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Neutrosophic Crisp Points & Neutrosophic Crisp Ideals

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Abstract. The purpose of this paper is to define the so called "neutrosophic crisp points" and "neutrosophic crisp ideals", and obtain their fundamental properties. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Crisp Point, Neutrosophic Crisp Ideal.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts. The idea of "neutrosophic set" was first given by Smarandache [12, 13]. In 2012 neutrosophic operations have been investigated by Salama at el. [4 - 10]. The fuzzy set was introduced by Zadeh [13]. The intuitionstic fuzzy set was introduced by Atanassov [1, 2, 3]. Salama at el. [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [11]. Here we shall present the crisp version of these concepts.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [12, 13], and Salama at el. [4 -10].

3 Neutrosophic Crisp Points

One can easily define a natural type of neutrosophic crisp set in X, called "neutrosophic crisp point" in X, corresponding to an element $p \in X$:

3.1 Definition

Let X be a nonempty set and $p \in X$. Then the neutrosophic crisp point p_N defined by $p_N = \left\langle \{p\}, \phi, \{p\}^c \right\rangle$ is called a neutrosophic crisp point (NCP for short) in X, where NCP is a triple ({only one element in X}, the empty set,{the complement of the same element in X}).

Neutrosophic crisp points in X can sometimes be inconvenient when expressing a neutrosophic crisp set in X in terms of neutrosophic crisp points. This situation will occur if $A = \left\langle A_1, A_2, A_3 \right\rangle$, and $p \notin A_1$, where A_1, A_2, A_3 are three subsets such that $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, $A_{21} \cap A_3 = \phi$. Therefore we define the vanishing neutrosophic crisp points as follows:

3.2 Definition

Let X be a nonempty set, and $p \in X$ a fixed element in X. Then the neutrosophic crisp set $p_{N_N} = \left\langle \phi, \{p\}, \{p\}^c \right\rangle$ is called "vanishing neutrosophic crisp point" (VNCP for short) in X, where VNCP is a triple (the empty set,{only one element in X},{the complement of the same element in X}).

3.1 Example

Let $X = \{a, b, c, d\}$ and $p = b \in X$. Then $p_N = \left\langle \{b\}, \phi, \{a, c, d\} \right\rangle$ Now we shall present some types of inclusions of a neutrosophic crisp point to a neutrosophic crisp set:

3.3 Definition

Let $p_N = \left\langle \{p\}, \phi, \{p\}^c \right\rangle$ be a NCP in X and $A = \left\langle A_1, A_2, A_3 \right\rangle$ a neutrosophic crisp set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) Let p_{N_N} be a VNCP in X, and $A = \left\langle A_1, A_2, A_3 \right\rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_3$.

3.1 Proposition

Let $\{D_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p_N \in \bigcap_{j \in J} D_j$ iff $p_N \in D_j$ for each $j \in J$.

(a_2) $p_{N_N} \in \bigcap_{j \in J} D_j$ iff $p_{N_N} \in D_j$ for each $j \in J$.

(b_1) $p_N \in \bigcup_{j \in J} D_j$ iff $\exists j \in J$ such that $p_N \in D_j$.

(b_2) $p_{N_N} \in \underset{j\in J}{\cap} D_j$ iff $\exists j \in J$ such that $p_{N_N} \in D_j$.

Proof

Straightforward**.**

3.2 Proposition

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$.

b) $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof

Obvious.

3.4 Proposition

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then

$A = (\cup\{p_N : p_N \in A\}) \cup (\cup\{p_{NN} : p_{NN} \in A\})$.

Proof

It is sufficient to show the following equalities:
$A_1 = (\cup\{p\ \} : p_N \in A\}) \cup (\cup\{\phi : p_{NN} \in A\})$, $A_3 = \phi$
and $A_3 = (\cap\{\{p\ \}^c : p_N \in A\}) \cap (\cap\{\{p\}^c : p_{NN} \in A\})$, which are fairly obvious.

3.4 Definition

Let $f : X \to Y$ be a function.

(a) Let p_N be a nutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$, is defined by $f(p_N) = \langle \{q\}, \phi, \{q\}^c \rangle$, where $q = f(p\ )$.

(b) Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \langle \phi, \{q\}, \{q\}^c \rangle$, where $q = f(p\ )$.

It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$, where $q = f(p\ )$, and it is exactly the same meaning of the image of a NCP under the function f.

$f(p_{NN})$ is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p\ )$.

3.4 Proposition

Any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup\{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup\{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then $\underset{N}{A} = \langle x, A_1, \phi, A_1^c \rangle$ and $\underset{NN}{A} = \langle x, \phi, A_2, A_3 \rangle$.

3.5 Proposition

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

4 Neutrosophic Crisp Ideal Subsets

4.1 Definition

Let X be non-empty set, and L a non–empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

ii. $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic crisp ideal L is called a σ-neutrosophic crisp ideal if $\{M_j\}_{j\in N} \leq L$, implies $\underset{j\in J}{\cup} M_j \in L$ (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set X are $\{\phi_N\}$ and the NSs on X. Also, NCL_f, NCL_c are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NCS : B \subseteq A\}$ is an NCL on X. This is called the principal NCL of all NCSs, denoted by NCL$\langle A \rangle$.

4.1 Remark

i. If $X_N \notin L$, then L is called neutrosophic proper ideal.

ii. If $X_N \in L$, then L is called neutrosophic improper ideal.

iii. $\varphi_N \in L$.

4.1 Example

Let $X=\{a,b,c\}$, $A=\langle\{a\},\{a,b,c\},\{c\}\rangle$, $B=\langle\{a\},\{a\},\{c\}\rangle$, $C=\langle\{a\},\{b\},\{c\}\rangle$, $D=\langle\{a\},\{c\},\{c\}\rangle$, $E=\langle\{a\},\{a,b\},\{c\}\rangle$, $F=\langle\{a\},\{a,c\},\{c\}\rangle$, $G=\langle\{a\},\{b,c\},\{c\}\rangle$. Then the family $L=\{\phi_N, A, B, D, E, F, G\}$ of NCSs is an NCL on X.

4.2 Definition

Let L_1 and L_2 be two NCLs on X. Then L_2 is said to be finer than L_1, or L_1 is coarser than L_2, if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1, or L_1 is strictly coarser than L_2.

Two NCLs said to be comparable, if one is finer than the other. The set of all NCLs on X is ordered by the relation: L_1 is coarser than L_2; this relation is induced the inclusion in NCSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear. $L_j=\langle A_{j1}, A_{j2}, A_{j3}\rangle$.

4.1 Proposition

Let $\{L_j : j\in J\}$ be any non - empty family of neutrosophic crisp ideals on a set X. Then $\bigcap_{j\in J} L_j$ and $\bigcup_{j\in J} L_j$ are neutrosophic crisp ideals on X, where

$\bigcap_{j\in J} L_j=\left\langle \bigcap_{j\in J} A_{j1}, \bigcap_{j\in J} A_{j2}, \bigcup_{j\in J} A_{j3}\right\rangle$ or

$\bigcap_{j\in J} L_j=\left\langle \bigcap_{j\in J} A_{j1}, \bigcup_{j\in J} A_{j2}, \bigcup_{j\in J} A_{j3}\right\rangle$ and

$\bigcup_{j\in J} L_j=\left\langle \bigcup_{j\in J} A_{j1}, \bigcup_{j\in J} A_{j2}, \bigcap_{j\in J} A_{j3}\right\rangle$ or

$\bigcup_{j\in J} L_j=\left\langle \bigcup_{j\in J} A_{j_1}, \bigcap_{j\in J} A_{j_2}, \bigcap_{j\in J} A_{j_3}\right\rangle$.

In fact, L is the smallest upper bound of the sets of the L_j in the ordered set of all neutrosophic crisp ideals on X.

4,2 Remark

The neutrosophic crisp ideal defined by the single neutrosophic set ϕ_N is the smallest element of the ordered set of all neutrosophic crisp ideals on X.

4.2 Proposition

A neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$ in the neutrosophic crisp ideal L on X is a base of L iff every member of L is contained in A.

Proof

(Necessity) Suppose A is a base of L. Then clearly every member of L is contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic crisp subsets in X contained in A coincides with L by the Definition 4.3.

4.3 Proposition

A neutrosophic crisp ideal L_1, with base $A=\langle A_1, A_2, A_3\rangle$, is finer than a fuzzy ideal L_2 with base $B=\langle B_1, B_2, B_3\rangle$, iff every member of B is contained in A.

Proof

Immediate consequence of the definitions.

4.1 Corollary

Two neutrosophic crisp ideals bases A, B, on X, are equivalent iff every member of A is contained in B and vice versa.

4.1 Theorem

Let $\eta=\langle A_{j1}, A_{j2}, A_{j3}\rangle : j\in J$ be a non-empty collection of neutrosophic crisp subsets of X. Then there exists a neutrosophic crisp ideal $L(\eta)=\left\{A\in NCS : A\subseteq \bigcup_{j\in J} A_j\right\}$ on X for some finite collection $\{A_j : j=1,2,\ldots,n \subseteq \eta\}$.

Proof

It's clear.

4.3 Remark

The neutrosophic crisp ideal $L(\eta)$ defined above is said to be generated by η and η is called sub-base of $L(\eta)$.

4.2 Corollary

Let L_1 be a neutrosophic crisp ideal on X and A ∈ NCSs, then there is a neutrosophic crisp ideal L_2 which is finer than L_1 and such that A∈ L_2 iff $A \cup B \in L_2$ for each B∈ L_1.

Proof

It's clear.

4.2 Theorem

If $L = \{\phi_N, \langle A_1, A_2, A_3 \rangle\}$ is a neutrosophic crisp ideals on X, then:

i) $[\ ]L = \{\phi_N, \langle A_1, A_2, A_3{}^c \rangle\}$ is a neutrosophic crisp ideals on X.

ii) $\langle\ \rangle L = \{\phi_N, \langle A_3, A_2, A_1{}^c \rangle\}$ is a neutrosophic crisp ideals on X.

Proof

Obvious.

4.3 Theorem

Let $A = \langle A_1, A_2, A_3 \rangle \in L_1$, and $B = \langle B_1, B_2, B_3 \rangle \in L_2$, where L_1 and L_2 are neutrosophic crisp ideals on X, then A*B is a neutrosophic crisp set:

$A * B = \langle A_1 * B_1, A_2 * B_2, A_3 * B_3 \rangle$ where

$A_1 * B_1 = \cup \{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$,

$A_2 * B_2 = \cap \{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$ and

$A_3 * B_3 = \cap \{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$.

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