University of New Mexico

Irregular Neutrosophic Graphs

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Abstract. The concepts of neighbourly irregular neutrosophic graphs, neighbourly totally irregular neutrosophic graphs, highly irregular neutrosophic graphs and highly totally irregular neutrosophic graphs are introduced. A criteria for neighbourly irregular and highly irregular neutrosophic graphs to be equivalent is discussed.

Keywords: Neutrosophic graphs, Irregular neutrosophic graphs, Neighbourly irregular neutrosophic graphs, Highly irregular neutrosophic graphs.

1 Introduction

Azriel Rosenfeld [16] introduced the notion of fuzzy graphs in 1975 which have many applications in modeling, Environmental sciences, social sciences, Geography and Linguistics. Some remarks on fuzzy graphs are given by P. Bhattacharya [4]. J. N. Mordeson and C. S. Peng defined different operations on fuzzy graphs in his paper [10]. The concept of bipolar fuzzy sets was initiated by Zhang [24]. A bipolar fuzzy set is an extension of the fuzzy set which has a pair of positive and negative membership values ranging in $[-1,1]$. In usual fuzzy sets, the membership degrees of elements range over the interval $[0,1]$. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set, and the membership degree 0 indicates that an element does not belong to the fuzzy set. The membership degrees on the interval $(0,1)$ indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. In Bipolar fuzzy sets membership degree range is enlarged from the interval $[0,1]$ to $[-1,1]$. In a bipolar valued fuzzy set, the membership degree 0 indicate that elements are irrelevant to the corresponding property, the membership degrees on $(0,1]$ shows that elements some what satisfy the property, and the membership degrees on $[-1,0)$ shows that elements somewhat satisfy the implicit counter-property. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. The first definition of bipolar fuzzy graphs was introduced by Akram [1]which are the extensions of fuzzy graphs. He defined different operations of union, intersection, complement, isomorphisms in his paper. Smarandache [21] introduced notion of neutrosophic set which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems. N. Shah and A. Hussain introduced the notion of soft neutrosophic graphs [17]. N. Shah introduced the notion of neutrosophic graphs and different operations like union, intersection, complement in his work [18]. Furthermore he defined different morphisms on neutrosophic graphs and proved related theorems. In the present paper the concepts of neighbourly irregular neutrosophic graphs, neighbourly totally irregular neutrosophic graphs, highly irregular neutrosophic graphs, highly totally irregular neutrosophic graphs and neutrosophic digraphs are introduced. Some results on irregularity of neutrosophic graphs are also proven.

In section 2, some basic concepts about graphs and neutrosophic sets are given. Section 3 is about neutrosophic graphs, their different operations and irregularity of neutrosophic graphs. Examples along with figures are also given to make the ideas clear.

2 PRILIMINARIES

In this section, we have given some definitions about graphs and neutrosophic sets. These will be helpful in later sections.

2.1 Definition *[22] A graph* G^* *consists of set of finite objects* $V = \{v_1, v_2, v_3 \ldots\ldots, v_n\}$ *called vertices (also called points or nodes) and other set* $E = \{e_1, e_2, e_3 \ldots\ldots e_n\}$ *whose element are called edges (also called lines or arcs).* Usually a graph is denoted as $G^* = (V,E)$. Let G^* be a graph and $\{u,v\}$ an edge of G^* . Since $\{u,v\}$ is 2-element set, we may write $\{v,u\}$ instead of $\{u,v\}$. It is often more convenient to represent this edge by uv or vu .

2.2 Definition *[15] The cardinality of* V, *i.e., the no.*

of vertices, is called the order of graph G^* *and denoted by* $|V|$. *The cardinality of* E *, i.e., the number of edges, is called the size of the graph and denoted by* $|E|$ *. Let* $V(G^*)=\{v_1,v_2,....v_n\}$ *and* $E(G^*)=\{e_1,e_2,...e_n\}$ *be the set of vertices and edges of a graph* G^*. *Each edge* $e_k \in E(G^*)$ *is identified with an unordered pair* (v_i,v_j) *of vertices. The vertices* v_i *and* v_j *are called the end vertices of* e_k .

2.3 Definition *[15] Two vertices joined with an edge are called adjacent vertices.*

2.4 Definition *[15] [20 An edge* e *of a graph* G^* *is said to be incident with a vertex* v *and vice versa if* v *is the end vertex of* e *. Any two non-parallel edges say* e_i *and* e_j *are said to be adjacent if* e_i *and* e_j *are incident with a vertex* v.

2.5 Definition *[15] The degree of any vertex* v *of* G^* *is the number of edges incident with vertex* v *. Each self-loop is counted twice. Degree of a vertex is always a positive number and is denoted as* $deg(v)$ *. The minimum degree and maximum degree of vertices in* $V(G^*)$ *are denoted by* $\delta(G^*)$ *and* $\Delta(G^*)$, *respectively* H^* .

2.6 Definition *[15] A vertex which is not incident with any edge is called an isolated vertex. In other words a vertex with degree zero is called an isolated vertex.*

2.7 Definition *[15] A graph without self-loops and parallel edges is called a simple graph.*

2.8 Definition *[15]A simple graph is said to be regular if all vertices of graph G are of equal degree. In other words if in a graph* G, $\delta(G^*)=\Delta(G^*)=r$ *i.e., each vertex having degree* r *then* G^* *is said to be regular of degree* r *, or simply* $r-regular$.

2.9 Definition *[15] A graph* $G_1^*=(V_1,E_1)$ *is called a subgraph of* $G^*=(V,E)$ *if* $V_1(G_1^*)\subseteq V(G^*)$ *and* $E_1(G_1^*)\subseteq E(G^*)$ *and each edge of* G_1^* *has the same end vertices in* G_1^* *as in* G^*.

2.10 Definition *[15] In a graph* G^*, *a finite alternating sequence of vertices and edges,* $v_1,e_1,v_2,e_2,......e_m,v_k$ *starting and ending with vertices such that each edge in the sequence is incident with the vertices following and preceding it, is called a walk. In a walk no edge appears more than once however a vertex may appear more than once.*

2.11 Definition *[22] In a multigraph no loop are allowed but more than one edge can join two vertices, these edges are called multiple edges or parallel edges and a graph is called multigraph*

2.12 Definition *[22] Let* $G_1^*=(V_1,E_1)$ *and* $G_2^*=(V_2,E_2)$ *be two graphs. A function* $f : V_1 \to V_2$ *is called Isomorphism if i)* f *is one to one and onto.*
ii) for all $a,b\in V_1, \{a,b\}\in E_1$ *if and only if* $\{f(a),f(b)\}\in E_2$ *when such a function exists,* G_1^* *and* G_2^* *are called isomorphic graphs and is written as* $G_1^*\cong G_2^*$. *In other words, two graph* G_1^* *and* G_2^* *are said to be isomorphic to each other if there is a one to one correspondence between their vertices and between edges such that incidence relationship is preserved.*

2.13 Definition *[21] A neutrosophic set* Λ *on the universe of discourse* X *is defined as* $\Lambda=\{<x,T_\Lambda(x),I_\Lambda(x),F_\Lambda(x)>,\ x\in X\}$, *where* $T,I,F : X \to]\bar{0},1^+[$ $\bar{0}\le$

$T_\Lambda(x)+I_\Lambda(x)+F_\Lambda(x)\le 3^+$. *Hence we consider the neutrosophic set which takes the values from the subset of* $]\bar{0},1^+[$.

2.14 Definition *Let* $\Lambda=\{<x,T_\Lambda(x),I_\Lambda(x),F_\Lambda(x)>,\ x\in X\}$ *and* $\Theta=\{<x,T_\Theta(x),I_\Theta(x),F_\Theta(x)>,\ x\in X\}$ *be two neutrosophic sets on universe of discourse* X *. Then* Θ *is called neutrosophic relation on* Λ *if*

$$T_\Theta(x,y)\le \min\{T_\Lambda(x),T_\Lambda(y)\}$$
$$I_\Theta(x,y)\le \min\{I_\Lambda(x),I_\Lambda(y)\}$$
$$F_\Theta(x,y)\ge \max\{F_\Lambda(x),F_\Lambda(y)\}$$

for all $x,y\in X$. *A neutrosophic relation* Θ *on* X *is called symmetric if* $T_\Theta(x,y)=T_\Theta(y,x)$, $I_\Theta(x,y)=I_\Theta(y,x)$, $F_\Theta(x,y)=F_\Theta(y,x)$ *for all* $x,y\in X$.

3 NEUTROSOPHIC GRAPHS AND IRREGULARITY

In this section we will study some basic definitions about neutrosophic graphs and different types of degrees of vertices will be discussed. Irregularity of neutrosophic graphs and related results are also proven in this section

3.1 Definition *[18] Let* $G^* = (V, E)$ *be a simple graph and* $E \subseteq V \times V$ *. Let* $T_\Lambda, I_\Lambda, F_\Lambda : V \to [0,1]$ *denote the truth-membership, indeterminacy-membership and falsity- membership of an element* $x \in V$ *and* $T_\Theta, I_\Theta, F_\Theta : E \to [0,1]$ *denote the truth-membership, indeterminacy-membership and falsity-membership of an element* $(x,y) \in E$*. By a neutrosophic graph, we mean a* 3 *-tuple* $G = (G^*, \Lambda, \Theta)$ *such that*

$$T_\Theta(x,y) \le \min\{T_\Lambda(x), T_\Lambda(y)\}$$
$$I_\Theta(x,y) \le \min\{I_\Lambda(x), I_\Lambda(y)\}$$
$$F_\Theta(x,y) \ge \max\{F_\Lambda(x), F_\Lambda(y)\}$$

for all $(x,y) \in E$.

3.2 Definition *[18] Let* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *be two simple graphs. The union of two neutrosophic graphs* $G_1 = (G_1^*, \Lambda_1, \Theta_1)$ *and* $G_2 = (G_2^*, \Lambda_2, \Theta_2)$ *is denoted by* $G = (G^*, \Lambda, \Theta), G^* = G_1^* \cup G_2^*$, $\Lambda = \Lambda_1 \cup \Lambda_2, \Theta = \Theta_1 \cup \Theta_2$, *where the truth-membership, indeterminacy-membership and falsity-membership of union are as follows*

$$T_\Lambda(x) = \begin{cases} T_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ T_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \max\{T_{\Lambda_1}(x), T_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases},$$

$$I_\Lambda(x) = \begin{cases} I_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ I_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \max\{I_{\Lambda_1}(x), I_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases}$$

$$F_\Lambda(x) = \begin{cases} F_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ F_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \min\{F_{\Lambda_1}(x), F_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases},$$

Also

$$T_\Theta(x,y) = \begin{cases} T_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ T_{\Theta_2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{T_{\Theta_1}(x,y), T_{\Theta_2}(x,y)\} & \text{if } (x,y) \in E_2 \end{cases}$$

$$I_\Theta(x,y) = \begin{cases} I_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ I_{\Theta_2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{I_{\Theta_1}(x,y), I_{\Theta_2}(x,y)\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

$$F_\Theta(x,y) = \begin{cases} F_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ F_{\Theta_1}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \min\{F_{\Theta_1}(x,y), F_{\Theta_1}(x,y))\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

3.3 Definition *[18] The intersection of two neutrosophic graphs* $G_1 = (G_1^*, \Lambda_1, \Theta_1)$ *and* $G_2 = (G_2^*, \Lambda_2, \Theta_2)$ *is denoted by* $G = (G^*, \Lambda, \Theta)$ *where* $G^* = G_1^* \cap G_2^*$, $\Lambda = \Lambda_1 \cap \Lambda_2, \Theta = \Theta_1 \cap \Theta_2, V = V_1 \cap V_2$ *and the truth-membership, indeterminacy-membership and falsity-membership of intersection are as follows*

$$T_\Lambda(x) = \{\min\{T_{\Lambda_1}(x), T_{\Lambda_2}(x)\} \quad ,$$
$$I_\Lambda(x) = \{\min\{\{I_{\Lambda_1}(x), I_{\Lambda_2}(x)\},$$
$$F_\Lambda(x) = \{\max\{\{F_{\Lambda_1}(x), F_{\Lambda_2}(x)\}$$

also

$$T_\Theta(x,y) = \{\min\{T_{\Theta_1}(x,y), T_{\Theta_2}(x,y)\}$$
$$I_\Theta(x,y) = \{\min\{I_{\Theta_1}(x,y), I_{\Theta_2}(x,y)\},$$
$$F_\Theta(x,y) = \{\max\{F_{\Theta_1}(x,y), F_{\Theta_2}(x,y)\}$$

3.4 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The nbhd of a vertex in is defined as* $N(x) = (N_T(x), N_I(x), N_F(x))$ *where* $N_T(x) = \{y \in V : T_\Theta(x,y) \le \min\{T_\Lambda(x), T_\Lambda(y)\}\}$, $N_I(x) = \{y \in V : I_\Theta(x,y) \le \min\{I_\Lambda(x), I_\Lambda(y)\}\}, N_F(x) = \{y \in V : F_\Theta(x,y) \ge \max\{F_\Lambda(x), F_\Lambda(y)\}\}$.

3.5 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a* neutrosophic graph. The nbhd degree of a vertex *x* in *G* defined by

$$\deg_T(x) = \sum_{y \in N_{T_g}(x)} T_\Lambda(y),$$

$$\deg_I(x) = \sum_{y \in N_{g_I}(x)} I_\Lambda(y), \deg_F(x) = \sum_{y \in N_{F_g}(x)} F_\Lambda(y).$$

3.6 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The closed nbhd degree of a vertex in is defined as* $\deg[x] = (\deg_T[x], \deg_I[x], \deg_F[x])$, *where*

$$\deg_T[x] = \sum_{y \in N_T(x)} T_\Lambda(y) + T_\Lambda(x),$$

, $$\deg_I[x] = \sum_{y \in N_I(x)} I_\Lambda(y) + I_\Lambda(x),$$

$$\deg_F[x] = \sum_{y \in N_F(x)} F_\Lambda(y) + F_\Lambda(x).$$

3.7 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The order of neutrosophic graph denoted by* $O(G)$ *is defined as*

$$O(G) = (O_T(G), O_I(G), O_F(G)), \text{ Where}$$

$$O_T(G) = \sum_{x \in V} T_\Lambda(x),$$

$$O_I(G) = \sum_{x \in V} I_\Lambda(x), O_F(G) = \sum_{x \in V} F_\Lambda(x).$$

The size of a neutrosophic graph $G = (G^*, \Lambda, \Theta)$ *is denoted by* $S(G)$ *and is defined as*

The size of a neutrosophic graph $G = (G^*, \Lambda, \Theta)$ *is denoted by* $S(G)$ *and is defined as*

$$S(G) = (S_T(G), S_I(G), S_F(G)), \text{ where}$$

$$S_T(G) = \sum_{(x.y) \in E} T_\Theta(x, y),$$

$$S_I(G) = \sum_{(x.y) \in E} I_\Theta(x, y),$$

$$S_F(G) = \sum_{(x.y) \in E} F_\Theta(x, y)$$

3.8 Definition *A neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called regular if all the vertices have the same open nbhd degree.*

3.9 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. If there is a vertex which is adjacent to vertices with distinct neighborhood degrees then is called a irregular neutrosophic graph.*

3.10 Example *Let* $G^* = (V, E)$ *be a simple graph with* $V = \{x_1, x_2, x_3\}$ *and* $E = \{(x_1, x_2)(x_{2,}, x_3), (x_1, x_3)\}$. *A neutrosophic graph G is given in table* 1 *below and* $T_\Theta(x_i, x_j) = 0$, $I_\Theta(x_i, x_j) = 0$ *and* $F_\Theta(x_i, x_j) = 1$ *for all* $(x_i, x_j) \in E \setminus \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$.

Table 1

Λ	x_1	x_2	x_3
T_Λ	0.1	0.1	0.2
I_Λ	0.3	0.3	0.4
F_Λ	0.4	0.3	0.6
Θ	(x_1, x_2)	$(x_{2,}, x_3)$	(x_1, x_3)
T_Θ	0.1	0.1	0.1
I_Θ	0.2	0.3	0.3
F_Θ	0.8	0.7	0.8

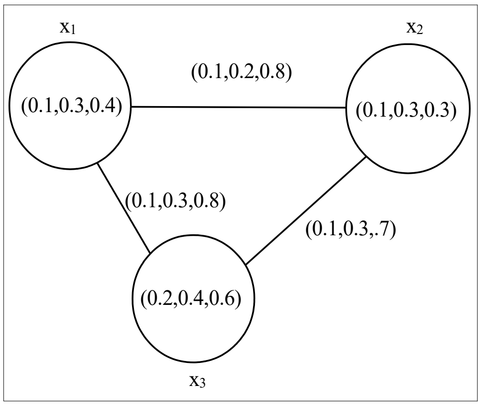

Figure 1

Here $\deg(x_1) = (0.3, 0.7, 0.9)$. *Similarly,*

$\deg(x_2) = (0.3, 0.7, 0.1), \deg(x_3) = (0.2, 0.6, 0.7).$

Clearly, G *is an irregular neutrosophic graph.*

3.11 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. If there is a vertex which is adjacent to vertices with distinct closed neighborhood degrees, then is called a totally irregular neutrosophic graph.*

3.12 Example *Consider a neutrosophic graph below with* $V = \{x_1, x_2, x_3, x_4, x_5\}$,

$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_2),$
$(x_1, x_3), (x_4, x_5), (x_4, x_1)\}$

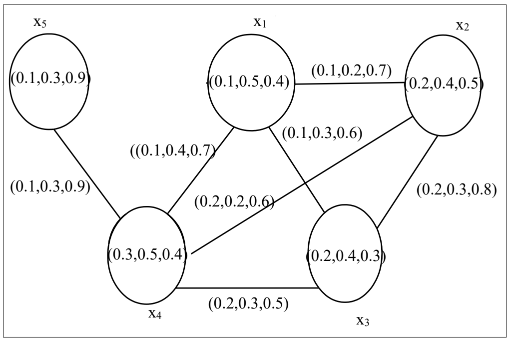

Figure 2

$\deg[x_1] = (0.8, 1.8, 1.6).$
$\deg[x_2] = (0.8, 1.8, 1.6), \deg[x_3] = (0.8, 1.8, 1.6),$
$\deg[x_4] = (0.9, 2.1, 2.5), \deg[x_5] = (0.4, 0.8, 1.3).$

Clearly G *is totally irregular neutrosophic graph.*

3.13 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a connected neutrosophic graph. If every two adjacent vertices of have distinct open neighborhood degrees, then is called neighbourly irregular neutrosophic graph*

3.14 Example *Consider a neutrosophic graph* G *below with* $V = \{x_1, x_2, x_3, x_4\}$,

$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_1)\}.$

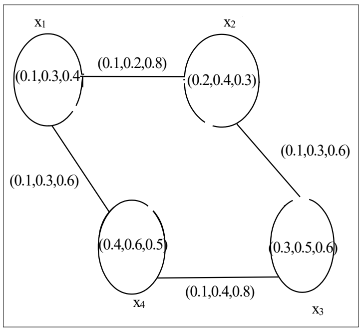

Figure 3

Here $\deg(x_1) = (0.6, 01, 0.8).$ *Similarly,*
$\deg(x_2) = (0.4, 0.8, 01), \deg(x_3) = (0.6, 01, 0.8).$
$\deg(x_4) = (0.4, 0.8, 01,),$ *Clearly*, G *is neighbourly irregular neutrosophic graph..*

3.15 Definition *A connected neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called neighbourly totally irregular neutrosophic graph if every two adjacent vertices of* G *have distinct closed neighborhood degrees.*

3.16 Example *An example of neighbourly totally irregular neutrosophic graph* G *is given below with*

$V = \{x_1, x_2, x_3, x_4\},$
$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_1)\}.$

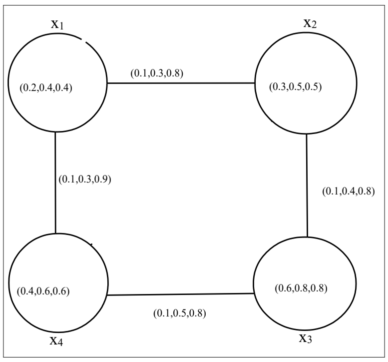

Figure 4

$\deg[x_1(=]0.9, 1.5, 1.5),$

$\deg[x_2(=]1.1, 1.7, 1.7),$

$\deg[x_3] = (1.3, 1.9, 1.9),$

$\deg[x_4] = (1.2, 1.8, 1.8)$

3.17 Definition *A connected neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called highly irregular neutrosophic graph if every vertex of* G *is adjacent to vertices with distinct neighborhood degrees.*

Note (i) A highly irregular neutrosophic graph may not be neighbourly irregular neutrosophic graph.

3.18 Example *From figure* 5 *below ,it can be seen that a highly irregular neutrosophic graph may not be neighbourly irregular neutrosophic graph.*

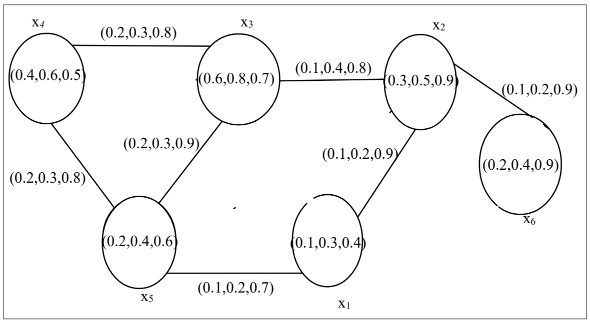

Figure 5

Here $x_2 \in V$, which is adjacent to the vertices x_1, x_3, x_6 with distinct nbhd degrees. But $\deg(x_2) = \deg(x_3)$.

So G is highly irregular neutrosophic graph but it is not a neighbourly irregular.

ii) A neighbourly irregular neutrosophic graph may not be highly irregular neutrosophic graph.

3.19 Example Consider the graph below

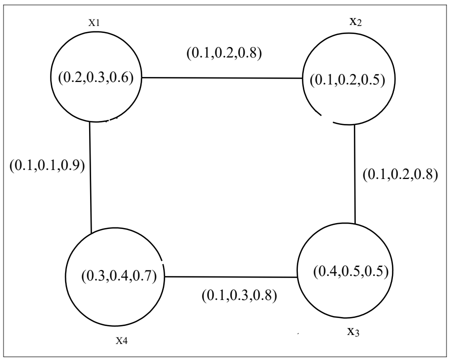

Figure 6

Here $\deg(x_1) = (0.4, 0.6, 1.2)$,

$\deg(x_2) = (0.6, 0.8, 1.1)$, $\deg(x_3) = (0.4, 0.6, 1.2)$,

$\deg(x_4) = (0.6, 0.8, 1.1)$

Clearly every two adjacent vertices have distinct nbhd degree, but x_2 is adjacent to x_1 and x_3 having same degree. Hence G is neighbourly irregular neutrosophic graph but not highly irregular neutrosophic graph.

(iii) A neighbourly irregular neutrosophic graph may not be a neighbourly totally irregular neutrosophic graph.

3.20 Example *Consider a neutrosophic graph such that* $V = \{x_1, x_2, x_3, x_4\}$,

$$E = \{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_1)\}.$$

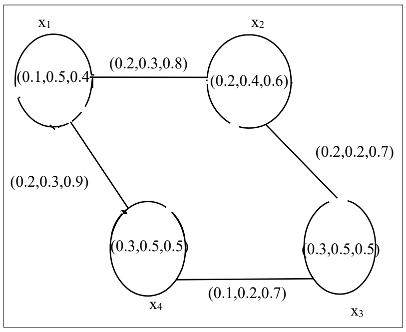

Figure 7

$\deg(x_1) = (0.5, 0.9, 1.1), \deg(x_2) = (0.6, 1, 0.9),$

$\deg(x_3) = (0.5, 0.9, 1.1), \deg(x_4) = (0.6, 1, 0.9)$

And

$\deg[x_1] = (0.6, 1.4, 1.5), \deg[x_2] = (0.6, 1.4, 1.5),$

$\deg[x_3] = (0.8, 1.4, 1.6), \deg[x_4] = (0.7, 1.5, 1.4).$

We see that $\deg[x_1] = \deg[x_2]$ *. Hence* G *is neighbourly irregular neutrosophic graph but not a neighbourly totally irregular neutrosophic graph.*

(iv) A neighbourly totally irregular neutrosophic graph may not be a neighbourly irregular neutrosophic graph.

3.21 Example *Consider a neutrosophic graph* G *such that*

$V = \{x_1, x_2, x_3, x_4\}$,

$E = \{x_1x_2, x_2x_3, x_3x_4, x_4x_1\}$.

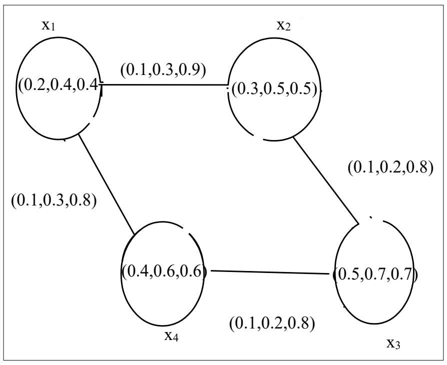

Figure 8

$\deg[x_1]=(0.9,1.5,1.5), \deg[x_2]=(1.0,1.6,1.6),$
$\deg[x_3]=(1.2,1.8,1.8), \deg[x_4]=(1.1,1.7,1.7).$

But

$\deg(x_1)=(0.7,1.1,1.1), \deg(x_2)=(0.7,1.1,1.1),$
$\deg(x_3)=(0.7,1.1,1.1), \deg(x_4)=(0.7,1.1,1.1).$

Hence $\deg(x_1)=\deg(x_2)=\deg(x_3)=\deg(x_4)$ *. So* G *is neighbourly totally irregular neutrosophic graph but not a neighbourly irregular neutrosophic graph.*

3.22 Proposition

Let be a Neutrosophic graph. Then G *is highly irregular neutrosophic graph and neighbourly irregular Neutrosophic graph iff the neighborhood degrees of all the vertices of* G *are distinct.*

Proof Let G be a neutrosophic graph with n-vertices $x_1, x_2, x_3, ..., x_n$. Suppose G is both highly irregular and neighbourly irregular neutrosophic graph. We want to show the neighborhood degrees of all vertices of are distinct. Let $\deg(x_i)=(p_i, q_i, r_i)$, $i = 1,2,\ldots,n.$

Let the adjacent vertices of x_1 are $x_2, x_3, \ldots, x_n$ with nbhd degrees (p_2, q_2, r_2) , $(p_3, q_3, r_3), ..., (p_n, q_n, r_n)$ respectively. Since G is highly irregular so $p_2 \neq p, \ldots, \neq p_n$, $q_2 \neq q_3 \neq, ..., \neq q_n$, $r_2 \neq r_3 \neq, ..., \neq r_n$. Also $p_1 \neq p_2 \neq p, ..., \neq p_n$, $q_1 \neq q_2 \neq q_3 \neq, ..., \neq q_n$, $r_1 \neq \ r_2 \neq r_3 \neq, ..., \neq r_n$ because G is neighbourly irregular. So $(p_1, q_1, r_1) \neq (p_2, q_2, r_2) \neq (p_3, q_3, r_3) \neq, ..., \neq (p_n, q_n, r_n).$

Hence the neighborhood degrees of all the vertices of are distinct.

Conversely, suppose that the neighborhood degrees of all the vertices are distinct. Now we want to show that is highly irregular and neighbourly irregular neutrosophic graph. Let $deg(x_i) = (p_i, q_i, r_i)$, $i = 1,2,\ldots,n$ given that $p_1 \neq p_2 \neq p, ..., \neq p_n$,

$q_1 \neq q_2 \neq q_3 \neq, \ldots, \neq q_n$, and $r_1 \neq r_2 \neq r_3 \neq, ..., \neq r_n$ $\Rightarrow$ Every two adjacent vertices have distinct neighborhood degrees and to every vertex, the adjacent vertices have distinct neighborhood degrees, which completes the proof.

3.23 Proposition

Let G *be a neutrosophic graph. If* G *is neighbourly irregular neutrosophic graph and* $(T_\Lambda, I_\Lambda, F_\Lambda)$ *is a constant function, then* G *is a neighbourly totally irregular neutrosophic graph.*

Proof Let G be neighbourly irregular neutrosophic graph. Let $x_i, x_j \in V$, where x_i and x_j are adjacent vertices with distinct neighborhood degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively. Let us assume that $(T_\Lambda(x_i), I_\Lambda(x_i), F_\Lambda(x_i)) = (T_\Lambda(x_j),$, where $I_\Lambda(x_j), F_\Lambda(x_j)) = (k_1, k_2, k_3)$

k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$. Therefore, $\deg_T[x_i] = \deg_T(x_i) + T_\Lambda(x_i) = p_1 + k_1,$

$deg_I[x_i] = deg_I(x_i) + I_\Lambda(x_i) = q_1 + k_2,$

$deg_F[x_i] = deg_F(x_i) + F_\Lambda(x_i) = r_1 + k_3,$

$deg_T[x_j] = deg_T(x_j) + T_\Lambda(x_j) = p_2 + k_1,$

$deg_I[x_j] = deg_I(x_j) + I_\Lambda(x_j) = q_2 + k_2,$

$deg_F[x_j] = deg_F(x_j) + F_\Lambda(x_j) = r_2 + k_3.$ We want to show

$degT[x_i \neq]degT[x_j], degI[x_i \neq]degI[x_j], degF[x_i \neq]degF[x_j$

Suppose that on contrary,

$\deg_T[x_i] = \deg_T[x_j] \Rightarrow p_1 + k_1 = p_2 + k_1$ Which $\Rightarrow p_1 - p_2 = k_1 - k_1 = 0 \Rightarrow p_1 = p_2$ is a contradiction because $p_1 \neq p_2$. Similarly

$deg_I[x_i] = \deg_I[x_j] \Rightarrow q_1 - q_2 = k_2 - k_2 = 0 \Rightarrow q_1 = q_2.$

Which is a contradiction since $q_1 \neq q_2$. Consider

$deg_F[x_i =]deg_F[x_j \Rightarrow]r_1 \ +k_3 \ =r_2 \ +k_3$

Which is a contradiction since $r_1 \neq r_2$. Therefore G is neighbourly totally irregular neutrosophic graph.

3.24 Proposition

A neutrosophic graph G of G^*, where G^* is a cycle with 3 vertices is neighbourly irregular and highly irregular iff the truth- membership, indeterminacy-membership and falsity- membership values of vertices between every pair of vertices are all distinct.

Proof Suppose that truth membership, indeterminacy and falsity membership between every pair of vertices are all distinct. Let $x_i, x_j, x_k \in V$ and

$$T_\Lambda(x_i) \neq T_\Lambda(x_j) \neq T_\Lambda(x_k),$$
$$I_\Lambda(x_i) \neq I_\Lambda(x_j) \neq I_\Lambda(x_k),$$
$$F_\Lambda(x_i) \neq F_\Lambda(x_j) \neq F_\Lambda(x_k).$$

Which implies that

$$\sum_{x \in N(x_i)} F_\Lambda(x_i) \neq \sum_{x \in N(x_j)} F_\Lambda(x_j) \neq \sum_{x \in N(x_k)} F_\Lambda(x_k),$$
$$\sum_{x \in N(x_i)} I_\Lambda(x_i) \neq \sum_{x \in N(x_j)} I_\Lambda(x_j) \neq \sum_{x \in N(x_k)} I_\Lambda(x_k),$$
$$\sum_{x \in N(x_i)} T_\Lambda(x_i) \neq \sum_{x \in N(x_j)} T_\Lambda(x_j) \neq \sum_{x \in N(x_k)} T_\Lambda(x_k).$$

That is, $\deg_T(x_i) \neq \deg_T(x_j) \neq \deg_T(x_k)$.
Similarly we can show
$\deg_I(x_i) \neq \deg_I(x_j) \neq \deg_I(x_k)$,
$\deg_F(x_i) \neq \deg_F(x_j) \neq \deg_F(x_k)$ showing that
$\deg(x_i) \neq \deg(x_j) \neq \deg(x_k)$. Hence is neighbourly irregular and highly irregular neutrosophic graph.

Conversely, suppose that is neighbourly irregular and highly irregular. Let $deg(x_i) = (p_i, q_i, r_i)$, $i = 1, 2, 3, \ldots, n$. Suppose that, truthfulness, falsity and indeterminacy of two vertices are same. . Let $x_1, x_2 \in V$ with $T_\Lambda(x_1) = T_\Lambda(x_2)$, $I_\Lambda(x_1) = I_\Lambda(x_2), F_\Lambda(x_1) = F_\Lambda(x_2)$. Then $\deg_T(x_1) = \deg_T(x_2)$, $\deg_I(x_1) = \deg_I(x_2)$, $\deg_F(x_1) = \deg_F(x_2)$. Which implies $\deg(x_1) = \deg(x_2)$. Since G^* is a cycle, so we have a contradiction to the fact that G is neighbourly irregular and highly irregular neutrosophic graph. Hence the truth-membership, indeterminacy-membership and falsity-membership values of vertices between every pair of vertices are all distinct.

3.25 Proposition

Let G be a neutrosophic graph. If G is neighbourly totally irregular neutrosophic graph and $(T_\Lambda, I_\Lambda, F_\Lambda)$ is a constant function, then G is a neighbourly irregular neutrosophic graph.

Proof We suppose G is neighbourly totally irregular neutrosophic graph. Then by definition, the closed neighborhood degree of every two adjacent are distinct. Let $x_i, x_j \in V$, where x_i and x_j are adjacent vertices with distinct degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively. Let us assume that

$$(T_\Lambda(x_i), I_\Lambda(x_i), F_\Lambda(x_i)) = (T_\Lambda(x_j), I_\Lambda(x_j), F_\Lambda(x_j)) = (k_1, k_2, k_3),$$

where k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$ and $\deg[x_i] \neq \deg[x_j]$. We want to show $\deg(x_i) \neq \deg(x_j)$. Since $\deg[x_i] \neq \deg[x_j]$, so $\deg_T[x_i] \neq \deg_T[x_j]$, $\deg_I[x_i] \neq \deg_I[x_j]$, $\deg_F[x_i] \neq \deg_F[x_j]$. Now

$$\deg_T[x_i] \neq \deg_T[x_j] \Rightarrow p_1 + k_1 \neq p_2 + k_1$$
$$\Rightarrow p_1 - p_2 \neq k_1 - k_1 = 0 \Rightarrow p_1 \neq p_2$$

Similarly

$$\deg_I[x_i] \neq \deg_I[x_j] \Rightarrow q_1 - q_2 \neq k_2 - k_2 = 0$$
$$\Rightarrow q_1 \neq q.$$
$$\deg_F[x_i] \neq \deg_F[x_j] \Rightarrow r_1 + k_3 \neq r_2 + k_3$$
$$\Rightarrow r_1 - r_2 \neq k_3 - k_3 = 0 \Rightarrow r_1 \neq r_2.$$

Hence the degrees of $x_i, x_j \in V$ are distinct. This is true for every pair of adjacent vertices in G. Therefore G is neighbourly irregular neutrosophic graph.

Conclusion

Neutrosophic sets are the generalization of the classical sets and of the fuzzy sets, and have many applications in real world problems when the data is imprecise, indeterminant or inconsistent. In this paper, we initiated the idea of the irregular neutrosophic graphs, and discussed different properties of such graphs. We have seen how neighbourly irregular and highly irregular neutrosophic graphs are equivalent. In future, we will extend our work to other graph theory areas by using neutrosophic graphs.

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Received: November 02, 2016. Accepted: December 25, 2016