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Neutrosophic crisp Sets via Neutrosophic crisp Topological Spaces $NCTS$

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Abstract: In this paper, the structure of some classes of neutrosophic crisp nearly open sets are investigated via topology, and some applications are given. Finally, we generalize the crisp topological and neutrosophic crisp studies to the notion of neutrosophic crisp set.

Keywords: Set Theory, Topology, Neutrosophic crisp set theory, Neutrosophic crisp topology, Neutrosophic crisp α-open set, Neutrosophic crisp $semi$-open set, Neutrosophic crisp continuous function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their crisp and fuzzy counterparts, such as a neutrosophic set theory in [9, 11, 10]. It followed the introduction of the neutrosophic set concepts in [13, 12, 14, 15, 5, 7, 8, 16, 17] and the fundamental definitions of neutrosophic set operations. Smarandache [9, 11] and Salama et al. in [13, 18] provide a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics.
In this paper, we introduce the concept of neutrosophic crisp sets. We investigate the properties of continuous, open and closed maps in the neutrosophic crisp topological spaces, also give relations between neutrosophic crisp pre-continuous mapping and neutrosophic crisp $semi$-precontinuous mapping and some other continuous mapping, and show that the category of intuitionistic fuzzy topological spaces is a bireflective full subcategory of neutrosophic crisp topological spaces.

2 Terminology

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9, 11, 10], and Salama et al. [13, 12, 14, 15, 5, 7, 8, 16, 17, 6]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\rfloor^{-}0, 1^{+}\lfloor$ is a non-standard unit interval. Hanafy and Salama et al.[8, 16] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations.

Definition 1 *[20] Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form $A = \{A_1, A_2, A_3\}$, where A_1, A_2, and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, and $A_2 \cap A_1 = \phi$.*

Remark 2 *[20] Neutrosophic crisp set $A = \{A_1, A_2, A_3\}$ can be identified as an ordered triple $\{A_1, A_2, A_3\}$ where A_1, A_2, and A_3 are subsets on X, and one can define several relations and operations between $NCSs$.*

Types of $NCSs$ ϕ_N and X_N [20] in X as follows:
1- ϕ_N may be defined in many ways as a NCS, as follows

1. $\phi_N = \langle \phi, \phi, X \rangle$ or
2. $\phi_N = \langle \phi, X, X \rangle$ or
3. $\phi_N = \langle \phi, X, \phi \rangle$ or
4. $\phi_N = \langle \phi, \phi, \phi \rangle$

2- X_N may be defined in many ways as a NCS, as follows

1. $X_N = \langle X, \phi, \phi \rangle$ or
2. $X_N = \langle X, X, \phi \rangle$ or
3. $X_N = \langle X, X, X \rangle$ or

Definition 3 *[20] Let X is a non-empty set, and the $NCSs$ A and B in the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$. then we may consider two possible definition for subsets $A \subseteq B$, may defined in two ways:*

1. *$A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \subseteq B_2$, and $A_3 \supseteq B_3$ or*
2. *$A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \supseteq B_2$, and $A_3 \supseteq B_3$*

Definition 4 *[20] Let X is a non-empty set, and the $NCSs$ A and B in the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$. Then*

1. $A \cap B$ may defined in two way:

   i) $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

   ii) $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2. $A \cup B$ may defined in two way:

   i) $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

   ii) $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$

3. $[\,]A = \langle A_1, A_2, A_1^c \rangle$

4. $<> A = \langle A_3^c, A_2, A_3 \rangle$

Definition 5 *[20] A neutrosophic crisp topology (NCT) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms.*

1. $\phi_N, X_N \in \Gamma$.

2. $A_1 \cap A_2 \in \Gamma$, *for any* A_1 *and* $A_2 \in \Gamma$.

3. $\cup A_j \in \Gamma, \forall \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is said to be a neutrosophic crisp topological space $(NCTS)$ in X. The elements in Γ are said to be neutrosophic crisp open sets $(NCOSs)$ in Y. A neutrosophic crisp set F is closed $(NCCS)$ if and only if its complement F^c is an open neutrosophic crisp set.

Remark 6 *[20] Neutrosophic crisp topological spaces are very natural generalizations of topological spaces and intuitionistic topological spaces, and they allow more general functions to be members of topology:*

$$TS \Rightarrow ITS \Rightarrow NCTS$$

Definition 7 *[20] Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by*
$NCcl(A) = \cap\{K : is\, a\, NCCS\, in\, X\, and\, A \subseteq K\}$
$NCint(A) = \cup\{G : G\, is\, a\, NCOS\, in\, X\, and\, G \subseteq A\}$,
where NCS is a neutrosophic crisp closed set, and $NCOS$ is a neutrosophic crisp open set. Note that for any NCS in (X, Γ), we have
(1) $NCcl(A^c) = (NCcl(A))^c$, and
(2) $NCint(A^c) = (NCint(A))^c$
It can be also shown that $NCcl(A)$ is $NCCS$ (neutrosophic crisp closed set) and $NCint(A)$ is a $CNOS$ in X.

1. *A is in X if and only if $NCcl(A) \supseteq A$.*

2. *A is a $NCOS$ in X if and only if $NCint(A) = A$.*

Definition 8 *[20] Let (X, Γ) be a $NCTS$ and A, B be a NCS in X, then the following properties hold:*

1. $NCint(A) \subseteq A$,

2. $A \subseteq NCcl(A)$.

3. $A \subseteq B \Longrightarrow NCint(A) \subseteq NCint(B)$,

4. $A \subseteq B \Longrightarrow NCcl(A) \subseteq NCcl(B)$,

5. $NCint(A \cap B) = NCint(A) \cap NCint(B)$,

6. $NCint(A \cup B) = NCint(A) \cup NCint(B)$,

7. $NCint(X_N) = X_N$, $NCcl(\phi N) = \phi_N$

Definition 9 *[21] Let (X, Γ) be a $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X, then A is said to be*

1. *Neutrosophic crisp α-open set $(NC\alpha OS)$ iff $A \subseteq NCint(NCcl(NCint(A)))$,*

2. *Neutrosophic crisp $semi$-open set $(NCSOS)$ iff $A \subseteq NCcl(NCint(A))$.*

3. *Neutrosophic crisp pre-open set $(NCPOS)$ iff $A \subseteq NCint(NCcl(A))$.*

The class of all neutrosophic crisp α-open sets $NC\Gamma^{\alpha}$ which is finer than $NC\Gamma$, the class of all neutrosophic crisp $semi$-open sets $NC\Gamma^{s}$, and the class of all neutrosophic crisp pre-open sets $NC\Gamma^{p}$.

Definition 10 *[20] Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the α-neutrosophic crisp closure of A ($\alpha NCCl(A)$ for short) and α-neutrosophic crisp interior ($\alpha NCInt(A)$ for short) of A are defined by*

1. $\alpha NCcl(A) = \cap\{K : is\, an\, NC\alpha CS\, in\, X\, and\, A \subseteq K\}$,

2. $\alpha NCint(A) = \cup\{G : G\, is\, an\, NC\alpha OS\, in\, X\, and\, G \subseteq A\}$,

Proposition 11 *[20] Let (X, Γ) be $NCTS$ and A, B be two neutrosophic crisp sets in . Then the following properties hold:*

1. $NCint(A) \subseteq A$,

2. $A \subseteq NCcl(A)$,

3. $A \subseteq B \Rightarrow NCint(A) \subseteq NCint(B)$,

4. $A \subseteq B \Rightarrow NCcl(A) \subseteq NCcl(B)$,

5. $NCint(A \cap B) = NCint(A) \cap NCint(B)$,

6. $NCcl(A \cup B) = NCcl(A) \cup NCcl(B)$

7. $NCint(X_N) = X_N$,

8. $NCcl(\phi_N) = \phi_N$

Example 12 *Let $X = \{a, b, c, d\}, \phi_N, X_N$ be any types of theuniversal and empty subsets, and A, B two neutrosophic crisp subsets on X defined by $A = \langle \{a\}, \{b, d\}, \{c\} \rangle$, $B = \langle \{a\}, \{b\}, \{c, d\} \rangle$ then the family $\Gamma = \{\phi_N, X_N, A, B\}$ is a neutrosophic crisp topology on X.*

3 Neutrosophic Crisp Open Set

In this section, we will present an equivalent definition to Neutrosophic crisp α-open set and prove many special properties of it. Moreover, we will explain the relationship between different classes of neutrosophic crisp open sets by diagram.

Definition 13 *Let (X, Γ) be a $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X, then A is said to be*

1. *Neutrosophic crisp feebly-open $(NCFOS)$if there is a Neutrosophic crisp open set U such that $U \subseteq A \subseteq sNCcl(U)$, where $sNCcl(U)$ is denote neutrosophic closure with respect to $NC\Gamma^s$, is defined by the intersection of all Neutrosophic crisp semi closed sets containing A.*

2. *Neutrosophic crisp β-open set $(NC\beta OS)$ iff $A \subseteq NCcl(NCint(NCcl(A)))$,*

3. *Neutrosophic crisp $semipre$-open set $(NCSPOs)$ iff there exists a neutrosophic crisp preopen set U such that $U \subseteq A \subseteq NCcl(U)$.*

4. *Neutrosophic crisp regular-open set $(NCROS)$ iff $A = NCint(NCcl(A))$.*

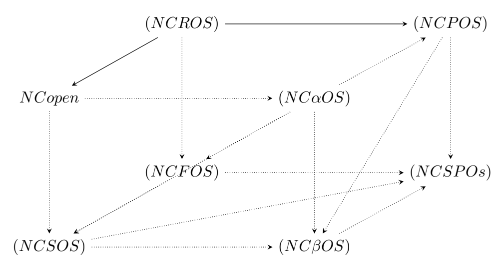
5. *Neutrosophic crisp semiα-open $(NCS\alpha OS)$ iff there exists a Neutrosophic crisp α-open set U such that $U \subseteq A \subseteq NCcl(U)$*

The class of all neutrosophic crisp feebly-open sets $NC\Gamma^{feebly}$, the calls all neutrosophic crisp β-open sets $NC\Gamma^{\beta}$, the class of all neutrosophic crisp semipre-open sets $NC\Gamma^{sp}$, the class of all neutrosophic crisp regular-open sets $NC\Gamma^{r}$, and the class of all neutrosophic crisp semiα-open sets $NC\Gamma^{s\alpha}$.

A neutrosophic crisp A is said to be a neutrosophic crisp semi-closed set, neutrosophic crisp α-closed set, neutrosophic crisp preclosed set, and neutrosophic crisp regular closed set, Neutrosophic crisp feebly-open, Neutrosophic crisp β-open set, Neutrosophic crisp $semipre$-open set, Neutrosophic crisp semiα-open respectively (($NCSCS$), ($NC\alpha CS$), ($NCPCS$),($NCRCS$),($NCFCS$),($NC\beta CS$),($NCSPCs$), ($NCS\alpha CS$), see the following table.

Table of Abbreviations	
Abbreviations	Neutrosophic crisp open sets
$NCFOS$	Neutrosophic crisp $feebly$-open
$NC\beta OS$	Neutrosophic crisp β-open
$NCSPOs$	Neutrosophic crisp $semipre$-open
$NCROS$	Neutrosophic crisp $regular$-open
$NCS\alpha OS$	Neutrosophic crisp semiα-open
$NC\alpha OS$	Neutrosophic crisp α-open set
$NCSOS$	Neutrosophic crisp $semi$-open
$NCPOS$	Neutrosophic crisp pre-open

Remark 14 *From above the following implication and none of these implications is reversible as shown by examples given below*



Example 15 *Let $X = \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subset, and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, then the family $\Gamma = \{\phi_N, X_N, A_1, A_2\}$ is a neutrosophic crisp topology on X. The NCS A_1 & and A_2 are neutrosophic crisp open ($NCOS$), then its neutrosophic crisp α-open sets i.e ($A \subseteq NCint(NCcl(NCint(A)))$) neutrosophic crisp pre-open sets i.e ($A \subseteq NCint(NCcl(A))$), neutrosophic crisp semi-open sets i.e ($A \subseteq NCcl(NCint(A))$). Also A_2 is neutrosophic crisp β-open sets, hence its Neutrosophic crisp $semipre$-open set.*
If $A_3 = \langle\{a\}, \{d\}, \{c\}\rangle$, then its clear A_3 neutrosophic crisp α-open set but not neutrosophic crisp open set.
If $A_4 = \langle\{a, b\}, \{c\}, \{d\}\rangle$, then A_4 neutrosophic crisp pre-open set but not neutrosophic crisp regular-open set, and we can see also that A_4 is neutrosophic crisp β-open but not neutrosophic crisp $semi$-open set.

Theorem 16 *An neutrosophic crisp A in a $NCTS$ (X, Γ) is a $NC\alpha OS$ if and only if it is both a $(NCSOS)$ and a $(NCPOS)$.*
Proof*. Necessity follows from the diagram given above. Suppose that A is both a $(NCSOS)$ and a $(NCPOS)$. Then*

$A \subseteq cl(int(A))$, and so

$$cl(A) \subseteq cl(cl(int(A))) = cl(int(A)).$$

It follows that $A \subseteq int(cl(A)) \subseteq int(cl(int(A)))$, so that A is a $(NC\alpha OS)$. We give condition(s) for a NCS to be a $(NC\alpha OS)$.

Theorem 17 *Let A be a NCS in a $NCTS$ (X, Γ). If B is a $(NCSOS)$ such that $B \subseteq A \subseteq int(cl(B))$, then A is a $(NC\alpha OS)$.*
Proof. *Since B is a $(NCSOS)$, we have $B \subseteq cl(int(B))$. Thus, $A \subset int(cl(B)) \subseteq int(cl(cl(int(B)))) \subseteq int(cl(int(B)))$,and so A is a $(NC\alpha OS)$.*

Lemma 18 *Any union of $(NC\alpha OS)$ (resp., $(NCPOS)$) is a $(NC\alpha OS)$ (resp., $(NCPOS)$).*
Proof. *The proof is straightforward.*

Definition 19 *Let $\langle a_{i1}, a_{i2}, a_{i3}\rangle \subseteq X$. A neutrosophic crisp point (NCP for short) $p(a_1, a_2, a_3)$ of X is a NCS of X defined by $a_{i1} \cap a_{i2} = \phi$, $a_{i1} \cap a_{i3} = \phi$, $a_{i2} \cap a_{i3} = \phi$. Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP of a $NCTS$ (X, Γ). An NCS A of X is said to be a neutrosophic crisp neighborhood (NCN) of $p(a_{i1}, a_{i2}, a_{i3})$ if there exists a $NCOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq A$.*

Theorem 20 *Let (X, Γ) be a $NCTS$. Then a neutrosophic crisp A of X is a neutrosophic crisp α-open (resp., neutrosophic crisp pre-open) if and only if for every $(NCP)\, P_{(a_{i1},a_{i2},a_{i3})} \in A$, there exists a $(NC\alpha OS)$ (resp., $(NCPOS)\, B_{p(\alpha,\beta)}$) such that $P_{(a_{i1},a_{i2},a_{i3})} \in B_{p(a_{i1},a_{i2},a_{i3})} \subseteq A$.*
Proof. *If A is a $(NC\alpha OS)$ (resp., $(NCPOS)$), then we may take $B_{p(\alpha,\beta} = A$ for every $P_{(a_{i1},a_{i2},a_{i3})} \in A$. Conversely assume that for every $(NCPOS)\, P_{(a_{i1},a_{i2},a_{i3})} \in A$, there exists a $(NC\alpha OS)$ (resp., $(NCPOS)$) $B_{p(a_{i1},a_{i2},a_{i3}}$ such that $P_{(a_{i1},a_{i2},a_{i3})} \in B_{p(a_{i1},a_{i2},a_{i3}} \subseteq A$. Then,*

$$A = \bigcup\{P_{(a_{i1},a_{i2},a_{i3})} | P_{(a_{i1},a_{i2},a_{i3})} \in A\} \subseteq \bigcup\{B_{p(a_{i1},a_{i2},a_{i3})} | P_{(a_{i1},a_{i2},a_{i3})} \in A\} \subseteq A$$

Theorem 21 *Let (X, Γ) be a $NCTS$,*

1. *If $V \in NCSOS(X)$ and $A \in NC\alpha OS(X)$, then $V \cap A \in NCSOS(X)$.*
2. *If $V \in NCPOS(X)$ and $A \in NC\alpha OS(X)$, then $V \cap A \in NCPOS(X)$.*

Proof. *(1) Let $V \in NCSOS(X)$ and $A \in NC\alpha OS(X)$. Then we obtain,*

$$\begin{aligned} V \cap A \subset & NCcl(NCint(V)) \cap NCnt(NCcl(NCint(A))) \\ & \subset NCcl[NCint(V) \cap NCint(NCcl(NCint(A)))] \\ & \subset NCcl[NCint(V) \cap NCcl(NCint(A))] \\ & \subset NCcl[NCcl[NCint(V) \cap NCint(A)]] \\ & \subset NCcl[NCint(V \cap A)]. \end{aligned}$$

This shows that $V \cap A \in NCSOS(X)$
(2) Let $V \in NCPOS(X)$ and $A \in NC\alpha OS(X)$. Then we obtain,

$$\begin{aligned} V \cap A \subset & NCint(NCcl(V)) \cap NCint(NCcl(NCint(A))) \\ & = NCint[NCint(V) \cap NCcl(NCint(A))] \\ & \subset NCint[NCcl(NCint(V) \cap NCint(A))] \\ & \subset NCint[NCcl[NCCL(V) \cap NCint(A)]] \\ & \subset NCint[NCcl[NCCL[V \cap NCint(A)]]] \\ & \subset NCint[NCcl[V \cap A]]. \end{aligned}$$

This shows that $V \cap A \in NCPOS(X)$.

Theorem 22 *Let A be a subset of a neutrosophic crisp topological space (X, Γ). Then the following properties hold:*

1. *A subset A of X is $NC\alpha OS$ if and only if it is $NCPOS$ and $NCSOS$,*
2. *If A is $NCSOS$, then A is $NC\beta OS$.*
3. *If A is $NCPOS$, then A is $NC\beta OS$.*

Proof. *(1) Necessity: This is obvious.*
Sufficiency: Let A be $NCSOS$ and $NCPOS$. Then we have

$$\begin{aligned} A & \subseteq NCint(NCcl(A)) \\ & \subseteq NCint(NCcl(NCcl(NCint(A)))) \\ & \subseteq NCint(NCcl(NCint(A))). \end{aligned}$$

This shows that A is $NC\alpha OS$.
(2) Since A is $NCSOS$, we have

$$\begin{aligned} A & \subseteq NCcl(NCint(A)) \\ & \subseteq NCcl(NCint(A)) \\ & \subseteq NCcl(NCint(c(A))) \end{aligned}$$

This shows that A is $NC\beta OS$.
(3) The proof is obvious.

Definition 23 *Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the $*$-neutrosophic crisp closure of A ($* - NCCl(A)$ for short) and $*$-neutrosophic crisp interior ($* - NCInt(A)$ for short) of A are defined by*

1. *$pNCcl(A) = \cap\{K : is\, a\, NCPCS\, in\, X\, and\, A \subseteq K\}$,*
2. *$pNCint(A) = \cup\{G : G\, is\, a\, NCPOS\, in\, X\, and\, G \subseteq A\}$,*
3. *$sNCcl(A) = \cap\{K : is\, a\, NCSCS\, in\, X\, and\, A \subseteq K\}$,*
4. *$sNCint(A) = \cup\{G : G\, is\, a\, NCSOS\, in\, X\, and\, G \subseteq A\}$,*
5. *$\beta NCcl(A) = \cap\{K : is\, a\, NC\beta CS\, in\, X\, and\, A \subseteq K\}$,*
6. *$\beta NCint(A) = \cup\{G : G\, is\, a\, NC\beta OS\, in\, X\, and\, G \subseteq A\}$,*

7. $rNCcl(A) = \cap\{K : is\, a\, NCRCS\, in\, X\, and\, A \subseteq K\}$,

8. $rNCint(A) = \cup\{G : G\, is\, a\, NCROS\, in\, X\, and\, G \subseteq A\}$,

Theorem 24 *For any neutrosophic crisp subset A of NCT S X. A is said to be neutrosophic crisp α-open set if and only if there exists a neutrosophic crisp open set G such that $G \subseteq A \subseteq NCint(NCcl(G))$.*
Proof. *Necessity : If A be a neutrosophic crisp α-open set $\Longrightarrow A \subseteq NCint(NCcl(A))$. Hence $G \subseteq A \subseteq NCint(NCcl(G))$, where $G = NCint(A)$*
Sufficiency : obvious.
This completes the proof of the theorem.

Theorem 25 *For any neutrosophic crisp subset of $NC\Gamma\ X$, the following properties are equivalent:*

1. $A \in NC\alpha OS(X)$.
2. *There exists a neutrosophic crisp open set say G such that $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$.*
3. $A \subseteq NCcl(NCint(NCcl(A)))$
4. $NCcl(A) = NCcl(NCint(NCcl(A)))$

Proof. $(1) \Rightarrow (2)$. *Let $A \in NC\alpha OS(X)$, there exists a neutrosophic crisp α-open set U in X such that $U \subseteq A \subseteq NCcl(U)$. Hence there exists G neutrosophic crisp open set such that $G \subseteq U \subseteq NCint(NCcl(G))$ (by Theorem 24). Therefore $NCcl(G) \subseteq NCcl(U) \subseteq NCcl(NCint(NCcl(G)))$. Then $G \subseteq U \subseteq A \subseteq NCcl(U) \subseteq NCcl(NCint(NCcl(G)))$. Therefore $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$ for some G neutrosophic crisp open sets.*

$(2) \Rightarrow (3)$. *Let there exists a neutrosophic crisp open set say G such that $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$. Hence $NCcl(G) \subseteq NCcl(NCint(A))$, then $NCint(NCcl(G)) \subseteq NCint(NCcl(NCint(A)))$. Therefore, $NCcl(NCint(NCcl(G))) \subseteq NCcl(NCint(NCcl(NCint(A))))$. Then (by hypothesis) $A \subseteq NCcl(NCint(NCcl(NCint(A))))$.*

$(3) \Rightarrow (4)$. *Obvious.*

$(4) \Rightarrow (1)$. *Let $NCcl(A) = NCcl(NCint(NCcl(NCint(A))))$. Then $A \subseteq NCcl(NCint(NCcl(NCint(A))))$. To prove $A \in NC\alpha OS(X)$. Since NCint $NCcl(NCint(A)) \subseteq NCint(NCcl(A))$, therefore $NCcl(NCint(NCcl(NCint(A)))) \subseteq NCcl(NCint(A)) \Rightarrow A \subseteq NCcl(NCint(A))$. let $U = NCint(A)$ Hence there exists a neutrosophic crisp open set U such that $U \subseteq A \subseteq NCcl(U)$. On othere hand, U is neutrosophic crisp α-open set. Hence $A \in NC\alpha OS(X)$.*

Proposition 26 *Let (X, Γ) be a $NCTS$, then arbitrary union of neutrosophic crisp α-open set is a neutrosophic crisp α-open set and arbitrary intersection neutrosophic crisp α-closed set is neutrosophic crisp α-closed set.*

Proof. *Let $A = \{A_i, A_i, A_i \mid i \in \Lambda\}$ be a collection of neutrosophic crisp α-open sets. Then, for each $i \in \Lambda$, $A_i \subseteq NCint(NCcl(NCint(A_i)))$. It follows that*

$$\begin{aligned}\bigcup A_i &\subseteq \bigcup NCint(NCcl(NCint(A_i)))\\ &\subseteq NCint(\bigcup NCcl(NCint(A_i)))\\ &= NCint(NCcl(\bigcup NCint(A_i)))\\ &\subseteq NCint(NCcl(NCint(\bigcup A_i)))\end{aligned}$$

Hence $\cup A_i$ is a neutrosophic crisp α-open set. The second part follows immediately from the first part by taking complements.

Having shown that arbitrary union of neutrosophic crisp α-open sets is a neutrosophic crisp α-open set, it is natural to consider whether or not the intersection of neutrosophic crisp α-open sets is a neutrosophic crisp α-open set, and the following example shows that the intersection of neutrosophic crisp α-open sets is not a neutrosophic crisp α-open set.

Example 27 *Let $X = \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subset, and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, then the family $\Gamma = \{\phi_N, X_N, A_1, A_2\}$ is a neutrosophic crisp topology on X. Let $A_3 = \langle\{b\}, \{c\}, \{d\}\rangle$ The NCS A_1 & and A_3 are neutrosophic crisp open ($NCOS$), then its sets neutrosophic crisp α-open sets i.e ($A \subseteq NCint(NCcl(NCint(A)))$). In fact, $A_1 \cap A_3$ is a NCS on X given by $A_1 \cap A_3 = \langle\phi, \phi, \{d, c\}\rangle$ or $A_2 \cap A_3 = \langle\phi, \{d, b, c\}, \{d, c\}\rangle$ and so $A_2 \cap A_3 \nsubseteq NCint(NCcl(NCint(A_2 \cap A_3)))$ and hence the intersection is not neutrosophic crisp α-open set.*

Proposition 28 *In a $NCTS$ (X, Γ), a NCS A is neutrosophic crisp α-closed if and only if $A = \alpha NCcI(A)$.*
Proof. *Assume that A is a neutrosophic crisp α-closed set. Obviously,*
$A \in \{B_i | B_i$ is a neutrosophic crisp α-closed set and $A \subseteq B_i\}$.
Also, $A \in \bigcap\{B_i | B_i$ is a neutrosophic crisp α-closed set and $A \subseteq B_i\} = NCcl(A)$.
Conversely, suppose that $A = \alpha NCcI(A)$, which shows that.
$A \in \{B_i | B_i$ is a neutrosophic crisp α-closed set and $A \subseteq B_i\}$
Hence A is a neutrosophic crisp α-closed set.

4 Neutrosophic Crisp Continuity

Definition 29 *Let (X, Γ_1) and (Y, Γ_2) be two $NCTS$ and let $f : X \to Y$ be a function then f is said to be*
(1) Continuous [20] iff the preimage of each NCS in Γ_2 is a NCS in Γ_1. i.e $f^{-1}(B)$ is neutrosophic crisp open set in X for each neutrosophic crisp open set B in Y where $B = \{B_1, B_2, B_3\}$, then the preimage of B under f, denoted by $f^{-1}(B)$, is neutrosophic crisp open in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), ^{-1}(B_2), f^{-1}(B_3)\rangle$.

(2) Open [20], iff the image of each NCS in Γ_1 is a NCS in Γ_2. i.e, if $A = \{A_1, A_2, A_3\}$ is a NCS in X, then the image of A under f denoted by $f(A)$ is NCS in Y defined by $f(A) = \{f(A_1), f(A_2), f(A_3)^c\}$.

Corollary 30 *[20] Let $A = \{A_i, i \in J\}$, be neutrosophic crisp sets in X, and $B = \{B_j, j \in K\}$ neutrosophic crisp sets in Y, and $: X \to Y$ be a function. Then*

1. $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, *and* $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
2. $f^{-1}(\bigcup B_i) = \bigcup f^{-1}(B_i)$, $f^{-1}(\bigcap B_i) = \bigcap f^{-1}(B_i)$,
3. $f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$,
4. $A \subseteq B \Rightarrow NCcl(A) \subseteq NCcl(B)$,
5. $A \subseteq f^{-1}(f(A))$, *and if f is surjective, then* $A = f^{-1}(f(A))$.

Definition 31 *Let $f : X \to Y$ be a function from a $NCTS$ (X, Γ_1) into a $NCTS$ (Y, Γ_2) is said to be*

1. *neutrosophic crisp α-continuous if $f^{-1}(B)$ is α-neutrosophic crisp open set in X for each neutrosophic crisp open set B in Y.*
2. *neutrosophic crisp pre-continuous if $f^{-1}(B)$ is neutrosophic crisp pre-open set in X for each neutrosophic crisp open set B in Y.*
3. *neutrosophic crisp $semi$-continuous if $f^{-1}(B)$ is neutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*
4. *neutrosophic crisp $semipre$-continuous if $f^{-1}(B)$ is neutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*
5. *neutrosophic crisp β-continuous if $f^{-1}(B)$ isneutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*

Theorem 32 *For a mapping f from a $NCTS$ (X, Γ_1) to a $NCTS$ (X, Γ_2), the following are equivalent.*

1. *f is neutrosophic crisp pre-continuouss.*
2. *$f^{-1}(B)$ is a $NCPCS$ in X for every $NCCS$ B in Y.*
3. *$NCcl(NCint(f^{-1}(A))) \subseteq f^{-1}(NCcl(A))$ for every NCS A in Y.*

Proof. $(1) \Rightarrow (1)$. *The proof is straightforward.*
$(2) \Rightarrow (3)$. *Let A be a NCS in Y. Then $cl(A)$ is neutrosophic crisp closed. It follows from 2 that $f^{-1}(NCcl(A))$ is a $NCPCS$ in X so that*

$$NCcl(NCint(f^{-1}(A))) \subseteq NCcl(NCint(f^{-1}(NCcl(A)))) \subseteq f^{-1}(NCcl(A))$$

$(3) \Rightarrow (1)$. *Let A be a $NCOS$ in Y. Then A is a $NCCS$ in Y, and so*

$$NCcl(NCint(f^{-1}(\overline{A}))) \subseteq f^{-1}(NCcl(\overline{A})) = f^{-1}(\overline{A}).$$

This implies

$$\begin{aligned}\overline{NCint(NCcl(f^{-1}(A)))} &= NCcl(\overline{NCcl(f^{-1}(A))})\\ &= NCcl(NCint(\overline{f^{-1}(A)}))\\ &= NCcl(NCint(f^{-1}(\overline{A})))\\ &\subseteq f^{-1}(\overline{A}) = f^{-1}(\overline{A}) = \overline{f^{-1}(A)},\end{aligned}$$

and thus $f^{-1}(A) \subseteq NCint(NCcl(f^{-1}(A)))$. Hence $f^{-1}(A)$ is a $NCPOS$ in X, and f is neutrosophic crisp pre-continuous.

Theorem 33 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp pre-continuous.*
2. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
3. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq A$.*

Proof. $(1) \Rightarrow (2)$. *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. Then there exists a $NCOS$ B in Y such that $f(p(a_{i1}, a_{i2}, a_{i3})) \in B \subseteq A$. Since f is neutrosophic crisp pre-continuous,*
we know that $f^{-1}(B)$ is a $NCPOS$ in X and

$$p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(f(p(a_{i1}, a_{i2}, a_{i3}))) \subseteq f^{-1}(B) \subseteq f^{-1}(A).$$

Thus (2) is valid.
$(2) \Rightarrow (3)$. *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. The condition (2) implies that there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$ so that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (3) is true.*

$(3) \Rightarrow (1)$ *Let B be a $NCOS$ in Y and let $p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(B)$. Then $f(p(a_{i1}, a_{i2}, a_{i3})) \in B$, and so B is a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$ since B is a $NCOS$. It follows from (3) that there exists a $NCPOS$ A in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in A$ and $f(A) \subseteq B$ so that.*

$$p(a_{i1}, a_{i2}, a_{i3}) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B).$$

Applying Theorem 20 induces that $f^{-1}(B)$ is a $NCPOS$ in X. Therefore, f is neutrosophic crisp pre-continuous.

Theorem 34 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2) that satisfies*

$$NCcl(NCint(f^{-1}(NCcl(B)))) \subseteq f^{-1}(NCcl(B))$$

for every NCS B in Y. Then f is neutrosophic crisp α-continuous.

Proof. *Let B be a $NCOS$ in Y. Then $\overline{B}$ is a $NCCS$ in Y, which implies from hypothesis that*

$$NCcl(NCint(f^{-1}(NCcl(\overline{B})))) \subseteq f^{-1}(NCcl(\overline{B})) = f^{-1}(\overline{B}).$$

Its follows

$$\begin{aligned} &\overline{NCint(NCcl(NCint(f^{-1}(B))))} \\ &= NCcl(\overline{NCcl(NCint(f^{-1}(B)))}) \\ &= NCcl(NCint(\overline{NCint(f^{-1}(B))})) \\ &= NCcl(NCint(NCcl(\overline{f^{-1}(B)}))) \\ &= NCcl(NCint(NCcl(f^{-1}\overline{(B)}))) \\ &\subseteq f^{-1}\overline{(B)} \\ &= \overline{f^{-1}(B)} \end{aligned}$$

so that $f^{-1}(B) \subseteq NCint(NCcl(NCint(f^{-1}(B))))$. This shows that $f^{-1}(B)$ is a $NC\alpha OS$ in X. Hence, f is neutrosophic crisp α-continuous.

Theorem 35 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp α-continuous.*
2. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
3. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq A$.*

Proof. (1) ⇒ (2). *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. Then there exists a $NCOS$ B in Y such that $f(p(a_{i1}, a_{i2}, a_{i3})) \in C \subseteq A$. Since f is neutrosophic crisp α-continuous,*
we know that $f^{-1}(B)$ is a $NC\alpha OS$ in X and

$$p(a_{i1}, a_{i2}, a_{i3}) \in {}^{-1}(f(p(a_{i1}, a_{i2}, a_{i3}))) \subseteq f^{-1}(C) = B \subseteq f^{-1}(A).$$

Thus (2) is valid.
(2) ⇒ (3). *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. The condition (2) implies that there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$, by (2). Thus, we have $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (3) is true.*

(3) ⇒ (1) *Let B be a $NCOS$ in Y and let $p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(B)$. Then $f(p(a_{i1}, a_{i2}, a_{i3})) \inf(f^{-1}(B)) \subseteq B$ and so B is a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$ since B is a $NCOS$. It follows from (3) that there exists a $NC\alpha OS$ A in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in A$ and $f(A) \subseteq B$ so that.*

$$p(a_{i1}, a_{i2}, a_{i3}) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B).$$

Using Theorem 20 induces that $f^{-1}(B)$ is a $NC\alpha OS$ in X. and hence f is neutrosophic crisp α-continuous.

Combining Theorems 35, 34, we have the following characterization of a neutrosophic crisp α-continuous mapping.

Theorem 36 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp α-continuous.*
2. *If C is a $NCCS$ in Y, then $f^{-1}(C)$ is a $NC\alpha CS$ in X.*
3. *$NCcl(NCint(f^{-1}(NCcl(B)))) \subseteq f^{-1}(NCcl(B))$ for every NCS B in Y.*
4. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
5. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq A$.*

Some aspects of neutrosophic crisp continuity, neutrosophic crisp α-continuity, neutrosophic crisp pre-continuity, neutrosophic crisp $semi$-continuity, and neutrosophic crisp β-continuity are studied in this paper and as well as in several papers, see [20]. The relation among these types of neutrosophic crisp continuity is given as follows, where NC means neutrosophic crisp.

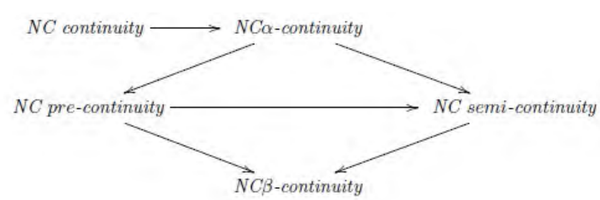


Figure 1: Diagram 2

Remark 37 *The reverse implications are not true in the above diagram in general.*

Example 38 *Let (X, Γ_0) and (Y, Ψ_0) be two $NCTS$. If $: X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of $f(A) = \{f(A_1), f(A_2), f(A_3)^c\}$. Here we* *consider the $NCTS$ on X and Y, respectively, as follows:* $\Gamma_1 = \{\langle G, \phi, G^c : G \in \Gamma_0\rangle\}$ *and* $\Gamma_2 = \{\langle H, \phi, H^c : H \in \Psi_0\rangle\}$*, in this case we have* $\langle H, \phi, H^c\rangle \in \Gamma_2$*,* $H \in \Psi_0$*,* $f^{-1}\langle H, \phi, H^c\rangle = \langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c)\rangle = \langle f^{-1}(H), f(\phi), (f(H))^c\rangle \in \Gamma_1$.

Example 39 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2), and let $X \doteq Y \doteq \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subset, and $A_1 \doteq \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 \doteq \langle\{a\}, \{b, d\}, \{c\}\rangle$, then the family $\Gamma_1 \doteq \Gamma_2 \doteq \{\phi_N, X_N, A_1, A_2\}$ is a neutrosophic crisp topology on X and Y. Then f is neutrosophic crisp continuous function, since $f^{-1}(A_1) \doteq A_1$ & and $f^{-1}(A_2) \doteq A_2$ are neutrosophic crisp open in X ($NCOS$), and hence its neutrosophic crisp α-continuous, since $f^{-1}(A_1), f^{-1}(A_2)$ is α-neutrosophic crisp open set in X.*

Example 40 *Let $X = \{a, b, c, d\}$, $Y = \{u, v, w\}$ and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$, $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, $A_3 = \langle\{u\}, \{v\}, \{w\}\rangle$. Then $\Gamma_1 = \{\phi_N, X_N, A_1, A_2\}$, $\Gamma_2 = \{\phi_N, X_N, A_3\}$ are neutrosophic crisp topology on X and Y respectively. Defined a mapping $f : (X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$ by $f(\{a\}) = \{u\}$, $f(\{d\}) = \{v\}$ and $f(\{c\}) = \{w\}$. Then f is neutrosophic crisp α-continuous function but not neutrosophic crisp continuous function.*

Example 41 *Let $X = \{a, b, c, d\}$, $Y = \{u, v, w\}$ and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$, $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, $A_3 = \langle\{u\}, \{v\}, \{w\}\rangle$. Then $\Gamma_1 = \{\phi_N, X_N, A_1, A_2\}$, $\Gamma_2 = \{\phi_N, X_N, A_3\}$ are neutrosophic crisp topology on X and Y respectively. Defined a mapping $f : (X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$ by $f(\{a\}) = f(\{b\}) = \{u\}$, $f(\{c\}) = \{v\}$ and $f(\{d\}) = \{w\}$. Then f is neutrosophic crisp pre-continuous function but not neutrosophic crisp regular-continuous function. Also f is neutrosophic crisp β-continuous function but not neutrosophic crisp $semi$-continuous function, since $f^{-1}(A_3) = \langle\{a, b\}, \{c\}, \{d\}\rangle$ is neutrosophic crisp pre-open set but not neutrosophic crisp regular-open set, also A_4 is neutrosophic crisp β-open but not neutrosophic crisp $semi$-open set.*

Theorem 42 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2). If f is both neutrosophic crisp pre-continuous and neutrosophic crisp $semi$-continuous, then it is neutrosophic crisp α-continuous.*

Proof. *Let B be a $NCOS$ in Y. Since f is both neutrosophic crisp pre-continuous and neutrosophic crisp $semi$-continuous, $f^{-1}(B)$ is both a $NCPOS$ and a $NCSOS$ in X. It follows from Theorem 17 that $f^{-1}(B)$ is a $NC\alpha OS$ in X so that f is ineutrosophic crisp α-continuous.*

5 Conclusions and Discussions

In this paper, we have introduced neutrosophic crisp β-open, Neutrosophic crisp semipre-open, Neutrosophic crisp regular-open, Neutrosophic crisp semiα-open sets and studied some of their basic properties. Also we study the relationship between the newly introduced sets and some of the Neutrosophic crisp open sets that already existed. In this paper, we also introduced Neutrosophic crisp closed sets and studied some of their basic properties. Finally, we introduced the definition of neutrosophic crisp continuous function, and studied some of its basic properties.

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