On New Types of Weakly Neutrosophic Crisp Closed Functions

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Abstract. This study utilizes some ideas of α-closed and semi-α-closed sets in neutrosophic crisp topological space to state roughly innovative categories of weakly neutrosophic crisp closed functions such as; neutrosophic crisp α*-closed functions, neutrosophic crisp α**-closed functions, neutrosophic crisp semi-α-closed functions, neutrosophic crisp semi-α*-closed and neutrosophic crisp semi-α**-closed functions. Moreover, the interactions among these kinds of feebly neutrosophic crisp closed functions and the suggestions for crisp closed functions are described. Furthermore, some theorems, properties, and remarks are debated.

Keywords: Neutrosophic crisp α*-closed, neutrosophic crisp α**-closed, neutrosophic crisp semi-α-closed, neutrosophic crisp semi-α*-closed and neutrosophic crisp semi-α**-closed functions.

1. Introduction

Smarandache [1,2] extended the view of sets by defending neutrosophic sets as a generality of Zadeh's fuzzy set concept, which states there is no accurate meaning for the set [3]. Soon after, the intuitionistic fuzzy set theory was submitted by Atanassov, such that he suggested that some elements have the degree of non-membership in the set [4]. The recently exhibited notions fascinated numerous scholars of conventional mathematics. Perhaps, fuzzy topology was set up by Chang [5] and Lowen [6] by redirecting the constructs from fuzzy sets to the traditional topological spaces. Additionally, the extraction of neutrosophic crisp topological space (shortly, NCTS) was announced by A. A. Salama et al. [7]. M. Abdel-Basset et al. [8-13] provided a new neutrosophic technique. The interpretation of neutrosophic crisp semi-α-closed sets was tendered by R. K. Al-Hamido et al. [14]. Some views of αgs continuity and αgs irresolute functions was examined by V. Banupriya et al. [15]. Some principles of neutrosophic α^m-continuity was demonstrated by R. Dhavaseelan et al. [16]. The gb-closed sets then gb-continuity was directed by C. Maheswari et al. [17]. The homeomorphism in neutrosophic topological spaces was generalized by M. H. PAGE et al. [18]. A weakly neutrosophic crisp continuity was established by Q. H. Imran et al. [19,20]. Recently, new types of open mappings in weakly neutrosophic crisp topology was defined by Al-Obaidi et al. [21].

The target of the study is to submit different categories of neutrosophic crisp closed functions in weakly forms, such as; neutrosophic crisp α*-closed, neutrosophic crisp α**-closed, neutrosophic crisp semi-α-closed, neutrosophic crisp semi-α*-closed and neutrosophic crisp semi-α**-closed functions. Additionally, the connections concerning these kinds of weakly neutrosophic crisp closed functions are illuminated, corresponding to the thoughts of neutrosophic crisp closed functions. As Well, some theorems, properties and remarks are demonstrated.

2. Preliminaries

Through this paper, $(\mathcal{I},\zeta),(\mathcal{J},\eta)$ and $(\mathcal{K},\gamma)$ (or in short $\mathcal{I},\mathcal{J}$ and $\mathcal{K}$) constantly imply NCTSs. Assume that $\mathcal{L}$ be a neutrosophic crisp set in a NCTS $\mathcal{I}$, then

- $\mathcal{L}^c = \mathcal{I} - \mathcal{L}$ signifies the neutrosophic crisp complement of $\mathcal{L}$.
- $NC\text{-}cl(\mathcal{L})$ refers to the neutrosophic crisp closure of $\mathcal{L}$.
- $NC\text{-}int(\mathcal{L})$ speaks of the neutrosophic crisp interior of $\mathcal{L}$.

Definition 2.1 [7]: Assume non-empty fixed set $\mathcal{L}$ is sample space. The object with form $\mathcal{N} = \langle \mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3 \rangle$ is called a neutrosophic crisp set, for short NC-set such that $\mathcal{N}_1, \mathcal{N}_2$ and $\mathcal{N}_3$ are subsets of $\mathcal{L}$ with the mutually disjoint property.

Definition 2.2: Let $\mathcal{L}$ be a NC-subset of a NCTS $\mathcal{I}$, then we have the following

i. NCα-CS is denoted as a neutrosophic crisp α-closed set [18] if $NC\text{-}cl(NC\text{-}int(NC\text{-}cl(\mathcal{L}))) \subseteq \mathcal{L}$.
ii. NCα-OSs is signified as a neutrosophic crisp α-open set (the complement of a NCα-CS) in $\mathcal{I}$.
iii. $\mathrm{NC\alpha C}(\mathcal{I})$ (resp. $\mathrm{NC\alpha O}(\mathcal{I})$) is represented as the collection of each NCα-CSs (resp. NCα-OSs) of $\mathcal{I}$.
iv. NCSα-CS is indicated as a neutrosophic crisp semi-α-closed set [14] if
$$NC\text{-}int(NC\text{-}cl(NC\text{-}int(NC\text{-}cl(\mathcal{L})))) \subseteq \mathcal{L}$$
or regularly if there exists a NCα-CS $\mathcal{D}$ in $\mathcal{I}$ such that $NC\text{-}int(\mathcal{D}) \subseteq \mathcal{L} \subseteq \mathcal{D}$.
v. NCSα-OS is designated as a neutrosophic crisp semi-α-open set (the complement of a NCSα-CS) in $\mathcal{I}$.
vi. NCSαC($\mathcal{I}$) (resp. NCSαO($\mathcal{I}$)) is shown as the collection of each NCSα-CSs (resp. NCSα-OSs) of $\mathcal{I}$.

Remark 2.3 [14,20]: In a NCTS $\mathcal{I}$, the resulting declarations hang on, and the reverse of each declaration is a fallacy:

i. Each NC-CS is a NCα-CS and NCSα-CS.
ii. Each NCα-CS is a NCSα-CS.

Theorem 2.4 [18]: For any NC-subset $\mathcal{L}$ of a NCTS $\mathcal{I}$, $\mathcal{L} \in \mathrm{NC\alpha C}(\mathcal{I})$ iff there exists a NC-CS $\mathcal{D}$ such that $NC\text{-}cl(NC\text{-}int(\mathcal{D})) \subseteq \mathcal{L} \subseteq \mathcal{D}$.

Definition 2.5: A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is called:

i. Neutrosophic crisp closed (briefly NC-closed) [7] iff for each NC-CS $\mathcal{L}$ in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NC-CS in $\mathcal{J}$.
ii. Neutrosophic crisp α-closed (briefly NCα-closed) [19] iff for each NC-CS $\mathcal{L}$ in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$.

Theorem 2.6 [7]:

i. A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NC-closed iff $NC\text{-}cl(\psi(\mathcal{L})) \subseteq \psi(NC\text{-}cl(\mathcal{L}))$, for every $\mathcal{L} \subseteq \mathcal{I}$.
ii. A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is neutrosophic crisp continuous (shortly NC-continuous) iff for each NC-CS $\mathcal{L}$ in $\mathcal{J}$, then $\psi^{-1}(\mathcal{L})$ is a NC-CS in $\mathcal{I}$.
iii. A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NC-continuous iff $NC\text{-}int(\psi(\mathcal{L})) \subseteq \psi(NC\text{-}int(\mathcal{L}))$, for every $\mathcal{L} \subseteq \mathcal{I}$.

3. Weakly Neutrosophic Crisp Closed Functions

Definition 3.1: Assume $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is a function, then ψ is named as the following:

i. Neutrosophic crisp α*-closed (briefly NCα*-closed) iff for each NCα-CS $\mathcal{L}$ in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$.

ii. Neutrosophic crisp α**-closed (briefly NCα**-closed) iff for each $\mathcal{L}$ NCα-CS in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NC-CS in $\mathcal{J}$.

Definition 3.2: A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is called:

i. Neutrosophic crisp semi-α-closed (briefly NCSα-closed) iff for each $\mathcal{L}$ NC-CS in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NCSα-CS in $\mathcal{J}$.

ii. Neutrosophic crisp semi-α*-closed (briefly NCSα*-closed) iff for each $\mathcal{L}$ NCSα-CS in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NCSα-CS in $\mathcal{J}$.

iii. Neutrosophic crisp semi-α**-closed (briefly NCSα**-closed) iff for each $\mathcal{L}$ NCSα-CS in $\mathcal{I}$, then $\psi(\mathcal{L})$ is a NC-CS in $\mathcal{J}$.

Theorem 3.3: A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NCSα-closed iff for every $\mathcal{L} \subseteq \mathcal{I}$, $NC\text{-}int(NC\text{-}cl(NC\text{-}int(NC\text{-}cl(\psi(\mathcal{L}))))) \subseteq \psi(NC\text{-}cl(\mathcal{L}))$.

Proof:

Necessity: For any $\mathcal{L} \subseteq \mathcal{I}$, $\psi(NC\text{-}cl(\mathcal{L}))$ is NCSα-CS in $\mathcal{J}$ this implies that

$$NC\text{-}int(NC\text{-}cl(NC\text{-}int(\psi(NC\text{-}cl(\mathcal{L}))))) \subseteq \psi(NC\text{-}cl(\mathcal{L})).$$

Hence, we have

$$NC\text{-}int(NC\text{-}cl(NC\text{-}int(NC\text{-}cl(\psi(\mathcal{L}))))) \subseteq \psi(NC\text{-}cl(\mathcal{L})).$$

Sufficiency: For any $\mathcal{L} \subseteq \mathcal{I}$, we have by hypothesis

$$NC\text{-}int(NC\text{-}cl(NC\text{-}int(NC\text{-}cl(\psi(\mathcal{L}))))) \subseteq \psi(NC\text{-}cl(\mathcal{L})).$$

So $\psi(\mathcal{L})$ is NCSα-CS in $\mathcal{J}$ and then we get that the function ψ is a NCSα-closed. ■

Theorem 3.4:

i. Any function NC-closed is NCα-closed, then it is NCSα-closed. Nonetheless, the inverse is generally a fallacy.

ii. Any function NCα-closed is NCSα-closed. Nonetheless, the inverse is generally a fallacy.

Proof:

i. Assume $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is a NC-closed function, and $\mathcal{L}$ is a NC-CS in $\mathcal{I}$. Then $\psi(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Because NC-CS is NCα-CS (NCSα-CS), $\psi(\mathcal{L})$ is a NCα-CS (NCSα-CS) in $\mathcal{J}$. Thus, the function ψ is NCα-closed (NCSα-closed).

ii. Assume $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is a NCα-closed function and $\mathcal{L}$ is a NC-CS in $\mathcal{I}$. Then $\psi(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Because NCα-CS is NCSα-CS, $\psi(\mathcal{L})$ is a NCSα-CS in $\mathcal{J}$. Thus, the function ψ is NCSα-closed. ■

Example 3.5: Assume $\mathcal{I} = \{p_1, p_2, p_3, p_4\}$. Then

$$\zeta_{\mathcal{I}} = \{\phi_N, \langle\{p_3\}, \phi, \phi\rangle, \langle\{p_1, p_3\}, \phi, \phi\rangle, \langle\{p_1, p_2, p_3\}, \phi, \phi\rangle, \mathcal{I}_N\}$$

is a NCTS. The collection of every NC-CSs of $\mathcal{I}$ is:

$$\text{NC-C}(\mathcal{I}) = \{\mathcal{I}_N, \langle\{p_1, p_2, p_4\}, \phi, \phi\rangle, \langle\{p_2, p_4\}, \phi, \phi\rangle, \langle\{p_4\}, \phi, \phi\rangle, \phi_N\}.$$

The collection of every NCα-CSs (NCSα-CSs) of $\mathcal{I}$ is:

$$\text{NCS}\alpha\text{C}(\mathcal{I}) = \text{NC}\alpha\text{C}(\mathcal{I}) = \text{NC-C}(\mathcal{I}) \cup \{\langle\{p_1, p_4\}, \phi, \phi\rangle, \langle\{p_1, p_2\}, \phi, \phi\rangle, \langle\{p_2\}, \phi, \phi\rangle, \langle\{p_1\}, \phi, \phi\rangle\}.$$

Define a function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ by

$$\psi(\langle\{p_1\}, \phi, \phi\rangle) = \langle\{p_1\}, \phi, \phi\rangle, \psi(\langle\{p_2\}, \phi, \phi\rangle) = \langle\{p_4\}, \phi, \phi\rangle,$$
$$\psi(\langle\{p_3\}, \phi, \phi\rangle) = \langle\{p_3\}, \phi, \phi\rangle, \psi(\langle\{p_4\}, \phi, \phi\rangle) = \langle\{p_2\}, \phi, \phi\rangle.$$

We observe ψ is a NCα-closed. It is NCSα-closed; nonetheless, it is not a NC-closed function because of $\langle\{p_4\}, \phi, \phi\rangle$ is NC-CS in $\mathcal{I}$ and $\psi(\langle\{p_4\}, \phi, \phi\rangle) = \langle\{p_2\}, \phi, \phi\rangle$ is not a NC-CS in $\mathcal{I}$.

Example 3.6: Let $\mathcal{I} = \{p_1, p_2, p_3, p_4\}$. Then

$$\zeta_{\mathcal{I}} = \{\phi_N, \langle\{p_1\},\phi,\phi\rangle, \langle\{p_2\},\phi,\phi\rangle, \langle\{p_1,p_2\},\phi,\phi\rangle, \langle\{p_1,p_2,p_3\},\phi,\phi\rangle, \mathcal{I}_N\}$$

is a NCTS. The collection of every NC-CSs of $\mathcal{I}$ is:

$$\text{NC-C}(\mathcal{I}) = \{\mathcal{I}_N, \langle\{p_2,p_3,p_4\},\phi,\phi\rangle, \langle\{p_1,p_3,p_4\},\phi,\phi\rangle, \langle\{p_3,p_4\},\phi,\phi\rangle, \langle\{p_4\},\phi,\phi\rangle, \phi_N\}.$$

The collection of every NCα-CSs of $\mathcal{I}$ is:

$$\text{NC}\alpha\text{C}(\mathcal{I}) = \text{NC-C}(\mathcal{I}) \cup \{\langle\{p_3\},\phi,\phi\rangle\}.$$

The collection of every NCSα-CSs of $\mathcal{I}$ is:

$$\text{NCS}\alpha\text{C}(\mathcal{I}) = \text{NC}\alpha\text{C}(\mathcal{I}) \cup \{\langle\{p_2,p_4\},\phi,\phi\rangle, \langle\{p_2,p_3\},\phi,\phi\rangle, \langle\{p_1,p_4\},\phi,\phi\rangle, \langle\{p_1,p_3\},\phi,\phi\rangle, \langle\{p_2\},\phi,\phi\rangle, \langle\{p_1\},\phi,\phi\rangle\}.$$

Define a function $\psi: \mathcal{I} \longrightarrow \mathcal{I}$ by

$$\psi(\langle\{p_1\},\phi,\phi\rangle) = \langle\{p_1\},\phi,\phi\rangle, \psi(\langle\{p_2\},\phi,\phi\rangle) = \langle\{p_2\},\phi,\phi\rangle,$$
$$\psi(\langle\{p_3\},\phi,\phi\rangle) = \psi(\langle\{p_4\},\phi,\phi\rangle) = \langle\{p_4\},\phi,\phi\rangle.$$

We observe that the function ψ is a NCSα-closed. Furthermore, it is not NCα-closed function because of $\langle\{p_1,p_3,p_4\},\phi,\phi\rangle$ is NC-CS in $\mathcal{I}$ and $\psi(\langle\{p_1,p_3,p_4\},\phi,\phi\rangle) = \langle\{p_1,p_4\},\phi,\phi\rangle$ is not a NCα-CS in $\mathcal{I}$.

Remark 3.7: There is no relation between the ideas of NC-closed and NCα*-closed functions, as the next two examples are displayed below.

Example 3.8: The function ψ in Example (3.5) is a NCα*-closed. Nonetheless, it is not NC-closed.

Example 3.9: Assume that $\mathcal{I} = \{p_1, p_2, p_3, p_4\}$ is a set. Then

$$\zeta_{\mathcal{I}} = \{\phi_N, \langle\{p_1\},\phi,\phi\rangle, \langle\{p_2\},\phi,\phi\rangle, \langle\{p_1,p_2\},\phi,\phi\rangle, \langle\{p_1,p_2,p_3\},\phi,\phi\rangle, \mathcal{I}_N\}$$

is a NCTS. The collection of each NC-CSs of $\mathcal{I}$ is:

$$\text{NC-C}(\mathcal{I}) = \{\mathcal{I}_N, \langle\{p_2,p_3,p_4\},\phi,\phi\rangle, \langle\{p_1,p_3,p_4\},\phi,\phi\rangle, \langle\{p_3,p_4\},\phi,\phi\rangle, \langle\{p_4\},\phi,\phi\rangle, \phi_N\}.$$

The family of all NCα-CSs of $\mathcal{I}$ is: $\text{NC}\alpha\text{C}(\mathcal{I}) = \text{NC-C}(\mathcal{I}) \cup \{\langle\{p_3\},\phi,\phi\rangle\}$. The collection of each NCSα-CSs of $\mathcal{I}$ is:

$$\text{NCS}\alpha\text{C}(\mathcal{I}) = \text{NC}\alpha\text{C}(\mathcal{I}) \cup \{\langle\{p_2,p_4\},\phi,\phi\rangle, \langle\{p_2,p_3\},\phi,\phi\rangle, \langle\{p_1,p_4\},\phi,\phi\rangle, \langle\{p_1,p_3\},\phi,\phi\rangle, \langle\{p_2\},\phi,\phi\rangle, \langle\{p_1\},\phi,\phi\rangle\}$$

Define a function $\psi: \mathcal{I} \longrightarrow \mathcal{I}$ by

$$\psi(\langle\{p_1\},\phi,\phi\rangle) = \psi(\langle\{p_2\},\phi,\phi\rangle) = \langle\{p_1\},\phi,\phi\rangle,$$
$$\psi(\langle\{p_3\},\phi,\phi\rangle) = \langle\{p_2\},\phi,\phi\rangle, \psi(\langle\{p_4\},\phi,\phi\rangle) = \langle\{p_3\},\phi,\phi\rangle.$$

We observe that the ψ is a NC-closed. Furthermore, it is not NCα*-closed function because of $\langle\{p_3\},\phi,\phi\rangle$ is NCα-CS in $\mathcal{I}$ and $\psi(\langle\{p_3\},\phi,\phi\rangle) = \langle\{p_2\},\phi,\phi\rangle$ is not a NCα-CS in $\mathcal{I}$.

Proposition 3.10:

i. A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NC-closed and NC-continuous, then this function is NCα*-closed.

ii. A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is a NCα*-closed iff $\psi: (\mathcal{I}, \text{NC}\alpha\text{O}(\mathcal{I})) \longrightarrow (\mathcal{J}, \text{NC}\alpha\text{O}(\mathcal{J}))$ is NC-closed.

Proof:

i. Assume that a function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NC-closed and NC-continuous. To verify the function ψ is NCα*-closed, we suppose that $\mathcal{L} \in \text{NC}\alpha\text{C}(\mathcal{J})$, then for some sets like NC-CS $\mathcal{N}$ with this fact $NC\text{-}cl(NC\text{-}int(\mathcal{N})) \subseteq \mathcal{L} \subseteq \mathcal{N}$ (by theorem (2.4)). Hence $\psi(NC\text{-}cl(NC\text{-}int(\mathcal{N}))) \subseteq \psi(\mathcal{L}) \subseteq \psi(\mathcal{N})$ but $NC\text{-}cl(\psi(NC\text{-}int(\mathcal{N}))) \subseteq \psi(NC\text{-}cl(NC\text{-}int(\mathcal{N})))$ (because the function ψ is NC-closed). Then $NC\text{-}cl(\psi(NC\text{-}int(\mathcal{N}))) \subseteq \psi(NC\text{-}cl(NC\text{-}int(\mathcal{N}))) \subseteq \psi(\mathcal{L}) \subseteq \psi(\mathcal{N})$. But $NC\text{-}cl(NC\text{-}int(\psi(\mathcal{N}))) \subseteq NC\text{-}cl(\psi(NC\text{-}int(\mathcal{N})))$ (because the function ψ is NC-continuous). Consequently, we get $NC\text{-}cl(NC\text{-}int(\psi(\mathcal{N}))) \subseteq \psi(\mathcal{L}) \subseteq \psi(\mathcal{N})$. However, $\psi(\mathcal{N})$ is a NC-CS in $\mathcal{J}$ (because the function ψ is NC-closed). Hence $\psi(\mathcal{L}) \in \text{NC}\alpha\text{C}(\mathcal{J})$ (it is clear from theorem (2.4)). Thus, the function ψ is NCα*-closed.

ii. Part (ii) is clear for proof. ■

Remark 3.11: It is understood that every function is defined as NCα*-closed, then it is NCα-closed as well as NCSα-closed. Nonetheless, the inverse is generally a fallacy, as the next example is displayed below.

Example 3.12: It is clear to note that the function ψ is a NCα-closed and NCSα-closed in Example (3.12). However, it is not NCα*-closed.

Remark 3.13: There is no relation between the ideas of NC-closed and NCSα*-closed functions, as the next two examples are displayed below.

Example 3.14: The function ψ in Example (3.5) is a NCSα*-closed. However, it is not NC-closed.

Example 3.15: Let $\mathcal{I} = \{p_1, p_2, p_3\}$. Then $\zeta = \{\phi_N, \langle\{p_1\}, \phi, \phi\rangle, \mathcal{I}_N\}$ is a NCTS. The collection of each NC-CSs of $\mathcal{I}$ is $\text{NC-C}(\mathcal{I}) = \{\mathcal{I}_N, \langle\{p_2, p_3\}, \phi, \phi\rangle, \phi_N\}$. The collection of each NCα-CSs (NCSα-CSs) of $\mathcal{I}$ is:

$$\text{NC}\alpha\text{C}(\mathcal{I}) = \text{NCS}\alpha\text{C}(\mathcal{I}) = \text{NC-C}(\mathcal{I}) \cup \{\langle\{p_3\}, \phi, \phi\rangle, \langle\{p_2\}, \phi, \phi\rangle\}.$$

Let $\mathcal{J} = \{q_1, q_2, q_3, q_4\}$. Then $\eta = \{\phi_N, \langle\{q_1\}, \phi, \phi\rangle, \langle\{q_2, q_3\}, \phi, \phi\rangle, \langle\{q_1, q_2, q_3\}, \phi, \phi\rangle, \mathcal{J}_N\}$ is a NCTS. The collection of each NC-CSs of $\mathcal{J}$ is:

$$\text{NC-C}(\mathcal{J}) = \{\mathcal{J}_N, \langle\{q_2, q_3, q_4\}, \phi, \phi\rangle, \langle\{q_1, q_4\}, \phi, \phi\rangle, \langle\{q_4\}, \phi, \phi\rangle, \phi_N\}.$$

The collection of each NCα-CSs of $\mathcal{J}$ is: $\text{NC}\alpha\text{C}(\mathcal{J}) = \text{NC-C}(\mathcal{J})$. The family of all NCSα-CSs of $\mathcal{J}$ is:

$$\text{NCS}\alpha\text{C}(\mathcal{J}) = \text{NC}\alpha\text{C}(\mathcal{J}) \cup \{\langle\{q_1\}, \phi, \phi\rangle\}.$$

Define a function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ by

$$\psi(\langle\{p_1\}, \phi, \phi\rangle) = \langle\{q_1\}, \phi, \phi\rangle, \psi(\langle\{p_2\}, \phi, \phi\rangle) = \langle\{q_2\}, \phi, \phi\rangle, \ \psi(\langle\{p_3\}, \phi, \phi\rangle) = \langle\{q_3\}, \phi, \phi\rangle.$$

Clearly, we can realize that the function ψ is a NC-closed; nonetheless, this function doesn't represent NCSα*-closed because of $\langle\{p_3\}, \phi, \phi\rangle \in \text{NCS}\alpha\text{C}(\mathcal{I})$ and $\psi(\langle\{p_3\}, \phi, \phi\rangle) = \langle\{q_3\}, \phi, \phi\rangle \notin \text{NCS}\alpha\text{C}(\mathcal{J})$.

Proposition 3.16: A function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is a NCSα*-closed iff this function $\psi: (\mathcal{I}, \text{NCS}\alpha\text{O}(\mathcal{I})) \longrightarrow (\mathcal{J}, \text{NCS}\alpha\text{O}(\mathcal{J}))$ is a NC-closed.

Proof: Understandable. ■

Remark 3.17: There is no relation between the ideas of NCα*-closed and NCSα*-closed functions, as the next two examples are displayed below.

Example 3.18: The function ψ in Example (3.9) is a NCSα*-closed. However, it is not NCα*-closed.

Example 3.19: Assume that the set $\mathcal{I} = \{p_1, p_2, p_3, p_4\}$. Then

$$\zeta = \{\phi_N, \langle\{p_1\}, \phi, \phi\rangle, \langle\{p_2, p_4\}, \phi, \phi\rangle, \langle\{p_1, p_2, p_4\}, \phi, \phi\rangle, \mathcal{I}_N\}$$

is a NCTS. The collection of each NC-CSs of $\mathcal{I}$ is:

$$\text{NC-C}(\mathcal{I}) = \{\mathcal{I}_N, \langle\{p_2, p_3, p_4\}, \phi, \phi\rangle, \langle\{p_1, p_3\}, \phi, \phi\rangle, \langle\{p_3\}, \phi, \phi\rangle, \phi_N\}.$$

Assume that the set $\mathcal{J} = \{q_1, q_2, q_3, q_4\}$. Then

$$\eta = \{\phi_N, \langle\{q_1\}, \phi, \phi\rangle, \langle\{q_2, q_4\}, \phi, \phi\rangle, \langle\{q_1, q_2, q_4\}, \phi, \phi\rangle, \mathcal{J}_N\}$$

is a NCTS. The collection of each NC-CSs of $\mathcal{J}$ is

$$\text{NC-C}(\mathcal{J}) = \{\mathcal{J}_N, \langle\{q_2, q_3, q_4\}, \phi, \phi\rangle, \langle\{q_1, q_3\}, \phi, \phi\rangle, \langle\{q_3\}, \phi, \phi\rangle, \phi_N\}.$$

Define a function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ by

$$\psi(\langle\{p_1\}, \phi, \phi\rangle) = \langle\{q_1\}, \phi, \phi\rangle,$$
$$\psi(\langle\{p_2\}, \phi, \phi\rangle) = \psi(\langle\{p_3\}, \phi, \phi\rangle) = \langle\{q_2\}, \phi, \phi\rangle,$$
$$\psi(\langle\{p_4\}, \phi, \phi\rangle) = \langle\{q_4\}, \phi, \phi\rangle.$$

Clearly, we can note that the function ψ is NCα*-closed. However, it is not NCSα*-closed.

Theorem 3.20: If a function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NCα*-closed and NC-continuous, then it is NCSα*-closed.

Proof: Assume that the function $\psi: \mathcal{I} \longrightarrow \mathcal{J}$ is NCα*-closed as well as NC-continuous. Moreover, suppose $\mathcal{L}$ is NCSα-CS in $\mathcal{I}$. Then for some sets, like NCα-CS $\mathcal{N}$ with this fact $NC\text{-}int(\mathcal{N}) \subseteq \mathcal{L} \subseteq \mathcal{N}$. Consequently, we have

$$NC\text{-}int(\psi(\mathcal{N})) \subseteq \psi(NC\text{-}int(\mathcal{N})) \subseteq \psi(\mathcal{L}) \subseteq \psi(\mathcal{N})$$

(this is because the function ψ is NC-continuous). However, the set $\psi(\mathcal{N}) \in \mathrm{NC\alpha C}(\mathcal{J})$ because the function ψ is NCα*-closed). Thus, the set $NC\text{-}int(\psi(\mathcal{N})) \subseteq \psi(\mathcal{L}) \subseteq \psi(\mathcal{N})$. Therefore, $\psi(\mathcal{L}) \in \mathrm{NCS\alpha C}(\mathcal{J})$. Thus, ψ is a NCSα*-closed function. ■

Theorem 3.21: The two functions $\psi_1: \mathcal{I} \longrightarrow \mathcal{J}$ and $\psi_2: \mathcal{J} \longrightarrow \mathcal{K}$ are satisfying the following

i. If a function ψ_1 is NC-closed and a function ψ_2 is NCα-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα-closed.
ii. If a function ψ_1 is NCα-closed and a function ψ_2 is NCα*-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα-closed.
iii. If functions ψ_1 and ψ_2 are NCα*-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα*-closed.
iv. If functions ψ_1 and ψ_2 are NCSα*-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCSα*-closed.
v. If functions ψ_1 and ψ_2 are NCα**-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα**-closed.
vi. If functions ψ_1 and ψ_2 are NCSα**-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCSα**-closed.
vii. If a function ψ_1 is NCα**-closed and a function ψ_2 is NCα*-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα*-closed.
viii. If a function ψ_1 is NCα-closed and a function ψ_2 is NCα**-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NC-closed.
ix. If a function ψ_1 is NCα**-closed and a function ψ_2 is NCα-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα*-closed.
x. If a function ψ_1 is NCα**-closed and a function ψ_2 is NC-closed, then a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is a NCα**-closed.

Proof:

i. Let $\mathcal{L}$ be a NC-CS in $\mathcal{I}$. Since ψ_1 is a NC-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since ψ_2 is a Nα-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCα-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is Nα-closed.
ii. Let $\mathcal{L}$ be a NC-CS in $\mathcal{I}$. Since ψ_1 is a NCα-closed function, $\psi_1(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Since ψ_2 is a NCα*-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCα-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα-closed.
iii. Let $\mathcal{L}$ be a NCα-CS in $\mathcal{I}$. Since ψ_1 is a NCα*-closed function, $\psi_1(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Since ψ_2 is a NCα*-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCα-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα*-closed.
iv. Let $\mathcal{L}$ be a NCSα-CS in $\mathcal{I}$. Since ψ_1 is a NCSα*-closed function, $\psi_1(\mathcal{L})$ is a NCSα-CS in $\mathcal{J}$. Since ψ_2 is a NCSα*-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCSα-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCSα*-closed.
v. Let $\mathcal{L}$ be a NCα-CS in $\mathcal{I}$. Since ψ_1 is a NCα**-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since any NC-CS is NCα-CS, $\psi_1(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Since ψ_2 is a NCα**-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NC-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα**-closed.

vi. Let $\mathcal{L}$ be a NCSα-CS in $\mathcal{I}$. Since ψ_1 is a NCSα**-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since any NC-CS is NCSα-CS, $\psi_1(\mathcal{L})$ is a NCSα-CS in $\mathcal{J}$. Since ψ_2 is a NCSα**-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NC-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCSα**-closed.

vii. Let $\mathcal{L}$ be a NCα-CS in $\mathcal{I}$. Since ψ_1 is a NCα**-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since any NC-CS is NCα-CS, $\psi_1(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Since ψ_2 is a NCα*-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCα-CS in $\mathcal{K}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα*-closed.

viii. Let $\mathcal{L}$ be a NC-CS in $\mathcal{I}$. Since ψ_1 is a NCα-closed function, $\psi_1(\mathcal{L})$ is a NCα-CS in $\mathcal{J}$. Since ψ_2 is a NCα**-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NC-CS in $\mathcal{J}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NC-closed.

ix. Let $\mathcal{L}$ be a NCα-CS in $\mathcal{J}$. Since ψ_1 is a NCα**-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since ψ_2 is a NCα-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NCα-CS in $\mathcal{J}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is a NCα*-closed.

x. Let $\mathcal{L}$ be a NCα-CS in $\mathcal{J}$. Since ψ_1 is a NCα**-closed function, $\psi_1(\mathcal{L})$ is a NC-CS in $\mathcal{J}$. Since ψ_2 is a NC-closed function, $\psi_2 \circ \psi_1(\mathcal{L}) = \psi_2(\psi_1(\mathcal{L}))$ is a NC-CS in $\mathcal{L}$. Thus, a function $\psi_2 \circ \psi_1: \mathcal{I} \longrightarrow \mathcal{K}$ is NCα**-closed. ■

Remark 3.22: The following diagram on the next page explains the relationship between weakly NC-closed functions.

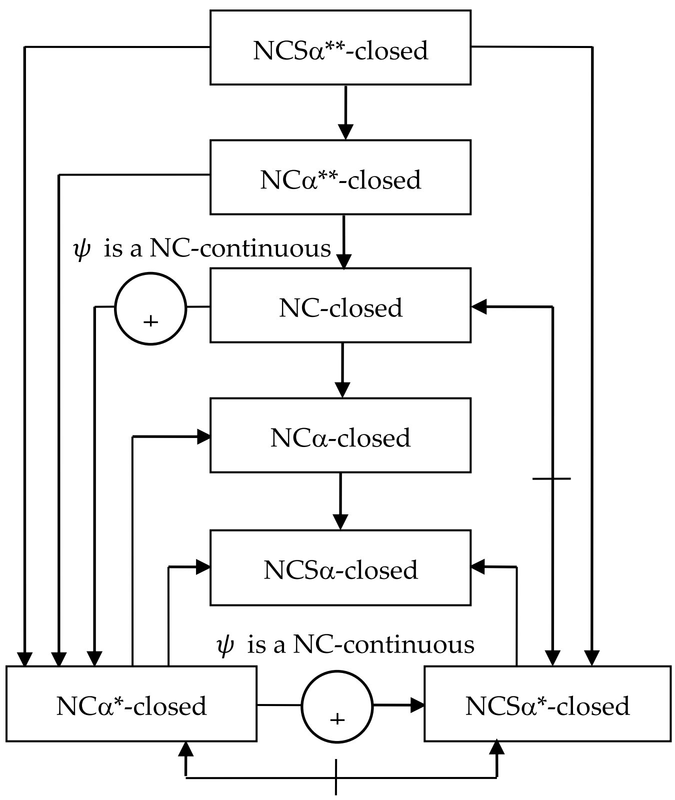

Fig. 3.1

4. Conclusion

We utilize the ideas of NCα-CSs and NCSα-CSs to identify some different kinds of weakly NC-closed functions, for instance; NCα*-closed, NCα**-closed, NCSα-closed, NCSα*-closed and NCSα**-closed functions. The most significant results are that the neutrosophic crisp semi-α**-closed maps are neutrosophic crisp α**-closed**,** neutrosophic crisp α*-closed and neutrosophic crisp semi-α*-closed. Moreover, neutrosophic crisp α**-closed map is neutrosophic crisp α*-closed and neutrosophic crisp closed. However, the neutrosophic crisp closed map is not neutrosophic crisp α*-closed and the latter is not neutrosophic crisp semi-α*-closed unless they are NC-continuous maps. Furthermore, neutrosophic crisp α*-closed and neutrosophic crisp closed maps are neutrosophic crisp α-closed and neutrosophic crisp semi-α-closed because crisp α-closed map is neutrosophic crisp semi-α-closed. Finally, neutrosophic crisp semi-α*-closed map is neutrosophic crisp semi-α-closed. As future works, the NCα-CSs and NCSα-CSs can be used to derive some neutrosophic crisp separation axioms, and we can generalize our results from the multivalued neutrosophic crisp closed.

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References

1. F. Smarandache, Neutrosophic Set, a Generalization of the Intuitionistic Fuzzy Sets, International Journal of Pure and Applied Mathematics, 24 (2005), 287-297.
2. F. Smarandache, Introduction to Neutrosophic Measure, Neutrosophic Integral and Neutrosophic Probability, Infinite Study, (2013).
3. L. A. Zadeh, Fuzzy Sets, Information and Control, 8 (1965), 338-353.
4. K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20 (1986), 87-96.
5. C.L.Chang, Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, 24 (1968), 182-190.
6. R. Lowen, Fuzzy Topological Spaces and Compactnees, Journal of Mathematical Analysis and Applications, 56 (1976), 621- 633.
7. A. A. Salama, F. Smarandache and V. Kroumov, Neutrosophic Crisp Sets and Neutrosophic Crisp Topological Spaces. Neutrosophic Sets and Systems, 2 (2014), 25-30.
8. M. Abdel-Basset, R. Mohamed, A. E. N. H. Zaied, & F. Smarandache, A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. Symmetry, 11 (2019), 903.
9. M. Abdel-Basset, N.A. Nabeeh, H.A. El-Ghareeb, & A. Aboelfetouh, Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems, (2019), 1-21.
10. M. Abdel-Baset, V. Chang, & A. Gamal, Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108 (2019), 210-220.
11. M. Abdel-Basset, M. Saleh, A. Gamal, & F. Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77 (2019), 438-452.
12. M. Abdel-Baset, V. Chang, A. Gamal, & F. Smarandache, An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. Computers in Industry, 106 (2019), 94-110.

13. M. Abdel-Basset, G. Manogaran, A. Gamal, & F. Smarandache, A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems, 43(2), (2019), 38.
14. R. K. Al-Hamido, Q. H. Imran, K. A. Alghurabi and T. Gharibah, On neutrosophic crisp semi-α-closed sets. Neutrosophic Sets and Systems, 21 (2018), 28-35.
15. V. Banupriya and S. Chandrasekar, Neutrosophic αgs Continuity and Neutrosophic αgs Irresolute Maps. Neutrosophic Sets and Systems, 28 (2019), 162-170.
16. R. Dhavaseelan, R. Narmada Devi, S. Jafari and Qays Hatem Imran, Neutrosophic α^{m}-continuity. Neutrosophic Sets and Systems, 27 (2019), 171-179.
17. C. Maheswari and S. Chandrasekar, Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity. Neutrosophic Sets and Systems, 29 (2019), 89-100.
18. Md. H. PAGE, Q. H. Imran, Neutrosophic Generalized Homeomorphism, Neutrosophic Sets and Systems, 35 (2020), 341–346.
19. Q. H. Imran, R. K. Al-Hamido, A. H. M. Al-Obaidi, On New Types of Weakly Neutrosophic Crisp Continuity, Neutrosophic Sets and Systems, 38 (2020), 179–187.
20. Q. H. Imran, R. Dhavaseelan, A. H. M. Al-Obaidi, M. H. Page, On neutrosophic generalized alpha generalized continuity, Neutrosophic Sets and Systems, 35 (2020), 511–521.
21. A. H. M. Al-Obaidi, Q. H. Imran, On New Types of Weakly Neutrosophic Crisp Open Mappings. Iraqi Journal of Science, 62 (2021), 2660-2666.
22. A. A. Salama, Basic structure of some classes of neutrosophic crisp nearly open sets & possible application to GIS topology. Neutrosophic Sets and Systems, 7 (2015), 18-22.
23. A. A. Salama, I. M. Hanafy, H. Elghawalby and M. S. Dabash, Some GIS Topological Concepts via Neutrosophic Crisp Set Theory. New Trends in Neutrosophic Theory and Applications, 2016.
24. W. Al-Omeri, Neutrosophic crisp sets via neutrosophic crisp topological spaces. Neutrosophic Sets and Systems, 13(2016), 96-104.

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