



Constant single valued neutrosophic graphs with applications

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Abstract. In this paper, we introduced a new concept of single valued neutrosophic graph (SVNG) known as constant single valued neutrosophic graph (CSVNG). Basically, SVNG is a generalization of intuitionistic fuzzy graph (IFG). More specifically, we described and explored some graph theoretic ideas related to the introduced concepts of CSVNG. An application of CSVNG in a Wi-Fi network system is discussed and a comparison of CSVNG with constant IFG is established showing the worth of the proposed work. Further, several terms like constant function and totally constant function are investigated in the frame-work of CSVNG and their characteristics are studied.

Keywords. Single valued neutrosophic graph. Constant single valued neutrosophic graph; constant function; totally constant function; Wi-Fi network.

1. Introduction

Dealing with uncertain situations and insufficient information requires some high potential mathematical tools. Graph theory is one of the mathematical tools which effectively deals with large data. If there are some of uncertainty factors, then fuzzy graph is the appropriate tool to be used. In addition to its ability of handling large data, graph theory has a special interest as it can be applied in several important areas including management sciences [19], social sciences [17], computer and information sciences [41], communication networks [18], description of group structures [39], database theory [26] and economics [25].

The concept of fuzzy set (FS) proposed by Zadeh [46] is among the famous tools dealing with uncertain situations and insufficient information. After, Kaufmann [20] introduced the notion of fuzzy graph. A comprehensive study on fuzzy graphs is done by Rosenfeld [40] in which he shown some of their basic properties. The work in the field of graph theory is exemplary during the past decades as its concepts are applied in many real-life problems such as cluster analysis [14,6,45,30], slicing [30], for solving fuzzy intersecting equations [31,29], in some theory of data base [26], in networking problems [27], in the structure of a group [43, 32], in chemistry [44], in air trafficking [35], in the control of traffic [34] etc. The worth of FG lies in its capability of handling with uncertainties and it has done so far better but Atanassov [1] proposed that FSs only deals with one sided uncertainties which is not enough as human nature isn't limited to only yes type or no type problems. Hence the logic of intuitionistic fuzzy set (IFS) have been developed sufficient to deal with uncertainties of both yes and no types. Atanassov's IFS gave rise to the theory of IFG proposed by Parvathi and Karunambigai [36]. The structure of IFG is advanced and is applied successfully social networks [13], clustering [23], radio coverage network [21] and shortest path problems [32] etc. Furthermore, Parvathi et al [36-28] did some work on constant IFGs and operations of IFGs. The concept of intuitionistic fuzzy hypergraphs (IFHG) was proposed by Parvathi et al. [37] which were applied in real life problems by Akram and Wieslaw [3]. Nagoor Gani and Shajitha [15] wrote about degree, order and size for IFG in 2010. Akram and Davvaz [2] gave the concept of strong IFG.

Smarandache in 1995 develop the neutrosophic logic which give rise to a novel theory of neutrosophic set (NS) [42] which give rise to the development of single/double and triple valued NSs [16,22,24]. Broumi et al initiated the concept of single-valued neutrosophic graph (SVNG) [7]. Work on the operations of SVNG can be found in [5]. Note on the degree, order and size of SVNG is present in [8]. Recently, Broumi et al [47] introduced a single-valued neutrosophic techniques for analysis of WIFI connection. The hypergraph i.e. single-valued neutrosophic hyper graph is introduced in [4]. Neutrosophic sets and graphs have been widely studied in recent decades. Various

real life applications are discussed using neutrosophic techniques. For development in neutrosophic sets and graphs and their applications, one is refer to [9-12, 48-67,68-71].

In this paper, we introduced the concept of CSVNG and investigated some graph theoretic ideas related to this introduced concept. An application of CSVNG in a Wi-Fi network system is discussed and a comparison of CSVNG with constant IFG is established in order to show the worth of the proposed concept.

The rest of the paper is organized as follows. In Section 2, we recalled the necessary basic concepts and properties of IFG, CIFG and SVNG. In section 3, the concept of CSVNG is described and some related graph theoretic ideas are explored. In Section 4, we discussed the characteristic of CSVNGs, while section 5 deals with an application of CSVNGs in Wi-Fi network system. Finally, advantages and concluding remarks are discussed.

2 Preliminaries

This section is basically about some very basic definitions. The concepts of IFG, CIFG and SVNG are discussed and explained with the help of some examples. For undefined terms and notions, we refer to [5, 8, 35, 36].

Definition 1 [36]. A Pair $G = (\tilde{V}, \tilde{E})$ is said to be *IFG* if

- (i) $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \dots, \tilde{v}_n\}$ are the set of vertices such that $\tilde{T}_1: \tilde{V} \rightarrow [0, 1]$ and $F_1: \tilde{V} \rightarrow [0, 1]$ represents the degree of membership and non-membership of the element $\tilde{v}_i \in \tilde{V}$ respectively with a condition that $0 \leq \tilde{T}_1(\tilde{v}_i) + F_1(\tilde{v}_i) \leq 1$ for all $\tilde{v}_i \in \tilde{V}$, ($i \in I$).
- (ii) $\tilde{E} \subseteq \tilde{V} \times \tilde{V}$ where $\tilde{T}_2: \tilde{V} \times \tilde{V} \rightarrow [0, 1]$ and $F_2: \tilde{V} \times \tilde{V} \rightarrow [0, 1]$ represents the degree of membership and non-membership of the element $(\tilde{v}_i, \tilde{v}_j) \in \tilde{E}$ such that $\tilde{T}_2(\tilde{v}_i, \tilde{v}_j) \leq \min\{\tilde{T}_1(\tilde{v}_i), \tilde{T}_1(\tilde{v}_j)\}$ and $F_2(\tilde{v}_i, \tilde{v}_j) \leq \max\{F_1(\tilde{v}_i), F_1(\tilde{v}_j)\}$ with a condition $0 \leq \tilde{T}_2(\tilde{v}_i, \tilde{v}_j) + F_2(\tilde{v}_i, \tilde{v}_j) \leq 1$ for all $(\tilde{v}_i, \tilde{v}_j) \in \tilde{E}$ ($i \in I$).

Example 1. Let $G = (\tilde{V}, \tilde{E})$ be an IFG where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3\}$ be the set of vertices and $\tilde{E} = \{\tilde{v}_1\tilde{v}_2, \tilde{v}_1\tilde{v}_3, \tilde{v}_2\tilde{v}_3\}$ be the set of edges. Then

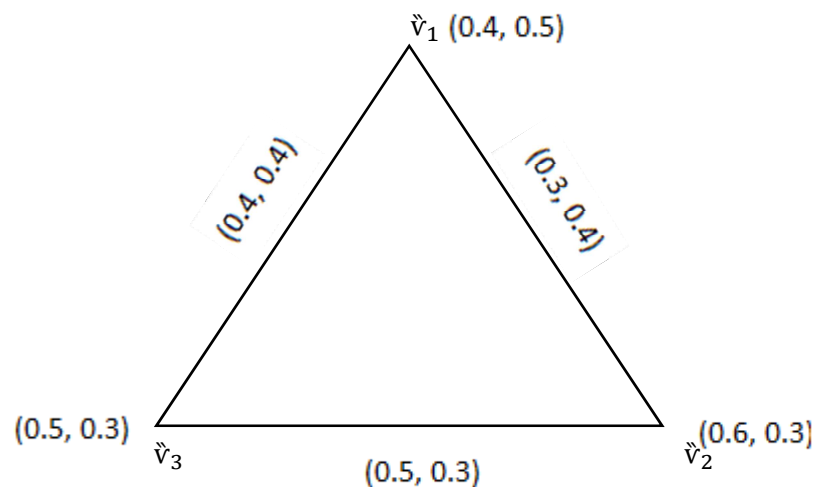


Figure 1 (IFG)

Definition 2 [28]. A pair $G = (\tilde{V}, \tilde{E})$ is said to be Constant-IFG of degree (k_i, k_j) or (k_i, k_j) -IFG. If

$$d_T(\tilde{v}_i) = k_i, d(F_j) = k_j \forall \tilde{v}_i, \tilde{v}_j \in \tilde{V}.$$

Example 2. Let $G = (\tilde{V}, \tilde{E})$ be an IFG where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4\}$ be the set of vertices and $\tilde{E} = \{\tilde{v}_1\tilde{v}_2, \tilde{v}_2\tilde{v}_3, \tilde{v}_3\tilde{v}_4, \tilde{v}_4\tilde{v}_1\}$ be the set of edges. Then

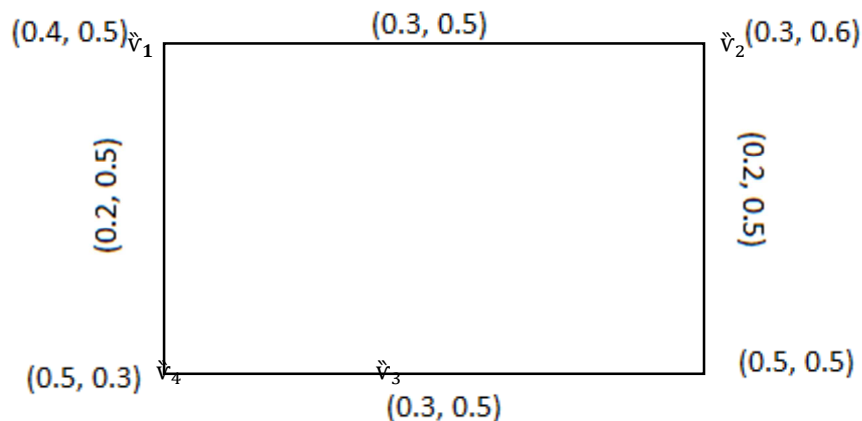


Figure 2 (Constant-IFG of degree (k_i, k_j))

The degree of $\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4$ is $(0.5, 1.0)$.

Definition 3 [7]. A pair $G = (\check{V}, \check{E})$ is said to be as *SVNG* if

- (i) $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \dots, \check{v}_n\}$ are the set of vertices such that $\check{T}_1: \check{V} \rightarrow [0, 1]$, $\check{I}_1: \check{V} \rightarrow [0, 1]$ and $\check{F}_1: \check{V} \rightarrow [0, 1]$ denote the degree of membership, indeterminacy and non-membership of the element $\check{v}_i \in \check{V}$ respectively with a condition that $0 \leq \check{T}_1 + \check{I}_1 + \check{F}_1 \leq 3$ for all $\check{v}_i \in \check{V}$, $(i \in I)$.
- (ii) $\check{E} \subseteq \check{V} \times \check{V}$ where $\check{T}_2: \check{V} \times \check{V} \rightarrow [0, 1]$, $\check{I}_2: \check{V} \times \check{V} \rightarrow [0, 1]$ and $\check{F}_2: \check{V} \times \check{V} \rightarrow [0, 1]$ denote the degree of membership, abstinance and non-membership of the element $(\check{v}_i, \check{v}_j) \in \check{E}$ such that $\check{T}_2(\check{v}_i, \check{v}_j) \leq \min\{\check{T}_2(\check{v}_i), \check{T}_2(\check{v}_j)\}$, $\check{I}_2(\check{v}_i, \check{v}_j) \geq \max\{\check{I}_2(\check{v}_i), \check{I}_2(\check{v}_j)\}$ and $\check{F}_2(\check{v}_i, \check{v}_j) \geq \max\{\check{F}_2(\check{v}_i), \check{F}_2(\check{v}_j)\}$ with a condition $0 \leq \check{T}_2(\check{v}_i, \check{v}_j) + \check{I}_2(\check{v}_i, \check{v}_j) + \check{F}_2(\check{v}_i, \check{v}_j) \leq 3$ for all $(\check{v}_i, \check{v}_j) \in \check{E}$, $(i \in I)$.

Example 3. Let $G = (\check{V}, \check{E})$ be a *SVNG* where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

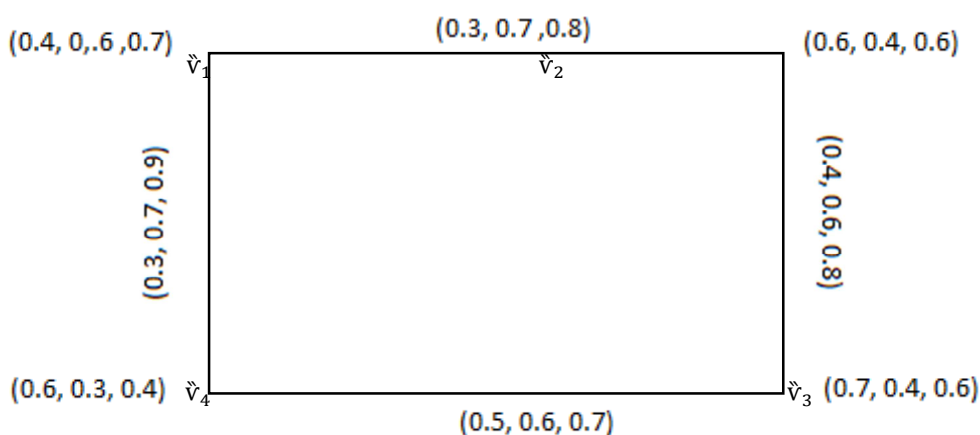


Figure 3 .SVNG

3 Constant single valued neutrosophic graph

In this section, the concept of CSVNG is introduced and supported with some examples. We discussed some related terms like completeness, total degree and constant function and exemplified them. Some results are also studied related to completeness and constant functions.

Definition 4. A pair $G = (\tilde{V}, \tilde{E})$ is said to be constant-SVNG of degree (k_i, k_j, k_k) or $(k_i, k_j, k_k) - SVNG$. If $d_T(\tilde{v}_i) = k_i, d_I(\tilde{v}_j) = k_j,$ and $d_F(\tilde{v}_k) = k_k \forall \tilde{v}_i, \tilde{v}_j, \tilde{v}_k \in \tilde{V}$.

Example 4. Let $G = (\tilde{V}, \tilde{E})$ be a SVNG where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4\}$ be the set of vertices and $\tilde{E} = \{\tilde{v}_1\tilde{v}_2, \tilde{v}_2\tilde{v}_3, \tilde{v}_3\tilde{v}_4, \tilde{v}_4\tilde{v}_1\}$ be the set of edges. Then CSVNG is shown in the below figure 4.

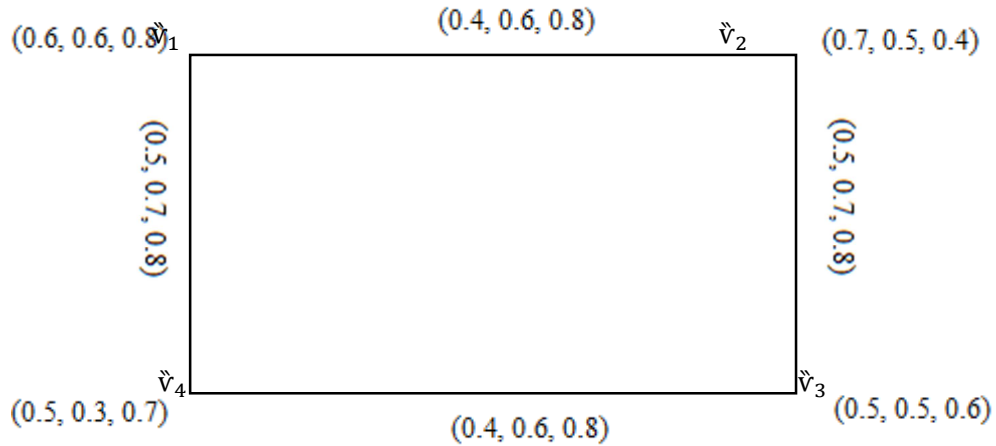


Figure 4 (Constant-SVNG of degree (k_i, k_j, k_k))

The degree of $\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4$ is $(0.9, 1.3, 1.6)$.

Remark 1. A complete SVNG may not be a constant-SVNG.

Example 5. Consider a graph $G = (\tilde{V}, \tilde{E})$ where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4\}$ be the set of vertices and $\tilde{E} = \{\tilde{v}_1\tilde{v}_2, \tilde{v}_2\tilde{v}_3, \tilde{v}_2\tilde{v}_4, \tilde{v}_1\tilde{v}_3, \tilde{v}_3\tilde{v}_4, \tilde{v}_4\tilde{v}_1\}$ be the set of edges. Then

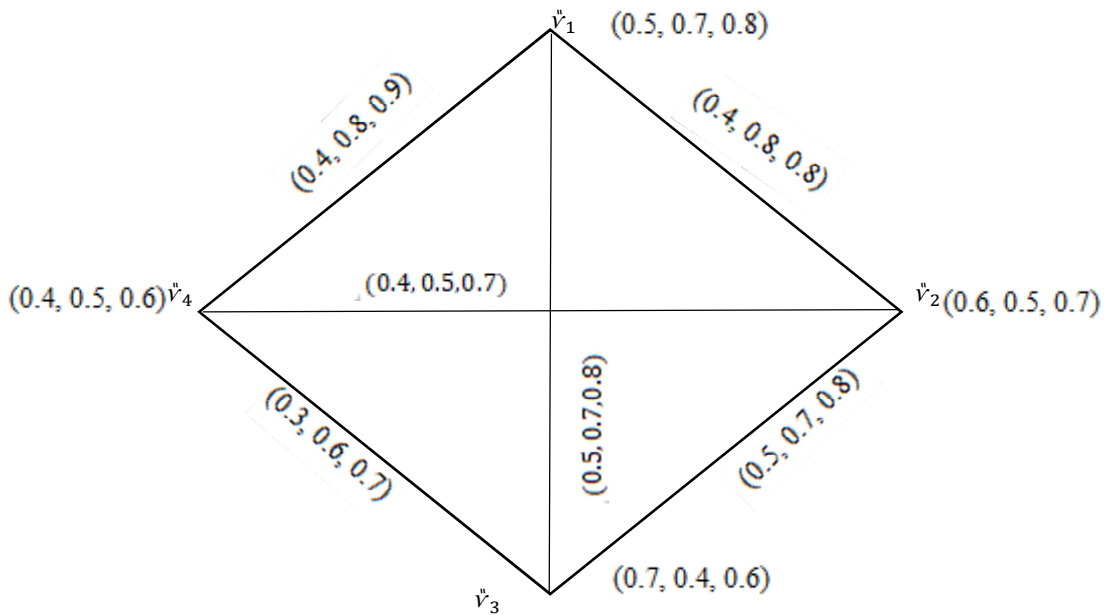


Figure 5 (G is complete but not Constant-SVNG)

Definition 5. The total degree of a vertex \check{v} in aSVNG is defined as

$$td(\check{v}) = \left[\sum_{\check{v} \in \check{E}} d_{T_2}(\check{v}) + \hat{T}_1(\check{v}), \sum_{\check{v} \in \check{E}} d_{I_2}(\check{v}) + \hat{I}_1(\check{v}), \sum_{\check{v} \in \check{E}} d_{F_2}(\check{v}) + F_1(\check{v}) \right]$$

If every vertex has the same total degree, then it is called SVNG of total degree or totally constant SVNG.

Example 6. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

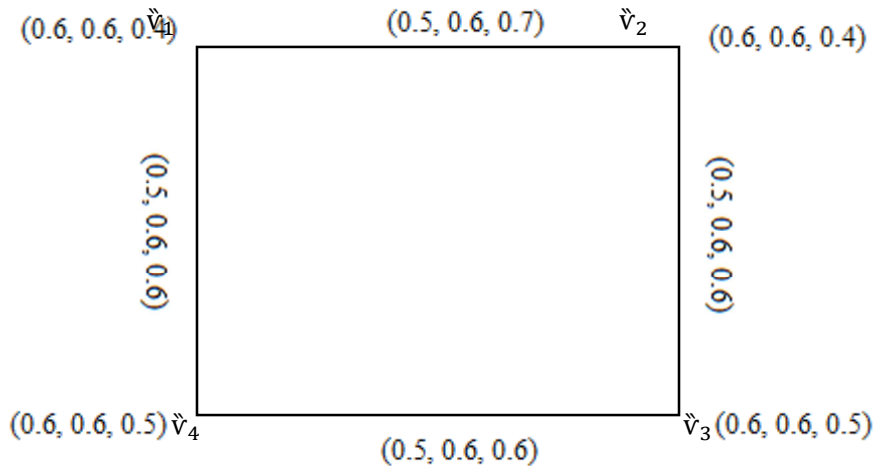


Figure 6 (SVNG)

Constant SVNG of total degree (1.6, 1.8, 1.7).

Theorem 1. If G be a SVNG. Then $(\hat{T}_1, \hat{I}_1, F_1)$ is a constant function iff the following are equivalent.

- (i) G is a constant SVNG.
- (ii) G is a totally constant SVNG.

Proof Let $(\hat{T}_1, \hat{I}_1, F_1)$ be a constant function and $\hat{T}_1(\check{v}) = \hat{c}_1, \hat{I}_1(\check{v}) = \hat{c}_2,$ and $F_1(\check{v}) = \hat{c}_3$ for all $\check{v}_i \in \check{V}$. Where \hat{c}_1, \hat{c}_2 and \hat{c}_3 are constants. Suppose that G is a (k_i, k_j, k_k) -CSVNG. Then $d_T(\check{v}_i) = k_1, d_I(\check{v}_i) = k_2$ and $d_F(\check{v}_i) = k_3$ for all $\check{v}_i \in \check{V}$. Therefore, $td_T(\check{v}_i) = d_T(\check{v}_i) + \hat{T}_1(\check{v}_i), td_I(\check{v}_i) = d_I(\check{v}_i) + \hat{I}_1(\check{v}_i)$ and $td_F(\check{v}_i) = d_F(\check{v}_i) + F_1(\check{v}_i)$ for all $\check{v}_i \in \check{V}, td_T(\check{v}_i) = k_1 + \hat{c}_1, td_I(\check{v}_i) = k_2 + \hat{c}_2$ and $td_F(\check{v}_i) = k_3 + \hat{c}_3$ for all $\check{v}_i \in \check{V}$. Hence G is a totally constant SVNG.

Now, Assume that G is a $(\hat{T}_1, \hat{I}_1, F_1)$ -totally constant SVNG. Then $td_T(\check{v}_i) = r_1, td_I(\check{v}_i) = r_2$ and $td_F(\check{v}_i) = r_3$ for all $\check{v}_i \in \check{V}$. $d_T(\check{v}_i) + \hat{T}_1(\check{v}_i) = r_1, d_I(\check{v}_i) + \hat{I}_1(\check{v}_i) = r_2, d_F(\check{v}_i) + F_1(\check{v}_i) = r_3$, similarly $d_T(\check{v}_i) = r_1 - \hat{c}_1, d_I(\check{v}_i) = r_2 - \hat{c}_2$ and $d_F(\check{v}_i) = r_3 - \hat{c}_3$. Therefore, G is a constant SVNG. Hence (i) and (ii) are equivalent.

Conversely, assume that (i) and (ii) are equivalent That is G is a constant SVNG iff G is a totally constant SVNG. Assume $(\hat{T}_1, \hat{I}_1, F_1)$ is not a constant function. Then $\hat{T}_1(\check{v}_1) \neq \hat{T}_1(\check{v}_2), \hat{I}_1(\check{v}_1) \neq \hat{I}_1(\check{v}_2)$ and $F_1(\check{v}_1) \neq F_1(\check{v}_2)$ for at least one pair of vertices $\check{v}_1, \check{v}_2 \in \check{V}$. Consider G be a (k_i, k_j, k_k) -SVNG. Then, $\hat{T}_1(\check{v}_1) = \hat{T}_1(\check{v}_2) = k_1, \hat{I}_1(\check{v}_1) = \hat{I}_1(\check{v}_2) = k_2$ and $F_1(\check{v}_1) = F_1(\check{v}_2) = k_3$. So, $td_T(\check{v}_1) = d_T(\check{v}_1) + \hat{T}_1(\check{v}_1) = k_1 + \hat{T}_1(\check{v}_1)$, and $td_T(\check{v}_2) = k_1 + \hat{T}_1(\check{v}_2)$. Similarly, $td_I(\check{v}_1) = k_2 + \hat{I}_1(\check{v}_1), td_I(\check{v}_2) = k_2 + \hat{I}_1(\check{v}_2)$ and $td_F(\check{v}_1) = k_3 + F_1(\check{v}_1), td_F(\check{v}_2) = k_3 + F_1(\check{v}_2)$. Since $\hat{T}_1(\check{v}_1) \neq \hat{T}_1(\check{v}_2), \hat{I}_1(\check{v}_1) \neq \hat{I}_1(\check{v}_2)$ and $F_1(\check{v}_1) \neq F_1(\check{v}_2)$. We have $td_T(\check{v}_1) \neq td_T(\check{v}_2), td_I(\check{v}_1) \neq td_I(\check{v}_2)$ and $td_F(\check{v}_1) \neq td_F(\check{v}_2)$. Therefore, G is not totally constant SVNG which is contradiction to our supposition.

Now, consider G be a totally constant SVNG. Then, $td_T(\check{v}_1) = td_T(\check{v}_2), d_T(\check{v}_1) + \hat{T}_1(\check{v}_1) = d_T(\check{v}_2) + \hat{T}_1(\check{v}_2), d_T(\check{v}_1) - d_T(\check{v}_2) = \hat{T}_1(\check{v}_2) - \hat{T}_1(\check{v}_1)$ (i.e. $\neq 0$) $d_T(\check{v}_1) \neq d_T(\check{v}_2)$. Similarly $d_I(\check{v}_1) \neq d_I(\check{v}_2)$ and $d_F(\check{v}_1) \neq d_F(\check{v}_2)$. G is not constant which is contradiction to our assumption. Hence $(\hat{T}_1, \hat{I}_1, F_1)$ is constant function.

Example 7. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

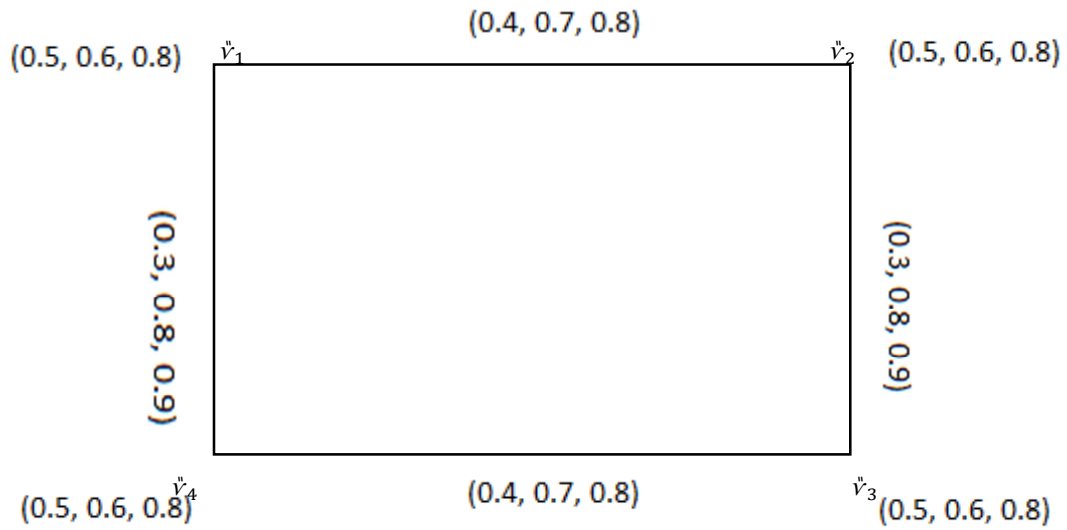


Figure 7. SVNG

$(\hat{T}_1, \hat{I}_1, F_1)$ is a constant function, then G is constant and totally constant.

Theorem 2. Let G is constant and totally constant then $(\hat{T}_1, \hat{I}_1, F_1)$ is a constant function.

Proof. Assume that G be a (k_i, k_j, k_k) –constant and (r_1, r_2, r_3) –totally constant SVNG. Therefore, $d_T(\check{v}_1) = k_1, d_I(\check{v}_1) = k_2$ and $d_F(\check{v}_1) = k_3$ for $\check{v}_1 \in \check{V}$ and $td_T(\check{v}_1) = r_1, td_I(\check{v}_1) = r_2$ and $td_F(\check{v}_1) = r_3$ for all $\check{v} \in \check{V}$. $\hat{T}_1(\check{v}) + k_1 = r_1$ for all $\check{v} \in \check{V}$. $\hat{T}_1(\check{v}) = r_1 - k_1$, for all $\check{v} \in \check{V}$. Hence $\hat{T}_1(\check{v}_1)$ is a constant function. Similarly $\hat{I}_1(\check{v}) = r_2 - k_2$ and $F_1(\check{v}) = r_3 - k_3$ for all $\check{v} \in \check{V}$.

Remark 2. Converse of the above theorem 2 is not true.

Example 8. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

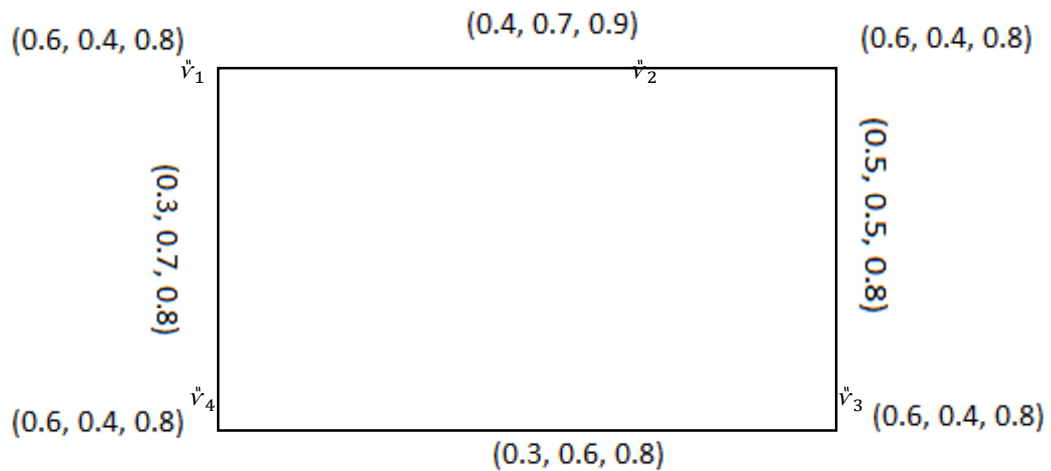


Figure 8. SVNG

$(\hat{T}_1, \hat{I}_1, F_1)$ is a constant function But neither constant SVNG nor totally constant SVNG.

4 Characterization of constant SVNG on a cycle

This section is based on some important results on even (odd) cycles, bridges in SVNGs and cut vertex of even (odd) cycle. The stated results are supported with some examples. .

Theorem 3. If G is an SVNG where crisp graph G is an odd cycle. Then G is constant SVNG iff $(\hat{T}_2, \hat{I}_2, F_2)$ is a constant function.

Proof. Suppose $(\hat{T}_2, \hat{I}_2, F_2)$ is constant function $\hat{T}_2 = \hat{c}_1, \hat{I}_2 = \hat{c}_2$, and $F_2 = \hat{c}_3$ for all $(\check{v}_i, \check{v}_j) \in \check{E}$. Then $d_{\hat{T}}(\check{v}_i) = 2\hat{c}_1, d_{\hat{I}}(\check{v}_i) = 2\hat{c}_2$ and $d_{F_2}(\check{v}_i) = 2\hat{c}_3$ for all $\check{v}_2 \in \check{V}$ SoG is constant SVNG.

Conversely, assume that G is (k_1, k_2, k_3) –regular SVNG. If $e_1, e_2, e_3 \dots e_{2n+1}$ be the edges of G in that order. If $\hat{T}_2(e_1) = \hat{c}_1, \hat{T}_2(e_2) = k_1 - \hat{c}_1, \hat{T}_2(e_3) = k_1 - (k_1 - \hat{c}_1) = \hat{c}_1, \hat{T}_2(e_4) = k_1 - \hat{c}_1$ and so on. Likewise, $\hat{I}_2(e_1) = \hat{c}_2, \hat{I}_2(e_2) = k_2 - \hat{c}_2, \hat{I}_2(e_3) = k_2 - (k_2 - \hat{c}_2) = \hat{c}_2, \hat{I}_2(e_4) = k_2 - \hat{c}_2$ and $F_2(e_1) = \hat{c}_3, F_2(e_2) = k_3 - \hat{c}_3, F_2(e_3) = k_3 - (k_3 - \hat{c}_3) = \hat{c}_3, F_2(e_4) = k_3 - \hat{c}_3$ and so on. Therefore

$$\hat{T}_2(e_i) = \begin{cases} \hat{c}_1, & \text{if } i \text{ is odd} \\ k_1 - \hat{c}_1, & \text{if } i \text{ is even} \end{cases}$$

Hence $\hat{T}_2(e_1) = \hat{T}_2(e_{2n+1}) = \hat{c}_1$. So, if e_1 and e_{2n+1} incident at a vertex \check{v}_1 , then $d_{\hat{T}}(\check{v}_1) = k_1, d(e_1) + d(e_{2n+1}) = k_1, \hat{c}_1 + \hat{c}_1 = k_1, 2\hat{c}_1 = k_1, \hat{c}_1 = \frac{k_1}{2}$.

Remark 3. The above theorem (3) is not true for totally constant SVNG.

Example 8. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_1\}$ be the set of edges. Then

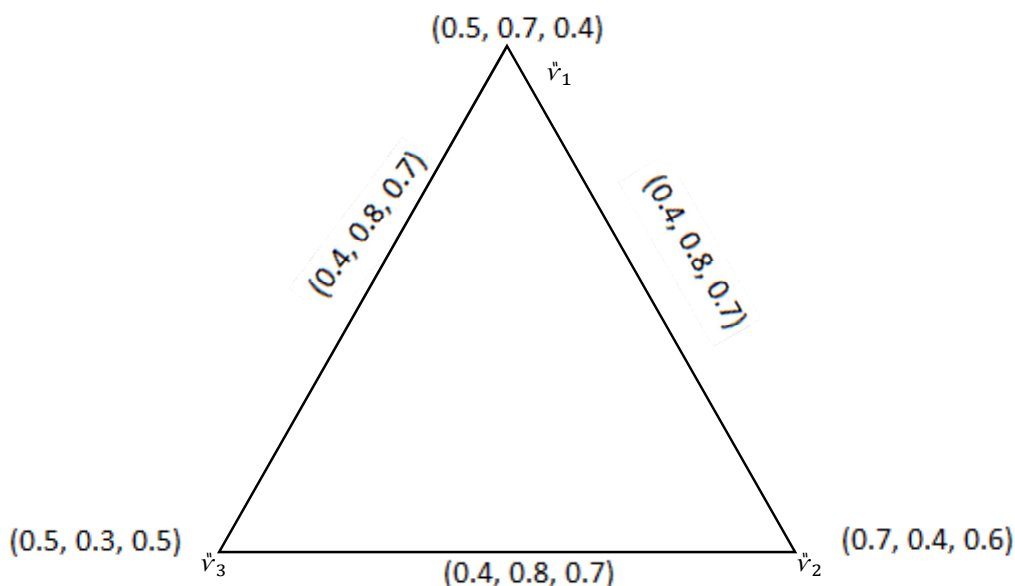


Figure 9. SVNG

$(\hat{T}_2, \hat{I}_2, F_2)$ is constant function but not totally constant.

Theorem 4. If G is an SVNG where crisp graph G is an even cycle. Then G is constant SVNG iff either $(\hat{T}_2, \hat{I}_2, F_2)$ is a constant function or alternative edges have same membership, indeterminacy and non-membership values.

Proof. If $(\hat{T}_2, \hat{I}_2, F_2)$ is a constant function then G is constant SVNG. Conversely, assume that G is (k_1, k_2, k_3) –constant SVNG. If $e_1, e_2, e_3 \dots e_{2n}$ be the edges of even cycle G in that order. By using the above theorem (3), $\hat{T}_2(e_i) = \begin{cases} \hat{c}_1, & \text{if } i \text{ is odd} \\ k_1 - \hat{c}_1, & \text{if } i \text{ is even} \end{cases}, \hat{I}_2(e_i) = \begin{cases} \hat{c}_2, & \text{if } i \text{ is odd} \\ k_2 - \hat{c}_2, & \text{if } i \text{ is even} \end{cases}$

And

$F_2(e_i) = \begin{cases} \hat{c}_3, & \text{if } i \text{ is odd} \\ k_3 - \hat{c}_3, & \text{if } i \text{ is even} \end{cases}$. If $\hat{c}_1 = k_1 - \hat{c}_1$, the $(\hat{T}_2, \hat{I}_2, F_2)$ is constant function. If $\hat{c}_1 \neq k_1 - \hat{c}_1$ then alternative edges have same membership, indeterminacy and non-membership values.

Remark 4. The above theorem (4) is not true for totally constant SVNG.

Example 9. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

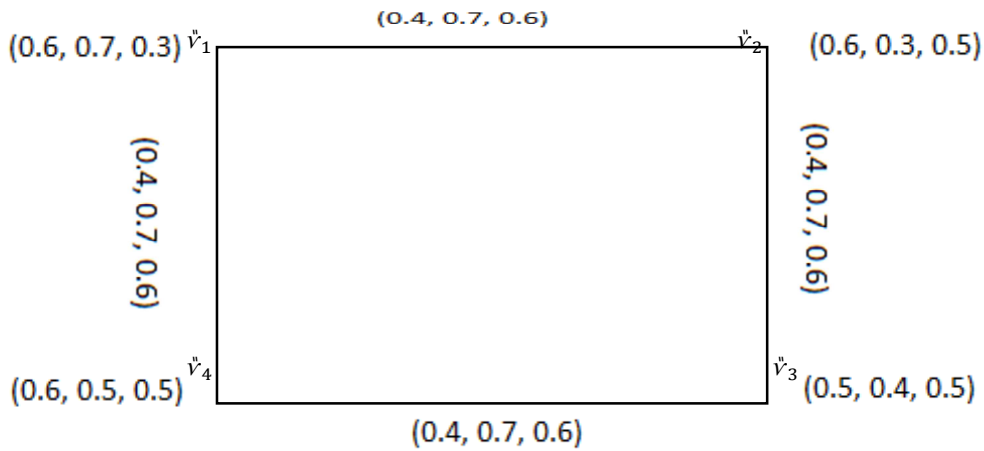


Figure 10.SVNG

$(\hat{T}_2, \hat{I}_2, F_2)$ is constant function, then G is constant SVNG. But not totally constant SVNG.

Theorem 5. If G is constant SVNG is an odd cycle does not have SVN bridge. Hence it does not have SVN cut-vertex.

Proof. Suppose G is constant SVNG is an odd cycle of its crisp graph. Then $(\hat{T}_2, \hat{I}_2, F_2)$ is constant function. Therefore removal any edge does not reduce the strength of connectedness between any pair of vertex. Therefore G has no SVN edge and Hence there is no SVN cut vertex.

Remark 5. For totally constant the above theorem (5) is not true.

Example 10. Consider a graph $G = (V, E)$ where $V = \{v_1, v_2, v_3\}$ be the set of vertices and $E = \{v_1v_2, v_2v_3, v_3v_1\}$ be the set of edges. Then

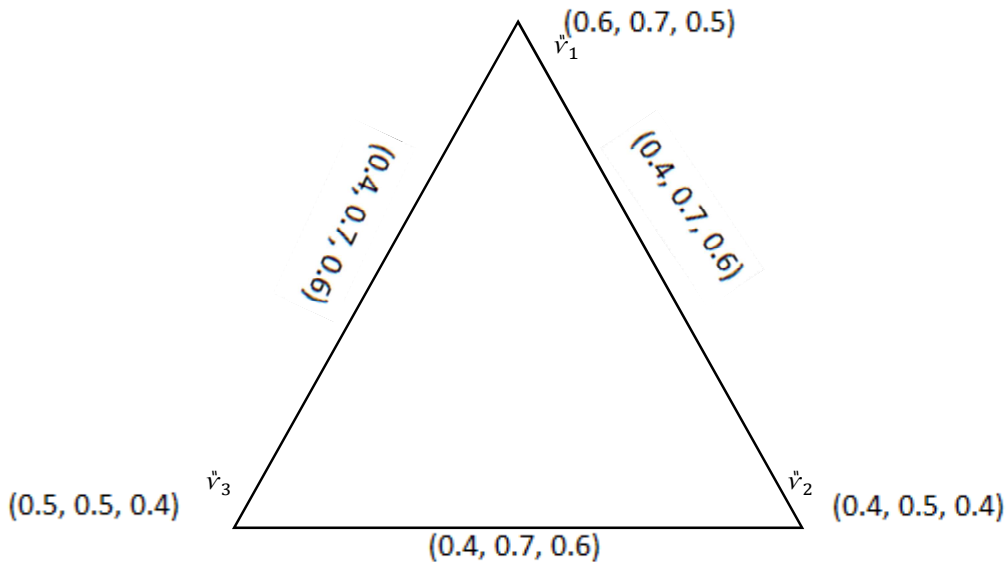


Figure 11 .SVNG

$(\hat{T}_2, \hat{I}_2, F_2)$ is constant function, but neither SVN bridge nor SVN cut vertex.

Theorem 6. If G is constant SVNG is an even cycle of its crisp graph. Then either G does not have SVN bridge also it does not have SVN cut vertex.

Proof. Straightforward.

Remark 6. For totally constant the above theorem (6) is not true.

Example 11. Consider a graph $G = (\check{V}, \check{E})$ where $\check{V} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ be the set of vertices and $\check{E} = \{\check{v}_1\check{v}_2, \check{v}_2\check{v}_3, \check{v}_3\check{v}_4, \check{v}_4\check{v}_1\}$ be the set of edges. Then

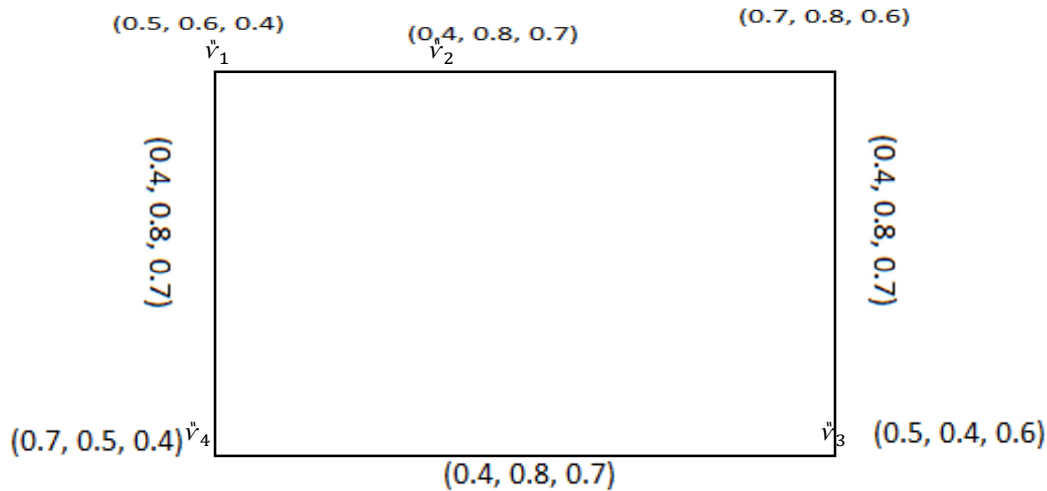


Figure 12.SVNG

$(\hat{T}_2, \hat{I}_2, F_2)$ is constant function, but neither SVN bridge nor SVN cut vertex.

5 Application

In this section, we applied the concept of CSVNG to model a Wi-Fi system. It is discussed how the concept of CSVNGs is useful in modelling such network.

The Wi-Fi technology that is connected to the internet can be employed to deliver access to devices which are within the range of a wireless networks. The coverage extension can be as small area as few rooms to large as many square kilometres among two or more interconnected access points. The dependency of Wi-Fi range is on frequency band, radio power production and modulation techniques. Paralleled to traditional wired network security which is wired networking, simplified access is basic problem with wireless network security, it is essential that one either gain access to building (connecting/ relating into interior web tangibly), or a break through an exterior firewall. To facilitate Wi-Fi, one essentially require to be within the range of Wi-Fi linkage. The solid Wi-Fi hotspot device is the internal coin Wi-Fi which is designed to aid all internal setting owners. Make available 100 meters Wi-Fi signal range to outdoor and 30 meters to indoor. With the help of CSVNG this type of Wi-Fi linkage is deliberated and demonstrated.

The CSVNG is useful to a Wi-Fi network. The purpose for doing this is that there are three values in aCSVNG. The first one signifies connectivity, the second one defined the technical error of the device such as device is in range but changes between the connected and disconnected state and the third value indicates the disconnectivity. The notion of IFG only permits us to model two states such as connected and disconnected, a Wi-Fi system cannot be demonstrated using this confined structure of IFG. Though the CSVNG deliberate more than these two similarities.

An outdoor Wi-Fi co-ordination, comprises four vertices which characterise the Wi-Fi devices in such a way that there is a block between each two routers and collectively both routers have been giving signals to the block, given away in figure (13). The devices can provide signal to each block with the help of CSVNG persistently.

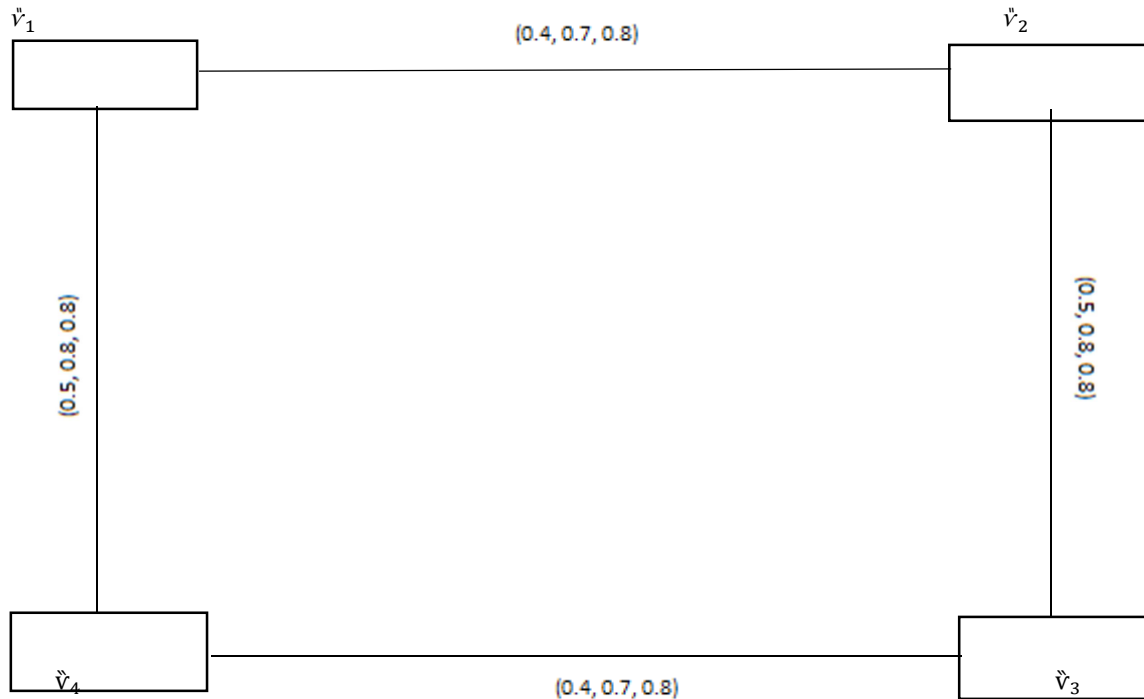


Figure 13.SVNG.

In figure 13, the four apexes denotes four different routers. The edge displays the signal strength of routers between each two routers. Each edge and apex take the single valued neutrosophic number form where the first value denotes the connectivity, the second one defined the technical error of the device, changes between the connected and disconnected state while the device is in range but, and the third value displays the disconnectivity. By using definition 4, the degree of every vertex is deliberated. In this situation which characterises that all router has been giving the same signal, so the degree of all routers is same. This also indicates that each router providing the same signal to the block. As a consequence, the concept of CSVNG displaying its importance, has been exercised to practical operations effectively.

Table 1 shows the degree of each vertex of figure 13.

vertex	Degree
\check{v}_1	(0.9, 1.5, 1.6)
\check{v}_2	(0.9, 1.5, 1.6)
\check{v}_3	(0.9, 1.5, 1.6)
\check{v}_4	(0.9, 1.5, 1.6)

Table 1 .vertex and its degree

Advantages:

The advantages of SVNGs over prevailing concepts of IFGs is due to the enhanced structure of SVNGs which allows us to deal with of more than two types ambiguous condition as it is done in the present situation of Wi-Fi

system. While the IFG allow only to deal with two states connected and disconnected which means that IFGs cannot be employed to model the Wi-Fi system.

Conclusion:

The conception of CSVNG has been developed in this paper. With the help of examples, basic graph theoretic ideas such as degree of CSVNG, constant functions, totally CSVNG and characterization of CSVNG on a cycle are proved. That notion of CSVNG have been applied to a real-world problem of Wi-Fi system and the consequences are deliberated. A comparison of CSVNG with CIFG have showed the worth of CSVNGs. Further, in the proposed frame work, implementations in the field of engineering and computer sciences can be considered in near future.

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