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N_ω –CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract. Neutrosophic set and Neutrosophic Topological spaces has been introduced by Salama[5]. Neutrosophic Closed set and Neutrosophic Continuous Functions were introduced by Salama et. al.. In this paper, we introduce the concept of Nω- closed sets and their properties in Neutrosophic topological spaces.

Keywords: Intuitionistic Fuzzy set, Neutrosophic set, Neutrosophic Topology, N_s-open set, N_s-closed set, N_ω- closed set, N_ω-open set and Nω-closure.

1. Introduction

Many theories like, Theory of Fuzzy sets[10], Theory of Intuitionistic fuzzy sets[1], Theory of Neutrosophic sets[8] and The Theory of Interval Neutrosophic sets[4] can be considered as tools for dealing with uncertainities. However, all of these theories have their own difficulties which are pointed out in[8].

In 1965, Zadeh[10] introduced fuzzy set theory as a mathematical tool for dealing with uncertainities where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov[1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of nonmembership of each element. The neutrosophic set was introduced by Smarandache[7] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set.

In 2012, Salama, Alblowi[5] introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element. In 2014 Salama, Smarandache and Valeri [6] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. In this paper, we introduce the concept of N_ω- closed sets and their properties in neutrosophic topological spaces.

2. Preliminaries

In this paper, X denote a topological space (X, τ_N) on which no separation axioms are assumed unless otherwise explicitly mentioned. We recall the following definitions, which will be used throughout this paper. For a subset A of X, Ncl(A), Nint(A) and A^c denote the neutrosophic closure, neutrosophic interior, and the complement of neutrosophic set A respectively.

Definition 2.1.[3] Let X be a non-empty fixed set. A neutrosophic set(NS for short) A is an object having the form $A = \{<x, \mu_A(x), \sigma_A(x), \upsilon_A(x)>: \text{for all } x \in X\}$. Where $\mu_A(x)$, $\sigma_A(x)$, $\upsilon_A(x)$ which represent the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element $x \in X$ to the set A.

Definition 2.2.[5] Let A and B be NSs of the form $A = \{<x, \mu_A(x), \sigma_A(x), \upsilon_A(x)>: \text{for all } x \in X\}$ and $B = \{<x, \mu_B(x), \sigma_B(x), \upsilon_B(x)>: \text{for all } x \in X\}$. Then

i. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\upsilon_A(x) \geq \upsilon_B(x)$ for all $x \in X$,
ii. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
iii. $A^c = \{<x, \upsilon_A(x), 1- \sigma_A(x), \mu_A(x)>: \text{for all } x \in X \}$,
iv. $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \upsilon_A(x) \wedge \upsilon_B(x): \text{for all } x \in X >\}$,
v. $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \upsilon_A(x) \vee \upsilon_B(x): \text{for all } x \in X >\}$.

Definition 2.3.[5] A neutrosophic topology(NT for short) on a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

i) $0_N, 1_N \in \tau$,
ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
iii) $\cup G_i \in \tau$, for all $G_i: i \in J \subseteq \tau$

In this pair (X, τ) is called a neutrosophic topological space (NTS for short) for neutrosophic set (NOS for short) τ in X. The elements of τ are called open neutrosophic sets. A neutrosophic set F is called closed if and only if the complement of F(F^c for short) is neutrosophic open.

Definition 2.4.[5] Let (X, τ) be a neutrosophic topological space. A neutrosophic set A in (X, τ) is said to be neutrosophic closed(N-closed for short) if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is neutrosophic open.

Definition 2.5.[5] The complement of N-closed set is N-open set.

Proposition 2.6.[6] In a neutrosophic topological space (X, T), T = $\mathfrak{I}$ (the family of all neutrosophic closed sets) iff every neutrosophic subset of (X, T) is a neutrosophic closed set.

3. N_ω -closed sets

In this section, we introduce the concept of N_ω-closed set and some of their properties. Throughout this paper (X, τ_N) represent a neutrosophic topological spaces.

Definition 3.1. Let (X, τ_N) be a neutrosophic topological space. Then A is called neutrosophic semi open set(N_s-open set for short) if A ⊂ Ncl(Nint(A)).

Definition 3.2. Let (X, τ_N) be a neutrosophic topological space. Then A is called neutrosophic semi closed set(N_s-closed set for short) if Nint(Ncl(A)) ⊂ A.

Definition 3.3. Let A be a neutrosophic set of a neutrosophic topological space (X, τ_N). Then,

i. The neutrosophic semi closure of A is defined as $N_s$cl(A) = ∩ {K: K is a N_s-closed in X and A ⊆ K}
ii. The neutrosophic semi interior of A is defined as $N_s$int(A) = ∪ {G: G is a N_s-open in X and G ⊆ A}

Definition 3.4. Let (X, τ_N) be a neutrosophic topological space. Then A is called Neutrosophic ω closed set(N_ω-closed set for short) if Ncl(A) ⊆ G whenever A ⊆ G and G is N_s-open set.

Theorem 3.5. Every neutrosophic closed set is N_ω-closed set, but the converse may not be true.
Proof: If A is any neutrosophic set in X and G is any N_s-open set containing A, then Ncl (A) ⊆ G. Hence A is N_ω-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let X = {a,b,c} and τ_N = {0_N,G_1, 1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological spaces. Take G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)>, A = <x, (0.2, 0.2, 0.1), (0, 1, 0.2), (0.8, 0.6,0.9)>. Then the set A is N_ω-closed set but A is not a neutrosophic closed.

Theorem 3.7. Every N_ω-closed set is N-closed set but not conversely.
Proof: Let A be any N_ω-closed set in X and G be any neutrosophic open set such that A ⊆ G. Then G is N_s-open, A ⊆ G and Ncl (A) ⊆ G. Thus A is N-closed.

The converse of the above theorem proved by the following example.

Example 3.8. Let X = {a,b,c} and τ_N = {0_N,G_1,1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological spaces. Let G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> and A = <x, (0.55, 0.45, 0.6), (0.11, 0.3, 0.1), (0.11, 0.25, 0.2)>. Then the set A is N-closed but A is not a N_ω-closed set.

Remark 3.9. The concepts of N_ω-closed sets and N_s-closed sets are independent.

Example 3.10. Let X = {a,b,c} and τ_N = {0_N,G_1, 1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological spaces. Take G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)>, A = <x, (0.2, 0.2, 0.1), (0, 1, 0.2), (0.8, 0.6,0.9)>. Then the set A is N_ω-closed set but A is not a N_s-closed set.

Example 3.11. Let X = {a,b} and τ_N = {0_N,G_1, G_2, 1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological spaces. Take G_1 = <x, (0.6, 0.7), (0.3, 0.2), (0.2, 0.1)> and A = <x, (0.3, 0.4), (0.6, 0.7), (0.9,0.9)>. Then the set A is N_s-closed set but A is not a N_ω-closed.

Theorem 3.12. If A and B are N_ω-closed sets, then A ∪ B is N_ω-closed set.
Proof: If A ∪ B ⊆ G and G is N_s-open set, then A ⊆ G and B ⊆ G. Since A and B are N_ω-closed sets, Ncl(A) ⊆ G and Ncl(B) ⊆ G and hence Ncl(A) ∪ Ncl(B) ⊆ G. This implies Ncl(A ∪ B) ⊆ G. Thus A ∪ B is N_ω-closed set in X.

Theorem 3.13. A neutrosophic set A is N_ω-closed set then Ncl(A) – A does not contain any nonempty neutrosophic closed sets.
Proof: Suppose that A is N_ω-closed set. Let F be a neutrosophic closed subset of Ncl(A) – A. Then A ⊆ F^c. But A is N_ω-closed set. Therefore Ncl(A) ⊆ F^c. Consequently F ⊆ $(Ncl(A))^c$. We have F ⊆ Ncl(A). Thus F ⊆ Ncl(A)∩$(Ncl(A))^c$ = ϕ. Hence F is empty.

The converse of the above theorem need not be true as seen from the following example.

Example 3.14. Let X = {a,b,c} and τ_N = {0_N,G_1, 1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological spaces. Take G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> and A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9,0.9)>. Then the set A is not a N_ω-closed set and Ncl(A) – A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9,0.9)> does not contain non-empty neutrosophic closed sets.

Theorem 3.15. A neutrosophic set A is N_ω-closed set if and only if Ncl(A) – A contains no non-empty N_s-closed set.

Proof: Suppose that A is N_ω-closed set. Let S be a N_s-closed subset of Ncl(A) – A. Then A ⊆ S^c. Since A is N_ω-closed set, we have Ncl(A) ⊆ S^c. Consequently S ⊆ $(Ncl(A))^c$. Hence S ⊆ Ncl(A)∩$(Ncl(A))^c$ = ϕ. Therefore S is empty.
Conversely, suppose that Ncl(A) – A contains no non-empty N_s-closed set. Let A ⊆ G and that G be N_s-open. If Ncl(A) ⊈ G, then Ncl(A) ∩ G^c is a non-empty N_s-closed subset of Ncl(A) – A. Hence A is N_ω-closed set.

Corollary 3.16. A N_ω-closed set A is N_s-closed if and only if $N_s$cl(A) – A is N_s-closed.
Proof: Let A be any N_ω-closed set. If A is N_s-closed set, then $N_s$cl(A) – A = ϕ. Therefore $N_s$cl(A) – A is N_s-closed set.
Conversely, suppose that Ncl(A) – A is N_s-closed set. But A is N_ω-closed set and Ncl(A) – A contains N_s-closed. By theorem 3.15, $N_s$cl(A) – A = ϕ. Therefore $N_s$cl(A) = A. Hence A is N_s-closed set.

Theorem 3.17. Suppose that B ⊆ A ⊆ X, B is a N_ω-closed set relative to A and that A is N_ω-closed set in X. Then B is N_ω-closed set in X.
Proof: Let B ⊆ G, where G is N_s-open in X. We have B ⊆ A∩G and A∩G is N_s-open in A. But B is a N_ω-closed set relative to A. Hence Ncl_A(B) ⊆ A∩G. Since Ncl_A(B) = A ∩ Ncl(B). We have A∩Ncl(B) ⊆ A∩G. It implies A ⊆ G∪$(Ncl(B))^c$ and G∪$(Ncl(B))^c$ is a N_s-open set in X. Since A is N_ω-closed in X, we have Ncl(A) ⊆ G∪$(Ncl(B))^c$. Hence Ncl(B) ⊆ G∪$(Ncl(B))^c$ and Ncl(B) ⊆ G. Therefore B is N_ω-closed set relative to X.

Theorem 3.18. If A is N_ω-closed and A ⊆ B ⊆ Ncl(A), then B is N_ω-closed.
Proof: Since B ⊆ Ncl(A), we have Ncl(B) ⊆ Ncl(A) and Ncl(B) – B ⊆ Ncl(A) – A. But A is N_ω-closed. Hence Ncl(A) – A has no non-empty N_s-closed subsets, neither does Ncl(B) – B. By theorem 3.15, B is N_ω-closed.

Theorem 3.19. Let A ⊆ Y ⊆ X and suppose that A is N_ω-closed in X. Then A is N_ω-closed relative to Y.
Proof: Let A ⊆ Y∩G where G is N_s-open in X. Then A ⊆ G and hence Ncl(A) ⊆ G. This implies, Y ∩ Ncl(A) ⊆ Y∩G. Thus A is N_ω-closed relative to Y.

Theorem 3.20. If A is N_s-open and N_ω-closed, then A is neutrosophic closed set.
Proof: Since A is N_s-open and N_ω-closed, then Ncl(A) ⊆ A. Therefore Ncl(A) =A. Hence A is neutrosophic closed.

4. N_ω-open sets

In this section, we introduce and study about N_ω-open sets and some of their properties.

Definition 4.1. A Neutrosophic set A in X is called N_ω-open in X if A^c is N_ω-closed in X.

Theorem 4.2. Let (X, τ_N) be a neutrosophic topological space. Then

(i) Every neutrosophic open set is N_ω-open but not conversely.

(ii) Every N_ω-open set is N-open but not conversely.

The converse part of the above statements are proved by the following example.

Example 4.3. Let X = {a,b,c} and τ_N = {0_N,G_1, 1_N} is a neutrosophic topology and (X, τ_N) is a neutrosophic topological space. Take G_1 = <x, (0.7, 0.6, 0.9), (0.6, 0.5, 0.8), (0.5, 0.6, 0.4)> and A = <x, (0.8, 0.6, 0.9), (1, 0, 0.8), (0.2, 0.2, 0.1)>. Then the set A is N_ω-open set but not a neutrosophic open and B = <x, (0.11, 0.25, 0.2), (0.89, 0.7, 0.9), (0.55, 0.45, 0.6)> is N-open but not a N_ω-open set.

Theorem 4.4. A neutrosophic set A is N_ω-open if and only if F ⊆ Nint(A) where F is N_s-closed and F ⊆ A.
Proof: Suppose that F ⊆ Nint(A) where F is N_s-closed and F ⊆ A. Let A^c ⊆ G where G is N_s-open. Then G^c ⊆ A and G^c is N_s-closed. Therefore G^c ⊆ Nint(A). Since G^c ⊆ Nint(A), we have $(Nint(A))^c$ ⊆ G. This implies Ncl($(A)^c$) ⊆ G. Thus A^c is N_ω-closed. Hence A is N_ω-open.
Conversely, suppose that A is N_ω-open, F ⊆ A and F is N_s-closed. Then F^c is N_s-open and A^c ⊆ F^c. Therefore Ncl($(A)^c$) ⊆ F^c. But Ncl($(A)^c$) = $(Nint(A))^c$. Hence F ⊆ Nint(A).

Theorem 4.5. A neutrosophic set A is N_ω-open in X if and only if G = X whenever G is N_s-open and (Nint(A)∪A^c) ⊆ G.
Proof: Let A be a N_ω-open, G be N_s-open and (Nint(A)∪A^c) ⊆ G. This implies G^c ⊆ $(Nint(A))^c$ ∩$((A)^c)^c$ = $(Nint(A))^c$ – A^c = Ncl($(A)^c$) – A^c. Since A^c is N_ω-closed and G^c is N_s-closed, by Theorem 3.15, it follows that G^c = ϕ. Therefore X = G.
Conversely, suppose that F is N_s-closed and F ⊆ A. Then Nint(A) ∪ A^c ⊆ Nint(A) ∪ F^c. This implies Nint(A) ∪ F^c = X and hence F ⊆ Nint(A). Therefore A is N_ω-open.

Theorem 4.6. If Nint(A) ⊆ B ⊆ A and if A is N_ω-open, then B is N_ω-open.
Proof: Suppose that Nint(A) ⊆ B ⊆ A and A is N_ω-open. Then A^c ⊆B^c ⊆ Ncl(A^c) and since A^c is N_ω-closed. We have by Theorem 3.15, B^c is N_ω-closed. Hence B is N_ω-open.

Theorem 4.7. A neutrosophic set A is N_ω-closed, if and only if Ncl(A) – A is N_ω-open.
Proof: Suppose that A is N_ω-closed. Let F ⊆ Ncl(A) – A Where F is N_s-closed. By Theorem 3.15, F = ϕ. Therefore F ⊆ Nint((Ncl(A) – A) and by Theorem 4.4, we have Ncl(A) – A is N_ω-open.

Conversely, let A ⊆ G where G is a N_s-open set. Then Ncl(A) ∩ G^c ⊆ Ncl(A) ∩ A^c = Ncl(A) – A. Since Ncl(A) ⊆ G^c is N_s-closed and Ncl(A) – A is N_ω-open. By Theorem 4.4, we have Ncl(A) ∩ G^c ⊆ Nint(Ncl(A) – A) = ϕ. Hence A is N_ω-closed.

Theorem 4.8. For a subset A⊆X the following are equivalent:

(i) A is N_ω-closed.

(ii) Ncl(A) – A contains no non-empty N_s-closed set.

(iii) Ncl(A) – A is N_ω-open set.

Proof: Follows from Theorem 3.15 and Theorem 4.7.

5. N_ω-closure and Properties of N_ω-closure

In this section, we introduce the concept of N_ω-closure and some of their properties.

Definition 5.1. The N_ω-closure (briefly $N_\omega$cl(A)) of a subset A of a neutrosophic topological space (X, τ_N) is defined as follows:

$N_\omega$cl(A) = ∩ { F ⊆ X / A ⊆ F and F is N_ω-closed in (X, τ_N)}.

Theorem 5.2. Let A be any subset of (X, τ_N). If A is N_ω-closed in (X, τ_N) then A = $N_\omega$cl(A).

Proof: By definition, $N_\omega$cl(A) = ∩ { F ⊆ X / A ⊆ F and F is a N_ω-closed in (X, τ_N)} and we know that A ⊆ A. Hence A = $N_\omega$cl(A).

Remark 5.3. For a subset A of (X, τ_N), A ⊆ $N_\omega$cl(A) ⊆ Ncl(A).

Theorem 5.4. Let A and B be subsets of (X, τ_N). Then the following statements are true:

i. $N_\omega$cl(A) = ϕ and $N_\omega$cl(A) = X.

ii. If A ⊆ B, then $N_\omega$cl(A) ⊆ $N_\omega$cl(B)

iii. $N_\omega$cl(A) ∪ $N_\omega$cl (B) ⊂ $N_\omega$cl (A ∪ B)

iv. $N_\omega$cl (A ∩ B) ⊂ $N_\omega$cl(A) ∩ $N_\omega$cl (B)

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