



# Bézier Surface Modeling for Neutrosophic Data Problems

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**Abstract.** The main goal of this paper is to construct Bézier surface modeling for neutrosophic data problems. We show how to build the surface model over a data sample from agriculture science after the theoretical

structure of the modeling is introduced. As a sampler application for agriculture systems, we give a visualization of Bézier surface model of an estimation of a given yield of bean seeds grown in a field over a period.

**Keywords:** Neutrosophic logic, neutrosophic data, neutrosophic geometry, Bézier surface, geometric design

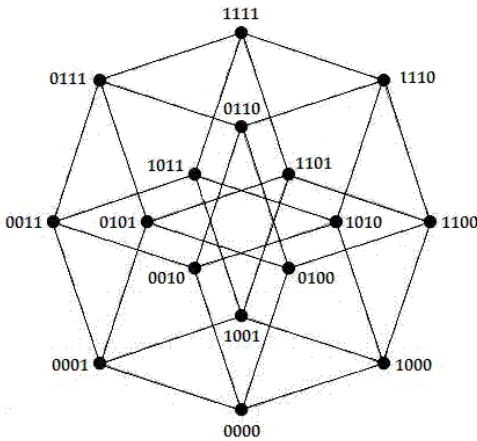
## 1 Introduction

The contribution of mathematical researches is fundamental and leading the science as today's technologies are rapidly developing. The geometrical improvements both model the mathematics of the objects and become geometrically most abstract concepts. In the future of science will be around the artificial intelligence. For the development of this technology, many branches of science work together and especially the topics such as logic, data mining, quantum physics, machine learning come to the forefront. Of course, the common place where these areas can cooperate is the computer environment. Data can be transferred in several ways. One of them is to transfer the data as a geometric model. The first method that comes to mind in terms of a geometric model is the Bézier technique. This method is generally used for curve and surface designs. In addition to this, it is used in many disciplines ranging from the solution of differential equations to robot motion planning.

The concretization state of obtaining meaning and mathematical results from uncertainty states (fuzzy) was introduced by Zadeh [1]. Fuzzy sets proposed by Zadeh provided a new dimension to the concept of classical sets. Atanassov introduced intuitionistic fuzzy sets dealing with membership and non-membership degrees [2]. Smarandache proposed neutrosophy as a mathematical application of the concept neutrality [3]. Neutrosophic set concept is defined with membership, non-membership and indeterminacy degrees. Neutrosophic set concept is separated from intuitionistic fuzzy set by the difference as follows: intuitionistic fuzzy sets are defined by degree of

membership and non-membership degree and, uncertainty degrees by the 1- (membership degree plus non-membership degree), while degree of uncertainty is considered independently of the degree of membership and non-membership in neutrosophic sets. Here, membership, non-membership, and uncertainty (indeterminacy) degrees can be evaluated according to the interpretation in the spaces to be used, such as truth and falsity degrees. It depends entirely on subject or topic space (discourse universe). In this sense, the concept of neutrosophic set is the solution and representation of the problems with various fields.

The paths of logic and geometry sometimes intersect and sometimes separate but both deal with information. Logic is related to information about the truth of statements, and geometry deals with information about location and visualization. Classical truth considers false and true, 0 and 1. It's geometrical interpretation with boolean connectives was represented as a boolean lattice by Miller [4-5]. Furthermore, a more geometrical representation was given by the 16 elements of the affine 4-space  $A$  over the two-element Galois field  $GF(2)$  [6] as can be seen in Figure 1. The affine space is created by 0,1 and 16 operators.



**Figure 1.** 16 elements of the affine 4-space A over the two-element Galois field GF(2).

Neutrosophic data has become an important instance of the expression "think outside the box" that goes beyond classical knowledge, accuracy and truth. Geometric approach to neutrosophic data which involve truth, falsity and indeterminacy values between the interval [0,1] provide rich mathematical structures. This paper presents an initial geometrical interpretation of neutrosophy theory.

Recently, geometric interpretations of data that have uncertain truth have presented by Wahab and friends [7-10]. They studied geometric models of fuzzy and intuitionistic fuzzy data and gave fuzzy interpolation and Bézier curve modeling. The authors of this paper presented Bézier curve modeling of neutrosophic data [11]. In this paper, we consider Bézier surface modeling of neutrosophic data problems and applications in real life.

**2. Preliminaries**

In this section, we first give some fundamental definitions dealing with Bézier curve and neutrosophic sets (elements). We then introduce new definitions needed to form a neutrosophic Bézier surface.

**Definition 1.** Let  $P_i, i = 0 \dots n$  are the set of points in 3-dimensional Euclidean space. Then the Bézier curve with degree  $n$  is defined by

$$B(t) = \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} t^i P_i, \quad t \in [0, 1].$$

where  $\binom{n}{i} = \frac{n!}{(n-i)!i!}$  and the points  $P_i$  are the control points of this Bézier curve.

**Definition 2.** Let  $P_{ij}, i = 0 \dots n, j = 0 \dots m$ , are the set of points in 3-dimensional Euclidean space. Then the

Bézier surface with degree  $n \times m$  is defined by

$$B(u, v) = \sum_{i=0}^n \sum_{j=0}^m \binom{n}{i} (1-u)^{n-i} u^i \binom{m}{j} (1-v)^{m-j} v^j P_{ij}, \quad t \in [0, 1].$$

where the points  $P_{ij}$  are the control points of this Bézier surface. The First-degree interpolation of these points forms a mesh and called the control polyhedron. These types of surfaces are called tensor product surfaces too. Therefore, one can show the matrix representation of a Bézier surface as

$$B(u, v) = [(1-u)^n \quad nu(1-u)^{n-1} \quad \dots \quad u^n] [P_{ij}] \begin{bmatrix} (1-v)^m \\ nu(1-u)^{m-1} \\ \vdots \\ v^m \end{bmatrix}$$

**Definition 3.** Let  $E$  be a universe and  $A \subseteq E$ .

$N = \{(x, T(x), I(x), F(x)): x \in A\}$  is a neutrosophic element where  $T_p: N \rightarrow [0, 1]$  (membership function),  $I_p: N \rightarrow [0, 1]$  (indeterminacy function) and  $F_p: N \rightarrow [0, 1]$  (non-membership function).

**Definition 4.** Let  $A^* = \{(x, T(x), I(x), F(x)): x \in A\}$

and  $B^* = \{(y, T(y), I(y), F(y)): y \in B\}$

be neutrosophic elements.

$NR = \{(x, y), T(x, y), I(x, y), F(x, y)): x \in A, y \in B\}$

is a neutrosophic relation on  $A^*$  and  $B^*$ .

### 2.1. Neutrosophic Bézier Model

**Definition 5.** Neutrosophic set of  $P^*$  in space  $N$  is  $NCP$  (neutrosophic control point) and

$P^* = \{P_i^*\}$  where  $i = 0, \dots, n$  is a set of NCPs where

there exists  $T_p: N \rightarrow [0, 1]$  as membership function,

$I_p: N \rightarrow [0, 1]$  as indeterminacy function and

$F_p: N \rightarrow [0, 1]$  as non-membership function with

$$T_p(P^*) = \begin{cases} 0 & \text{if } P_i \in N \\ a \in (0, 1) & \text{if } P_i \notin N \\ 1 & \text{if } P_i \in N \end{cases}$$

$$I_p(P^*) = \begin{cases} 0 & \text{if } P_i \in N \\ b \in (0, 1) & \text{if } P_i \notin N \\ 1 & \text{if } P_i \in N \end{cases}$$

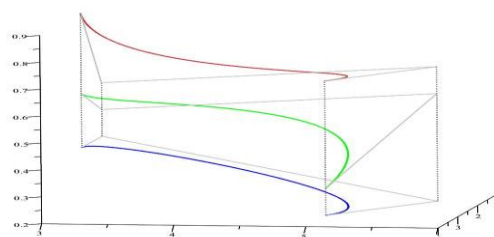
$$F_p(P^*) = \begin{cases} 0 & \text{if } P_i \in N \\ c \in (0, 1) & \text{if } P_i \notin N \\ 1 & \text{if } P_i \in N \end{cases}$$

**Definition 6.** A neutrosophic Bézier curve with degree  $n$  was defined by Taş and Topal [11].

$$NB(t) = \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} t^i NR_{P_i}, t \in [0, 1]$$

One can see there are three Bézier curves (Fig 1). The  $i$ th ( $i=0 \dots n$ ) control points of these curves are on the

same straight line. Line geometry shows us that if we interpolate these straight lines then we get a developable (cylindrical) ruled surface. Therefore, these curves belong to a developable ruled surface that is a surface that can be transformed to a plane without tearing or stretching (Figure 2). As a result, we can say that a neutrosophic Bézier curve corresponds to a cylindrical ruled surface.



**Figure 2.** Neutrosophic Bézier curve: membership (green curve), non-membership (orange curve), and indeterminacy (blue curve).

**Definition 7.** Neutrosophic Bézier surfaces are generated by the control points from one of

$TC = \{(x, y, T(x, y)): x \in A, y \in B\}$ ,

$IC = \{(x, y, I(x, y)): x \in A, y \in B\}$ ,

$FC = \{(x, y, F(x, y)): x \in A, y \in B\}$  sets. Thus, there

will be three different Bézier surface models for a neutrosophic relation and variables  $x$  and  $y$ . A neutrosophic control point relation can be defined as a set of  $(n+1)(m+1)$  points that shows a position and coordinate of a location and is used to describe three surface which are denoted by

$$NR_{P_{ij}} = \{NR_{P_{00}}, NR_{P_{01}}, \dots, NR_{P_{nm}}\}$$

and can be written as quadruples

$$\{(x_i, y_j), T(x_i, y_j), I(x_i, y_j), F(x_i, y_j)\}: i = 0, \dots, n, j = 0, \dots, m\}$$

in order to control the shape of a curve from a neutrosophic data.

**Definition 7.** A neutrosophic Bézier surface with degree  $n \times m$  is defined by

$$NB(u, v) = \sum_{i=0}^n \sum_{j=0}^m \binom{n}{i} (1-u)^{n-i} u^i \binom{m}{j} (1-v)^{m-j} v^j NR_{p_{ij}}$$

Every set of **TC**, **IC** and **FC** determines a Bézier surface. Thus, we obtain three Bézier surfaces. A neutrosophic Bézier surface is defined by these three surfaces. So it is a set of surfaces as in its definition.

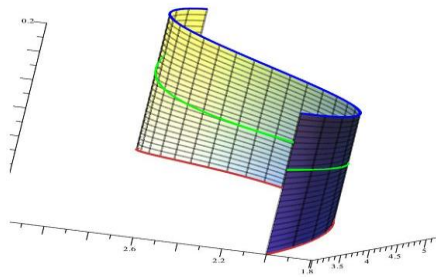
As an illustrative example, we can consider a neutrosophic data in Table 1. One can see there are three Bézier surfaces.

**Example 1.** Suppose that a field is a subset of two-dimensional space. By choosing a starting point (origin point) we seed certain point bean seeds. Depending on the reasons such as irrigation, rocky soil and so on, this is an estimate of the length of time that these seeds will arrive after a certain period of time. For example, we estimate each of the bean poles to reach 100 cm in length (Table 1). So we are trying to predict which parts of the land are more productive without planting yet. A yield map of the field with the data presented is obtained from the surface map of the plant.

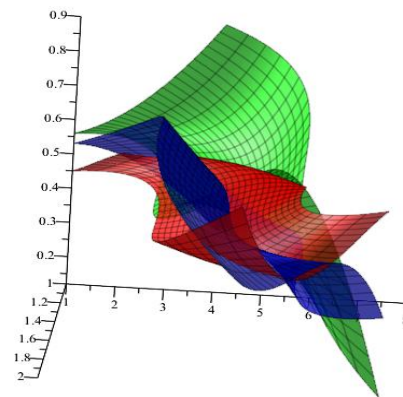
**Table 1.** Neutrosophic data

Bean seeds in coordinate system	Truth	Indeterminacy	Falsity
$P_{00}=(1,1)$	0.53	0.45	0.56
$P_{01}=(1,2)$	0.53	0.5	0.6
$P_{02}=(1,3)$	0.45	0.65	0.72
$P_{03}=(1,4)$	0.3	0.24	0.9
$P_{10}=(1,5)$	0.72	0.5	0.6
$P_{11}=(1,6)$	0.5	0.4	0.5
$P_{12}=(1,7)$	0.25	0.6	0.19
$P_{13}=(1,8)$	0.42	0.6	0.7
$P_{20}=(2,1)$	0.91	0.33	0.4
$P_{21}=(2,2)$	0.7	0.59	0.6
$P_{22}=(2,3)$	0.53	0.45	0.5
$P_{23}=(2,4)$	0.28	0.55	0.67
$P_{30}=(2,5)$	0.43	0.65	0.7
$P_{31}=(2,6)$	0.32	0.25	0.9
$P_{32}=(2,7)$	0.7	0.54	0.6
$P_{33}=(2,8)$	0.35	0.66	0.12

Neutrosophic Bézier surface of data in Table 1 can be illustrated in Figure 4. The surface can be turned to neutrosophic data because these surfaces are connected to the control points.



**Figure 3.** Neutrosophic Bézier curve and cylindrical ruled surface.



**Figure 4.** Neutrosophic Bézier surface according to data in Table 1.

### 3. Conclusions

Visualization or geometric modeling of data plays a significant role in data mining, databases, stock market, economy, stochastic processes and engineering. In this article, we have used a strong tool, the Bézier surface technique for visualizing neutrosophic data which belongs to agriculture systems. This surface model also is appropriate for statisticians, data scientists, economists and engineers. Furthermore, the differential geometric properties of this model can be investigated for classification of neutrosophic data. On the other hand, transforming the images of objects into neutrosophic data is an important problem [12]. In our model, the surface and the data can be transformed into each other by the blossoming method, which can be used in neutrosophic image processing. This and similar applications should be studied in the future.

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