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Compactness on Single-Valued Neutrosophic Ideal Topological Spaces

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Abstract: In the current paper, particular acheivments of single-valued neutrosophic continuity on a single-valued neutrosophic topological space $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}}, \tilde{\tau}^{\tilde{\eta}}, \tilde{\tau}^{\tilde{\mu}})$ are introduced. Some necessary implications between them are illustrated. The theories of *r*-single-valued neutrosophic compact, *r*-single-valued neutrosophic ideal compact, *r*-single-valued neutrosophic quasi H-closed and *r*-single-valued neutrosophic compact modulo an single-valued neutrosophic ideal $\tilde{\mathcal{J}}$ are presented and investigated.

Keywords: single-valued neutrosophic (almost; weakly) continuous mapping; single-valued neutrosophic ideal (compact; quasi H-closed) and *r*-single-valued neutrosophic compact modulo.

1. Introduction

Using a fuzzy ideal $\tilde{\mathcal{J}}$ defined on a fuzzy topological space (FTS) $(\tilde{\mathfrak{T}}, \tilde{\tau})$, a fuzzy ideal topological space (FITS) $(\tilde{\mathfrak{T}}, \tilde{\tau}, \tilde{\mathcal{J}})$ is generated. It is a way of generalizing so many notions and results in $(\tilde{\mathfrak{T}}, \tilde{\tau})$. The main definition of fuzzy topology that is related to the results in this article was established by Šostak in [1]. The notion of fuzzy ideal was created in [2]. Tripathy et al. in [3 - 6] introduced different valuble research studies on (FITS) and gave several forms of fuzzy continuities. Saber and others [7 - 11] have considered several *r*-fuzzy compactnesses in (FITS) $(\tilde{\mathfrak{T}}, \tilde{\tau}, \tilde{\mathcal{J}})$ and several types of fuzzy continuity.

Smarandache established the idea of the neutrosophic sets [12] in 1998. In terms of neutrosophic sets, there are a membership score $(\tilde{\gamma})$, an indeterminacy score $(\tilde{\eta})$ and a non-membership score $(\tilde{\mu})$ and a neutrosophic value is in the form $(\tilde{\gamma}, \tilde{\eta}, \tilde{\mu})$. In other meaning, in explaining an event or finding of a solution to a problem, a condition is handled according to its truth, not truth and resolution. Hence, the study of neutrosophic sets and neutrosophic logic are useful for decision-making applications in neutrosophic theories and led to too many researches and studies in the field as in [12-25]. It also gives the opportunity to others to establish some approaches in decision-making for neutrosophic theory as in [26-31]. Wang et al, [32] and Kim et al, [33] presented the theory of the neutrosophoic equivalence relation single-valued. Single-valued neutrosophic ideal ($\mathcal{SVNI}$) aspects in single-valued neutrosophic topological spaces ($\mathcal{SVNTS}$), have been introduced and considered by several authors from diverse viewpoints such as in [34-37].

In this research, we foreground the idea of r-single-valued neutrosophic (compact, ideal compact and quasi H-closed) in ($\mathcal{SVNTS}$) in the sense of Šostak. We are working on getting some of its important characteristics and results. Moreover, we investigate some properties of single-valued neutrosophic continuous mappings. Finally, some fascinating application of neutrosophic topology in reverse logistics arises could be found as in Abdel-Baset paper articles and others [38-41].

2. Preliminaries

Definition 2.1 [22] Suppose that $\widetilde{\mathfrak{T}}$ is a non-empty set. We mean by a neutrosophic set (briefly, $\mathcal{NS}$) A the objects having the form

$$\mathcal{S} = \{\langle \omega, \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}} \rangle : \omega \in \widetilde{\mathfrak{T}}\}.$$

Anywhere $\tilde{\mu}_{\mathcal{S}}$, $\tilde{\eta}_{\mathcal{S}}$ *and* $\tilde{\gamma}_{\mathcal{S}}$ indicate the degree of non-membership, the degree of indeterminacy, and the degree of membership, respectively of any element $\omega \in \widetilde{\mathfrak{T}}$ to the set $\mathcal{S}$.

Definition 2.2 [32] Suppose that $\widetilde{\mathfrak{T}}$ is a universal set. For $\forall \omega \in \widetilde{\mathfrak{T}}$, $0 \leq \tilde{\gamma}_{\mathcal{S}}(\omega) + \tilde{\eta}_{\mathcal{S}}(\omega) + \tilde{\mu}_{\mathcal{S}}(\omega) \leq 3$, by the meanings $\tilde{\gamma}_{\mathcal{S}}: \mathcal{S} \to [0.1]$, $\tilde{\eta}_{\mathcal{S}}: \mathcal{S} \to [0.1]$ and $\tilde{\mu}_{\mathcal{S}}: \mathcal{S} \to [0.1]$, a single-valued neutrosophic set (briefly, $\mathcal{SVNS}$) on $\widetilde{\mathfrak{T}}$ is defined by

$$\mathcal{S} = \{\langle \omega, \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}} \rangle : \omega \in \widetilde{\mathfrak{T}}\}.$$

Now, $\tilde{\mu}_{\mathcal{S}}$, $\tilde{\eta}_{\mathcal{S}}$ and $\tilde{\gamma}_{\mathcal{S}}$ are the degrees of falsity, indeterminacy and trueness of $\omega \in \widetilde{\mathfrak{T}}$, respectively. We will convey the set of all $\mathcal{SVNS}$s in $\mathcal{S}$ as $I^{\widetilde{\mathfrak{T}}}$.

Definition 2.3 [32] The accompaniment of a $\mathcal{SVNS}$ $\mathcal{S}$ is indicated by $\mathcal{S}^c$ and is cleared by

$$\tilde{\gamma}_{\mathcal{S}^c}(\omega) = \tilde{\mu}_{\mathcal{S}}(\omega), \quad \tilde{\eta}_{\mathcal{S}^c}(\omega) = 1 - \tilde{\eta}_{\mathcal{S}}(\omega) \text{ and } \tilde{\mu}_{\mathcal{S}^c}(\omega) = \tilde{\gamma}_{\mathcal{S}}(\omega).$$

for any $\omega \in \widetilde{\mathfrak{T}}$,

Definition 2.4 [41] Let $\mathcal{S}, \mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$. Then,

1. $\mathcal{S} \subseteq \mathcal{E}$, if, for every $\omega \in \widetilde{\mathfrak{T}}$,
$$\tilde{\gamma}_{\mathcal{S}}(\omega) \leq \tilde{\gamma}_{\mathcal{E}}(\omega), \qquad \tilde{\eta}_{\mathcal{S}}(\omega) \geq \tilde{\eta}_{\mathcal{E}}(\omega), \qquad \tilde{\mu}_{\mathcal{S}}(\omega) \geq \tilde{\mu}_{\mathcal{E}}(\omega)$$
2. $\mathcal{S} = \mathcal{E}$ if $\mathcal{S} \subseteq \mathcal{E}$ and $\mathcal{S} \supseteq \mathcal{E}$.
3. $\tilde{0} = \langle 0,1,1 \rangle$ and $\tilde{1} = \langle 1,0,0 \rangle$

Definition 2.5 [42] Let $\mathcal{S}, \mathcal{E} \in I^{\widetilde{\mathfrak{T}}}$. Then,

1. $\mathcal{S} \cap \mathcal{E}$ is a $\mathcal{SVNS}$ in $\widetilde{\mathfrak{T}}$ defined as:
$$\mathcal{S} \cap \mathcal{E} = (\tilde{\gamma}_{\mathcal{S}} \cap \tilde{\gamma}_{\mathcal{E}},\ \tilde{\eta}_{\mathcal{S}} \cup \tilde{\eta}_{\mathcal{E}},\ \tilde{\mu}_{\mathcal{S}} \cup \tilde{\mu}_{\mathcal{E}}).$$

Where, $(\tilde{\mu}_{\mathcal{S}} \cup \tilde{\mu}_{\mathcal{E}})(\omega) = \tilde{\mu}_{\mathcal{S}}(\omega) \cup \tilde{\mu}_{\mathcal{E}}(\omega)$ and $(\tilde{\gamma}_{\mathcal{S}} \cap \tilde{\gamma}_{\mathcal{E}})(\omega) = \tilde{\gamma}_{\mathcal{S}}(\omega) \cap \tilde{\gamma}_{\mathcal{E}}(\omega)$, for all $\omega \in \widetilde{\mathfrak{T}}$,

1. $\mathcal{S} \cup \mathcal{E}$ is an $\mathcal{SVNS}$ on $\widetilde{\mathfrak{T}}$ defined as:

$$\mathcal{S}\cup\mathcal{E}=(\tilde{\gamma}_{\mathcal{S}}\cup\tilde{\gamma}_{\mathcal{E}},\ \tilde{\eta}_{\mathcal{S}}\cap\tilde{\eta}_{\mathcal{E}},\ \tilde{\mu}_{\mathcal{S}}\cap\tilde{\mu}_{\mathcal{E}}).$$

Definition 2.6 [21] Suppose that $\widetilde{\mathfrak{T}}$ is a nonempty set and $\mathcal{S}\in I^{\widetilde{\mathfrak{T}}}$ is having the form $\mathcal{S}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\widetilde{\mathfrak{T}}\}$ on $\widetilde{\mathfrak{T}}$. Then,

1. $\left(\bigcap_{j\in\triangle}\mathcal{S}_j\right)(\omega)=\left(\bigcap_{j\in\triangle}\tilde{\gamma}_{\mathcal{S}_j}(\omega),\ \ \bigcup_{j\in\triangle}\tilde{\eta}_{\mathcal{S}_j}(\omega),\ \ \bigcup_{j\in\triangle}\tilde{\mu}_{\mathcal{S}_j}(\omega)\right)$,

2. $\left(\bigcup_{j\in\triangle}\mathcal{S}_j\right)(\omega)=\left(\bigcup_{j\in\triangle}\tilde{\gamma}_{\mathcal{S}_j}(\omega),\ \bigcap_{j\in\triangle}\tilde{\eta}_{\mathcal{S}_j}(\omega),\ \bigcap_{j\in\triangle}\tilde{\mu}_{\mathcal{S}_j}(\omega)\right)$.

Definition 2.7 [34] Let $s,t,k\in I_0$ and $s+t+k\leq 3$. A single-valued neutrosophic point ($\mathcal{SVNP}$) $x_{s,t,k}$ of $\widetilde{\mathfrak{T}}$ is the $\mathcal{SVNS}$ in $I^{\widetilde{\mathfrak{T}}}$ for every $\omega\in\mathcal{S}$, defined by

$$x_{s,t,k}(\omega)=\begin{cases}(s,t,k), & if\ x=\omega,\\ (0,1,1), & if\ x\neq\omega.\end{cases}$$

A $\mathcal{SVNP}$ $x_{s,t,k}$ is supposed to belong to a $\mathcal{SVNS}$ $\mathcal{S}=\{\langle\omega,\tilde{\gamma}_{\mathcal{S}},\tilde{\eta}_{\mathcal{S}},\tilde{\mu}_{\mathcal{S}}\rangle:\omega\in\widetilde{\mathfrak{T}}\}\in I^{\widetilde{\mathfrak{T}}}$, (notion: $x_{s.t.p}\in\mathcal{S}$ iff $s<\tilde{\gamma}_{\mathcal{S}}$, $t\geq\tilde{\eta}_{\mathcal{S}}$ and $k\geq\tilde{\mu}_{\mathcal{S}}$), and the set off all $\mathcal{SVNP}$ in $\widetilde{\mathfrak{T}}$ indicated by $\mathcal{SVNP}(\widetilde{\mathfrak{T}})$. $x_{s,t,k}\in\mathcal{SVNP}(\widetilde{\mathfrak{T}})$ quasi-coincident with a $\mathcal{SVNS}$ $\mathcal{S}\in I^{\widetilde{\mathfrak{T}}}$ denoted by $x_{s,t,k}q\mathcal{S}$, if

$$s+\tilde{\gamma}_{\mathcal{S}}>1,\ t+\tilde{\eta}_{\mathcal{S}}\leq 1\ ,\ k+\tilde{\mu}_{\mathcal{S}}\leq 1.$$

For every $\mathcal{S},\mathcal{E}\in I^{\widetilde{\mathfrak{T}}}$ $\mathcal{S}$ is quasi-coincident with $\mathcal{E}$ indicated by $\mathcal{S}q\mathcal{E}$, if there exists $x_{s,t,k}\in I^{\widetilde{\mathfrak{T}}}$ s.t

$$\tilde{\gamma}_{\mathcal{E}}+\tilde{\gamma}_{\mathcal{S}}>1,\ \tilde{\eta}_{\mathcal{E}}+\tilde{\eta}_{\mathcal{S}}\leq 1\ \text{and}\ \tilde{\mu}_{\mathcal{E}}+\tilde{\mu}_{\mathcal{S}}\leq 1.$$

Definition 2.8 [25] Let $\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}}:I^{\widetilde{\mathfrak{T}}}\to I$ be mappings satisfying the following conditions:

1. $\tilde{\tau}^{\tilde{\gamma}}(\underline{0})=\tilde{\tau}^{\tilde{\gamma}}(\underline{1})=1$ and $\tilde{\tau}^{\tilde{\eta}}(\underline{0})=\tilde{\tau}^{\tilde{\eta}}(\underline{1})=\tilde{\tau}^{\tilde{\mu}}(\underline{0})=\tilde{\tau}^{\tilde{\mu}}(\underline{1})=0$,
2. $\tau^{\tilde{\gamma}}(\mathcal{S}\cap\mathcal{E})\geq\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S})\cap\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E})$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}\cap\mathcal{E})\leq\tau^{\tilde{\eta}}(\mathcal{S})\cup\tilde{\tau}^{\tilde{\eta}}(\mathcal{E})$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}\cap\mathcal{E})\leq\tilde{\tau}^{\tilde{\mu}}(\mathcal{S})\cup\tilde{\tau}^{\tilde{\mu}}(\mathcal{E})$, for every $\mathcal{S},\mathcal{E}\in I^{\widetilde{\mathfrak{T}}}$,
3. $\tilde{\tau}^{\tilde{\gamma}}(\cup_{j\in\Gamma}\mathcal{S}_j)\geq\cap_{j\in\Gamma}\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j)$, $\tilde{\tau}^{\tilde{\eta}}(\cup_{i\in\Gamma}\mathcal{S}_j)\leq\cup_{j\in\Gamma}\tau^{\tilde{\eta}}(\mathcal{S}_j)$ and $\tilde{\tau}^{\tilde{\mu}}(\cup_{j\in\Gamma}\mathcal{S}_j)\leq\cup_{j\in\Gamma}\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)$, for every $\{\mathcal{S}_j,\ j\in\Gamma\}\in I^{\widetilde{\mathfrak{T}}}$.

Then $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ is called single valued neutrosophic topology $\mathcal{SVNT}$. Usually, we will write $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ for $(\tilde{\tau}^{\tilde{\gamma}},\tilde{\tau}^{\tilde{\eta}},\tilde{\tau}^{\tilde{\mu}})$ and it will cause no indistinctness.

Definition 2.9 [34] Let $(\widetilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$. Then, for all $\mathcal{S}\in I^{\widetilde{\mathfrak{T}}}$ and $r\in I_0$, the single valued neutrosophic (closure and interior) of $\mathcal{S}$ are define by:

$$C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}.r)=\bigcap\{\mathcal{E}\in I^{\widetilde{\mathfrak{T}}}:\ \mathcal{S}\leq\mathcal{E},\quad \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}^c)\geq r,\quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}^c)\leq 1-r,\quad \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}^c)\leq 1-r\}$$

$$int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}.r)=\bigcup\{\mathcal{E}\in I^{\widetilde{\mathfrak{T}}}:\ \mathcal{S}\geq\mathcal{E},\quad \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E})\geq r,\quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{E})\leq 1-r,\quad \tilde{\tau}^{\tilde{\mu}}(\mathcal{E})\leq 1-r\}.$$

Definition 2.10 [34] A mapping $\tilde{\mathcal{J}}^{\tilde{\gamma}},\tilde{\mathcal{J}}^{\tilde{\eta}},\tilde{\mathcal{J}}^{\tilde{\mu}}:I^{\widetilde{\mathfrak{T}}}\to I$ is said to be $\mathcal{SVNJ}$ on $\widetilde{\mathfrak{T}}$ if it satisfies the next three conditions for $\mathcal{S},\mathcal{E}\in I^{\widetilde{\mathfrak{T}}}$:

1. $\tilde{\mathcal{J}}^{\tilde{\eta}}(\tilde{0})=\tilde{\mathcal{J}}^{\tilde{\mu}}(\tilde{0})=0$, $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\tilde{0})=1$,
2. If $\mathcal{S}\leq\mathcal{E}$ then $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E})\geq\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S})$, $\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{E})\geq\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S})$ and $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{E})\leq\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S})$.
3. $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E})\cup\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E})$, $\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S})\cup\tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{E})$ and $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S}\cup\mathcal{E})\geq\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S})\cap\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{E})$.

Then, $(\widetilde{\mathfrak{T}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is said to be a single-valued neutrosophic ideal topological space ($\mathcal{SVNITS}$).

Definition 2.12 [36] A mapping $f: (\widetilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}) \to (\widetilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ from an $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is said to be single-valued neutrosophic continuous (briefly, $\mathcal{SVN}$-continuous) if and only if $\tilde{\tau}_2^{\widetilde{\gamma}}(\mathcal{S}) \leq \tilde{\tau}_1^{\widetilde{\gamma}}(f^{-1}(\mathcal{S}))$, $\tilde{\tau}_2^{\widetilde{\eta}}(\mathcal{S}) \geq \tilde{\tau}_1^{\widetilde{\eta}}(f^{-1}(\mathcal{S}))$ and $\tilde{\tau}_2^{\widetilde{\mu}}(\mathcal{S}) \geq \tilde{\tau}_1^{\widetilde{\mu}}(f^{-1}(\mathcal{S}))$, for every $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_2}$.

3. Single-Valued Neutrosophic (almost , weakly) Continuous Mappings

This section is dedicated to present the concepts of the single-valued neutrosophic (almost and weakly) mappings (briefly $\mathcal{SVN}-$ almost continuous, $\mathcal{SVN} - \textit{weakly continuous}$) mappings, respectively. It is also devoted to mark out the concepts of single-valued neutrosophic (preopen , regular-open) sets (briefly, $r - SVNPO$, $r - SVNRO$) sets, respectively.

Definition 3.1. Let $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be an $\mathcal{SVNTS}$ and $r \in I_0$. Then, $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}}$ is said to be:

1. $r - SVNPO\ set$ iff $\mathcal{S} \leq int_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\big(C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}, r), r\big)$,
2. $r - SVNRO$ set if $\mathcal{S} = int_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\big(C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}, r), r\big)$.

The complement of $r - SVNPO$ (resp, $r - SVNRO$) are said to be $r - SVNPC$ (resp, $r - SVNRC$), respectively.

Remark 3.2. Let $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be an $\mathcal{SVNTS}$ and $r \in I_0$, if $\mathcal{S}$ is an $r - SVNRO$ set, then $\mathcal{S}$ is $r - SVNPO$.

Example 3.3. Let $\widetilde{\mathfrak{X}} = \{a, b\}$. Define $\mathcal{E}_1, \mathcal{E}_2 \in I^{\widetilde{\mathfrak{X}}}$ as follows:

$\mathcal{E}_1 = \langle(0\cdot5, 0.4, 0\cdot5), (0\cdot5, 0.4, 0\cdot5), (0\cdot5, 0.5, 0\cdot5)\rangle$, $\mathcal{E}_2 = \langle(0\cdot4, 0\cdot4, 0.4), (0\cdot5, 0\cdot4, 0.4), (0\cdot5.0\cdot5, .4)\rangle$.

Define $\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}} : I^{\widetilde{\mathfrak{X}}} \to I$ as follows:

$$\tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \tilde{0}, \\ 1, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_1, \\ 0, & otherwise \end{cases} \qquad \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & otherwise \end{cases}$$

$$\tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & otherwise \end{cases}$$

Let, $\mathcal{E}_3 = \{\langle\omega, (0\cdot5, 0.5, 0\cdot1), (0\cdot6, 0.3, 0\cdot1), (0\cdot6, 0.3, 0\cdot1)\rangle : \omega \in \widetilde{\mathfrak{X}}\}$. Then, $\mathcal{E}_3$ is $\frac{1}{2} - SVNPO$ set but it is not $\frac{1}{2} - SVNRO$ set because, $\mathcal{E}_3 \neq int_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}\left(\mathcal{E}_3, \frac{1}{2}\right), \frac{1}{2}\right) = \tilde{1}$.

Lemma 3.4. Let $\mathcal{S}$ be an $\mathcal{SVNS}$ in an $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$. Then, for each $r \in I_0$.

1. If $\mathcal{S}$ is $r - SVNRO$ set (resp, $r - SVNRC\ set$), then $[\tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}) \geq r,\ \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}) \leq 1 - r\ ,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}) \leq 1 - r]$ (resp, $[\tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}^c) \geq r,\ \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}^c) \leq 1 - r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}^c) \leq 1 - r]$),
2. $\mathcal{S}$ is $r - SVNRO$ set if and only if $\mathcal{S}^c$ is $r - SVNRC$ set.

Proof. Follows directly from Definition 3.1.

Lemma 3.5. Let $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be an $\mathcal{SVNTS}$. Then,

1. the union of two $r - SVNRC$ sets is $r - SVNRC$,
2. the intersection of two $r - SVNRO$ sets, is $r - SVNRO$.

Proof. (1) Let $\mathcal{S},\mathcal{E}$ be any two $r-SVNRC$ sets. By Lemma 3.4, $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\leq 1-r]$ and $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}^c)\geq r,\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}^c)\leq 1-r]$. Then,

$$\tilde{\tau}^{*\tilde{\gamma}}(\mathcal{S}\cup\mathcal{E})\geq\tilde{\tau}^{*\tilde{\gamma}}(\mathcal{S})\cap\tilde{\tau}^{*\tilde{\gamma}}(\mathcal{E}),\ \tilde{\tau}^{*\tilde{\eta}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\tau}^{*\tilde{\eta}}(\mathcal{S})\cup\tilde{\tau}^{*\tilde{\eta}}(\mathcal{E}),\ \ \tilde{\tau}^{*\tilde{\mu}}(\mathcal{S}\cup\mathcal{E})\leq\tilde{\tau}^{*\tilde{\mu}}(\mathcal{S})\cup\tilde{\tau}^{*\tilde{\mu}}(\mathcal{E}),$$

but $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r})\leq\mathcal{S}\cup\mathcal{E}$, this suggests that

$$C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r}),\mathrm{r}\big)\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},r)=\mathcal{S}\cup\mathcal{E}.$$

Now,

$$\mathcal{S}=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big)\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r}),\mathrm{r}\big),$$

and

$$\mathcal{E}=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E},\mathrm{r}),\mathrm{r}\big)\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r}),\mathrm{r}\big).$$

Thus, $\mathcal{S}\cup\mathcal{E}\leq C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r}),\mathrm{r}\big)$. So, $\mathcal{S}\cup\mathcal{E}=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}\cup\mathcal{E},\mathrm{r}),\mathrm{r}\big)$. Hence, $\mathcal{S}\cup\mathcal{E}$ $r-SVNRC$ set.

(2) It can be ascertained by the same method.

Theorem 3.6. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$, Then,

1. If $\mathcal{S}\in I^{\tilde{\mathfrak{X}}}$ s.t, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\leq 1-r$, then, $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})$ is $r-SVNRO$ set,
2. If $\mathcal{S}\in I^{\tilde{\mathfrak{X}}}$ s.t, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S})\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S})\leq 1-r$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S})\leq 1-r$, then, $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})$ is $r-SVNRC$ set.

Proof. (1) Suppose that $\mathcal{S}\in I^{\tilde{\mathfrak{X}}}$ such that, $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\leq 1-r$. Clearly,

$$int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})\leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big),$$

this denotes that, $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})\leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big),r)$. Now, since,

$$\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\leq 1-r,$$

then $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big)\leq\mathcal{S}$; therefore,

$$int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})\geq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big),r).$$

Then, $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})=int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r}\big),r)$. Hence, $int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r})$ is $r-SVNRO$ set.

(2) Similar to the proof of (1).

Definition 3.7. A mapping $f:(\tilde{\mathfrak{X}}_1,\ \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})\rightarrow(\tilde{\mathfrak{X}}_2,\ \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ from an $\mathcal{SVNTS}$ $(\tilde{\mathfrak{X}}_1,\ \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\tilde{\mathfrak{X}}_2,\ \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is called:

1. $SVN-$ *almost continuous* iff $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{S}))\geq\mathrm{r},\ \tilde{\tau}_1^{\tilde{\eta}}\big(f^{-1}(\mathcal{S})\big)\leq 1-r,\ \tilde{\tau}_1^{\tilde{\mu}}\big(f^{-1}(\mathcal{S})\big)\leq 1-\mathrm{r}$, for each $r-SVNRO$ set $\mathcal{S}$ of $\tilde{\mathfrak{X}}_2$,
2. $SVN-$ *weakly continuous* iff $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S})\geq\mathrm{r},\ \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S})\leq 1-r$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S})\leq 1-\mathrm{r}$, implies $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{S}))\geq\mathrm{r}$, $\tilde{\tau}_1^{\tilde{\eta}}\big(f^{-1}(\mathcal{S})\big)\leq 1-r,\ \tilde{\tau}_1^{\tilde{\mu}}\big(f^{-1}(\mathcal{S})\big)\leq 1-\mathrm{r}$, , for each $\mathcal{S}\in I^{\tilde{\mathfrak{X}}_2}$.

Remark 3.8. From Definition 3.7, it is clear that the next implications are correct for $r\in I_0$:

$$SVN-\ almost\ continuous\ mapping$$
$$\Uparrow$$
$$SVN-\ continuous\ mapping$$
$$\Downarrow$$
$$SVN-\ weakly\ continuous\ mapping$$

However, the one-sided suggestions are not correct in general, as presented by the next example.

Example 3.9. Suppose that $\widetilde{\mathfrak{T}} = \{a, b, c\}$. Define $\mathcal{E}_1, \mathcal{E}_2 \in I^{\widetilde{\mathfrak{T}}}$ as follows:

$\mathcal{E}_1 = \langle(0\cdot 5, 0.4, 0\cdot 5), (0\cdot 5, 0.4, 0\cdot 5), (0\cdot 5, 0.5, 0\cdot 5)\rangle$, $\mathcal{E}_2 = \langle(0\cdot 5, 0\cdot 4, 0.4), (0\cdot 5, 0\cdot 4, 0.4), (0\cdot 5, 0\cdot 5, .4)\rangle$,

$\mathcal{E}_3 = \langle(0\cdot 3, 0.6, 0\cdot 5), (0\cdot 3, 0.6, 0\cdot 5), 0\cdot 3, 0.6, 0\cdot 5\rangle$, $\mathcal{E}_4 = \langle(0\cdot 4, 0\cdot 4, 0.4), (0\cdot 5, 0\cdot 4, 0.4), (0\cdot 5.0\cdot 5, .4)\rangle$.

We difine an $\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}} : I^{\widetilde{\mathfrak{T}}} \to I$ as follows:

$$\tilde{\tau}_1^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \tilde{0}, \\ 1, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \mathcal{E}_2, \\ 0, & otherwise \end{cases} \qquad \tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \tilde{0}, \\ 1, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 0, & otherwise \end{cases}$$

$$\tilde{\tau}_1^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & otherwise \end{cases} \qquad \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 1, & otherwise \end{cases}$$

$$\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_3\}, \\ 1, & otherwise \end{cases} \qquad \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0}, \\ 0, & if\ \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & if\ \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 1, & otherwise \end{cases}$$

Then, the identity mapping, $f: (\widetilde{\mathfrak{T}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{T}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $\mathcal{SVN}-$ almost continuous, but it is not $\mathcal{SVN}-$ continuou. Since, $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_4) = \frac{1}{2}$ and $\mathcal{E}_4$ is not $\frac{1}{2} - SVNO$ set in $\widetilde{\mathfrak{T}}_1$, because, $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_4)) = 0 \ngeq \frac{1}{2}$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_4)) = 1 \nleq \frac{1}{2}$ and $\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_4)) = 1 \ngeq \frac{1}{2}$. Hence, [$\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_4) = \frac{1}{2} \nleq 0 = \tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_4))$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_4) = \frac{1}{2} \ngeq 1 = \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_4))$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_4)\frac{1}{2} \ngeq 1 = \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_4))$].

Theorem 3.10. Let $f: (\widetilde{\mathfrak{T}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \to (\widetilde{\mathfrak{T}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a mapping from an $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{T}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{T}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$. Then the next statements are equivalent:

1. f is $\mathcal{SVN}-$ almost continuous,
2. $\tilde{\tau}_1^{\tilde{\gamma}}\left(\left(f^{-1}(\mathcal{S})\right)^c\right) \geq$ r, $\tilde{\tau}_1^{\tilde{\eta}}\left(\left(f^{-1}(\mathcal{S})\right)^c\right) \leq 1-r$, $\tilde{\tau}_1^{\tilde{\mu}}\left(\left(f^{-1}(\mathcal{S})\right)^c\right) \leq 1-$ r, for any $r - SVNRC$ set $\mathcal{S}$ of $\widetilde{\mathfrak{T}}_2$,
3. $f^{-1}(\mathcal{S}) \leq int_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, \mathrm{r}), \mathrm{r})), r)$, for any $\mathcal{S}$ of $\widetilde{\mathfrak{T}}_2$ such that $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq$ r, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1-$ r,
4. $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, \mathrm{r}), \mathrm{r})), r) \leq f^{-1}(\mathcal{S})$, for any $\mathcal{S}$ of $\widetilde{\mathfrak{T}}_2$ such that $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq$ r, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1-$ r.

Proof. (1)⇒(2). Let $\mathcal{S}$ be an $r - SVNRC$ set of $\widetilde{\mathfrak{T}}_2$ Then by Lemma 3.4, $\mathcal{S}^c$ is $r - SVNRO$ set in $\widetilde{\mathfrak{T}}_2$. By (1), we obtain

$$\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\gamma}}((f^{-1}(\mathcal{S}))^c) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\eta}}((f^{-1}(\mathcal{S}))^c) \leq 1-r,$$
$$\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\mu}}((f^{-1}(\mathcal{S}))^c) \leq 1-r.$$

(2)⇒(1). It is analogous to the proof of (1)⇒(2).

(1)⇒(3). Since, [$\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1-r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1-r$], then, $\mathcal{S} = int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r) \leq int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)$, and hence, $\mathrm{f}^{-1}(\mathcal{S}) = \mathrm{f}^{-1}(\mathrm{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathrm{C}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, \mathrm{r}), \mathrm{r}))$, since

$$\tilde{\tau}_2^{\widetilde{\gamma}}\left([C_{\tilde{\tau}_2^{\widetilde{\gamma}}}(\mathcal{S},r)]^c\right) \geq r\,, \quad \tilde{\tau}_2^{\widetilde{\eta}}\left([C_{\tilde{\tau}_2^{\widetilde{\eta}}}(\mathcal{S},r)]^c\right) \leq 1-r\,, \quad \tilde{\tau}_2^{\widetilde{\mu}}\left([C_{\tilde{\tau}_2^{\widetilde{\mu}}}(\mathcal{S},r)]^c\right) \leq 1-r\,,$$

then by Theorem 3.6 $int_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r),r)$ is $r-SVNRO$ set. So,

$\tilde{\tau}_1^{\widetilde{\gamma}}(f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\gamma}}}(C_{\tilde{\tau}_2^{\widetilde{\gamma}}}(\mathcal{S},r),r))) \geq r,\ \tilde{\tau}_1^{\widetilde{\eta}}(f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\eta}}}(C_{\tilde{\tau}_2^{\widetilde{\eta}}}(\mathcal{S},r),r))) \leq 1-r,\ \tilde{\tau}_1^{\widetilde{\mu}}(f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\mu}}}(C_{\tilde{\tau}_2^{\widetilde{\mu}}}(\mathcal{S},r),r))) \leq 1-r.$

Therefore, $f^{-1}(\mathcal{S}) \leq f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C_{\tilde{\tau}_2^{\widetilde{\gamma}}}(\mathcal{S},r),r)) = int_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},r),r)).$

(3)⇒(1). Let $\mathcal{S}$ be an $r-SVNRO$ set of $\widetilde{\mathfrak{X}}_2$. Then, we get

$$f^{-1}(\mathcal{S}) \leq int_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(int_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r}),\mathrm{r})),r) = int_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r});$$

this suggests that, $f^{-1}(\mathcal{S}) = int_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})$, then

$$\tilde{\tau}_1^{\widetilde{\gamma}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\widetilde{\gamma}}(int_{\tilde{\tau}_1^{\widetilde{\gamma}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \geq r, \quad \tilde{\tau}_1^{\widetilde{\eta}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\widetilde{\eta}}(int_{\tilde{\tau}_1^{\widetilde{\eta}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq 1-r,$$

$$\tilde{\tau}_1^{\widetilde{\mu}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\widetilde{\mu}}(int_{\tilde{\tau}_1^{\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq 1-r.$$

Therefore, f is $\mathcal{SVN}-$ *almost continuous*.

(2)⇔(4). Can be proved similarly.

Theorem 3.11. Let $f:(\widetilde{\mathfrak{X}}_1,\ \tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}) \to (\widetilde{\mathfrak{X}}_2,\ \tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be a map from an $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}_1,\ \tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}_2,\ \tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$. Then the following are equivalent:

1. f is $\mathcal{SVN}-$ *weakly continuous,*
2. $f(C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r})) \leq C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f(\mathcal{S}),\mathrm{r})$ for each $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_1}$

Proof. (1) ⇒ (2). : Let $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_1}$. Then,

$$f^{-1}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f(\mathcal{S}),\mathrm{r}))) = f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\widetilde{\mathfrak{X}}_2}: \tilde{\tau}_2^{\widetilde{\gamma}}(\mathcal{E}^c) \geq r, \tilde{\tau}_2^{\widetilde{\eta}}(\mathcal{E}^c) \leq 1-r, \tilde{\tau}_2^{\widetilde{\mu}}(\mathcal{E}^c) \leq 1-r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right]$$

$$\geq f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\widetilde{\mathfrak{X}}_2}: \tilde{\tau}_1^{\widetilde{\gamma}}(f^{-1}(\mathcal{E}^c)) \geq r, \tilde{\tau}_1^{\widetilde{\eta}}(f^{-1}(\mathcal{E}^c)) \leq 1-r, \tilde{\tau}_1^{\widetilde{\mu}}(f^{-1}(\mathcal{E}^c)) \leq 1-r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right]$$

$$\geq f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\widetilde{\mathfrak{X}}_2}: \tilde{\tau}_1^{\widetilde{\gamma}}\left((f^{-1}(\mathcal{E}))^c\right) \geq r, \tilde{\tau}_1^{\widetilde{\eta}}\left((f^{-1}(\mathcal{E}))^c\right) \leq 1-r, \tilde{\tau}_1^{\widetilde{\mu}}\left((f^{-1}(\mathcal{E}))^c\right) \leq 1-r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right]$$

$$\geq \bigcap\left\{f^{-1}(\mathcal{E}) \in I^{\widetilde{\mathfrak{X}}_1}: \tilde{\tau}_1^{\widetilde{\gamma}}\left((f^{-1}(\mathcal{E}))^c\right) \geq r, \tilde{\tau}_1^{\widetilde{\eta}}\left((f^{-1}(\mathcal{E}))^c\right) \leq 1-r, \tilde{\tau}_1^{\widetilde{\mu}}\left((f^{-1}(\mathcal{E}))^c\right) \leq 1-r, \quad f^{-1}(\mathcal{E}) \geq \mathcal{S}\right\}$$

$$\geq \bigcap\left\{\mathcal{D} \in I^{\widetilde{\mathfrak{X}}_1}: \tilde{\tau}_1^{\widetilde{\gamma}}(\mathcal{D}^c) \geq r, \tilde{\tau}_1^{\widetilde{\eta}}(\mathcal{D}^c) \leq 1-r, \tilde{\tau}_1^{\widetilde{\mu}}(\mathcal{D}^c) \leq 1-r, \ \ \mathcal{D} \geq \mathcal{S}\right\} = C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r}).$$

Hence, $f(C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r})) \leq f(f^{-1}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f(\mathcal{S}),\mathrm{r}))) \leq C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f(\mathcal{S}),\mathrm{r}).$

(2)⇒(1). It is similar to that of (1)⇒(2).

Corollary 3.12. Let $f:\widetilde{\mathfrak{X}}_1 \to \widetilde{\mathfrak{X}}_2$ be an $\mathcal{SVN}-$ *continuous* mapping with respect to the $\mathcal{SVNT}$s $\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}$ and $\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}$ respectively. Then , for each $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_1}$, $f(C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r})) \leq C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f(\mathcal{S}),\mathrm{r}).$

Theorem 3.13. Let $f:\widetilde{\mathfrak{X}}_1 \to \widetilde{\mathfrak{X}}_2$ be an $\mathcal{SVN}-$ *continuous* mapping with respect to the $\mathcal{SVNT}$ $\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}$ and $\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}$, respectively. Then , for any $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_2}$, $C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq f^{-1}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}),\mathrm{r})).$

Proof. Let $\mathcal{S} \in I^{\widetilde{\mathfrak{X}}_2}$. We get from Theorem 3.12, $C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq f^{-1}(f(C_{\tilde{\tau}_1^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq f^{-1}(C_{\tilde{\tau}_2^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S},\mathrm{r})).$

Hence, $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}),\mathrm{r})) \leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S},\mathrm{r}))$, for every $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_2}$.

4. Compactness on Single-Valued Neutrosophic Ideal Topological Spaces

This section aims to establish new notions of r-single-valued neutrosophic aspects called (compact, ideal compact, ideal quasi H-closed, compact modulo an single-valued neutrosophic ideal) (briefly, $r-\mathcal{SVN}-compact$, $r-\mathcal{SVNJ}-compact$, $r-\mathcal{SVNJ}-\ quasi\ H-closed$, $r-\mathcal{SVNC}(\mathcal{J})-compact$) in $\mathcal{SVNJTS}$.

Definition 4.1. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},)$ be an $\mathcal{SVNTS}$ and $r \in I_0$. Then $\tilde{\mathfrak{T}}$ is called $r-\mathcal{SVN}-compact$ iff for every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\bigcup_{j\in\Gamma_0}\mathcal{S}_j = \tilde{1}$.

Definition 4.2. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$ and $r \in I_0$. Then,

(1) $\tilde{\mathfrak{T}}$ is called $r-\mathcal{SVNJ}-compact$ (resp., $r-\mathcal{SVNJ}-\ quasi\ H-closed$) iff every family, $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$ (resp., $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)\right]^c\right) \leq 1-r$).

(2) $\tilde{\mathfrak{T}}$ is called $r-\mathcal{SVNC}(\mathcal{J})-$compact if for any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ and every family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j\in\Gamma}\mathcal{E}_j$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j,r)\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r$.

Definition 4.3. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$ and $\mathcal{S} \in I^{\tilde{\mathfrak{T}}}$. Then $\mathcal{S}$ is called $r-\mathcal{SVNJ}-compact$ iff every family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j\in\Gamma}\mathcal{E}_j$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \leq 1-r$.

Theorem 4.4. Let $(\tilde{\mathfrak{T}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$ and $r \in I_0$. Then,

(1) $r-\mathcal{SVN}-compact \Rightarrow r-\mathcal{SVNJ}-compact$,

(2) $r-\mathcal{SVNJ}-compact \Rightarrow r-\mathcal{SVNC}(\mathcal{J})-compact$,

(3) $r-\mathcal{SVNJ}-compact \Rightarrow r-SVNI-\ quasi\ H-closed$.

Proof. (1) For every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. By $\mathrm{r}-\mathcal{SVN}-$compactness of $\tilde{\mathfrak{T}}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\bigcup_{j\in\Gamma_0}\mathcal{S}_j = \tilde{1}$. Now, since $\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c = \tilde{0}$, we have $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$.

(2) For every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ and evrey family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j\in\Gamma}\mathcal{E}_j$. By $\mathrm{r}-\mathcal{SVNJ}-$compactness of $\mathcal{S}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right) \leq 1-r$. Since, $\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c \geq \mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r)\right]^c$, we have

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j,r)\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r, \quad \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r$$

Hence, $\widetilde{\mathfrak{X}}$ is $r-\mathcal{SVNC}(\mathcal{J})-compact$.

(3) Let $\{\mathcal{S}_j \in I^{\widetilde{\mathfrak{X}}}: \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j) \leq 1-r: j \in \Gamma\}$ be a family such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. By $r-\mathcal{SVNJ}-$compactness of $(\widetilde{\mathfrak{X}}, \widetilde{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}})$, there exists a finite subfamily $\Gamma_0 \subseteq \Gamma$ such that $\mathcal{J}^{\widetilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \geq r$, $\mathcal{J}^{\widetilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$, $\mathcal{J}^{\widetilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$. Since, $\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c \geq \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}_j, r)\right]^c$, we have

$$\mathcal{J}^{\widetilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}_j, r)\right]^c\right) \geq r, \quad \mathcal{J}^{\widetilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}_j, r)\right]^c\right) \leq 1-r, \quad \mathcal{J}^{\widetilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{S}_j, r)\right]^c\right) \leq 1-r$$

Hence, $\widetilde{\mathfrak{X}}$ is $r-SVNI-\ quasi\ H-closed$.

Theorem 4.5. The next statements are equivalent in an $\mathcal{SVNJTS}$ $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$:

(1) $\widetilde{\mathfrak{X}}$ is $\mathrm{r}-\mathcal{SVNJ}-$compact,

(2) For any family $\{\mathcal{S}_j \in I^{\widetilde{\mathfrak{X}}}:\ \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j^c) \geq r,\ \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j \in \Gamma\}$ with $\bigcap_{j\in\Gamma}\mathcal{S}_j = \tilde{0}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ with $\mathcal{J}^{\widetilde{\gamma}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \geq r$, $\mathcal{J}^{\widetilde{\eta}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \leq 1-r$, $\mathcal{J}^{\widetilde{\mu}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \leq 1-r$.

Proof. (1)⇒(2). For each family $\{\mathcal{S}_j \in I^{\widetilde{\mathfrak{X}}}:\ \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j^c) \geq r,\ \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j \in \Gamma\}$ with $\bigcap_{j\in\Gamma}\mathcal{S}_j = \tilde{0}$. Then, $\bigcup_{j\in\Gamma}\mathcal{S}_j^c = \tilde{1}$. By $r-\mathcal{SVNJ}-$compactness of $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\mathcal{J}^{\widetilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j^c\right]^c\right) \geq r$, $\mathcal{J}^{\widetilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j^c\right]^c\right) \leq 1-r$, $\mathcal{J}^{\widetilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j^c\right]^c\right) \leq 1-r$, this implies that,

$$\mathcal{J}^{\widetilde{\gamma}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \geq r, \quad \mathcal{J}^{\widetilde{\eta}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \leq 1-r, \quad \mathcal{J}^{\widetilde{\mu}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j\right) \leq 1-r.$$

(2)⇒(1). Let $\{\mathcal{S}_j \in I^{\widetilde{\mathfrak{X}}}: \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j \in \Gamma\}$ be a family such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. Then, $\bigcap_{j\in\Gamma}\mathcal{S}_j^c = \tilde{0}$, by (2), there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\mathcal{J}^{\widetilde{\gamma}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j^c\right) \geq r$, $\mathcal{J}^{\widetilde{\eta}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j^c\right) \leq 1-r$, $\mathcal{J}^{\widetilde{\mu}}\left(\bigcap_{j\in\Gamma_0}\mathcal{S}_j^c\right) \leq 1-r$ this implies that $\mathcal{J}^{\widetilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \geq r$, $\mathcal{J}^{\widetilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$, $\mathcal{J}^{\widetilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0}\mathcal{S}_j\right]^c\right) \leq 1-r$. Therefore $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is $r-\mathcal{SVNJ}-compact$.

Remark 4.6. Let $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be an $\mathcal{SVNJTS}$. The simplest $\mathcal{SVNJ}$ on $\widetilde{\mathfrak{X}}$ is $\mathcal{J}_0^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}: I^{\widetilde{\mathfrak{X}}} \longrightarrow I$, where

$$\mathcal{J}_0^{\widetilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & if\ \mathcal{S} = \tilde{0} \\ 0, & otherwise, \end{cases} \qquad \mathcal{J}_0^{\widetilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0} \\ 1, & otherwise, \end{cases} \qquad \mathcal{J}_0^{\widetilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & if\ \mathcal{S} = \tilde{0} \\ 1, & otherwise, \end{cases}$$

If $\mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}} = \mathcal{J}_0^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}$ then $r-\mathcal{SVN}-$compact and $r-\mathcal{SVNJ}-$compact are equivalent

Definition 4.7. An $\mathcal{SVNTS}$ $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is said to be *r*-single-valued neutrosophic regular ($r-\mathcal{SVN}-$regular) iff for every $\tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}) \leq 1-r$, $\tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}) \leq 1-r$ and $r \in I_0$,

$$\mathcal{S} = \bigcup\{\mathcal{E} \in I^{\widetilde{\mathfrak{X}}}: \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{E}) \geq r, \quad \tilde{\tau}^{\widetilde{\eta}}(\mathcal{E}) \leq 1-r, \quad \tilde{\tau}^{\widetilde{\mu}}(\mathcal{E}) \leq 1-r, \quad C_{\tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}}(\mathcal{E}, \mathrm{r}) = \mathcal{S}\}.$$

Theorem 4.8. Let $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ be an $r-\mathcal{SVNJ}-$ quasi H $-$ closed and $r-\mathcal{SVN}-$regular . Then $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$ is $r-\mathcal{SVNJ}-$compact.

Proof. For every family $\{\mathcal{S} \in I^{\widetilde{\mathfrak{X}}}: \tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j \in \Gamma\}$ such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. By $r-\mathcal{SVN}-$regularity of $(\widetilde{\mathfrak{X}}, \tilde{\tau}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}}, \mathcal{J}^{\widetilde{\gamma}\widetilde{\eta}\widetilde{\mu}})$, for any $\tilde{\tau}^{\widetilde{\gamma}}(\mathcal{S}_j) \geq r$, $\tilde{\tau}^{\widetilde{\eta}}(\mathcal{S}_j) \leq 1-r$, $\tilde{\tau}^{\widetilde{\mu}}(\mathcal{S}_j) \leq 1-r$, we have

$$\mathcal{S}_j = \bigcup_{j_\Delta \in \Delta_j} \{\mathcal{S}_{j_\Delta}:\ \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_{j_\Delta}) \geq r, \quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_{j_\Delta}) \leq 1-r, \quad \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_{j_\Delta}) \leq 1-r, \quad C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_{j_\Delta}, \mathrm{r}) \leq \mathcal{S}_j\}.$$

Thus, $\cup_{j\in\Gamma}(\cup_{j_\Delta\in\Delta_j}\mathcal{S}_{j_\Delta}) = \tilde{1}$. Since $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}$- quasi H-closed, there exists a finite subset $K \times \Delta_K$ such that

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \geq r,\ \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \leq 1-r, \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \leq 1-r.$$

For each $k \in K$, since $\cup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_{k_\Delta}, r) \leq \mathcal{S}_k$. It implies that $\left[\cup_{k\in K}(\cup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_{k_\Delta}, r))\right]^c \geq [\cup_{k\in K}\ \mathcal{S}_k]^c$. Thus,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{k\in K}\mathcal{S}_k\right]^c\right) \geq \tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{k\in K}\mathcal{S}_k\right]^c\right) \leq \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \leq 1-r$$

$$\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{k\in K}\mathcal{S}_k\right]^c\right) \leq \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{k\in K}(\bigcup_{k_\Delta\in\Delta_k} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_{k_\Delta}, r))\right]^c\right) \leq 1-r.$$

Hence, $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}-$ compact.

Definition 4.9. A family $\{\mathcal{S}_j\}_{j\in\Gamma}$ in $\tilde{\mathfrak{X}}$ has the finite intersection property $(\boldsymbol{I}-\boldsymbol{FIP})$ iff the intersection of no finite sub-family $\Gamma_0 \subseteq \Gamma$ s.t $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\cap_{j\in\Gamma_0}\mathcal{S}_j) \geq r,\ \tilde{\mathcal{J}}^{\tilde{\eta}}(\cap_{j\in\Gamma_0}\mathcal{S}_j) \leq 1-r,\ \tilde{\mathcal{J}}^{\tilde{\mu}}(\cap_{j\in\Gamma_0}\mathcal{S}_j) \leq 1-r$.

Theorem 4.10. An $\mathcal{SVNJTS}$ $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $\mathrm{r}-\mathcal{SVNJ}-compact$, iff every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c) \geq r,$ $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j \in \Gamma\}$ having the finite intersection property $(\boldsymbol{I}-\boldsymbol{FIP})$ has a non-empty intersection.

Proof. Obvious.

Theorem 4.11. Suppose that $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is an $\mathcal{SVNJTS}$, $\mathcal{S}$ is $r-\mathcal{SVNJ}-compact$. Then for every collection $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{E}_j \leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r),\ j \in \Gamma\}$ with $\mathcal{S} \leq \cup_{j\in\Gamma}\mathcal{E}_j$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ s.t,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r), r)\right]^c\right) \leq 1-r$$

$$\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)\right]^c\right) \leq 1-r.$$

Proof. Let $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{E}_j \leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r),\ j \in \Gamma\}$ with $\mathcal{S} \leq \cup_{j\in\Gamma}\mathcal{E}_j$.Then, $\mathcal{S} \leq \cup_{j\in\Gamma} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)$, $[\tilde{\tau}^{\tilde{\gamma}}\left(int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r), r)\right) \geq r,\ \tilde{\tau}^{\tilde{\eta}}\left(int_{\tilde{\tau}^{\tilde{\eta}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r), r)\right) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}\left(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r), r)\right) \leq 1-r]$. By $\mathrm{r}-\mathcal{SVNJ}$ $-compactness$ of $\mathcal{S}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ s.t,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r), r)\right]^c\right) \leq 1-r$$

$$\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S}\cap\left[\bigcup_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r)\right]^c\right)\leq 1-r.$$

Definition 4.12. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$ and $\mathcal{S}\in I^{\tilde{\mathfrak{X}}}$. Then $\mathcal{S}$ is called r-single-valued neutrosophic locally closed iff $\mathcal{S}=\mathcal{E}\cap\mathcal{D}$ where $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E})\geq r,\tilde{\tau}^{\tilde{\eta}}(\mathcal{E})\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E})\leq 1-r]$, $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{D}^c)\geq r,\tilde{\tau}^{\tilde{\eta}}(\mathcal{D}^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{D}^c)\leq 1-r]$.

Lemma 4.13. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$ and $\mathcal{S}\in I^{\tilde{\mathfrak{X}}}$. Then $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S})\geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S})\leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S})\leq 1-r$ iff $\mathcal{S}$ both r-single-valued neutrosophic locally closed and $r-SVNPO$ set.

Proof. It is trivial.

Lemma 4.14. If $\mathcal{S}$ is $r-\mathcal{SVNJ}-$ compact, then for every collection $\{\mathcal{E}_j\in I^{\tilde{\mathfrak{X}}}\colon \mathcal{E}_j\ is\ both\ r-SVNPO\ and\ r-single-valued\ neutrosophic\ locally\ closed\ sets,\ j\in\Gamma\}$ with $\mathcal{S}\leq\bigcup_{j\in\Gamma}(\mathcal{E}_j)$, there exists a finite subfamily $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S}\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S}\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S}\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r$.

Proof. Follows from Lemma 4.13.

Theorem 4.15. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$, $\mathcal{S}_1$ and $\mathcal{S}_2$ are $\mathrm{r}-\mathcal{SVNJ}-compact$. Then, $\mathcal{S}\cup\mathcal{E}$ is $\mathrm{r}-\mathcal{SVNJ}-compact$ subset relative to $\tilde{\mathfrak{X}}$.

Proof. Let $\{\mathcal{E}_j\in I^{\tilde{\mathfrak{X}}}\colon \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j)\geq r,\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j)\leq 1-r,\ j\in\Gamma\}$ be a family such that $\mathcal{S}_1\cup\mathcal{S}_2\leq\bigcup_{j\in\Gamma}\mathcal{E}_j$. Then $\mathcal{S}_1\leq\bigcup_{j\in\Gamma}\mathcal{E}_j$ and $\mathcal{S}_2\leq\bigcup_{j\in\Gamma}\mathcal{E}_j$. Since $\mathcal{S}_1$ and $\mathcal{S}_2$ are $r-\mathcal{SVNJ}-compact$, there exists a finite subset $\Gamma_0\subseteq\Gamma$ such that

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S}_k\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\geq r,\qquad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S}_k\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r,\qquad \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S}_k\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r,$$

for $k=1,2$, since $\left(\mathcal{S}_1\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\cup\left(\mathcal{S}_2\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)=(\mathcal{S}_1\cup\mathcal{S}_2)\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c$. Then,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left((\mathcal{S}_1\cup\mathcal{S}_2)\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\geq r,\qquad \tilde{\mathcal{J}}^{\tilde{\eta}}\left((\mathcal{S}_1\cup\mathcal{S}_2)\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r,\qquad \tilde{\mathcal{J}}^{\tilde{\mu}}\left((\mathcal{S}_1\cup\mathcal{S}_2)\cap\left[\bigcup_{j\in\Gamma_0}\mathcal{E}_j\right]^c\right)\leq 1-r.$$

This shown that $(\mathcal{S}_1\cup\mathcal{S}_2)$ is $r-\mathcal{SVNJ}-$ compact.

Theorem 4.16. Suppose $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$, $r\in I_0$. Then the next statements are equivalent:

(1) $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}-\ quasi\ H-closed$,

(2) For every collection $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}\colon \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c)\leq 1-r,\ j\in\Gamma\}$ with $\bigcap_{j\in\Gamma}\mathcal{S}_j=\tilde{0}$, there exists $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r))\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r))\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r))\leq 1-r$,

(3) $\bigcap_{j\in\Gamma}\mathcal{S}_j\neq\tilde{0}$, holds for any collection $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}\colon \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c)\leq 1-r,\ j\in\Gamma\}$ such that $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r)\colon \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c)\leq 1-r,\ j\in\Gamma\}$ has the $\boldsymbol{I-FIP}$,

(4) For any collection $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}\colon \mathcal{S}_j \text{ is } r-SVNRO\ sets,\ j\in\Gamma\}$ such taht $\bigcup_{j\in\Gamma}\mathcal{S}_j=\tilde{1}$, there exists $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)\right]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)\right]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)\right]^c\right)\leq 1-r$,

(5) For every collection $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{S}_j \text{ is } r-SVNRC\ set,\ j \in \Gamma\}$ such taht $\bigcap_{j\in\Gamma}\mathcal{S}_j = \tilde{0}$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\mathfrak{J}^{\tilde{\gamma}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r)) \geq r$, $\mathfrak{J}^{\tilde{\eta}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)) \leq 1-r$, $\mathfrak{J}^{\tilde{\mu}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)) \leq 1-r$,

(6) $\bigcap_{j\in\Gamma}\mathcal{S}_j \neq \tilde{0}$, holds for every collection $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{S}_j \text{ is } r-SVNRC\ set,\ j \in \Gamma\}$ such taht $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r): \mathcal{S}_j \text{ is } r-SVNRC\ set,\ j\in\Gamma\}$ has the $\boldsymbol{I-FIP}$.

Proof. (1)⇒(2). Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j\in\Gamma\}$ be a family with $\bigcap_{j\in\Gamma}\mathcal{S}_j = \tilde{0}$. Then, $\bigcup_{j\in\Gamma}\mathcal{S}_j^c = \tilde{1}$. Since, $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \mathfrak{J}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}-quasi\ H-closed$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\mathfrak{J}^{\tilde{\gamma}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j^c, r)]^c) \geq r$, $\mathfrak{J}^{\tilde{\eta}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j^c, r)]^c) \leq 1-r$, $\mathfrak{J}^{\tilde{\mu}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)]^c) \leq 1-r$. Since, $[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c, r)]^c = \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r)$, we have

$$\mathfrak{J}^{\tilde{\gamma}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)\right) \geq r, \quad \mathfrak{J}^{\tilde{\eta}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)\right) \leq 1-r, \quad \mathfrak{J}^{\tilde{\mu}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)\right) \leq 1-r.$$

(2)⇒(1). Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r, j\in\Gamma\}$ be a family s.t $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. Then, $\bigcap_{j\in\Gamma}\mathcal{S}_j^c = \tilde{0}$ and by hypothesis, there exists $\Gamma_0 \subseteq \Gamma$ s.t, $\mathfrak{J}^{\tilde{\gamma}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)) \geq r$, $\mathfrak{J}^{\tilde{\eta}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)) \leq 1-r$, $\mathfrak{J}^{\tilde{\mu}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)) \leq 1-r$. Since, $\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c, r) = [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r)]^c$,

$$\mathfrak{J}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r)\right]^c\right) \geq r, \quad \mathfrak{J}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)\right]^c\right) \leq 1-r, \quad \mathfrak{J}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)\right]^c\right) \leq 1-r.$$

Thus, $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \mathfrak{J}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}$- *quasi H-closed,*

(1) ⇒ (3). For any family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j\in\Gamma\}$ such that $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r): \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1-r,\ j\in\Gamma\}$ has the $\boldsymbol{I-FIP}$. If $\bigcap_{j\in\Gamma}\mathcal{S}_j = \tilde{0}$, then $\bigcup_{j\in\Gamma}\mathcal{S}_j^c = \tilde{1}$. Since $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \mathfrak{J}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}$- *quasi H-closed*, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that

$$\mathfrak{J}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j^c, r)\right]^c\right) \geq r, \quad \mathfrak{J}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j^c, r)\right]^c\right) \leq 1-r, \quad \mathfrak{J}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)\right]^c\right) \leq 1-r.$$

Since, $[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c, r)]^c = \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)$, we have

$$\mathfrak{J}^{\tilde{\gamma}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r)\right) \geq r, \quad \mathfrak{J}^{\tilde{\eta}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)\right) \leq 1-r, \quad \mathfrak{J}^{\tilde{\mu}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)\right) \leq 1-r.$$

Which is a contradiction.

(3)⇒(1). For any family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j\in\Gamma\}$ such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$, with the property that for no finite $\Gamma_0 \subseteq \Gamma$ such that $\mathfrak{J}^{\tilde{\gamma}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r)]^c) \geq r$, $\mathfrak{J}^{\tilde{\eta}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)]^c) \leq 1-r$, $\mathfrak{J}^{\tilde{\mu}}([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)]^c) \leq 1-r$. Since,

$$\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r)\right]^c = \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c, r).$$

The family $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c, r): \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1-r,\ j\in\Gamma\}$ has the $\boldsymbol{I-FIP}$ .By (3). $\bigcap_{j\in\Gamma}\mathcal{S}_j^c \neq \tilde{0}$, Then, $\bigcup_{j\in\Gamma}\mathcal{S}_j \neq \tilde{1}$. It is a contradiction.

(1)⇒(4). Let $\{\mathcal{S}_j\}_{j\in\Gamma}$ be a family of $r-SVNRO$ set such that $\bigcup_{j\in\Gamma}\mathcal{S}_j = \tilde{1}$. Then, $\bigcup_{j\in\Gamma} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j, r), \mathrm{r}) = \tilde{1}$, since, $\tilde{\tau}^{\tilde{\gamma}}(int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r), \mathrm{r})) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(int_{\tilde{\tau}^{\tilde{\eta}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r), \mathrm{r})) \leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r), \mathrm{r})) \leq 1-r$ and $\tilde{\mathfrak{X}}$ is $r-\mathcal{SVNJ}$- *quasi H-closed*, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r),\mathrm{r}),r)\right]^c\right)\geq r,\quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}\left(int_{\tilde{\tau}^{\tilde{\eta}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r),\mathrm{r})\right)\right]^c\right)\leq 1-r,$$

$$\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r),\mathrm{r}),r)\right]^c\right)\leq 1-r.$$

Since, for $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j)\geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j)\leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)\leq 1-r$ we have $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r),\mathrm{r}),r)=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r)$. Hence, $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$.

(4)⇒(5). Let $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}:\ j\in\Gamma\}$ be a family of $r-SVNRC$ sets such that $\bigcap_{j\in\Gamma}\mathcal{S}_j=\tilde{0}$. Then, $\bigcup_{j\in\Gamma}\mathcal{S}_j^c=\tilde{1}$, and $\{\mathcal{S}_j^c\in I^{\tilde{\mathfrak{X}}}:\ j\in\Gamma\}$ is a family of $r-SVNRO$ sets. By (4), there will be a finite subset $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j^c,r)]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j^c,r)]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c,r)]^c\right)\leq 1-r$, Thus,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)\right)\geq r,\quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)\right)\leq 1-r,\quad \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)\right)\leq 1-r.$$

(5)⇒(1). Let $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)\leq 1-r,\ j\in\Gamma\}$ be a family such that $\bigcup_{j\in\Gamma}\mathcal{S}_j=\tilde{1}$. Then, $\bigcup_{j\in\Gamma} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r),\mathrm{r})=\tilde{1}$. Thus, $\bigcap_{j\in\Gamma} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c,r),\mathrm{r})=\tilde{0}$ and $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c,r),\mathrm{r})$ is $r-SVNRC$. For the hypothesis, there exists $\Gamma_0\subseteq\Gamma$ such that

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(C_{\tilde{\tau}^{\tilde{\gamma}}}(int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j^c,r),\mathrm{r}),r)\right)\geq r,\quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j^c,r),\mathrm{r}),r)\right)\leq 1-r,$$

$$\tilde{\mathcal{J}}^{\tilde{\mu}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c,r),\mathrm{r}),r)\right)\leq 1-r$$

Since, for $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j)\geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j)\leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)\leq 1-r$ we have $C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r),\mathrm{r}),r)=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r)$, and hence, $\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j^c,r),\mathrm{r}),r)=[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,r)]^c$. Therefore, $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$). Hence, $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}-quasi\ H-closed$,

(6)⇔(4) is proved similarly like (3)⇔(1).

Theorem 4.17. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$ and $r\in I_0$, Then the next statements are equivalent:

(1) $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNJ}-\ quasi\ H-closed$,

(2) For any family $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}: \mathcal{S}_j\leq int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,\mathrm{r}),\mathrm{r})\}$ with $\bigcup_{j\in\Gamma}\mathcal{S}_j=\tilde{1}$, there exists a finite subset $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r)]^c\right)\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}\left([\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r)]^c\right)\leq 1-r$),

(3) For any family $\{\mathcal{S}_j\in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j^c)\geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c)\leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c)\leq 1-r,\ j\in\Gamma\}$ such that $\bigcap_{j\in\Gamma}\mathcal{S}_j=\tilde{0}$, there exists a finite subset $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j,r))\geq r$, $\tilde{\mathcal{J}}^{\tilde{\eta}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j,r))\leq 1-r$, $\tilde{\mathcal{J}}^{\tilde{\mu}}(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j,r))\leq 1-r$).

Proof. Obvious.

Theorem 4.18. Let $(\tilde{\mathfrak{X}},\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNJTS}$ and $r\in I_0$, Then the next statements are equivalent:

(1) $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r-\mathcal{SVNC}(\mathcal{J})-compact$,

(2) For each family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1-r,\ j \in \Gamma\}$ and every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ with $\bigcap_{j\in\Gamma} \mathcal{E}_j\, \bar{q}\mathcal{S}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)) \geq r,\ \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)) \leq 1-r,\ \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)) \leq 1-r$.

(3) $\bigcap_{j\in\Gamma} \mathcal{E}_j\, q\mathcal{S}$ holds for each family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1-r,\ j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ with $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)\ \mathrm{q}\mathcal{S},\ j \in \Gamma\}$ has the $\boldsymbol{I-FIP}$,

(4) For each family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{E}_j\ is\ r-SVNRO,\ j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r.\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$ with $\mathcal{S} \leq \bigcup_{j\in\Gamma} \mathcal{E}_j$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right]^c\right) \geq r, \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1-r, \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1-r.$$

(5) For each family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{E}_j$ is $\mathrm{r}-SVNRC,\ j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, with $\bigcap_{j\in\Gamma} \mathcal{E}_j\, \bar{q}\mathcal{S}$, there exists $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \leq 1-r, \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \leq 1-r,$$

(6) $\bigcap_{j\in\Gamma} \mathcal{E}_j\, q\mathcal{S}$ holds for each family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \mathcal{E}_j\ is\ r-SVNRC,\ j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ such taht $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}:\ j \in \Gamma\}$ has the $\boldsymbol{I-FIP}$.

Proof. (1)⇒(2). Let $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1-r,\ j \in \Gamma\}$and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$ with $\bigcap_{j\in\Gamma} \mathcal{E}_j\, \bar{q}\mathcal{S}$. Then, $\tilde{\gamma}_{\bigcap_{j\in\Gamma}\mathcal{E}_j} + \tilde{\gamma}_{\mathcal{S}} \leq 1,\ \tilde{\eta}_{\bigcap_{j\in\Gamma}\mathcal{E}_j} + \tilde{\eta}_{\mathcal{S}} \geq 1,\ \tilde{\mu}_{\bigcap_{j\in\Gamma}\mathcal{E}_j} + \tilde{\mu}_{\mathcal{S}} \geq 1$. It implies that $\mathcal{S} \leq \bigcup_{j\in\Gamma} \mathcal{E}_j^c$. By $r-\mathcal{SVNC}(\mathcal{J})-compactness$ of $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j^c, r)\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j^c, r)\right]^c\right) \leq 1-r, \quad \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c, r)\right]^c\right) \leq 1-r.$$

Since, $\mathcal{S} \cap [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c, r)]^c = \mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)$. Then

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right) \leq 1-r, \quad \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \bigcap_{j\in\Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right) \leq 1-r.$$

(2)⇒(3). It is trivial.

(3) ⇒ (1). Let $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1-r,\ j \in \Gamma\}$ be a family and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r$ such that $\mathcal{S} \leq \bigcup_{j\in\Gamma} \mathcal{E}_j$ with property that for no finite subfamily Γ_0 of Γ one has, $\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)]^c\right) \geq r,\ \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)]^c\right) \leq 1-r,\ \tilde{\mathcal{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)]^c\right) \leq 1-r$. Since, $\mathcal{S} \cap [\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)]^c = \bigcap_{j\in\Gamma_0}\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j^c, r) \cap \mathcal{S}$, the family $\{\bigcap_{j\in\Gamma}\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j^c, r) \cap \mathcal{S},\ \ j \in \Gamma\}$ has the $\boldsymbol{I-FIP}$, By (3), $\bigcap_{j\in\Gamma} \mathcal{E}_j^c\, q\mathcal{S}$ implies that $\bigcup_{j\in\Gamma} \mathcal{E}_j \leq \mathcal{S}$. It is a contradiction.

(1)⇒(4). Let $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}:\ j \in \Gamma\}$ be a family of $r-SVNRO\ sets$ and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1-r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1-r$ with $\mathcal{S} \leq \bigcup_{j\in\Gamma} \mathcal{E}_j$. Then, $\mathcal{S} \leq \bigcup_{j\in\Gamma} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)$. By $r-\mathcal{SVNC}(\mathcal{J})-compactness$ of $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\mathcal{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r), r), r)\right]^c\right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j\in\Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(int_{\tilde{\tau}^{\tilde{\eta}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r), r), r)\right]^c\right) \leq 1-r,$$

$$\tilde{\mathcal{I}}^{\tilde{\mu}}\left(\mathcal{S}\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r),r),r)\right]^c\right)\le 1-r$$

Since, for $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j)\ge r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j)\le 1-r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j)\le 1-r,\ C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}\big(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),\mathrm{r}),r\big)=C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r)$. Therefore, $\tilde{\mathcal{I}}^{\tilde{\gamma}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j,r)]^c\big)\ge r,\ \tilde{\mathcal{I}}^{\tilde{\eta}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j,r)]^c\big)\le 1-r,\ \tilde{\mathcal{I}}^{\tilde{\mu}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r)]^c\big)\le 1-r$.

(4)⇒(1). It is trivial.

(4)⇒(5). Let $\{\mathcal{E}_j\}_{j\in\Gamma}$ be a family of $r-SVNRC$ sets and every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\ge r,\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\le 1-r\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\le 1-r$ such that $\bigcap_{j\in\Gamma}\mathcal{E}_j\,\bar{q}\mathcal{S}$. Then, $\mathcal{S}\le\bigcup_{j\in\Gamma}\mathcal{E}_j^c$ and $\{\mathcal{E}_j^c\in I^{\tilde{\mathfrak{X}}}: j\in\Gamma\}$ be a family of $r-SVNRO$ sets. By (4), there exists a finite subset $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{I}}^{\tilde{\gamma}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j^c,r)]^c\big)\ge r,\ \tilde{\mathcal{I}}^{\tilde{\eta}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j^c,r)]^c\big)\le 1-r$, $\tilde{\mathcal{I}}^{\tilde{\mu}}\big(\mathcal{S}\cap[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c,r)]^c\big)\le 1-r$ implies that

$$\tilde{\mathcal{I}}^{\tilde{\gamma}}\left(\mathcal{S}\cap\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j,r)\right)\ge r,\qquad \tilde{\mathcal{I}}^{\tilde{\eta}}\left(\mathcal{S}\cap\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j,r)\right)\le 1-r,\qquad \tilde{\mathcal{I}}^{\tilde{\mu}}\left(\mathcal{S}\cap\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r)\right)\le 1-r.$$

(5)⇒(6). Let $\{\mathcal{E}_j\}_{j\in\Gamma}$ be a family of $r-SVNRC\ sets$ and every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c)\ge r,\ \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c)\le 1-r,\ \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c)\le 1-r$ such taht $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r)\cap\mathcal{S}: j\in\Gamma\}$ has the $\boldsymbol{I-FIP}$. If $\bigcap_{j\in\Gamma}\mathcal{E}_j\,\bar{q}\mathcal{S}$. By (5), there exists a finite subset $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{I}}^{\tilde{\gamma}}(\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j,r)\cap\mathcal{S})\ge r,\ \tilde{\mathcal{I}}^{\tilde{\eta}}(\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j,r)\cap\mathcal{S})\le 1-r,\ \tilde{\mathcal{I}}^{\tilde{\mu}}(\bigcap_{j\in\Gamma_0}int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j,r)\cap\mathcal{S})\le 1-r$. It is a contradiction.

(6)⇒(4). It is trivial.

Theorem 4.19. Let $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$, $\big(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ be two $\mathcal{SVNITS}'s$ and $f:\tilde{\mathfrak{X}}_1\longrightarrow\tilde{\mathfrak{X}}_2$ a surjective $\mathcal{SVN}$-continuous. If $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ is $r-\mathcal{SVNI}_1-compact$ and $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}(\mathcal{S})\le\tilde{\mathcal{I}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\eta}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\mu}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\mu}}(f(\mathcal{S}))$. Then, $\big(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ is $r-\mathcal{SVNI}_2-compact$.

Proof. Let $\big\{\mathcal{E}_j\in I^{\tilde{\mathfrak{X}}}:\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_j)\ge r,\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j)\le 1-r,\ \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_j)\le 1-r,\ j\in\Gamma\big\}$ be a family such that $\bigcup_{j\in\Gamma}\mathcal{E}_j=\tilde{1}$. Then, $\bigcup_{j\in\Gamma}f^{-1}(\mathcal{E}_j)=\tilde{1}$. Since, f is $\mathcal{SVN}-continuous$, for each $j\in\Gamma$, $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_j))\ge r$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_j))\le 1-r$, $\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_j))\le 1-r$. By $r-\mathcal{SVNI}_1-compactness$ of $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$, there exists a finite $\Gamma_0\subseteq\Gamma$ such that $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}\big([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c\big)\ge r,\ \tilde{\mathcal{I}}_1^{\tilde{\eta}}\big([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c\big)\le 1-r,\ \tilde{\mathcal{I}}_1^{\tilde{\mu}}\big([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c\big)\le 1-r$. Since $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}(\mathcal{S})\le\tilde{\mathcal{I}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\eta}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\mu}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\mu}}(f(\mathcal{S}))$, for $j\in\Gamma_0$, $\tilde{\mathcal{I}}_2^{\tilde{\gamma}}\big(f([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c)\big)\ge r$, $\tilde{\mathcal{I}}_2^{\tilde{\eta}}\big(f([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c)\big)\le 1-r$, $\tilde{\mathcal{I}}_2^{\tilde{\mu}}\big(f([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c)\big)\le 1-r$. From the surjectively of f we obtain $f\big([\bigcup_{j\in\Gamma_0}f^{-1}(\mathcal{E}_j)]^c\big)=[\bigcup_{j\in\Gamma_0}\mathcal{E}_j]^c$. Hence, $\tilde{\mathcal{I}}_2^{\tilde{\gamma}}\big([\bigcup_{j\in\Gamma_0}\mathcal{E}_j]^c\big)\ge r,\ \tilde{\mathcal{I}}_2^{\tilde{\eta}}\big([\bigcup_{j\in\Gamma_0}\mathcal{E}_j]^c\big)\le 1-r,\ \tilde{\mathcal{I}}_2^{\tilde{\mu}}\big([\bigcup_{j\in\Gamma_0}\mathcal{E}_j]^c\big)\le 1-r$. Thus, $\big(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ is $r-\mathcal{SVNI}_2-compact$.

Theorem 4.20. Let $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$, $\big(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ be two $\mathcal{SVNITS}'s$ and $f:\tilde{\mathfrak{X}}_1\longrightarrow\tilde{\mathfrak{X}}_2$ a surjective $\mathcal{SVN}$-continuous. If $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ is $r-\mathcal{SVNC}(\mathcal{I})_1-compact$ and $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}(\mathcal{S})\le\tilde{\mathcal{I}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\eta}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\mu}}(\mathcal{S})\ge\tilde{\mathcal{I}}_2^{\tilde{\mu}}(f(\mathcal{S}))$. Then, $\big(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$ is $r-\mathcal{SVNC}(\mathcal{I})_2-compact$.

Proof. Let $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S})\ge r,\ \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S})\le 1-r,\ \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S})\le 1-r$ and every family $\{\mathcal{E}_j\in I^{\tilde{\mathfrak{X}}}:\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_j)\ge r,\ \ \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j)\le 1-r\}$ with $\mathcal{S}\le\bigcup_{j\in\Gamma}\mathcal{E}_j$. Then, $f^{-1}(\mathcal{S})\le\bigcup_{j\in\Gamma}f^{-1}(\mathcal{E}_j)$. Since, f is $\mathcal{SVN}$- continuous for each $j\in\Gamma$, $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_j))\ge r$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_j))\le 1-r,\ \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_j))\le 1-r$. By $r-\mathcal{SVNC}(\mathcal{I})_1-compactness$ of $\big(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\big)$, there exists a finite $\Gamma_0\subseteq\Gamma$ such that

$$\tilde{\mathcal{J}}_1^{\tilde{\gamma}}\left(f^{-1}(\mathcal{S})\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_1^{\tilde{\gamma}}}(f^{-1}(\mathcal{E}_j),\mathrm{r})\right]^c\right)\geq r,\quad \tilde{\mathcal{J}}_1^{\tilde{\eta}}\left(f^{-1}(\mathcal{S})\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_1^{\tilde{\eta}}}(f^{-1}(\mathcal{E}_j),\mathrm{r})\right]^c\right)\leq 1-r,$$

$$\tilde{\mathcal{J}}_1^{\tilde{\mu}}\left(f^{-1}(\mathcal{S})\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_1^{\tilde{\mu}}}(f^{-1}(\mathcal{E}_j),\mathrm{r})\right]^c\right)\leq 1-r.$$

Since, f is $\mathcal{SVN}$ - continuous mapping, $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}_j,\mathrm{r})\leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j,\mathrm{r}))$ for every $\mathcal{S}\in I^{\tilde{\mathfrak{X}}_2}$. Therefore, $f^{-1}(\mathcal{S})\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{E}_j,\mathrm{r})\right]^c=f^{-1}(\mathcal{S})\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,\mathrm{r}))\right]^c$. Hence,

$$\tilde{\mathcal{J}}_1^{\tilde{\gamma}}\left(f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S},\mathrm{r}))\right]^c\right)\geq r,\quad \tilde{\mathcal{J}}_1^{\tilde{\eta}}\left(f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S},\mathrm{r}))\right]^c\right)\leq 1-r,$$

$$\tilde{\mathcal{J}}_1^{\tilde{\mu}}\left(f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S},\mathrm{r}))\right]^c\right)\leq 1-r.$$

Since, $\tilde{\mathcal{J}}_1^{\tilde{\gamma}}(\mathcal{S})\leq\tilde{\mathcal{J}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{J}}_1^{\tilde{\eta}}(\mathcal{S})\geq\tilde{\mathcal{J}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{J}}_1^{\tilde{\mu}}(\mathcal{S})\geq\tilde{\mathcal{J}}_2^{\tilde{\mu}}(f(\mathcal{S}))$, for each $j\in\Gamma_0$ we have,

$$\tilde{\mathcal{J}}_2^{\tilde{\gamma}}\left(f[f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}\left(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S},\mathrm{r})\right)\right]^c]\right)\geq r,\quad \tilde{\mathcal{J}}_2^{\tilde{\eta}}\left(f[f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}\left(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S},\mathrm{r})\right)\right]^c]\right)\leq 1-r,$$

$$\tilde{\mathcal{J}}_2^{\tilde{\mu}}\left(f[f^{-1}(\mathcal{S}_j)\cap\left[\bigcup_{j\in\Gamma_0}f^{-1}\left(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S},\mathrm{r})\right)\right]^c]\right)\leq 1-r.$$

Since, f is surjective,

$$\tilde{\mathcal{J}}_2^{\tilde{\gamma}}\left(\mathcal{S}_j\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S},\mathrm{r})\right]^c\right)\geq r,\quad \tilde{\mathcal{J}}_2^{\tilde{\eta}}\left(\mathcal{S}_j\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S},\mathrm{r})\right]^c\right)\leq 1-r,\quad \tilde{\mathcal{J}}_2^{\tilde{\mu}}\left(\mathcal{S}_j\cap\left[\bigcup_{j\in\Gamma_0}C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S},\mathrm{r})\right]^c\right)\leq 1-r.$$

Thus, $\left(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right)$ is $r-\mathcal{SVN}(\mathcal{I})_2-compact$.

Theorem 4.21. The image of an $r-\mathcal{SVNI}_1-compact$ under a surjective $\mathcal{SVN}-almost\ continuous$ mapping and $\tilde{\mathcal{J}}_1^{\tilde{\gamma}}(\mathcal{S})\leq\tilde{\mathcal{J}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{J}}_1^{\tilde{\eta}}(\mathcal{S})\geq\tilde{\mathcal{J}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{J}}_1^{\tilde{\mu}}(\mathcal{S})\geq\tilde{\mathcal{J}}_2^{\tilde{\mu}}(f(\mathcal{S}))$ is $r-\mathcal{SVNC}(\mathcal{I})_2-compact$.

Proof. Let $\mathcal{S}\in \mathrm{I}^{\tilde{\mathfrak{X}}_1}$ be an $r-\mathcal{SVNI}_1-compact$ in $\left(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right)$ and $f:\left(\tilde{\mathfrak{X}}_1,\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right)\to\left(\tilde{\mathfrak{X}}_2,\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}},\tilde{\mathcal{J}}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right)$ a surjective $\mathcal{SVN}-almost\ continuous$. If $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}^c)\geq r,\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}^c)\leq 1-r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}^c)\leq 1-r$ and each family $\{\mathcal{E}_j\in I^{\tilde{\mathfrak{X}}}$: $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_j)\geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j)\leq 1-r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_j)\leq 1-r\}$ with $f(\mathcal{S})\leq\bigcup_{j\in\Gamma}\mathcal{E}_j$, then $f(\mathcal{S})\leq\bigcup_{j\in\Gamma}int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r)$ and

since for $j\in\Gamma$,

$$int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r),r),\mathrm{r})=int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r).$$

By $\mathcal{SVN}-almost\ continuous$ of f we have $\mathcal{S}\leq\bigcup_{j\in\Gamma}f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r))$ and

$$\tilde{\tau}_1^{\tilde{\gamma}}\left(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{E}_j,r),r))\right)\geq r,\quad \tilde{\tau}_2^{\tilde{\eta}}\left(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\eta}}}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{E}_j,r),r))\right)\leq 1-r,$$

$$\tilde{\tau}_1^{\tilde{\mu}}\left(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{E}_j,r),r))\right) \leq 1-r.$$

By $r-\mathcal{SVNI}_1-compactness$ of $\mathcal{S}$ in $\left(\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{I}}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}\right)$, there exists a finite $\Gamma_0 \subseteq \Gamma$ such that

$$\tilde{\mathcal{I}}_1^{\tilde{\gamma}}\left(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{E}_j,r),r))\right]^c\right) \geq r, \quad \tilde{\mathcal{I}}_1^{\tilde{\eta}}\left(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\eta}}}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{E}_j,r),r))\right]^c\right) \leq 1-r,$$

$$\tilde{\mathcal{I}}_1^{\tilde{\mu}}\left(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{E}_j,r),r))\right]^c\right) \leq 1-r.$$

Since $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{\mathcal{I}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{\mathcal{I}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{\mathcal{I}}_2^{\tilde{\mu}}(f(\mathcal{S}))$, we have

$$\tilde{\mathcal{I}}_2^{\tilde{\gamma}}\left(f(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{E}_j,r),r))\right]^c)\right) \geq r, \quad \tilde{\mathcal{I}}_2^{\tilde{\eta}}\left(f(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\eta}}}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{E}_j,r),r))\right]^c)\right) \leq 1-r,$$

$$\tilde{\mathcal{I}}_2^{\tilde{\mu}}\left(f(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{E}_j,r),r))\right]^c)\right) \leq 1-r.$$

By surjectively of f, $f(\mathcal{S}_j \cap \left[\bigcup_{j\in\Gamma_0} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r),r))\right]^c) = f(\mathcal{S}_j) \cap \left[\bigcup_{j\in\Gamma_0}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j,r)\right]^c$. Thus,

$$\tilde{\mathcal{I}}_2^{\tilde{\gamma}}\left(f(\mathcal{S}_j) \cap \left[\bigcup_{j\in\Gamma_0}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{E}_j,r)\right]^c\right) \geq r, \quad \tilde{\mathcal{I}}_2^{\tilde{\eta}}\left(f(\mathcal{S}_j) \cap \left[\bigcup_{j\in\Gamma_0}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r,$$

$$\tilde{\mathcal{I}}_2^{\tilde{\mu}}\left(f(\mathcal{S}_j) \cap \left[\bigcup_{j\in\Gamma_0}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{E}_j,r)\right]^c\right) \leq 1-r.$$

and hence, $f(\mathcal{S})$ is $\mathrm{r}-\mathcal{SVNC}(\mathcal{I})_2-compact$.

Theorem 4.22. The image of an $r-\mathcal{SVNI}_1-compact$ under a surjective $\mathcal{SVN}-weakly\ continuous$ mapping and $\tilde{\mathcal{I}}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{\mathcal{I}}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{\mathcal{I}}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\mathcal{I}}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{\mathcal{I}}_2^{\tilde{\mu}}(f(\mathcal{S}))$, is $r-\mathcal{SVNI}_2$ −quasi H-closed.

Proof. Similar to proof of Theorem 4.21.

5. Conclusions

In the current research paper, we found some results of single-valued neutrosophic continuous mappings called almost continuous and weakly continuous. These instances are kinds of some generalizations of fuzzy continuity in view of the definition of $\tilde{S}$ostak. We brought counterexamples whenever such properties fail to be preserved. We also introduced and studied several kinds of *r*-single-valued neutrosophic compactness defined on the single-valued neutrosophic ideal topological spaces.

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