HSSM- MADM Strategy under SVPNS environment

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Abstract
In the present paper, we propose the Hyperbolic Sine Similarity Measure (HSSM) for pentapartitioned neutrosophic sets which is based on hyperbolic sine function. We also establish some properties of the similarity measures by providing some suitable examples. Further we develop an MADM (Multi-Attribute-Decision-Making) model for single valued pentapartitioned neutrosophic set (SVPNS) environment based on the similarity measure which we call HSSM-MADM strategy. We also validate our proposed model by solving a numerical example.
Keywords: *MADM*; *Neutrosophic Set*; *Pentapartitioned Neutrosophic Set*; *Similarity Measure*.

1. Introduction:

Smarandache grounded the idea of Neutrosophic Set (NS) [1] as an extension of Fuzzy Set (FS) [2], and Intuitionistic Fuzzy Set (IFS) [3] to deal with incomplete and indeterminate information. In NS theory, truth-membership, indeterminacy-membership, and falsity-membership values are independent of each other. The concept of Single Valued NS (SVNS) was presented by Wang et al. [4], which is the subclass of an NS. By using SVNS, we can represent incomplete, imprecise, and indeterminate information that helps in decision making in the real- world problems. NS and the various extensions of NSs were studied and used for model/algorithm in different areas of research such as medical diagnosis ([5-7], social problems [8], conflict resolution [9], decision making [10-27], etc. Detail theoretical development and applications of NS and its extensions can be found in the studies [28-37].

Chatterjee et al. [38] defined the Quadripartitioned SVNS (QSVNS) by introducing contradiction and ignorance membership functions in place of indeterminacy membership function. Mallick and Pramanik [39] defined Pentapartitioned Neutrosophic Set (PNS) by introducing unknown membership function in QSVNS to handle uncertainty and indeterminacy comprehensively.

Similarity measures [40-68] were defined in various NS environments and were utilized for decision, medical diagnosis, etc. Mondal and Pramanik [69] proposed Hyperbolic Sine Similarity Measure (HSSM) and proved their basic properties in SVNS environment. Receiving motivation from the work of Mondal and Pramanik [70], we extend the HSSM for Single Valued PNSs (SVPNSs) and prove their basic properties. Based on HSSM, we propose an HSSM based MADM strategy which we call the HSSM-MADM model under SVPNS environment. Also, we validate our model by solving an illustrative example of an MADM problem.

The remaining part of this paper is divided into several sections:

In section 2, we recall PNS, and some relevant properties of PNSs. In section 3, we introduce the notion of SVPNS and HSSM between them. In section 4, we develop the SVPNS- MADM strategy. In section 5, we validate the proposed strategy by solving an illustrative MADM problem. In section 6, we conclude the paper by stating the future scope of research.

2. Some Relevant Definitions:

Definition 2.1. [4] An SVNS K over a non-empty set L is defined as follows:

$K = \{(u, T_K(u), I_K(u), F_K(u)): u \in L\}$, where T_K, I_K, F_K are truth, indeterminacy, and falsity membership mappings from L to$]^-0,1^+[$, and $^-0 \leq T_K(u) + I_K(u) + F_K(u) \leq 3^+$.

Example 2.1. Let $L= \{q, w, e\}$ be a universe of discourse. Then $\{(q, 0.9, 0.6, 0.4), (w, 0.4, 0.6, 0.7), (e, 0.2, 0.7, 0.7)\}$ is an SVNS over L.

Definition 2.2. [4] Suppose that L be a universe of discourse. Then P, a pentapartitioned neutrosophic set (P-NS) over L is denoted as follows:

$P= \{(u, T_P(q), C_P(u), G_P(u), U_P(u), F_P(u)): u \in L\}$, where T_P, C_P, G_P, U_P, $F_P : L \rightarrow\ ]0,1[$ are the truth, contradiction, ignorance, unknown, falsity membership functions and so $0 \leq T_P(q)+C_P(q)+G_P(q)+U_P(q)+F_P(q) \leq 5$.

Example 2.2. Let $L = \{q, w\}$ be a universe of discourse. Then $\{(q, 0.9, 0.6, 0.4, 0.3, 0.5), (w, 0.4, 0.6, 0.7, 0.8, 0.2)\}$ is a PNS over L.

Definition 2.3.[4] Assume that $X= \{(q, T_X(q), C_X(q), G_X(q), U_X(q), F_X(q)): q \in W\}$ and $Y = \{(q, T_Y(q), C_Y(q), G_Y(q), U_Y(q), F_Y(q)): q \in W\}$ be two PNSs over W. Then $X \subseteq Y \Leftrightarrow T_X(q) \leq T_Y(q), C_X(q) \leq C_Y(q), G_X(q) \geq G_Y(q), U_X(q) \geq U_Y(q), F_X(q) \geq F_Y(q)$, for all $q \in W$.

Example 2.3. Let $L= \{q, w\}$ be a universe of discourse. Consider two PNSs $X=\{(q, 0.5, 0.6, 0.5, 0.7, 0.3), (w, 0.8, 0.8, 0.3, 0.3, 0.3)\}$ and $Y=\{(q, 0.9, 0.9, 0.3, 0.3, 0.3), (w, 1.0, 0.8, 0.2, 0.1, 0.3)\}$ over L. Then $X \subseteq Y$.

Definition 2.4.[4] Suppose that $X= \{(u, T_X(u), C_X(u), G_X(u), U_X(u), F_X(u)): u \in L\}$ and $Y= \{(u, T_Y(u), C_Y(u), G_Y(u), U_Y(u), F_Y(u)): u \in L\}$ be two PNSs over L. Then $X \cup Y= \{(u, \max\{T_X(u), T_Y(u)\}, \max\{C_X(u), C_Y(u)\}, \min\{G_X(u), G_X(u)\}, \min\{U_X(u), U_X(u)\}, \min\{F_X(u), F_X(u)\}): u \in L\}$.

Example 2.4. Suppose that $L= \{q, w\}$. Consider two PNSs $X= \{(q, 0.7, 0.5, 0.5, 0.7 ,0.7), (w, 0.5, 0.6, 0.7, 0.7, 0.6)\}$ and $Y= \{(q,1.0, 0.6, 0.8, 0.7, 0.7), (w, 0.6, 0.7, 0.8, 0.4, 0.6)\}$ over L. Then $X \cup Y=$ $\{(q,1.0, 0.6, 0.5, 0.7, 0.7), (w, 0.6, 0.7, 0.7, 0.4, 0.6)\}$.

Definition 2.5.[4] Suppose that $X= \{(u, T_X(u), C_X(u), G_X(u), U_X(u), F_X(u)): u \in W\}$ and $Y= \{(u, T_Y(u), C_Y(u), G_Y(u), U_Y(u), F_Y(u)): u \in L\}$ are two PNSs over L. Then $X^c= \{(u, F_X(u), U_X(u), 1\text{-}G_X(u), C_X(u), T_X(u)): u \in L\}$.

Example 2.5. Suppose that L= {q, w} be a universe of discourse and X= {(q, 0.5, 0.7, 0.7, 0.6, 1.0), (w, 1.0,0.5, 0.5, 0.5, 1.0)} be a PNS over L. Then X^c= {(q, 1.0, 0.6, 0.3, 0.7, 0.5), (w,1.0, 0.5, 0.5, 0.5, 1.0)}.

Definition 2.6.[4] Suppose that X= {(u, $T_X(u)$, $C_X(u)$, $G_X(u)$, $U_X(u)$, $F_X(u)$): $u\in L$} and Y= {(u, $T_Y(u)$, $C_Y(u)$, $G_Y(u)$, $U_Y(u)$, $F_Y(u)$): $u\in L$} be two PNSs over L. Then $X \cap Y$ = {(u, min {$T_X(u)$, $T_Y(u)$}, min {$C_X(u)$, $C_Y(u)$}, max {$G_X(u)$, $G_X(u)$}, max {$U_X(u)$, $U_X(u)$}, max {$F_X(u)$, $F_X(u)$}): $u\in L$}.

Example 2.6. Suppose that X and Y be two PNSs over a non-empty set L, as shown in Example 2.4. Then $X \cap Y$= {(q, 0.7, 0.5, 0.8, 0.7, 0.7), (w, 0.5, 0.6, 0.8, 0.7, 0.6)}.

Definition 2.7. [4] The null PNS (0_{PN}) and the absolute PNS (1_{PN}) over L are defined by

(i) 0_{PN} = {(u, 0, 0, 1, 1, 1): $u\in L$};

(ii) 1_{PN} = {(u,1, 1, 0, 0, 0): $u\in L$}.

3. Single Valued Pentapartitioned Neutrosophic Set (SVPNS):

Definition 3.1. [39] Assume that L be a universe of discourse. An *SVPNS Y* over L is characterized by a truth-membership function T_Y, a contradiction-membership function C_Y, an ignorance-membership function G_Y, an unknown-membership function U_Y, a falsity-membership function F_Y. For each element $u\in L$, $T_Y(u)$, $C_Y(u)$, $G_Y(u)$, $U_Y(u)$, $F_Y(u)\in[0,1]$.

The *SVPNS Y* is denoted as follows:

Y= {(u, $T_Y(u)$, $C_Y(u)$, $G_Y(u)$, $U_Y(u)$, $F_Y(u)$): $u\in L$}.

Definition 3.2. [39] Suppose that B= {(u, $T_B(u)$, $C_B(u)$, $G_B(u)$, $U_B(u)$, $F_B(u)$): $u\in L$} and A= {(u, $T_A(u)$, $C_A(u)$, $G_A(u)$, $U_A(u)$, $F_A(u)$): $u\in L$} be any two *SVPNSs* over L. Then

(i) $B=A \Leftrightarrow T_B(u) = T_A(u)$, $C_B(u) = C_A(u)$, $G_B(u) = G_A(u)$, $U_B(u) = U_A(u)$, $F_B(u) = F_A(u)$, for each $u\in L$;

(ii) $B\subseteq Y \Leftrightarrow T_B(u) \le T_A(u)$, $C_B(u) \le C_A(u)$, $G_B(u) \ge G_A(u)$, $U_B(u) \ge U_A(u)$, $F_B(u) \ge F_A(u)$, for each $u\in L$.

Definition 3.3. Suppose that M= {(u, $T_M(u)$, $C_M(u)$, $G_M(u)$, $U_M(u)$, $F_M(u)$): $u\in L$} and W = {(u, $T_W(u)$, $C_W(u)$, $G_W(u)$, $U_W(u)$, $F_W(u)$): $u\in L$} are any two *SVPNSs* over L. Then the hyperbolic sine similarity measure between M and W is defined by:

HSSM(M, W)=

$$1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) \quad (1)$$

Definition 3.4. Suppose that M = {(u, $T_M(u)$, $C_M(u)$, $G_M(u)$, $U_M(u)$, $F_M(u)$): $u\in L$} and W = {(u, $T_W(u)$, $C_W(u)$, $G_W(u)$, $U_W(u)$, $F_W(u)$): $u\in L$} be any two SVPNSs over L. Then the weighted hyperbolic sine similarity measure between M and W is defined by:

WHSSM(M, W)=

$$1- \frac{1}{n}\sum_{i=1}^{n} w_i\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) \quad (2)$$

where $0 \le w_i \le 1$ and $\sum_{i=}^{n} w_i$=1.

Theorem 3.1. Assume that HSSM(M, W) is the hyperbolic sine similarity measure between two SVPNSs M and W. Then $0 \le$ HSSM(M, W) ≤ 1.

Proof. Suppose that M={(u, $T_M(u)$, $C_M(u)$, $G_M(u)$, $U_M(u)$, $F_M(u)$): $u\in L$} and W={(u, $T_W(u)$, $C_W(u)$, $G_W(u)$, $U_W(u)$, $F_W(u)$): $u\in L$} aew any two SVPNSs over L.

Now $0\le T_M(u_i)$, $C_M(u_i)$, $G_M(u_i)$, $U_M(u_i)$, $F_M(u_i)$, $T_W(u_i)$, $C_W(u_i)$, $G_W(u_i)$, $U_W(u_i)$, $F_W(u_i)\le 1$.

$\Rightarrow 0 \leq |T_M(u_i)\text{-} T_W(u_i)| + |C_M(u_i)\text{-} C_W(u_i)| + |G_M(u_i)\text{-} G_W(u_i)| + |U_M(u_i)\text{-} U_W(u_i)| + |F_M(u_i)\text{-} F_W(u_i)| \leq 5.$

$\Rightarrow 0 \leq \left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) \leq 1.$

$\Rightarrow 0 \leq 1\text{-} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) \leq 1.$

$\Rightarrow 0 \leq$ HSSM$(M, W) \leq 1$.

Theorem 3.2. Assume that HSSM(M, W) is the hyperbolic sine similarity measure between two SVPNSs M and W. Then HSSM$(M, W) = 1$ if $M = W$.

Proof. Suppose that $M= \{(u, T_M(u), C_M(u), G_M(u), U_M(u), F_M(u)): u\in L\}$ and $W= \{(u, T_W(u), C_W(u), G_W(u), U_W(u), F_W(u)): u\in L\}$ are any two SVPNSs over L such that $M=W$.

So $T_M(u_i) = T_W(u_i)$, $C_M(u_i)= C_W(u_i)$, $G_M(u_i)= G_W(u_i)$, $U_M(u_i)= U_W(u_i)$, $F_M(u_i)= F_W(u_i)$ for each $u_i\in L$.

$\Rightarrow |T_M(u_i)\text{-} T_W(u_i)|=0, |C_M(u_i)\text{-} C_W(u_i)|=0, |G_M(u_i)\text{-} G_W(u_i)|=0, |U_M(u_i)\text{-} U_W(u_i)|=0, |F_M(u_i)\text{-} F_W(u_i)|=0$ for each $u_i\in L$.

$\Rightarrow sinh\begin{pmatrix}|T_M(u_i) - T_W(u_i)| + |C_M(u_i) - C_W(u_i)| + |G_M(u_i) - G_W(u_i)| \\ +|U_M(u_i) - U_W(u_i)| + |F_M(u_i) - F_W(u_i)|\end{pmatrix} = 0.$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) = 0$

$\Rightarrow 1\text{-} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right) = 1$

$\Rightarrow$ HSSM$(M, W) = 1$.

Theorem 3.3. Assume that HSSM(M, W) is the hyperbolic sine similarity measure between two SVPNSs M and W. Then HSSM(M, W) = HSSM(W, M).

Proof. Suppose that $M = \{(u, T_M(u), C_M(u), G_M(u), U_M(u), F_M(u)): u\in L\}$ and $W = \{(u, T_W(u), C_W(u), G_W(u), U_W(u), F_W(u)): u\in L\}$ any two SVPNSs over L.

Now HSSM(M, W)=

$1\text{-} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right)$

$=1\text{-} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_W(u_i)-T_M(u_i)| + |C_W(u_i)-C_M(u_i)| + |G_W(u_i)-G_M(u_i)| + |U_W(u_i)-U_M(u_i)| + |F_W(u_i)-F_M(u_i)|)}{75}\right)$

= HSSM(W, M).

Therefore HSSM(M, W)= HSSM(M, W).

Theorem 3.4. Assume that SSM(M, W) is the hyperbolic sine similarity measure between the SVPNSs M and W. If Q is an SVPNS over L such that $M\subseteq W\subseteq Q$, then HSSM$(M, W)\geq$ HSSM(M, Q), HSSM$(W, Q) \geq$ HSSM(M, Q).

Proof. Suppose that $M= \{(u, T_M(u), C_M(u), G_M(u), U_M(u), F_M(u)): u\in L\}$ and $W= \{(u, T_W(u), C_W(u), G_W(u), U_W(u), F_W(u)): u\in L\}$ are any two SVPNSs over L. Let Q be an SVPNS over L such that $M\subseteq W\subseteq Q$. Since $M\subseteq W\subseteq Q$, so $|T_M(u_i)\text{-}T_W(u_i)|\leq|T_M(u_i)\text{-}T_Q(u_i)|$, $|C_M(u_i)\text{-}C_W(u_i)|\leq|C_M(u_i)\text{-}C_Q(u_i)|$, $|G_M(u_i)\text{-}G_W(u_i)|\leq |G_M(u_i)\text{-}G_Q(u_i)|$, $|U_M(u_i)\text{-}U_W(u_i)|\leq|U_M(u_i)\text{-}U_Q(u_i)|$, $|F_M(u_i)\text{-}F_W(u_i)|\leq|F_M(u_i)\text{-}F_Q(u_i)|$.

Now HSSM(M, W)=

$1\text{-} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_W(u_i)| + |C_M(u_i)-C_W(u_i)| + |G_M(u_i)-G_W(u_i)| + |U_M(u_i)-U_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{75}\right)$

$$\geq 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_Q(u_i)| + |C_M(u_i)-C_Q(u_i)| + |G_M(u_i)-G_Q(u_i)| + |U_M(u_i)-U_Q(u_i)| + |F_M(u_i)-F_Q(u_i)|)}{75}\right)$$

= HSSM(M, Q).

Therefore, HSSM$(M, W) \geq$ HSSM(M, Q).

Again, from $M \subseteq W \subseteq Q$, we can say that $|T_W(u_i)-T_Q(u_i)| \leq |T_M(u_i)-T_Q(u_i)|$, $|C_W(u_i)-C_Q(u_i)| \leq |C_M(u_i)-C_Q(u_i)|$, $|G_W(u_i)-G_Q(u_i)| \leq |G_M(u_i)-G_Q(u_i)|$, $|U_W(u_i)-U_Q(u_i)| \leq |U_M(u_i)-U_Q(u_i)|$, $|F_M(u_i)-F_W(u_i)| \leq |F_M(u_i)-F_Q(u_i)|$.

Now, HSSM(W, Q)=

$$1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_W(u_i)-T_Q(u_i)| + |C_W(u_i)-C_Q(u_i)| + |G_W(u_i)-G_Q(u_i)| + |U_W(u_i)-U_Q(u_i)| + |F_W(u_i)-F_Q(u_i)|)}{75}\right)$$

$$\geq 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{sinh(|T_M(u_i)-T_Q(u_i)| + |C_M(u_i)-C_Q(u_i)| + |G_M(u_i)-G_Q(u_i)| + |U_M(u_i)-U_Q(u_i)| + |F_M(u_i)-F_Q(u_i)|)}{75}\right)$$

= HSSM(M, Q).

Therefore, HSSM$(M, W) \geq$ HSSM(M, Q).

4. SVPNS- MADM Strategy

Suppose that $Q = \{Q_1, Q_2, ..., Q_n\}$ is a finite set of possible alternatives from which a decision maker needs to choose the best alternative. Let $P = \{P_1, P_2, ..., P_m\}$ be the finite collection of attributes for every alternative. A decision maker provides their evaluation information of each alternative Q_i (i = 1, 2,..., n) against the attribute P_j (j = 1, 2, ..., m) in terms of single valued pentapartitioned numbers. The whole evaluation information of all alternatives can be expressed by a decision matrix. The steps of proposed HSSM-MADM strategy (see figure 1) are described as follows:

Step-1: Construct the decision matrix

The whole evaluation information of each alternative Q_i (i = 1, 2,..., n) based on the attributes P_j (j = 1, 2, ..., m) is expressed in terms of SVPNS E_{Q_i}= $\{(P_j, T_{ij}(Q_i, P_j), C_{ij}(Q_i, P_j), G_{ij}(Q_i, P_j), U_{ij}(Q_i, P_j), F_{ij}(Q_i, P_j))$: $P_j \in P\}$, where $(T_{ij}(Q_i, P_j), C_{ij}(Q_i, P_j), G_{ij}(Q_i, P_j), U_{ij}(Q_i, P_j), F_{ij}(Q_i, P_j))$ denotes the evaluation assessment of Q_i (i = 1, 2,..., n) against P_j (j = 1, 2, ..., m).

Then the Decision Matrix (DM$[Q|P]$) can be expressed as:

DM$[Q|P]$ =

	P_1	P_2	……. ….	……..	P_m
Q_1	<$T_{11}(Q_1, P_1)$,$C_{11}(Q_1, P_1)$, $G_{11}(Q_1, P_1)$,$U_{11}(Q_1, P_1)$, $F_{11}(Q_1, P_1)$>	<$T_{12}(Q_1, P_2)$,$C_{12}(Q_1, P_2)$, $G_{12}(Q_1, P_2)$,$U_{12}(Q_1, P_2)$, $F_{12}(Q_1, P_2)$>	……. ..	…….	<$T_{1m}(Q_1, P_m)$,$C_{1m}(Q_1, P_m)$, $G_{1m}(Q_1, P_m)$,$U_{1m}(Q_1, P_m)$, $F_{1m}(Q_1, P_m)$>
Q_2	<$T_{21}(Q_2, P_1)$,$C_{21}(Q_2, P_1)$, $G_{21}(Q_2, P_1)$,$U_{21}(Q_2, P_1)$, $F_{21}(Q_2, P_1)$>	<$T_{22}(Q_2, P_2)$,$C_{22}(Q_2, P_2)$, $G_{22}(Q_2, P_2)$,$U_{22}(Q_2, P_2)$, $F_{22}(Q_2, P_2)$>	… … …..	……..	<$T_{2m}(Q_2, P_m)$,$C_{2m}(Q_2, P_m)$, $G_{2m}(Q_2, P_m)$,$U_{2m}(Q_2, P_m)$, $F_{2m}(Q_2, P_m)$>
. .	. .	. .	. .	. .	. .

.	.	.	.	.	
Q_n	< $T_{n1}(Q_n, P_1)$, $C_{n1}(Q_n, P_1)$, $G_{n1}(Q_n, P_1)$, $U_{n1}(Q_n, P_1)$, $F_{n1}(Q_n, P_1)$ >	< $T_{n2}(Q_n, P_2)$, $C_{n2}(Q_n, P_2)$, $G_{n2}(Q_n, P_2)$, $U_{n2}(Q_n, P_2)$, $F_{n2}(Q_n, P_2)$ >	… … …..	…… ….	< $T_{nm}(Q_n, P_m)$, $C_{nm}(Q_n, P_m)$, $G_{nm}(Q_n, P_m)$, $U_{nm}(Q_n, P_m)$, $F_{nm}(Q_n, P_m)$ >

Step-2: Determine the weights of the attributes

In an MADM strategy, the weights of the attributes play an important role in taking decision. When the weights of the attributes are totally unknown to the decision makers, then the attribute weights can be determined by using the compromise function defined in equation (3).

Compromise Function: The compromise function of Q is defined by:

$$\Omega_j=\sum_{i=1}^{n} (3+T_{ij}(Q_i, P_j)+C_{ij}(Q_i, P_j)-G_{ij}(Q_i, P_j)-U_{ij}(Q_i, P_j)-F_{ij}(Q_i, P_j))/5 \quad (3)$$

Then the desired weight of the jth attribute is defined by $w_j=\frac{\Omega_j}{\sum_{j=1}^{m}\Omega_j}$ (4)

Here $\sum_{j=1}^{m} w_j=1$.

Step-3: Determination of ideal solution

In every MADM process, the attributes chosen by the decision maker can be split into two different types. One is "benefit type" attribute and the other is "cost type" attribute. In our proposed SVPNS-MADM model, an ideal alternative can be identified by the decision maker using the following operators:

(i) For the cost type attributes (P_j), we use the maximum operator to determine the best value (P_j*) of each attribute among all the alternatives. The best value (P_j*) is defined by:

$$P_j^* = (\max\ T_{11}(Q_1, P_1), \max\ C_{11}(Q_1, P_1), \min\ G_{11}(Q_1, P_1), \min\ U_{11}(Q_1, P_1), \min\ F_{11}(Q_1, P_1)) \quad (5)$$

where j=1, 2, ….., m.

(ii) For the benefit type attributes (P_j), we use the minimum operator to determine the best value (P_j*) of each attribute among all the alternatives. The best value (P_j*) is defined by:

$$P_j^* = (\min\ T_{11}(Q_1, P_1), \min\ C_{11}(Q_1, P_1), \max\ G_{11}(Q_1, P_1), \max\ U_{11}(Q_1, P_1), \max\ F_{11}(Q_1, P_1)) \quad (6)$$

where j=1, 2, ….., m.

Then we define an ideal solution as follows:

$Q^*= \{P_1^*, P_2^*, ……., P_m^*\}$, which is also an SVPNS.

Step-4: Determination of hyperbolic sine similarity value.

After the formation of ideal solution in step-3, by using eq (1), we calculate the HSSM values for every alternative between the ideal solutions and the corresponding SVPNS from decision matrix DM[$Q|P$].

Step-5: Ranking order of the alternatives.

The rank of the alternatives Q_1, Q_2, ……., Q_n is determined based on the ascending order of hyper sine similarity values. The alternative with lowest hyper sine similarity value is the best alternative among the set of possible alternatives.

Step-6: End.

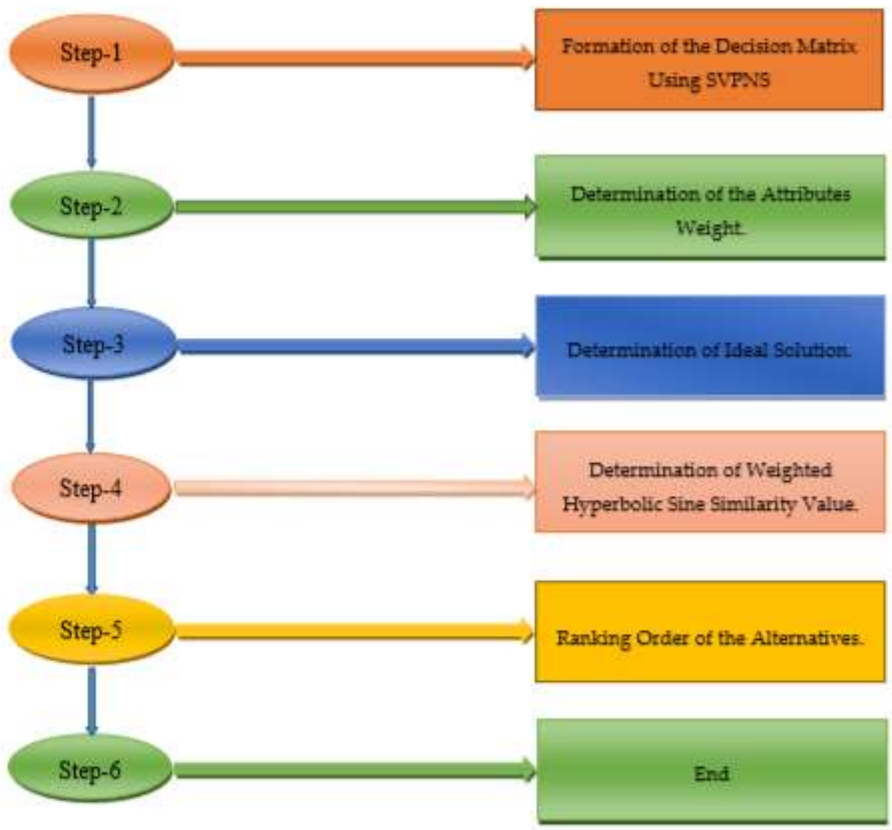

Figure 1: Flow chart of the SVPNS- MADM strategy

4. Validation of the Proposed Model:

In this section, we validate our proposed model / strategy by giving a numerical example.

4.1. Numerical example:

In this section, we demonstrate a numerical example as a real- life application of our proposed strategy. In our daily life time management is very important for everyone. Suppose a passenger needs to travel from the city-X to the city-Y by road. The passenger wants to book a car (alternative) by an online App to reach his/her destination. The selection of car by the passenger can be done based on some attributes, namely, Charges(P_1), Payment mode (P_2), AC / Non-AC(P_3), Rating(P_4). So, the selection of affordable car (for travelling) by an online App can be considered as a MADM approach.

Then the MADM strategy is presented by using the following steps.

Step-1: Construct the decision matrix under single valued pentapartitioned neutrosophic environment.

The decision matrix is shown in table 1.

Table-1: Decision matrix

	P_1	P_2	P_3	P_4
Q_1	(0.7,0.3,0.1,0.3,0.4)	(0.8,0.4,0.2,0.3,0.8)	(0.8,0.2,0.5,0.7,0.3)	(0.8,0.4,0.2,0.3,0.6)
Q_2	(0.7,0.4,0.3,0.6,0.2)	(0.7,0.4,0.4,0.7,0.5)	(0.6,0.2,0.4,0.5,0.7)	(0.9,0.3,0.9,0.2,0.3)
Q_3	(0.5,0.4,0.6,0.3,0.4)	(0.6,0.4,0.4,0.7,0.9)	(0.5,0.3,0.4,0.5,0.6)	(0.7,0.5,0.7,0.3,0.8)

Step-2: Determine the weights of attributes.
By using the eq. (3) and (4), we have the weight vector as follows:
(w_1, w_2, w_3, w_4) = (0.279, 0.234, 0.222, 0.263).

Step-3: Determine the ideal solution.
In this problem, the attribute P_1 is cost type attribute and P_2, P_3, P_4 are the benefit type attributes. The ideal solution is given in the table 2:

Table-2: The ideal solution

	${P_1}^*$	${P_2}^*$	${P_3}^*$	${P_4}^*$
Q^*	(0.7,0.4,0.1,0.3,0.2)	(0.6,0.4,0.4,0.4,0.7,0.9)	(0.5,0.2,0.5,0.7,0.7)	(0.7,0.3,0.9,0.7,0.8)

Step-4: Determine the weighted hyperbolic sine similarity values.
By using eq. (2), we calculate the similarity measure values for each alternative. The weighted hyperbolic sine similarity values are:
WHSSM(Q_1, Q^*) = 0.996488;
WHSSM(Q_2, Q^*) = 0.997482;
WHSSM(Q_3, Q^*) = 0.997881.

Step-5: Ranking the alternatives.
From the above step, we see that WHSSM(Q_1, Q^*) < WHSSM(Q_2, Q^*) < WHSSM(Q_3, Q^*). Therefore, Q_1 is the best suitable alternative (car) for the passenger to book for travelling.

5. Conclusions:
In the study, we propose a hyperbolic sine similarity measure and weighted hyperbolic sine similarity measures for single valued pentapartitioned neutrosophic set and prove some of their basic properties. We develop a novel HSSM-MADM strategy based on the proposed weighted hyperbolic sine similarity measure to solve MADM problems. We also validate the proposed strategy by solving an illustrative MADM problem to demonstrate the effectiveness of the proposed SVPNS-MADM strategy.
The proposed SVPNS-MADM strategy can also be used to deal with other decision-making problems such as teacher selection [71], weaver selection [72], brick selection [73], logistic center location selection [74], personnel selection [75], etc.

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