given as follows [9,26,27]:

$$m_{1\oplus 2}(X_i) = \sum_{\substack{Y,Z\in G^{\Theta} \text{ and } Y,Z\neq\emptyset \\ Y \mathrm{I} Z = X_i}} m_1(Y)\cdot m_2(Z) \tag{6}$$

$$m_{PCR5}(X_i) = \begin{cases} m_{1\oplus 2} + \sum\limits_{\substack{X_j\in G^{\Theta} \text{ and } i\neq j \\ X_i \mathrm{I} X_j=\emptyset}} \left[\dfrac{m_1(X_i)^2\cdot m_2(X_j)}{m_1(X_i)+m_2(X_j)} + \dfrac{m_2(X_i)^2\cdot m_1(X_j)}{m_2(X_i)+m_1(X_j)}\right] & X_i\in G^{\Theta} \text{ and } X_i\neq\emptyset \\ 0 & X_i=\emptyset \end{cases}$$

where all denominators are more than zero, otherwise the fraction is discarded, and where G^{Θ} can be regarded as a general power set which is equivalent to the power set 2^{Θ}, the hyper-power set D^{Θ} and the super-power set S^{Θ}, if discernment of the fusion problem satisfies the Shafer's model, the hybrid DSm model, and the minimal refinement Θ^{ref} of Θ respectively [9,26,27].

Although PCR5 rule can get more reasonable fusion results than the combination rule of DST, it still has two disadvantages, first, it is not associative which means that the fusion sequence of multiple (more than 2) sources of evidences can influence the fusion results, second, with the increment of the focal element number in discernment frame, the computational complexity increases exponentially.

It is pointed out in [24] that importances and reliabilities of multisources in evidence fusion are different. The reliability of a source in DSmT framework represents the ability of describing the given problem by its real-time evidence which is the same as the notion in DST framework. The importances of sources in DSmT framework represent the weight that the fusion system designer assigns to the sources. Since the notions of importances and reliabilities of sources make no difference in DST framework, Shafer's discounting method can not be applied to evidence fusion of multisources with unequal importances.

The importance of a source in DSmT framework [24] can be characterized by an importance discounting factor, denoted β in [0,1]. The importance discounting factor β is not related with the reliability discounting factor α which is defined the same as DST framework. β can be any value in [0,1] chosen by the fusion system designer for his or her experience. The main difference of importance discounting method and reliability discounting method lies in the importance discounted mass beliefs of evidences are transferred to the empty set rather than the total ignorance Θ. The importance discounting method in DSmT framework can be mathematically defined as

$$\begin{cases} m_{\beta}(X) = \beta\cdot m(X), \text{for } X\neq\emptyset \\ m_{\beta}(\emptyset) = \beta(\emptyset) + (1-\beta) \end{cases} \tag{7}$$

where the importance discounting factor is denoted by β and $0\leq\beta\leq 1, X$ denotes the focal element which is not the empty set, $m(.)$ denotes the original bba of evidence, $m_{\beta}(.)$ denotes the bba after importance discounting. The empty set $\emptyset$ of Equation (7) is particular in DSmT discounted framework which is not the representation of unknown elements under the open-world assumption (Smets model), but only the meaning of the discounted importance of a source. Obviously, the importance discounted mass beliefs are transferred to the empty set in DSmT discounted framework which leads to the Dempster combination rule is not suitable to solve this type of fusion problems. The fusion rule with importance discounting factors in DSmT framework for 2 sources is considered as the extension of PCR5 rule, defined as follows [24]:

$$m_{PCR5_{\emptyset}}(A) = \sum_{\substack{X_1,X_2\in G^{\Theta} \\ X_1 \mathrm{I} X_2 = A}} m_1(X_1)m_2(X_2) + \sum_{\substack{X\in G^{\Theta} \\ X \mathrm{I} A=\emptyset}} \left[\frac{m_1(A)^2\cdot m_2(X)}{m_1(A)+m_2(X)} + \frac{m_2(A)^2\cdot m_1(X)}{m_2(A)+m_1(X)}\right] \tag{8}$$

The fusion rules with importance discounting factors considered as the extension of PCR6 and the fusion rule for multisources $(s > 2)$ as the extension of PCR5 can be seen referred in [24].

3. An Evidence Fusion Method with Importance Discounting Factors Based on Neutrosopic Probability Analysis in DSMT Framework

An evidence fusion method with importance discounting factors based on neutrosophic probability analysis in DSmT framework is proposed in this section. First, the reasonable evidence sources are selected out based on the statistical analysis of the pignistic probability functions of single focal elements. Secondly, the neutrosophic probability analysis is conducted based on the similarities of the pignistic probability functions from the prior evidence knowledge of the reasonable evidence sources. Thirdly, the importance discounting factors of the reasonable evidence sources are obtained based on the neutrosophic probability analysis and the reliability discounting factors of the real-time evidences are calculated based on probabilistic-based distances. Fourthly, the real-time evidences are discounted by the importance discounting factors and then the evidences with the mass assignments of neutrosophic empty sets are discounted by the reliability discounting factors. Finally, DSmT+PCR5 of importance discounted evidences is applied.

3.1. The reasonable evidence sources are selected out

Definition 1: Extraction function for extracting focal elements from the the pignistic probability functions of single focal elements.

$$\chi\big(P(a_i)\big) = a_i, a_i \in \{a_1, a_2, L, a_2\} \quad (11)$$

Definition 2: Reasonable sources.

The evidence sources are defined as reasonable sources if and only if the focal element which has the maximum mean value of the pignistic probability functions of all single focal elements is the element a_j which is known in prior knowledge, denoted by

$$\chi\big(P(\theta)\big) = \max\overline{\big(P(a)\big)} = a_j, 1 \le i \le z \quad (12)$$

where θ represents that the focal element which has the maximum mean value of the pignistic probability functions of all single focal elements.

Based on *Definition 2* and the prior evidence knowledge, reasonable sources are selected out. The unreasonable sources are not suggested to be considered in the following procedure for they are imprecise and unbelievable.

3.2. The neutrosophic probability analysis of the sources and the importance discounting factors in DSmT framework

The neutrosophic probability theory is proposed by Smarandache [30]. In this section, the neutrosophic probability analysis is conducted based on the similarities of the pignistic probability functions from the prior evidence knowledge of the reasonable evidence sources.

Definition 3: Similarity measure of the pignistic probability functions (SMPPF).

Assume that the distribution characteristics of pignistic probability functions of the focal elements $a_i, 1 \le i \le z$ and $a_k, k \ne i, 1 \le k \le z$ are denoted by:

$\boldsymbol{P}(a_i): \{\overline{P(a_i)}, \sigma(a_i)\}$, $\boldsymbol{P}(a_k): \{\overline{P(a_k)}, \sigma(a_k)\}$.

The similarity measure of the pignistic probability functions(SMPPF) is the function satisfying the following conditions:

(1) Symmetry:

$\forall a_i, a_k \in \Theta, Sim\big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_k)\big) = Sim\big(\boldsymbol{P}(a_k), \boldsymbol{P}(a_i)\big)$;

(2) Consistency:

$\forall a_i \in \Theta, Sim\big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_i)\big) = Sim\big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_i)\big) = 1$;

(3) Nonnegativity:

$\forall a_i, a_k \in \Theta, Sim\big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_k)\big) > 0$.

We will say that $\boldsymbol{P}(a_i)$ is more similar to $\boldsymbol{P}(a_k)$ than $\boldsymbol{P}(a_g)$ if and only if:

$Sim\big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_k)\big) > Sim\Big(\boldsymbol{P}(a_i), \boldsymbol{P}(a_g)\Big)$.

The similarity measure of the pignistic probability functions based on the distribution characteristics of the pignistic probability functions is defined as follows:

$$similarity(a_i, a_k) = \exp\left\{-\frac{|\overline{P(a_i)} - \overline{P(a_k)}|}{2[\sigma(a_i)+\sigma(a_k)]}\right\} \quad (13)$$

Assume that a_j is known in prior knowledge, the diagram for the similarity of the pignistic probability functions of focal elements a_j and a_k which has the largest SMPPF to a_j is shown in *Fig. 1*. $\boldsymbol{P}(a_j)$ is mapped to a circle in which $\overline{P(a_j)}$ is the center and $\sigma(a_j)$ is the radius. Similarly, $\boldsymbol{P}(a_k)$ is mapped to a circle in which $\overline{P(a_k)}$ is the center and $\sigma(a_k)$ is the radius. All the evidences in the prior knowledge from the reasonable source are mapped to the drops in any circle which means that the mapping from drops in the circle of $\boldsymbol{P}(a_j)$ to the prior evidences is one-to-one mapping and similarly the mapping from drops in the circle of $\boldsymbol{P}(a_k)$ to the prior evidences is also one-to-one mapping. If $\boldsymbol{P}(a_j)$ is very similar to $\boldsymbol{P}(a_k)$, the shadow accounts for a large proportion of $\boldsymbol{P}(a_j)$ or $\boldsymbol{P}(a_k)$. If $\boldsymbol{P}(a_j)$ or $\boldsymbol{P}(a_k)$ has the random values in the shadow of the diagram, the evidences of the reasonable source can not totally and correctly support decision-making for there are two possibilities which are $P(a_j) > P(a_k)$ and $P(a_j) \le P(a_k)$. If $P(a_j) \le P(a_k)$ in the evidences, the decisions are wrong. However, if $\boldsymbol{P}(a_j)$ or $\boldsymbol{P}(a_k)$ has the random values in the blank of the diagram, there is only one possibility which is $P(a_j) > P(a_k)$ for the sources are reasonable and the decisions by these evidences are totally correct. So, we define the neutrosophic probability and the absolutely right probability of the reasonable evidence source as probability of $\boldsymbol{P}(a_j)$ in the shadow and blank of the diagram.

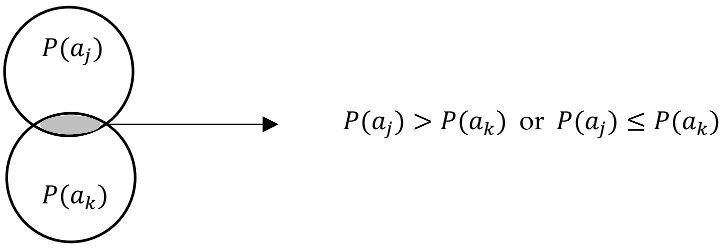

Figure 1. The diagram for the similarity.

Based on the above analysis, the neutrosophic probability and the absolutely right probability of the reasonable evidence source can be obtained by the similarity from the prior evidences for the mapping of the SMPPF of $\boldsymbol{P}(a_j)$ and $\boldsymbol{P}(a_k)$ to the probability of $\boldsymbol{P}(a_j)$ in the shadow is one-to-one mapping.

As $\forall a_i, a_k \in \Theta, 0 < similarity(\boldsymbol{P}(a_i)\ \mathrm{P}(a_k)) \le \mathbf{1}$, iff $a_i = a_k, similarity(\boldsymbol{P}(a_i))$, we define that the probability of $\boldsymbol{P}(a_j)$ in the shadow is the same as $similarity(\boldsymbol{P}(a_i)\ \mathrm{P}(a_k))$.

Assume there are reasonable evidence sources for evidence fusion, denoted by $S_k, k = 1,2,\mathrm{L},h$. So, the neutrosophic probability of the the reasonable evidence source in the prior condition that a_j is known can be calculated as follows:

$$P(S_k \text{ is neutral } |a_i) = \max_{1<j<n, j\ne i}[similarity(\boldsymbol{P}(a_i)\ \mathrm{P}(a_k))] \quad (14)$$

Then, the absolutely right probability of the reasonable evidence source in the prior condition that a_j is known can be calculated as follows:

$$(S_k \text{ is absolutely right}|a_i) = 1 - P(S_k \text{ is neutral } |a_i) = 1 - \max_{1<j<n, j\ne i}[similarity(\boldsymbol{P}(a_i)\ \mathrm{P}(a_k))] \quad (15)$$

So, if the prior probability of each focal element can be obtained accurately, the absolutely right probability of the reasonable evidence source can be calculated by the equation

$$P(S_k \text{ is absolutely right}) = \sum_{a_i\in\Theta, i=1,2,\mathrm{L},n} P\ (S_k \text{ is absolutely right}|a_i) g P(a_i). \quad (16)$$

If the prior probability of each focal element can not be obtained accurately and any focal element has no advantage in the prior knowledge, denoted by $P(a_1) = P(a_2) = \mathrm{L} = P(a_n)$, the absolutely right probability of the reasonable evidence source can be calculated as follows:

$$P(S_k \text{ is absolutely right}) = \frac{\sum_{a_i\in\Theta, i=1,2,\mathrm{L},n}(S_k \text{ is absolutely right } |a_i)}{n} \quad (17)$$

We define the discounting factors of importances in DSmT framework $\alpha_{SIG}(S_k)$ as the normalization of the absolutely right probabilities of the the reasonable evidence sources P(S_k is right), $k = 1,2,\mathrm{L},h$, denoted by

$$\alpha_{SIG}(S_k) = \frac{P(S_k \text{ is absolutely right})}{\max_{k=1,2,\mathrm{L},h}[P(S_k \text{ is absolutely right})]} \quad (18)$$

3.3. The reliablility discounting factors based on probabilistic-based distances

The Classical Pignistic Transformation(CPT) [9,10,11] is introduced briefly as follows:

$$P(A) = \sum_{X\in 2^\Theta} \frac{|X I A|}{|X|} m(X) \quad (19)$$

Based on CPT, if the mass assignments of the single focal elements which consist of the union set of single focal elements are equal divisions of the mass assignment of the union set of single focal elements in two evidences, the pignistic probability of two evidences are equal and the decisions of the two evidences based on CPT are also the same. From the view of decision, it is a good way to measure the similarity of the real-time evidences based on pignistic probability of evidences. Probabilistic distance based on Minkowski's distance [25] is applied in this paper to measure the similarity of real-time evidences. The method for calculating the reliability discounting factors based on Minkowski's distance [25] ($t = 1$) is given as follows.

Assume that there are h evidence sources, denoted by $S_k, k = 1,2,\mathrm{L}, h$, the real-time 2 evidences from S_i and S_j, $i \neq j$ are denoted by $\boldsymbol{m}_i$, $\boldsymbol{m}_j$ the discernment framework of the sources is $\{\theta_1, \theta_2, L, \theta_n\}$, the pignistic probabilities of single focal elements from S_i are denoted by $P_{S_i}(\theta_w), 1 < w < n$ and the pignistic probabilities of single focal elements from S_j are denoted by $P_{S_j}(\theta_w), 1 < w < n$.

1) Minkowski's distance ($t = 1$) between two real-time evidences is calculated as follows:

$$DistP(\boldsymbol{m}_i, \boldsymbol{m}_j) = \frac{1}{2} \sum_{\substack{\theta_w\in\Theta \\ |\theta_w|=1}} \left| P_{S_i}(\theta_w) - P_{S_j}(\theta_w) \right|. \quad (20)$$

2) The similarity of the real-time evidences is obtained by

$$similary(\boldsymbol{m}_i, \boldsymbol{m}_j) = 1 - DistP(\boldsymbol{m}_i, \boldsymbol{m}_j). \quad (21)$$

3) The similarity matrix of the real-time evidences from $S_k, k = 1,2,\mathrm{L}, h$ is given

$$S = \begin{bmatrix} 1 & similarly(\boldsymbol{m}_1, \boldsymbol{m}_2) & \mathrm{L} & similarly(\boldsymbol{m}_1, \boldsymbol{m}_h) \\ similarly(\boldsymbol{m}_2, \boldsymbol{m}_1) & 1 & \mathrm{L} & similarly(\boldsymbol{m}_2, \boldsymbol{m}_h) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ similarly(\boldsymbol{m}_h, \boldsymbol{m}_1) & similarly(\boldsymbol{m}_h, \boldsymbol{m}_2) & \mathrm{L} & 1 \end{bmatrix} \quad (22)$$

The average similarity of the real-time evidences from $S_k, k = 1,2,\mathrm{L}, h$ is given

$$\overline{similarly(S_k)} = \frac{\sum_{i=1,2,\mathrm{L},h, i\neq k} similarly(\boldsymbol{m}_i, \boldsymbol{m}_k)}{h-1} \quad (23)$$

4) The reliability discounting factors of the real-time evidences from $S_k, k = 1,2,\mathrm{L}, h$ is given

$$\alpha_{REL}(S_k) = \frac{\overline{similarly(S_k)}}{\max_{k=1,2,\mathrm{L},h} \left[\overline{similarly(S_k)}\right]} \quad (24)$$

3.4. The discounting method with both importance and reliability discounting factors in DSmT framework

1) Discounting evidences based on the discounting factors of importance.

Assume that the real-time evidence from the reasonable evidence source s_k is denoted by:

$\boldsymbol{m}_k = \{m(A), A \subseteq D^\Theta\}$, $G^\Theta = \{a_1 \mathrm{L}, a_2, a_1 \mathrm{I}\ \mathrm{L}\ \mathrm{I}\ a_2, a_1\ \mathrm{UL}\ \mathrm{U} a_2\}$.

Based on the discounting factors of importances in DSmT framework $\alpha_{SIG}(s_k)$, the new evidence $\boldsymbol{m}_k^{SIG}$ after importance-discounting by $\alpha_{SIG}(s_k)$ can be calculated by:

$$\boldsymbol{m}_K^{SIG} = \begin{cases} m^{\alpha SIG}(A) = \alpha_{SIG}(S_K) g(m(A)), A \subseteq G^\Theta \\ m^{\alpha SIG}(\emptyset) = 1 - \alpha_{SIG}(S_K) \end{cases} \quad (25)$$

where $m^{\alpha SIG}(A)$ are the mass assignments to all focal elements of the original evidence and $m^{\alpha SIG}(\emptyset)$ is the neutrosophic probability of the source, which represents the mass assignment of paradox.

2) Discounting the real-time evidences based on reliability discounting factors after importance discounting.

As the property of the neutrosophic probability of the source, the pignistic probabilities of single focal elements are not changed after importance-discounting the real-time evidences in DSmT framework and the mass assignments of neutrosophic empty focal element Ø which represent the importances degree of sources is added to the new evidences. If some real-time evidence has larger conflict with the other real-time evidences, the evidence should be not reliable and the mass assignments of the focal elements of the evidence should be discounted based on the discounting factors of reliabilities. As one focal element of the new evidence, the mass assignment of neutrosophic empty focal element Ø of the unreliable evidence should also be discounted. So the new discounting method based on the discounting factors of reliabilities after discounting by the discounting factors of importances is given as follows

$$\boldsymbol{m}_K^{SIG}=\begin{cases} m^{\alpha SIG}(A)=\alpha_{REL}(S_k)g\alpha_{SIG}(S_k)g\big(m(A)\big), A\subseteq G^{\Theta} \\ m^{\alpha SIG}(\emptyset)=\alpha_{REL}(S_k)g[1-\alpha_{SIG}(S_k)] \\ m^{\alpha SIG}(\Theta)=1-\alpha_{REL}(S_k) \end{cases} \quad (26)$$

3.5. The fusion method of PCR5Ø in DSmT framework is applied

After applying the new discounting method to the real-time evidences, the new evidences with the mass assignments of both the neutrosophic empty focal element Ø and the total ignorance focal elements Θ are obtained. The classic Dempster fusion rules can not be sufficient to process these evidences in DSmT framework and PCR5Ø for 2 sources in DSmT framework is applied as our fusion method as follows:

$$m_{PCR5_\emptyset}(A)=\sum_{\substack{X_1,X_2\in G^\Theta \\ X_1 I X_2=A}} m_1(X_1)m_2(X_2)+\sum_{\substack{X\in G^\Theta \\ XIA=\emptyset}}\left[\frac{m_1(A)^2\cdot m_2(X)}{m_1(A)+m_2(X)}+\frac{m_2(A)^2\cdot m_1(X)}{m_2(A)+m_1(X)}\right], A\in G^\Theta \text{ or } \emptyset \quad (27)$$

The mass assignment of the neutrosophic empty focal element Ø is included in the fusion results, which is not meaningful to decision. According to the principle of proportion, $m_{PCR5_\emptyset}(\emptyset)$ in the fusion result is redistributed to the other focal elements of the fusion result as follows:

$$\begin{aligned} m'_{PCR5_\emptyset}(A)&=m_{PCR5_\emptyset}(A)+\frac{m_{PCR5_\emptyset}(A)}{\sum_{A\in G^\Theta} m_{PCR5_\emptyset}(A)}\cdot m_{PCR5_\emptyset}(\emptyset), A\in G^\Theta \\ m'_{PCR5_\emptyset}(\emptyset)&=0 \end{aligned} \quad (28)$$

where $m'_{PCR5_\emptyset}(A), A\in G^\Theta$ is the final fusion results of our method.

4. Simulation Experiments

The Monto Carlo simulation experiments of recognition fusion are carried out. Through the simulation experiment results comparison of the proposed method and the existed methods, included PCR5 fusion method, the method in [25] and PCR5 fusion method with the reliability discounting factors, the superiority of the proposed method is testified. (In this paper, all the simulation experiments are implemented by Matlab simulation in the hardware condition of Pentimu(R) Dual-Core CPU E5300 2.6GHz 2.59GHz, memory 1.99GB. Abscissas of the figures represent that the ratio of the the standard deviation of Gauss White noise to the maximum standard deviation of the pignistic probabilities of focal elements in prior knowledge of the evidence sources, denoted by 'the ratio of the standard deviation of GWN to the pignistic probabilities of focal elements'.)

Assume that the prior knowledge of the evidence sources is counted as the random distributions of the pignistic probability when different focal element occurs. The prior knowledge is shown in *Tabel 3* and the characteristics of random distributions are denoted by *P*(.): (mean value, variance).

Table 3. Prior knowledge of evidence sources.

Evidence sources	Prior knowledge when *a* occurs	Prior knowledge when *b* occurs
s_1	$P_1(a)$ ~ (0.6,0.3)	$P_1(a)$ ~ (0.46,0.3)
	$P_1(b)$ ~ (0.4,0.3)	$P_1(b)$ ~ (0.54,0.3)
s_2	$P_2(a)$ ~ (0.6,0.3)	$P_2(a)$ ~ (0.4,0.3)
	$P_2(b)$ ~ (0.4,0.3)	$P_2(b)$ ~ (0.6,0.3)
s_3	$P_3(a)$ ~ (0.8,0.05)	$P_3(a)$ ~ (0.2,0.05)
	$P_3(b)$ ~ (0.2,0.05)	$P_3(b)$ ~ (0.8,0.05)

5.1.1 Simulation experiments in the condition that importance discounting factors of most evidence sources are low

Assume that there are 3 evidence sources, denoted by s_1, s_2, s_3, and the discernment framework of the sources is 2 types of targets, denoted by $\{a,b\}$. The prior knowledge is shown in *Table 3*. Assume that the pignistic probabilities of the focal elements are normally distributed. The real-time evidences of 3 sources are random selected out 1000 times based on the prior knowledge in *Table 3* in the condition that a occurs and b occurs respectively. The Moto-carlo simulation experiments of recognition fusion based on the proposed method and the existed methods are carried out. With the increment of the standard deviation of Gauss White noise in the mass assignments of evidences, the fusion results comparisons in different conditions are shown in *Fig. 3* and *Fig. 4*, and the mean value of the correct recognition rates and computation time are show in *Table 11* and *Table 12*.

The fusion results comparisons in the condition that importance discounting factors of most evidence sources are low show that:

1) The method proposed in this paper has the highest correct recognition rates among the existed methods. PCR5 fusion method has the secondly highest correct recognition rates, PCR5 fusion method with reliability discounting factors has the thirdly highest correct recognition rates, the method in [25] has the lowest correct recognition rates.

2) The method proposed in this paper has the largest computation time among the existed methods. the method in [25] has the secondly largest computation time, PCR5 fusion method with reliability discounting factors has the thirdly largest computation time, PCR5 fusion method has the lowest computation time.

Table 11. The mean value of correct recognition rates.

Prior conditions	The proposed method	PCR5 fusion method	The method in [25]	PCR5 fusion method with realibility-discounting factors
a	98.9%	88.6%	80.5%	84.3%
b	98.9%	87.6%	79.0%	82.9%

Table 12. The mean value of computation time.

Prior conditions	The proposed method	PCR5 fusion method	The method in [25]	PCR5 fusion method with realibility-discounting factors
a	1.47×10^{-4}	0.48×10^{-4}	0.88×10^{-4}	0.67×10^{-4}
b	1.46×10^{-4}	0.47×10^{-4}	0.89×10^{-4}	0.66×10^{-4}

Table 13. Prior knowledge of evidence sources.

Evidence sources	Prior knowledge when a occurs	Prior knowledge when b occurs
s_1	$P_1(a)$ ~ (0.6,0.3) $P_1(b)$ ~ (0.4,0.3)	$P_1(a)$ ~ (0.46,0.3) P_1(b) ~ (0.54,0.3)
s_2	$P_2(a)$ ~ (0.8,0.05) $P_2(b)$ ~ (0.2,0.05)	$P_2(a)$ ~ (0.2,0.05) P_2(b) ~ (0.8,0.05)
s_3	$P_3(a)$ ~ (0.8,0.05) $P_3(b)$ ~ (0.2,0.05)	$P_3(a)$ ~ (0.2,0.05) $P_3(b)$ ~ (0.8,0.05)

5.1.2 Simulation experiments in the condition that importance discounting factors of most evidence sources are high

Assume that there are 3 evidence sources, denoted by s_1, s_2, s_3, and the discernment framework of the sources is 2 types of targets, denoted by $\{a,b\}$. The prior knowledge is shown in *Table 13*. Assume that the pignistic probabilities of the focal elements are normally distributed. The Moto-carlo simulation experiments are carried out similarly to the *Section 4.3.1*. With the increment of the standard deviation of Gauss White noise in the mass assignments of evidences, the fusion results comparisons in different conditions are shown in *Fig. 5* and *Fig. 6*, and the mean value of the correct recognition rates and computation time are show in *Table 14* and *Table 15*. The importance factors of the evidences are calculated by Equation (18). The importance factor of s_1 is 0.19, the importance factor of s_2 and s_3 is 1.

Table 14. The mean value of correct recognition rates.

Prior conditions	The proposed method	PCR5 fusion method	The method in [25]	PCR5 fusion method with realibility-discounting factors
a	99.0%	98.8%	99.0%	99.0%
b	99.0%	98.8%	99.0%	99.0%

Table 15. The mean value of computation time.

Prior conditions	The proposed method	PCR5 fusion method	The method in [25]	PCR5 fusion method with realibility-discounting factors
a	1.45×10^{-4}	0.47×10^{-4}	0.86×10^{-4}	0.67×10^{-4}
b	1.46×10^{-4}	0.47×10^{-4}	0.87×10^{-4}	0.65×10^{-4}

The fusion results comparisons in the condition that importance discounting factors of most evidence sources are high show that:

1) The correct recognition rates of four methods are similarly closed, PCR5 fusion method has the lowest correct recognition rates among four methods.

2) The method proposed in this paper has the largest computation time among the existed methods. the method in [25] has the secondly largest computation time, PCR5 fusion method with reliability discounting factors has the thirdly largest computation time, PCR5 fusion method has the lowest computation time.

5. Conclusions

Based on the experiments results, we suggest that the fusion methods should be chosen based on the following conditions:

1) Judge whether the evidences are simple.

2) The importance discounting factors of most evidences are low or not high, the method in this paper is chosen.

The importance discounting factors of most evidences are high, PCR5 fusion method with reliability discounting factors is chosen

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Received: July 31, 2017. Accepted: August 18, 2017.