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Neutrosophic Sets and Systems

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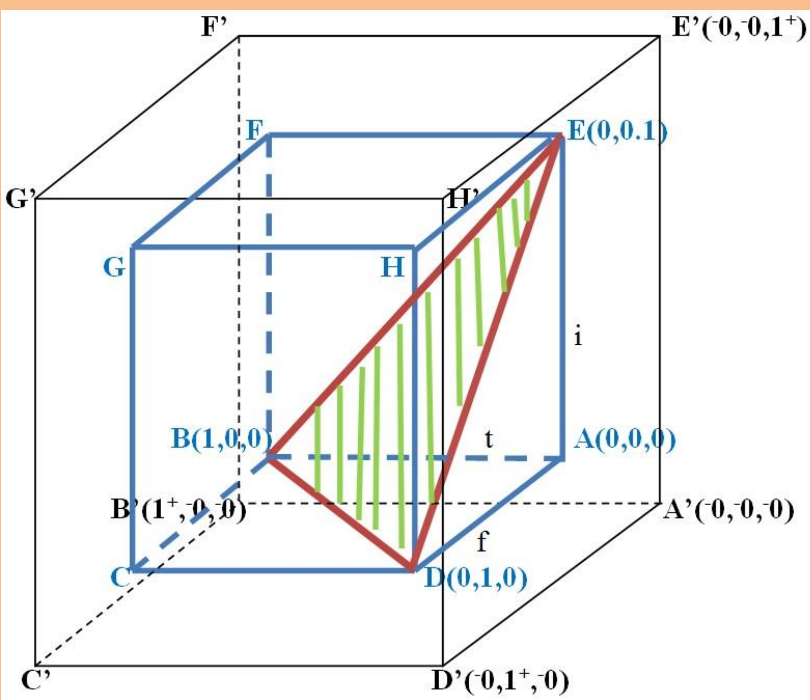

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*"*Neutrosophic Sets and Systems*"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeter

minacy (*I*), and a degree of falsity (*F*), where *T*, *I*, *F* are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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NSS

An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)

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Abstract. In this paper, finding - a maximal solution is introduced to (V, Λ) fuzzy neutrosophic relation equation. the notion of fuzzy relation equation was first investigated by Sanchez in 1976, while Florentin Smarandache put forward a fuzzy neutrosophic relation equations in 2004 with innovative investigation. This paper is first attempt to establish the structure of solution set on model. The NRE have a wide applications in various real world problems like flow rate in chemical plants, transportation problem, study of bounded labor problem, study of interrelations among HIV/AIDS affected patients and use of genetic algorithms in chemical problems .

Keyword Neutrosophic Logic, Neutrosophic Relation Equations (NRE),Integral Neutrosophic lattices, Fuzzy Integral Neutrosophic Matrices, Maximal Solution, Fuzzy Geometric Programming (FGP).

Introduction

The analysis of most of the real world problems involves the concept of indeterminacy. One cannot establish or cannot rule out the possibility of some relation but says that cannot determine the relation or link; this happens in legal field, medical diagnosis even in the construction of chemical flow in industries and more chiefly in socio economic problems prevailing in various countries.[4], as well as the importance of geometric programming and the fuzzy neutrosophic relation equations in theory and application, I have proposed a new structure for maximum solution in FNRE with geometric programming.

1.1 Definition [4]

Let T, I, F be real standard or non-standard subsets of $]^{-}0,1^{+}[$, with

$sup\, T = t\_sup, inf\, T = t\_inf, sup\, I = i\_sup, inf\, I = i\_inf, sup\, F = f\_sup, inf\, F = f\_inf, and\, n\_sup = t\_sup + i\_sup + f\_sup, n\_inf = t\_inf + i\_inf + f\_inf. Let\, U$

be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set $M\, as\, x(T, I, F)$ and belongs to M in the following way: It is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F. Statically T, I, F are subsets, but dynamically T, I, F are functions operators depending on many known or unknown parameters.

1.2 Physics Example for Neutrosophic Logic [4]

For example the Schrodinger's Theory says that the quantum state of a photon can basically be in more than one place in the same time , which translated to the neutrosophic set means that an element (quantum state) belongs and does not belong to a set (one place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of “alternative worlds” theory very well represented by the neutrosophic set theory. In Schroedinger’s Equation on the behavior of electromagnetic waves and “matter waves” in quantum theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points). Don’t we better describe, using the attribute “neutrosophic” than “fuzzy” or any others, a quantum particle that neither exists nor non-exists?

1.3 Application for Neutrosophic Logic [4]

A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set. (e.g. there are a kind of separated water props, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud). Also, we are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That's why the percent of indeterminacy is required and the neutrosophic probability (using subsets - not numbers - as components) should be used for better modeling: it is a more organic, smooth, and especially accurate estimation. Indeterminacy is the zone of ignorance of a proposition's value, between truth and falsehood. From the intuitionistic logic, paraconsistent logic, dialetheism, fallibilism, paradoxes, pseudoparadoxes, and tautologies we transfer the "adjectives" to the sets, i.e. to intuitionistic set (set incompletely known), paraconsistent set, dialetheist set, faillibilist set (each element has a percentage of indeterminacy), paradoxist set (an element may belong and may not belong in the same time to the set), pseudoparadoxist set, and tautological set respectively. hence, the neutrosophic set generalizes:

- the intuitionistic set, which supports incomplete set theories $(for\ 0 < n < 1, 0 [t, i, f [1)$ and incomplete known elements belonging to a set;

- the fuzzy set (for $n = 1\ and\ i = 0, and\ 0 [t, i, f [1)$;
- the classical set (for $n = 1\ and\ i = 0$, with $t, f\ either\ 0\ or\ 1$);
- the paraconsistent set (for $n > 1, with\ all\ t, i, f < 1^{+}$);
- the faillibilist set ($i > 0$);
- the dialetheist set, a set M whose at least one of its elements also belongs to its complement $C(M)$; thus, the intersection of some disjoint sets is not empty
- the paradoxist set ($t = f = 1$);
- the pseudoparadoxist set ($0 < i < 1, t = 1\ and\ f > 0\ or\ t > 0\ and\ f = 1$);
- the tautological set ($i, f < 0$).

Compared with all other types of sets, in the neutrosophic set each element has three components which are subsets (not numbers as in fuzzy set) and considers a subset, similarly to intuitionistic fuzzy set, of "indeterminacy" - due to unexpected parameters hidden in some sets, and let the superior limits of the components to even boil over 1 (over flooded) and the inferior limits of the components to even freeze under 0 (under dried). For example: an element in some tautological sets may have $t > 1$, called "over included". Similarly, an element in a set may be "over indeterminate" (for $i > 1$, in some paradoxist sets), "over excluded" (for $f > 1$, in some unconditionally false appurtenances); or "under true" (for $t < 0$, in some unconditionally false appurtenances), "under indeterminate" (for $i < 0$, in some unconditionally true or false appurtenances), "under fals some unconditionally true appurtenances). This is because we should make a distinction between unconditionally true $(t > 1, and\ f < 0\ or\ i < 0)$ and conditionally true appurtenances $(t [1, and\ f [1\ or\ i [1)$. In a rough set RS, an element on its boundary-line cannot be classified neither as a member of RS nor of its complement with certainty. In the neutrosophic set a such element may be characterized by $x(T, I, F)$, with corresponding set-values for T, I, F]-0,1+[. One first presents the evolution of sets from fuzzy set to neutrosophic set. Then one introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]-0,1+[is the non-standard unit interval, and thus one defines the neutrosophic set.[4]

2 Basic Concepts for NREs

2.1 Definition [3].

A Brouwerian lattice L in which, for any given elements a & b the set of all $x \in L$ such that $a \Lambda x \leq b$ contains a greatest element, denoted $a \propto b$, the relative pseudocomplement of $a\ in\ b$ [san].

2.2 Remark [4]

If $L = [0, 1]$, then it is easy to see that for any given $a, b \in L$,

$$a \propto b = \begin{cases} 1 & a \leq b \\ b & a > b \end{cases}$$

2.3 Definition [4]

Let $N = L \cup \{I\}$ *where* L *is any lattice and* I *an indeterminate.*

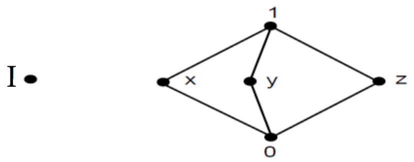
Define the max, min operation on N as follows

$Max\ \{x, I\} = I$ for $all\ x \in L \setminus \{1\}$

$Max\,\{1, I\} = 1$
$Min\,\{x, I\} = I$ for $all\ x \in L \setminus \{0\}$
$Min\,\{0, I\} = 0$
We know if $x, y \in L$ then max and min are well defined in L. N is called the integral neutrosophic lattice.

2.3 Example [4]

Let $N = L \cup \{I\}$ given by the following diagram:



Clearly [4]

$Min\,\{x, I\} = I \quad for\ all\ x \in L \setminus \{0\}$
$Min\,\{0, I\} = 0$
$Max\,\{x, I\} = I \quad for\ all\ x \in L \setminus \{1\}$
$Max\,\{1, I\} = 1$
We see N is an integral neutrosophic lattice and clearly the order of N is 6.

2.4 Remark [4]

1- If L is a lattice of order n and $\mathrm{N} = \mathrm{L} \cup \{\mathrm{I}\}$ be an integral neutrosophic lattice then order of N is $\mathrm{n} + 1$.
2. For an integral neutrosophic lattice N also $\{0\}$ is the minimal element and $\{1\}$ is the maximal element of N.

2.5 Conventions About Neutrosophic Sets [4]

Let $A, B \in N(X)\ i.e.,$
$A : X \to [0,1] \cup I \ , B : X \to [0,1] \cup I$
$(A \cap B)(x) = min\,\{A(x), B(x)\}$,if
$A(x) = I$ or $B(x) = I$ then $(A \cap B)(x)$ is defined to be I i.e., $min\,\{A(x), B(x))\} = \mathrm{I}_9$
$I\ (A \cup B)(x) = max\,\{A(x), B(x)\}$ if one of $A(x) = I\ or\ B(x) = I\ then\ (A \cup B)(x) = I$ i.e., $max\,\{A(x), B(x)\} = I$.
Thus it is pertinent to mention here that if one of $A(x) = I\ or\ B(x) = I$ then $(A \cup B)(x) = (A \cap B)(x)$. i.e., is the existence of indeterminacy $max\,\{A(x), B(x)\} = min\,\{A(x), B(x)\} = I$
$\bar{A}(x) = 1 - A(x);\ if\ A(x) = I$
$then \quad \bar{A}(x) = A(x) = I.$

2.6 Definition [4]

Let $N = [0,1] \cup I$ where I is the indeterminacy. The $m \times n$ matrices $M_{m \times n} = \{(a_{ij}) \,/\, a_{ij} \in [0,1] \cup I\}$ is called the fuzzy integral neutrosophic matrices. Clearly the class of $m \times n$ matrices is contained in the class of fuzzy integral neutrosophic matrices.

2.7 Example [4]

Let $A = \begin{pmatrix} I & 0.1 & 0 \\ 0.9 & 1 & I \end{pmatrix}$

A is a 2×3 integral fuzzy neutrosophic matrix. We define operation on these matrices. An integral fuzzy neutrosophic row vector is a $1 \times n$ integral fuzzy neutrosophic matrix. Similarly an integral fuzzy neutrosophic column vector is a $m \times 1$ integral fuzzy neutrosophic matrix.

2.8 Example [4]

$A = (0.1, 0.3, 1, 0, 0, 0.7, I, 0.002, 0.01, I, 1, 0.12)$ is a integral row vector or a 1×12, integral fuzzy neutrosophic matrix.

2.9 Example [4]

$B = (1, 0.2, 0.111, I, 0.32, 0.001, I, 0, 1)^T$ is an integral neutrosophic column vector or B is a 9×1 integral fuzzy neutrosophic matrix.
We would be using the concept of fuzzy neutrosophic column or row vector in our study.

2.10 Definition [4]

Let $\mathrm{P} = (\mathrm{p_{ij}})$ be a $\mathrm{m} \times \mathrm{n}$ integral fuzzy neutrosophic matrix and $\mathrm{Q} = (\mathrm{q_{ij}})$ be a $\mathrm{n} \times \mathrm{p}$ integral fuzzy neutrosophic matrix. The composition map P o Q is defined by R $(\mathrm{r_{ij}})$ which is a $\mathrm{m} \times \mathrm{p}$ matrix where $\left(\mathrm{r_{ij}} = \max_{\mathrm{k}} \min(\mathrm{p_{ik}}\ \mathrm{q_{kj}})\right)$ with the assumption $\max(\mathrm{p_{ij}}, \mathrm{I}) = \mathrm{I}$ and $\min(\mathrm{p_{ij}}, \mathrm{I}) = \mathrm{I}$ where pij $\epsilon(0,1)$. $\min(0, \mathrm{I}) = 0$ and $\max(1, \mathrm{I}) = 1$.

2.11 Example [4]

Let $p = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix}$, $Q = (0.1, I, 0)^T$ be two integral fuzzy neutrosophic matrices.

$$\mathrm{P \ o \ Q} = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix} \mathrm{o} \begin{bmatrix} 0.1 \\ I \\ 0 \end{bmatrix} =$$
$(I\ , I\ , 0.1)^{\mathrm{T}}$

3 Structure of the Maximal Solution Set

B.Y.Cao proposed the structure set of maximal and minimal solution for FRGP (fuzzy relation geometric programming) with $(\vee, \wedge)$ operator [optimal models & meth with fuzzy quantities 2010] [1-2]. Its useful and necessary to call back the following ideas

3.1 Definition

If $X(A,b) \neq \emptyset$ it can be completely determined by a unique maximum solution and a finite number of minimal solution. The maximum solution can be obtained by applying the following operation:-

$\widehat{x_j} = \wedge\{b_i | b_i < a_{ij}\}$ $(1 \leq i \leq m, 1 \leq j \leq n)$
Stipulate that set $\{\wedge\emptyset = 1\}$. If $\hat{x}_j = (\hat{x}_1, \hat{x}_2, \ldots_n, \hat{x})^T$ is a solution to $AoX = b$. then $\hat{x}$ must be the greatest solution.

3.2 An original notion to find maximal solution

The most important question:

What is the structure of the maximum element for any fuzzy neutrosophic relation equations in the interval $[0,1] \cup I$??

We know that

$\max\{0, I\} = I$ & $\max\{x, I\} = I \quad \forall x \in [0,1) \cup I$.

Depending upon the definition (3.1), the stipulation that $\{\wedge\emptyset = 1\}$ will be fixed.

Also by Sanchez (1976) we have

$$a\alpha b = \begin{cases} 1 & a \leq b \\ b & a > b \end{cases}$$

Don't forget that α is relative psedo complement of $a \; in \; b$. On the other hand, Florentin was deffind the neutrosophic lattice see ref. [4] p.235

So if we want to establish the maximum solution for any FNRE in the interval $[0,1] \cup I$, we must redefine $a\alpha b$ where $a\alpha b = a \wedge x \leq b$; $a, x, b \in [0,1] \cup I$.

Note that, all matrices in this work are an integral fuzzy neutrosophic matrices. It is obvious that a or b are either belonging to $[0,1]$ or equal to I , so we have that following status:

1- If $a \in [0,1]$ & $b \in I$, $a\alpha b = x = I$ where $a \neq 0$ therefore $a \in (0,1]$, here we must remember that $\min(I, x) = I \; \forall \; x \in (0,1] \cup I$.
2- If $a = I$ & $b \in [0,1]$, then $a\alpha b = x = 0$,here $\min(I, x) = I \; \forall \; x \in (0,1] \cup I$ also $\min(I, 0) = 0$.
3- At a&$b \in [0,1]$, the solution will back to the same case that stated by Sanchez ,i.e.
$$a\alpha b = \begin{cases} 1 & a \leq b \\ b & a > b \end{cases}$$
4- At $a = b = I$,this implies that $a\alpha b = 1$.

Note that , $a \wedge x \leq b \;\; \rightarrow \min(a, x) \leq b \;\; \rightarrow \min(I, x) \leq I \;\; \rightarrow x = 1$

Consequently :

$$\widehat{x_j} = a\alpha b = \begin{cases} 1 & a_{ij} \leq b_i \; or \; a_{ij} = b_{ij} = I \\ b_i & a_{ij} > b_i \\ 0 & a_{ij} = I \; and \; b_{ij} = [0,1] \\ I & b_i = I \; and \; a_{ij} = (0,1] \\ not\; comp. & a_{ij} = 0 \; and \; b_{ij} = I \end{cases}$$

3.3 Lemma

If $a_{ij} = 0$ and $b_i = I$ then $AoX = b$ is not compatible.

Proof

Let $a_{ij} = 0$, $b_i = I$

What is the value of $x_j \in [0,1] \cup I$ satisfying

$\vee_{ij}^{n} (a_{ij} \wedge x_j) = b_i \quad \forall \;\; 1 \leq i \leq m$?

We have

1- $a\alpha b = a_{ij} \wedge x_j \leq b_i$
2- $\min(0, x_j) = 0 \quad \forall \; x_j \in [0,1] \cup I$

So $a\alpha b = a_{ij} \wedge x_j = \min(0, x_j) = 0$ not equal nor less than to I

We know that the incomparability occurs only when $x \in FN \; and \; y \in [0,1]$ see ref. [4] p.233

$\therefore AoX = b$ is not compatible

Without loss of generality , suppose $b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$ when we rearranged the components of b in decreasing order we also adjusted A, x and $f(x)$ accordingly b.

Now, in fuzzy neutrosophic numbers, how can we classify numbers to rearrange them? For more details see ref. [5] page 245**.**

3.4 Example

Rearranged the following matrices in decreasing order

1) $b = \begin{bmatrix} 0.85 \\ I \\ 0.5 \\ I \end{bmatrix}$ 2) $b = \begin{bmatrix} I \\ I \\ 0.5 \\ 0.1 \end{bmatrix}$ 3) $b = \begin{bmatrix} I \\ 0.6 \\ I \\ 0.1 \end{bmatrix}$

Solution.

1) $b = \begin{bmatrix} I \\ I \\ 0.85 \\ 0.5 \end{bmatrix}$ 2) $b = \begin{bmatrix} I \\ I \\ 0.5 \\ 0.1 \end{bmatrix}$ 3) $b = \begin{bmatrix} I \\ I \\ 0.6 \\ 0.1 \end{bmatrix}$

4 Numerical Examples:-

Find the maximum solution for the following FNREGP problems:-

1)

$$minf(x) = 5x_1^{-.5}x_2^{-1.5}x_3^{2}x_4^{-2}x_5^{-1}$$

$$s.t. \begin{bmatrix} I & 0.8 & 0.6 & I & 0 \\ 0.8 & 0.7 & 0.8 & 1 & 0.8 \\ 0.6 & 0.9 & 0.8 & 0.9 & 0.5 \\ I & 0.2 & I & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.5 & 0.2 & 0.1 \\ I & 0.1 & 0.2 & 0.3 & I \end{bmatrix} o \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} I \\ I \\ I \\ .5 \\ .5 \\ .4 \end{bmatrix}$$

Solution :- The greatest solution is

$\hat{x}_1 = \wedge\{1, I, I, 0,1,0\} = 0$

$\hat{x}_2 = \wedge\{I, I, I, 1,1,1\} = I$

$\hat{x}_3 = \wedge\{I, I, I, 0,1,1\} = 0$

$\hat{x}_4 = \wedge\{1, I, I, .5,1,1\} = I$

$\hat{x}_5 = \wedge\{not\ comparable, \ldots \ldots .\}$

Therefore by lemma (3.3) the system $Aox = b$ is not comparable.

2)

$$\min f(x) = (1.5I\Lambda x_1^{.5})V(2I\Lambda x_2)V(.8\Lambda x_3^{-.5})V(4\Lambda x_4^{-1})$$

s.t. $Aox = b$ where

$$A = \begin{bmatrix} I & .2 & .85 & .9 \\ .8 & .2 & I & .1 \\ .9 & .1 & I & .6 \\ I & .8 & .1 & I \end{bmatrix}$$

$b = (I, .6, .5, I)$, $0 \leq x_i \leq 1$, $1 \leq i \leq 4$

$\hat{x}_1 = \wedge(1,1,.5,1) = 0.5$

$\hat{x}_2 = \wedge(I, 1,1, I) = I$

$\hat{x}_3 = \wedge(I, 0,0, I) = 0$

$\hat{x}_4 = \wedge(I, 1, .5,1) = I$

$\therefore\ \hat{x} = (0.5, I, 0, I)^T$

Conclusion

In this article, the basic notion for finding maximal solution in a geometric programming subject to a system of fuzzy neotrosophic relational equation with max-min composition was introduced. In 1976,Sanchez gave the formula of the maximal solution for fuzzy relation equation concept and describing in details its structure. Some numerical examples have shown that the proposed method is betimes step to enter in this kind of problems to search for minimal solutions which remains as unfathomable issue.

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Rough Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis

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Abstract. This paper presents rough netrosophic multi-attribute decision making based on grey relational analysis. While the concept of neutrosophic sets is a powerful logic to deal with indeterminate and inconsistent data, the theory of rough neutrosophic sets is also a powerful mathematical tool to deal with incompleteness. The rating of all alternatives is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Weight of each attribute is partially known to decision maker. We extend the neutrosophic grey relational analysis method to rough neutrosophic grey relational analysis method and apply it to multi-attribute decision making problem. Information entropy method is used to obtain the partially known attribute weights. Accumulated geometric operator is defined to transform rough neutrosophic number (neutrosophic pair) to single valued neutrosophic number. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal rough neutrosophic estimates reliability solution and the ideal rough neutrosophic estimates un-reliability solution. Then rough neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, a numerical example is provided to illustrate the applicability and efficiency of the proposed approach.

Keywords: Neutrosophic set, Rough Neutrosophic set, Single-valued neutrosophic set, Grey relational analysis, Information Entropy, Multi-attribute decision making.

Introduction

The notion of rough set theory was originally proposed by Pawlak [1, 2]. The concept of rough set theory [1, 2, 3, 4] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful tool for dealing with uncertainty or imprecision information. It has been successfully applied in the different fields such as artificial intelligence [5], pattern recognition [6, 7], medical diagnosis [8, 9, 10, 11], data mining [12, 13, 14], image processing [15], conflict analysis [16], decision support systems [17,18], intelligent control [19], etc. In recent years, the rough set theory has caught a great deal of attention and interest among the researchers. Various notions that combine the concept of rough sets [1], fuzzy sets [20], vague set [21], grey set [22, 23] intuitionistic fuzzy sets [24], neutrosophic sets [25] are developed such as rough fuzzy sets [26], fuzzy rough sets [27, 28, 29], generalized fuzzy rough sets [30, 31], vague rough set [32], rough grey set [33, 34, 35, 36] rough intuitionistic fuzzy sets [37], intuitionistic fuzzy rough sets [38], rough neutrosophic sets [39, 40]. However neutrosophic set [41, 42] is the generalization of fuzzy set, intuitionistic fuzzy set, grey set, and vague set. Among the hybrid concepts, the concept of rough neutrosophic sets [39, 40] is recently proposed and very interesting. Literature review reveals that only two studies on rough neutrosophic sets [39, 40] are done.

Neutrosophic sets and rough sets are two different concepts. Literature review reflects that both are capable of handing uncertainty and incomplete information. New hybrid intelligent structure called "rough neutrosophic sets" seems to be very interesting and applicable in realistic problems. It seems that the computational techniques based on any one of these structures alone will not always provide the best results but a fusion of two or more of them can often offer better results [40].

Decision making process evolves through crisp environment to the fuzzy and uncertain and hybrid environment. Its dynamics, adaptability, and flexibility continue to exist and reflect a high degree of survival value. Approximate reasoning, fuzziness, greyness, neutrosophics and dynamic readjustment characterize this process. The decision making paradigm evolved in modern society must be strategic, powerful and pragmatic rather than retarded. Realistic model cannot be constructed without genuine understanding of the most advanced decision making model evolved so far i.e. the human decision making process. In order to perform this, very new hybrid concept such as rough neutrosophic set must be introduced in decision making model.

Decision making that includes more than one measure of performance in the evaluation process is termed as multi-attribute decision making (MADM). Different methods of MADM are available in the literature. Several methods of MADM have been studied for crisp, fuzzy, intuitionistic fuzzy, grey and neutrosophic environment. Among these, the most popular MADM methods are Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang & Yoon [43], Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) proposed by Brans et al. [44], VIšekriterijumsko KOmpromisno Rangiranje (VIKOR) developed by Opricovic & Tzeng [45], ELimination Et Choix Traduisant la REalité (ELECTRE) studied by Roy [46], ELECTRE II proposed by Roy and Bertier [47], ELECTREE III proposed by(Roy [48], ELECTRE IV proposed by Roy and Hugonnard [49), Analytical Hierarchy Process(AHP) developed by Satty [50], fuzzy AHP developed by Buckley [51], Analytic Network Process (ANP) studied by Mikhailov [52], Fuzzy TOPSIS proposed by Chen [53], single valued neutrosophic multi criteria decision making studied by Ye [54, 55, 56], neutrosophic MADM studied by Biswas et al. [57], Entropy based grey relational analysis method for MADM studied by Biswas et al. [58]. A small number of applications of neutrosophic MADM are available in the literature. Mondal and Pramanik [59] used neutrosophic multicriteria decision making for teacher selection in higher education. Mondal and Pramanik [60] also developed model of school choice using neutrosophic MADM based on grey relational analysis. However, MADM in rough neutrosophic environment is yet to appear in the literature. In this paper, an attempt has been made to develop rough neutrosophic MADM based on grey relational analysis.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and rough neutrosophic sets. Section 3 is devoted to present rough neutrosophic multi-attribute decision-making based on grey relational analysis. Section 4 presents a numerical example of the proposed method. Finally, section 5 presents concluding remarks and direction of future research.

2 Mathematical Preliminaries

2.1 Definitions on neutrosophic Set

The concept of neutrosophy set is originated from the new branch of philosophy, namely, neutrosophy. Neutrosophy [25] gains very popularity because of its capability to deal with the origin, nature, and scope of neutralities, as well as their interactions with different conceptional spectra.

Definition2.1.1: Let E be a space of points (objects) with generic element in E denoted by y. Then a neutrosophic set $N1$ in E is characterized by a truth membership function T_{N1} , an indeterminacy membership function I_{N1} and a falsity membership function F_{N1}. The functions T_{N1} and F_{N1} are real standard or non-standard subsets of $]^-0,1^+[$ that is T_{N1}: $E \rightarrow]^-0,1^+[$; I_{N1}: $E \rightarrow]^-0,1^+[$; F_{N1}: $E \rightarrow]^-0,1^+[$.

It should be noted that there is no restriction on the sum of $T_{N1}(y)$, $I_{N1}(y)$, $F_{N1}(y)$ i.e.

$^-0 \le T_{N1}(y) + I_{N1}(y) + F_{N1}(y) \le 3^+$

Definition2.1.2: (complement) The complement of a neutrosophic set A is denoted by $N1^c$ and is defined by

$$T_{N1^c}(y) = \{1^+\} - T_{N1}(y)\,;\; I_{N1^c}(y) = \{1^+\} - I_{N1}(y)$$
$$F_{N1^c}(y) = \{1^+\} - F_{N1}(y)$$

Definition2.1.3: (Containment) A neutrosophic set $N1$ is contained in the other neutrosophic set $N2$, $N1 \subseteq N2$ if and only if the following result holds.

$$\inf T_{N1}(y) \le \inf T_{N2}(y),\ \sup T_{N1}(y) \le \sup T_{N2}(y)$$
$$\inf I_{N1}(y) \ge \inf I_{N2}(y),\ \sup I_{N1}(y) \ge \sup I_{N2}(y)$$
$$\inf F_{N1}(y) \ge \inf F_{N2}(y),\ \sup F_{N1}(y) \ge \sup F_{N2}(y)$$

for all y in E.

Definition2.1.4: (Single-valued neutrosophic set). Let E be a universal space of points (objects) with a generic element of E denoted by y.

A single valued neutrosophic set [61] S is characterized by a truth membership function $T_N(y)$, a falsity membership function $F_N(y)$ and indeterminacy membership function $I_N(y)$ with $T_N(y)$, $F_N(y)$, $I_N(y) \in [0,1]$ for all y in E.

When E is continuous, a SNVS S can be written as follows:

$$S = \int_y \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in E$$

and when E is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(y), F_S(y), I_S(y) \rangle / y, \forall y \in E$$

It should be observed that for a SVNS S,

$0 \le \sup T_S(y) + \sup F_S(y) + \sup I_S(y) \le 3,\ \forall y \in E$

Definition2.1.5: The complement of a single valued neutrosophic set S is denoted by S^c and is defined by

$$T_S^c(y) = F_S(y)\,;\; I_S^c(y) = 1 - I_S(y)\,;\; F_S^c(y) = T_S(y)$$

Definition2.1.6: A SVNS S_{N1} is contained in the other SVNS S_{N2} , denoted as $S_{N1} \subseteq S_{N2}$ iff, $T_{S_{N1}}(y) \le T_{S_{N2}}(y)$; $I_{S_{N1}}(y) \ge I_{S_{N2}}(y)$; $F_{S_{N1}}(y) \ge F_{S_{N2}}(y)$, $\forall y \in E$.

Definition2.1.7: Two single valued neutrosophic sets S_{N1} and S_{N2} are equal, i.e. S_{N1}= S_{N2}, iff, $S_{N1} \subseteq S_{N2}$ and $S_{N1} \supseteq S_{N2}$

Definition2.1.8: (Union) The union of two SVNSs S_{N1} and S_{N2} is a SVNS S_{N3} , written as $S_{N3} = S_{N1} \cup S_{N2}$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_{N1} and S_{N2} as follows:

$T_{S_{N3}}(y) = max(T_{S_{N1}}(y), T_{S_{N2}}(y))$;

$I_{S_{N3}}(y) = max(I_{S_{N1}}(y), I_{S_{N2}}(y))$;

$F_{S_{N3}}(y) = min(F_{S_{N1}}(y), F_{S_{N2}}(y))$ for all y in E.

Definition2.1.9: (Intersection) The intersection of two SVNSs $N1$ and $N2$ is a SVNS $N3$, written as $N3 = N1 \cap N2$. Its truth membership, indeterminacy membership and falsity membership functions are related to $N1$ an $N2$ as follows:

$T_{S_{N3}}(y) = \min(T_{S_{N1}}(y), T_{S_{N2}}(y))$;

$I_{S_{N3}}(y) = \max(I_{S_{N1}}(y), I_{S_{N2}}(y))$;

$F_{S_{N3}}(y) = \max(F_{S_{N1}}(y), F_{S_{N2}}(y))$, $\forall y \in E$.

Distance between two neutrosophic sets.

The general SVNS can be presented in the follow form

$S = \{(y/(T_S(y), I_S(y), F_S(y))) : y \in E\}$

Finite SVNSs can be represented as follows:

$$S = \begin{Bmatrix} (y_1/(T_S(y_1), I_S(y_1), F_S(y_1))), \cdots, \\ (y_m/(T_S(y_m), I_S(y_m), F_S(y_m))) \end{Bmatrix}, \forall y \in E \tag{1}$$

Definition 2.1.10: Let

$$S_{N1} = \begin{Bmatrix} (y_1/(T_{S_{N1}}(y_1), I_{S_{N1}}(y_1), F_{S_{N1}}(y_1))), \cdots, \\ (y_n/(T_{S_{N1}}(y_n), I_{S_{N1}}(y_n), F_{S_{N1}}(y_n))) \end{Bmatrix} \tag{2}$$

$$S_{N2} = \begin{Bmatrix} (x_1/(T_{S_{N2}}(y_1), I_{S_{N2}}(y_1), F_{S_{N2}}(y_1))), \cdots, \\ (x_n/(T_{S_{N2}}(y_n), I_{S_{N2}}(y_n), F_{S_{N2}}(y_n))) \end{Bmatrix} \tag{3}$$

be two single-valued neutrosophic sets, then the Hamming distance [57] between two SNVS $N1$and $N2$ is defined as follows:

$$d_S(S_{N1}, S_{N2}) = \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_{N1}}(y) - T_{S_{N2}}(y)| + \\ |I_{S_{N1}}(y) - I_{S_{N2}}(y)| + \\ |F_{S_{N1}}(y) - F_{S_{N2}}(y)| \end{matrix} \right\rangle \tag{4}$$

and normalized Hamming distance [58] between two SNVSs S_{N1} and S_{N2} is defined as follows:

$$^N d_S(S_{N1}, S_{N2}) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_{N1}}(y) - T_{S_{N2}}(y)| + \\ |I_{S_{N1}}(y) - I_{S_{N2}}(y)| + \\ |F_{S_{N1}}(y) - F_{S_{N2}}(y)| \end{matrix} \right\rangle \tag{5}$$

with the following properties

1. $0 \le d_S(S_{N1}, S_{N2}) \le 3n$ (6)
2. $0 \le {}^N d_S(S_{N1}, S_{N2}) \le 1$ (7)

2.2 Definitions on rough neutrosophic set

There exist two basic components in rough set theory, namely, crisp set and equivalence relation, which are the mathematical basis of RSs. The basic idea of rough set is based on the approximation of sets by a couple of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [39, 40] is the generalization of rough fuzzy set [26] and rough intuitionistic fuzzy set [37].

Definition2.2.1: Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle, \tag{8}$$

$$\overline{N}(P) = \left\langle \begin{matrix} < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle \tag{9}$$

Here, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R T_P(z)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R F_P(z)$,

$T_{\overline{N}(P)}(x) = \vee_z \in [x]_R T_P(z)$, $I_{\overline{N}(P)}(x) = \vee_z \in [x]_R T_P(z)$,

$F_{\overline{N}(P)}(x) = \vee_z \in [x]_R I_P(z)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ present the "max" and the "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ present the membership, indeterminacy and non-membership of z with respect to P. It is very easy to observe that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z. Therefore, the NS mapping $\underline{N}$, $\overline{N}$: $N(Z) \rightarrow N(Z)$ presents the lower and upper rough NS approximation operators. The pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [40] in (Z, R).

Based on the above definition, it is observed that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence clases of R if $\underline{N}(P) = \overline{N}(P)$

i.e $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$, $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$.

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets [40].

Definition2.2.2 If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (E, R) , the rough complement of $N(P)$ [40] is the rough neutrosophic set denoted by $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$,where $\underline{N}(P)^c, \overline{N}(P)^c$ represent the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$$\underline{N}(P)^c = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x) > / \\ , x \in Z \end{matrix} \right\rangle, \text{ ,and}$$

$$\overline{N}(P)^c = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x) > / \\ , x \in Z \end{matrix} \right\rangle \quad (10)$$

Definition2.2.3. If $N(P_1)$ *and* $N(P_2)$ are two rough neutrosophic sets of the neutrosophic sets respectively in Z, then Bromi et al. [40] defined the following definitions.

$$N(P_1) = N(P_2) \Leftrightarrow \underline{N}(P_1) = \underline{N}(P_2) \wedge \overline{N}(P_1) = \overline{N}(P_2)$$

$$N(P_1) \subseteq N(P_2) \Leftrightarrow \underline{N}(P_1) \subseteq \underline{N}(P_2) \wedge \overline{N}(P_1) \subseteq \overline{N}(P_2)$$

$$N(P_1) \cup N(P_2) = < \underline{N}(P_1) \cup \underline{N}(P_2),\ \overline{N}(P_1) \cup \overline{N}(P_2) >$$

$$N(P_1) \cap N(P_2) = < \underline{N}(P_1) \cap \underline{N}(P_2),\ \overline{N}(P_1) \cap \overline{N}(P_2) >$$

$$N(P_1) + N(P_2) = < \underline{N}(P_1) + \underline{N}(P_2),\ \overline{N}(P_1) + \overline{N}(P_2) >$$

$$N(P_1)\ .\ N(P_2) = < \underline{N}(P_1)\ .\ \underline{N}(P_2),\ \overline{N}(P_1)\ .\ \overline{N}(P_2) >$$

If N, M, L are rough neutrosophic sets in (Z, R), then the following proposition [40] are stated from definitions

Proposition i:

1. $\sim N(\sim N) = N$
2. $N \cup M = M \cup N, M \cup N = N \cup M$
3. $(L \cup M) \cup N = L \cup (M \cup N)$,
   $(L \cap M) \cap N = L \cap (M \cap N)$
4. $(L \cup M) \cap N = (L \cup M) \cap (L \cup N)$,
   $(L \cap M) \cup N = (L \cap M) \cup (L \cap N)$

Proposition ii:

De Morgan's Laws are satisfied for neutrosophic sets

1. $\sim (N(P_1) \cup N(P_2)) = (\sim N(P_1)) \cap (\sim N(P_2))$
2. $\sim (N(P_1) \cap N(P_2)) = (\sim N(P_1)) \cup (\sim N(P_2))$

Proposition iii:

If P_1 and P_2 are two neutrosophic sets in U such that $P_1 \subseteq P_2$ *, then* $N(P_1) \subseteq N(P_2)$

1. $N(P_1 \cap P_2) \subseteq N(P_2) \cap N(P_2)$
2. $N(P_1 \cup P_2) \supseteq N(P_2) \cup N(P_2)$

Proposition iv:

1. $\underline{N}(P) = \sim \overline{N}(\sim P)$
2. $\overline{N}(P) = \sim \underline{N}(\sim P)$
3. $\underline{N}(P) \subseteq \overline{N}(P)$

rough neutrosophic multi-attribute decision-making based on grey relational analysis

3. Rough neutrosophic multi-attribute decision-making based on grey relational analysis

We consider a multi-attribute decision making problem with m alternatives and n attributes. Let A_1, A_2, ..., A_m and C_1, C_2, ..., C_n represent the alternatives and attributes respectively.

The rating reflects the performance of the alternative A_i against the attribute C_j. For MADM weight vector $W = \{w_1, w_2,...,w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of the attribute C_j (j = 1, 2, ..., m) to the decision making process. The weights of the attributes are usually determined on subjective basis. They represent the opinion of a single decision maker or accumulate the opinions of a group of experts using group decision technique. The values associated with the alternatives for MADM problems are presented in the Table 1.

Table1: Rough neutrosophic decision matrix

$$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m \times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle \underline{d}_{11}, \overline{d}_{11} \right\rangle & \left\langle \underline{d}_{12}, \overline{d}_{12} \right\rangle & \ldots & \left\langle \underline{d}_{1n}, \overline{d}_{1n} \right\rangle \\ A_2 & \left\langle \underline{d}_{21}, \overline{d}_{21} \right\rangle & \left\langle \underline{d}_{22}, \overline{d}_{22} \right\rangle & \ldots & \left\langle \underline{d}_{2n}, \overline{d}_{2n} \right\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \left\langle \underline{d}_{m1}, \overline{d}_{m1} \right\rangle & \left\langle \underline{d}_{m2}, \overline{d}_{m2} \right\rangle & \ldots & \left\langle \underline{d}_{mn}, \overline{d}_{mn} \right\rangle \end{array} \quad (11)$$

Where $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is rough neutrosophic number according to the i-th alternative and the j-th attribute.

Grey relational analysis [GRA] [62] is a method of measuring degree of approximation among sequences according to the grey relational grade. Grey system theory deals with primarily on multi-input, incomplete, or uncertain information. GRA is suitable for solving problems with complicated relationships between multiple factors and variables. The theories of grey relationalanalysis have already caught much attention and interest among the researchers [63, 64]. In educational field, Pramanik and

Mukhopadhyaya [65] studied grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Rough neutrosophic multi-attribute decision-making based on grey relational analysis is presented by the following steps**.**

Step1: Determination the most important criteria.

Many attributes may be involved in decision making problems. However, all attributes are not equally important. So it is important to select the proper criteria for decision making situation. The most important criteria may be selected based on experts' opinions.

Step2: Data pre-processing

Considering a multiple attribute decision making problem having m alternatives and n attributes, the general form of decision matrix can be presented as shown in Table-1. It may be mentioned here that the original GRA method can effectively deal mainly with quantitative attributes. There exists some complexity in the case of qualitative attributes. In the case of a qualitative attribute (i.e. quantitative value is not available), an assessment value is taken as rough neutrosophic number.

Step3: Construction of the decision matrix with rough neutrosophic form

For multi-attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed to be rough neutrosophic sets. It can be represented with the following form.

$$A_i=\left[\begin{array}{l} C_1\Big/\left\langle \underline{N}_1\left(\underline{T}_{i1}\,\underline{I}_{i1}\,\underline{F}_{i1}\right),\overline{N}_1\left(\overline{T}_{i1}\,\overline{I}_{i1}\,\overline{F}_{i1}\right)\right\rangle, \\ C_2\Big/\left\langle \underline{N}_2\left(\underline{T}_{i2}\,\underline{I}_{i2}\,\underline{F}_{i2}\right),\overline{N}_2\left(\overline{T}_{i2}\,\overline{I}_{i2}\,\overline{F}_{i2}\right)\right\rangle,\cdots, \\ C_n\Big/\left\langle \underline{N}_n\left(\underline{T}_{in}\,\underline{I}_{in}\,\underline{F}_{in}\right),\overline{N}_n\left(\overline{T}_{in}\,\overline{I}_{in}\,\overline{F}_{in}\right)\right\rangle : C_j\in C \end{array}\right]$$

$$=\left[C_j\Big/\left\langle \underline{N}_j\left(\underline{T}_{ij}\,\underline{I}_{ij}\,\underline{F}_{ij}\right),\overline{N}_j\left(\overline{T}_{ij}\,\overline{I}_{ij}\,\overline{F}_{ij}\right)\right\rangle : C_j\in C\right] \textit{ for } \quad (12)$$

$$j=1,2,\cdots,n$$

Here $\overline{N}$ and $\underline{N}$ are neutrosophic sets with

$\left\langle \overline{T}_{ij},\overline{I}_{ij},\overline{F}_{ij}\right\rangle$ *and* $\left\langle \underline{T}_{ij},\underline{I}_{ij},\underline{F}_{ij}\right\rangle$

are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i satisfying the attribute C_j, respectively where

$$0\le \underline{T}_{ij},\overline{T}_{ij}\le 1,\qquad 0\le \underline{I}_{ij},\overline{I}_{ij}\le 1,\qquad 0\le \underline{F}_{ij},\overline{F}_{ij}\le 1$$

$$0\le \underline{T}_{ij}+\underline{I}_{ij}+\underline{F}_{ij}\le 3,\ 0\le \overline{T}_{ij}+\overline{I}_{ij}+\overline{F}_{ij}\le 3$$

The rough neutrosophic decision matrix (see Table 2) can be presented in the following form:

Table 2. Rough neutrosophic decision matrix

$$d_{\tilde{N}}=\left\langle \underline{N}_{ij}(F),\overline{N}_{ij}(F)\right\rangle_{m\times n}=$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \dots & C_n \\ \hline A_1 & \left\langle \underline{N}_{11},\overline{N}_{11}\right\rangle & \left\langle \underline{N}_{12},\overline{N}_{12}\right\rangle & \dots & \left\langle \underline{N}_{1n},\overline{N}_{1n}\right\rangle \\ A_2 & \left\langle \underline{N}_{21},\overline{N}_{21}\right\rangle & \left\langle \underline{N}_{22},\overline{N}_{22}\right\rangle & \dots & \left\langle \underline{N}_{2n},\overline{N}_{2n}\right\rangle \\ . & \dots & \dots & \dots & \dots \\ . & \dots & \dots & \dots & \dots \\ A_m & \left\langle \underline{N}_{n1},\overline{N}_{n1}\right\rangle & \left\langle \underline{N}_{n2},\overline{N}_{n2}\right\rangle & \dots & \left\langle \underline{N}_{mn},\overline{N}_{mn}\right\rangle \end{array} \quad (13)$$

Where $\underline{N}_{ij}$ *and* $\overline{N}_{ij}$ are lower and upper approximations of the neutrosophic set *P*.

Step4: Determination of the accumulated geometric operator.

Let us consider a rough neutrosophic set as $\left\langle \underline{N}_{ij}\left(\underline{T}_{ij},\underline{I}_{ij},\underline{F}_{ij}\right),\overline{N}_{ij}\left(\overline{T}_{ij},\overline{I}_{ij},\overline{F}_{ij}\right)\right\rangle$

We transform the rough neutrosophic number to SVNSs by the following operator. The Accumulated Geometric Operator (AGO) is defined in the following form:

$$N_{ij}\left\langle T_{ij},I_{ij},F_{ij}\right\rangle=$$

$$N_{ij}\left\langle \left(\underline{T}_{ij}.\overline{T}_{ij}\right)^{0.5},\left(\underline{I}_{ij}.\overline{I}_{ij}\right)^{0.5},\left(\underline{F}_{ij}.\overline{F}_{ij}\right)^{0.5},\right\rangle \quad (14)$$

The decision matrix (see Table 3) is transformed in the form of SVNSs as follows:

Table 3. Transformed decision matrix in the form SVNS

$$d_S=\left\langle T_{ij},I_{ij},F_{ij}\right\rangle_{m\times n}=$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \dots & C_n \\ \hline A_1 & \left\langle T_{11},I_{11},F_{11}\right\rangle & \left\langle T_{12},I_{12},F_{12}\right\rangle & \dots & \left\langle T_{1n},I_{1n},F_{1n}\right\rangle \\ A_2 & \left\langle T_{21},I_{21},F_{21}\right\rangle & \left\langle T_{22},I_{22},F_{22}\right\rangle & \dots & \left\langle T_{2n},I_{2n},F_{2n}\right\rangle \\ . & \dots & \dots & \dots & \dots \\ . & \dots & \dots & \dots & \dots \\ A_m & \left\langle T_{m1},I_{m1},F_{m1}\right\rangle & \left\langle T_{m2},I_{m2},F_{m2}\right\rangle & \dots & \left\langle T_{mn},I_{mn},F_{mn}\right\rangle \end{array}$$

(15)

Step5: Determination of the weights of criteria.

During decision-making process, decision makers may often encounter with partially known or unknown attribute weights. So, it is crucial to determine attribute weight for proper decision making. Many methods are available in the literatre to determine the unknown attribute weight such as maximizing deviation method proposed by Wu and Chen [66], entropy method proposed by Wei and Tang [67], and Xu and Hui [68]), optimization method proposed by Wang and Zhang [69], Majumder and Samanta [70].

Biswas et al. [57] used entropy method [70] for single valed neutrosophic MADM.

In this paper we use an entropy method for determining attribute weight. According to Majumder and Samanta [70], the entropy measure of a SVNS

$S_{N1}=\left\langle T_{S_{N1}}(x_i), I_{S_{N1}}(x_i), F_{S_{N1}}(x_i)\right\rangle$

$$En_i(S_{N1}) = 1-\frac{1}{n}\sum_{i=1}^{m}(T_{S_{N1}}(x_i)+F_{S_{N1}}(x_i))\left|I_{S_{N1}}(x_i)-I^c{}_{S_{N1}}(x_i)\right| \quad (16)$$

which has the following properties:

1. $En_i(S_{N1})=0 \Rightarrow S_{N1}$ *is a crisp set and* $I_{S_{N1}}(x_i)=0\ \forall x\in E.$
2. $En_i(S_{N1})=1 \Rightarrow \left\langle T_{S_{N1}}(x_1), I_{S_{N1}}(x_1), F_{S_{N1}}(x_1)\right\rangle = \left\langle 0.5, 0.5, 0.5\right\rangle \forall x\in E.$
3. $En_i(S_{N1})\geq En_i(S_{N2}) \Rightarrow$ $(T_{S_{N1}}(x_1)+F_{S_{N1}}(x_1)\leq (T_{S_{N2}}(x_1)+F_{S_{N2}}(x_1)$ *and* $\left|I_{S_{N1}}(x_1)-I^c{}_{S_{N1}}(x_1)\right| \leq \left|I_{S_{N2}}(x_1)-I^c{}_{S_{N2}}(x_1)\right|$
4. $En_i(S_{N1})=En_i(S_{N1^c})\ \forall x\in E.$

In order to obtain the entropy value En_j of the j-th attribute C_j (j = 1, 2,…, n), equation (16) can be written as follows:

$$En_j=1-\frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i)+F_{ij}(x_i))\left|I_{ij}(x_i)-I_{ij}^C(x_i)\right|$$

For i = 1, 2, …, n; j = 1, 2, …, m (17)

It is observed that $E_j \in [0,1]$. Due to Hwang and Yoon [71], and Wang and Zhang [69] the entropy weight of the j-th attibute C_j is presented as:

$$W_j=\frac{1-En_j}{\sum_{j=1}^{n}\left(1-En_j\right)} \quad (18)$$

We have weight vector W= (w_1, w_2,…,w_n)^T of attributes C_j (j = 1, 2, …, n) with $w_j\geq 0$ and $\sum_{i=1}^{n} w_j=1$

Step6: Determination of the ideal rough neutrosophic estimates reliability solution (IRNERS) and the ideal rough neutrosophic estimates un-reliability solution (IRNEURS) for rough neutrosophic decision matrix.

Based on the concept of the neutrosophic cube [72], maximum reliability occurs when the indeterminacy membership grade and the degree of falsity membership grade reach minimum simultaneously. Therefore, the ideal neutrosophic estimates reliability solution (INERS) $R_S^+=\left[r_{S_1}^+, r_{S_2}^+, \cdots, r_{S_n}^+\right]$ is defined as the solution in which every component $r_{S_j}^+=\left\langle T_j^+, I_j^+, F_j^+\right\rangle$ is defined as follows: $T_j^+=\max_i\{T_{ij}\}$, $I_j^+=\min_i\{I_{ij}\}$ and $F_j^+=\min_i\{F_{ij}\}$ in the neutrosophic decision matrix $D_S=\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see the Table 1) for i = 1, 2, …, n and j = 1, 2, …, m.

Based on the concept of the neutrosophic cube [72], maximum un-reliability occurs when the indeterminacy membership grade and the degree of falsity membership grade reach maximum simultaneously. So, the ideal neutrosophic estimates unreliability solution (INEURS) $R_S^-=\left[r_{S_1}^-, r_{S_2}^-, \cdots, r_{S_n}^-\right]$ is the solution in which every component $r_{S_j}^-=\left\langle T_j^-, I_j^-, F_j^-\right\rangle$ is defined as follows: $T_j^-=\max_i\{T_{ij}\}$, $I_j^-=\min_i\{I_{ij}\}$ *and* $F_j^-=\min_i\{F_{ij}\}$ in the neutrosophic decision matrix $D_S=\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see the Table 1)for i = 1, 2, …, n and j = 1, 2, …, m.

For the rough neutrosophic decision making matrix $D = \left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle_{m\times n}$ (see Table 1), T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i of A satisfying the attribute C_j of C.

Step7: Calculation of the rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS.

Rough grey relational coefficient of each alternative from IRNERS is:

$$G_{ij}^+=\frac{\min_i\min_j\Delta_{ij}^+ + \rho\max_i\max_j\Delta_{ij}^+}{\Delta_{ij}^+ + \rho\max_i\max_j\Delta_{ij}^+}, \text{ where}$$

$\Delta_{ij}^+=d\left(q_{S_j}^+, q_{S_{ij}}\right)$, i=1, 2, …,m and j=1, 2, ….,n (19)

Rough grey relational coefficient of each alternative from IRNEURS is:

$$G_{ij}^-=\frac{\min_i\min_j\Delta_{ij}^- + \rho\max_i\max_j\Delta_{ij}^-}{\Delta_{ij}^- + \rho\max_i\max_j\Delta_{ij}^-}, \text{ where}$$

$\Delta_{ij}^-=d\left(q_{S_{ij}}, q_{S_{ij}}^-\right)$, i=1, 2, …,m and j=1, 2, ….,n (20)

$\rho\in[0,1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho=1$, the comparison environment is unchanged; when $\rho=0$, the comparison environment disappears. Smaller value of distinguishing coefficient reflests the large range of grey relational coefficient. Generally, $\rho=0.5$ is fixed for decision making .

Step8: Calculation of the rough neutrosophic grey relational coefficient.

Rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS are defined respectively as follows:

$$G_i^+ = \sum_{j=1}^{n} w_j G_{ij}^+ \quad \text{for } i=1, 2, \ldots, m \tag{21}$$

$$G_i^- = \sum_{j=1}^{n} w_j G_{ij}^- \quad \text{for } i=1, 2, \ldots, m \tag{22}$$

Step9: Calculation of the rough neutrosophic relative relational degree.

Rough neutrosophic relative relational degree of each alternative from Indeterminacy Trthfullness Falsity Positive Ideal Soltion (ITFPIS) is defined as follows:

$$\Re_i = \frac{G_i^+}{G_i^- + G_i^+}, \text{ for } i=1, 2, \ldots, m \tag{23}$$

Step 10: Ranking the alternatives.

The ranking order of all alternatives can be determined according to the decreasing order of the rough relative relational degree. The highest value of $\Re_i$ indicates the best alternative.

4 Numerical example

In this section, rough neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider a decision-making problem stated as follows. Suppose there is a conscious guardian, who wants to admit his/her child to a suitable school for proper education. There are three schools (possible alternatives) to admit his/her child: (1) A_1 is a Christian Missionary School; (2) A_2 is a Basic English Medium School; (3) A_3 is a Bengali Medium Kindergarten. The guardian must take a decision based on the following four criteria: (1) C_1 is the distance and transport; (2) C_2 is the cost; (3) C_3 is stuff and curriculum; and (4) C_4 is the administration and other facilites. We obtain the following rough neutrosophic decision matrix (see the Table 4) based on the experts' assessment:

Table 4. Decision matrix with rough neutrosophic number

$d_S = \langle \underline{N}_{ij}(P), \overline{N}_{ij}(P) \rangle_{3\times 4} =$

	C_1	C_2	C_3	C_4
A_1	$\langle(.6,.3,.4),(.8,.2,.2)\rangle$	$\langle(.6,.4,.3),(.8,.2,.2)\rangle$	$\langle(.6,.4,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.4),(.9,.2,.2)\rangle$
A_2	$\langle(.7,.4,.3),(.8,.1,.2)\rangle$	$\langle(.6,.3,.3),(.8,.1,.1)\rangle$	$\langle(.7,.2,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.4),(.8,.2,.2)\rangle$
A_3	$\langle(.7,.2,.3),(.8,.2,.1)\rangle$	$\langle(.7,.4,.3),(.8,.2,.2)\rangle$	$\langle(.7,.2,.2),(.9,.2,.1)\rangle$	$\langle(.8,.3,.2),(.9,.1,.1)\rangle$

(23)

Step2: Determination of the decision matrix in the form SVNS

Using accumulated geometric operator (AGO) from equation (13) we have the decision matrix in SVNS form is presented as follows:

	C_1	C_2	C_3	C_4
A_1	$\langle .6928, .2449, .2828 \rangle$	$\langle .6928, .2828, .2449 \rangle$	$\langle .6928, .2828, .1732 \rangle$	$\langle .7937, .2828, .2828 \rangle$
A_2	$\langle .7483, .2000, .2449 \rangle$	$\langle .6928, .1732, .1732 \rangle$	$\langle .7483, .2000, .1414 \rangle$	$\langle .7483, .2449, .2000 \rangle$
A_3	$\langle .7483, .2000, .1732 \rangle$	$\langle .7483, .2828, .2449 \rangle$	$\langle .7937, .2000, .1414 \rangle$	$\langle .8485, .1732, .1414 \rangle$

(24)

Step3: Determination of the weights of attribute

Entropy value En_j of the j-th (j = 1, 2, 3) attributes can be determined from the decision matrix d_S (15) and equation (17) as: En_1= 0.4512, En_2 = 0.5318, En_3 = 0.5096, En_4= 0.4672.

Then the corresponding entropy weights w_1, w_2, w_3, w_4 of all attributes according to equation (17) are obtained by w_1 = 0.2700, w_2 =0.2279, w_3 =0.2402, w_4 =0.2619 such that $\sum_{j=1}^{n} w_j = 1$

Step4: Determination of the ideal rough neutrosophic estimates reliability solution (IRNERS):

$$Q_S^+ = \langle q_{S1}^+, q_{S2}^+, q_{S3}^+, q_{S4}^+ \rangle =$$

$$\begin{bmatrix} \langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\} \rangle, \\ \langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\} \rangle, \\ \langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\} \rangle, \\ \langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\} \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.7483, 0.2000, 0.1732 \rangle, \langle 0.7483, 0.1732, 0.1732 \rangle, \\ \langle 0.7937, 0.2000, 0.1414 \rangle, \langle 0.8485, 0.1732, 0.1414 \rangle \end{bmatrix}$$

Step5: Determination of the ideal rough neutrosophic estimates un-reliability solution (IRNEURS):

$$Q_S^- = \langle q_{S1}^-, q_{S2}^-, q_{S3}^-, q_{S4}^- \rangle =$$

$$\begin{bmatrix} \langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\} \rangle, \\ \langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\} \rangle, \\ \langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\} \rangle, \\ \langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\} \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.6928, 0.2449, 0.2828 \rangle, \langle 0.6928, 0.2828, 0.2449 \rangle, \\ \langle 0.6928, 0.2828, 0.1732 \rangle, \langle 0.7483, 0.2828, 0.2828 \rangle \end{bmatrix}$$

Step 6: Calculation of the rough neutrosophic grey relational coefficient of each alternative from IRNERS and IRNEURS.

By using Equation (19) the rough neutrosophic grey relational coefficient of each alternative from IRNERS can be obtained

$$\left[G_{ij}^{+}\right]_{3\times 4}=\begin{bmatrix} 0.3333 & 0.6207 & 0.6341 & 0.5544 \\ 0.7645 & 0.8075 & 0.8368 & 0.6305 \\ 1.0000 & 0.6399 & 1.0000 & 1.0000 \end{bmatrix} \quad (25)$$

Similarly, from Equation (20) the rough neutrosophic grey relational coefficient of each alternative from IRNEURS is

$$\left[G_{ij}^{-}\right]_{3\times 4}=\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 0.5403 \\ 0.5948 & 0.4752 & 0.5314 & 0.5266 \\ 0.4755 & 0.6812 & 0.4690 & 0.3333 \end{bmatrix} \quad (26)$$

Step7: Determine the degree of rough neutrosophic grey relational co-efficient of each alternative from INERS and IRNEURS. The required rough neutrosophic grey relational co-efficient corresponding to IRNERS is obtained by using equations (20) as:

$$G_1^{+}=0.5290,\ G_2^{+}=0.7566,\ G_3^{+}=0.9179 \quad (27)$$

and corresponding to IRNEURS is obtained with the help of equation (21) as:

$$G_1^{-}=0.8796,\ G_2^{-}=0.5345,\ G_3^{-}=0.4836 \quad (28)$$

Step8: Thus rough neutrosophic relative degree of each alternative from IRNERS can be obtained with the help of equation (22) as:

$$\Re_1=0.3756,\ \Re_2=0.5860,\ \Re_3=0.6549 \quad (29)$$

Step9: The ranking order of all alternatives can be determined according to the value of rough neutrosophic relational degree i.e. $\Re_3 > \Re_2 > \Re_1$. It is seen that the highest value of rough neutrosophic relational degree is R_3 therefore A_3 (Bengali Medium Kindergarten) is the best alternative (school) to admit the child.

Conclusion

In this paper, we introduce rough neutrosophic multi-attribute decision-making based on modified GRA. The concept of rough set, netrosophic set and grey system theory are fused to conduct the study first time. We define the Accumulated Geometric Operator (AGO) to transform rough neutrosophic matrix to SVNS. Here all the attribute weights information are partially known. Entropy based modified GRA analysis method is introduced to solve this MADM problem. Rough neutrosophic grey relation coefficient is proposed for solving multiple attribute decision-making problems. Finally, an illustrative example is provided to show the effectiveness and applicability of the proposed approach.

However, we hope that the concept presented here will open new approach of research in current rough neutrosophic decision-making field. The main thrsts of the paper will be in the field of practical decision-making, pattern recognition, medical diagnosis and clustering analysis.

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Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology

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Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [30, 31, 32] and Salama et al. in [4-29]. In Geographical information systems (GIS) there is a need to model spatial regions with indeterminate boundary and under indeterminacy. In this paper the structure of some classes of neutrosophic crisp nearly open sets are investigated and some applications are given. Finally we generalize the crisp topological and intuitioistic studies to the notion of neutrosophic crisp set. Possible applications to GIS topological rules are touched upon.

Keywords: Neutrosophic Crisp Set; Neutrosophic Crisp Topology; Neutrosophic Crisp Open Set; Neutrosophic Crisp Nearly Open Set; Neutrosophic GIS Topology.

1 Introduction

The fundamental concepts of neutrosophic set, introduced by Smarandache [30, 31, 32] and Salama et al. in [4-29]., provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 20, 21, 22, 23, 34] such as a neutrosophic set theory. In this paper the structure of some classes of neutrosophic crisp sets are investigated, and some applications are given. Finally we generalize the crisp topological and intuitioistic studies to the notion of neutrosophic crisp set.

2 2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [30, 31, 32] and Salama et al. [4-29]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^{-}0, 1^{+}\right[$ is non-standard unit interval. Salama et al. [9, 10, 13, 14, 16, 17] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following. Salama extended the concepts of topological space and intuitionistic topological space to the case of neutrosophic crisp sets.

Definition1. 2 [13]

A neutrosophic crisp topology (NCT for short) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms

i) $\phi_{N,} X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma \quad \forall \ \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is called a neutrosophic crisp topological space ($NCTS$ for short) in X. The elements in Γ are called neutrosophic crisp open sets (NCOSs for short) in X. A neutrosophic crisp set F is closed if and only if its complement F^C is an open neutrosophic crisp set.

Let (X, Γ) be a NCTS (identified with its class of neutrosophic crisp open sets), and NCint and NCcl denote neutrosophic interior crisp set and neutrosophic crisp closure with respect to neutrosophic crisp topology

3 Nearly Neutrosophic Crisp Open Sets

Definition3. 1

Let (X,Γ) be a NCTS and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X, then A is called

Neutrosophic crisp $\alpha-$open set iff $A\subseteq NC\operatorname{int}(NCcl(NC\operatorname{int}(A)))$,

Neutrosophic crisp $\beta-$open set iff $A\subseteq NCcl(NC\operatorname{int}(A))$.

Neutrosophic crisp semi-open set iff $A\subseteq NC\operatorname{int}(NCcl(A))$.

We shall denote the class of all neutrosophic crisp $\alpha-$ open sets $NC\Gamma^{\alpha}$, the calls all neutrosophic crisp $\beta-$ open sets $NC\Gamma^{\beta}$, and the class of all neutrosophic crisp semi-open sets $NC\Gamma^{s}$.

Remark 3.1

A class consisting of exactly all a neutrosophic crisp $\alpha-$ structure (resp. NC $\beta-$ structure). Evidently $NC\Gamma\subseteq NC\Gamma^{\alpha}\subseteq NC\Gamma^{\beta}$.

We notice that every non- empty neutrosophic crisp $\beta-$open has NC $\alpha-$nonempty interior. If all neutrosophic crisp sets the following $\{B_i\}_{i\in I}$ are NC $\beta-$ open sets, then $\{\bigcup_{i\in I} B_i\}_{i\in I}\subset NCcl(NC\operatorname{int}(B_i))\subset NCcl(NC\operatorname{int}(B_i))$, that is A NC $\beta-$structure is a neutrosophic closed with respect to arbitrary neutrosophic crisp unions.

We shall now characterize $NC\Gamma^{\alpha}$ in terms $NC\Gamma^{\beta}$

Theorem 3.1

Let (X,Γ) be a NCTS. $NC\Gamma^{\alpha}$ Consists of exactly those neutrosophic crisp set A for which $A\cap B\in NC\Gamma^{\beta}$ for $B\in NC\Gamma^{\beta}$

Proof

Let $A\in NC\Gamma^{\alpha}$, $B\in NC\Gamma^{\beta}$, $p\in A\cap B$ and U be a neutrosophic crisp neighbourhood (for short NCnbd)of p. Clearly $U\cap NC\operatorname{int}(NCcl(NC\operatorname{int}(A)))$, too is a neutrosophic crisp open neighbourhood of p, so $V=(U\cap NC\operatorname{int}(NCcl(NC\operatorname{int}(A))))\cap NC\operatorname{int}(B)$ is non-empty. Since $V\subset NCcl(NC\operatorname{int}(A))$ this implies $(U\cap NC\operatorname{int}(A)\cap NC\operatorname{int}(B))=V\cap NC\operatorname{int}(A)=\phi_N$. It follows that $A\cap B\subset NCcl(Nc\operatorname{int}(A)\cap NC\operatorname{int}(B))=NCcl(NC\operatorname{int}(A\cap B))$ i.e. $A\cap B\in NC\Gamma^{\beta}$. Conversely, let $A\cap B\in NC\Gamma^{\beta}$ for all $B\in NC\Gamma^{\beta}$ then in particular $A\in NC\Gamma^{\beta}$. Assume that $p\in A\cap(NC\operatorname{int}(NCcl(A)\cap NC\operatorname{int}(A)))^{c}$. Then $p\in NCcl(B)$, where $(NCcl(NC\operatorname{int}(A)))^{c}$. Clearly $\{p\}\cup B\in NC\Gamma^{\beta}$, and consequently $A\cap\{\{p\}\cup B\}\in NC\Gamma^{\beta}$. But $A\cap\{\{p\}\cup B\}=\{p\}$. Hence $\{p\}$ is a neutrosophic crisp open. As $p\in(NCcl(NC\operatorname{int}(A))$ this implies $p\in NC\operatorname{int}(NCcl(NC\operatorname{int}(A)))$, contrary to assumption. Thus $p\in A$ implies $p\in NCcl(NC\operatorname{int}(A))$ and $A\in NC\Gamma^{\alpha}$. This completes the proof. Thus we have found that $NC\Gamma^{\alpha}$ is complete determined by $NC\Gamma^{\beta}$ i.e. all neutrosophic crisp topologies with the same NC β -structure also determined the same NC α -structure, explicitly given Theorem 3.1.

We shall that conversely all neutrosophic crisp topologies with the same NC α -structure, so that $NC\Gamma^{\beta}$ is completely determined by $NC\Gamma^{\alpha}$.

Theorem 3.2

Every neutrosophic crisp NC α -structure is a neutrosophic crisp topology.

Proof

$NC\Gamma^{\beta}$ Contains the neutrosophic crisp empty set and is closed with respect to arbitrary unions. A standard result gives the class of those neutrosophic crisp sets A for which $A\cap B\in NC\Gamma^{\beta}$ for all $B\in NC\Gamma^{\beta}$ constitutes a neutrosophic crisp topology, hence the theorem. Hence forth we shall also use the term NC α -topology for NC α -structure two neutrosophic crisp topologies deterring the same NC α -structure shall be called NC α -equivalent , and the equivalence classes shall be called NC α -classes.

We may now characterize $NC\Gamma^{\beta}$ in terms of $NC\Gamma^{\alpha}$ in the following way.

Proposition 3.1

Let (X,Γ) be a NCTS. Then $NC\Gamma^{\beta}=NC\Gamma^{\alpha\beta}$ and hence NC α -equivalent topologies determine the same NC β -structure.

Proof

Let NC $\alpha-cl$ and NC $\alpha-\operatorname{int}$ denote neutrosophic closure and Neutrosophic crisp interior with respect to

$NC\Gamma^{\alpha}$. If $p \in B \in NC\Gamma^{\beta}$ and $p \in B \in NC\Gamma^{\alpha}$, then $(NC\,\text{int}(NCcl(NC\,\text{int}(A))) \cap NC\,\text{int}(B)) \neq \phi_N$. Since $NC\,\text{int}(NCcl(NC\,\text{int}(A)))$ is a crisp neutrosophic neighbourhood of point p. So certainly $NC\,\text{int}(B)$ meets $NCcl(NC\,\text{int}(A))$ and therefore (bing neutrosophic open) meets $NC\,\text{int}(A)$, proving $A \cap NC\,\text{int}(B) \neq \phi_N$ this means $B \subset NC\alpha cl(NC\,\text{int}(B))$ i.e. $B \in NC\Gamma^{\alpha\beta}$ on the other hand let $A \in NC\Gamma^{\alpha\beta}$, $p \in A$ and $p \in V \in NC\Gamma$. As $V \in NC\Gamma^{\alpha}$ and $p \in NCcl(NC\,\text{int}(A))$ we have $V \cap NC\,\text{int}(A) \neq \phi_N$ and there exist a nutrosophic crisp set $W \in \Gamma$ such that $W \subset V \cap NC\alpha\,\text{int}(A) \subset A$.
In other words $V \cap NC\,\text{int}(A) \neq \phi_N$ and $p \in NCcl(NC\,\text{int}(A))$. Thus we have verified $NC\Gamma^{\alpha\beta} \subset NC\Gamma^{\alpha}$, and the proof is complete combining Theorem 1 and Proposition 1. We get $NC\Gamma^{\alpha\alpha} = NC\Gamma^{\alpha}$.

Corollary 3.2

A neutrosophic crisp topology $NC\Gamma$ a $NC\alpha-$topology iff $NC\Gamma = NC\Gamma^{\alpha}$. Thus an $NC\alpha-$ topology belongs to the $NC\alpha-$class if all its determining a Neutrosophic crisp topologies, and is the finest topology of finest neutrosophic topology of this class. Evidently $NC\Gamma^{\beta}$ is a neutrosophic crisp topology iff $NC\Gamma^{\alpha} = NC\Gamma^{\beta}$. In this case $NC\Gamma^{\beta\beta} = NC\Gamma^{\alpha\beta} = NC\Gamma^{\beta}$.

Corollary 3.3

$NC\beta-$Structure B is a neutrosophic crisp topology, then $B = B^{\alpha} = B^{\beta}$.

We proceed to give some results an the neutrosophic structure of neutrosophic crisp $NC\alpha-$topology

Proposition 3.4

The $NC\alpha-$open with respect to a given neutrosophic crisp topology are exactly those sets which may be written as a difference between a neutrosophic crisp open set and neutrosophic crisp nowhere dense set

If $A \in NC\Gamma^{\alpha}$ we have $A = (NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap (NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap A^C)^C$, where $(NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap A^C)$ clearly is neutrosophic crisp nowhere dense set , we easily see that $B \subset NCcl(NC\,\text{int}(A))$ and consequently $A \subset B \subset NC\,\text{int}(NCcl(NC\,\text{int}(A))$ so the proof is complete.

Corollary 3.4

A neutrosophic crisp topology is a $NC\alpha-$topology iff all neutrosophic crisp nowhere dense sets are neutrosophic crisp closed.

For a neutrosophic crisp $NC\alpha-$topology may be characterized as neutrosophic crisp topology where the difference between neutrosophic crisp open and neutrosophic crisp nowhere dense set is again a neutrosophic crisp open, and this evidently is equivalent to the condition stated.

Proposition 3.5

Neutrosophic crisp topologies which are $NC\alpha-$ equivalent determine the same class of neutrosophic crisp nowhere dense sets.

Definition 3.2

We recall a neutrosophic crisp topology a neutrosophic crisp extremely disconnected if the neutrosophic crisp closure of every neutrosophic crisp open set is a neutrosophic crisp open .

Proposition 3.6

If $NC\alpha-$Structure B is a neutrosophic crisp topology, all a neutrosophic crisp topologies Γ for which $\Gamma^{\beta} = B$ are neutrosophic crisp extremely disconnected.

In particular: Either all or none of the neutrosophic crisp topologies of a $NC\alpha-$class are extremely disconnected.

Proof

Let $\Gamma^{\beta} = B$ and suppose there is a $A \in \Gamma$ such that $NCcl(A) \notin \Gamma$ Let $p \in NCcl(A) \cap (NC\,\text{int}(NCcl(A))^C$ with $B = \{p\} \cup NC\,\text{int}(NCcl(A))$, $M = (NC\,\text{int}(NCcl(A)))^C$ we have $\{p\} \subset M = (NC\,\text{int}(NCcl(A)))^C = NCcl(NC\,\text{int}(M)$, $\{p\} \subset NCcl(A) = NCcl(NC\,\text{int}(NCcl(A))$ $\subset NCcl(NC\,\text{int}(B))$. Hence both B and M are in Γ^{β} . The intersection $B \cap M = \{p\}$ is not neutrosophic crisp open since $p \in NCcl(A) \cap M^C$, hence not $NC\beta-$ open so. $\Gamma^{\beta} = B$ is not a neutrosophic crisp topology. Now suppose B is not a topology, and $\Gamma^{\beta} = B$ There is a $B \in \Gamma^{\beta}$ such that $B \notin \Gamma^{\alpha}$. Assume that $NCcl(NC\,\text{int}(B) \in \Gamma.$ Then $B \subset NCcl(NC\,\text{int}(B) = NC\,\text{int}(NCcl(NC\,\text{int}(B))$ .i.e. $B \in \Gamma^{\alpha}$, contrary to assumption. Thus we have produced a neutrosophic crisp open set whose neutrosophic crisp closure is not neutronsophic crisp open, which completes the proof.

Corollary 3.5

A neutrosophic crisp topology Γ is a neutrosophic crisp extremely disconnected if and only if Γ^{β} is a neutrosophic crisp topology.

4 Conclusion and future work

Neutrosophic set is well equipped to deal with missing data. By employing NSs in spatial data models, we can express a hesitation concerning the object of interest. This article has gone a step forward in developing methods that can be used to define neutrosophic spatial regions and their relationships. The main contributions of the paper can be described as the following: Possible applications have been listed after the definition of NS. Links to other models have been shown. We are defining some new operators to describe objects, describing a simple neutrosophic region. This paper has demonstrated that spatial object may profitably be addressed in terms of neutrosophic set. Implementation of the named applications is necessary as a proof of concept.

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Interval Neutrosophic Rough Set

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Abstract: This Paper combines interval- valued neutrouphic sets and rough sets. It studies roughness in interval- valued neutrosophic sets and some of its properties. Finally we propose a Hamming distance between lower and upper approximations of interval valued neutrosophic sets.

Keywords: interval valued neutrosophic sets, rough sets, interval valued neutrosophic sets.

1.Introduction

Neutrosophic set (NS for short), a part of neutrosophy introduced by Smarandache [1] as a new branch of philosophy, is a mathematical tool dealing with problems involving imprecise, indeterminacy and inconsistent knowledge. Contrary to fuzzy sets and intuitionistic fuzzy sets, a neutrosophic set consists of three basic membership functions independently of each other, which are truth, indeterminacy and falsity. This theory has been well developed in both theories and applications. After the pioneering work of Smarandache, In 2005, Wang [2] introduced the notion of interval neutrosophic sets (INS for short) which is another extension of neutrosophic sets. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval neutrosophic (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and Interval Neutrosophic Set (INS) provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information. The interval neutrosophic set generalize, the classical set ,fuzzy set [3] , the interval valued fuzzy set [4], intuitionistic fuzzy set [5] , interval valued intuitionstic fuzzy set [6] and so on. Many scholars have performed studies on neutrosophic sets , interval neutrosophic sets and their properties [7,8,9,10,11,12,13]. Interval neutrosophic sets have also been widely applied to many fields [14,15,16,17,18,19].

The rough set theory was introduced by Pawlak [20] in 1982, which is a technique for managing the uncertainty and imperfection, can analyze incomplete information effectively. Therefore, many models have been built upon different aspect, i.e, univers, relations, object, operators by many scholars [21,22,23,24,25,26] such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough, rough intuitionistic fuzzy set. intuitionistic fuzzy rough sets [27]. It has been successfully applied in many fields such as attribute reduction [28,29,30,31], feature selection [32,33,34], rule extraction [35,36,37,38] and so on. The rough sets theory approximates any subset of objects of the universe by two sets, called the lower and upper approximations. It focuses on the ambiguity caused by the limited discernibility of objects in the universe of discourse.

More recently, S.Broumi et al [39] combined neutrosophic sets with rough sets in a new hybrid mathematical structure called “rough neutrosophic sets” handling incomplete and indeterminate information . The concept of rough neutrosophic sets generalizes fuzzy rough sets and intuitionistic fuzzy rough sets. Based on the equivalence relation on the universe of discourse, A.Mukherjee et al [40] introduced lower and upper approximation of interval valued intuitionistic fuzzy set in Pawlak’s approximation space . Motivated by this ,we extend the interval intuitionistic fuzzy lower and upper approximations to the case of interval valued neutrosophic set. The concept of interval valued neutrosophic rough set is introduced by coupling both interval neutrosophic sets and rough sets.

The organization of this paper is as follow : In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3 , basic concept of rough approximation of an interval valued neutrosophic sets and their properties are presented. In section 4, Hamming distance between lower approximation and upper approximation of interval neutrosophic set is introduced, Finally, we concludes the paper.

2.Preliminaries

Throughout this paper, We now recall some basic notions of neutrosophic sets , interval valued neutrosophic sets , rough set theory and intuitionistic fuzzy rough sets. More can found in ref [1, 2,20,27].

Definition 1 [1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= {< x: $\boldsymbol{\mu}_{A(x)}$, $\boldsymbol{\nu}_{A(x)}$, $\boldsymbol{\omega}_{A(x)}$ >,x ∈ U}, where the functions $\boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\omega}$: U→]$^{-}$0,1^{+}[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+}. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^{-}$0,1^{+}[.so instead of]$^{-}$0,1^{+}[we need to take the interval [0,1] for technical applications, because]$^{-}$0,1^{+}[will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 [2]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x) \in [0,1]$.

For two IVNS, A= {<x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | x ∈ X } (2)

And B= {<x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1)A ⊆ Bif and only if $\mu_A^L(x) \leq \mu_B^L(x), \mu_A^U(x) \leq \mu_B^U(x)$, $\nu_A^L(x) \geq \nu_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$, $\omega_A^L(x) \geq \omega_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$

(2)A = B if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any x ∈ X

The complement of A_{IVNS} is denoted by A_{IVNS}^{o} and is defined by

A^{o}={ <x , $[\omega_A^L(x), \omega_A^U(x)]$> , $[1-\nu_A^U(x), 1-\nu_A^L(x)]$, $[\mu_A^L(x), \mu_A^U(x)]$ | x ∈ X }

A∩B ={ <x , [min($\mu_A^L(x)$,$\mu_B^L(x)$), min($\mu_A^U(x)$,$\mu_B^U(x)$)], [max($\nu_A^L(x)$,$\nu_B^L(x)$), max($\nu_A^U(x)$,$\nu_B^U(x)$], [max($\omega_A^L(x)$,$\omega_B^L(x)$), max($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

A∪B ={ <x , [max($\mu_A^L(x)$,$\mu_B^L(x)$), max($\mu_A^U(x)$,$\mu_B^U(x)$)], [min($\nu_A^L(x)$,$\nu_B^L(x)$), min($\nu_A^U(x)$,$\nu_B^U(x)$], [min($\omega_A^L(x)$,$\omega_B^L(x)$), min($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

0_N = {<x, [0, 0] ,[1 , 1], [1 ,1] >| x ∈ X}, denote the neutrosophic empty set ϕ

1_N = {<x, [0, 0] ,[0 , 0], [1 ,1] >| x ∈ X}, denote the neutrosophic universe set U

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse U={x1, x2, x3}, where x1characterizes the capability, x2characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x1, x2 and x3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,

A = {< x1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 3 [20]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R^*X and R_*X and are defined by

$R_*X = \{x \in U: [x]_R \subseteq X\}$,

$R^*X = \{x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R^*X = R_*X$, then X is called definable; otherwise X is called a rough set.

Definition 4 [27]

Let U be a universe and X , a rough set in U. An IF rough set A in U is characterized by a membership function μ_A :U→ [0, 1] and non-membership function ν_A :U→ [0 , 1] such that

$\mu_A(\underline{R}X) = 1$, $\nu_A(\underline{R}X) = 0$

Or $[\mu_A(x), \nu_A(x)] = [1, 0]$ if $x \in (\underline{R}X)$ and $\mu_A(U - \overline{R}X) = 0$, $\nu_A(U - \overline{R}X) = 1$

Or $[\mu_A(x), \nu_A(x)] = [0, 1]$ if $x \in U - \overline{R}X$,

$0 \leq \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \leq 1$

Example 2: Example of IF Rough Sets

Let U= {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult, Senior, Elderly} be a universe.

Let the equivalence relation R be defined as follows:

R*= {[Child, Pre-Teen], [Teen, Youth, Teenager], [Young-Adult, Adult],[Senior, Elderly]}.

Let X = {Child, Pre-Teen, Youth, Young-Adult} be a subset of univers U.

We can define X in terms of its lower and upper approximations:

$\underline{R}X$ = {Child, Pre-Teen}, and $\overline{R}X$ = {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult}.

The membership and non-membership functions μ_A:U→] 1 , 0 [and ν_A: U→] 1 , 0 [on a set A are defined as follows:

μ_AChild) = 1, μ_A (Pre-Teen) = 1 and μ_A (Child) = 0, μ_A(Pre-Teen) = 0

μ_A (Young-Adult) = 0, μ_A (Adult) = 0, μ_A(Senior) = 0, μ_A (Elderly) = 0

3.Basic Concept of Rough Approximations of an Interval Valued Neutrosophic Set and their Properties.

In this section we define the notion of interval valued neutrosophic rough sets (in brief ivn- rough set) by combining both rough sets and interval neutrosophic sets. IVN- rough sets are the generalizations of interval valued intuitionistic fuzzy rough sets, that give more information about uncertain or boundary region.

Definition 5 : Let (U,R) be a pawlak approximation space ,for an interval valued neutrosophic set

$A = \{<x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)]> \mid x \in X\}$ be interval neutrosophic set. The lower approximation $\underline{A}_R$ and $\overline{A}_R$ upper approximations of A in the pawlak approwimation space (U, R) are defined as:

$\underline{A}_R = \{<x, [\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}], [\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}], [\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}]>: x \in U\}$.

$\overline{A}_R = \{<x, [\vee_{y \in [x]_R}\{\mu_A^L(y)\}, \vee_{y \in [x]_R}\{\mu_A^U(y)\}], [\wedge_{y \in [x]_R}\{\nu_A^L(y)\}, \wedge_{y \in [x]_R}\{\nu_A^U(y)], [\wedge_{y \in [x]_R}\{\omega_A^L(y)\}, \wedge_{y \in [x]_R}\{\omega_A^U(y)\}]: x \in U\}$.

Where " ∧ " means " min" and " ∨ " means " max", R denote an equivalence relation for interval valued neutrosophic set A.

Here $[x]_R$ is the equivalence class of the element x. It is easy to see that

$[\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}] \subset [0,1]$

$[\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}] \subset [0,1]$

$[\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}] \subset [0,1]$

And

$\mathbf{0} \leq \wedge_{y \in [x]_R}\{\mu_A^U(y)\} + \vee_{y \in [x]_R}\{\nu_A^U(y)\} + \vee_{y \in [x]_R}\{\omega_A^U(y)\} \leq \mathbf{3}$

Then, $\underline{A}_R$ is an interval neutrosophic set

Similarly , we have

$[\vee_{y\in[x]_R}\{\mu_A^L(y)\},\ \vee_{y\in[x]_R}\{\mu_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{\nu_A^L(y)\},\ \wedge_{y\in[x]_R}\{\nu_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{\omega_A^L(y)\},\ \wedge_{y\in[x]_R}\{\omega_A^U(y)\}] \subset [0,1]$

And

$\mathbf{0} \le \vee_{y\in[x]_R}\{\mu_A^U(y)\} + \wedge_{y\in[x]_R}\{\nu_A^U(y)\} + \wedge_{y\in[x]_R}\{\omega_A^U(y)\} \le \mathbf{3}$

Then, $\overline{A}_R$ is an interval neutrosophic set

If $\underline{A}_R = \overline{A}_R$,then A is a definable set, otherwise A is an interval valued neutrosophic rough set, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval valued neutrosophic set with respect to approximation space (U, R), respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$.

In the following , we employ an example to illustrate the above concepts

Example:

Theorem 1. Let A, B be interval neutrosophic sets and $\underline{A}$ and $\overline{A}$ the lower and upper approximation of interval –valued neutrosophic set A with respect to approximation space (U, R) ,respectively. $\underline{B}$ and $\overline{B}$ the lower and upper approximation of interval –valued neutrosophic set B with respect to approximation space (U,R) ,respectively.Then we have

i. $\underline{A} \subseteq A \subseteq \overline{A}$
ii. $\overline{A \cup B} = \overline{A} \cup \overline{B}$, $\underline{A \cap B} = \underline{A} \cap \underline{B}$
iii. $\underline{A} \cup \underline{B} = \underline{A \cup B}$, $\overline{A \cap B} = \overline{A} \cap \overline{B}$
iv. $\underline{(\overline{A})} = \overline{(\overline{A})} = \overline{A}$, $\underline{(\underline{A})} = \overline{(\underline{A})} = \underline{A}$
v. $\underline{U} = U$; $\overline{\phi} = \phi$
vi. If A ⊆ B ,then $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$
vii. $\underline{A^c} = (\overline{A})^c$, $\overline{A^c} = (\underline{A})^c$

Proof: we prove only i,ii,iii, the others are trivial

(i)

Let $A = \{<x , [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] >| x \in X \}$ be interval neutrosophic set

From definition of $\underline{A}_R$ and $\overline{A}_R$, we have

Which implies that

$\mu_{\underline{A}}^L(x) \le \mu_A^L(x) \le \mu_{\overline{A}}^L(x)$; $\mu_{\underline{A}}^U(x) \le \mu_A^U(x) \le \mu_{\overline{A}}^U(x)$ for all $x \in X$

$\nu_{\underline{A}}^L(x) \ge \nu_A^L(x) \ge \nu_{\overline{A}}^L(x)$; $\nu_{\underline{A}}^U(x) \ge \nu_A^U(x) \ge \nu_{\overline{A}}^U(x)$ for all $x \in X$

$\omega_{\underline{A}}^L(x) \ge \omega_A^L(x) \ge \omega_{\overline{A}}^L(x)$; $\omega_{\underline{A}}^U(x) \ge \omega_A^U(x) \ge \omega_{\overline{A}}^U(x)$ for all $x \in X$

$([\mu_{\underline{A}}^L , \mu_{\underline{A}}^U], [\nu_{\underline{A}}^L , \nu_{\underline{A}}^U], [\omega_{\underline{A}}^L , \omega_{\underline{A}}^U]) \subseteq ([\mu_A^L , \mu_A^U], [\nu_A^L , \nu_A^U], [\omega_A^L , \omega_A^U]) \subseteq ([\mu_{\overline{A}}^L , \mu_{\overline{A}}^U], [\nu_{\overline{A}}^L , \nu_{\overline{A}}^U], [\omega_{\overline{A}}^L , \omega_{\overline{A}}^U])$ .Hence $\underline{A}_R \subseteq A \subseteq \overline{A}_R$

(ii) Let $A = \{<x , [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] >| x \in X \}$ and

$B = \{<x, [\mu_B^L(x), \mu_B^U(x)], [\nu_B^L(x), \nu_B^U(x)] , [\omega_B^L(x), \omega_B^U(x)] >| x \in X \}$ are two intervalvalued neutrosophic set and

$\overline{A \cup B} = \{<x , [\mu_{\overline{A \cup B}}^L(x), \mu_{\overline{A \cup B}}^U(x)] , [\nu_{\overline{A \cup B}}^L(x), \nu_{\overline{A \cup B}}^U(x)] , [\omega_{\overline{A \cup B}}^L(x), \omega_{\overline{A \cup B}}^U(x)] >| x \in X \}$

$\overline{A} \cup \overline{B} = \{x, [\max(\mu_{\overline{A}}^L(x) , \mu_{\overline{B}}^L(x)) ,\max(\mu_{\overline{A}}^U(x) , \mu_{\overline{B}}^U(x))],[\min(\nu_{\overline{A}}^L(x) , \nu_{\overline{B}}^L(x)) ,\min(\nu_{\overline{A}}^U(x) , \nu_{\overline{B}}^U(x))],[\min(\omega_{\overline{A}}^L(x) , \omega_{\overline{B}}^L(x)) ,\min(\omega_{\overline{A}}^U(x) , \omega_{\overline{B}}^U(x))]$

for all $x \in X$

$$\mu_{\overline{A \cup B}}^L(x) = \vee\{ \mu_{A \cup B}^L(y) | \ y \in [x]_R\}$$

$$= \vee \{\mu_A^L(y) \vee \mu_B^L(y) \mid y \in [x]_R\}$$

$$= (\vee \mu_A^L(y) \mid y \in [x]_R) \vee (\vee \mu_A^L(y) \mid y \in [x]_R)$$

$$=(\mu_{\overline{A}}^L \vee \mu_{\overline{B}}^L)(x)$$

$$\mu_{\overline{A \cup B}}^U(x) = \vee\{ \mu_{A \cup B}^u(y) | \ y \in [x]_R\}$$

$$= \vee \{\mu_A^U(y) \vee \mu_B^U(y) \mid y \in [x]_R\}$$

$= (\vee\ \mu^{u}_{A}(y) \mid y \in [x]_R) \vee (\vee\ \mu^{U}_{A}(y) \mid y \in [x]_R)$

$=(\mu^{U}_{\overline{A}} \vee \mu^{U}_{\overline{B}})(x)$

$\nu^{L}_{\overline{A \cup B}}(x)=\wedge\{ \nu^{L}_{A \cup B}(y) \mid y \in [x]_R\}$

$= \wedge \{\nu^{L}_{A}(y) \wedge \nu^{L}_{B}(y) \mid y \in [x]_R\}$

$= (\wedge\ \nu^{L}_{A}(y) \mid y \in [x]_R) \wedge (\wedge\ \nu^{L}_{B}(y) \mid y \in [x]_R)$

$=(\nu^{L}_{\overline{A}} \wedge \nu^{L}_{\overline{B}})(x)$

$\nu^{U}_{\overline{A \cup B}}(x)=\wedge\{ \nu^{U}_{A \cup B}(y) \mid y \in [x]_R\}$

$= \wedge \{\nu^{U}_{A}(y) \wedge \nu^{U}_{B}(y) \mid y \in [x]_R\}$

$= (\wedge\ \nu^{U}_{A}(y) \mid y \in [x]_R) \wedge (\wedge\ \nu^{U}_{B}(y) \mid y \in [x]_R)$

$=(\nu^{U}_{\overline{A}}(y) \wedge \nu^{U}_{\overline{B}}(y))(x)$

$\omega^{L}_{\overline{A \cup B}}(x)=\wedge\{ \omega^{L}_{A \cup B}(y) \mid y \in [x]_R\}$

$= \wedge \{\omega^{L}_{A}(y) \wedge \omega^{L}_{B}(y) \mid y \in [x]_R\}$

$= (\wedge\ \omega^{L}_{A}(y) \mid y \in [x]_R) \wedge (\wedge\ \omega^{L}_{B}(y) \mid y \in [x]_R)$

$=(\omega^{L}_{\overline{A}} \wedge \omega^{L}_{\overline{B}})(x)$

$\omega^{U}_{\overline{A \cup B}}(x)=\wedge\{ \omega^{U}_{A \cup B}(y) \mid y \in [x]_R\}$

$= \wedge \{\omega^{U}_{A}(y) \wedge \nu^{U}_{B}(y) \mid y \in [x]_R\}$

$= (\wedge\ \omega^{U}_{A}(y) \mid y \in [x]_R) \wedge (\wedge\ \omega^{U}_{B}(y) \mid y \in [x]_R)$

$=(\omega^{U}_{\overline{A}} \wedge \omega^{U}_{\overline{B}})(x)$

Hence, $\overline{A \cup B} = \overline{A} \cup \overline{B}$

Also for $\underline{A \cap B} = \underline{A} \cap \underline{B}$ for all x ∈ A

$\mu^{L}_{\underline{A \cap B}}(x) = \wedge\{ \mu^{L}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \wedge \{\mu^{L}_{A}(y) \wedge \mu^{L}_{B}(y) \mid y \in [x]_R\}$

$= \wedge\ (\mu^{L}_{A}(y) \mid y \in [x]_R) \wedge (\vee \mu^{L}_{B}(y) \mid y \in [x]_R)$

$=\mu^{L}_{\underline{A}}(x) \wedge \mu^{L}_{\underline{B}}(x)$

$=(\mu^{L}_{\underline{A}} \wedge \mu^{L}_{\underline{B}})(x)$

Also

$\mu^{U}_{\underline{A \cap B}}(x) = \wedge\{ \mu^{U}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \wedge \{\mu^{U}_{A}(y) \wedge \mu^{U}_{B}(y) \mid y \in [x]_R\}$

$= \wedge\ (\mu^{U}_{A}(y) \mid y \in [x]_R) \wedge (\vee \mu^{U}_{B}(y) \mid y \in [x]_R)$

$=\mu^{U}_{\underline{A}}(x) \wedge \mu^{U}_{\underline{B}}(x)$

$=(\mu^{U}_{\underline{A}} \wedge \mu^{U}_{\underline{B}})(x)$

$\nu^{L}_{\underline{A \cap B}}(x) = \vee\{ \nu^{L}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \vee \{\nu^{L}_{A}(y) \vee \nu^{L}_{B}(y) \mid y \in [x]_R\}$

$= \vee\ (\nu^{L}_{A}(y) \mid y \in [x]_R) \vee (\vee \nu^{L}_{B}(y) \mid y \in [x]_R)$

$=\nu^{L}_{\underline{A}}(x) \vee \nu^{L}_{\underline{B}}(x)$

$=(\nu^{L}_{\underline{A}} \vee \nu^{L}_{\underline{B}})(x)$

$\nu^{U}_{\underline{A \cap B}}(x) = \vee\{ \nu^{U}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \vee \{\nu^{U}_{A}(y) \vee \nu^{U}_{B}(y) \mid y \in [x]_R\}$

$= \vee\ (\nu^{U}_{A}(y) \mid y \in [x]_R) \vee (\vee \nu^{U}_{B}(y) \mid y \in [x]_R)$

$=\nu^{U}_{\underline{A}}(x) \vee \nu^{U}_{\underline{B}}(x)$

$=(\nu^{U}_{\underline{A}} \vee \nu^{U}_{\underline{B}})(x)$

$\omega^{L}_{\underline{A \cap B}}(x) = \vee\{ \omega^{L}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \vee \{\omega^{L}_{A}(y) \vee \omega^{L}_{B}(y) \mid y \in [x]_R\}$

$= \vee\ (\omega^{L}_{A}(y) \mid y \in [x]_R) \vee (\vee \omega^{L}_{B}(y) \mid y \in [x]_R)$

$=\omega^{L}_{\underline{A}}(x) \vee \nu\omega^{L}_{\underline{B}}(x)$

$=(\omega^{L}_{\underline{A}} \vee \omega^{L}_{\underline{B}})(x)$

$\omega^{U}_{\underline{A \cap B}}(x) = \vee\{ \omega^{U}_{A \cap B}(y) \mid y \in [x]_R\}$

$= \vee \{\omega^{U}_{A}(y) \vee \omega^{U}_{B}(y) \mid y \in [x]_R\}$

$= \vee\ (\omega^{U}_{A}(y) \mid y \in [x]_R) \vee (\vee \omega^{U}_{B}(y) \mid y \in [x]_R)$

$=\omega^{U}_{\underline{A}}(x) \vee \omega^{U}_{\underline{B}}(x)$

$=(\omega^{U}_{\underline{A}} \vee \omega^{U}_{\underline{B}})(x)$

(iii)

$\mu^{U}_{\overline{A\cap B}}(x) = \vee\{ \mu^{U}_{A\cap B}(y) \mid y \in [x]_R\}$

$= \vee \{\mu^{U}_{A}(y) \wedge \mu^{U}_{B}(y) \mid y \in [x]_R\}$

$=(\vee (\mu^{U}_{A}(y) \mid y \in [x]_R)) \wedge (\vee (\mu^{U}_{A}(y) \mid y \in [x]_R))$

$= \mu^{U}_{\overline{A}}(x) \vee \mu^{U}_{\overline{B}}(x)$

$=(\mu^{U}_{\overline{A}} \vee \mu^{U}_{\overline{B}})(x)$

$\nu^{U}_{\overline{A\cap B}}(x) = \wedge\{ \nu^{U}_{A\cap B}(y) \mid y \in [x]_R\}$

$= \wedge \{\nu^{U}_{A}(y) \wedge \nu^{U}_{B}(y) \mid y \in [x]_R\}$

$=(\wedge (\nu^{U}_{A}(y) \mid y \in [x]_R)) \vee (\wedge (\nu^{U}_{A}(y) \mid y \in [x]_R))$

$= \nu^{U}_{\overline{A}}(x) \vee \nu^{U}_{\overline{B}}(x)$

$=(\nu^{U}_{\overline{A}} \vee \nu^{U}_{\overline{B}})(x)$

$\omega^{U}_{\overline{A\cap B}}(x) = \wedge\{ \omega^{U}_{A\cap B}(y) \mid y \in [x]_R\}$

$= \wedge \{\omega^{U}_{A}(y) \wedge \omega\nu^{U}_{B}(y) \mid y \in [x]_R\}$

$=(\wedge (\omega^{U}_{A}(y) \mid y \in [x]_R)) \vee (\wedge (\omega^{U}_{A}(y) \mid y \in [x]_R))$

$= \omega^{U}_{\overline{A}}(x) \vee \omega^{U}_{\overline{B}}(x)$

$=(\omega^{U}_{\overline{A}} \vee \omega^{U}_{\overline{B}})(x)$

Hence follow that $\overline{A\cap B} = \overline{A} \cap \overline{B}$ .we get $\underline{A} \cup \underline{B} = \underline{A \cup B}$ by following the same procedure as above.

Definition 6:

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U.

If $\underline{A} = \underline{B}$,then A and B are called interval valued neutrosophic lower rough equal.

If $\overline{A}=\overline{B}$, then A and B are called interval valued neutrosophic upper rough equal.

If $\underline{A} = \underline{B}$, $\overline{A}=\overline{B}$, then A and B are called interval valued neutrosophic rough equal.

Theorem 2 .

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U. then

1. $\underline{A} = \underline{B} \Leftrightarrow \underline{A \cap B} = \underline{A}$, $\underline{A \cap B} = \underline{B}$
2. $\overline{A}=\overline{B} \Leftrightarrow \overline{A \cup B} = \overline{A}$, $\overline{A \cup B} = \overline{B}$
3. If $\overline{A} = \overline{A'}$ and $\overline{B} = \overline{B'}$,then $\overline{A \cup B} = \overline{A' \cup B'}$
4. If $\underline{A} = \underline{A'}$ and $\underline{B} = \underline{B'}$,Then
5. If $A \subseteq B$ and $\underline{B} = \underline{\phi}$,then $\underline{A} = \underline{\phi}$
6. If $A \subseteq B$ and $\underline{B} = \underline{U}$,then $\underline{A} = \underline{U}$
7. If $\underline{A} = \underline{\phi}$ or $\underline{B} = \underline{\phi}$ or then $\underline{A \cap B} = \underline{\phi}$
8. If $\overline{A} = \overline{U}$ or $\overline{B} = \overline{U}$,then $\overline{A \cup B} = \overline{U}$
9. $\overline{A} = \overline{U} \Leftrightarrow A = U$
10. $\overline{A} = \overline{\phi} \Leftrightarrow A = \phi$

Proof: the proof is trial

4.Hamming distance between Lower Approximation and Upper Approximation of IVNS

In this section , we will compute the Hamming distance between lower and upper approximations of interval neutrosophic sets based on Hamming distance introduced by **Ye [41]** of interval neutrosophic sets.

Based on Hamming distance between two interval neutrosophic set A and B as follow:

$$d(A,B)=\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu^{L}_{A}(x_i)-\mu^{L}_{B}(x_i)\right|+\left|\mu^{U}_{A}(x_i)-\mu^{U}_{B}(x_i)\right|+\left|\nu^{L}_{A}(x_i)-\nu^{L}_{B}(x_i)\right|+\left|\nu^{U}_{A}(x_i)-\nu^{U}_{B}(x_i)\right|+\left|\omega^{L}_{A}(x_i)-\omega^{L}_{B}(x_i)\right|+\left|\omega^{L}_{A}(x_i)-\nu^{U}_{B}(x_i)\right|\right]$$

we can obtain the standard hamming distance of $\underline{A}$ and $\overline{A}$ from

$$d_H(\underline{A},\overline{A})=\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu^{L}_{\underline{A}}(x_j)-\mu^{L}_{\overline{A}}(x_j)\right|+\left|\mu^{U}_{\underline{A}}(x_j)-\mu^{U}_{\overline{A}}(x_j)\right|+\left|\nu^{L}_{\underline{A}}(x_j)-\nu^{L}_{\overline{A}}(x_j)\right|+\left|\nu^{U}_{\underline{A}}(x_j)-\nu^{U}_{\overline{A}}(x_j)\right|+\left|\omega^{L}_{\underline{A}}(x_j)-\omega^{L}_{\overline{A}}(x_j)\right|+\left|\omega^{U}_{\underline{A}}(x_j)-\omega^{U}_{\overline{A}}(x_j)\right|\right]$$

Where

$\underline{A}_R$={<x, [$\wedge_{y\in[x]_R}\{\mu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\mu_A^U(y)\}$], [$\vee_{y\in[x]_R}\{\nu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\nu_A^U(y)\}$], [$\vee_{y\in[x]_R}\{\omega_A^L(y)\}$, $\vee_{y\in[x]_R}\{\omega_A^U(y)\}$]>:x ∈ U}.

$\overline{A}_R$={<x, [$\vee_{y\in[x]_R}\{\mu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\mu_A^U(y)\}$], [$\wedge_{y\in[x]_R}\{\nu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\nu_A^U(y)\}$], [$\wedge_{y\in[x]_R}\{\omega_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\omega_A^U(y)\}$]:x ∈ U}.

$\mu_{\underline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\mu_A^L(y)\}$; $\mu_{\underline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\mu_A^U(y)\}$

$\nu_{\underline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\nu_A^L(y)\}$; $\nu_{\underline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\nu_A^U(y)\}$

$\omega_{\underline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\omega_A^L(y)\}$; $\omega_{\underline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\omega_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\mu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\mu_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\nu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\nu_A^U(y)\}$

$\omega_{\overline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\omega_A^L(y)\}$; $\omega_{\overline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\omega_A^U(y)\}$

Theorem 3. Let (U, R) be approximation space, A be an interval valued neutrosophic set over U . Then

(1) If d ($\underline{A}$, $\overline{A}$) = 0, then A is a definable set.

(2) If 0 < d($\underline{A}$, $\overline{A}$) < 1, then A is an interval-valued neutrosophic rough set.

Theorem 4. Let (U, R) be a Pawlak approximation space, and A and B two interval-valued neutrosophic sets over U . Then

1. d ($\underline{A}$, $\overline{A}$) ≥ d ($\underline{A}$, A) and d ($\underline{A}$, $\overline{A}$) ≥ d (A , $\overline{A}$);
2. d ($\overline{A \cup B}$, $\overline{A} \cup \overline{B}$) = 0, d ($\underline{A \cap B}$, $\underline{A} \cap \underline{B}$) = 0.
3. d ($\underline{A} \cup \underline{B}$, A ∪ B) ≥ d($\underline{A} \cup \underline{B}$, $\underline{A \cup B}$) and d($\underline{A} \cup \underline{B}$, A ∪ B) ≥ d($\underline{A \cup B}$, A ∪ B) ; and d(A ∩ B, $\overline{A} \cap \overline{B}$) ≥ d(A ∩ B, $\overline{A \cap B}$) and d(A ∩ B, $\overline{A} \cap \overline{B}$) ≥ $d(\overline{A \cap B}, \overline{A} \cap \overline{B})$
4. d($\overline{(\overline{A})}$, $\underline{(\overline{A})}$)= 0 , d($\overline{(\overline{A})}$, $\overline{A}$) = 0 , d($\underline{(\overline{A})}$, $\overline{A}$)= 0; d($\overline{(\underline{A})}$, $\underline{(\overline{A})}$) = 0 , d($\underline{(\underline{A})}$, , $\overline{A}$) = 0 , d($\underline{(\overline{A})}$, $\underline{A}$) = 0,
5. d ($\underline{U}$, U) =0 , d($\overline{\phi}$, ϕ) = 0
6. if A B ,then d($\overline{A}$,B) ≥ d($\underline{A}$, $\underline{B}$) and d($\underline{A}$, B) ≥ d($\underline{B}$,B)

   d(A , $\overline{B}$) ≥d(A, $\overline{A}$) and d(A, $\overline{B}$)= ≥d($\overline{A}$, $\overline{B}$)
7. d($\underline{A^c}$,$(\overline{A})^c$)= 0, d($\overline{A^c}$,$(\underline{A})^c$) = 0

5-Conclusion

In this paper we have defined the notion of interval valued neutrosophic rough sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and interval valued neutrosophic set theory. Further, we have introduced the Hamming distance between two interval neutrosophic rough sets. We hope that our results can also be extended to other algebraic system.

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Examples of Neutrosophic Probability in Physics

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Abstract: This paper re-discusses the problems of the so-called "law of nonconservation of parity" and "accelerating expansion of the universe", and presents the examples of determining Neutrosophic Probability of the experiment of Chien-Shiung Wu et al in 1957, and determining Neutrosophic Probability of accelerating expansion of the partial universe.

Keywords: Neutrosophic Probability, law of nonconservation of parity, accelerating expansion of the universe .

1 Introduction

According to reference [1], Neutrosophic probability is a generalization of the classical and imprecise probabilities. Several classical probability rules are adjusted in the form of neutrosophic probability rules. In some cases, the neutrosophic probability is extended to n-valued refined neutrosophic probability.

The neutrosophic probability is a generalization of the classical probability because, when the chance of indeterminacy of a stochastic process is zero, these two probabilities coincide.

The Neutrosophic Probability that an event A occurs is

$$NP(A) = (ch(A), ch(neutA), ch(antiA)) = (T, I, F)$$

where T, I, F are standard or nonstandard subsets of the nonstandard unitary interval $]^-0, 1^+[$, and T is the chance that A occurs, denoted $ch(A)$; I is the indeterminate chance related to A, $ch(neutA)$; and F is the chance that A does not occur, $ch(antiA)$.

This paper presents some examples of Neutrosophic Probability in physics.

2 Determining Neutrosophic Probability of the experiment of Chien-Shiung Wu et al in 1957

One of the reasons for 1957 Nobel Prize for physics is "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles", and according to the experiment of Chien-Shiung Wu et al in 1957, the so-called "law of nonconservation of parity" is established. While, according to the viewpoint of Neutrosophic Probability, this conclusion should be re-discussed.

Supposing that event A denotes parity is conservation, *antiA* denotes parity is nonconservation, and *neutA* denotes indeterminacy.

In the experiment of Chien-Shiung Wu et al in 1957, they found that the number of the electrons that exiting angle $\theta > 90°$ is 40% more than that of $\theta < 90°$ (the ratio is 1.4:1.0). For this result, we cannot simply say that parity is conservation or nonconservation. The correct way of saying should be that, besides indeterminacy, the chance of conservation of parity is as follows

$$ch(A) = 1.0/1.4 = 71\%$$

and the chance of nonconservation of parity is as follows

$$ch(antiA)) = (1.4 - 1.0)/1.4 = 29\%$$

Thus, the Neutrosophic Probability that "parity is conservation" is as follows

$$NP(A) = (ch(A), ch(neutA), ch(antiA)) = (71\%, 0, 29\%)$$

It should be noted that, for the reason that we cannot know the indeterminacy, so we suppose that it is equal to 0.

In reference [2] we point out that, the essential reason for the phenomena of nonconservation (including nonconservation of parity, momentum, angular momentum and the like) is that so far only the "law of conservation of energy" can be considered as the unique truth in physics. As for other "laws", they are correct only in the cases that they are not contradicted with law of conservation of energy or they can be derived by law of conservation of energy.

Similarly, the Neutrosophic Probability for other laws of conservation should be determined by law of conservation of energy or experiment (currently for most cases the Neutrosophic Probability can only be determined by experiment, like the experiment of Chien-Shiung Wu et al in 1957).

3 Determining Neutrosophic Probability of accelerating expansion of the partial universe

One of the reasons for 2011 Nobel Prize for physics is "for the discovery of the accelerating expansion of the universe through observations of distant supernovae". But "the accelerating expansion of the universe" is debatable, and Neutrosophic Probability of the accelerating expansion of the partial universe should be determined.

In 1929, Hubble, an astronomer of the United States, found the famous Hubble's law. According to Hubble's law, some scholars reach the conclusion of the accelerating expansion of the universe. But "the accelerating expansion of the universe" is debatable. Due to the observation of distance is limited and the observation time is also limited, at most we can say: "partial universe is in the state of expansion (including accelerating expansion) for limited time."

Firstly we discuss the unreasonable results caused by Hubble’s Law.

Hubble's law reads

$$V = H_0 \times D \qquad (1)$$

where： V —（galaxy’s） far away speed, unit: km/s； H_0 —Hubble's Constant, unit: km/(s . Mpc)； D —（galaxy’s） far away distance, unit: Mpc.

According to Hubble's law, we have

$$V = \frac{dD(t)}{dt} = H_0 \times D(t) \qquad (2)$$

From this differential equation, it gives

$$D = ke^{H_0 t} = k\exp(H_0 t) \qquad (3)$$

where： k — a constant to be determined; if we assume that the distance is positive, then its value is positive too.

It gives the far away speed as follows

$$V = kH_0 \exp(H_0 t) \qquad (4)$$

The far away acceleration is as follows

$$a = dV/dt = kH_0^2 \exp(H_0 t) \qquad (5)$$

According to Newton's second law, the force acted on this galaxy is as follows

$$F = ma = mkH_0^2 \exp(H_0 t) \qquad (6)$$

Based on these equations, apparently we can reach the unreasonable conclusions: as time tends to infinity, all of the values will tend to infinity too.

If Hubble’s law needs to be amended, the conclusion of "the accelerating expansion of the universe" also needs to be amended. At least it should be amended as "the accelerating expansion of the partial universe."

Secondly we discuss the states of contraction and the like of the partial universe.

Many scholars have presented the state of contraction of the universe (or partial universe). Here we stress that partial universe (such as the area nearby a black hole) is in the state of contraction.

As well-known, the mass of black hole (or similar black hole) is immense, and it produces a very strong gravitational field, so that all matters and radiations (including the electromagnetic wave or light) will be unable to escape if they enter to a critical range around the black hole.

The viewpoint of "the accelerating expansion of the universe" unexpectedly turns a blind eye to the fact that partial universe (such as the area nearby a black hole) is in the state of contraction.

To sum up, considering all possible situations, the correct conclusion is that there exist at least seven states of accelerating expansion and contraction and the like in the universe, namely "partial universe is in the state of accelerating expansion, partial universe is accelerating contraction, partial universe is uniform expansion, partial universe is uniform contraction, partial universe is decelerating expansion, partial universe is decelerating contraction, and partial universe is neither expansion nor contraction (this may be the static state)". As for the detailed study for these seven states, it will be the further topic in future.

Besides these seven states, due to the limitations of human knowledge and the like, there may be other unknown states or indeterminacy states.

Supposing that the chance of getting indeterminacy ch(indeterm) = 9%, and the chance of accelerating expansion of the partial universe is equal to the chance of other states, thus the Neutrosophic Probability that “accelerating expansion of the partial universe (*A*)” is as follows

NP (*A*) = (*ch*(*A*) ,*ch*(*neutA*) ,*ch*(*antiA*)) = (13*%, 9%, 78%*)

While, according to the classical probability, the probability that “accelerating expansion of the partial universe” is equal to 1/7 (14.2857%).

4 Conclusions

The problems of the so-called “law of nonconservation of parity” and “the accelerating expansion of the universe” should be re-discussed. The Neutrosophic Probability that “parity is conservation” is (*71%, 0, 29%*), and the Neutrosophic Probability that “accelerating expansion of the partial universe” is (13*%, 9%, 78%*).

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A New Approach to Multi-spaces Through the Application of Soft Sets

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Abstract. Multi-space is the notion combining different fields in to a unifying field, which is more applicable in our daily life. In this paper, we introduced the notion of multi-soft space which is the approximated collection of the multi-subspaces of a multi-space . Further, we defined some basic operations such as union, intersection, AND, OR etc. We also investigated some properties of multi-soft spaces.

Keywords: Multi-space, soft set, multi-soft space.

1. Introduction

Multi-spaces [24] were introduced by Smarandache in 1969 under the idea of hybrid structures: combining different fields into a unifying field [23] that are very effective in our real life. This idea has a wide range of acceptance in the world of sciences. In any domain of knowledge a Smarandache multi-space is the union of n different spaces with some different for an integer $n \geq 2$. Smarandache multi-space is a qualitative notion as it is too huge which include both metric and non-metric spaces. This multi-space can be used for both discrete or connected spaces specially in spacetimes and geometries in theoretical physics. Multi-space theory has applied in physics successfully in the Unified Field Theory which unite the gravitational, electromagnetic, weak and strong interactions or in the parallel quantum computing or in the mu-bit theory etc. Several multi-algebraic structures have been introduced such as multi-groups, multi-rings, multi-vector spaces, multi-metric spaces etc. Literature on multi-algebraic structures can be found in [17].

Molodtsov [20] proposed the theory of soft sets. This mathematical framework is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. Soft set theory has been applied successfully in many areas such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability thoery. Soft sets gained much attention of the researchers recently from its appearance and some literature on soft sets can be seen in [1]-[16]. Some other properties and algebras may be found in [18,19,20]. Some other concepts together with fuzzy set and rough set were shown in [21,22,23].

In section 2, we review some basic concepts and notions on multi-spaces and soft sets. In section 3, we define multi-subspac. Then multi-soft spaces has been introduced in the current section. Multi-soft space is a parameterized collection of multi-subspaces. We also investigated some properties and other notions of multi-soft spaces.

2. Basic Concepts

In this section, we review some basic material of multi-spaces and soft sets.

Definition 2.1 [24]. For any integer i, $1 \le i \le n$, let M_i be a set with ensemble of law L_i, and the intersection of k sets $M_{i_1}, M_{i_2}, \ldots, M_{i_k}$ of them constrains the law $I \; M_{i_1}, M_{i_2}, \ldots, M_{i_k}$. Then the union of M_i, $1 \le i \le n$

$$M = \bigcup_{i=1}^{n} M_i$$

is called a multi-space.

Let U be an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 2.2 [20]. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F \; a$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Example 2.3. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \begin{Bmatrix} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{Bmatrix}$$

Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{a_1, a_2, a_3\}$ be the set of parameters where

a_1 stands for the parameter 'high rent,

a_2 stands for the parameter 'normal rent,

a_3 stands for the parameter 'in good condition.

Suppose that

$$F(a_1) = \{s_1, s_4\},$$
$$F(a_2) = \{s_2, s_5\},$$
$$F(a_3) = \{s_3\}.$$

The soft set (F, A) is an approximated family $\{F(a_i), i = 1,2,3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F, A) is a soft set as a collection of approximations over U, where

$$F(a_1) = high\ rent = \{s_1, s_2\},$$
$$F(a_2) = normal\ rent = \{s_2, s_5\},$$
$$F(a_3) = in\ good\ condition = \{s_3\}.$$

Definition 2.4 [19]. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, \mathrm{B})$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, \mathrm{B})$.

Definition 2.5 [19]. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.6 [19]. Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (\mathrm{G}, \mathrm{B}) = (H, \mathrm{C})$ where (H, C) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in C = A \cap \mathrm{B}$.

Definition 2.7 [12]. The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c)=\begin{cases} F(c) & \text{if } c \in A-B, \\ G(c) & \text{if } c \in B-A, \\ F(c)\cap G(c) & \text{if } c \in A\cap B. \end{cases}$$

We write $(F,A)\cap_{\varepsilon}(G,B)=(H,C)$.

Definition 2.8 [19]. The restricted union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,B), where $C = A\cup B$, and for all $c\in C$, $H(c)$ is defined as $H(c)=F(c)\cup G(c)$ for all $c\in C$. We write it as $(F,A)\cup_R (G,B)=(H,C)$.

Definition 2.9 [12]. The extended union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,B), where $C = A\cup B$, and for all $c\in C$, $H(c)$ is defined as

$$H(c)=\begin{cases} F(c) & \text{if } c \in A-B \\ G(c) & \text{if } c \in B-A, \\ F(c)\cup G(c) & \text{if } c \in A\cap B. \end{cases}$$

We write $(F,A)\cup_{\varepsilon}(G,B)=(H,C)$.

In the next section, we introduced multi-soft spaces.

3. Multi-Soft Space and Its Properties

In this section, first we introduced the definition of multi-subspace. Further, we introduced multi-soft spaces and their core properties.

Definition 3.1. Let M be a multi-space and $M^{'}\subseteq M$. Then $M^{'}$ is called a multi-subspace if $M^{'}$ is a multi-space under the operations and constaints of M .

Definition 3.2. Let $A_1=\{a_j : j\in J\}$, $A_2=\{a_k : k\in K\}$,..., $A_n=\{a_n : n\in L\}$ be n-set of parameters. Let $(F_1,A_1),(F_2,A_2),...,(F_n,A_n)$ are soft set over the distinct universes $M_1,M_2,...,M_n$ respectively. Then (H,C) is called a multi-soft space over $M=M_1\cup M_2\cup...\cup M_n$, where $(H,C)=(F_1,A_1)\cup_E(F_2,A_2)\cup_E,...,\cup_E(F_n,A_n)$ such that $C=A_1\cup A_2\cup....\cup A_n$ and for all $c\in C$, $H(c)$ is defined by

$$H(c)=F_{i_1}(c)\cup F_{i_2}(c)\cup...\cup F_{i_k}(c)$$

if

$$c\in(A_{i1}\cap A_{i_2}\cap...\cap A_{ik})-(A_{i_{k+1}}\cup A_{i_{k+2}}\cup...\cup A_{i_n}) ,$$

where $(i_1,i_2,...,i_k,i_{k+1},...,i_n)$ are all possible permutations of the indexes $(1,2,...,n)$ *k = 1, 2, ..., n*. There are 2^{n-1} pieces of the piece-wise function (H,C).

Proposition 3.3. Let M be a universe of discourse and (F,A) is a soft set over M . Then (F,A) is a multi-soft space over M if and only if M is a multi-space.

Proof: Suppose that M is a multi-space and $F: A\rightarrow P(M)$ be a mapping. Then clearly for each $a\in A$, then $F(a)$ is a subset of M which is a multi-subspace. Thus each $F(a)$ is a multi-subspace of M and so the soft set (F,A) is the parameterized collection of multi-subspaces of M . Hence (F,A) is a multi-soft space over M .

For converse, suppose that (F,A) is a multi-soft space over M . This implies that $F(a)$ is a multi-subspace of M for all $a\in A$. Therefore, M is a mutli-space.

This situation can be illustrated in the following Example.

Example 3.4. Let $M=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7\}$ be an initial universe such that M is a multi-space. Let $A_1=\{a_1,a_2,a_3,a_8\}$, $A_2=\{a_2,a_4,a_5,a_6,a_8\}$ and $A_3=\{a_5,a_7,a_8\}$ are set of parameters. Let $(F_1,A_1),(F_2,A_2)$ and (F_3,A_3) respectively be the soft sets over M as following:

$$F_1(a_1)=\{m_1,m_2,m_3\},$$

$$F_1(a_2)=\{m_4,m_5\},$$

$$F_1(a_3)=\{m_1,m_4,m_6,m_7\},$$

$$F_1(a_8)=\{m_2,m_4,m_6,m_7\}.$$

and

$$F_2(a_2)=\{m_1,m_2,m_3,m_6,m_7\},$$

$$F_2(a_4)=\{m_3,m_4,m_5,m_6\},$$

$$F_2(a_5)=\{m_2,m_4,m_5\},$$

$$\mathrm{F}_2(a_6) = \{\mathrm{m}_1, \mathrm{m}_7\},$$

$$F_2(a_8) = \{\mathrm{m}_1, \mathrm{m}_3, \mathrm{m}_5, \mathrm{m}_7\}.$$

Also

$$F_3(a_5) = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5,\},$$

$$F_3(a_7) = \{\mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_7\},$$

$$F_3(a_8) = \{\mathrm{m}_2\}.$$

Let $A = A_1 \cup A_2 \cup A_3 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$. Then the multi-soft space of $(F_1, A_1), (\mathrm{F}_2, A_2)$ and (F_3, A_3) is (F, A), where $(F, A) = (F_1, A_1) \cup_E (F_2, A_2) \cup_E (F_3, A_3)$ such that

$$F(a_1) = F_1(a_1) = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3\}, \text{ as } a_1 \in A_1 - A_2 \cup A_3,$$

$$F(a_2) = F_1(a_2) \cup \mathrm{F}_2(a_2) = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_6, \mathrm{m}_7\}, \text{ as } a_2 \in A_1 \cap A_2 - A_3,$$

$$F(a_3) = F_1(a_3) = \{\mathrm{m}_1, \mathrm{m}_4, \mathrm{m}_6, \mathrm{m}_7\} \text{ as } a_3 \in A_1 - A_2 \cup A_3,$$

$$F(a_4) = F_2(a_4) = \{\mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_6\}, \text{ as } a_4 \in A_2 - A_1 \cup A_3,$$

$$F(a_5) = F_2(a_5) \cup F_3(a_5) = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5,\}, \text{ as } a_5 \in A_2 \cap A_3 - A_1,$$

$$\mathrm{F}(a_6) = \mathrm{F}_2(a_6) = \{\mathrm{m}_1, \mathrm{m}_7\} \text{ as } a_6 \in A_2 - A_1 \cup A_3,$$

$$F(a_7) = F_3(a_7) = \{\mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_7\} \text{ as } a_7 \in A_3 - A_1 \cup A_2,$$

$$F(a_8) = F_1(a_8) \cup F_2(a_8) \cup F_3(a_8) = \{\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4, \mathrm{m}_5, \mathrm{m}_6, \mathrm{m}_7\} \text{ as } a_8 \in A_1 \cap A_2 \cap A_3.$$

Definition 3.5. Let (F, A) and (H, B) be two multi-soft spaces over $M_1 \cup M_2 \cup ... \cup M_n$. Then (F, A) is called a multi-soft subspace of (H, B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $a \in A$.

This can be denoted by $(F, A) \subset (H, B)$.

Similarly (F, A) is called a multi-soft superspace of (F, A) if (F, A) is a multi-soft subspace of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 3.6. Two multi-soft spaces (F, A) and (H, B) over $M_1 \cup M_2 \cup ... \cup M_n$ are called multi-soft multi-equal if (F, A) is a multi-soft subspace of (H, B) and (H, B) is a multi-soft subspace of (F, A).

Proposition 3.6. Let (F, A) and (K, B) be two multi-soft spaces over $M_1 \cup M_2 \cup ... \cup M_n$ such that $A \cap B \neq \phi$. Then their restricted intersection $(F, A) \cap_R (K, B) = (H, C)$ is also a multi-soft space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.7. The extended intersection of two multi-soft multi-spaces (F, A) and (K, B) over $M_1 \cup M_2 \cup ... \cup M_n$ is again a multi-soft multi-space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.8. Let (F, A) and (K, B) be two multi-soft multi-spaces over $M_1 \cup M_2 \cup ... \cup M_n$ such that $A \cap B \neq \phi$. Then their restricted union $(F, A) \cup_R (K, B) = (H, C)$ is also a multi-soft muti-space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.9. The extended union of two multi-soft multi-spaces (F, A) and (K, B) over $M_1 \cup M_2 \cup ... \cup M_n$ is again a multi-soft multi-space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.10. The AND operation of two multi-soft multi-spaces (F,A) and (K,B) over $M_1 \cup M_2 \cup ... \cup M_n$ is again a multi-soft mulit-space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.11. The OR operation of two multi-soft multi-spaces (F,A) and (K,B) over $M_1 \cup M_2 \cup ... \cup M_n$ is again a multi-soft multi-space over $M_1 \cup M_2 \cup ... \cup M_n$.

Proposition 3.12. The complement of a multi-soft space over a multi-space M is again a multi-soft space over M.

Prof. This is straightforward.

Definition 3.13. A multi-soft multi-space (F,A) over $M_1 \cup M_2 \cup ... \cup M_n$ is called absolute multi-soft multi-space if $F(a) = M_1 \cup M_2 \cup ... \cup M_n$ for all $a \in A$.

Proposition 3.14. Let (F,A), (G,B) and (H,C) are three multi-soft multi-spaces over $M_1 \cup M_2 \cup ... \cup M_n$. Then

1. $(F,A) \cup_E (G,B) \cup_E (H,C) = (F,A) \cup_E (G,B) \cup_E (H,C)$,
2. $(F,A) \cap_R (G,B) \cap_R (H,C) = (F,A) \cap_R (G,B) \cap_R (H,C)$.

Proposition 3.15. Let (F,A), (G,B) and (H,C) are three multi-soft multi-spaces over $M_1 \cup M_2 \cup ... \cup M_n$. Then

1. $(F,A) \wedge (G,B) \wedge (H,C) = (F,A) \wedge (G,B) \wedge (H,C)$,
2. $(F,A) \vee (G,B) \vee (H,C) = (F,A) \vee (G,B) \vee (H,C)$.

Conclusion

In this paper, we introduced multi-soft spaces which is a first attempt to study the multi-spaces in the context of soft sets. Multi-soft spaces are more rich structure than the multi-spaces which represent different fields in an approximated unifying field. We also studied some properties of multi-soft spaces. A lot of further research can do in the future in this area. In the future, one can define the algebraic structures of multi-soft spaces.

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The Neutrosophic Entropy and its Five Components

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Abstract. This paper presents two variants of penta-valued representation for neutrosophic entropy. The first is an extension of Kaufman's formula and the second is an extension of Kosko's formula.

Based on the primary three-valued information represented by the degree of truth, degree of falsity and degree of neutrality there are built some penta-valued representations that better highlights some specific features of neutrosophic entropy. Thus, we highlight five features of neutrosophic uncertainty such as ambiguity, ignorance, contradiction, neutrality and saturation. These five features are supplemented until a seven partition of unity by adding two features of neutrosophic certainty such as truth and falsity.

The paper also presents the particular forms of neutrosophic entropy obtained in the case of bifuzzy representations, intuitionistic fuzzy representations, paraconsistent fuzzy representations and finally the case of fuzzy representations.

Keywords: Neutrosophic information, neutrosophic entropy, neutrosophic uncertainty, ambiguity, contradiction, neutrality, ignorance, saturation.

1 Introduction

Neutrosophic representation of information was proposed by Smarandache [10], [11], [12] as an extension of fuzzy representation proposed by Zadeh [16] and intuitionistic fuzzy representation proposed by Atanassov [1], [2]. Primary neutrosophic information is defined by three parameters: degree of truth μ, degree of falsity ν and degree of neutrality ω.

Fuzzy representation is described by a single parameter, degree of truth μ, while the degree of falsity ν has a default value calculated by negation formula :

$$\nu = 1-\mu \tag{1.1}$$

and the degree of neutrality has a default value that is $\omega = 0$.

Fuzzy intuitionistic representation is described by two explicit parameters, degree of truth μ and degree of falsity ν, while the degree of neutrality has a default value that is $\omega = 0$.

Atanassov considered the incomplete variant taking into account that $\mu+\nu \le 1$. This allowed defining the index of ignorance (incompleteness) with the formula:

$$\pi = 1-\mu-\nu \tag{1.2}$$

thus obtaining a consistent representation of information, because the sum of the three parameters is 1, namely:

$$\mu+\pi+\nu = 1 \tag{1.3}$$

Hence, we get for neutrality the value $\omega = 0$.

For paraconsistent fuzzy information where $\mu+\nu \ge 1$, the index of contradiction can be defined:

$$\kappa = \mu+\nu-1 \tag{1.4}$$

and for neutrality it results: $\omega = 0$.

For bifuzzy information that is defined by the pair (μ,ν), the net truth, the index of ignorance (incompleteness), index of contradiction and index of ambiguity can be defined, by:

$$\tau = \mu-\nu \tag{1.5}$$

$$\pi = 1-\min(\mu+\nu,1) \tag{1.6}$$

$$\kappa = \max(\mu+\nu,1)-1 \tag{1.7}$$

$$\alpha = 1-|\mu-\nu|-|\mu+\nu-1| \tag{1.8}$$

Among of these information parameters the most important is the construction of some measures for information entropy or information uncertainty. This paper is dedicated to the construction of neutrosophic entropy formulae.

In the next, section 2 presents the construction of two variants of the neutrosophic entropy. This construction is based on two similarity formulae; Section 3 presents a penta-valued representation of neutrosophic entropy based on ambiguity, ignorance, contradiction, neutrality and saturation; Section 4 outlines some conclusions.

2 The Neutrosophic Entropy

For neutrosophic entropy, we will trace the Kosko idea for fuzziness calculation [5]. Kosko proposed to measure this information feature by a similarity function between the distance to the nearest crisp element and the distance to the farthest crisp element. For neutrosophic information the two crisp elements are $(1,0,0)$ and $(0,0,1)$. We consider the following vector: $V = (\mu-\nu, \mu+\nu-1, \omega)$. For $(1,0,0)$ and $(0,0,1)$ it results: $V_T = (1,0,0)$ and $V_F = (-1,0,0)$. We will compute the distances:

$$D(V,V_T) = |\mu-\nu-1| + |\mu+\nu-1| + \omega \qquad (2.1)$$

$$D(V,V_F) = |\mu-\nu+1| + |\mu+\nu-1| + \omega \qquad (2.2)$$

The neutrosophic entropy will be defined by the similarity between these two distances.

Using the Czekanowskyi formula [3] it results the similarity S_C and the neutrosophic entropy E_C:

$$S_C = 1 - \frac{|D(V,V_T) - D(V,V_F)|}{D(V,V_T) + D(V,V_F)} \qquad (2.3)$$

$$E_C = 1 - \frac{|\mu-\nu|}{1+|\mu+\nu-1|+\omega} \qquad (2.4)$$

or in terms of $\tau, \omega, \pi, \kappa$:

$$E_C = 1 - \frac{|\tau|}{1+\pi+\kappa+\omega} \qquad (2.5)$$

The neutrosophic entropy defined by (2.4) can be particularized for the following cases:

For $\omega = 0$, it result the bifuzzy entropy, namely:

$$E_C = 1 - \frac{|\mu-\nu|}{1+|\mu+\nu-1|} \qquad (2.6)$$

For $\omega = 0$ and $\mu+\nu \le 1$ it results the intuitionistic fuzzy entropy proposed by Patrascu [8], namely:

$$E_C = 1 - \frac{|\mu-\nu|}{1+\pi} \qquad (2.7)$$

For $\omega = 0$ and $\mu+\nu \ge 1$ it results the paraconsistent fuzzy entropy, namely:

$$E_C = 1 - \frac{|\mu-\nu|}{1+\kappa} \qquad (2.8)$$

For $\mu+\nu = 1$ and $\omega = 0$, it results the fuzzy entropy proposed by Kaufman [4], namely:

$$E_C = 1 - |2\mu-1| \qquad (2.9)$$

Using the Ruzicka formula [3] it result the formulae for the similarity S_R and the neutrosophic entropy E_R:

$$S_R = 1 - \frac{|D(V,V_T) - D(V,V_F)|}{\max\left(D(V,V_T), D(V,V_F)\right)} \qquad (2.10)$$

$$E_R = 1 - \frac{2|\mu-\nu|}{1+|\mu-\nu|+|\mu+\nu-1|+\omega} \qquad (2.11)$$

or its equivalent form:

$$E_R = \frac{1-|\mu-\nu|+|\mu+\nu-1|+\omega}{1+|\mu-\nu|+|\mu+\nu-1|+\omega} \qquad (2.12)$$

or in terms of $\tau, \omega, \pi, \kappa$:

$$E_R = 1 - \frac{2|\tau|}{1+|\tau|+\pi+\kappa+\omega} \qquad (2.13)$$

The neutrosophic entropy defined by (2.12) can be particularized for the following cases:

For $\omega = 0$, it results the bifuzzy entropy proposed by Patrascu [7], namely:

$$E_R = \frac{1-|\mu-\nu|+|\mu+\nu-1|}{1+|\mu-\nu|+|\mu+\nu-1|} \qquad (2.14)$$

For $\omega = 0$ and $\mu+\nu \le 1$ it results the intuitionistic fuzzy entropy proposed by Szmidt and Kacprzyk [14], [15], explicitly:

$$E_R = \frac{1-|\mu-\nu|+\pi}{1+|\mu-\nu|+\pi} \qquad (2.15)$$

For $\omega = 0$ and $\mu+\nu \ge 1$ it results the paraconsistent fuzzy entropy, explicitly :

$$E_R = \frac{1-|\mu-\nu|+\kappa}{1+|\mu-\nu|+\kappa} \qquad (2.16)$$

For $\mu+\nu = 1$ and $\omega = 0$, it results the fuzzy entropy proposed by Kosko [5], namely:

$$E_R = \frac{1-|2\mu-1|}{1+|2\mu-1|} \qquad (2.17)$$

We notice that the neutrosophic entropy is a strictly decreasing function in $|\mu-\nu|$ and non-decreasing in ω and in $|\mu+\nu-1|$.

The neutrosophic entropy verify the following conditions:

(i) $E(\mu,\omega,\nu)=0$ if (μ,ω,ν) is a crisp value, namely if $(\mu,\omega,\nu)\in\{(1,0,0),(0,0,1)\}$.

(ii) $E(\mu,\omega,\nu)=1$ if $\mu=\nu$.

(iii) $E(\mu,\omega,\nu)=E(\nu,\omega,\mu)$

(iv) $E(\mu,\omega_1,\nu)\le E(\mu,\omega_2,\nu)$ if $\omega_1\le\omega_2$.

(v) $E(\tau_1,\omega,\pi,\kappa)<E(\tau_2,\omega,\pi,\kappa)$ if $|\tau_1|>|\tau_2|$.

(vi) $E(\tau,\omega_1,\pi,\kappa)\le E(\tau,\omega_2,\pi,\kappa)$ if $\omega_1\le\omega_2$.

(vii) $E(\tau,\omega,\pi_1,\kappa)\le E(\tau,\omega,\pi_2,\kappa)$ if $\pi_1\le\pi_2$.

(viii) $E(\tau,\omega,\pi,\kappa_1)\le E(\tau,\omega,\pi,\kappa_2)$ if $\kappa_1\le\kappa_2$.

3 Penta-valued Representation of Neutrosophic Entropy

The five components of neutrosophic entropy will be: ambiguity a, ignorance u, contradiction c, neutrality n and saturation s [6]. We construct formulas for these features both for the variant defined by formula (2.4) and for the variant defined by formula (2.12). For each decomposition, among the four components u,n,s,c, always two of them will be zero.

In the neutrosophic cube, we consider the entropic rectangle defined by the points: $U=(0,0,0)$, $N=(0,1,0)$, $S=(1,1,1)$, $C=(1,0,1)$.

In addition, we consider the point $A=(0.5,0,0.5)$ which is located midway between the point U (*unknown*) and the point C (*contradiction*).

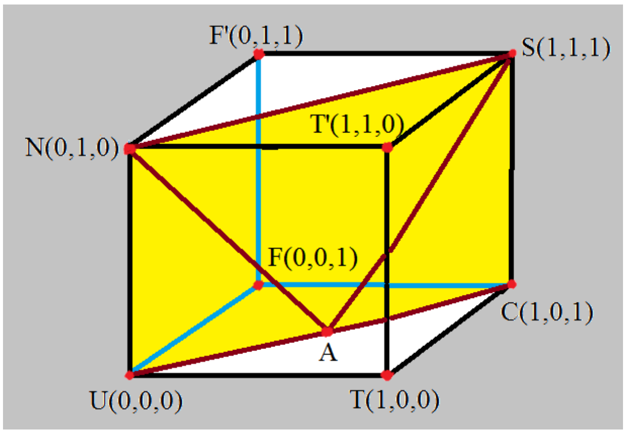

Figure 1. The neutrosophic cube TUFCT'NF'S and its entropic rectangle UNSC.

Also, this point is located midway between the point F (*false*) and the point T (*true*). In other words, the point A (*ambiguous*) represents the center of the Belnap square TUFC (*true-uknown-false-contradictory*).

If the projection of the point (μ,ω,ν) on the rectangle *UNSC* (*unknown-neutral-saturated-contradictory*) is inside the triangle *UNA* (*unknown-neutral-ambiguous*) then saturation and contradiction will be zero $(s=c=0)$, if the projection is inside the triangle *ANS* (*ambigouous-neutral-saturated*) then ignorance and contradiction will be zero $(u=c=0)$ and if the projection is inside the triangle *ASC* (*ambiguous-saturated-contradictory*) then ignorance and neutrality will be zero $(u=n=0)$.

We consider first the first version defined by formula (2.4), namely:

$$E_C=1-\frac{|\mu-\nu|}{1+|\mu+\nu-1|+\omega} \tag{3.1}$$

The five features must verify the following condition:

$$a+u+c+n+s=E_C \tag{3.2}$$

First, we start with ambiguity formula defined by:

$$a=\frac{1-|\mu-\nu|-|\mu+\nu-1|}{1+|\mu+\nu-1|+\omega} \tag{3.3}$$

The prototype for the ambiguity is the point $(0.5,0,0.5)$ and the formula (3.3) defines the similarity between the points (μ,ω,ν) and $(0.5,0,0.5)$.

Then, the other four features will verify the condition:

$$u+c+n+s=\frac{2|\mu+\nu-1|+\omega}{1+|\mu+\nu-1|+\omega} \tag{3.4}$$

We analyze three cases depending on the order relation among the three parameters ω,π,κ where π is the bifuzzy *ignorance and* κ is the bifuzzy *contradiction* [9]. It is obvious that:

$$\pi\cdot\kappa=0 \tag{3.5}$$

Case (I)

$$\pi\ge\omega\ge\kappa=0 \tag{3.6}$$

It results for ignorance and contradiction these formulas:

$$u=\frac{2\pi-2\omega}{1+\omega+\pi+\kappa} \tag{3.7}$$

$$c=0 \tag{3.8}$$

and for saturation and neutrality:

$$s=0 \tag{3.9}$$

$$n = \frac{3\omega}{1+\omega+\pi+\kappa} \tag{3.10}$$

Case (II)

$$\kappa \ge \omega \ge \pi = 0 \tag{3.11}$$

It results for ignorance and contradiction these formulas:

$$u = 0 \tag{3.12}$$

$$c = \frac{2\kappa - 2\omega}{1+\omega+\pi+\kappa} \tag{3.13}$$

then for neutrality and saturation it results:

$$n = 0 \tag{3.14}$$

$$s = \frac{3\omega}{1+\omega+\pi+\kappa} \tag{3.15}$$

Case (III)

$$\omega \ge \max(\pi, \kappa) \tag{3.16}$$

It results for ignorance and contradiction these values:

$$u = 0 \tag{3.17}$$

$$c = 0 \tag{3.18}$$

Next we obtain:

$$s + n = \frac{(\omega-\pi-\kappa)+3\pi+3\kappa}{1+\omega+\pi+\kappa} \tag{3.19}$$

The sum (3.19) can be split in the following manner:

$$n = \frac{\frac{\omega-\pi-\kappa}{2}+3\pi}{1+\omega+\pi+\kappa} \tag{3.20}$$

$$s = \frac{\frac{\omega-\pi-\kappa}{2}+3\kappa}{1+\omega+\pi+\kappa} \tag{3.21}$$

Combining formulas previously obtained, it results the final formulas for the five components of the neutrosophic entropy defined by (3.1):

ambiguity

$$a = \frac{1-|\mu-\nu|-\pi-\kappa}{1+\omega+\pi+\kappa} \tag{3.22}$$

ignorance

$$u = 2\cdot\frac{\max(\pi,\omega)-\omega}{1+\omega+\pi+\kappa} \tag{3.23}$$

The prototype for *ignorance* is the point (0,0,0) and formula (3.23) defines the similarity between the points (μ,ω,ν) and $(0,0,0)$.

contradiction

$$c = 2\cdot\frac{\max(\kappa,\omega)-\omega}{1+\omega+\pi+\kappa} \tag{3.24}$$

The prototype for *contradiction* is the point (1,0,1) and formula (3.24) defines the similarity between the points (μ,ω,ν) and $(1,0,1)$.

neutrality

$$n = \frac{\frac{\max(\omega-\pi-\kappa,0)}{2}+3\min(\omega,\pi)}{1+\omega+\pi+\kappa} \tag{3.25}$$

The prototype for *neutraliy* is the point $(0,1,0)$ and formula (3.25) defines the similarity between the points (μ,ω,ν) and $(0,1,0)$.

saturation

$$s = \frac{\frac{\max(\omega-\pi-\kappa,0)}{2}+3\min(\omega,\kappa)}{1+\omega+\pi+\kappa} \tag{3.26}$$

The prototype for *saturation* is the point (1,1,1) and formula (3.26) defines the similarity between the points (μ,ω,ν) and $(1,1,1)$.

In addition we also define:

The index of truth

$$t = \frac{\max(\mu,\nu)-\nu}{1+\omega+\pi+\kappa} \tag{3.27}$$

The prototype for the *truth* is the point $(1,0,0)$ and formula (3.27) defines the similarity between the points (μ,ω,ν) and $(1,0,0)$.

The index of falsity

$$f = \frac{\max(\nu,\mu)-\mu}{1+\omega+\pi+\kappa} \tag{3.28}$$

The prototype for the *falsity* is the point (0,0,1) and formula (3.28) defines the similarity between the points (μ,ω,ν) and $(0,0,1)$.

Thus, we get the following hepta-valued partition for the neutrosophic information:

$$t + f + a + u + c + n + s = 1 \tag{3.29}$$

Formula (3.29) shows that neutrosophic information can be structured so that it is related to a logic where the information could be: true, false, ambiguous, unknown, contradictory, neutral or saturated [6] (see Figure 2).

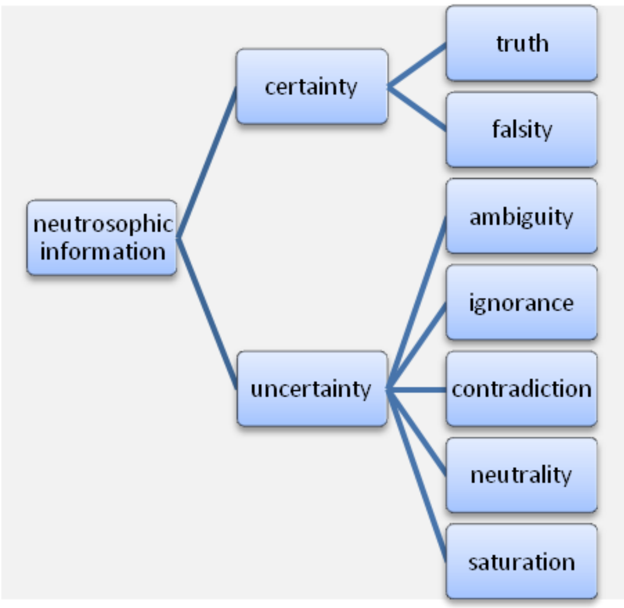

Figure 2. The structure of the neutrosophic information.

Next, we also deduce the five components for the variant defined by the formula (2.12).

Using (1.5) formula (2.12) becomes:

$$E_R = 1 - \frac{2|\tau|}{1+|\tau|+\pi+\kappa+\omega} \tag{3.30}$$

or

$$E_R = \frac{1-|\tau|+\pi+\kappa+\omega}{1+|\tau|+\pi+\kappa+\omega} \tag{3.31}$$

In this case, the five formulas are obtained from formulas (3.22) - (3.26) by changing the denominator, thus:

ambiguity

$$a = \frac{1-|\tau|-\pi-\kappa}{1+|\tau|+\omega+\pi+\kappa} \tag{3.32}$$

ignorance

$$u = \frac{2\max(\pi-\omega,0)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.33}$$

contradiction

$$c = \frac{2\max(\kappa-\omega,0)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.34}$$

neutrality

$$n = \frac{\frac{\max(\omega-\pi-\kappa,0)}{2}+3\min(\omega,\pi)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.35}$$

saturation

$$s = \frac{\frac{\max(\omega-\pi-\kappa,0)}{2}+3\min(\omega,\kappa)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.36}$$

From (3.27) and (3.28) it results:

Index of truth

$$t = \frac{2\max(\tau,0)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.37}$$

Index of falsity

$$f = \frac{2\max(-\tau,0)}{1+|\tau|+\omega+\pi+\kappa} \tag{3.38}$$

Also in this case, the 7 parameters verify the condition of partition of unity defined by formula (3.29).

For bifuzzy information when $\omega = 0$, neutrality and saturation are zero and formula (3.29) becomes:

$$t+f+a+u+c=1 \tag{3.39}$$

Thus, we obtained for bifuzzy information a penta-valued representation. We can conclude that the bifuzzy information is related to a penta-valued logic where the information could be: true, false, ambiguous, unknown and contradictory [6] (see Figure 3).

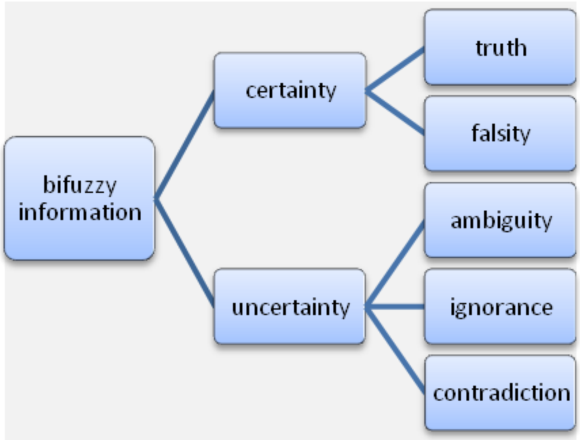

Figure 3. The structure of the bifuzzy information.

For intuitionistic fuzzy information when $\omega = 0$ and $\mu + \nu \le 1$, neutrality, saturation and contradiction are zero and formula (3.29) becomes:

$$t + f + a + u = 1 \tag{3.40}$$

Thus, we obtained for intuitionistic fuzzy information a tetra-valued representation. We can conclude that the intuitionistic fuzzy information is related to a tetra-valued logic where the information could be: true, false, ambiguous and unknown [6] (see Figure 4).

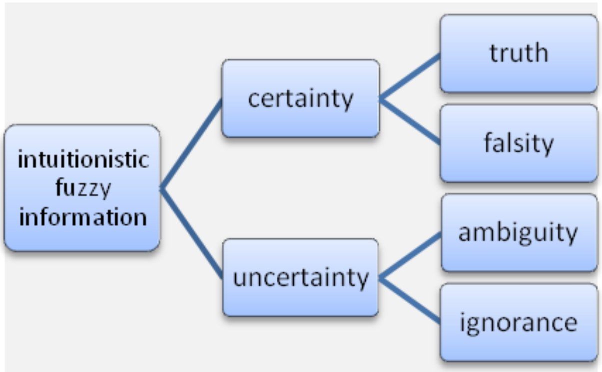

Figure 4. The structure of the intuitionistic fuzzy information.

For paraconsistent fuzzy information when $\omega = 0$ and $\mu + \nu > 1$, neutrality, saturation and ignorance are zero and formula (3.29) becomes:

$$t + f + a + c = 1 \tag{3.41}$$

The paraconsistent fuzzy information is related to a tetra-valued logic where the information could be: true, false, ambiguous and contradictory [6] (see Figure 5).

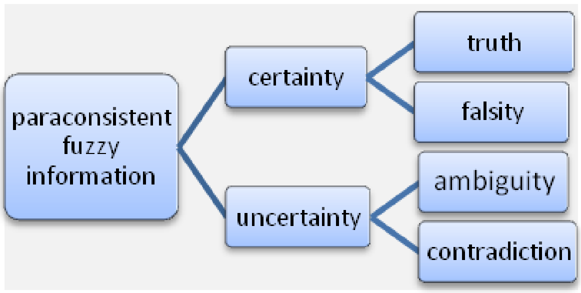

Figure 5. The structure of the paraconsistent fuzzy information.

For fuzzy information when $\omega = 0$ and $\mu + \nu = 1$, neutrality, saturation ignorance and contradiction are zero and formula (3.29) becomes:

$$t + f + a = 1 \tag{3.42}$$

Thus, we obtained for fuzzy information a three-valued representation. We can conclude that the fuzzy information is related to a three-valued logic where the information could be: true, false and ambiguous [6] (see Figure 6).

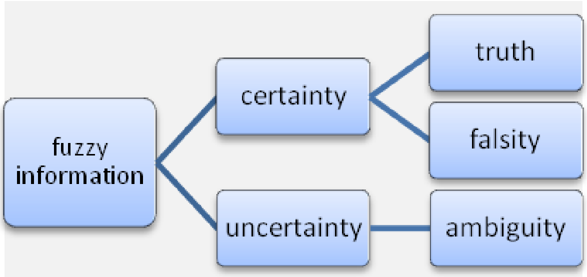

Figure 6. The structure of the fuzzy information.

4 Conclusion

In this article, there are constructed two formulas for neutrosophic entropy calculating. For each of these, five components are defined and they are related to the following features of neutrosophic uncertainty: ambiguity, ignorance, contradiction, neutrality and saturation. Also, two components are built for neutrosophic certainty: truth and falsity. All seven components, five of the uncertainty and two of certainty form a partition of unity. Building these seven components, primary neutrosophic information is transformed in a more nuanced representation .

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Medical Diagnosis Using Distance-Based Similarity Measures of Single Valued Neutrosophic Multisets

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Abstract. This paper proposes a generalized distance measure and its similarity measures between single valued neutrosophic multisets (SVNMs). Then, the similarity measures are applied to a medical diagnosis problem with incomplete, indeterminate and inconsistent information. This diagnosis method can deal with the diagnosis problem with indeterminate and inconsistent information which cannot be handled by the diagnosis method based on intuitionistic fuzzy multisets (IFMs).

Keywords: *Single valued neutrosophic multiset, distance measure, similarity measure, medical diagnosis*.

1 Introduction

The vagueness or uncertainty representation of imperfect knowledge becomes a crucial issue in the areas of computer science and artificial intelligence. To deal with the uncertainty, the fuzzy set proposed by Zadeh [1] allows the uncertainty of a set with a membership degree between 0 and 1. Then, Atanassov [2] introduced an intuitionistic Fuzzy set (IFS) as a generalization of the Fuzzy set. The IFS represents the uncertainty with respect to both membership and non-membership. However, it can only handle incomplete information but not the indeterminate and inconsistent information which exists commonly in real situations. Therefore, Smarandache [3] proposed a neutrosophic set. It can independently express truth-membership degree, indeterminacy-membership degree, and false-membership degree and deal with incomplete, indeterminate, and inconsistent information. After that, Wang et al [4] introduced a single valued neutrosophic set (SVNS), which is a subclass of the neutrosophic set. SVNS is a generalization of the concepts of the classic set, fuzzy set, and IFS. The SVNS should be used for better representation as it is a more natural and justified estimation [4]. All the factors described by the SVNS are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world. For example, for a given proposition "Movie X would be hit", in this situation human brain certainly cannot generate precise answers in terms of yes or no, as indeterminacy is the sector of unawareness of a proposition's value between truth and falsehood. Obviously, the neutrosophic components are best fit in the representation of indeterminacy and inconsistent information. Recently, Ye [5-7] proposed some similarity measures of SVNSs and applied them to decision making and clustering analysis.

Based on multiset theory, Yager [8] introduced a fuzzy multiset concept, which allows the repeated occurrences of any element. Thus, the fuzzy multiset can occur more than once with the possibility of the same or different membership values. Then, Shinoj and Sunil [9] extended the fuzzy multiset to the intuitionistic fuzzy multiset (IFM) and presented some basic operations and a distance measure for IFMs, and then applied the distance measure to medical diagnosis problem. Rajarajeswari and Uma [10] put forward the Hamming distance-based similarity measure for IFMs and its application in medical diagnosis. Recently, Ye et al. [11] presented a single valued neutrosophic multiset (SVNM) as a generalization of IFM and the Dice similarity measure between SVNMs, and then applied it to medical diagnosis. Based on SVNMs, this paper further develops a generalized distance measure and the distance-based similarity measures between SVNMs, and then applies the similarity measures to medical diagnosis. To do so, the rest of the article is organized as follows. Section 2 introduces some concepts and basic operations of SVNSs and SVNMSs. Sections 3 presents a generalized distance and its similarity measures between SVNMs and investigates their properties. In Section 4, the similarity measures are applied to medicine diagnosis. Conclusions and further research are contained in Section 5.

2 Preliminaries

2.1 Some concepts of SVNSs

Smarandache [3] originally presented the concept of a neutrosophic set. A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \to\ ]^-0, 1^+[$, $I_A(x): X \to\ ]^-0, 1^+[$, and $F_A(x)$:

$X \to]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is no restriction, i.e. $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

However, Smarandache [3] introduced the neutrosophic set from philosophical point of view. Therefore, it is difficult to apply the neutrosophic set to practical problems. To easily apply in science and engineering areas, Wang et al. [4] introduced the concept of SVNs, which is a subclass of the neutrosophic set and gave the following definition.

Definition 1 [4]. Let X be a universal set. A SVNs A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by the following form:

$$\langle \quad _A \quad _A \quad _A \rangle \quad ,$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For two SVNs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$, there are the following relations [4]:

(1) Complement:

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\};$$

(2) Inclusion:

$A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;

(3) Equality:

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:

$$A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle \mid x \in X\};$$

(5) Intersection:

$$A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)\rangle \mid x \in X\};$$

(6) Addition:

$$A + B = \left\{\left\langle \begin{array}{l} x, T_A(x) + T_B(x) - T_A(x)T_B(x), \\ I_A(x)I_B(x), F_A(x)F_B(x) \end{array}\right\rangle \mid x \in X\right\};$$

(7) Multiplication:

$$A \times B = \left\{\left\langle \begin{array}{l} x, T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), \\ F_A(x) + F_B(x) - F_A(x)F_B(x) \end{array}\right\rangle \mid x \in X\right\}.$$

2.2 Some concepts of SVNMs

As a generalization of the concept of IFM, a concept of SVNM and some basic operational relations for SVNMs [11] are introduced below.

Definition 2 [11]. Let X be a nonempty set with generic elements in X denoted by x. A single valued neutrosophic multiset (SVNM) A drawn from X is characterized by three functions: count truth-membership of CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x): X \to Q$, $CI_A(x): X \to Q$, $CF_A(x): X \to Q$ for $x \in X$, where Q is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by

$$A = \left\{\left\langle \begin{array}{l} x, (T_A^1(x), T_A^2(x), \ldots, T_A^q(x)), \\ (I_A^1(x), I_A^2(x), \ldots, I_A^q(x)), \\ (F_A^1(x), F_A^2(x), F_A^q(x)) \end{array}\right\rangle \mid x \in X\right\},$$

where the truth-membership sequence $(T_A^1(x), T_A^2(x), \ldots, T_A^q(x))$, the indeterminacy-membership sequence $(I_A^1(x), I_A^2(x), \ldots, I_A^q(x))$, and the falsity-membership sequence $(F_A^1(x), F_A^2(x), \ldots, F_A^q(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x)$, $I_A^i(x)$, $F_A^i(x) \in [0, 1]$ satisfies the condition $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, a SVNM A can be denoted by the following simplified form:

$$A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x)\rangle \mid x \in X, i = 1, 2, \ldots, q\}.$$

Definition 3 [11]. The length of an element x in a SVNM is defined as the cardinality of $CT_A(x)$ or $CI_A(x)$, or $CF_A(x)$ and is denoted by $L(x: A)$. Then $L(x: A) = |CT_A(x)| = |CI_A(x)| = |CF_A(x)|$.

Definition 4 [11]. Let A and B be two SVNMs in X, then the length of an element x in A and B is denoted by $l_x = L(x: A, B) = \max\{L(x: A), L(x: B)\}$.

Example 1. Consider two SVNMs in the set $X = \{x, y, z\}$:

A = {<x, (0.3, 0.2), (0.4, 0.3), (0.6, 0.8)>, <y, (0.5, 0.4, 0.3), (0.1, 0.2, 0.3), (0.3, 0.4, 0.5)>},

B = {<x, (0.3), (0.4), (0.6) >, <z, (0.5, 0.4, 0.3, 0.2), (0.0, 0.1, 0.2, 0.3), (0.2, 0.3, 0.4, 0.5)>}.

Thus, there are $L(x: A) = 2$, $L(y: A) = 3$, $L(z: A) = 0$; $L(x: B) = 1$, $L(y: B) = 0$, $L(z: B) = 4$, $l_x = L(x: A, B) = 2$, $l_y = L(y: A, B) = 3$, and $l_z = L(z: A, B) = 4$.

For convenient operation between SVNMs A and B in X, one can make $L(x: A) = L(x: B)$ by appending sufficient minimal number for the truth-membership value and sufficient maximum number for the indeterminacy-membership and falsity-membership values.

Definition 5 [11]. Let $A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x, T_B^i(x), I_B^i(x), F_B^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ be any two SVNMs in X. Then, there are the following relations:

(1) Inclusion: $A \subseteq B$ if and only if $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$, $F_A^i(x) \geq F_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

(2) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(3) Complement:
$$A^c = \left\{\left\langle x, F_A^i(x), \left(1 - I_A(x)\right)^i, T_A^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\};$$

(4) Union:
$$A \cup B = \left\{\left\langle \begin{array}{l} x, T_A^i(x) \vee T_B^i(x), \\ I_A^i(x) \wedge I_B^i(x), \\ F_A^i(x) \wedge F_B^i(x) \end{array}\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\};$$

(5) Intersection:
$$A \cap B = \left\{\left\langle \begin{array}{l} x, T_A^i(x) \wedge T_B^i(x), \\ I_A^i(x) \vee I_B^i(x), \\ F_A^i(x) \vee F_B^i(x) \end{array}\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\}.$$

For convenience, we can use $a = \langle (T^1, T^2, \ldots, T^q), (I^1, I^2, \ldots, I^q), (F^1, F^2, \ldots, F^q)\rangle$ to represent an element in a SVNM A and call it a single valued neutrosophic multiset value (SVNMV).

Definition 6. Let $a_1 = \langle (T_1^1, T_1^2, \ldots, T_1^q), (I_1^1, I_1^2, \ldots, I_1^q), (F_1^1, F_1^2, \ldots, F_1^q)$ and $a_2 = \langle (T_2^1, T_2^2, \ldots, T_2^q), (I_2^1, I_2^2, \ldots, I_2^q), (F_2^1, F_2^2, \ldots, F_2^q)$ be two SVNMVs and $\lambda \geq 0$, then the operational rules of SVNMVs are defined as follows:

(1)
$$a_1 \oplus a_2 = \left\langle \begin{array}{l} (T_1^1 + T_2^1 - T_1^1 T_2^1, \\ T_1^2 + T_2^2 - T_1^2 T_2^2, \\ \ldots, T_1^q + T_2^q - T_1^q T_2^q), \\ (I_1^1 I_2^1, I_1^2 I_2^2, \ldots, I_1^q I_2^q), \\ (F_1^1 F_2^1, F_1^2 F_2^2, \ldots, F_1^q F_2^q) \end{array} \right\rangle;$$

(2)
$$a_1 \otimes a_2 = \left\langle \begin{array}{l} (T_1^1 T_2^1, T_1^2 T_2^2, \ldots, T_1^q T_2^q), \\ (I_1^1 + I_2^1 - I_1^1 I_2^1, I_1^2 + I_2^2 - I_1^2 I_2^2, \\ \ldots, I_1^q + I_2^q - I_1^q I_2^q), \\ (F_1^1 + F_2^1 - F_1^1 F_2^1, F_1^2 + F_2^2 - F_1^2 F_2^2, \\ \ldots, F_1^q + F_2^q - F_1^q F_2^q) \end{array} \right\rangle;$$

(3)
$$\lambda a_1 = \left\langle \begin{array}{l} \left(1 - \left(1 - T_1^1\right)^\lambda, 1 - \left(1 - T_1^2\right)^\lambda, \ldots, 1 - \left(1 - T_1^q\right)^\lambda\right), \\ \left(\left(I_i^1\right)^\lambda, \left(I_i^2\right)^\lambda, \ldots, \left(I_i^q\right)^\lambda\right), \left(\left(F_i^1\right)^\lambda, \left(F_i^2\right)^\lambda, \ldots, \left(F_i^q\right)^\lambda\right) \end{array} \right\rangle;$$

(4)
$$a_1^\lambda = \left\langle \begin{array}{l} \left(\left(T_i^1\right)^\lambda, \left(T_i^2\right)^\lambda, \ldots, \left(T_i^q\right)^\lambda\right), \\ \left(1 - \left(1 - I_1^1\right)^\lambda, 1 - \left(1 - I_1^2\right)^\lambda, \ldots, 1 - \left(1 - I_1^q\right)^\lambda\right), \\ \left(1 - \left(1 - F_1^1\right)^\lambda, 1 - \left(1 - F_1^2\right)^\lambda, \ldots, 1 - \left(1 - F_1^q\right)^\lambda\right) \end{array} \right\rangle.$$

3 Distance and similarity measures of SVNMs

The distance measure and similarity measure are usually used in real science and engineering applications. Therefore, the section proposes a generalized distance measure between SVNMs and the distance-based similarity measures between SVNMs. However, the distance and similarity measures in SVNSs are considered for truth-membership, indeterminacy-membership, and falsity-membership functions only once, while the distance and similarity measures in SVNMs should be considered more than once because their functions are multi-values.

Definition 7. Let $A = \{\langle x_j, T_A^i(x_j), I_A^i(x_j), F_A^i(x_j) \mid x_j \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x_j, T_B^i(x_j), I_B^i(x_j), F_B^i(x_j) \mid x_j \in X, i = 1, 2, \ldots, q\}$ be any two SVNMs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, we define the following generalized distance measure between A and B:

$$D_p(A, B) = \left[\frac{1}{n}\sum_{j=1}^{n}\frac{1}{3l_j}\sum_{i=1}^{l_j}\left(\begin{array}{l} \left|T_A^i(x_j) - T_B^i(x_j)\right|^p + \\ \left|I_A^i(x_j) - I_B^i(x_j)\right|^p + \\ \left|F_A^i(x_j) - F_A^i(x_j)\right|^p \end{array}\right)\right]^{1/p}, \quad (1)$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$. If $p = 1, 2$, Eq. (1) reduces to the Hamming distance and the Euclidean distance, which are usually applied to real science and engineering areas.

Then, the defined distance measure has the following Proposition 1:

Proposition 1. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the generalized distance measure $D_p(A, B)$ should satisfy the following properties (D1-D4):

(D1) $0 \le D_p(A, B) \le 1$;

(D2) $D_p(A, B) = 0$ if and only if $A = B$;

(D3) $D_p(A, B) = D_p(B, A)$;

(D4) If C is a SVNM in X and $A \subseteq B \subseteq C$, then $D_p(A, C) \le D_p(A, B) + D_p(B, C)$ for $p > 0$.

Proofs:

(D1) Proof is straightforward.

(D2) If $A = B$, then there are $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for $i = 1, 2, \ldots, l_j$, $j = 1, 2, \ldots, n$, and $x_j \in X$. Hence $\left|T_A^i(x_j) - T_B^i(x_j)\right|^p = 0$, $\left|I_A^i(x_j) - I_B^i(x_j)\right|^p = 0$, and $\left|F_A^i(x_j) - F_B^i(x_j)\right|^p = 0$. Thus $D_p(A, B) = 0$. When $D_p(A, B) = 0$, there are $\left|T_A^i(x_j) - T_B^i(x_j)\right|^p = 0$, $\left|I_A^i(x_j) - I_B^i(x_j)\right|^p = 0$, and $\left|F_A^i(x_j) - F_B^i(x_j)\right|^p = 0$. Then, one can obtain $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for $i = 1, 2, \ldots, l_j$, $j = 1, 2, \ldots, n$, and $x_j \in X$. Hence $A = B$.

(D3) Proof is straightforward.

(D4) Since $T_A^i(x_j) - T_c^i(x_j) = T_A^i(x_j) - T_B^i(x_j) + T_B^i(x_j) - T_c^i(x_j)$, It is obvious that

$$\left|T_A^i(x_j) - T_c^i(x_j)\right| \le \left|T_A^i(x_j) - T_B^i(x_j)\right| + \left|T_B^i(x_j) - T_C^i(x_j)\right|,$$
$$\left|I_A^i(x_j) - I_c^i(x_j)\right| \le \left|I_A^i(x_j) - I_B^i(x_j)\right| + \left|I_B^i(x_j) - I_C^i(x_j)\right|,$$
$$\left|F_A^i(x_j) - F_c^i(x_j)\right| \le \left|F_A^i(x_j) - F_B^i(x_j)\right| + \left|F_B^i(x_j) - F_C^i(x_j)\right|.$$

For $p > 0$, we have

$$\left|T_A^i(x_j) - T_c^i(x_j)\right|^p \le \left|T_A^i(x_j) - T_B^i(x_j)\right|^p + \left|T_B^i(x_j) - T_C^i(x_j)\right|^p,$$
$$\left|I_A^i(x_j) - I_c^i(x_j)\right|^p \le \left|I_A^i(x_j) - I_B^i(x_j)\right|^p + \left|I_B^i(x_j) - I_C^i(x_j)\right|^p,$$
$$\left|F_A^i(x_j) - F_c^i(x_j)\right|^p \le \left|F_A^i(x_j) - F_B^i(x_j)\right|^p + \left|F_B^i(x_j) - F_C^i(x_j)\right|^p.$$

Considering the above inequalities and Eq. (1), one can obtain that $D_p(A, C) \le D_p(A, B) + D_p(B, C)$ for $p > 0$.

Therefore, the proofs of these properties are completed. □

Based on the relationship between the distance measure and the similarity measure, we can introduce two distance-based similarity measures between A and B:

$$S_1(A,B) = 1 - D_p(A,B)$$
$$= 1 - \left[\frac{1}{n}\sum_{j=1}^{n}\frac{1}{3l_j}\sum_{i=1}^{l_j}\begin{pmatrix}\left|T_A^i(x_j) - T_B^i(x_j)\right|^p + \\ \left|I_A^i(x_j) - I_B^i(x_j)\right|^p + \\ \left|F_A^i(x_j) - F_A^i(x_j)\right|^p\end{pmatrix}\right]^{1/p}, \quad (2)$$

$$S_2(A,B) = \frac{1 - D_p(A,B)}{1 + D_p(A,B)}$$
$$= \frac{1 - \left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{3l_j}\sum_{j=1}^{l_j}\begin{pmatrix}\left|T_A^j(x_i) - T_B^j(x_i)\right|^p + \\ \left|I_A^j(x_i) - I_B^j(x_i)\right|^p + \\ \left|F_A^j(x_i) - F_A^j(x_i)\right|^p\end{pmatrix}\right]^{1/p}}{1 + \left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{3l_j}\sum_{j=1}^{l_j}\begin{pmatrix}\left|T_A^j(x_i) - T_B^j(x_i)\right|^p + \\ \left|I_A^j(x_i) - I_B^j(x_i)\right|^p + \\ \left|F_A^j(x_i) - F_A^j(x_i)\right|^p\end{pmatrix}\right]^{1/p}}. \quad (3)$$

According to Proposition 1 for the defined distance measure and the relationship between the distance measure and the similarity measure, it is easy to obtain the following Proposition 2 for the distance-based similarity measures.

Proposition 2. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the distance-based similarity measure $S_k(A, B)$ ($k = 1, 2$) should satisfy the following properties (S1-S4):

(S1) $0 \le S_k(A, B) \le 1$;

(S2) $S_k(A, B) = 1$ if and only if $A = B$;

(S3) $S_k(A, B) = S_k(B, A)$;

(S4) If C is a SVNM in X and $A \subseteq B \subseteq C$, , then $S_k(A, C) \le S_k(A, B)$ and $S_k(A, C) \le S_k(B, C)$.

By the similar proofs of Proposition 1 and the relationship between the distance and the similarity measure, Proofs are straightforward.

Example 2: Let A and B be two SVNMs in $X = \{x_1, x_2\}$, which are given as follows:

A = {<x_1, (0.7, 0.8), (0.1, 0.2), (0.2, 0.3)>, <x_2, (0.5, 0.6), (0.2, 0.3), (0.4, 0.5)>},

B = {<x_1, (0.5, 0.6), (0.1, 0.2), (0.4, 0.5)>, <x_2, (0.6, 0.7), (0.1, 0.2), (0.7, 0.8)>}.

The calculational process of the similarity measures between A and B is shown as follows:

(1) Using Hamming distance (p = 1):

By using Eq. (1) we obtain:

$D_1(A, B)$ = [(|0.7 – 0.5|+|0.1 – 0.1| + |0.2 – 0.4| + |0.8 – 0.6| + |0.2 – 0.2| + |0.3 – 0.5|)/6 + (|0.5 – 0.6| + |0.2 – 0.1| +|0.4 – 0.7| + |0.6 – 0.7| + |0.3 – 0.2| + |0.5 – 0.8|)/6]/2 = 0.15.

Then, by applying Eqs. (2) and (3) we have the following result:

$S_1(A, B) = 1 – D_1(A, B) = 1 – 0.15 = 0.85$ and $S_2(A, B) = [1 – D_1(A, B)]/[1+ D_1(A, B)] = 0.7391$.

(2) Using the Euclidean distance (p = 2):

By using Eq. (1) we can obtain the following result:

$D_2(A, B) = \{[(|0.7 – 0.5|^2 + |0.1 – 0.1|^2 + |0.2 – 0.4|^2 + |0.8 – 0.6|^2 + |0.2 – 0.2|^2 +| 0.3 – 0.5|^2)/6 + (|0.5 – 0.6|^2 + |0.2 – 0.1|^2 + |0.4 – 0.7|^2 + |0.6 – 0.7|^2 + |0.3 – 0.2|^2 + |0.5 – 0.8|^2)/6]/2\}^{1/2} = 0.178$.

Then, by applying Eqs. (2) and (3) we have the following result:

$S_1(A, B) = 1 – D_2(A, B) = 1 – 0.178 = 0.822$ and $S_2(A, B) = [1 – D_2(A, B)]/[1 + D_2(A, B)] = 0.6979$.

4 Medical diagnosis using the similarity measure

Due to more and more complexity of real medical diagnosis, a lot of information available to physicians from modern medical technologies is often incomplete, indeterminate and inconsistent information. Then, the SVNS proposed by Wang et al. [4] can be better to express this kind of information, but fuzzy sets and intuitionistic fuzzy sets cannot handle indeterminate and inconsistent information. However, by only taking one time inspection, we wonder whether we can obtain a conclusion from a particular person with a particular decease or not. Sometimes he/she may also show the symptoms of different diseases. Then, how can we give a proper conclusion? One solution is to examine the patient at different time intervals (e.g. two or three times a day). Thus, we present SVNMs as a better tool for reasoning such a situation. The details of a typical example (adapted from [9]) are given below.

Let $P = \{P_1, P_2, P_3, P_4\}$ be a set of four patients, $D = \{D_1, D_2, D_3, D_4\}$ = {Viral fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases, and $S = \{S_1, S_2, S_3, S_4, S_5\}$ = {Temperature, Cough, Throat pain, Headache, Body pain} be a set of symptoms. Table 1 shows the characteristics between symptoms and the considered diseases represented by the form of single valued neutrosophic values (SVNVs).

In the medical diagnosis, if we have to take three different samples in three different times in a day (e.g., morning, noon and night), we can construct Table 2, in which the characteristics between patients and the indicated symptoms are represented by SVNMVs.

Then, by using Eqs. (1) and (2) and taking p = 2, we can obtain the similarity measure between each patient P_i (i = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in Table 3.

Similarly, by using Eqs. (1) and (3) and taking p = 2, we can obtain the similarity measure between each patient P_i (i = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in Table 4.

In Tables 3 and 4, the largest similarity measure indicates the proper diagnosis. Patient P_1 suffers from viral fever, Patient P_2 suffers from tuberculosis, Patient P_3 suffers from typhoid, and Patient P_4 also suffers from typhoid.

Table 1 Characteristics between symptoms and the considered diseases represented by SVNVs

	Temperature (S_1)	Cough (S_2)	Throat pain (S_3)	Headache (S_4)	Body pain (S_5)
Viral fever (D_1)	<0.8, 0.1, 0.1>	<0.2, 0.7, 0.1>	<0.3, 0.5, 0.2>	(0.5, 0.3, 0.2)	<0.5, 0.4, 0.1>
Tuberculosis (D_2)	<0.2, 0.7, 0.1>	<0.9, 0.0, 0.1>	<0.7, 0.2, 0.1>	(0.6, 0.3, 0.1)	<0.7, 0.2, 0.1>
Typhoid (D_3)	<0.5, 0.3, 0.2>	<0.3, 0.5, 0.2>	<0.2, 0.7, 0.1>	(0.2, 0.6, 0.2)	<0.4, 0.4, 0.2>
Throat disease(D_4)	<0.1, 0.7, 0.2>	<0.3, 0.6, 0.1>	<0.8, 0.1, 0.1>	(0.1, 0.8, 0.1)	<0.1, 0.8, 0.1>

Table 2 Characteristics between patients and the indicated symptoms represented by SVNMVs

	Temperature (S_1)	Cough (S_2)	Throat pain (S_3)	Headache (S_4)	Body pain (S_5)
P_1	<(0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>	<(0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.4)>	<(0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>	<(0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>	<(0.4, 0.3, 0.2), (0.6, 0.5, 0.5), (0.6, 0.4, 0.4)>
P_2	<(0.5, 0.4, 0.3), (0.3, 0.3, 0.2), (0.5, 0.4, 0.4)>	<(0.9, 0.8, 0.7), (0.2, 0.1, 0.1), (0.2, 0.2, 0.1)>	<(0.6, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.3, 0.3)>	<(0.6, 0.4, 0.3), (0.3, 0.1, 0.1), (0.7, 0.7, 0.3)>	<(0.8, 0.7, 0.5), (0.4, 0.3, 0.1), (0.3, 0.2, 0.1)>
P_3	<(0.2, 0.1, 0.1), (0.3, 0.2, 0.2), (0.8, 0.7, 0.6)>	<(0.3, 0.2, 0.2), (0.4, 0.2, 0.2), (0.7, 0.6, 0.5)>	<(0.8, 0.8, 0.7), (0.2, 0.2, 0.2), (0.1, 0.1, 0.0)>	<(0.3, 0.2, 0.2), (0.3, 0.3, 0.3), (0.7, 0.6, 0.6)	<(0.4, 0.4, 0.3), (0.4, 0.3, 0.2), (0.7, 0.7, 0.5)>
P_4	<(0.5, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.4, 0.3)>	<(0.4, 0.3, 0.1), (0.4, 0.3, 0.2), (0.7, 0.5, 0.3)>	<(0.2, 0.1, 0.0), (0.4, 0.3, 0.3), (0.7, 0.7, 0.6)>	<(0.6, 0.5, 0.3), (0.2, 0.2, 0.1), (0.6, 0.4, 0.3)>	<(0.5, 0.4, 0.4), (0.3, 0.3, 0.2), (0.6, 0.5, 0.4)>

Table 3 Similarity measure values of $S_1(P_i, D_j)$

	Viral fever (D_1)	Tuberculosis (D_2)	Typhoid (D_3)	Throat disease(D_4)
P_1	**0.7358**	0.6101	0.7079	0.5815
P_2	0.6884	**0.7582**	0.6934	0.5964
P_3	0.6159	0.6141	**0.6620**	0.6294
P_4	0.7199	0.6167	**0.7215**	0.5672

Table 4 Similarity measure values of $S_2(P_i, D_j)$

	Viral fever (D_1)	Tuberculosis (D_2)	Typhoid (D_3)	Throat disease(D_4)
P_1	**0.5821**	0.4390	0.5478	0.4100
P_2	0.5248	**0.6106**	0.5307	0.4249
P_3	0.4450	0.4431	**0.4948**	0.4592
P_4	0.5624	0.4459	**0.5643**	0.3958

6 Conclusion

This paper proposed the generalized distance and its two similarity measures. Then, the two similarity measures of SVNMs were applied to medical diagnosis to demonstrate the effectiveness of the developed measure methods. The medical diagnosis shows that the new measures perform well in the case of truth-membership, indeterminacy-membership, and falsity-membership functions and the example depicts that the proposed measure is effective with the three representatives of SVNMV – truth-membership, indeterminacy-membership and falsity-membership values. Therefore, the measures of SVNMs make them possible to handle the diagnosis problems with indeterminate and inconsistent information, which cannot be handled by the measures of IFMs because IFMs cannot express and deal with the indeterminate and inconsistent information.

In further work, it is necessary and meaningful to extend SVNMs to propose interval neutrosophic multisets and their operations and measures and to investigate their applications such as decision making, pattern recognition, and medical diagnosis.

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Algebraic Structures Of Neutrosophic Soft Sets

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Abstract: In this paper, we study the algebraic operations of neutrosophic soft sets and their basic properties associated with these opertaions. And also define the associativity and distributivity of these operations. We discusse different algebraic structures, such as monoids, semiring and lattices, of neutrosophic soft sets.

Keywords: Soft sets, Neutrosophic soft sets, monoid, semiring, lattices.

1 Introduction

During recent years soft set theory has gained popularity among the researchers due to its applications in various areas. Number of publications related to soft sets has risen exponentially. Theory of soft sets is proposed by Moldtsov in [16]. Basic aim of this theory is to introduce a mathematical model with enough parameters to handle uncertainty. Prior to soft set theory, probability theory, fuzzy set theory, rough set theory and interval mathematics were common tools to discuss uncertainty. But unfortunately difficulties were attached with these theories, for details see [11, 16]. As mentioned above soft set theory has enough number of parameters, so it is free from difficulties associated with other theories. Soft set theory has been applied to various fields very successfully.
The concept of neutrosophic set was introduced by Smarnandache [20]. The traditional neutrosophic sets is characterized by the truth value, indeterminate value and false value. Neutrosophic set is a mathematically tool for handling problems involving imprecise, indeterminacy inconsistent data and inconsistent information which exits in belief system.
Maji et al. proposed the concept of "Fuzzy Soft Sets" [13] and later on applied the theories in decision making problem [14, 15]. Different algebraic structures and their applications have also been studied in soft and fuzzy soft context [2, 19]. In [12] Maji proposed the concept of "Neutrosophic soft set" and applied the theories in decision making problem. Later Broumi and Smarandache defined the concepts of interval valued neutrosophic soft set and inituitionistic neutrosophic soft set in [3, 5]. Recently Sahin and Kucuk applied the concept of neutrosophic soft set in decision making problems [17,18]. Different algebraic structures and their application can be study in neutrosophic soft set context [4,7, 8, 9, 10]. In this paper we define some new operations on the neutrosophic soft set and modified results and laws are established. And also define the associativity and distributivity of these operations. The paper is organized in five sections. First we have given preliminaries on the theories of soft sets and neutrosophic sets. Section 3 completely describes for what new and modified operations define on neutrosophic soft set. In section 4 we have used new and modified definitions and operations to discuss the properties of associativity and distributivity of these operations for neutrosophic soft sets. Counter examples are provided to show the converse of proper inclusion is not true in general. In section 5, monoids, semiring and lattices of neutrosophic soft sets associated with new operations have been determined completely.

2 Preliminaries

In this section we present the theory of neutrosophic sets and soft sets, taken from [1,20]， and some definitions and notions about algebraic structures are given.
Let X be a universe of discourse and a neutrosophic

set A on X is defined as
$A=\{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle,\ x\in X\}$
where $T,\ I,\ F\ :X\to\ ]^-0,1^+[$ and
$^-0\le T_A(x)+I_A(x)+F_A(x)\le 3^+.$
Philosophical point of view, neutrosophic set takes the value from real standard or non standard subsets of $]^-0,1^+[$. But it is difficult to use neutrosophic set with value from real standard or non standard subsets of $]^-0,1^+[$ in real life application like scientific and engineering problems.

Definition 2.1:

A neutrosophic set A is contained in another neutrosophic set B *i.e.* $A\subseteq B$ if
$T_A(x)\le T_B(x),\ I_A(x)\le I_A(x),\ F_A(x)\ge F_B(x)$
$\forall\ x\in X.$

***Example 2.2*:**

Mr. X and his father wants to purchase a laptop. They have their expectations and perceptions. Based on these, they identify three criteria x_1, x_2, x_3 which are as follows
x_1=Performance, x_2=Size of laptop, x_3=Price of laptop
It may be assumed that the values of x_1, x_2, x_3 are in $[0,\ 1]$. The buyer consults with experts and also collects data from his own survey. The experts may impose their opinion in three components viz, the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set of $X=\{x_1,x_2,x_3\}$ such that

$$A=\left\{\begin{matrix}\langle x_1,0.8,0.4,0.5\rangle,\langle x_2,0.7,0.2,0.4\rangle,\\ \langle x_3,0.8,0.3,0.4\rangle\end{matrix}\right\}.$$

Where the degree of goodness of performance (x_1) is 0.8 degree of indeterminacy of performance (x_1) is 0.4 and the degree of poorness of performance is 0.5 etc.

Definition 2.3:

Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denote the power set of U and let A be a non-empty subset of E. A pair $(F,\ A)$ is called soft set over $U,$ where F is mapping given by $F\ :\ A\to P(U).$

Definition 2.4:

For two soft sets $(F,\ A)$ and $(G,\ B)$ over a common universe U, we say that $(F,\ A)$ is a soft subset of $(G,\ B)$ if
(i) $A\subseteq B,$
(ii) $F(e)\subseteq G(e)\ \forall\ e\in A.$
We write $(F,\ A)\subseteq(G,\ B).$

Definition 2.5:

Two soft sets $(F,\ A)$ and $(G,\ B)$ over a common universe U are said to be soft equal if $(F,\ A)$ is a soft subset of $(G,\ B)$ and $(G,\ B)$ is a soft subset of $(F,\ A).$

Definition 2.6:

Extended union of two soft sets (F, A) and (G, B) over the common universe U is the soft set $(H,\ C)$, where $C=A\cup B$ and for all $e\in C,$

$$H(e)=\begin{cases}F(e) & \text{if } e\in A-B\\ G(e) & \text{if } e\in B-A\\ F(e)\cup G(e) & \text{if } e\in A\cap B.\end{cases}$$

We write $(F,\ A)\cup_{E}(G,\ B)=(H,\ C).$
Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over the same universe U, such that $A\cap B\neq\emptyset$. The restricted union of $(F,\ A)$ and $(G,\ B)$ is denoted by $(F,\ A)\cup_{\mathcal{R}}(G,\ B)$ and is defined as $(F,\ A)\cup_{\mathcal{R}}(G,\ B)=(H,\ C),$ where $C=A\cap B$ and for all $e\in C,$ $H(e)=F(e)\cup G(e).$

2.7 Definition:

The extended intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{E} (G, B) = (H, C)$.

Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \Phi$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_{R} (G, B)$ and is defined as $(F, A) \cap_{R} (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

A semigroup $(S,*)$ is a non-empty set with an associative binary operation $*$. We use usual algebraic practice and write xy instead of $x * y$. If there exists an element e in S such that $ex = xe = x$ for all x in S then we say that S is a monoid and e is called the identity element. An element $x \in S$ is called idempotent if $xx = x$. If every element of S is idempotent then we say that S is idempotent.

A semiring is an algebraic structure consisting of a non-empty set R together with two associative binary operations, addition "+" and multiplication "." such that "." distributes over "+" from both sides. Semirings which are regarded as a generalization of rings. By a hemiring, we mean a semiring with a zero and with a commutative addition.

A Lattice $(L,\vee,\wedge)$ is a non-empty set with two binary operations $\vee$ and $\wedge$ such that

(1) $(L,\vee)$ is a commutative, idempotent semigroup,

(2) $(L,\wedge)$ is a commutative, idempotent semigroup,

(3) Absorption laws $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a$ hold for all $a, b \in L$.

If a lattice has identity elements with respect to both the operations then we say that it is bounded. Usually identity element of L with respect to operation $\wedge$ is denoted by 0 and whereas the identity element with respect to binary operation $\vee$ is denoted by 1. If a lattice L has identities and for each $a \in L$ there exists an element b such that $a \wedge b = 0$ and $a \vee b = 1$, then L is called complemented. If distributive laws hold in a lattice then it is called a distributive lattice.

3 Neutrosophic soft set

Definition 3.1[12]:

Let U be an initial universe set and E be the set of all parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. A pair (F, A) is termed to be the neutrosophic soft set (NSS) over U, where F is mapping given by $F : A \to P(U)$.

Example 3.2:

Let U be the set of calculators under consideration and E is the set of parameters. Consider

$$U = \left\{ \begin{array}{l} c_1 = scientific,\ c_2 = programmable, \\ c_3 = four \text{ function} \end{array} \right\}$$

and

$$E = \{ e_1 = \text{performance}, e_2 = size,\ e_3 = price \}$$

suppose that

$$F(e_1) = \left\{ \begin{array}{l} \langle c_1, 0.7, 0.4, 0.5 \rangle, \langle c_2, 0.8, 0.5, 0.3 \rangle \\ , \langle c_3, 0.4, 0.6, 0.8 \rangle \end{array} \right\},$$

$$F(e_2)=\left\{\begin{array}{l}\langle c_1,0.6,0.7,0.8\rangle,\langle c_2,0.2,0.4,0.7\rangle,\\ \langle c_3,0.8,0.4,0.6\rangle\end{array}\right\},$$

$$F(e_3)=\left\{\begin{array}{l}\langle c_1,0.8,0.1,0.7\rangle,\langle c_2,0.5,0.8,0.9\rangle,\\ \langle c_3,1,0.3,0.4\rangle\end{array}\right\}.$$

The neutrosophic soft set $(F,\ E)$ is a parametrized family $\{F(e_i),\ i=1,2,3\}$ of all neutrosophic sets of U and describes a collection of approximation of an object.

To store a neutrosophic soft set in computer, we could present it in the form of a table as shown below. In this table the entries are a_{ij} corresponding to the calculator c_i and the parameter e_j where

$$a_{ij}=\begin{pmatrix}\text{true - membership value of } c_i, \\ \text{indeterminacy - membership value of } c_i, \\ \text{falsity - membership value of } c_i\end{pmatrix}$$

in $F(e_j)$. The neutrosophic soft set $(F,\ E)$ in tabular representation is as follow:

U	e_1 =performance	e_2 =size of calculator	e_3 =price of calculator
c_1	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.1, 0.7\rangle$
c_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.5, 0.8, 0.9\rangle$
c_3	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.3, 0.4\rangle$

Tabular form of the neutrosophic soft set $(F,\ E)$.

Definition 3.3 [14]:

For two neutrosophic soft sets $(H,\ A)$ and $(G,\ B)$ over the common universe U . We say that $(H,\ A)$ is a neutrosophic soft subset of $(G,\ B)$ if

(i) $A\subseteq B,$

(ii) $T_{H(e)}(x)\leq T_{G(e)}(x),\quad I_{H(e)}(x)\leq I_{G(e)}(x),$
$F_{H(e)}(x)\geq F_{G(e)}(x)$

for all $e\in A$ and $x\in U.$ We write $(H,A)\subseteq(G,B).$

Definition 3.4:

For two neutrosophic soft sets $(H,\ A)$ and $(G,\ B)$ over the common universe U . We say that $(H,\ A)$ is a neutrosophic soft twisted subset of $(G,\ B)$ if

(i) $A\subseteq B$

(ii) $T_{H(e)}(x)\geq T_{G(e)}(x),\ I_{H(e)}(x)\geq I_{G(e)}(x),$
$F_{H(e)}(x)\leq F_{G(e)}(x)$

for all $e\in A$ and $x\in U.$ We write $(H,\ A)\overset{\frown}{\subseteq}(G,\ B).$

Definition 3.5:

(1) $(H,\ A)$ is called relative null neutrosophic soft set (with respect to parameter A), if $T_{H(e)}(x)=0,\ I_{H(e)}(x)=0,\ F_{H(e)}(x)=1\ \forall\, e\in A \text{ and } x\in U.$ It is denoted by Φ_A.

(2) $(G,\ A)$ is called relative whole neutrosophic soft set (with respect to parameter A) if $T_{G(e)}(x)=1,\ I_{G(e)}(x)=1,\ F_{G(e)}(x)=0\ \forall\, e\in A \text{ and } x\in U.$ It is denoted by $\cup_A$.

Similarly we define absolute neutrosophic soft set over U , and it is denoted by $\cup_E$, and null neutrosophic soft set over U, it is denoted by Φ_E.

Definition 3.6:

Let $E=\{e_1,e_2,e_3,\ldots e_n\}$ be a set of parameters. The not set of E is denoted by $\neg E$ and defined as $\neg E=\{\neg e_1,\neg e_2,\neg e_3,\ldots\neg e_n\}$, where $\neg\ e_i=$ not e_i, $\forall i.$

Definition 3.7 [14]:

Complement of a neutrosophic soft set $(G,\ A)$ denoted by $(G,\ A)^c$ and is defined as

$(G,\ A)^c = (G^c,\ \neg A)$ where

$G^c\ : \neg A \to P(U)$ is a mapping given by

$G^c(\neg e) =$ neutrosophic soft compliment with

$T_{G^c(\neg e)} = F_{G(e)}, \quad I_{G^c(\neg e)} = I_{G(e)}$ and

$F_{G^c(\neg e)} = T_{G(e)}.$

Definition 3.8:

Let $(H,\ A)$ and $(G,\ B)$ be two NSS,s over the common universe U. Then the extended union of $(H,\ A)$ and $(G,\ B)$ is denoted by $(H,\ A) \cup_{\mathcal{E}} (G,\ B)$ and defined as $(H,\ A) \cup_{\mathcal{E}} (G,\ B) = (K,\ C)$, where $C = A \cup B,$ and the truth-membership, indeterminacy-membership and falsity-membership of $(K,\ C)$ are as follows

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e \in A - B \\ T_{G(e)}(x) & \text{if } e \in B - A \\ \max\left(T_{H(e)}(x), T_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e \in A - B \\ I_{G(e)}(x) & \text{if } e \in B - A \\ \max\left(I_{H(e)}(x), I_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x) & \text{if } e \in A - B \\ F_{G(e)}(x) & \text{if } e \in B - A \\ \min\left(F_{H(e)}(x), F_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

and the restricted union of $(H,\ A)$ and $(G,\ B)$ is denoted and defined as $(H,\ A) \cup_{\mathcal{R}} (G,\ B) = (K,\ C)$ where $C = A \cap B$ and

$T_{K(e)}(x) = \max\left(T_{H(e)}(x), T_{G(e)}(x)\right)$ if $e \in A \cap B$

$I_{K(e)}(x) = \max(I_{H(e)}(x), I_{G(e)}(x))$ if $e \in A \cap B$

$F_{K(e)}(x) = \min\left(F_{H(e)}(x), F_{G(e)}(x)\right)$ if $e \in A \cap B$

If $A \cap B = \emptyset,$ then $(H,\ A) \cup_{\mathcal{R}} (G,\ B) = \Phi_{\Phi}.$

Definition 3.9:

Let $(H,\ A)$ and $(G,\ B)$ be two NSS,s over the common universe U. Then the extended intersection of $(H,\ A)$ and $(G,\ B)$ is denoted by $(H,\ A) \cap_{\mathcal{E}} (G,\ B)$ and defined as $(H,\ A) \cap_{\mathcal{E}} (G,\ B) = (K,\ C),$ where $C = A \cup B,$ and the truth-membership, indeterminacy-membership and falsity-membership of $(K,\ C)$ are as follows

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e \in A - B \\ T_{G(e)}(x) & \text{if } e \in B - A \\ \min\left(T_{H(e)}(x), T_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e \in A - B \\ I_{G(e)}(x) & \text{if } e \in B - A \\ \min\left(I_{H(e)}(x), I_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x) & \text{if } e \in A - B \\ F_{G(e)}(x) & \text{if } e \in B - A \\ \max\left(F_{H(e)}(x), F_{G(e)}(x)\right) & \text{if } e \in A \cap B \end{cases}$$

and the restricted intersection of $(H,\ A)$ and $(G,\ B)$ is denoted and defined as $(H,\ A) \cap_{\mathcal{R}} (G,\ B) = (K,\ C)$ where $C = A \cap B$ and

$T_{K(e)}(x) = \min(T_{H(e)}(x), T_{G(e)}(x))$ if $e \in A \cap B$
$I_{K(e)}(x) = \min(I_{H(e)}(x), I_{G(e)}(x))$ if $e \in A \cap B$
$F_{K(e)}(x) = \max(F_{H(e)}(x), F_{G(e)}(x))$ if $e \in A \cap B$

If $A \cap B = \Phi$ then $(H, A) \cap_{\mathbf{R}} (G, B) = \Phi_{\Phi}$.

4 Distributive laws for neutrosophic soft sets

In this section we present distributive laws on the collection of neutrosophic soft set. It is interesting to see that the equality does not hold in some assertions and counter example is given to show it.

Let U be an initial universe and E be the set of parameters. We denote the collection as follows:

NSS$(U)^E$: The collection of all neutrosophic soft sets over U.

NSS$(U)_A$: The collection of all those neutrosophic soft sets over U with a fixed parameter set A.

Theorem 4.1:

Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then

(i) $(H, A) \cup_R (H, A) = (H, A)$ and $(H, A) \cap_R (H, A) = (H, A)$,

(ii) $(H, A) \cap_R \Phi_A = \Phi_A$,

(iii) $(H, A) \cup_R \Phi_A = (H, A)$,

(iv) $(H, A) \cup_R \mathsf{U}_A = \mathsf{U}_A$,

(v) $(H, A) \cap_R \mathsf{U}_A = (H, A)$,

(vi) $((H, A) \cup_R (G, B))^c = (H, A)^c \cap_R (G, B)^c$,

(vii) $((H, A) \cap_R (G, B))^c = (H, A)^c \cup_R (G, B)^c$.

Proof: Straightforward.

Remark 4.2:

Let $\alpha, \beta \in \{\cup_{\mathbf{R}}, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}, \cap_{\mathbf{E}}\}$, if

$(H, A)\alpha((G, B)\beta(K, C)) =$
$((H, A)\alpha(G, B))\beta((H, A)\alpha(K, C))$

holds, then we have 1 otherwise 0 in the following table

	$\cap_{\mathbf{E}}$	$\cup_{\mathbf{E}}$	$\cap_{\mathbf{R}}$	$\cup_{\mathbf{R}}$
$\cap_{\mathbf{E}}$	1	0	0	1
$\cup_{\mathbf{E}}$	0	1	1	0
$\cap_{\mathbf{R}}$	1	1	1	1
$\cup_{\mathbf{R}}$	1	1	1	1

Distributive law for neutrosophic soft sets

Proofs in the cases where equality holds can be followed by definition of respective operations. For which the equality does not hold, see the following example.

Example 4.3:

Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word. Consider $U = \{h_1, h_2, h_3, h_4, h_5\}$ and $E = \{$ beautiful, wooden , costly, green surroundings, good repair, cheap, expensive $\}$.

Suppose that $A = \{$ beautiful, wooden , costly, green surroundings $\}$ $B = \{$ costly, good repair, green surroundings $\}$ and $C = \{$ costly,good repair,beautiful $\}$. Let (F, A), (G, B) and (H, C) be the neutrosophic soft sets over U , which are defined as follows:

Neutrosophic soft set (G, B)

U	*costly*	*good repair*	*green surroundings*
h_1	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.6, 0.2, 0.7\rangle$
h_2	$\langle 0.7, 0.3, 0.6\rangle$	$\langle 0.6, 0.6, 0.8\rangle$	$\langle 0.4, 0.5, 0.2\rangle$
h_3	$\langle 0.8, 0.6, 0.3\rangle$	$\langle 1, 0.7, 0.5\rangle$	$\langle 0.6, 0.4, 0.7\rangle$

Neutrosophic soft set $(F,\ A)$

U	beautiful	wooden	costly	green surroundings
h_1	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.1, 0.7\rangle$	$\langle 0.5, 0.3, 0.7\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.5, 0.8, 0.9\rangle$	$\langle 0.6, 0.2, 0.3\rangle$
h_3	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.3, 0.4\rangle$	$\langle 0.7, 0.1, 0.5\rangle$

and Neutrosophic soft set $(H,\ C)$ is

U	costly	good repair	beautiful
h_1	$\langle 0.4, 0.4, 0.6\rangle$	$\langle 0.5, 0.7, 0.4\rangle$	$\langle 0.6, 0.8, 0.4\rangle$
h_2	$\langle 0.3, 0.7, 0.4\rangle$	$\langle 0.3, 0.7, 0.4\rangle$	$\langle 0.5, 0.3, 0.3\rangle$
h_3	$\langle 0.2, 0.4, 0.3\rangle$	$\langle 0.4, 0.5, 0.3\rangle$	$\langle 0.8, 0.5, 0.8\rangle$

Let

$(F,\ A)\cup_{\mathbf{E}}((G,\ B)\cup_{\mathbf{R}}(H,\ C)) = (I,\ A\cup(B\cap C))$ and

$((F,\ A)\cup_{\mathbf{E}}(G,\ B))\cup_{\mathbf{R}}((F,\ A)\cup_{\mathbf{E}}(H,\ C)) = (J,\ (A\cup B)\cap(A\cup C))$

Then

Neutrosophic soft set $(I,\ A\cup(B\cap C))$

U	beautiful	wooden	costly	green surroundings	good repair
h_1	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.7, 0.7, 0.4\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.6, 0.7, 0.4\rangle$
h_3	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.6, 0.3\rangle$	$\langle 0.7, 0.1, 0.5\rangle$	$\langle 1, 0.7, 0.3\rangle$

Neutrosophic soft set $(J,\ (A\cup B)\cap(A\cup C))$

U	beautiful	wooden	costly	green surroundings	good repair
h_1	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.7, 0.7, 0.4\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.6, 0.7, 0.4\rangle$
h_3	$\langle 0.8, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.6, 0.3\rangle$	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 1, 0.7, 0.3\rangle$

Thus

$$(F,\ A)\cup_{\mathbf{E}}((G,\ B)\cup_{\mathbf{R}}(H,\ C)) \neq$$
$$((F,\ A)\cup_{\mathbf{E}}(G,\ B))\cup_{\mathbf{R}}((F,\ A)\cup_{\mathbf{E}}(H,\ C)).$$

Similarly we can show that

$$(F,\ A)\cap_{\mathbf{E}}((G,\ B)\cap_{\mathbf{R}}(H,\ C)) \neq$$
$$((F,\ A)\cap_{\mathbf{E}}(G,\ B))\cap_{\mathbf{R}}((F,\ A)\cap_{\mathbf{E}}(H,\ C)),$$

and

$$(F,\ A)\cup_{\mathbf{E}}((G,\ B)\cap_{\mathbf{E}}(H,\ C)) \neq$$
$$((F,\ A)\cup_{\mathbf{E}}(G,\ B))\cap_{\mathbf{E}}((F,\ A)\cup_{\mathbf{E}}(H,\ C)),$$
$$(F,\ A)\cap_{\mathbf{E}}((G,\ B)\cup_{\mathbf{E}}(H,\ C)) \neq$$
$$((F,\ A)\cap_{\mathbf{E}}(G,\ B))\cup_{\mathbf{E}}((F,\ A)\cap_{\mathbf{E}}(H,\ C)).$$

5 Algebraic structures associated with neutrosophic soft sets

In this section, we initiate the study of algebraic structures associated with single and double binary operations, for the set of all neutrosophic soft sets over the universe U, and the set of all neutrosophic soft sets with a fixed set of parameters. Recall that, let U be an initial universe and E be the set of parameters. Then we have:

$\mathbf{NSS}(U)^E$: The collection of all neutrosophic soft sets over U.

$\mathbf{NSS}(U)_A$: The collection of all those neutrosophic soft sets over U with a fixed parameter set A.

5.1 Commutative monoids

From Theorem 4.1, it is clear that $(\mathbf{NSS}(U)^E,\ \alpha)$ are idempotent, commutative, semigroups for $\alpha \in \{\cup_{\mathcal{R}}, \cap_{\mathcal{R}}, \cup_{\mathcal{E}}, \cup_{\mathcal{E}}\}$.

(1) $(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{R}})$ is a monoid with Φ_E as an identity element, $(\mathbf{NSS}(U)_A,\ \cup_{\mathbf{R}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{R}})$.

(2) $(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{R}})$ is a monoid with $\mathfrak{U}_E$ as an identity element, $(\mathbf{NSS}(U)_A,\ \cap_{\mathbf{R}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{R}})$.

(3) $(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{E}})$ is a monoid with Φ_Φ as an identity element, $(\mathbf{NSS}(U)_A,\ \cup_{\mathbf{E}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{E}})$.

(4) $(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{E}})$ is a monoid with Φ_Φ as an identity element, $(\mathbf{NSS}(U)_A,\ \cap_{\mathbf{E}})$ is a

subsemigroup of $\left(\mathbf{NSS}(U)^E, \cap_{\mathbf{E}}\right)$.

5.2 Semirings:

(1) $\left(\mathbf{NSS}(U)^E, \cup_{\mathbf{R}}, \cap_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with U_E as an identity element.

(2) $\left(\mathbf{NSS}(U)^E, \cup_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(3) $\left(\mathbf{NSS}(U)^E, \cup_{\mathbf{R}}, \cap_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(4) $\left(\mathbf{NSS}(U)^E, \cap_{\mathbf{R}}, \cup_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with Φ_E as an identity element.

(5) $\left(\mathbf{NSS}(U)^E, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(6) $\left(\mathbf{NSS}(U)^E, \cap_{\mathbf{R}}, \cap_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(7) $\left(\mathbf{NSS}(U)^E, \cup_{\mathbf{E}}, \cap_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with $\mathfrak{U}_E$ as an identity element.

(8) $\left(\mathbf{NSS}(U)^E, \cap_{\mathbf{E}}, \cup_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with Φ_E as an identity element.

5.3 Lattices:

In this subsection we study what type of lattice structure is associated with the neutrosophic soft sets.

Remark 5.3.1:

Let $\alpha, \beta \in \{\cup_{\mathbf{R}}, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}, \cap_{\mathbf{E}}\}$. if the absorption law

$(F,\ A)\alpha((F,\ A)\beta(G,\ B)) = (F,\ A)$

holds we write 1 otherwise 0 in the following table.

	$\cap_{\mathbf{E}}$	$\cup_{\mathbf{E}}$	$\cap_{\mathbf{R}}$	$\cup_{\mathbf{R}}$
$\cap_{\mathbf{E}}$	0	0	0	1
$\cup_{\mathbf{E}}$	0	0	1	0
$\cap_{\mathbf{R}}$	0	1	0	0
$\cup_{\mathbf{R}}$	1	0	0	0

Absorption law for neutrosophic soft sets

(1) $\left(\mathbf{NSS}(U)^E, \Phi_\Phi, \mathsf{U}_E, \cup_{\mathbf{R}}, \cap_{\mathbf{E}}\right)$ and $\left(\mathbf{NSS}(U)^E, \mathsf{U}_E, \Phi_\Phi, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ are lattices with $\left(\mathbf{NSS}(U)_A, \Phi_A, \mathsf{U}_A, \cup_{\mathbf{R}}, \cap_{\mathbf{E}}\right)$ and $\left(\mathbf{NSS}(U)_A, \mathsf{U}_A, \Phi_A, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ as their sublattices respectively.

(2) $\left(\mathbf{NSS}(U)^E, \Phi_\Phi, \mathsf{U}_E, \cup_{\mathbf{E}}, \cap_{\mathbf{R}}\right)$ and $\left(\mathbf{NSS}(U)^E, \mathsf{U}_E, \Phi_\Phi, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ are lattices with $\left(\mathbf{NSS}(U)_A, \Phi_A, \mathsf{U}_A, \cup_{\mathbf{E}}, \cap_{\mathbf{R}}\right)$ and $\left(\mathbf{NSS}(U)_A, \mathsf{U}_A, \Phi_A, \cap_{\mathbf{R}}, \cup_{\mathbf{E}}\right)$ as their sublattices respectively.

The above mentioned lattices and sublattices are bounded distributive lattices.

Proposition 5.3.2 :

For the lattice of neutrosophic soft set $\left(\mathbf{NSS}(U)^E, \Phi_\Phi, \mathsf{U}_E, \cup_{\mathbf{R}}, \cap_{\mathbf{E}}\right)$ for any $(H,\ A)$ and $(G,\ B) \in \mathbf{NSS}(U)^E$, then

(1) $(H,\ A) \hat{\subset} (G,\ B)$ if and only if $(H,\ A) \cup_{\mathbf{R}} (G,\ B) = (H,\ A)$.

(2) $(H,\ A) \hat{\subset} (G,\ B)$ if and only if $(H,\ A) \cap_{\mathcal{E}} (G,\ B) = (G,\ B)$.

Proof: Straightforward.

Proposition 5.3.3:

For the lattice of neutrosophic soft set $\left(\mathbf{NSS}(U)^E, \Phi_\Phi, \mathsf{U}_E, \cup_{\mathbf{E}}, \cap_{\mathbf{R}}\right)$ for any

$(H,\ A)$ and $(G,\ B) \in \mathbf{NSS}(U)^E$, then

(1) $(H,\ A) \subset (G,\ B)$ if and only if $(H,\ A) \cup_{\mathbf{E}} (G,\ B) = (G,\ B)$.

(2) $(H,\ A) \subset (G,\ B)$ if and only if $(H,\ A) \cap_{\mathbf{R}} (G,\ B) = (H,\ A)$.

Proof: Straightforward.

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Neutrosophic Decision Making Model of School Choice

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Abstract The purpose of this paper is to present single valued neutrosophic decision making model of school choice. Childhood is a crucial stage in terms of a child's physical, intellectual, emotional and social development i.e. all round development of a child. Mental and physical abilities of children grow at an increasing rate. Children particularly need high quality personal care and learning experiences.

Children begin learning from the moment the child takes his/her birth and continues on throughout his/her life. Babies and toddlers need positive early learning experiences for their mental and physical development and this lays the foundation for later school success. So it is necessary to select the best school for the children among all the feasible alternatives by which all needs of the children are fulfilled.

A large number of parents have an increasing array of options in choosing the best school for their children among all alternatives. Those options vary from place to place. In this paper, neutrosophic multi-attribute decision-making with interval weight information is used to form a decision-making model for choosing the best school for the children. A numerical example is developed based on expert opinions from english medium schools of Nadia districts, West Bengal, India. The problem is solved to show the effectiveness of the proposed single valued neutrosophic decision making model.

Keywords: Neutrosophic multi-attribute decision-making, Grey relational analysis, School choice.

1 Introduction

Decision-making is a challenging act of choosing between two or more possible alternatives. Decision makers have to make decision based on complete or incomplete information. That's why new scientific strategies must be introduced for improvement the quality of decisions.

Rational choice theory [1, 2, 3] suggests that parents are utility maximizer in decision making who make decisions from clear value preferences based on the costs, benefits, and probabilities of success of various options. The options [1, 2, 3] are namely, ability to fulfil the demand effectively from local schools and teachers, and pursuing the best interests of their children.

Literature review for school choice [4, 5], however, reflects that the context of parental decision-making is more complex than the result of individual rational calculations of the economic return of their investment. Parental choice is a part of social process influenced by salient properties of social class and networks of social relationships [6-9]. Coleman [6], Bauch and Goldring [7], Bosetti [8], Reay and Lucey [9] explain that when an individual comes in close contact with important decision making situation, a rational actor will engage in a search for information before making a decision. However, According to Ball [10], parents seem to use a mixture of rationalities involving an element of the fortuitous and haphazard.

For school choice, parents generally depend on their personal values and judgment as well as others within their social and professional networks in order to collect required information. Parents prefer to choice private schools because they think their children will have better opportunities in private schools. To perform this optimally, parents need to have a clear understanding of school administration and the rules of the school admission process and engage in strategic school choice. In this challenging and demanding process, parents may make technical errors about the rules as well as in judgment in selecting and ordering the schools. Abdulkadiroglu and Sonmez [11] mentioned that the open enrollment school choice programs in Boston, Minneapolis, and Seattle ask parents to make complex school choice decisions, which can result in an inefficient allocation of school seats. In order to deal school choice problem, new model is urgently needed.

Most of the study on school choice is based on assumptions at the theoretical level with little practical situation. Most of the research on school choice is done in crisp environment. Radhakrishnan and Kalaichelvi [12] studied school choice problem based on fuzzy analytic hierarchy process. However, in fuzzy environment degree of indeterminacy is not included. So neutrosophic set theoretic based approach may be helpful to deal this type of problems. Literature review indicates that no research on school choce is done in neutrosophic environment.

Decision making is oriented in every sphere of human activities. However, human being realizes problems in making decision on many normal activities such as education for children, quality of food, transportation, purchasing, selection of partner, healthcare, selection of shelter, etc. A small number of studies are done on edcational problems based on the concept of fuzzy set, neutrosophic set and grey system theory. Pramanik and Mukhopadhyaya [13] presented grey relational analysis based on intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Mondal and Pramanik [14] presented multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. In this paper we present a methodological approach to choose the best elementary school for children among particular alternatives to their designated neighbourhood using neutrosophic multi-attribute decision-making with interval weight information based on grey relational analysis. .

A numerical example is developed based on expert opinions from english medium schools of Nadia districts, West Bengal, India . The problem is solved to demonstrate the effectiveness of the proposed neutrosophic decision making model.

Rest of the paper is organized as follows: Section 2 presents preliminaries of neutrosophic sets. Section 3 describes single valued neutrosophic multiple attribute decision making problem based on GRA with interval weight information. Section 4 is devoted to propose neutrosophic decision making model of school choice. Finally, Section 5 presents concluding remarks.

2 Neutrosophic preliminaries

2.1 Definition on neutrosophic sets

The concept of neutrosophic set is originated from neutrosophy, a new branch of philosophy. According to Smarandache [15] ‘‘Neutrosophy studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra``.

Definition1: Let ξ be a space of points (objects) with generic element in ξ denoted by x. Then a neutrosophic set α in ξ is characterized by a truth membership function T_α an indeterminacy membership function I_α and a falsity membership function F_α. The functions T_α and F_α are real standard or non-standard subsets of $]0^-,1^+[$ that is T_α: $\xi \to]0^-,1^+[$; I_α: $\xi \to]0^-,1^+[$; F_α: $\xi \to]0^-,1^+[$.

It should be noted that there is no restriction on the sum of $T_\alpha(x)$, $I_\alpha(x)$, $F_\alpha(x)$ i.e.

$0^- \le T_\alpha(x) + I_\alpha(x) + F_\alpha(x) \le 3^+$

Definition2: The complement of a neutrosophic set α is denoted by α^c and is defined by

$$T_{\alpha^c}(x) = \{1^+\} - T_\alpha(x);\ I_{\alpha^c}(x) = \{1^+\} - I_\alpha(x)$$

$$F_{\alpha^c}(x) = \{1^+\} - F_\alpha(x)$$

Definition3: (Containment) A neutrosophic set α is contained in the other neutrosophic set β, $\alpha \subseteq \beta$ if and only if the following result holds.

$$\inf T_\alpha(x) \le \inf T_\beta(x),\ \sup T_\alpha(x) \le \sup T_\beta(x)$$

$$\inf I_\alpha(x) \ge \inf I_\beta(x),\ \sup I_\alpha(x) \ge \sup I_\beta(x)$$

$$\inf F_\alpha(x) \ge \inf F_\beta(x),\ \sup F_\alpha(x) \ge \sup F_\beta(x)$$

for all x in ξ.

Definition4: (Single-valued neutrosophic set). Let ξ be a universal space of points (objects) with a generic element of ξ denoted by x.

A single-valued neutrosophic set S is characterized by a truth membership function $T_s(x)$, an indeterminacy membership function $I_s(x)$, and a falsity membership function $F_s(x)$ with $T_s(x)$, $I_s(x)$, $F_s(x) \in [0, 1]$ for all x in ξ . When ξ is continuous, a SNVS can be written as follows:

$$S = \int_x \langle T_s(x), F_s(x), I_s(x) \rangle / x, \forall x \in \xi$$

and when ξ is discrete, a SVNSs S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in \xi$$

It should be noted that for a SVNS S,

$$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3, \forall x \in \xi$$

and for a neutrosophic set, the following relation holds:

$$0^- \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3^+, \forall x \in \xi$$

Definition5: The complement of a neutrosophic set S is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x);\ I_S{}^c(x) = 1 - I_S(x);\ F_S{}^c(x) = T_S(x)$$

Definition6: A SVNS S_α is contained in the other SVNS S_β , denoted as $S_\alpha \subseteq S_\beta$ iff, $T_{S_\alpha}(x) \le T_{S_\beta}(x)$; $I_{S_\alpha}(x) \ge I_{S_\beta}(x)$; $F_{S_\alpha}(x) \ge F_{S_\beta}(x)$, $\forall x \in \xi$.

Definition7: Two single valued neutrosophic sets S_α and S_β are equal, i.e. $S_\alpha = S_\beta$, if and only if $S_\alpha \subseteq S_\beta$ and $S_\alpha \supseteq S_\beta$

Definition8: (Union) The union of two SVNSs S_α and S_β is a SVNS S_γ , written as $S_\gamma = S_\alpha \cup S_\beta$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_α and S_β as follows:

$$T_{S_\gamma}(x) = \max\left(T_{S_\alpha}(x), T_{S_\beta}(x)\right);$$

$$I_{S_\gamma}(x) = \min\left(I_{S_\alpha}(x), I_{S_\beta}(x)\right);$$

$F_{S_\gamma}(x) = \min\left(F_{S_\alpha}(x), F_{S_\beta}(x)\right)$ for all x in ξ

Definition 9: (intersection) The intersection of two SVNSs, S_α and S_β is a SVNS S_δ, written as $S_\delta = S_\alpha \cap S_\beta$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_α an S_β as follows:

$T_{S_\delta}(x) = \min\left(T_{S_\alpha}(x), T_{S_\beta}(x)\right)$;

$I_{S_\delta}(x) = \max\left(I_{S_\alpha}(x), I_{S_\beta}(x)\right)$;

$F_{S_\delta}(x) = \max\left(F_{S_\alpha}(x), F_{S_\beta}(x)\right), \forall x \in \xi$

3. Distance between two neutrosophic sets

The general SVNS can be written by the following form:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in \xi\}$

Finite SVNSs can be represented by the ordered tetrads:

$$S = \begin{Bmatrix} (x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, \\ (x_m/(T_S(x_m), I_S(x_m), F_S(x_m))) \end{Bmatrix}, \forall x \in \xi \quad (1)$$

Definition 10: Let

$$S_\alpha = \begin{Bmatrix} (x_1/(T_{S_\alpha}(x_1), I_{S_\alpha}(x_1), F_{\alpha S}(x_1))), \cdots, \\ (x_n/(T_{S_\alpha}(x_n), I_{S_\alpha}(x_n), F_{S_\alpha}(x_n))) \end{Bmatrix} \quad (2)$$

$$S_\beta = \begin{Bmatrix} (x_1/(T_{S_\beta}(x_1), I_{S_\beta}(x_1), F_{S_\beta}(x_1))), \cdots, \\ (x_n/(T_{S_\beta}(x_n), I_{S_\beta}(x_n), F_{S_\beta}(x_n))) \end{Bmatrix} \quad (3)$$

be two single-valued neutrosophic sets (SVNSs) in $x = \{x_1, x_2, x_3, \ldots, x_n\}$

Then the Hamming distance between two SVNSs as S_α and S_β is defined as follows:

$$d_S(S_\alpha, S_\beta) = \sum_{i=1}^{n} \left\langle \begin{matrix} \left|T_{S_\alpha}(x) - T_{S_\beta}(x)\right| + \\ \left|I_{S_\alpha}(x) - I_{S_\beta}(x)\right| + \\ \left|F_{S_\alpha}(x) - F_{S_\beta}(x)\right| \end{matrix} \right\rangle \quad (4)$$

and normalized Hamming distance between two SNVS S_α and S_β is defined as follows:

$$^N d_S(S_\alpha, S_\beta) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} \left|T_{S_\alpha}(x) - T_{S_\beta}(x)\right| + \\ \left|I_{S_\alpha}(x) - I_{S_\beta}(x)\right| + \\ \left|F_{S_\alpha}(x) - F_{S_\beta}(x)\right| \end{matrix} \right\rangle \quad (5)$$

with the following two properties

1. $0 \le d_S(S_\alpha, S_\beta) \le 3n \quad (6)$
2. $0 \le {}^N d_S(S_\alpha, S_\beta) \le 1 \quad (7)$

Definition11: From the neutrosophic cube [16], it can be stated that the membership grade represents the estimates reliability. Ideal neutrosophic reliability solution INERS[17]

$Q_S^+ = \left\langle q_{S_1}^+, q_{S_2}^+, \cdots, q_{S_n}^+ \right\rangle$ is a solution in which every component is presented by $q_{S_j}^+ = \left\langle T_j^+, I_j^+, F_j^+ \right\rangle$ where $T_j^+ = \max_i \{T_{ij}\}$, $I_j^+ = \min_i \{I_{ij}\}$ and $F_j^+ = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$

Definition 12: In the neutrosophic cube [16] maximum un-reliability occurs when the indeterminacy membership grade and the degree of falsity membership reaches maximum simultaneously. Therefore, the ideal neutrosophic estimates un-reliability solution (INEURS) [17]

$Q_S^- = \left\langle q_{S_1}^-, q_{S_2}^-, \cdots, q_{S_n}^- \right\rangle$ is a solution in which every component is represented by $q_{S_j}^- = \left\langle T_j^-, I_j^-, F_j^- \right\rangle$ where $T_j^- = \min_i \{T_{ij}\}$, $I_j^- = \max_i \{I_{ij}\}$ and $F_j^- = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$

3. Single valued neutrosophic multiple attribute decision-making problems based on GRA with interval weight information [17]

A multi-criteria decision making problem with m alternatives and n attributes is here considered. Let $A_1, A_2, \ldots, A_m$ be a discrete set of alternatives, and $C_1, C_2, \ldots, C_n$ be the set of criteria. The decision makers provide the ranking of alternatives. The ranking presents the performances of alternatives A_i against the criteria C_j. The values associated with the alternatives for MADM problem can be presented in the following decision matrix (see Table 1).

Table 1: Decision matrix

$$D = \left\langle \delta_{ij} \right\rangle_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \delta_{11} & \delta_{12} & \ldots & \delta_{1n} \\ A_2 & \delta_{21} & \delta_{22} & \ldots & \delta_{2n} \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \delta_{m1} & \delta_{m2} & \ldots & \delta_{mn} \end{array} \quad (8)$$

The weight $\omega_j \in [0, 1]$ (j = 1, 2, ..., n) represents the relative importance of criteria C_j (j = 1, 2, ..., m) to the decision-making process such that $\sum_{j=1}^{n} \omega_j = 1$. S is the set of partially known weight information that can be represented by the following forms due to Kim and Ahn[18] and Park [19].

Form1. A weak ranking: $\omega_i \ge \omega_j$ for $i \ne j$;

Form2. A strict ranking: $\omega_i - \omega_j \ge \psi_i$, $\psi_i > 0$, for $i \ne j$;

Form3. A ranking of differences: $\omega_i - \omega_j \ge \omega_k - \omega_1$, for $j \ne k \ne 1$;

Form4. A ranking with multiples: $\omega_i \ge \sigma_j \omega_j$, $\sigma_j \in [0, 1]$, for $i \ne j$;

Form5. An interval form $\delta_i \le \omega_i \le \delta_i + \varepsilon_i$, $0 \le \delta_i < \delta_i + \varepsilon_i \le 1$

The steps of single valued neutrosophic multiple attribute decision-making based on GRA under SVNS due to Biswal et al.[20] can be presented as follows.

Step1. Construction of the decision matrix with SVNSs

Consider the above mention multi attribte decision making problem(8). The general form of decision matrix as shown in Table1 can be presented after data pre-processing. Here, the ratings of alternatives A_i (i = 1, 2, … m) with respect to attributes C_j (j = 1, 2,…n) are considered as SVNSs. The neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix (see Table 2):

Table2:Decision matrix with SVNS

$$\delta_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \ldots & C_n \\ \hline A_1 & \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \ldots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ A_2 & \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \ldots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \ldots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{array} \quad (9)$$

In the matrix $\mathrm{d_S} = \langle \mathrm{T_{ij}}, \mathrm{I_{ij}}, \mathrm{F_{ij}} \rangle_{\mathrm{m\times n}}$ T_{ij} I_{ij} and F_{ij} denote the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i with respect to attribute C_j. These three components for SVNS satisfy the following properties:

1. $0 \le T_{ij} \le 1,\ \ 0 \le I_{ij} \le 1,\ \ 0 \le F_{ij} \le 1$ (10)
2. $0 \le T_{ij} + I_{ij} + F_{ij} \le 3$ (11)

Step2. Determination of the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS).

The ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS)for single valued neutrosophic decision matrix can be determined from the defintion 11 and 12.

Step3. Calculation of the neutrosophic grey relational coefficient.

Grey relational coefficient of each alternative from INERS can be defined as follows:

$$G_{ij}^{+} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}}{\Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}} \quad (12)$$

where $\Delta_{ij}^{+} = d\left(q_{S_j}^{+}, q_{S_{ij}}\right)$, i = 1, 2,…,m. and j = 1, 2,…,n.

Grey relational coefficient of each alternative from INEURS can be defined as follows:

$$G_{ij}^{-} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}}{\Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}} \quad (13)$$

where $\Delta_{ij}^{-} = d\left(q_{S_{ij}}, q_{S_j}^{-}\right)$, i = 1, 2,…,m. and j =1, 2,…, n.

$\rho \in [0,1]$ is the distinguishing coefficient or the identification coefficient,. Smaller value of distinguishing coefficient reflects the large range of grey relational coefficient. Generally, ρ = 0.5 is set for decision-making situation.

Step4. Determination of the weights of the criteria

The grey relational coefficient between INERS and itself is (1, 1, …, 1). Similarly, the grey relational coefficient between INEURS and itself is also (1, 1, …, 1). The corresponding deviations are presented as follows:

$$d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j \quad (14)$$

$$d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{-}\right)w_j \quad (15)$$

A satisfactory weight vector W= (w_1, w_2,…, w_n) is determined by making smaller all the distances $d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j$ and $d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j$

Using the max-min operator [21] to integrate all the distances $d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j$ for i = 1, 2, …, m and $d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j$ for i = 1, 2, …, m, Biswas et al. [20] formulated the following programming model:

$$Model: 1a: \begin{cases} \min z^{+} \\ subject\ to: \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_{ij} \le z^{+} \end{cases} \quad (16)$$

For i=1, 2, …, m

$$Model: 1b: \begin{cases} \min z^{-} \\ subject\ to: \sum_{j=1}^{n}\left(1 - G_{ij}^{-}\right)w_{ij} \le z^{-} \end{cases} \quad (17)$$

$W \in S$

Here $z^{+} = \max\limits_i \left\langle \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j \right\rangle$ and

$z^{-} = \max\limits_i \left\langle \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j \right\rangle$ for i =1, 2, …, m

Solving these two models (*Model-1a*) and (*Model-1b*), the optimal solutions W^{+}= (w_1^{+}, w_2^{+}, …, w_n^{+}) and W^{-} = (w_1^{-}, w_2^{-}, …, w_n^{-}) can be obtained. Combination of these two optimal solutions provides the weight vector of the criterion i.e.

W= tW^{+}+ (1-t)W^{-} for t ∈]0, 1[. (18)

Step5. Calculation of the neutrosophic grey relational coefficient (NGRC)

The degree of neutrosophic grey relational coefficient of each alternative from Indeterminacy Truthfullness Falsity Positive Ideal Solution (ITFPIS) and Indeterminacy Truthfullness Falsity Negative Ideal Solution (ITFNIS) are obtaoinrd using the following relationss:

$$G_i^{+} = \sum_{j=1}^{n} w_j G_{ij}^{+} \quad (19)$$

$$G_i^{-} = \sum_{j=1}^{n} w_j\, G_{ij}^{-} \quad (20)$$

Step6. Calculation of the neutrosophic relative relational degree (NRD)

Neutrosophic relative relational degree of each alternative from ITFPIS can be obtained by employing the following equation:

$$R_i = \frac{G_i^+}{G_i^+ + G_i^-} \quad (21)$$

Step7. **Ranking of the alternatives**

The highest value of neutrosophic relative relational degree R_i reflects the most desired alternative.

4. Single valued neutrosophic decision making model of school choice

Based on the field study, five major criteria for are identified by domain experts for developing a model for the selection of the best school by the parents for their children. The details are presented as follows.

1) Facility of transportation (C_1): It includes the cost of transportation facility availed by the child provided by school administration from child's house to the school.

2) Cost (C_2): It includes reasonable admission fees and other fees stipulated by the school administration.

3) Staff and curriculums (C_3): The degree of capability of the school administration in providing good competent staff, teaching and coaching, and extra curricular activities.

4) Healthy environmnet and medical facility(C_4): The degree of providing modern infrastructure, campus discipline, security, and medical facilities to the students by the school administration..

5) Administration(C_5): The degree of capability of administration in dealing with academic performance, staff and student welfare, reporting to parents.

After the initial screening, three schools listed below were considered as alternatives and an attempt has been made to develop a model to select the best one based on the above mentioned criteria.

A₁: Ananda Niketan Nursery & KG School, Santipur

A₂:Krishnagar Academy English Medium Public School, Krishnagar

A₃: Sent Mary's English School, Ranaghat

We obtain the following single-valued neutrosophic decision matrix (see Table 3) based on the experts' assessment:

Table3: Decision matrix with SVNS

$\delta_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{3\times 5} =$

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle .8,.1,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .8,.3,.3\rangle$	$\langle .7,.2,.4\rangle$	$\langle .7,.4,.3\rangle$
A_2	$\langle .7,.2,.2\rangle$	$\langle .8,.2,.3\rangle$	$\langle .8,.2,.3\rangle$	$\langle .7,.3,.4\rangle$	$\langle .8,.3,.3\rangle$
A_3	$\langle .8,.2,.3\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.1,.3\rangle$	$\langle .8,.3,.3\rangle$	$\langle .9,.1,.2\rangle$

(22)

Information of the attribute weights is partially known. The known weight information is given as follows:

$0.16 \le w_1 \le .22,\quad 0.15 \le w_2 \le 0.25,\quad 0.19 \le w_3 \le 0.3,$

$0.13 \le w_4 \le 0.21,\quad 0.17 \le w_5 \le 0.2,$

$\sum_{j=1}^{5} w_j = 1$

and $w_j \ge 0$ for $j = 1, 2, 3, 4, 5$

The problem is solved by the following steps:

Step1: Determination of the ideal neutrosophic estimates reliability solution

The ideal neutrosophic estimates reliability solution (INERS) from the given decision matrix (see Table 3) can be obtained as follows:

$$Q_S^+ = \left[q_{S_1}^+, q_{S_2}^+, q_{S_3}^+, q_{S_4}^+, q_{S_5}^+\right] = \begin{bmatrix} \left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\} \right\rangle, \\ \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\} \right\rangle, \\ \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{I3}\} \right\rangle, \\ \left\langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\} \right\rangle, \\ \left\langle \max_i\{T_{i5}\}, \min_i\{I_{i5}\}, \min_i\{F_{i5}\} \right\rangle \end{bmatrix} \quad (23)$$

$$= \begin{bmatrix} \langle .8,.1,.2\rangle, \langle .8,.2,.2\rangle, \langle .8,.1,.3\rangle, \\ \langle .8,.2,.3\rangle, \langle .9,.1,.2\rangle \end{bmatrix}$$

Step2. **Determination of the ideal neutrosophic estimates un-reliability solution**

The ideal neutrosophic estimates un-reliability solution can be obtained as follows:

$$Q_S^- = \left[q_{S_1}^-, q_{S_2}^-, q_{S_3}^-, q_{S_4}^-, q_{S_5}^-\right] = \begin{bmatrix} \left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\} \right\rangle, \\ \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\} \right\rangle, \\ \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\} \right\rangle, \\ \left\langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\} \right\rangle, \\ \left\langle \min_i\{T_{i5}\}, \max_i\{I_{i5}\}, \max_i\{F_{i5}\} \right\rangle \end{bmatrix} \quad (24)$$

$$= \begin{bmatrix} \langle .7,.2,.3\rangle, \langle .7,.2,.3\rangle, \langle .7,.3,.3\rangle, \\ \langle .7,.3,.4\rangle, \langle .7,.4,.3\rangle \end{bmatrix}$$

Step3. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS and INEURS

Using equation (12), the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as follows:

$\left\langle G_{ij}^{+} \right\rangle_{3\times5} =$

$$\begin{bmatrix} 1.0000 & 0.5692 & 0.4805 & 0.5692 & 0.3333 \\ 0.5692 & 0.6491 & 0.6491 & 0.5211 & 0.4253 \\ 0.5692 & 0.6491 & 0.6491 & 0.6491 & 1.0000 \end{bmatrix} \quad (25)$$

and from equation (13), the neutrosophic grey relational coefficient of each alternative from INEUS is obtained as follows:

$\left\langle G_{ij}^{-} \right\rangle_{3\times5} =$

$$\begin{bmatrix} 0.5211 & 1.0000 & 0.6491 & 0.6411 & 1.0000 \\ 0.6411 & 0.6411 & 0.5692 & 1.0000 & 0.5692 \\ 0.6411 & 0.6411 & 0.4805 & 0.5692 & 0.3333 \end{bmatrix} \quad (26)$$

Step4. Determination of the weights of attribute

Case1. Using the model (*Model-1a*) and (*Model-2b*), the single objective LPP models is formulated as follows:

Case1a:

Min z^{+}

Subject to,

$0.4308w_2+0.5195w_3+0.4308w_4+0.6667w_5 \le z^{+}$;

$0.4308w_1+0.3509w_2+0.3509w_3+0.4789w_4+0.5747w_5 \le z^{+}$;

$0.4308w_1+0.3509w_2+0.3509w_3+0.3509w_4 \le z^{+}$;

$0.16 \le w_1 \le 0.22$;

$0.15 \le w_2 \le 0.25$;

$0.19 \le w_3 \le 0.30$;

$0.13 \le w_4 \le 0.21$;

$0.17 \le w_5 \le 0.20$;

$w_1+ w_2+ w_3+ w_4+ w_5=1$;

$w_j \ge 0$, j= 1, 2, 3, 4, 5

Case1b:

Min z^{-}

Subject to,

$0.4789w_1+0.3509w_3+0.3589w_4 \le z^{-}$;

$0.3509w_1+0.3509w_2+0.4308w_3+0.4308w_5 \le z^{-}$;

$0.3589w_1+0.3509w_2+0.5195w_3+0.4308w_4 + 0.6667w_5 \le z^{-}$;

$0.16 \le w_1 \le .22$;

$0.15 \le w_2 \le 0.25$;

$0.19 \le w_3 \le 0.3$;

$0.13 \le w_4 \le 0.21$;

$0.17 \le w_5 \le .2$;

$w_1+ w_2+ w_3+ w_4+ w_5=1$;

$w_j \ge 0$, j= 1, 2, 3, 4, 5

After solving Case1a and Case1b separately, we obtain the solution set W^{+} = (0.1602, 0.15, 0.30, 0.21, 0.1798), W^{-} = (0.16, 0.15, 0.30, 0.21, 0.18) Therefore, the obtained weight vector of criteria is W = (0.1601, 0.15, 0.30, 0.21, 0.1799).

Step5. Determination of the degree of neutrosophic grey relational co-efficient (NGRC) of each alternative from INERS and INEUS.

The required neutrosophic grey relational co-efficient of each alternative from INERS is determined using equation (19). Tthe corresponding obtained weight vector W for Case-1 and Case-2 is presented in the Table 4. Similarly, the neutrosophic grey relational co-efficient of each alternative from INEURS is obtained with the help of equation (20) (see the Table 4).

Step6. Calculation of the neutrosophic relative relational degree (NRD)

Neutrosophic relative degree (NRD) of each alternative from INERS is obtained with the help of equation (21) (see the Table 4).

Table4: Ranking of the alternatives

Weight vector	(0.1601, 0.15, 0.30, 0.21, 0.1799)
NGRC from INERS	(0.5691, 0.5692, 0.6994)
NGRC from INEURS	(0.7427, 0.6820, 0.5224)
NRD from INERS	(0.4338, 0.4549, 0.5724)
Ranking Result	$R_3 > R_2 > R_1$
Selection	$\boldsymbol{R_3}$

Step7. Ranking of the alternatives

From Table4, we observe that $R_3 > R_2 > R_1$ i.e. Sent Mary's English School, Ranaghat ($\boldsymbol{A_3}$) is the best school for admission of children.

Conclusion

In this paper, we showed the application of single valued neutrosophic decision making model on school choice based on hybridization of grey system theory and single valed neutrosophic set. Five criteria are used to modeling the school choice problem in neutrosophic environment which are realistic in nature. New criterion can be easily incorporated in the model for decision making if it is needed. Application of the single-valued neutrosophic multiple attribte decision-making in real life problems helps the people to take a correct decision from the available alternatives in grey and neutrosophic hybrid environment. The concept presented in this paper can also be easily extended when the weight information are incomplete.

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Soft Interval –Valued Neutrosophic Rough Sets

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Abstract: In this paper, we first defined soft interval-valued neutrosophic rough sets(SIVN- rough sets for short) which combines interval valued neutrosophic soft set and rough sets and studied some of its basic properties. This concept is an extension of soft interval valued intuitionistic fuzzy rough sets(SIVIF- rough sets). Finally an illustartive example is given to verfy the developped algorithm and to demonstrate its practicality and effectiveness.

Keywords: Interval valued neutrosophic soft sets, rough set, soft Interval valued neutrosophic rough sets

1. Introduction

In 1999, Florentin Smarandache introduced the concept of neutrosophic set (NS) [13] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The concept of neutrosophic set is the generalization of the classical sets, conventional fuzzy set [27], intuitionistic fuzzy set [24] and interval valued fuzzy set [45] and so on. A neutrosophic sets is defined on universe U. x= x(T, I, F) ∈ A with T, I and F being the real standard or non –standard subset of $]0^{-},1^{+}[$, T is the degree of truth membership of A, I is the degree of indeterminacy membership of A and F is the degree of falsity membership of A. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and false –membership are independent.

Recently, works on the neutrosophic set theory is progressing rapidly. M. Bhowmik and M. Pal [28, 29] defined the concept “intuitionistic neutrosophic set”. Later on A. A. Salam and S. A.Alblowi [1] introduced another concept called “generalized neutrosophic set”. Wang et al [18] proposed another extension of neutrosophic set called ”single valued neutrosophic sets”. Also, H.Wang et al. [17] introduced the notion of interval valued neutrosophic sets (IVNSs) which is an instance of neutrosophic set. The IVNSs is characterized by an interval membership degree,interval indeterminacy degree and interval non-membership degree. K.Geogiev [25] explored some properties of the neutrosophic logic and proposed a general simplification of the neutrosophic sets into a subclass of theirs, comprising of elements of R^3. Ye [20, 21] defined similarity measures between interval neutrosophic sets and their multicriteria decision-making method. P. Majumdar and S.K. Samant [34] proposed some types of similarity and entropy of neutrosophic sets. S.Broumi and F. Smarandache [38,39,40] proposed several similarity measures of neutrosophic sets. P. Chi and L. Peid [33] extended TOPSIS to interval neutrosophic sets.

In 1999, Molodtsov [8]initiated the concept of soft set theory as proposed a new mathematical for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields including game theory, operations research, Riemmann integration, Perron integration. Recently, I. Deli [10] combined the concept of soft set and interval valued neutrosophic sets together by introducing anew concept called “ interval valued neutrosophic soft sets” and gave an application of interval valued neutrosophic soft sets in decision making. This concept generalizes the concept of the soft sets, fuzzy soft sets [35], intuitionistic fuzzy soft sets [36], interval valued intuitionistic fuzzy soft sets [22], the concept of neutrosophic soft sets [37] and intuitionistic neutrosophic soft sets [41].

The concept of rough set was originally proposed by Pawlak [50] as a formal tool for modeling and processing incomplete information in information systems. Rough set theory has been conceived as a tool to conceptualize, organize and analyze various types of data, in particular, to deal with inexact, uncertain or vague knowledge in applications related to artificial intelligence technique. Therefore, many models have been built upon different aspect, i.e, universe, relations, object, operators by many scholars [6, 9, 23, 48, 49, 51] such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough, rough intuitionistic fuzzy set, intuitionistic fuzzy rough sets [26]. The rough sets has been successfully applied in many fields such as attribute reduction [19, 30, 31, 46], feature selection [11, 18, 44], rule extraction [5, 7, 12, 47] and so on. The rough sets theory approximates any subset of objects of the universe by two sets, called the lower and upper approximations. The lower approximation of a given set is the union of all the equivalence classes which are subsets of the set, and the upper approximation is the union of all the equivalence classes which have a non empty intersection with the set.

Moreover, many new rough set models have also been established by combining the Pawlak rough set with other uncertainty theories such as soft set theory. Feng et al [14] provided a framework to combine fuzzy sets, rough sets, and soft sets all together, which gives rise to several interesting new concepts such as rough soft sets, soft rough sets, and soft rough fuzzy sets. The combination of hybrid structures of soft sets and rough sets models was also discussed by some researchers [15,32,43]. Later on, J. Zhang, L. Shu, and S. Liao [22] proposed the notions of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets, which can be seen as two new generalized soft rough set models, and investigated some properties of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets in detail. A.Mukherjee and A. Saha [3] proposed the concept of interval valued intuitionistic fuzzy soft rough sets. Also A. Saha and A. Mukherjee [4] introduced the concept of Soft interval valued intuitionistic fuzzy rough sets.

More recently, S.Broumi et al. [42] combined neutrosophic sets with rough sets in a new hybrid mathematical structure called "rough neutrosophic sets" handling incomplete and indeterminate information . The concept of rough neutrosophic sets generalizes rough fuzzy sets and rough intuitionistic fuzzy sets. Based on the equivalence relation on the universe of discourse, A. Mukherjee et al. [3] introduced soft lower and upper approximation of interval valued intuitionistic fuzzy set in Pawlak's approximation space. Motivated by the idea of soft interval valued intuitionistic fuzzy rough sets introduced in [4], we extend the soft interval intuitionistic fuzzy rough to the case of an interval valued neutrosophic set. The concept of soft interval valued neutrosophic rough set is introduced by coupling both the interval valued neutrosophic soft sets and rough sets.

The paper is structured as follows. In Section 2, we first recall the necessary background on soft sets, interval neutrosophic sets, interval neutrosophic soft sets, rough set, rough neutrosophic sets and soft interval valued intuitionistic fuzzy rough sets. Section 3 presents the concept of soft interval neutrosophic rough sets and examines their respective properties. Section 4 presents a multiciteria group decision making scheme under soft interval –valued neutrosophic rough sets. Section 5 presents an application of multiciteria group decision making scheme regarding the candidate selection problem . Finally we concludes the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of soft sets, interval neutrosophic setsinterval neutrosophic soft set, rough set, rough neutrosophic sets and soft interval valued intuitionistic fuzzy rough sets. For more details the reader may refer to [4, 8, 10, 13, 17, 50, 42].

Definition 2.1 [13] : Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x)$>,x $\in$ U}, where the functions $\mathbf{\mu_A(x)}$, $\mathbf{\nu_A(x)}, \mathbf{\omega_A(x)}$: U→]$^-$0,1$^+$[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x $\in$ X to the set A with the condition.

$$^-0 \leqslant sup\ \mu_A(x) + \sup\nu_A(x) + sup\omega_A(x) \leqslant 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0,1$^+$[. So instead of]$^-$0,1$^+$[we need to take the interval [0,1] for technical applications, because]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.3 [13]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeterminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x) \in$ int([0 ,1]).

For two IVNS, A_{IVNS}= {<x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | x ∈ X } (2)

And B_{IVNS}= {<x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1)$A_{IVNS} \subseteq B_{IVNS}$if and only if $\mu_A^L(x) \le \mu_B^L(x)$, $\mu_A^U(x) \le \mu_B^U(x)$,$\nu_A^L(x) \ge \nu_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$,$\omega_A^U(x) \ge \omega_B^U(x)$.

(2)$A_{\mathrm{IVNS}} = B_{\mathrm{IVNS}}$ if and only if, $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$

The complement of A_{IVNS} is denoted by A^o_{IVNS} and is defined by

A^o_{IVNS}={ <x , $[\omega^L_A(x), \omega^U_A(x)]$, $[1-\nu^U_A(x), 1-\nu^L_A(x)]$, $[\mu^L_A(x), \mu^U_A(x)]$ | x ∈ X }

A∩B ={ <x , [min($\mu^L_A(x)$,$\mu^L_B(x)$), min($\mu^U_A(x)$,$\mu^U_B(x)$)], [max($\nu^L_A(x)$,$\nu^L_B(x)$), max($\nu^U_A(x)$,$\nu^U_B(x)$)], [max($\omega^L_A(x)$,$\omega^L_B(x)$), max($\omega^U_A(x)$,$\omega^U_B(x)$)] >: x ∈ X }

A∪B ={ <x , [max($\mu^L_A(x)$,$\mu^L_B(x)$), max($\mu^U_A(x)$,$\mu^U_B(x)$)], [min($\nu^L_A(x)$,$\nu^L_B(x)$),min($\nu^U_A(x)$,$\nu^U_B(x)$)], [min($\omega^L_A(x)$,$\omega^L_B(x)$), min($\omega^U_A(x)$,$\omega^U_B(x)$)] >: x ∈ X }

As an illustration, let us consider the following example.

Example 2.4. Assume that the universe of discourse U={x_1, x_2, x_3}, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval valued neutrosophic set (IVNS) of U, such that, A = {< x_1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x_2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x_3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is [0.3, 0.4], degree of indeterminacy of capability is[0.5, 0.6] and degree of falsity of capability is [0.4, 0.5] etc.

Definition 2.5 . [8]

Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K : A → P(U).

As an illustration, let us consider the following example.

Example 2.6 . Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

A={e_1,e_2,e_3,e_4,e_5};

K(e_1) = {h_2, h_3, h_5}, K(e_2) = {h_2, h_4}, K(e_3) = {h_1}, K(e_4) = U, K(e_5) = {h_3, h_5}.

Definition 2.7. [10]

Let U be an initial universe set and A ⊂ E be a set of parameters. Let IVNS (U) denote the set of all interval valued neutrosophic subsets of U. The collection (K, A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K: A → IVNS(U).

The interval valued neutrosophic soft set defined over an universe is denoted by IVNSS.

Here,

1. ϒ is an ivn-soft subset of Ψ, denoted by ϒ ⋐ Ψ, if K(e) ⊆L(e) for all e∈E.
2. ϒ is an ivn-soft equals to Ψ, denoted by ϒ = Ψ, if K(e)=L(e) for all e∈E.
3. The complement of ϒ is denoted by $ϒ^c$, and is defined by $ϒ^c$ = {(x, K^o (x)): x∈E}
4. The union of ϒ and Ψ is denoted by ϒ ∪" Ψ, if K(e) ∪L(e) for all e∈E.
5. The intersection of ϒand Ψ is denoted by ϒ ∩" Ψ,if K(e) ∪ L(e) for all e∈E.

Example 2.8 :

Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is an interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, moderate, expensive }. In this case, to define an interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are four houses in the universe U given by, U = {h_1,h_2,h_3,h_4 } and the set of parameters A = {e_1,e_2,e_3}, where each e_i is a specific criterion for houses:

e_1 stands for 'beautiful',

e_2 stands for 'costly',

e_3 stands for 'moderate',

Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >}

.K(costly)={< h_1,[0.3, 0.6], [0.2 0.7], [0.1, 0.4]>,< h_2,[0.3, 0.5], [0.6 ,0.8], [0.2, 0.6] >, < h_3,[0.3, 0.7],[0.1 ,0.3],[0.3, 0.6] >,< h_4,[0.6 ,0.8],[0.2, 0.4],[0.2, 0.5 >}

K(moderate)={< h_1,[0.5, 0.8], [0.4, 0.7], [0.3, 0.6]>,< h_2,[0.3, 0.5], [0.7 ,0.9], [0.2, 0.4] >, < h_3,[0.1, 0.7],[0.3 ,0.3],[0.3, 0.6] >,< h_4,[0.3,0.8],[0.2, 0.4],[0.3, 0.6] >}.

Defintion.2.9 [50]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R_*X and $\boldsymbol{R}^*$X and are defined by

$R_*X=\{x \in U: [x]_R \subseteq X\}$,

$R^*X=\{ x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R_*X = R^*$ X, then X is called definable; otherwise X is called a rough set.

Definition 2.10 [42]

Let U be a non-null set and R be an equivalence relation on U. Let F be neutrosophic set in U with the membership function μ_F, indeterminacy function ν_F and non-membership function ω_F. Then, the lower and upper rough approximations of F in (U, R) are denoted by $\underline{R}$ (F) and $\overline{R}$(F) and respectively defined as follows:

$\overline{R}(F) =\{ <x, \mu_{\overline{R}(F)}(x) , \nu_{\overline{R}(F)}(x) , \omega_{\overline{R}(F)}(x)> | \ x\in U\}$,

$\underline{R}(F) =\{ <x, \mu_{\underline{R}(F)}(x) , \nu_{\underline{R}(F)}(x) , \omega_{\underline{R}(F)}(x)> | \ x\in U\}$,

Where:

$\mu_{\overline{R}(F)}(x) =\vee_{y \in [x]_R} \mu_F(y)$, $\nu_{\overline{R}(F)}(x)=\wedge_{y \in [x]_R} \nu_F(y)$, $\omega_{\overline{R}(F)} =\wedge_{y \in [x]_R} \omega_F(y)$,

$\mu_{\underline{R}(F)}(x)=\wedge_{y \in [x]_R} \mu_F(y)$, $\nu_{\underline{R}(F)}(x)=\vee_{y \in [x]_R} \nu_F(y)$, $\omega_{\underline{R}(F)} =\vee_{y \in [x]_R} \omega_F(y)$,

It is easy to observe that $\overline{R}$(F) and $\underline{R}$(F) are two neutrosophic sets in U, thus NS mapping

$\overline{R}$, $\underline{R}$:R(U) → R(U) are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair ($\underline{R}$(F), $\overline{R}$(F)) is called the rough neutrosophic set.

Definition 2.11[4] . Let us consider an interval-valued intuitionstic fuzzy set σ defined by

σ = {x, $\mu_\sigma(x)$, $\nu_\sigma(x)$: x ∈ U} where $\mu_\sigma(x)$, $\nu_\sigma(x)$, ∈ int ([0, 1]) for each x ∈ U and

$0 \leq \mu_\sigma(x)+ \nu_\sigma(x) \leq 1$

Now Let Θ=(f,A) be an interval-valued intuitionstic fuzzy soft set over U and the pair SIVIF= (U, Θ) be the soft interval-valued intuitionistic fuzzy approximation space.

Let f:A→ $IVIFS^U$ be defined f(a) ={ x, $\mu_{f(a)}(x)$, $\nu_{f(a)}(x)$: x ∈ U } for each a ∈ A. Then , the lower and upper soft interval-valued intuitionistic fuzzy rough approximations of σ with respect to SIVIF are denoted by $\downarrow Apr_{SIVIF}(\sigma)$ and $\uparrow Apr_{SIVIF}(\sigma)$ respectively, which are interval valued intuitionistic fuzzy sets in U given by:

$\downarrow Apr_{SIVIF}(\sigma)$ **={< x,**
[$\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_\sigma(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_\sigma(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_\sigma(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_\sigma(x)]$>: x ∈ U }

$\uparrow Apr_{SIVIF}(\sigma)$ **={< x, [** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_\sigma(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_\sigma(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_\sigma(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_\sigma(x)]$ >: x ∈ U}

The operators $\downarrow Apr_{SIVIF}(\sigma)$ and $\uparrow Apr_{SIVIF}(\sigma)$ are called the lower and upper soft interval-valued intuitionistic fuzzy rough approximation operators on interval valued intuitionistic fuzzy sets. If $\downarrow Apr_{SIVIF}(\sigma) = \uparrow Apr_{SIVIF}(\sigma)$, then σ is said to be soft interval valued intuitionistic fuzzy definable; otherwise is called a soft interval valued intuitionistic fuzzy rough set.

Example 3.3 . Let U={x, y) and A={a, b}. Let (f, A) be an interval –valued intuitionstic fuzzy soft set over U where f:A→ $IVIFS^U$ be defined

f(a)= **{** <x,[0.2, 0. 5], [0.3, 0.4]>, <y, [0.6, 0.7],[0.1, 0.2] >}

f(b)= **{** <x,[0.1, 0. 3], [0.4, 0.5>, <y, [0.5, 0.8],[0.1, 0.2] >}

Let σ = **{** <x,[0.3, 0.4], [0.3, 0.4]>, <y, [0.2, 0.4],[0.4, 0.5] >}. Then

$\downarrow Apr_{SIVIF}(\sigma)$= **{** <$x$,[0.1, 0.3],[0.3, 0.4] >, <y,[0.2, 0.4],[0.4, 0.5]>}

$\uparrow Apr_{SIVIF}(\sigma)$ = **{** <x,[0.3, 0.4],[0.3, 0.4] >, <y,[0.5, 0.7],[0.1, 0.2]>}. Then σ is a soft interval-valued intuitionstic fuzzy rough set.

3. Soft Interval Neutrosophic Rough Set.

A. Saha and A. Mukherjee [4] used the interval valued intuitioinstic fuzzy soft set to granulate the universe of discourse and obtained a mathematical model called soft interval –valued intuitionistic fuzzy rough set. Because the soft interval –valued intuitionistic fuzzy rough set cannot deal with indeterminate and inconsistent data, in this section, we attempt to develop an new concept called soft interval –valued neutrosophic rough sets.

Definition 3.1. Let us consider an interval-valued neutrosophic set σ defined by

σ = {x, $\mu_\sigma(x)$, $\nu_\sigma(x)$, $\omega_\sigma(x)$: x ∈ U} where $\mu_\sigma(x)$, $\nu_\sigma(x)$, $\omega_\sigma(x)$ ∈ int ([0, 1]) for each x ∈ U and

$$0 \leq \mu_\sigma(x)+ \nu_\sigma(x) + \omega_\sigma(x) \leq 3$$

Now Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and the pair SIVN= (U, Θ) be the soft interval-valued neutrosophic approximation space.

Let f:A→ $IVNS^U$ be defined f(a) ={ x, $\mu_{f(a)}(x)$, $\nu_{f(a)}(x)$, $\omega_{f(a)}(x)$: x ∈ U } for each a ∈ A. Then , the lower and upper soft interval-valued neutrosophic rough approximations of σ with respect to SIVN are denoted by $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ and $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ respectively, which are interval valued neutrosophic sets in U given by:

$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ **={<x,**
[$\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x)]$, **[** $\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))$, $\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma}(x)]$>: $x \in U$ }

$\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ **={< x, [** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_{\sigma}(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_{\sigma}(x)]$,
[$\wedge_{a \in A}(\inf \omega_{f(a)}(x) \wedge \inf \omega_{\sigma}(x))$, $\wedge_{a \in A}(\sup \omega_{f(a)}(x) \wedge \sup \omega_{\sigma}(x)]$>: $x \in U$}

The operators $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ and $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ are called the lower and upper soft interval-valued neutrosophic rough approximation operators on interval valued neutrosophic sets. If $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$, then σ is said to be soft interval valued neutrosophic definable; otherwise is called a soft interval valued neutrosophic rough set.

Remark 3.2: it is to be noted that if $\mu_{\sigma}(x)$, $\nu_{\sigma}(x)$, $\omega_{\sigma}(x) \in$ int ([0, 1]) and $0 \leq \mu_{\sigma}(x) + \nu_{\sigma}(x) + \omega_{\sigma}(x) \leq 1$, then soft interval valued neutrosophic rough sets becomes soft interval valued intuitionistic fuzzy rough sets.

Example 3.3 . Let U={x, y) and A={a, b}. Let (f, A) be an interval –valued neutrosophic soft se over U where f:A→ $IVNS^{U}$ be defined

f(a)= **{**<x,[0.2, 0. 5],[0.3, 0.4],[0.4, 0.5]>,<y,[0.6, 0.7],[0.1, 0.2],[0.3 0.4]>}
f(b)=**{**<x,[0.1, 0. 3],[0.4, 0.5],[0.1, 0.2]>, <y,[0.5, 0.8],[0.1, 0.2],[0.1 0.2]>}
Let σ =**{**<x,[0.3, 0.4],[0.3, 0.4],[0.1, 0.2]>, <y,[0.2, 0.4],[0.4, 0.5],[0.2 0.3]>}. Then

$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$= **{** <$x$,[0.1, 0.3],[0.3, 0.4],[0.1, 0.2]>, <y,[0.2, 0.4],[0.4, 0.5],[0.2, 0.3]>}
$\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ = **{** <x,[0.3, 0.4],[0.3, 0.4],[0.1, 0.2]>, <y,[0.5, 0.7],[0.1, 0.2],[0.1, 0.2]>}. Then σ is a soft interval-valued neutrosophic rough set.

Theorem 3.4

Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and SIVN= (U, Θ) be the soft interval-valued neutrosophic approximation space. Then for $\sigma, \lambda \in$ $\mathrm{IVNS}^{\mathrm{U}}$, we have

1) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\emptyset) = \emptyset = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\emptyset)$
2) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(U) = U = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(U)$
3) $\sigma \subseteq \lambda \Longrightarrow \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$
4) $\sigma \subseteq \lambda \Longrightarrow \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$
5) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.
6) $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$**.**
7) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$.
8) $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$

Proof .(1)-(4) are straight forward.
(5) We have
σ={<x,[$\inf \mu_{\sigma}(x)$, $\sup \mu_{\sigma}(x)$], [$\inf \nu_{\sigma}(x)$, $\sup \nu_{\sigma}(x)$], [$\inf \omega_{\sigma}(x)$, $\sup \omega_{\sigma}(x)$]>:x∈ U},
λ ={<x,[$\inf \mu_{\lambda}(x)$, $\sup \mu_{\lambda}(x)$], [$\inf \nu_{\lambda}(x)$, $\sup \nu_{\lambda}(x)$], [$\inf \omega_{\lambda}(x)$, $\sup \omega_{\lambda}(x)$]>:x∈ U}
and

$\sigma \cap \lambda$= {<x,[$\inf \mu_{\sigma \cap \lambda}(x)$, $\sup \mu_{\sigma \cap \lambda}(x)$], [$\inf \nu_{\sigma \cap \lambda}(x)$, $\sup \nu_{\sigma \cap \lambda}(x)$], [$\inf \omega_{\sigma \cap \lambda}(x)$, $\sup \omega_{\sigma \cap \lambda}(x)$]>:x∈ U},
Now
$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda)$=**{< x, [** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma \cap \lambda}(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma \cap \lambda}(x)]$,
[$\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma \cap \lambda}(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma \cap \lambda}(x)]$, **[** $\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma \cap \lambda}(x))$, $\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma \cap \lambda}(x)]$>: $x \in U$ }
=**{< x, [** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \min(\inf \mu_{\sigma}(x), \inf \mu_{\lambda}(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \min(\sup \mu_{\sigma}(x), \sup \mu_{\lambda}(x))]$,
[$\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \max(\inf \nu_{\sigma}(x), \inf \nu_{\lambda}(x)))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \max(\sup \nu_{\sigma}(x), \sup \nu_{\lambda}(x))]$,
[$\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \max(\inf \omega_{\sigma}(x), \inf \omega_{\lambda}(x)))$, $\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \max(\sup \omega_{\sigma}(x), \sup \omega_{\lambda}(x)]$>: $x \in U$ }

Now $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.

= **{< x, [min (** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))$, $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\lambda}(x))$), **min (** $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\lambda}(x))$)] ,[**max (** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))$, $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\lambda}(x))$), **max(** $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\lambda}(x))$)], [**max (** $\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))$

, $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ inf ω_{λ} (x))), **max(** $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) $\vee$ sup ω_{σ} (x)) , $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) $\vee$ sup ω_{λ} (x)))]> : x$\in$ U}.

Since min(inf μ_{σ} (y), infμ_{λ} (y)) $\leq$ inf μ_{σ}(y)
and min(inf μ_{σ} (y), infμ_{λ} (y)) $\leq$ inf μ_{λ}(y)
we have
$\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ min(inf μ_{σ} (x) , inf μ_{λ}(x)) $\leq$ $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ inf μ_{σ} (x))
and $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ min(inf μ_{σ} (x) , inf μ_{λ}(x)) $\leq$ $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ inf μ_{λ} (x))

Hence $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ min(inf μ_{σ} (x) , inf μ_{λ}(x)) $\leq$ **min** ($\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ inf μ_{σ} (x)) , $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) $\wedge$ inf μ_{λ} (x)))

Similarly
$\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) $\wedge$ min(sup μ_{σ} (x) , sup μ_{λ}(x)) $\leq$ **min** ($\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) $\wedge$ sup μ_{σ} (x)) , $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) $\wedge$ sup μ_{λ} (x)))
Again since
max(inf ν_{σ} (y), infν_{λ} (y)) $\geq$ inf ν_{σ}(y)
and max(inf ν_{σ} (y), infν_{λ} (y)) $\geq$ inf ν_{λ}(y)

we have
$\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ max(inf ν_{σ} (x) , inf ν_{λ}(x)) $\geq$ $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ inf ν_{σ} (x))
and $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ max(inf ν_{σ} (x) , inf ν_{λ}(x)) $\geq$ $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ inf ν_{λ} (x))

Hence $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ max(inf ν_{σ} (x) , inf ν_{λ}(x)) $\geq$ **max** ($\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ inf ν_{σ} (x)) , $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) $\vee$ inf ν_{λ} (x)))

Similarly
$\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) $\vee$ max(sup ν_{σ} (x) , sup ν_{λ}(x)) $\geq$ **max** ($\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) $\vee$ sup ν_{σ} (x)) , $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) $\vee$ sup ν_{λ} (x)))

Again since
max(inf ω_{σ} (y), infω_{λ} (y)) $\geq$ inf ω_{σ}(y)
And max(inf ω_{σ} (y), infω_{λ} (y)) $\geq$ inf ω_{λ}(y)

we have
$\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ max(inf ω_{σ} (x) , inf ω_{λ}(x)) $\geq$ $\wedge_{a \in A}$ (inf $\nu_{\omega f(a)}$(x) $\vee$ inf ω_{σ} (x))
and $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ max(inf ω_{σ} (x) , inf ω_{λ}(x)) $\geq$ $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\wedge$ inf ω_{λ} (x))

Hence
$\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ max(inf ω_{σ} (x) , inf ν_{λ}(x)) $\geq$ **max** ($\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ inf ω_{σ} (x)) , $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) $\vee$ inf ω_{λ} (x)))

Similarly
$\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) $\vee$ max(sup ω_{σ} (x) , sup ω_{λ}(x)) $\geq$ **max** ($\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) $\vee$ sup ω_{σ} (x)) , $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) $\vee$ sup ω_{λ} (x)))
Consequently,
$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.

(6) Proof is similar to (5).
(7) we have
σ={<x,[inf μ_{σ}(x), sup μ_{σ}(x)] ,[inf ν_{σ}(x), sup ν_{σ}(x)] ,[inf ω_{σ}(x), sup ω_{σ}(x)]>:x$\in$ U},
λ ={<x,[inf μ_{λ}(x), sup μ_{λ}(x)],[inf ν_{λ}(x), sup ν_{λ}(x)] ,[inf ω_{λ}(x), sup ω_{λ}(x)]>:x$\in$ U}
And

$\sigma \cup \lambda$= {<x,[inf $\mu_{\sigma \cup \lambda}$(x), sup $\mu_{\sigma \cup \lambda}$(x)], [inf $\nu_{\sigma \cup \lambda}$(x), sup $\nu_{\sigma \cup \lambda}$(x)], [inf $\omega_{\sigma \cup \lambda}$(x), sup $\omega_{\sigma \cup \lambda}$(x)]>:x∈ U},
$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$=**{< x, [** $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf $\mu_{\sigma \cup \lambda}$(x)) , $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ sup $\mu_{\sigma \cup \lambda}$(x)],
[$\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf $\nu_{\sigma \cup \lambda}$(x)) , $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ sup $\nu_{\sigma \cup \lambda}$(x)], **[** $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ inf $\omega_{\sigma \cup \lambda}$(x)) ,
$\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ sup $\omega_{\sigma \cup \lambda}$(x)]>: x ∈ U }
=**{< x, [** $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ max(inf μ_σ (x) , inf μ_λ(x)) , $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ max(sup μ_σ (x) , sup μ_λ(x))],
[$\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ min(inf ν_σ (x) , inf ν_λ(x))) , $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ min(sup ν_σ (x) , sup ν_λ(x))],
[$\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ min(inf ω_σ (x) , inf ω_λ(x))) , $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ min(sup ω_σ (x) , sup ω_λ(x)]>: x ∈ U }

Now $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.

= **{< x, [max (** $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_σ (x)) , $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_λ (x))), **max(** $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ sup μ_σ (x)) , $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ sup μ_λ (x)))], [**min (** $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_σ (x)) , $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_λ (x))), **min (** $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ sup ν_σ (x)) , $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ sup ν_λ (x)))], [**min (** $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ inf ω_σ (x)) , $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ inf ω_λ (x))), **min (** $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ sup ω_σ (x)) , $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ sup ω_λ (x)))]> : x∈ U}

Since max(inf μ_σ (y), infμ_λ (y)) ≥ inf μ_σ(y)
and max(inf μ_σ (y), infμ_λ (y)) ≥ inf μ_λ(y)
we have
$\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ max(inf μ_σ (x) , inf μ_λ(x)) ≥ $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_σ (x))
and $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ max(inf μ_σ (x) , inf μ_λ(x)) ≥ $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_λ (x))

Hence $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ max(inf μ_σ (x) , inf μ_λ(x)) ≥ **max** ($\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_σ (x)) , $\wedge_{a \in A}$ (inf $\mu_{f(a)}$(x) ∧ inf μ_λ (x)))

Similarly
$\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ max(sup μ_σ (x) , sup μ_λ(x)) ≥ **max** ($\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ sup μ_σ (x)) , $\wedge_{a \in A}$ (sup $\mu_{f(a)}$(x) ∧ sup μ_λ (x)))
Again since
min(inf ν_σ (y), infν_λ (y)) ≤ inf ν_σ(y)
and min(inf ν_σ (y), infν_λ (y)) ≤ inf ν_λ(y)

we have
$\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ min(inf ν_σ (x) , inf ν_λ(x)) ≤ $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_σ (x))
and $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ min(inf ν_σ (x) , inf ν_λ(x)) ≤ $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_λ (x))

Hence $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ min(inf ν_σ (x) , inf ν_λ(x)) ≤ **min** ($\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_σ (x)) , $\wedge_{a \in A}$ (inf $\nu_{f(a)}$(x) ∨ inf ν_λ (x)))

Similarly
$\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ min(sup ν_σ (x) , sup ν_λ(x)) ≤ **minx** ($\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ sup ν_σ (x)) , $\wedge_{a \in A}$ (sup $\nu_{f(a)}$(x) ∨ sup ν_λ (x)))

Again since
min(inf ω_σ (y), infω_λ (y)) ≤ inf ω_σ(y)
And min(inf ω_σ (y), infω_λ (y)) ≤ inf ω_λ(y)

we have
$\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ min(inf ω_σ (x) , inf ω_λ(x)) ≤ $\wedge_{a \in A}$ (inf $\nu_{\omega f(a)}$(x) ∨ inf ω_σ (x))
and $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ min(inf ω_σ (x) , inf ω_λ(x) ≤ $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ inf ω_λ (x))

Hence $\wedge_{a \in A}(\inf\omega_{f(a)}(x) \vee \min(\inf\omega_{\sigma}(x), \inf\nu_{\lambda}(x)) \leq \mathbf{min}(\wedge_{a \in A}(\inf\omega_{f(a)}(x) \vee \inf\omega_{\sigma}(x)), \wedge_{a \in A}(\inf\omega_{f(a)}(x) \vee \inf\omega_{\lambda}(x)))$

Similarly
$\wedge_{a \in A}(\sup\omega_{f(a)}(x) \vee \min(\sup\omega_{\sigma}(x), \sup\omega_{\lambda}(x)) \leq \mathbf{min}(\wedge_{a \in A}(\sup\omega_{f(a)}(x) \wedge \sup\omega_{\sigma}(x)), \wedge_{a \in A}(\sup\omega_{f(a)}(x) \wedge \sup\omega_{\lambda}(x)))$
Consequently,
$\downarrow Apr_{SIVN}(\sigma) \cup \downarrow Apr_{SIVN}(\lambda) \subseteq \downarrow Apr_{SIVN}(\sigma \cup \lambda)$
(8) Proof is similar to (7).

Theorem 3.5. Every soft interval-valued neutrosophic rough set is an interval valued neutrosophic soft set.
Proof. Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and SIVN=(U, Θ) be the soft interval-valued neutrosophic approximation space. Let σ be a soft interval-valued neutrosophic rough set. Let us define an interval-valued neutrosophic set χ by:

$$\chi = \{(x, [\frac{\wedge_{a \in A}(\inf\mu_{f(a)}(x) \wedge \inf\mu_{\sigma}(x))}{\wedge_{a \in A}(\inf\mu_{f(a)}(x) \vee \inf\mu_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup\mu_{f(a)}(x) \wedge \sup\mu_{\sigma}(x))}{\wedge_{a \in A}(\sup\mu_{f(a)}(x) \vee \sup\mu_{\sigma}(x))}], [\frac{\wedge_{a \in A}(\inf\nu_{f(a)}(x) \wedge \inf\nu_{\sigma}(x))}{\wedge_{a \in A}(\inf\nu_{f(a)}(x) \vee \inf\nu_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup\nu_{f(a)}(x) \wedge \sup\nu_{\sigma}(x))}{\wedge_{a \in A}(\sup\nu_{f(a)}(x) \vee \sup\nu_{\sigma}(x))}], [\frac{\wedge_{a \in A}(\inf\omega_{f(a)}(x) \wedge \inf\omega_{\sigma}(x))}{\wedge_{a \in A}(\inf\omega_{f(a)}(x) \vee \inf\omega_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup\omega_{f(a)}(x) \wedge \sup\mu\omega_{\sigma}(x))}{\wedge_{a \in A}(\sup\omega_{f(a)}(x) \vee \sup\omega_{\sigma}(x))}]): x \in U \}$$

Now, for $\theta \in [0, 1]$, we consider the following six sets:

$F_1(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\inf\mu_{f(a)}(x) \wedge \inf\mu_{\sigma}(x))}{\wedge_{a \in A}(\inf\mu_{f(a)}(x) \vee \inf\mu_{\sigma}(x))} \geq \theta\}$

$F_2(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\sup\mu_{f(a)}(x) \wedge \sup\mu_{\sigma}(x))}{\wedge_{a \in A}(\sup\mu_{f(a)}(x) \vee \sup\mu_{\sigma}(x))} \geq \theta\}$

$F_3(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\inf\nu_{f(a)}(x) \wedge \inf\nu_{\sigma}(x))}{\wedge_{a \in A}(\inf\nu_{f(a)}(x) \vee \inf\nu_{\sigma}(x))} \geq \theta\}$

$F_4(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\sup\nu_{f(a)}(x) \wedge \sup\nu_{\sigma}(x))}{\wedge_{a \in A}(\sup\nu_{f(a)}(x) \vee \sup\nu_{\sigma}(x))} \geq \theta\}$

$F_5(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\inf\omega_{f(a)}(x) \wedge \inf\omega_{\sigma}(x))}{\wedge_{a \in A}(\inf\omega_{f(a)}(x) \vee \inf\omega_{\sigma}(x))} \geq \theta\}$

$F_6(\theta) = \{ x \in U : \frac{\wedge_{a \in A}(\sup\omega_{f(a)}(x) \wedge \sup\mu\omega_{\sigma}(x))}{\wedge_{a \in A}(\sup\omega_{f(a)}(x) \vee \sup\omega_{\sigma}(x))} \geq \theta\}$

Then $\psi(\theta)$= { (x, [inf{ θ: x ∈ $F_1(\theta)$}, inf{ θ: x ∈ $F_2(\theta)$}], [inf{ θ: x ∈ $F_3(\theta)$}, inf{ θ: x ∈ $F_4(\theta)$}], [inf{ θ: x ∈ $F_5(\theta)$}, inf{ θ: x ∈ $F_6(\theta)$}]) :x ∈ U}is an interval –valued neutrosophic set over U for each $\theta \in [0, 1]$. Consequently (ψ, θ) is an interval-valued neutrosophic soft set over U.

4.A Multi-criteria Group Decision Making Problem

In this section, we extend the soft interval –valued intuitionistic fuzzy rough set based multi-criteria group decision making scheme [4] to the case of the soft interval-valued neutrosophic rough set.

Let U={$o_1, o_2, o_3,\dots, o_r$} be a set of objects and E be a set of parameters and A = {$e_1, e_2, e_3,\dots, e_m$} ⊆ E and S=(F, A) be an interval- neutrosophic soft set over U. Let us assume that we have an expert group G = {$T_1, T_2, T_3,\dots, T_n$} consisting of n specialists to evaluate the objects in U. Each specialist will examine all the objects in U and will point out his/her evaluation result. Let X_i denote the primary evaluation result of the specialist T_i. It is easy to see that the primary evaluation result of the whole expert group G can be represented as an interval valued neutrosophic evaluation soft set S^* = (F^*, G) over U, where $F^*: G \longrightarrow IVNS^U$ is given by $F^*(T_i)$= X_i, for i=1,2,..n.

Now we consider the soft interval valued neutrosophic rough approximations of the specialist T_i's primary evaluation result X_i w.r.t the soft interval valued neutrosophic approximation space SIVN = (U, S). Then we obtain two other interval valued neutrosophic soft sets $\downarrow S^*$= ($\downarrow F^*$,G) and $\uparrow S^*$= ($\uparrow F^*$,G) over U, where $\downarrow S^*$: $G \longrightarrow IVNS^U$ is given by $\downarrow F^*$ =$\downarrow X_i$ and $\uparrow F^*: G \longrightarrow IVNS^U$ is given by $\uparrow F^*(T_i)$= =$\uparrow X_i$, for i=1,2,..n. Here $\downarrow S^*$ can be considered as the evaluation result for the whole expert group G with 'low confidence', $\uparrow S^*$ can be considered as the evaluation result for the whole expert group G with 'high confidence' and S^* can be considered as the evaluation result for the whole expert group G with 'middle confidence' Let us define two interval valued neutrosophic sets $IVNS_{\downarrow S^*}$ **and** $IVNS_{\uparrow S^*}$ by

$IVNS_{\downarrow S^*}$ ={⟨o_k, [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\mu_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\mu_{\downarrow F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\nu_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\nu_{\downarrow F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\omega_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\omega_{\downarrow F^*(T_j)}(o_k)$]>: $k = 1,2,..r$}

And
$IVNS_{\uparrow S^*}$ ={⟨o_k, **[**$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\mu_{\uparrow F^*(T_i)}(o_k)$**,** $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\mu_{\uparrow F^*(T_i)}(o_k)$**],** **[**$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\nu_{\uparrow F^*(T_i)}(o_k)$**,** $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\nu_{\uparrow F^*(T_i)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\omega_{\uparrow F^*(T_i)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\omega_{\uparrow F^*(T_i)}(o_k)$]>: $k = 1,2,..r$}

Now we define another interval valued neutrosophic set $IVNS_{S^*}$ by

$IVNS_{S^*}$ $=\{\langle o_k, [\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\mu_{F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\mu_{F^*(T_j)}(o_k)], [\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\nu_{F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\nu_{F^*(T_j)}(o_k)], [\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\omega_{F^*(T_j)}(o_k), \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\omega_{F^*(T_j)}(o_k)]\rangle : k = 1,2,..r\}$

Then clearly,

$IVNS_{\downarrow S^*} \subseteq IVNS_{S^*} \subseteq IVNS_{\uparrow S^*}$

Let C={L (low confidence), M (middle confidence), H (high confidence)} be a set of parameters. Let us consider the interval valued neutrosophic soft set S^{**}= (f, C) over U, where f: $C \longrightarrow IVNS^U$ is given by f(L)= $IVNS_{\downarrow S^*}$, f(M)= $IVNS_{S^*}$, f(H)= $IVNS_{\uparrow S^*}$. Now given a weighting vector W= (ω_L , ω_M , ω_H) such that ω_L , ω_M , $\omega_H \in$ [0, 1], we define $\alpha: U \longrightarrow P(U) by\ \alpha(o_k) = \omega_L \diamond s_{f(L)}(o_k) + \omega_M \diamond s_{f(M)}(o_k) + \diamond s_{f(H)}(o_k)$, $o_k \in$ U ($\diamond$ represents ordinary multiplication) where

$s_{f(L)}(o_k) = \frac{\boldsymbol{inf}\mu_{\downarrow F^*(T_j)} + \boldsymbol{sup}\mu_{\downarrow F^*(T_j)} - \boldsymbol{inf}\,\nu_{\downarrow F^*(T_j)} . \boldsymbol{sup}\,\nu_{\downarrow F^*(T_j)} - \boldsymbol{inf}\omega_{\downarrow F^*(T_j)} . \boldsymbol{sup}\omega_{\downarrow F^*(T_j)}}{2}$

denotes the score function, the same as $s_{f(M)}(o_k)$ and $s_{f(H)}(o_k)$. Here $\alpha(o_k)$ is called the weighted evaluation value of the alternative $o_k \in$ U. Finally, we can select the object o_p =max{ $\alpha(o_k)$}:k=1,2,…,r} as the most preferred alternative.

Algorithm:

(1) Input the original description Interval valued neutrosophic soft set (F, A).

(2) Construct the interval valued neutrosophic evaluation soft set S^* =(F^*, G)

(3) Compute the soft interval valued neutrosophic rough approximations and then construct the interval valued neutrosophic soft sets $\downarrow S^*$ **and** $\uparrow S^*$

(4) Construct the interval valued neutrosophic $IVNS_{\downarrow S^*}$, $IVNS_{S^*}$, $IVNS_{\uparrow S^*}$

(5) Construct the interval valued neutrosophic soft set S^{**}.

(6) Input the weighting vector W and compute the weighted evaluation values of each alternative $\alpha(o_k)$ of each alternative $o_k \in$ U.

(7) Select the object o_p such that object o_p =max{ $\alpha(o_k)$}:k=1,2,…,r} as the most preferred alternative.

5.An illustrative example

The following example is adapted from **[4]** with minor changes.

Let us consider a staff selection problem to fill a position in a private company.

Let U = {c_1, c_2, c_3, c_4, c_5} is the universe set consisting of five candidates. Let us consider the soft set S=(F, A), which describes the "quality of the candidates", where A={e_1 (experience), e_2 (computer knowledge), e_3 (young and efficient), e_4 (good communication skill)}. Let the tabular representation of the interval valued neutrosophicsoft set (F, A) be:

	c_1	c_2	c_3	c_4	c_5
e_1	([.2, .3],[.4, .5],[.3, .4])	([.5, .7],[.1, .3],[.2, .3])	([.4, .5],[.2, .4],[.2, .5])	([.1, .2],[.1, .3],[.1, .2])	([.3, .5],[.3, .4],[.1, .2])
e_2	([.3, .6],[.1, .2],[.2, .3])	([.1, .3],[.2, .3],[.2, .4])	([.3, .6],[.2, .4],[.2, .4])	([.5, .6],[.2, .3],[.2, .4])	([.1, .3],[.3, .6],[.2, .5])
e_3	([.4, .5],[.2, .3],[.4, .5])	([.2, .4],[.2, .5],[.1, .2])	([1, .3],[.4, .6],[.3, .5])	([.3, .4],[.3, .4],[.4, .6])	([.4, .6],[.1, .3],[.2, .3])
e_4	([.2, .4],[.6, .7],[.6, .7])	([.6, .7],[.1, .2],[.4, .5])	([.3, .4],[.3, .4],[.1, .2])	([.2, .4],[.4, .6],[.1, .2])	([.5, .7],[.1, .2],[.1, .5])

Let G = {T_1, T_2, T_3, T_4, T_4} be the set of interviewers to judge the quality of the candidate in U. Now if X_i denote the primary evaluation result of the interviewer T_i (for i=1, 2, 3, 4,5), then the primary evaluation result of the whole expert group G can be represented as an interval valued neutrosophic evaluation soft set S^*= (F^*,G) over U, where $F^*: G \longrightarrow IVNS^U$ is given by F^* (T_i) = X_i for i=1, 2, 3, 4,5.

Let the tabular representation of S^* be given as:

	c_1	c_2	c_3	c_4	c_5
T_1	([.4, .6],[.4, .5],[.3, .4])	([.3, .4],[.1, .2],[.2, .3])	([.2, 3],[.2, .3],[.2, .5])	([.6, .8],[.1, .2],[.1, .2])	([.1, .4],[.2, .3],[.1, .2])
T_2	([.3, .5],[.2, .4],[.2, .3])	([.5, .7],[.1, .3],[.2, .4])	([.4, .6],[.1, .3],[.2, .4])	([.3, .5],[.1, .3],[.2, .4])	([.4, .5],[.2, .3],[.2, .5])
T_3	([.1, .3],[.5, .6],[.4, .5])	([.2, .3],[.4, .5],[.1, .2])	([.1, .4],[.2, .4],[.3, .5])	([.2, .3],[.5, .6],[.4, .6])	([.3, .6],[.2, .3],[.2, .3])
T_4	([.2, .3],[.3, .4],[.6, .7])	([.4, .7],[.1, .2],[.4, .5])	([.3, .5],[.4, .5],[.1, .2])	([.4, .5],[.2, .4],[.1, .2])	([.5, .7],[.1, .2],[.1, .5])
T_5	([.6, .7],[.1, .2],[.6, .7])	([.3, .5],[.3, .4],[.4, .6])	([.5, .6],[.3, .4],[.2, .3])	([.1, .3],[.3, .6],[.4, .6])	([.1, .2],[.6, .8],[.2, .5])

Let us choose P=(U, S) as the soft interval valued neutrosophic approximation space. Let us consider the interval valued neutrosophic evaluation soft sets.

$\downarrow S^*$ = ($\downarrow F^*$, G) and $\uparrow S^*$ = ($\uparrow F^*$, G) over U.
Then the tabular representation of these sets are:

$\downarrow S^*$ = ($\downarrow F^*$, G):

	c_1	c_2	c_3	c_4	c_5
T_1	([.2, .3],[.1, .2],[.3, .4])	([.1, .3],[.3, .4],[.2, .3])	([.1, .3],[.2, .4],[.2, .5])	([.1, .2],[.1, .3],[.1, .2])	([.1, .3],[.2, .4],[.1, .2])
T_2	([.2, .3],[.2, .4],[.2, .3])	([.1, .3],[.1, .3],[.2, .4])	([.1, 3],[.2, .4],[.2, .4])	([.1, .2],[.1, .3],[.2, .4])	([.1, .3],[.2, .3],[.2, .5])
T_3	([.1, .3],[.5, .6],[.4, .5])	([.1, .3],[.4, .5],[.1, .2])	([.1, .3],[.2, .4],[.3, .5])	([.1, .2],[.5, .6],[.4, .6])	([.1, .3],[.2, .3],[.2, .3])
T_4	([.2, .3],[.3, .4],[.6, .7])	([.1, .3],[.1, .2],[.4, .5])	([.1, .3],[.4, .5],[.1, .2])	([.1, .2],[.2, .4],[.1, .2])	([.1, .3],[.1, .2],[.1, .5])
T_5	([.2, .3],[.1, .2],[.6, .7])	([.1, .3],[.2, .5],[.4, .6])	([.1, .3],[.3, .4],[.2, .3])	([.1, .2],[.3, 6],[.4, .6])	([.1, .2],[.6, .8],[.2, .5])

$\uparrow S^*$ = ($\uparrow F^*$, G)

	c_1	c_2	c_3	c_4	c_5
T_1	([.4, .6],[.1, .2],[.2, .3])	([.3, .4],[.1, .2],[.1, .2])	([.2, .3],[.2, .3],[.1, .2])	([.6, .8],[.1, .2],[.1, .2])	([.1, .4],[.1, .2],[.1, .2])
T_2	([.3, .5],[1, .2],[.2, .3])	([.5, .7],[.1, .2],[.1, .2])	([.4, .6],[.1, .3],[.1, .2])	([.3, .5],[.1, .3,[.1, .2])	([.4, .5],[.1, .2],[.1, .2])
T_3	([.2, .3],[.1, .2],[.2, .3])	([.2, .3],[.1, .2],[.1, .2])	([.1, .4],[.2, .4],[.1, .2])	([.2, .3],[.1 .3],[.1, .2])	([.3, .6],[.1, .2],[.1, .2])
T_4	([.2, .3],[.1, .2],[.2, .3])	([.4, .7],[.1, .2],[.1, .2])	([.3, .5],[.2, .4],[.1, .2])	([.4, .5],[.1, .3],[.1, .2])	([.5, .7],[.1, .2],[.1, .2])
T_5	([.6, .7],[.1, .2],[.2, .3])	([.3, .5],[.1, .2],[.1, .2])	([.5, .6],[.2, .4],[.1, .2])	([.1, .3],[.1, 3],[.1, .2])	([.1, .3],[.1, .2],[.1, .2])

Here, $\downarrow S^* \subseteq S^* \subseteq \uparrow S^*$

$IVNS_{\downarrow S^*}$ = **{** <c_1,[0.15, 0.35],[0.4, 0.625],[0.42, 0.52]> <c_2,[0.175, 0.325],[0.375, 0.575],[0.26, 0.4]>, <c_3,[0.175, 0.375],[0.375, 0.575],[0.2, 0.38]>, <c_4,[0.175, 0.375],[0.375, 0.575],[0.24, 0.4]>, <c_5,[0.175, 0.375],[0.375, 0.575],[0.16, 0.4]>}.

$IVNS_{\uparrow S^*}$= **{** <c_1,[0.575, 0.75],[0.125, 0.225],[0.2, 0.3]> <c_2,[0.575, 0.75],[0.125, 0.225], [0.1, 0.2]>, <c_3,[0.575, 0.725],[0.125, 0.225],[0.1, 0.2]>, <c_4,[0.525, 0.700],[0.125, 0.225],[0.1, 0.2]>, <c_5,[0.55, 0.700],[0.125, 0.225],[0.1, 0.2]>}.

$IVNS_{S^*}$= **{** <c_1,[0.25, 0.45],[0.375, 0.475],[0.42, 0.52]> <c_2,[0.375, 0.525],[0.225, 0.35], [0.26, 0.4]>, <c_3,[0.350, 0.525],[0.2, 0.4],[0.2, 0.38]>, <c_4,[0.4, 0.6],[0.20, 0.35],[0.24, 0.4]>, <c_5,[0.35, 0.55],[0.15, 0.375],[0.16, 0.4]>}.

Here, $IVNS_{\downarrow S^*} \subseteq IVNS_{S^*} \subseteq IVNS_{\uparrow S^*}$ **. Let** C={ L (low confidence), M (middle confidence),H(high confidence)} be a set of parameters. Let us consider the interval valued neutrosophic soft set S^{**}= (f, C) over U, where f: $C \longrightarrow IVNS^U$ is given by f(L) = $IVNS_{\downarrow S^*}$, f(M) = $IVNS_{S^*}$, f(H) = $IVNS_{\uparrow S^*}$. Now assuming the weighting vector W =(ω_L, ω_M, ω_H) such that ω_L= 0.7 ω_M=0.6, ω_H=0.8, we have ,

$\alpha(c_1)$ = 0.7 ⋄ 0.0158 +0.6 ⋄ 0.15174 +0.8 ⋄ 0.6184
=0.5968
$\alpha(c_2)$= 0.7 ⋄ 0.0901 +0.6 ⋄ 0.3586 +0.8 ⋄ 0.6384
= 0.7890
$\alpha(c_3)$= 0.7 ⋄ 0.1041 +0.6 ⋄ 0.3595 +0.8 ⋄ 0.6384
=0.7993
$\alpha(c_4)$= 0.7 ⋄ 0.1191 +0.6 ⋄ 0.4170 +0.8 ⋄ 0.6134
=0.8243
$\alpha(c_5)$= 0.7 ⋄ 0.1351 +0.6 ⋄ 0.3898 +0.8 ⋄ 0.600
=0.8093

Since max($\alpha(c_1)$, $\alpha(c_2)$, $\alpha(c_3)$, $\alpha(c_4)$, $\alpha(c_5)$} = 0.8243, so the candidate c_4 will be selected as the most preferred alternative.

5.Conclusions

In this paper we have defined, for the first time, the notion of soft interval valued neutrosophic rough sets which is a combination of interval valued neutrosophic rough sets and soft sets. We have studied some of their basic properties. Thus our work is a generalization of SIVIF-rough sets. We hope that this paper will promote the future study on soft interval valued neutrosophic rough sets to carry out a general framework for their application in practical life.

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Neutrosophic LA-Semigroup Rings

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Abstract. Neutrosophic LA-semigroup is a midway structure between a neutrosophic groupoid and a commutative neutrosophic semigroup. Rings are the old concept in algebraic structures. We combine the neutrosophic LA-semigroup and ring together to form the notion of neutrosophic LA-semigroup ring. Neutrosophic LA-semigroup ring is defined analogously to neutrosophic group ring and neutrosophic semigroup ring.

Keywords: Neutrosophic LA-semigroup, ring, neutrosophic LA-semigroup ring.

1. Introduction

Smarandache [13] in 1980 introduced neutrosophy which is a branch of philosophy that studies the origin and scope of neutralities with ideational spectra. The concept of neutrosophic set and logic came into being due to neutrosophy, where each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. This mathematical tool is used to handle problems with imprecise, indeterminate, incomplete and inconsistent etc. Kandasamy and Smarandache apply this concept in algebraic structures in a slight different manner by using the indeterminate/unknown element I, which they call neutrosophic element. The neutrosophic element I is then combine to the elements of the algebraic structure by taking union and link with respect to the binary operation * of the algebraic stucutre. Therefore, a neutrosophic algebraic structure is generated in this way. They studied several neutrosophic algebraic structure [3,4,5,6]. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup denoted as LA-semigroup is an algebraic structure which was studied by Kazim and Naseeruddin [7] in 1972. An LA-semigroup is basically a midway structure between a groupoid and a commutative semigroup. It is also termed as Able-Grassmann's groupoid shortly AG-groupoid [11]. LA-semigroup is a non-associative and non-commutative algebraic structure which closely matching with commutative semigroup. LA-semigroup is a generalization to semigroup theory which has wide applications in collaboration with semigroup. Mumtaz et al.[1] introduced neutrosophic left almost semigroup in short neutrosophic LA-semigroup which is basically generated by an LA-semigroup and the neutrosophic element I. Mumtaz et al.[1] also studied their generalization and other properties. Neutrosophic group rings [5] and neutrosophic semigroup rings [5] are defined analogously to group rings and semigroup rings respectively. In fact these are generalization of group ring and semigroup ring ring. The notion of neutrosophic matrix ring have been successfully applied and used in the neutrosophic models such as neutrosophic cognitive maps (NCMs), neutrosophic relational maps (NRMs) etc.

In this paper, we introduced neutrosophic LA-semigroup rings owing to neutrosophic semigroup rings. Neutrosophic LA-semigroup rings are generalization of neutrosophic semigroup rings. These neutrosophic LA-semigroup rings are defined analogously to neutrosophic group rings and neutrosophic semigroup rings. We also studied some of their basic properties and other related notions in this paper. In section 2, we after reviewing the literature, we presented some basic concepts of neutrosophic LA-semigroup and rings. In section 3, neutrosophic LA-semigroup rings are introduced and studied some of their properties.

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2. Basic Concepts

Definition 2.1 [1]: Let $(S,*)$ be an LA-semigroup and let $\langle S\cup I\rangle=\{a+bI : a,b\in S\}$. The neutrosophic

LA-semigroup is generated by S and I under the operation $*$ which is denoted as $N(S)=\{\langle S\cup I\rangle,*\}$, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I=0$.

I^{-1}, the inverse of I is not defined and hence does not exist.

Definition 2.2 [1]: Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition 2.3 [1]: Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if

$$N(S)N(K)\subseteq N(K),\ \{N(K)N(S)\subseteq N(K)\}.$$

If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition 2.4 [5]: Let $(R,+,\cdot)$ be a set with two binary operations $+$ and $\cdot$. Then $(R,+,\cdot)$ is called a ring if the following conditions are hold.

1. $(R,+)$ is a commutative group under the operation of $+$.
2. $(R,\cdot)$ is a semigroup under the operation of $\cdot$.
3. $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$ for all $a,b,c\in R$.

Definition 2.5 [5]: Let $(R,+,\cdot)$ be a ring and $(R_1,+,\cdot)$ be a proper subset of $(R,+,\cdot)$. Then $(R_1,+,\cdot)$ is called a subring if $(R_1,+,\cdot)$ itself is a ring under the operation of R.

Definition 2.6 [5]: Let R be a ring. The neutrosophic ring $\langle R\cup I\rangle$ is also a ring generated by R and I under the operation of R, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I=0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 2.8: Let $\mathbb{Z}$ be the ring of integers. Then $\langle \mathbb{Z}\cup I\rangle$ is the neutrosophic ring of integers.

Definition 2.8 [5]: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A proper subset P of $\langle R\cup I\rangle$ is called a neutosophic subring if P itself a neutrosophic ring under the operation of $\langle R\cup I\rangle$.

Definition 2.9 [5]: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A non-empty set P of $\langle R\cup I\rangle$ is called a neutrosophic ideal of $\langle R\cup I\rangle$ if the following conditions are satisfied.

1. P is a neutrosophic subring of $\langle R\cup I\rangle$, and
2. For every $p\in P$ and $r\in\langle R\cup I\rangle$, pr and $rp\in P$.

3. Neutrosophic LA-semigroup Rings

In this section, we introduced neutosophic LA-semigroup rings and studied some of their basic properties and types.

Definition 3.1: Let $\langle S\cup I\rangle$ be any neutrosophic LA-semigroup. R be any ring with 1 which is commutative or field. We define the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ of the neutrosophic LA-semigroup $\langle S\cup I\rangle$ over the ring R as follows:

1. $R\langle S\cup I\rangle$ consists of all finite formal sum of the form $\alpha=\sum_{i=1}^{n} r_i g_i$, $n<\infty$, $r_i\in R$ and $g_i\in\langle S\cup I\rangle$ $\left(\alpha\in R\langle S\cup I\rangle\right)$.

2. Two elements $\alpha=\sum_{i=1}^{n} r_i g_i$ and $\beta=\sum_{i=1}^{m} s_i g_i$ in $R\langle S\cup I\rangle$ are equal if and only if $r_i=s_i$ and $n=m$.

3. Let $\alpha=\sum_{i=1}^{n} r_i g_i,\beta=\sum_{i=1}^{m} s_i g_i\in R\langle S\cup I\rangle$;
$\alpha+\beta=\sum_{i=1}^{n}(\alpha_i+\beta_i)g_i\in R\langle S\cup I\rangle$, as $\alpha_i,\beta_i\in R$, so $\alpha_i+\beta_i\in R$ and $g_i\in\langle S\cup I\rangle$.

4. $0=\sum_{i=1}^{n} 0g_i$ serve as the zero of $R\langle S\cup I\rangle$.

5. Let $\alpha=\sum_{i=1}^{n} r_i g_i\in R\langle S\cup I\rangle$ then
$-\alpha=\sum_{i=1}^{n}(-\alpha_i)g_i$ is such that

$$\alpha+(-\alpha)=0$$
$$=\sum_{i=1}^{n}(\alpha_i+(-\alpha_i))g_i$$
$$=\sum 0g_i$$

Thus we see that $R\langle S\cup I\rangle$ is an abelian group under $+$.

6. The product of two elements α,β in $R\langle S\cup I\rangle$ is follows:

Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$. Then

$$\alpha.\beta=\sum_{\substack{1\le i\le n\\1\le j\le m}}^{n}\alpha_i.\beta_j g_i h_j \;=\sum_k y_k t_k$$

where $y_k=\sum\alpha_i\beta_j$ with $g_i h_j=t_k$, $t_k\in\langle S\cup I\rangle$ and $y_k\in R$.

Clearly $\alpha.\beta\in R\langle S\cup I\rangle$.

7. Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$ and $\gamma=\sum_{k=1}^{p}\delta_k l_k$.

Then clearly $\alpha(\beta+\gamma)=\alpha\beta+\alpha\gamma$ and $(\beta+\gamma)\alpha=\beta\alpha+\gamma\alpha$ for all $\alpha,\beta,\gamma\in R\langle S\cup I\rangle$, that is the distributive law holds.
Hence $R\langle S\cup I\rangle$ is a ring under the binary operations $+$ and $\bullet$. We call $R\langle S\cup I\rangle$ as the neutrosophic LA-semigroup ring.

Similarly on the same lines, we can define neutrosophic Right Almost semigroup ring abbrivated as neutrosophic RA-semigroup ring.

Example 3.2: Let $\mathbb{R}$ be the ring of real numbers and let $N(S)=\{1,2,3,4,1I,2I,3I,4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Then $\mathbb{R}\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring.

Theorem 3.3: Let $\langle S\cup I\rangle$ be a neutrosophic LA-semigroup and $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring such that $R\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring over R. Then $\langle S\cup I\rangle\subseteq R\langle S\cup I\rangle$.

Proposition 3.4: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring over the ring R. Then $R\langle S\cup I\rangle$ has non-trivial idempotents.

Remark 3.5: The neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ is commutative if and only if $\langle S\cup I\rangle$ is commutative neutrosophic LA-semigroup.

Remark 3.6: The neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ has finite number of elements if both R and $\langle S\cup I\rangle$ are of finite order.

Example 3.7: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example (1). Then $R\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of infinite order.

Example 3.8: Let $\langle S\cup I\rangle=\{1,2,3,4,5,1I,2I,3I,4I,5I\}$ with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
1	4	5	1	2	3	$4I$	$5I$	$1I$	$2I$	$3I$
2	3	4	5	1	2	$3I$	$4I$	$5I$	$1I$	$2I$
3	2	3	4	5	1	$2I$	$3I$	$4I$	$5I$	$1I$
4	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
5	5	1	2	3	4	$5I$	$1I$	$2I$	$3I$	$4I$
$1I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$
$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$
$3I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$
$4I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$
$5I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$

Let $\mathbb{Z}_2$ be the ring of two elements. Then $\mathbb{Z}_2\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of finite order.

Theorem 3.9: Every neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ contains atleast one proper subset which is an LA-semigroup ring.

Proof: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. Then clearly $RS\subseteq R\langle S\cup I\rangle$. Thus $R\langle S\cup I\rangle$ contains an LA-semigroup ring.

Definition 3.10: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let P be a proper subset of $R\langle S\cup I\rangle$. Then P is called a subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$ if $P=R\langle H\cup I\rangle$ or $Q\langle S\cup I\rangle$ or $T\langle H\cup I\rangle$. In $P=R\langle H\cup I\rangle$, R is a ring and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$ or in $Q\langle S\cup I\rangle$, Q is a proper subring with 1 of R and $\langle S\cup I\rangle$ is a neutrosophic LA-semigroup and if $P=T\langle H\cup I\rangle$, T is a subring of R with unity and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$.

Example 3.11: Let $\langle S\cup I\rangle$ and $\mathbb{R}\langle S\cup I\rangle$ be as in Example 3.2. Let $H_1=\{1,3\}$, $H_2=\{1,1I\}$ and $H_3=\{1,3,1I,3I\}$ are neutrosophic sub LA-semigroups. Then $\mathbb{Q}\langle S\cup I\rangle$, $\mathbb{R}H_1$, $\mathbb{Z}\langle H_2\cup I\rangle$ and $\mathbb{Q}\langle H_3\cup I\rangle$ are all subneutrosophic LA-semigroup rings of $\mathbb{R}\langle S\cup I\rangle$.

Definition 3.12: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a neutrosophic subring if $P=\langle S_1\cup I\rangle$ where S_1 is a subring of RS or R.

Example 3.13: Let $R\langle S\cup I\rangle=\mathbb{Z}_2\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example 3.8. Then clearly $\langle\mathbb{Z}_2\cup I\rangle$ is a neutrosophic subring of $\mathbb{Z}_2\langle S\cup I\rangle$.

Theorem 3.14: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring R. Then $R\langle S\cup I\rangle$ always has a nontrivial neutrosophic subring.

Proof: Let $\langle R\cup I\rangle$ be the neutrosophic ring which is generated by R and I. Clearly $\langle R\cup I\rangle \subseteq R\langle S\cup I\rangle$ and this guaranteed the proof.

Definition 3.15: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset T of $R\langle S\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring of $R\langle S\cup I\rangle$.

Example 3.16: Let $\mathbb{Z}_6\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup $\langle S\cup I\rangle$ over $\mathbb{Z}_6$. Then $T=\{0,3I\}$ is a proper subset of $\mathbb{Z}_6\langle S\cup I\rangle$ which is a pseudo neutrosophic subring of $\mathbb{Z}_6\langle S\cup I\rangle$.

Definition 3.17: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a sub LA-semigroup ring if $P=R_1H$ where R_1 is a subring of R and H is a sub LA-semigroup of S. SH is the LA-semigroup ring of the sub LA-semigroup H over the subring R_1.

Theorem 3.18: All neutrosophic LA-semigroup rings have proper sub LA-semigroup rings.

Definition 3.19: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a subring but P should not have the LA-semigroup ring structure and is defined to be a subring of $R\langle S\cup I\rangle$.

Definition 3.20: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a neutrosophic ideal of $R\langle S\cup I\rangle$,

1. if P is a neutrosophic subring or subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle S\cup I\rangle$, αp and $p\alpha\in P$.

One can easily define the notions of left or right neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$.

Example 3.21: Let $\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup with the following table.

$*$	1	2	3	$1I$	$2I$	$3I$
1	3	3	3	$3I$	$3I$	$3I$
2	3	3	3	$3I$	$3I$	$3I$
3	1	3	3	$1I$	$3I$	$3I$
$1I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$2I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$3I$	$1I$	$3I$	$3I$	$1I$	$3I$	$3I$

Let $R=\mathbb{Z}$ be the ring of integers. Then $\mathbb{Z}\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring $\mathbb{Z}$. Thus clearly $P=2\mathbb{Z}\langle S\cup I\rangle$ is a neutrosophic ideal of $R\langle S\cup I\rangle$.

Definition 3.22: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a pseudo neutrosophic ideal of $R\langle S\cup I\rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle S\cup I\rangle$, αp and $p\alpha\in P$.

Definition 3.23: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S\cup I\rangle$ such that R_1 is an ideal over P i.e,

$rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S \cup I \rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S \cup I \rangle$.

Definition 3.24: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 3.25: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S \cup I \rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 3.26: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

Definition 3.27: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S \cup I \rangle$ such that R_1 is an ideal over P i.e, $rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S \cup I \rangle$ relative to P. If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S \cup I \rangle$.

Definition 3.28: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S \cup I \rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 3.29: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

One can define pseudo quasi neutrosophic ideal, pseudo loyal quasi neutrosophic ideal and pseudo bounded quasi neutrosophic ideals of a neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

4. LA-semigroup Neutrosophic Ring

In this section, LA-semigroup Neutrosophic ring is introduced and studied some of their basic properties.

Definition 4.1: Let S be an LA-semigroup and $\langle R \cup I \rangle$ be a commutative neutrosophic ring with unity.

$\langle R \cup I \rangle [S]$ is defined to be the LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle R \cup I \rangle$ and $s_i \in S$. This LA-semigroup neutrosophic ring is defined analogous to the group ring or semigroup ring.

Example 4.2: Let $\langle \mathbb{Z}_2 \cup I \rangle = \{0, 1, I, 1+I\}$ be the neutrosophic ring and let $S = \{1, 2, 3\}$ be an LA-semigroup with the following table:

$*$	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Then $\langle \mathbb{Z}_2 \cup I \rangle [S]$ is an LA-semigroup neutrosophic ring.

Definition 4.3: Let $\langle S \cup I \rangle$ be a neutrosophic LA-semigroup and $\langle K \cup I \rangle$ be a neutrosophic field or a commutative neutrosophic ring with unity. $\langle K \cup I \rangle [\langle S \cup I \rangle]$ is defined to be the neutrosophic LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle K \cup I \rangle$ and $s_i \in S$.

Example 4.4: Let $\langle \mathbb{Z} \cup I \rangle$ be the ring of integers and let $N(S) = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Then $\langle \mathbb{Z} \cup I \rangle \langle S \cup I \rangle$ is a neutrosophic LA-semigroup neutrosophic ring.

Theorem 4.5: Every neutrosophic LA-semigroup neutrosophic ring contains a proper subset which is a neutrosophic LA-semigroup ring.

Proof: Let $\langle R \cup I \rangle \langle S \cup I \rangle$ be a neutrosophic LA-semigroup neutrosophic ring and let $T = R\langle S \cup I \rangle$ be a proper subset of $\langle R \cup I \rangle \langle S \cup I \rangle$. Thus clearly $T = R\langle S \cup I \rangle$ is a neutrosophic LA-semigroup ring.

Conclusion

In this paper, we introduced neutosophic LA-semigroup rings which are more general concept than neutrosophic semigroup rings. These neutrosophic LA-semigroup rings are defined analogously to neutrosophic semigroup rings. We have studiesd several properties of neutrosophic LA-semigroup rings and also define different kind of neutrosophic LA-semigroup rings.

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