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A Study of Neutrosophic Cubic Hesitant Fuzzy Hybrid Geometric Aggregation Operators and its Application to Multi Expert Decision Making System

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Abstract: Hesitancy is an imperative part of belief system. In order to counter the hesitancy in neutrosophic cubic set (NCS), the notion of neutrosophic cubic hesitant fuzzy set (NCHFS) is presented. NCHFS couple NCS with hesitant fuzzy set (HFS). Operational laws in NCHFS are developed with examples. To meet the challenges of decision making problems, neutrosophic cubic hesitant fuzzy geometric (NCHFG) aggregation operators, neutrosophic cubic hesitant fuzzy Einstein geometric (NCHFEG) aggregation operators, neutrosophic cubic hesitant fuzzy hybrid geometric (NCHFHEG) aggregation operators are developed in the current study. At the end a multi expert decision making (MEDM) process is proposed and furnished upon numerical data of a company as applications.

Keywords: Neutrosophic Cubic Fuzzy Hesitant Set (NCHFS), Neutrosophic Cubic Hesitant Fuzzy Weighted geometric (NCHFWG) operator, Neutrosophic Cubic Hesitant Fuzzy Einstein Geometric (NCHFEG) operator, Neutrosophic Cubic Hesitant Fuzzy Einstein Hybrid Geometric (NCHFEHG) Operator. multi expert decision making (MEDM)

1. Introduction

We are in different mental states of acceptance, hesitancy and refusal while taking decisions in life. Many methods in MADM ignore the uncertainty and hence yields the results which are unreliable. The role of expert in decision making (DM) is vital. The participation of more than one expert in a DM process reduce the uncertainty. Zadeh proposed the notion of fuzzy set (FS) [1] as a function from a given set of objects to [0,1] called membership. Later Zadeh extended the idea to interval valued fuzzy set (IVFS) [2]. An IVFS a function from a given set of objects to the subintervals of [0,1]. The FS theory has many applications in artificial intelligence, robotics, computer networks, engineering and DM [3,4]. Different researchers [5-8] established similarity measures and other important concepts and successfully apply their models to medical diagnosis and selection criteria. R.A. Krohling and V.C. Campanharo, M. Xia and Z. Xu, M.K. Mehlawat and P.A. Guptal established different useful techniques to sort out MADM problems [9-11]. K. Atanassov introduced non-membership degree and presented the idea of intuitionistic fuzzy set (IFS) [12] which consist of both membership and non-membership degree within [0,1]. An extension of IFS was proposed and named as interval value intuitionistic fuzzy set (IVIFS) [13]. IVIFS contains membership and non-membership in the form of subintervals of [0,1]. This characteristic of intuitionistic fuzzy set made it more applicable then previous versions and attracted researchers [14-16] to apply it to the fields of science, engineering and daily life problems. Jun et al., combined IVFS and FS and proposed cubic set (CS). The CS is the generalization of IFS and IVIFS. CS become vital tool to deal the vague data. Several researchers explored algebraic aspects and apparently define ideal theory in CS [17-20]. F. Smarandache initiated the concept of indeterminacy and describe the notion of neutrosophic set (NS) [21]. NS consist of three components truth, indeterminacy and falsehood and all are independent. This characteristic of neutrosophic set enabled researchers to work with inconsistent and vague data more effictively. Wang *et al* proposed single valued neutrosophic set (SNVS) [22] by restricting components of NS to [0,1]. The NS was further extended to interval neutrosophic set (INS) [23]. After the appearance of NS, researchers put their contributions in theoretical as well as technological developments of the set [24-27]. Several researchers use neutrosophic and interval valued neutrosophic environments to construct MADM methods [28-32]. Zhan *et al.*,define aggregation operators and furnished some applications in MADM [33]. Torra define hesitant fuzzy set (HFS) [34] in contrast of FS. HFS on X is a function that maps every object of X into a subset of [0, 1]. Jun *et al.*, presented the notion of NCS [35] which consist of both INS and NS. These characteristics of NCS make it a powerful tool to deal the vague and inconsistent data more efficiently. Soon after its exploration it attracted the researcher to work in many fields like medicine, algebra, engineering and DM. Later the idea of cubic hesitant fuzzy set was introduced by Tahir *et al.*,[36]. Ye [37] establish similarity measure in neutrosophic hesitant fuzzy sets (NHFS) and established MADM method using these measures. Liu et. Al [38] proposed hybrid geometric aggregation operators in interval neutrosophic hesitant fuzzy sets (INHFS)and discuss its applications in MADM. Zhu et al. [39] proposed the method of β-normalization to enlarge a HFE, which is a useful technique in case of different cardinalities.

The remaining of the paper is formulated as follows. In section 2, we reviewed some basic definitions used later on. Section 3 deals with NCHFS, algebraic and Einstein operational laws in NCHFS. In section 4 we introduced aggregation operators in NCHFS. Section 5 concern with establishing a MEDM method based on NCHFG operators and use this method in MEDM problem.

2. Preliminaries

Definition 2.1: [1] A fuzzy set (FS)on a nonempty set W is a mapping $\Gamma: W \to [0,1]$.

Definition 2.2: [12] The cubic set (CS) on a nonempty set Z is defined by $\mu = \langle x; I(x), \delta(x)/x \in X\rangle$, where $I(x)$ is an IVFS on Z and $\delta(x)$ is an FS on Z.

Definition 2.3: [22] A neutrosophic set associated with a crisp set S, is a set of the form $\mu = \langle e; \xi_T(e), \xi_I(e), \xi_F(e)/e \in S\rangle$ where $\xi_T, \xi_I, \xi_F: S \to [0,1]$ respectively called a truth membership function, a non-membership function and a false membership function.

Definition 2.4: [34] A hesitant fuzzy set on a crisp set W is a mapping which assigns a set of values in [0,1], to each element of W.

Definition 2.5: [35] A neutrosophic cubic set in a nonempty set E is defined as a pair (B,μ) where $B = \langle x; B_T(e), B_I(e), B_F(e)/e \in E\rangle$ is an INS and $\mu = \langle x; \mu_T(e), \mu_I(e), \mu_F(e)/e \in X\rangle$ is a NS.

Definition 2.6: [39] A neutrosophic hesitant fuzzy set a nonempty set E is described as $\mu = \langle x; \mu_T(e), \mu_I(e), \mu_F(e)/e \in E\rangle$where $\mu_T(e), \mu_I(e), \mu_F(e)$ are three HFSs such that $\mu_T(e) + \mu_I(e) + \mu_F(e) \leq 3$.

Definition2.7: [40] The object $\zeta = \langle x; \xi_T(x), \xi_I(x), \xi_F(x)/x \in X\rangle$, s called an INHFS on X, where$\xi_T(x), \xi_I(x), \xi_F(x)$ are IHFSs.

Zhu et al. proposed the following β-normalization method to enlarge a hesitant fuzzy element, which is a useful technique in case of different cardinalities.

Definition 2.8:[41] Let m^+ and m^- be the maximum and minimum elements of an hesitant fuzzy set H and $\zeta(0 \le \zeta \le 1)$ an optimized parameter. We call $m = \zeta m^+ + (1-\zeta)m^-$ an added element.

3. NCHFS and operational Laws in NCHFS

Definition3.1: Let X be a nonempty set. A neutrosophic cubic hesitant fuzzy set in X is a pair $\alpha = \langle A, \lambda\rangle$ where $A = \langle x; A_T(x), A_I(x), A_F(x)/x \in X\rangle$ is an interval-valued neutrosophic hesitant set in X and $\lambda = \langle x; \lambda_T(x), \lambda_I(x), \lambda_F(x)/x \in X\rangle$ is a neutrosophic hesitant set in X. Furthermore $A_T = \{[A^L_{j_T}, A^U_{j_T}]; j = 1, \ldots, l\}, A_I = \{[A^L_{j_I}, A^U_{j_I}]; j = 1, \ldots, m\}, A_F = \{[A^L_{j_F}, A^U_{j_F}]; j = 1, \ldots, n\}$ are some interval values in [0,1] and $\lambda_T = \{\lambda_{j_T}; j = 1, \ldots, r\}, \lambda_I = \{\lambda_{j_I}; j = 1, \ldots, s\}, \lambda_F = \{\lambda_{j_F}; j = 1, \ldots, t\}$ are some values in [0,1].

Example 3.2: Let$X = \{u, v, w\}$ The pair $\alpha = \langle A, \lambda\rangle$ with

$$A_T(u) = \{[0.1, 0.5], [0.2, 0.7]\}, \lambda_T(u) = \{0.3, 0.5, 0.7\}, A_I(u) = \{[0.2, 0.4], [0.3, 0.6]\}, \lambda_T(u) = \{0.1, 0.4, 0.7\}, A_F(u) = \{[0.1, 0.4], [0, 0.3], [0.6, 0.8]\}, \lambda_F(u) = \{0.4, 0.6\}$$

$$A_T(v) = \{[0.1, 0.5], [0.2, 0.7]\}, \lambda_T(v) = \{0.3, 0.5\}, A_I(v) = \{[0.2, 0.3], [0.1, 0.6]\}, \lambda_I(v) = \{0.7, 0.8\}, A_F(v) = \{[0.1, 0.4], [0, 0.3]\}, \lambda_F(v) = \{0.4, 0.6\}$$

$$A_T(w) = \{[0.1, 0.5], [0.2, 0.7]\}, \lambda_T(w) = \{0.3, 0.5\}, A_I(w) = \{[0.2, 0.3], [0.1, 0.6]\}, \lambda_I(w) = \{0.7, 0.8\}, A_F(z) = \{[0.1, 0.4], [0, 0.3]\}, \lambda_F(w) = \{0.4, 0.6\}$$

is a NCHFS.

Definition 3.3: The sum of two NCHFSs $\alpha = \langle A, \lambda\rangle, \beta = \langle B, \mu\rangle$ is defined as

$$\alpha \oplus \beta = \left\langle \begin{array}{c} x, \left\{\left[A^L_{j_T} + B^L_{j_T} - A^L_{j_T} B^L_{j_T}, A^U_{j_T} + B^U_{j_T} - A^U_{j_T} B^U_{j_T}\right]\right\}, \left\{\left[A^L_{j_I} + B^L_{j_I} - A^L_{j_I} B^L_{j_I}, A^U_{j_I} + B^U_{j_I} - A^U_{j_I} B^U_{j_I}\right]\right\}, \left\{[A^L_{j_F} B^L_{j_F}, A^U_{j_F} B^U_{j_F}]\right\}, \\ \left\{\lambda_{j_T} + \mu_{j_T} - \lambda_{j_T}\mu_{j_T}\right\}, \left\{\lambda_{j_I} + \mu_{j_I} - \lambda_{j_I}\mu_{j_I}\right\}, \left\{\lambda_{j_F}\right\} \end{array} \right\rangle,$$

Moreover the β-normalization is used in case of different cardinalities.

Example3.4: If

$$\alpha = \langle \{[0.1, 0.5], [0.2, 0.7]\}, \{[0.2, 0.3], [0.1, 0.6]\}, \{[0.1, 0.4], [0.0, 0.3]\}, \{0.1, 0.2\}, \{0.3, 0.5, 0.7\}, \{0.4, 0.8\} \rangle,$$

and $\beta = \langle \{[0.4, 0.5], [0.3, 0.4]\}, \{[0.1, 0.3], [0.2, 0.5]\}, \{[0.1, 0.4], [0.7, 0.8]\}, \{0.3, 0.4, 0.5\}, \{0.7, 0.8\}, \{0.4, 0.6\} \rangle$,

then using above definition and β-normalization with parameter $\xi = 0.5$ we have

$$\alpha \oplus \beta = \langle \{[0.46, 0.75], [0.44, 0.82]\}, \{[0.28, 0.51], [0.28, 0.8]\}, \{[0.01, 0.16], [0, 0.24]\}, \{0.03, 0.06, 0.1\}, \{0.21, 0.375, 0.56\}, \{0.64, 0.92\} \rangle.$$

Definition 3.5: The Product of two NCHFSs $\alpha = \langle A, \lambda\rangle, \beta = \langle B, \mu\rangle$ is defined by

$$\alpha \otimes \beta = \left\langle \begin{array}{c} x, \left\{[A^L_{j_T} B^L_{j_T}, A^U_{j_T} B^U_{j_T}]\right\}, \left\{[A^L_{j_I} B^L_{j_I}, A^U_{j_I} B^U_{j_I}]\right\}, \left\{[A^L_{j_F} + B^L_{j_F} - A^L_{j_F} B^L_{j_F}, A^U_{j_F} + B^U_{j_F} - A^U_{j_F} B^U_{j_F}]\right\}, \\ \left\{\lambda_{j_T}\mu_{j_T}\right\}, \left\{\lambda_{j_I}\mu_{j_I}\right\}, \left\{\lambda_{j_F} + \mu_{j_F} - \lambda_{j_F}\mu_{j_F}\right\} \end{array} \right\rangle.$$

Moreover the β-normalization is used in case of different cardinalities.

Example3.6: If

$\alpha=\left\langle\{[0.1,0.5],[0.2,0.7]\},\{[0.2,0.3],[0.1,0.6]\},\{[0.1,0.4],[0.0,0.3]\},\{0.1,0.2\},\{0.3,0.5,0.7\},\{0.4,0.8\}\right\rangle$, and

$\beta=\left\langle\{[0.4,0.5],[0.3,0.4]\},\{[0.1,0.3],[0.2,0.5]\},\{[0.1,0.4],[0.7,0.8]\},\{0.3,0.4,0.5\},\{0.7,0.8\},\{0.4,0.6\}\right\rangle$, then using above definition and β-normalization with parameter $\xi = 0.5$ we have

$$\alpha\otimes\beta=\left\langle\begin{array}{c}\{[0.04,0.25],[0.06,0.28]\},\{[0.02,0.09],[0.02,0.3]\},\{[0.19,0.64],[0.7,0.86]\},\\ \{0.37,0.49,0.6\},\{0.79,0.875,0.94\},\{0.16,0.48\}\end{array}\right\rangle.$$

Definition 3.7: The scalar multiplication of a scalar q with a NCHFS $\alpha = \langle A,\lambda\rangle$ is defined by

$$q\alpha=\left\langle\begin{array}{c}\left\{\left[1-\left(1-A^L_{j_T}\right)^q,1-\left(1-A^U_{j_T}\right)^q\right]\right\},\left\{\left[1-\left(1-A^L_{j_I}\right)^q,1-\left(1-A^U_{j_I}\right)^q\right]\right\},\left\{\left[\left(A^L_{j_F}\right)^q,\left(A^U_{j_F}\right)^q\right]\right\},\\ \left\{1-\left(1-\lambda_{j_T}\right)^q\right\},\left\{1-\left(1-\lambda_{j_I}\right)^q\right\},\left\{\left(\lambda_{j_F}\right)^q\right\}\end{array}\right\rangle.$$

Example3.8: If

$\alpha=\left\langle\{[0.1,0.5],[0.2,0.7]\},\{[0.2,0.3],[0.1,0.6]\},\{[0.1,0.4],[0.0,0.3]\},\{0.1,0.2\},\{0.3,0.5\},\{0.4,0.8\}\right\rangle$,

then using above definition with q=3 we have

$$3\alpha=\left\langle\begin{array}{c}\{[0.271,0.875],[0.488,0.973]\},\{[0.488,0.657],[0.271,0.936]\},\{[0.001,0.64],[0,0.27]\},\\ \{0.001,0.008\},\{0.27,0.125\},\{0.784,0.992\}\end{array}\right\rangle.$$

Definition 3.9: For NCHFS $\alpha = \langle A,\lambda\rangle$ and a scalar q

$$\alpha^q=\left\langle\begin{array}{c}x,\left\{\left[\left(A^L_{p_T}\right)^q,\left(A^U_{p_T}\right)^q\right]\right\},\left\{\left[\left(A^L_{p_I}\right)^q,\left(A^U_{p_I}\right)\right]^q\right\},\left\{\left[1-\left(1-A^L_{p_F}\right)^q,1-\left(1-A^U_{p_F}\right)^q\right]\right\},\\ \left\{\left(\lambda_{p_T}\right)^q\right\},\left\{\left(\lambda_{p_I}\right)^q\right\},\left\{1-\left(1-\lambda_{p_F}\right)^q\right\}\end{array}\right\rangle$$

where $\alpha^q = \alpha\otimes\alpha\otimes\ldots\otimes\alpha(q-times)$ moreover α^q is a NCHF value for every $q > 0$.

Definition 3.10: The Einstein sum of two NCHFSs $\alpha = \langle A,\lambda\rangle, \beta = \langle B,\mu\rangle$ is defined by

$$\alpha\oplus_E\beta=\left\langle\begin{array}{c}\left\{\left[\frac{A^L_{j_T}+B^L_{j_T}}{1+A^L_{j_T}B^L_{j_T}},\frac{A^U_{j_T}+B^U_{j_T}}{1+A^U_{j_T}B^U_{j_T}}\right]\right\},\left\{\left[\frac{A^L_{j_I}+B^L_{j_I}}{1+A^L_{j_I}B^L_{j_I}},\frac{A^U_{j_I}+B^U_{j_I}}{1+A^U_{j_I}B^U_{j_I}}\right]\right\},\left\{\left[\frac{A^L_{j_F}B^L_{j_F}}{1+\left(1-A^L_{j_F}\right)\left(1-B^L_{j_F}\right)},\frac{A^U_{j_F}B^U_{j_F}}{1+\left(1-A^U_{j_F}\right)\left(1-B^U_{j_F}\right)}\right]\right\},\\ \left\{\frac{\lambda_{j_T}+\mu_{j_T}}{1+\lambda_{j_T}\mu_{j_T}}\right\},\left\{\frac{\lambda_{j_I}+\mu_{j_I}}{1+\lambda_{j_I}\mu_{j_I}}\right\},\left\{\frac{\lambda_{j_F}\mu_{j_F}}{1+\left(1-\lambda_{j_F}\right)\left(1-\mu_{j_F}\right)}\right\}\end{array}\right\rangle.$$

Moreover the β-normalization is used in case of different cardinalities.

Definition 3.11: The Einstein product of two NCHFSs $\alpha = \langle A,\lambda\rangle, \beta = \langle B,\mu\rangle$ is defined by

$$\alpha\otimes_E\beta=\left\langle\begin{array}{c}\left\{\left[\frac{A^L_{j_T}B^L_{j_T}}{1+\left(1-A^L_{j_T}\right)\left(1-B^L_{j_T}\right)},\frac{A^U_{j_T}B^U_{j_T}}{1+\left(1-A^U_{j_T}\right)\left(1-B^U_{j_T}\right)}\right]\right\},\left\{\left[\frac{A^L_{j_I}B^L_{j_I}}{1+\left(1-A^L_{j_I}\right)\left(1-B^L_{j_I}\right)},\frac{A^U_{j_I}B^U_{j_I}}{1+\left(1-A^U_{j_I}\right)\left(1-B^U_{j_I}\right)}\right]\right\},\\ \left\{\left[\frac{A^L_{j_F}+B^L_{j_F}}{1+A^L_{j_F}B^L_{j_F}},\frac{A^U_{j_F}+B^U_{j_F}}{1+A^U_{j_F}B^U_{j_F}}\right]\right\},\left\{\frac{\lambda_{j_T}\mu_{j_T}}{1+\left(1-\lambda_{j_T}\right)\left(1-\mu_{j_T}\right)}\right\},\left\{\frac{\lambda_{j_I}\mu_{j_I}}{1+\left(1-\lambda_{j_I}\right)\left(1-\mu_{j_I}\right)}\right\},\left\{\frac{\lambda_{j_F}+\mu_{j_F}}{1+\lambda_{j_F}\mu_{j_F}}\right\}\end{array}\right\rangle.$$

Moreover the β-normalization is used in case of different cardinalities.

Definition 3.12: The Einstein scalar multiplication of a scalar q with a NCHFS $\alpha = \langle A, \lambda \rangle$ is defined by

$$q_{_E}\alpha = \left\langle \begin{array}{c} \left\{\left[\frac{\left(1+A^L_{j_T}\right)^q-\left(1-A^L_{j_T}\right)^q}{\left(1+A^L_{j_T}\right)^q+\left(1-A^L_{j_T}\right)^q},\frac{\left(1+A^U_{j_T}\right)^q-\left(1-A^U_{j_T}\right)^q}{\left(1+A^U_{j_T}\right)^q+\left(1-A^U_{j_T}\right)^q}\right]\right\},\left\{\left[\frac{\left(1+A^L_{j_I}\right)^q-\left(1-A^L_{j_I}\right)^q}{\left(1+A^L_{j_I}\right)^q+\left(1-A^L_{j_I}\right)^q},\frac{\left(1+A^U_{j_I}\right)^q-\left(1-A^U_{j_I}\right)^q}{\left(1+A^U_{j_I}\right)^q+\left(1-A^U_{j_I}\right)^q}\right]\right\}, \\ \left\{\left[\frac{2\left(A^L_{j_F}\right)^q}{\left(2-A^L_{j_F}\right)^q+\left(A^L_{j_F}\right)^q},\frac{2\left(A^U_{j_F}\right)^q}{\left(2-A^U_{j_F}\right)^q+\left(A^U_{j_F}\right)^q}\right]\right\}, \\ \left\{\frac{\left(1+\lambda_{j_T}\right)^q-\left(1-\lambda_{j_T}\right)^q}{\left(1+\lambda_{j_T}\right)^q+\left(1-\lambda_{j_T}\right)^q}\right\},\left\{\frac{\left(1+\lambda_{j_I}\right)^q-\left(1-\lambda_{j_I}\right)^q}{\left(1+\lambda_{j_I}\right)^q+\left(1-\lambda_{j_I}\right)^q}\right\},\left\{\frac{2\left(\lambda_{j_F}\right)^q}{\left(2-\lambda_{j_F}\right)^q+\left(\lambda_{j_F}\right)^q}\right\} \end{array} \right\rangle.$$

Theorem 3.13: For a scalar q and a *NCHFS* $\alpha = \langle A, \lambda \rangle$ we have

$$\alpha^{E^q} = \left\langle \begin{array}{c} \left\{\left[\frac{2\left(A^L_{j_T}\right)^q}{\left(2-A^L_{j_T}\right)^q+\left(A^L_{j_T}\right)^q},\frac{2\left(A^U_{j_T}\right)^q}{\left(2-A^U_{j_T}\right)^q+\left(A^U_{j_T}\right)^q}\right]\right\},\left\{\left[\frac{2\left(A^L_{j_I}\right)^q}{\left(2-A^L_{j_I}\right)^q+\left(A^L_{j_I}\right)^q},\frac{2\left(A^U_{j_I}\right)^q}{\left(2-A^U_{j_I}\right)^q+\left(A^U_{j_I}\right)^q}\right]\right\} \\ \left\{\left[\frac{\left(1+A^L_{j_F}\right)^q-\left(1-A^L_{j_T}\right)^q}{\left(1+A^L_{j_F}\right)^q+\left(1-A^L_{j_F}\right)^q},\frac{\left(1+A^U_{j_F}\right)^q-\left(1-A^U_{j_F}\right)^q}{\left(1+A^U_{j_F}\right)^q+\left(1-A^U_{j_F}\right)^q}\right]\right\},\left\{\frac{2\left(\lambda_{j_T}\right)^q}{\left(2-\lambda_{j_T}\right)^q+\left(\lambda_{j_T}\right)^q}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)^q}{\left(2-\lambda_{j_I}\right)^q+\left(\lambda_{j_I}\right)^q}\right\}, \\ \left\{\frac{\left(1+\lambda_{j_F}\right)^q-\left(1-\lambda_{j_F}\right)^q}{\left(1+\lambda_{j_F}\right)^q+\left(1-\lambda_{j_F}\right)^q}\right\} \end{array} \right\rangle.$$

where $\alpha^{E^q} = \alpha \otimes_E \alpha \otimes_E \ldots \otimes_E \alpha(q - times)$ moreover α^{E^q} is a NCHF value for every $q > 0$.

Proof: Using induction on q. for *q=1* we have

$$\alpha^{E^1} = \left\langle \begin{array}{c} \left\{\left[\frac{2\left(A^L_{j_T}\right)}{\left(2-A^L_{j_T}\right)+\left(A^L_{j_T}\right)},\frac{2\left(A^U_{j_T}\right)}{\left(2-A^U_{j_T}\right)^q+\left(A^U_{j_T}\right)}\right]\right\},\left\{\left[\frac{2\left(A^L_{j_I}\right)}{\left(2-A^L_{j_I}\right)+\left(A^L_{j_I}\right)},\frac{2\left(A^U_{j_I}\right)}{\left(2-A^U_{j_I}\right)+\left(A^U_{j_I}\right)}\right]\right\} \\ \left\{\left[\frac{\left(1+A^L_{j_F}\right)-\left(1-A^L_{j_T}\right)}{\left(1+A^L_{j_F}\right)+\left(1-A^L_{j_F}\right)},\frac{\left(1+A^U_{j_F}\right)-\left(1-A^U_{j_F}\right)}{\left(1+A^U_{j_F}\right)+\left(1-A^U_{j_F}\right)}\right]\right\},\left\{\frac{2\left(\lambda_{j_T}\right)}{\left(2-\lambda_{j_T}\right)+\left(\lambda_{j_T}\right)}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)}{\left(2-\lambda_{j_I}\right)+\left(\lambda_{j_I}\right)}\right\}, \\ \left\{\frac{\left(1+\lambda_{j_F}\right)-\left(1-\lambda_{j_F}\right)}{\left(1+\lambda_{j_F}\right)+\left(1-\lambda_{j_F}\right)}\right\} \end{array} \right\rangle$$

Assuming that result is true for *q=m*.

$$\alpha^{E^m} = \left\langle \begin{array}{c} \left\{\left[\frac{2\left(A^L_{j_T}\right)^m}{\left(2-A^L_{j_T}\right)^m+\left(A^L_{j_T}\right)^m},\frac{2\left(A^U_{j_T}\right)^m}{\left(2-A^U_{j_T}\right)^m+\left(A^U_{j_T}\right)^m}\right]\right\},\left\{\left[\frac{2\left(A^L_{j_I}\right)^m}{\left(2-A^L_{j_I}\right)^m+\left(A^L_{j_I}\right)^m},\frac{2\left(A^U_{j_I}\right)^m}{\left(2-A^U_{j_I}\right)^m+\left(A^U_{j_I}\right)^m}\right]\right\} \\ \left\{\left[\frac{\left(1+A^L_{j_F}\right)^m-\left(1-A^L_{j_T}\right)^m}{\left(1+A^L_{j_F}\right)^m+\left(1-A^L_{j_F}\right)^m},\frac{\left(1+A^U_{j_F}\right)^m-\left(1-A^U_{j_F}\right)^m}{\left(1+A^U_{j_F}\right)^m+\left(1-A^U_{j_F}\right)^m}\right]\right\},\left\{\frac{2\left(\lambda_{j_T}\right)^m}{\left(2-\lambda_{j_T}\right)^m+\left(\lambda_{j_T}\right)^m}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)^m}{\left(2-\lambda_{j_I}\right)^m+\left(\lambda_{j_I}\right)^m}\right\}, \\ \left\{\frac{\left(1+\lambda_{j_F}\right)^m-\left(1-\lambda_{j_F}\right)^m}{\left(1+\lambda_{j_F}\right)^m+\left(1-\lambda_{j_F}\right)^m}\right\} \end{array} \right\rangle$$

α^{E^m} is neutrosophic cubic hesitant fuzzy value. Using assumption, we have

$$\alpha^{E^{m}}\otimes\alpha^{E^{1}}=\left\langle\begin{array}{c}
\left\{\left[\frac{2\left(A^{L}_{j_T}\right)^{m}}{\left(2-A^{L}_{j_T}\right)^{m}+\left(A^{L}_{j_T}\right)^{m}},\frac{2\left(A^{U}_{j_T}\right)^{m}}{\left(2-A^{U}_{j_T}\right)^{m}+\left(A^{U}_{j_T}\right)^{m}}\right]\right\},\\
\left\{\left[\frac{2\left(A^{L}_{j_I}\right)^{m}}{\left(2-A^{L}_{j_I}\right)^{m}+\left(A^{L}_{j_I}\right)^{m}},\frac{2\left(A^{U}_{j_I}\right)^{m}}{\left(2-A^{U}_{j_I}\right)^{m}+\left(A^{U}_{j_I}\right)^{m}}\right]\right\}\\
\left\{\left[\frac{\left(1+A^{L}_{j_F}\right)^{m}-\left(1-A^{L}_{j_T}\right)^{m}}{\left(1+A^{L}_{j_F}\right)^{m}+\left(1-A^{L}_{j_F}\right)^{m}},\frac{\left(1+A^{U}_{j_F}\right)^{m}-\left(1-A^{U}_{j_F}\right)^{m}}{\left(1+A^{U}_{j_F}\right)^{m}+\left(1-A^{U}_{j_F}\right)^{m}}\right]\right\},\\
\left\{\frac{2\left(\lambda_{j_T}\right)^{m}}{\left(2-\lambda_{j_T}\right)^{m}+\left(\lambda_{j_T}\right)^{m}}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)^{m}}{\left(2-\lambda_{j_I}\right)^{m}+\left(\lambda_{j_I}\right)^{m}}\right\},\\
\left\{\frac{\left(1+\lambda_{j_F}\right)^{m}-\left(1-\lambda_{j_F}\right)^{m}}{\left(1+\lambda_{j_F}\right)^{m}+\left(1-\lambda_{j_F}\right)^{m}}\right\}
\end{array}\right\rangle\otimes_{E}\left\langle\begin{array}{c}
\left\{\left[\frac{2\left(A^{L}_{j_T}\right)}{\left(2-A^{L}_{j_T}\right)+\left(A^{L}_{j_T}\right)},\frac{2\left(A^{U}_{j_T}\right)}{\left(2-A^{U}_{j_T}\right)^{q}+\left(A^{U}_{j_T}\right)}\right]\right\},\\
\left\{\left[\frac{2\left(A^{L}_{j_I}\right)}{\left(2-A^{L}_{j_I}\right)+\left(A^{L}_{j_I}\right)},\frac{2\left(A^{U}_{j_I}\right)}{\left(2-A^{U}_{j_I}\right)+\left(A^{U}_{j_I}\right)}\right]\right\}\\
\left\{\left[\frac{\left(1+A^{L}_{j_F}\right)-\left(1-A^{L}_{j_T}\right)}{\left(1+A^{L}_{j_F}\right)+\left(1-A^{L}_{j_F}\right)},\frac{\left(1+A^{U}_{j_F}\right)-\left(1-A^{U}_{j_F}\right)}{\left(1+A^{U}_{j_F}\right)+\left(1-A^{U}_{j_F}\right)}\right]\right\},\\
\left\{\frac{2\left(\lambda_{j_T}\right)}{\left(2-\lambda_{j_T}\right)+\left(\lambda_{j_T}\right)}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)}{\left(2-\lambda_{j_I}\right)+\left(\lambda_{j_I}\right)}\right\},\\
\left\{\frac{\left(1+\lambda_{j_F}\right)-\left(1-\lambda_{j_F}\right)}{\left(1+\lambda_{j_F}\right)+\left(1-\lambda_{j_F}\right)}\right\}
\end{array}\right\rangle$$

$$=\left\langle\begin{array}{c}
\left\{\left[\frac{\frac{4(A^{L}_{j_T})^{m+1}}{\left((2-A^{L}_{j_T})^{m}+(A^{L}_{jT})^{m}\right)\left((2-A^{L}_{j_T})^{1}+(A^{L}_{jT})^{1}\right)}}{1+\left(1-\frac{2(A^{L}_{jT})^{m}}{\left(2-A^{L}_{jT}\right)^{m}+(A^{L}_{jT})^{m}}\right)\left(1-\frac{2(A^{L}_{jT})^{1}}{\left(2-A^{L}_{jT}\right)^{1}+(A^{L}_{jT})^{1}}\right)},\frac{\frac{4(A^{U}_{j_T})^{m+1}}{\left((2-A^{U}_{j_T})^{m}+(A^{U}_{jT})^{m}\right)\left((2-A^{U}_{j_T})^{1}+(A^{U}_{jT})^{1}\right)}}{1+\left(1-\frac{2(A^{U}_{jT})^{m}}{\left(2-A^{U}_{jT}\right)^{m}+(A^{U}_{jT})^{m}}\right)\left(1-\frac{2(A^{U}_{jT})^{1}}{\left(2-A^{U}_{jT}\right)^{1}+(A^{U}_{jT})^{1}}\right)}\right]\right\},\\
\left\{\left[\frac{\frac{4(A^{L}_{j_I})^{m+1}}{\left((2-A^{L}_{j_I})^{m}+(A^{L}_{jI})^{m}\right)\left((2-A^{L}_{j_I})^{1}+(A^{L}_{jI})^{1}\right)}}{1+\left(1-\frac{2(A^{L}_{jI})^{m}}{\left(2-A^{L}_{jI}\right)^{m}+(A^{L}_{jI})^{m}}\right)\left(1-\frac{2(A^{L}_{jI})^{1}}{\left(2-A^{L}_{jI}\right)^{1}+(A^{L}_{jI})^{1}}\right)},\frac{\frac{4(A^{U}_{j_I})^{m+1}}{\left((2-A^{U}_{j_I})^{m}+(A^{U}_{j_I})^{m}\right)\left((2-A^{U}_{j_I})^{1}+(A^{U}_{j_I})^{1}\right)}}{1+\left(1-\frac{2(A^{U}_{jI})^{m}}{\left(2-A^{U}_{jI}\right)^{m}+(A^{U}_{jI})^{m}}\right)\left(1-\frac{2(A^{U}_{jI})^{1}}{\left(2-A^{U}_{jI}\right)^{1}+(A^{U}_{jI})^{1}}\right)}\right]\right\},\\
\left\{\left[\frac{\left(\frac{(1+A^{L}_{jF})^{m}-(1-A^{L}_{jF})^{m}}{(1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}}\right)+\left(\frac{(1+A^{L}_{jF})^{1}-(1-A^{L}_{jF})^{1}}{(1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}}\right)}{1+\left(\frac{(1+A^{L}_{jF})^{m}-(1-A^{L}_{jF})^{m}}{(1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}}\right)\left(\frac{(1+A^{L}_{jF})^{1}-(1-A^{L}_{jF})^{1}}{(1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}}\right)},\frac{\left(\frac{(1+A^{U}_{jF})^{m}-(1-A^{U}_{jF})^{m}}{(1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}}\right)+\left(\frac{(1+A^{U}_{jF})^{1}-(1-A^{U}_{jF})^{1}}{(1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}}\right)}{1+\left(\frac{(1+A^{U}_{jF})^{m}-(1-A^{U}_{jF})^{m}}{(1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}}\right)\left(\frac{(1+A^{U}_{jF})^{1}-(1-A^{U}_{jF})^{1}}{(1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}}\right)}\right]\right\},\\
\left\{\frac{\frac{4(\lambda_{j_T})^{m+1}}{\left((2-\lambda_{j_T})^{m}+(\lambda_{j_T})^{m}\right)\left((2-\lambda_{j_T})^{1}+(\lambda_{jF})^{1}\right)}}{1+\left(1-\frac{2(\lambda_{j_T})^{m}}{\left(2-\lambda_{j_T}\right)^{m}+(\lambda_{j_T})^{m}}\right)\left(1-\frac{2(\lambda_{j_T})^{1}}{\left(2-\lambda_{j_T}\right)^{1}+(\lambda_{j_T})^{1}}\right)}\right\},\left\{\frac{\frac{4(\lambda_{j_I})^{m+1}}{\left((2-\lambda_{j_I})^{m}+(\lambda_{j_I})^{m}\right)\left((2-\lambda_{j_I})^{1}+(\lambda_{j_I})^{1}\right)}}{1+\left(1-\frac{2(\lambda_{j_I})^{m}}{\left(2-\lambda_{j_I}\right)^{m}+(\lambda_{j_I})^{m}}\right)\left(1-\frac{2(\lambda_{j_I})^{1}}{\left(2-\lambda_{j_I}\right)^{1}+(\lambda_{j_I})^{1}}\right)}\right\},\\
\left\{\frac{\frac{(1+\lambda_{j_F})^{m}-(1-\lambda_{j_F})^{m}}{(1+\lambda_{j_F})^{m}+(1-\lambda_{j_F})^{m}}+\frac{(1+\lambda_{j_F})^{1}-(1-\lambda_{j_F})^{1}}{(1+\lambda_{j_F})^{1}+(1-\lambda_{j_F})^{1}}}{1+\left(\frac{(1+\lambda_{j_F})^{m}-(1-\lambda_{j_F})^{m}}{(1+\lambda_{j_F})^{m}+(1-\lambda_{j_F})^{m}}\right)\left(\frac{(1+\lambda_{j_F})^{1}-(1-\lambda_{j_F})^{1}}{(1+\lambda_{j_F})^{1}+(1-\lambda_{j_F})^{1}}\right)}\right\}
\end{array}\right\rangle$$

$$
=\left\langle
\begin{array}{c}
\left\{\left[\dfrac{\dfrac{4(A^{L}_{j_T})^{m+1}}{\left((2-A^{L}_{j_T})^{m}+(A^{L}_{jT})^{m}\right)\left((2-A^{L}_{j_T})^{1}+(A^{L}_{jT})^{1}\right)}}{1+\left(\dfrac{\left(2-A^{L}_{jT}\right)^{m}+(A^{L}_{jT})^{m}-2(A^{L}_{jT})^{m}}{\left(2-A^{L}_{jT}\right)^{m}+(A^{L}_{jT})^{m}}\right)\left(\dfrac{\left(2-A^{L}_{jT}\right)^{1}+(A^{L}_{jT})^{1}-2(A^{L}_{jT})^{1}}{\left(2-A^{L}_{jT}\right)^{1}+(A^{L}_{jT})^{1}}\right)},\dfrac{\dfrac{4(A^{U}_{j_T})^{m+1}}{\left((2-A^{U}_{j_T})^{m}+(A^{U}_{jT})^{m}\right)\left((2-A^{U}_{j_T})^{1}+(A^{U}_{jT})^{1}\right)}}{1+\left(\dfrac{\left(2-A^{U}_{jT}\right)^{m}+(A^{U}_{jT})^{m}-2(A^{U}_{jT})^{m}}{\left(2-A^{U}_{jT}\right)^{m}+(A^{U}_{jT})^{m}}\right)\left(\dfrac{\left(2-A^{U}_{jT}\right)^{1}+(A^{U}_{jT})^{1}-2(A^{U}_{jT})^{1}}{\left(2-A^{U}_{jT}\right)^{1}+(A^{U}_{jT})^{1}}\right)}\right]\right\},\\[2ex]
\left\{\left[\dfrac{\dfrac{4(A^{L}_{j_I})^{m+1}}{\left((2-A^{L}_{j_I})^{m}+(A^{L}_{jI})^{m}\right)\left((2-A^{L}_{j_I})^{1}+(A^{L}_{jI})^{1}\right)}}{1+\left(\dfrac{\left(2-A^{L}_{jI}\right)^{m}+(A^{L}_{jI})^{m}-2(A^{L}_{jI})^{m}}{\left(2-A^{L}_{jI}\right)^{m}+(A^{L}_{jI})^{m}}\right)\left(\dfrac{\left(2-A^{L}_{jI}\right)^{1}+(A^{L}_{jI})^{1}-2(A^{L}_{jI})^{1}}{\left(2-A^{L}_{jI}\right)^{1}+(A^{L}_{jI})^{1}}\right)},\dfrac{\dfrac{4(A^{U}_{j_I})^{m+1}}{\left((2-A^{U}_{j_I})^{m}+(A^{U}_{jI})^{m}\right)\left((2-A^{U}_{j_I})^{1}+(A^{U}_{jI})^{1}\right)}}{1+\left(\dfrac{\left(2-A^{U}_{jI}\right)^{m}+(A^{U}_{jI})^{m}-2(A^{U}_{jI})^{m}}{\left(2-A^{U}_{jI}\right)^{m}+(A^{U}_{jI})^{m}}\right)\left(\dfrac{\left(2-A^{U}_{jI}\right)^{1}+(A^{U}_{jI})^{1}-2(A^{U}_{jI})^{1}}{\left(2-A^{U}_{jI}\right)^{1}+(A^{U}_{jI})^{1}}\right)}\right]\right\},\\[2ex]
\left\{\left[\begin{array}{c}\dfrac{\dfrac{\left((1+A^{L}_{jF})^{m}-(1-A^{L}_{jF})^{m}\right)\left((1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}\right)+\left((1+A^{L}_{jF})^{1}-(1-A^{L}_{jF})^{1}\right)\left((1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}\right)}{\left((1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}\right)\left((1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}\right)}}{\dfrac{\left((1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}\right)\left((1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}\right)+\left((1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}\right)\left((1+A^{L}_{jF})^{m}-(1-A^{L}_{jF})^{m}\right)}{\left((1+A^{L}_{jF})^{m}+(1-A^{L}_{jF})^{m}\right)\left((1+A^{L}_{jF})^{1}+(1-A^{L}_{jF})^{1}\right)}},\\ \dfrac{\dfrac{\left((1+A^{U}_{jF})^{m}-(1-A^{U}_{jF})^{m}\right)\left((1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}\right)+\left((1+A^{U}_{jF})^{1}-(1-A^{U}_{jF})^{1}\right)\left((1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}\right)}{\left((1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}\right)\left((1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}\right)}}{\dfrac{\left((1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}\right)\left((1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}\right)+\left((1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}\right)\left((1+A^{U}_{jF})^{m}-(1-A^{U}_{jF})^{m}\right)}{\left((1+A^{U}_{jF})^{m}+(1-A^{U}_{jF})^{m}\right)\left((1+A^{U}_{jF})^{1}+(1-A^{U}_{jF})^{1}\right)}}\end{array}\right]\right\},\\[2ex]
\left\{\dfrac{\dfrac{4(\lambda_{jT})^{m+1}}{\left((2-\lambda_{jT})^{m}+(\lambda_{jT})^{m}\right)\left((2-\lambda_{jF})^{1}+(\lambda_{jT})^{1}\right)}}{1+\left(\dfrac{\left(2-\lambda_{jT}\right)^{m}+(\lambda_{jT})^{m}-2(\lambda_{jT})^{m}}{\left(2-\lambda_{jT}\right)^{m}+(\lambda_{jT})^{m}}\right)\left(\dfrac{\left(2-\lambda_{jT}\right)^{1}+(\lambda_{jT})^{1}-2(\lambda_{jT})^{1}}{\left(2-\lambda_{jT}\right)^{1}+(\lambda_{jT})^{1}}\right)}\right\},\\[2ex]
\left\{\dfrac{\dfrac{4(\lambda_{jI})^{m+1}}{\left((2-\lambda_{jI})^{m}+(\lambda_{jI})^{m}\right)\left((2-\lambda_{jI})^{1}+(\lambda_{jI})^{1}\right)}}{1+\left(\dfrac{\left(2-\lambda_{jI}\right)^{m}+(\lambda_{jI})^{m}-2(\lambda_{jI})^{m}}{\left(2-\lambda_{jI}\right)^{m}+(\lambda_{jI})^{m}}\right)\left(\dfrac{\left(2-\lambda_{jI}\right)^{1}+(\lambda_{jI})^{1}-2(\lambda_{jI})^{1}}{\left(2-\lambda_{jI}\right)^{1}+(\lambda_{jI})^{1}}\right)}\right\},\\[2ex]
\left\{\dfrac{\dfrac{\left((1+\lambda_{jF})^{m}-(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}+(1-\lambda_{jF})^{1}\right)+\left((1+\lambda_{jF})^{m}+(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}-(1-\lambda_{jF})^{1}\right)}{\left((1+\lambda_{jF})^{m}+(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}+(1-\lambda_{jF})^{1}\right)}}{\dfrac{\left((1+\lambda_{jF})^{m}+(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}+(1-\lambda_{jF})^{1}\right)+\left((1+\lambda_{jF})^{m}-(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}-(1-\lambda_{jF})^{1}\right)}{\left((1+\lambda_{jF})^{m}+(1-\lambda_{jF})^{m}\right)\left((1+\lambda_{jF})^{1}+(1-\lambda_{jF})^{1}\right)}}\right\}
\end{array}
\right\rangle
$$

$$
= \left\langle \begin{array}{c}
\left\{ \left[\frac{4\left(A^L_{j_T}\right)^{m+1}}{2\left(\left(2-A^L_{j_T}\right)^{m+1}+\left(A^L_{j_T}\right)^{m+1}\right)}, \frac{4\left(A^U_{j_T}\right)^{m+1}}{2\left(\left(2-A^U_{j_T}\right)^{m+1}+\left(A^U_{j_T}\right)^{m+1}\right)} \right] \right\}, \\
\left\{ \left[\frac{4\left(A^L_{j_I}\right)^{m+1}}{2\left(\left(2-A^L_{j_I}\right)^{m+1}+\left(A^L_{j_I}\right)^{m+1}\right)}, \frac{4\left(A^U_{j_I}\right)^{m+1}}{2\left(\left(2-A^U_{j_I}\right)^{m+1}+\left(A^U_{j_I}\right)^{m+1}\right)} \right] \right\}, \\
\left\{ \left[\frac{\left(1+A^L_{j_F}\right)^{m+1}-\left(1-A^L_{j_T}\right)^{m+1}}{\left(1+A^L_{j_F}\right)^{m+1}+\left(1-A^L_{j_F}\right)^{m+1}}, \frac{\left(1+A^U_{j_F}\right)^{m+1}-\left(1-A^U_{j_F}\right)^{m+1}}{\left(1+A^U_{j_F}\right)^{m+1}+\left(1-A^U_{j_F}\right)^{m+1}} \right] \right\}, \\
\left\{ \frac{4\left(\lambda_{j_T}\right)^{m+1}}{2\left(\left(2-\lambda_{j_T}\right)^{m+1}+\left(\lambda_{j_T}\right)^{m+1}\right)} \right\}, \left\{ \frac{4\left(\lambda_{j_I}\right)^{m+1}}{2\left(\left(2-\lambda_{j_I}\right)^{m+1}+\left(\lambda_{j_I}\right)^{m+1}\right)} \right\}, \\
\left\{ \frac{\left(1+\lambda_{j_F}\right)^{m+1}-\left(1-\lambda_{j_F}\right)^{m+1}}{\left(1+\lambda_{j_F}\right)^{m+1}+\left(1-\lambda_{j_F}\right)^{m+1}} \right\}
\end{array} \right\rangle
$$

$$
= \left\langle \begin{array}{c}
\left\{ \left[\frac{2\left(A^L_{j_T}\right)^{m+1}}{\left(2-A^L_{j_T}\right)^{m+1}+\left(A^L_{j_T}\right)^{m+1}}, \frac{2\left(A^U_{j_T}\right)^{m+1}}{\left(2-A^U_{j_T}\right)^{m+1}+\left(A^U_{j_T}\right)^{m+1}} \right] \right\}, \left\{ \left[\frac{2\left(A^L_{j_I}\right)^{m+1}}{\left(2-A^L_{j_I}\right)^{m+1}+\left(A^L_{j_I}\right)^{m+1}}, \frac{2\left(A^U_{j_I}\right)^{m+1}}{\left(2-A^U_{j_I}\right)^{m+1}+\left(A^U_{j_I}\right)^{m+1}} \right] \right\} \\
\left\{ \left[\frac{\left(1+A^L_{j_F}\right)^{m+1}-\left(1-A^L_{j_T}\right)^{m+1}}{\left(1+A^L_{j_F}\right)^{m+1}+\left(1-A^L_{j_F}\right)^{m+1}}, \frac{\left(1+A^U_{j_F}\right)^{m+1}-\left(1-A^U_{j_F}\right)^{m+1}}{\left(1+A^U_{j_F}\right)^{m+1}+\left(1-A^U_{j_F}\right)^{m+1}} \right] \right\}, \left\{ \frac{2\left(\lambda_{j_T}\right)^{m+1}}{\left(2-\lambda_{j_T}\right)^{m+1}+\left(\lambda_{j_T}\right)^{m+1}} \right\}, \left\{ \frac{2\left(\lambda_{j_I}\right)^{m+1}}{\left(2-\lambda_{j_I}\right)^{m+1}+\left(\lambda_{j_I}\right)^{m+1}} \right\}, \\
\left\{ \frac{\left(1+\lambda_{j_F}\right)^{m+1}-\left(1-\lambda_{j_F}\right)^{m+1}}{\left(1+\lambda_{j_F}\right)^{m+1}+\left(1-\lambda_{j_F}\right)^{m+1}} \right\}
\end{array} \right\rangle
$$

Definition 3.14: The Score, accuracy and certainty of a *NCHFS*$\alpha = \langle A, \lambda \rangle$ where $A = \langle A_T, A_I, A_F \rangle, A_T = \{[A^L_{j_T}, A^U_{j_T}]; j = 1, \dots, l\}, A_I = \{[A^L_{j_I}, A^U_{j_I}]; j = 1, \dots, m\}, A_F = \{[A^L_{j_F}, A^U_{j_F}]; i = 1, \dots, n\}$and $\lambda = \langle \lambda_T, \lambda_I, \lambda_F \rangle, \lambda_T = \{\lambda_{j_T}; j = 1, \dots, r\}, \lambda_I = \{\lambda_{j_I}; j = 1, \dots, s\}, \lambda_F = \{\lambda_{j_F}; j = 1, \dots, t\}$ are defined as:

$$
S(\alpha) = \frac{1}{2} \left\{ \begin{array}{c}
\frac{1}{6}\left(\frac{1}{l}\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_F}\right) + \frac{1}{m}\sum_{j=1}^{m}\left(A^L_{j_I} + A^U_{j_I}\right) + \frac{1}{n}\sum_{j=1}^{n}\left(2-\left(A^L_{j_T} + A^U_{j_F}\right)\right) \right) \\
+\frac{1}{3}\left(\frac{1}{r}\sum_{j=1}^{r}\lambda_{j_T} + \frac{1}{s}\sum_{j=1}^{s}\lambda_{j_I} + \frac{1}{t}\sum_{j=1}^{t}\left(1-\lambda_{j_F}\right) \right)
\end{array} \right\},
$$

$$
H(\alpha) = \tfrac{1}{9} \left\{ \begin{array}{c}
\tfrac{1}{l}\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_T}\right) + \tfrac{1}{m}\sum_{j=1}^{m}\left(A^L_{j_I} + A^U_{j_I}\right) + \tfrac{1}{n}\sum_{j=1}^{n}\left(A^U_{j_F} + A^L_{j_F}\right) \\
+\tfrac{1}{r}\sum_{j=1}^{r}\lambda_{j_T} + \tfrac{1}{s}\sum_{j=1}^{s}\lambda_{j_I} + \tfrac{1}{t}\sum_{j=1}^{t}\lambda_{j_F}
\end{array} \right\},
$$

$$
C(\alpha) = \tfrac{1}{3}\left\{ \tfrac{1}{l}\sum_{j=1}^{l}\left(A^L_{j_T} + A^U_{j_T}\right) + \tfrac{1}{r}\sum_{j=1}^{r}\lambda_{j_T} \right\}.
$$

If $\alpha = \left\langle \{[0.1, 0.5], [0.2, 0.7]\}, \{[0.2, 0.3], [0.1, 0.6]\}, \{[0.1, 0.4], [0, 0.3]\}, \{0.1, 0.2\}, \{0.3, 0.5\}, \{0.4, 0.8\} \right\rangle$, and

$\beta=\left\langle\{[0.4,0.5],[0.3,0.4]\},\{[0.1,0.3],[0.2,0.5]\},\{[0.1,0.4],[0.7,0.8]\},\{0.3,0.5\},\{0.7,0.8\},\{0.4,0.6\}\right\rangle$, then $S(\alpha)=0.404167, S(\beta)=0.470833, H(\alpha)=0.3222, H(\beta)=0.4444, C(\alpha)=0.15, C(\beta)=0.4.$

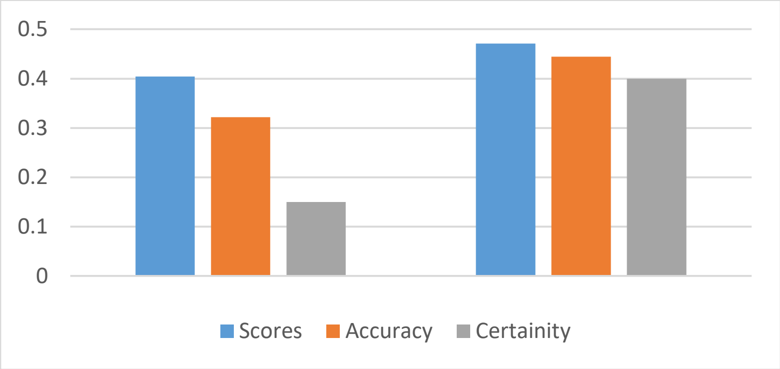

Figure 1: Scores, Accuracy and Certainty of above NHFSs

Definition 3.15: Let $\alpha=\langle A,\lambda\rangle, \beta=\langle B,\mu\rangle$ are two NCHFSs. We say that $\alpha>\beta$ if $S(\alpha)>S(\beta)$. If $S(\alpha)=S(\beta)$, then $\alpha>\beta$ if $A(\alpha)>A(\beta)$. If $A(\alpha)=A(\beta)$, then $\alpha>\beta$ if $C(\alpha)>C(\beta)$. If $S(\alpha)=S(\beta), A(\alpha)=A(\beta), C(\alpha)>C(\beta)$, then $\alpha=\beta$.

In the next section we define aggregation operators on neutrosophic cubic hesitant fuzzy set and prove some elegant results.

4. Aggregation Operators

Definition 4.1: The Neutrosophic cubic hesitant fuzzy weighted geometric operator is defined as

$NCHWG(\alpha_1,\dots,\alpha_n)=\bigotimes_{j=1}^{n}\alpha_j^{w_j}$, where $\alpha_{j_{(k)}}$ are neutrosophic cubic hesitant fuzzy values taken in descending order with corresponding weight vector $w=(w_1,\dots,w_n)^t$.

Definition 4.2: Neutrosophic cubic hesitant fuzzy order weighted geometric operator is defined as:

$NCHFOWG(\alpha_1,\dots,\alpha_n)=\bigotimes_{j=1}^{n}(\alpha_{(k)_j})^{w_j}$ where $\alpha_{j_{(k)}}$ are neutrosophic cubic hesitant fuzzy values taken in descending order with corresponding weight vector $w=(w_1,\dots,w_n)^t$.

Definition 4.3: The Neutrosophic cubic hesitant fuzzy Einstein weighted geometric operator is defined as:

$NCEHWG(\alpha_1,\dots,\alpha_n)=\bigotimes_{j=1}^{n}(\alpha_j^E)^{w_j}$ where $\alpha_{j_{(k)}}$ are neutrosophic cubic hesitant fuzzy values taken in descending order with corresponding weight vector $w=(w_1,\dots,w_n)^t$

Definition 4.4: Neutrosophic cubic hesitant fuzzy Einstein ordered weighted geometric operator is defined as:

$NCHEOWG(\alpha_1,\dots,\alpha_n)=\bigotimes_{j=1}^{n}\left(\alpha_j^E\right)^{w_j}$ where $\alpha_{j_{(k)}}$ are neutrosophic cubic hesitant fuzzy values taken in descending order with corresponding weight vector $w=(w_1,\dots,w_n)^t$

Theorem 4.5: Let $\left\{\alpha_{(k)} = \left\langle A_{(k)}, \lambda_{(k)} \right\rangle\right\}$ the set of neutrosophic cubic hesitant with corresponding weight vector $w = (w_1, \dots)^t$ fuzzy values, then

$$NCHFWG(\alpha_1,\dots,\alpha_m) = \left\langle \begin{array}{c} \left\{\left[\prod_{k=1}^{m}\left(A^L_{p_{T(k)}}\right)^{w_k}, \prod_{k=1}^{m}\left(A^U_{p_{T(k)}}\right)^{w_k}\right]\right\}, \left\{\left[\prod_{k=1}^{m}\left(A^L_{p_{I(k)}}\right)^{w_k}, \prod_{k=1}^{m}\left(A^U_{p_{I(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\prod_{k=1}^{m}\left(1-A^L_{p_{F(k)}}\right)^{w_k}, 1-\prod_{k=1}^{m}\left(1-A^U_{p_{F(k)}}\right)^{w_k}\right]\right\}, \left\{\prod_{k=1}^{m}\left(\lambda_{p_{T(k)}}\right)^{w_k}\right\}, \\ \left\{\prod_{k=1}^{m}\left(\lambda_{p_{I(k)}}\right)^{w_k}\right\}, \left\{1-\prod_{k=1}^{m}\left(1-\lambda_{p_{F(k)}}\right)^{w_k}\right\} \end{array} \right\rangle$$

Proof: Using induction for m=2

$$NCHFWG(\alpha_1,\alpha_2) = \bigotimes_{k=1}^{2}\alpha_k^{w_k}$$

$$= \left\langle \begin{array}{c} \left\{\left[\prod_{k=1}^{2}\left(A^L_{p_{T(k)}}\right)^{w_k}, \prod_{k=1}^{2}\left(A^U_{p_{T(k)}}\right)^{w_k}\right]\right\}, \left\{\left[\prod_{k=1}^{2}\left(A^L_{p_{I(k)}}\right)^{w_k}, \prod_{k=1}^{2}\left(A^U_{p_{I(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\prod_{k=1}^{2}\left(1-A^L_{p_{F(k)}}\right)^{w_k}, 1-\prod_{k=1}^{2}\left(1-A^U_{p_{F(k)}}\right)^{w_k}\right]\right\}, \left\{\prod_{k=1}^{2}\left(\lambda_{p_{T(k)}}\right)^{w_k}\right\}, \\ \left\{\prod_{k=1}^{2}\left(\lambda_{p_{I(k)}}\right)^{w_k}\right\}, \left\{1-\prod_{k=1}^{2}\left(1-\lambda_{p_{F(k)}}\right)^{w_k}\right\} \end{array} \right\rangle$$

For $m = q$ we have

$$NCHFWG\left(\alpha_1,\dots,\alpha_q\right) = \left\langle \begin{array}{c} \left\{\left[\prod_{k=1}^{q}\left(A^L_{p_{I(k)}}\right)^{w_k}, \prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}\right]\right\}, \left\{\left[\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}, \prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}, 1-\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\prod_{k=1}^{q}\left(\lambda^L_{p_{T(k)}}\right)^{w_k}\right\}, \left\{\prod_{k=1}^{q}\left(\lambda^L_{p_{I(k)}}\right)^{w_k}\right\}, \left\{1-\prod_{k=1}^{q}\left(1-\lambda^L_{p_{F(k)}}\right)^{w_k}\right\} \end{array} \right\rangle$$

we prove for m=q+1

$$\bigotimes_{k=1}^{q}\left(\alpha_k\right)^{w_k} \otimes \left(\alpha_{q+1}\right)^{w_{q+1}} = \left\langle \begin{array}{c} \left\{\left[\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}, \prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\left[\prod_{k=1}^{q}\left(A^L_{p_{I(k)}}\right)^{w_k}, \prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}, 1-\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}\right]\right\}, \\ \left\{\prod_{k=1}^{q}\left(\lambda_{p_{T(k)}}\right)^{w_k}\right\}, \left\{\prod_{k=1}^{q}\left(\lambda_{p_{I(k)}}\right)^{w_k}\right\}, \\ \left\{1-\prod_{k=1}^{q}\left(1-\lambda_{p_{F(k)}}\right)^{w_k}\right\} \end{array} \right\rangle \otimes \left\langle \begin{array}{c} \left\{\left[\left(A^L_{p_{T(q+1)}}\right)^{w_{q+1}}, \left(A^U_{p_{T(k)}}\right)^{w_{q+1}}\right]\right\}, \\ \left\{\left[\left(A^L_{p_{I(q+1)}}\right)^{w_{q+1}}, \left(A^U_{p_{I(q+1)}}\right)^{w_{q+1}}\right]\right\}, \\ \left\{\left[1-\left(1-A^L_{p_{F(q+1)}}\right)^{w_{q+1}}, 1-\left(1-A^U_{p_{F(q+1)}}\right)^{w_{q+1}}\right]\right\}, \\ \left\{\left(\lambda_{p_{T(q+1)}}\right)^{w_{q+1}}\right\}, \\ \left\{\left(\lambda_{p_{I(q+1)}}\right)^{w_{q+1}}\right\}, \left\{1-\left(1-\lambda_{p_{F(q+1)}}\right)^{w_{q+1}}\right\} \end{array} \right\rangle$$

$$
=\left\langle\begin{array}{c}
\left\{\left[\prod_{k=1}^{q}\left(A^{L}_{p_{T_{(k)}}}\right)^{w_k}\left(A^{L}_{p_{T_{(q+1)}}}\right)^{w_{q+1}},\prod_{k=1}^{q}\left(A^{U}_{p_{T_{(k)}}}\right)^{w_k}\left(A^{U}_{p_{T_{(q+1)}}}\right)^{w_{q+1}}\right]\right\},\left\{\left[\prod_{k=1}^{q}\left(A^{L}_{p_{I_{(k)}}}\right)^{w_k}\left(A^{L}_{p_{I_{(q+1)}}}\right)^{w_{q+1}},\prod_{k=1}^{q}\left(A^{U}_{p_{I_{(k)}}}\right)^{w_k}\left(A^{U}_{p_{I_{(q+1)}}}\right)^{w_{q+1}}\right]\right\},\\
\left\{\left[\begin{array}{c}
1-\prod_{k=1}^{q}\left(1-A^{L}_{p_{F_{(k)}}}\right)^{w_k}+1-\left(1-A^{L}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}-\left(\left(1-\prod_{k=1}^{q}\left(1-A^{L}_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-\left(1-A^{L}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}\right)\right),\\
1-\prod_{k=1}^{q}\left(1-A^{U}_{p_{F_{(k)}}}\right)^{w_k}+1-\left(1-A^{U}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}-\left(\left(1-\prod_{k=1}^{q}\left(1-A^{U}_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-\left(1-A^{U}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}\right)\right)
\end{array}\right]\right\},\\
\left\{\prod_{k=1}^{q}\left(\lambda_{p_{T_{(k)}}}\right)^{w_k}\left(\lambda_{p_{T_{(q+1)}}}\right)^{w_{q+1}}\right\},\left\{\prod_{k=1}^{q}\left(\lambda_{p_{I_{(k)}}}\right)^{w_k}\left(\lambda_{p_{I_{(q+1)}}}\right)^{w_{q+1}}\right\},\\
\left\{1-\prod_{k=1}^{q}\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_k}+1-\left(1-\lambda_{p_{F_{(q+1)}}}\right)^{w_{q+1}}-\left(\left(1-\prod_{k=1}^{q}\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-\left(1-\lambda_{p_{F_{(q+1)}}}\right)^{w_{q+1}}\right)\right)\right\}
\end{array}\right\rangle
$$

$$
=\left\langle\begin{array}{c}
\left\{\left[\prod_{k=1}^{q+1}\left(A^{L}_{p_{T_{(k)}}}\right)^{w_k},\prod_{k=1}^{q+1}\left(A^{U}_{p_{T_{(k)}}}\right)^{w_k}\right]\right\},\left\{\left[\prod_{k=1}^{q+1}\left(A^{L}_{p_{I_{(k)}}}\right)^{w_k},\prod_{k=1}^{q+1}\left(A^{U}_{p_{I_{(k)}}}\right)^{w_k}\right]\right\},\\
\left\{\left[\begin{array}{c}
2-\prod_{k=1}^{q+1}\left(1-A^{L}_{p_{F_{(k)}}}\right)^{w_k}-1+\prod_{k=1}^{q}\left(1-A^{L}_{p_{F_{(k)}}}\right)+\left(1-A^{L}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}-\left(\prod_{k=1}^{q}\left(1-A^{L}_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-A^{L}_{p_{F_{(q+1)}}}\right)^{w_{q+1}},\\
2-\prod_{k=1}^{q+1}\left(1-A^{U}_{p_{F_{(k)}}}\right)^{w_k}-1+\prod_{k=1}^{q}\left(1-A^{U}_{p_{F_{(k)}}}\right)+\left(1-A^{U}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}-\left(\prod_{k=1}^{q}\left(1-A^{U}_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-A^{U}_{p_{F_{(q+1)}}}\right)^{w_{q+1}}
\end{array}\right]\right\},\\
\left\{\prod_{k=1}^{q+1}\left(\lambda_{p_{T_{(k)}}}\right)^{w_k}\right\},\left\{\prod_{k=1}^{q+1}\left(\lambda_{p_{I_{(k)}}}\right)^{w_k}\right\},\\
\left\{\begin{array}{c}
2-\prod_{k=1}^{q+1}\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_k}-1+\prod_{k=1}^{q}\left(1-\lambda_{p_{F_{(k)}}}\right)+\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_{q+1}}\\
-\left(\prod_{k=1}^{q}\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_k}\right)\left(1-\lambda_{p_{F_{(k)}}}\right)^{w_{q+1}}
\end{array}\right\}
\end{array}\right\rangle
$$

$$= \left\langle \begin{array}{c} \left\{ \left[\prod_{k=1}^{q+1} \left(A^{L}_{p_{T_{(k)}}} \right)^{w_k}, \prod_{k=1}^{q+1} \left(A^{U}_{p_{T_{(k)}}} \right)^{w_k} \right] \right\}, \left\{ \left[\prod_{k=1}^{q+1} \left(A^{L}_{p_{I_{(k)}}} \right)^{w_k}, \prod_{k=1}^{q+1} \left(A^{U}_{p_{I_{(k)}}} \right)^{w_k} \right] \right\}, \\ \left\{ \left[1-\prod_{k=1}^{q+1} \left(1-A^{L}_{p_{F_{(k)}}} \right)^{w_k}, 1-\prod_{k=1}^{q+1} \left(1-A^{U}_{p_{F_{(k)}}} \right)^{w_k} \right] \right\} \left\{ \prod_{k=1}^{q+1} \left(\lambda_{p_{T_{(k)}}} \right)^{w_k} \right\}, \\ \left\{ \prod_{k=1}^{q+1} \left(\lambda_{p_{I_{(k)}}} \right)^{w_k} \right\}, \left\{ 1-\prod_{k=1}^{q+1} \left(1-\lambda_{p_{F_{(k)}}} \right)^{w_k} \right\} \end{array} \right\rangle$$

Theorem 4.6: Let $\left\{ \alpha_{(k)} = \left\langle A_{(k)}, \lambda_{(k)} \right\rangle \right\}$ the set of neutrosophic cubic hesitant fuzzy values**,** then

i) Idempotency: If $\alpha_k = \alpha, k = 1,...m$ then $NCHFWG(\alpha_1,...,\alpha_m) = \alpha.$

ii) Monotonicity: If $S(\alpha_q) \geq S(\alpha_l)$ then $NCHFWG(\alpha_q) \leq NCHFWG(\alpha_l)$.

Theorem 4.7: Let $\left\{ \alpha_{(k)} = \left\langle A_{(k)}, \lambda_{(k)} \right\rangle \right\}$ the set of neutrosophic cubic hesitant with corresponding weight vector $w = (w_1, \dots)^t$ fuzzy values, then

$$NCHFEWG\left(\alpha_1,\dots,\alpha_m\right)=\left\langle\begin{array}{c}
\left\{\left[\frac{2\prod_{k=1}^{m}\left(A_{p_{T(k)}}^{L}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-A_{p_{T(k)}}^{L}\right)^{w_k}+\prod_{k=1}^{m}\left(A_{p_{T(k)}}^{L}\right)^{w_k}},\frac{2\prod_{k=1}^{m}\left(A_{p_{T(k)}}^{U}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-A_{p_{T(k)}}^{U}\right)^{w_k}+\prod_{k=1}^{m}\left(A_{p_{T(k)}}^{U}\right)^{w_k}}\right]\right\},\\
\left\{\left[\frac{2\prod_{k=1}^{m}\left(A_{p_{I(k)}}^{L}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-A_{p_{I(k)}}^{L}\right)^{w_k}+\prod_{k=1}^{m}\left(A_{p_{T(k)}}^{L}\right)^{w_k}},\frac{2\prod_{k=1}^{m}\left(A_{p_{I(k)}}^{U}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-A_{p_{I(k)}}^{U}\right)^{w_k}+\prod_{k=1}^{m}\left(A_{p_{I(k)}}^{U}\right)^{w_k}}\right]\right\},\\
\left\{\left[\frac{\prod_{k=1}^{m}\left(1+A_{p_{F(k)}}^{L}\right)^{w_k}-\prod_{k=1}^{m}\left(1-A_{p_{F(k)}}^{L}\right)^{w_k}}{\prod_{k=1}^{m}\left(1+A_{p_{F(k)}}^{L}\right)^{w_k}+\prod_{k=1}^{m}\left(1-A_{p_{F(k)}}^{L}\right)^{w_k}},\frac{\prod_{k=1}^{m}\left(1+A_{p_{F(k)}}^{U}\right)^{w_k}-\prod_{k=1}^{m}\left(1-A_{p_{F(k)}}^{U}\right)^{w_k}}{\prod_{k=1}^{m}\left(1+A_{p_{F(k)}}^{U}\right)^{w_k}+\prod_{k=1}^{m}\left(1-A_{p_{F(k)}}^{U}\right)^{w_k}}\right]\right\},\\
\left\{\frac{2\prod_{k=1}^{m}\left(\lambda_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-\lambda_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{m}\left(\lambda_{p_{T(k)}}\right)^{w_k}}\right\},\left\{\frac{2\prod_{k=1}^{m}\left(\lambda_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{m}\left(2-\lambda_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{m}\left(\lambda_{p_{I(k)}}\right)^{w_k}}\right\},\\
\left\{\frac{\prod_{k=1}^{m}\left(1+\lambda_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{m}\left(1-\lambda_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{m}\left(1+\lambda_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{m}\left(1-\lambda_{p_{F(k)}}\right)^{w_k}}\right\}
\end{array}\right\rangle$$

Proof: we use induction.

$$\alpha^{E^{w_1}}=\left\langle\begin{array}{c}
\left\{\left[\frac{2\left(A_{j_T}^{L}\right)^{w_1}}{\left(2-A_{j_T}^{L}\right)^{w_1}+\left(A_{j_T}^{L}\right)^{w_1}},\frac{2\left(A_{j_T}^{U}\right)^{w_1}}{\left(2-A_{j_T}^{U}\right)^{w_1}+\left(A_{j_T}^{U}\right)^{w_1}}\right]\right\},\left\{\left[\frac{2\left(A_{j_I}^{L}\right)^{w_1}}{\left(2-A_{j_I}^{L}\right)^{w_1}+\left(A_{j_I}^{L}\right)^{w_1}},\frac{2\left(A_{j_I}^{U}\right)^{w_1}}{\left(2-A_{j_I}^{U}\right)^{w_1}+\left(A_{j_I}^{U}\right)^{w_1}}\right]\right\}\\
\left\{\left[\frac{\left(1+A_{j_F}^{L}\right)^{w_1}-\left(1-A_{j_T}^{L}\right)^{w_1}}{\left(1+A_{j_F}^{L}\right)^{w_1}+\left(1-A_{j_F}^{L}\right)^{w_1}},\frac{\left(1+A_{j_F}^{U}\right)^{w_1}-\left(1-A_{j_F}^{U}\right)^{w_1}}{\left(1+A_{j_F}^{U}\right)^{w_1}+\left(1-A_{j_F}^{U}\right)^{w_1}}\right]\right\},\left\{\frac{2\left(\lambda_{j_T}\right)^{w_1}}{\left(2-\lambda_{j_T}\right)^{w_1}+\left(\lambda_{j_T}\right)^{w_1}}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)^{w_1}}{\left(2-\lambda_{j_I}\right)^{w_1}+\left(\lambda_{j_I}\right)^{w_1}}\right\},\\
\left\{\frac{\left(1+\lambda_{j_F}\right)^{w_1}-\left(1-\lambda_{j_F}\right)^{w_1}}{\left(1+\lambda_{j_F}\right)^{w_1}+\left(1-\lambda_{j_F}\right)^{w_1}}\right\}
\end{array}\right\rangle$$

and $$\alpha^{E^{w_2}}=\left\langle\begin{array}{c}\left\{\left[\frac{2\left(A^L_{j_T}\right)^{w_2}}{\left(2-A^L_{j_T}\right)^{w_2}+\left(A^L_{j_T}\right)^{w_2}},\frac{2\left(A^U_{j_T}\right)^{w_2}}{\left(2-A^U_{j_T}\right)^{w_2}+\left(A^U_{j_T}\right)^{w_2}}\right]\right\},\left\{\left[\frac{2\left(A^L_{j_I}\right)^{w_2}}{\left(2-A^L_{j_I}\right)^{w_2}+\left(A^L_{j_I}\right)^{w_2}},\frac{2\left(A^U_{j_I}\right)^{w_2}}{\left(2-A^U_{j_I}\right)^{w_2}+\left(A^U_{j_I}\right)^{w_2}}\right]\right\} \\ \left\{\left[\frac{\left(1+A^L_{j_F}\right)^{w_2}-\left(1-A^L_{j_T}\right)^{w_2}}{\left(1+A^L_{j_F}\right)^{w_2}+\left(1-A^L_{j_F}\right)^{w_2}},\frac{\left(1+A^U_{j_F}\right)^{w_2}-\left(1-A^U_{j_F}\right)^{w_2}}{\left(1+A^U_{j_F}\right)^{w_2}+\left(1-A^U_{j_F}\right)^{w_1}}\right]\right\},\left\{\frac{2\left(\lambda_{j_T}\right)^{w_2}}{\left(2-\lambda_{j_T}\right)^{w_2}+\left(\lambda_{j_T}\right)^{w_2}}\right\},\left\{\frac{2\left(\lambda_{j_I}\right)^{w_2}}{\left(2-\lambda_{j_I}\right)^{w_2}+\left(\lambda_{j_I}\right)^{w_2}}\right\}, \\ \left\{\frac{\left(1+\lambda_{j_F}\right)^{w_2}-\left(1-\lambda_{j_F}\right)^{w_2}}{\left(1+\lambda_{j_F}\right)^{w_2}+\left(1-\lambda_{j_F}\right)^{w_2}}\right\}\end{array}\right\rangle$$

$$NCHFEWG(\alpha_1,\alpha_2)=\bigotimes_{k=1}^{2}\alpha_k^{E^{w_k}}$$

$$=\left\langle\begin{array}{c}\left\{\left[\frac{2\prod_{k=1}^{2}\left(A^L_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-A^L_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{2}\left(A^U_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-A^U_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(A^U_{p_{T(k)}}\right)^{w_k}}\right]\right\},\left\{\left[\frac{2\prod_{k=1}^{2}\left(A^L_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-A^L_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{2}\left(A^U_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-A^U_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(A^U_{p_{I(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\frac{\prod_{k=1}^{2}\left(1+A^L_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{2}\left(1-A^L_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(1+A^L_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(1-A^L_{p_{F(k)}}\right)^{w_k}},\frac{\prod_{k=1}^{2}\left(1+A^U_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{2}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{2}\left(1+A^U_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{2}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\frac{2\prod_{k=1}^{2}\left(\lambda_{pT(k)}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-\lambda_{pT(k)}\right)^{w_k}+\prod_{k=1}^{2}\left(\lambda_{pT(k)}\right)^{w_k}}\right\},\left\{\frac{2\prod_{k=1}^{2}\left(\lambda_{pI(k)}\right)^{w_k}}{\prod_{k=1}^{2}\left(2-\lambda_{pI(k)}\right)^{w_k}+\prod_{k=1}^{2}\left(\lambda_{pI(k)}\right)^{w_k}}\right\},\left\{\frac{\prod_{k=1}^{2}\left(1+\lambda_{pF(k)}\right)^{w_k}-\prod_{k=1}^{2}\left(1-\lambda_{pF(k)}\right)^{w_k}}{\prod_{k=1}^{2}\left(1+\lambda_{pF(k)}\right)^{w_k}+\prod_{k=1}^{2}\left(1-\lambda_{pF(k)}\right)^{w_k}}\right\}\end{array}\right\rangle$$

For m=q we have

$$NCHFEWG\left(\alpha_1,\ldots,\alpha_q\right)=\left\langle\begin{array}{c}\left\{\left[\frac{2\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^L_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^U_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\frac{2\prod_{k=1}^{q}\left(A^L_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^L_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^U_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}}\right]\right\} \\ \left\{\left[\frac{\prod_{k=1}^{q}\left(1+A^L_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+A^L_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}},\frac{\prod_{k=1}^{q}\left(1+A^U_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+A^U_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\frac{2\prod_{k=1}^{q}\left(\lambda_{pT(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-\lambda_{pT(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pT(k)}\right)^{w_k}}\right\},\left\{\frac{2\prod_{k=1}^{q}\left(\lambda_{pI(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-\lambda_{pI(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pI(k)}\right)^{w_k}}\right\},\left\{\frac{\prod_{k=1}^{q}\left(1+\lambda_{pF(k)}\right)^{w_k}-\prod_{k=1}^{q}\left(1-\lambda_{pF(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+\lambda_{pF(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pF(k)}\right)^{w_k}}\right\}\end{array}\right\rangle$$

Using assumption, we have

$$NCHFEWG\left(\alpha_1,\ldots,\alpha_{q+1}\right)=\underset{k=1}{\overset{q}{\otimes_E}}\left(\alpha_k^E\right)^{w_k}\otimes_E\left(\alpha_{q+1}^E\right)^{w_{q+1}}$$

$$=\left\langle\begin{array}{c}
\left\{\left[\dfrac{2\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^L_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}},\dfrac{2\prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^U_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^U_{p_{T(k)}}\right)^{w_k}}\right]\right\},\\
\left\{\left[\dfrac{2\prod_{k=1}^{q}\left(A^L_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^L_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^L_{p_{T(k)}}\right)^{w_k}},\dfrac{2\prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-A^U_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(A^U_{p_{I(k)}}\right)^{w_k}}\right]\right\}\\
\left\{\left[\dfrac{\prod_{k=1}^{q}\left(1+A^L_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+A^L_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(1-A^L_{p_{F(k)}}\right)^{w_k}},\dfrac{\prod_{k=1}^{q}\left(1+A^U_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+A^U_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}\right]\right\},\\
\left\{\dfrac{2\prod_{k=1}^{q}\left(\lambda_{pT(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-\lambda_{pT(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pT(k)}\right)^{w_k}}\right\},\\
\left\{\dfrac{2\prod_{k=1}^{q}\left(\lambda_{pI(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(2-\lambda_{pI(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pI(k)}\right)^{w_k}}\right\},\\
\left\{\dfrac{\prod_{k=1}^{q}\left(1+\lambda_{pF(k)}\right)^{w_k}-\prod_{k=1}^{q}\left(1-\lambda_{pF(k)}\right)^{w_k}}{\prod_{k=1}^{q}\left(1+\lambda_{pF(k)}\right)^{w_k}+\prod_{k=1}^{q}\left(\lambda_{pF(k)}\right)^{w_k}}\right\}
\end{array}\right\rangle\otimes_E\left\langle\begin{array}{c}
\left\{\left[\dfrac{2(A^L_{p_T})^{w_{q+1}}}{(2-A^L_{p_T})^{w_{q+1}}+(A^L_{p_T})^{w_{q+1}}},\dfrac{2(A^U_{p_T})^{w_{q+1}}}{(2-A^U_{p_T})^{w_{q+1}}+(A^U_{p_T})^{w_{q+1}}}\right]\right\},\\
\left\{\left[\dfrac{2(A^L_{p_I})^{w_{q+1}}}{(2-A^L_{p_I})^{w_{q+1}}+(A^L_{p_I})^{w_{q+1}}},\dfrac{2(A^U_{p_I})^{w_{q+1}}}{(2-A^U_{p_I})^{w_{q+1}}+(A^U_{p_I})^{w_{q+1}}}\right]\right\},\\
\left\{\left[\dfrac{(1+A^L_{p_F})^{w_{q+1}}-(1-A^L_{p_F})^{w_{q+1}}}{(1+A^L_{p_F})^{w_{q+1}}+(1-A^L_{p_F})^{w_{q+1}}},\dfrac{(1+A^U_{p_F})^{w_{q+1}}-(1-A^U_{p_F})^{w_{q+1}}}{(1+A^U_{p_F})^{w_{q+1}}+(1-A^U_{p_F})^{w_{q+1}}}\right]\right\},\\
\left\{\dfrac{2(\lambda_{p_T})^{w_{q+1}}}{(2-\lambda_{p_T})^{w_{q+1}}+(\lambda_{p_T})^{w_{q+1}}}\right\},\\
\left\{\dfrac{2(\lambda_{p_I})^{w_{q+1}}}{(2-\lambda_{p_I})^{w_{q+1}}+(\lambda_{p_I})^{w_{q+1}}}\right\},\\
\left\{\dfrac{(1+\lambda_{p_F})^{w_{q+1}}-(1-\lambda_{p_F})^{w_{q+1}}}{(1+\lambda_{p_F})^{w_{q+1}}+(1-\lambda_{p_F})^{w_{q+1}}}\right\}
\end{array}\right\rangle$$

$$=\left\langle \begin{array}{c} \left\{\left[\frac{2\prod_{k=1}^{q+1}\left(A^L_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-A^L_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{q+1}\left(A^U_{p_{T(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-A^U_{p_{T(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(A^U_{p_{T(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\frac{2\prod_{k=1}^{q+1}\left(A^L_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-A^L_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(A^L_{p_{T(k)}}\right)^{w_k}},\frac{2\prod_{k=1}^{q+1}\left(A^U_{p_{I(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-A^U_{p_{I(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(A^U_{p_{I(k)}}\right)^{w_k}}\right]\right\} \\ \left\{\left[\frac{\prod_{k=1}^{q+1}\left(1+A^L_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q+1}\left(1-A^L_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(1+A^L_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(1-A^L_{p_{F(k)}}\right)^{w_k}},\frac{\prod_{k=1}^{q+1}\left(1+A^U_{p_{F(k)}}\right)^{w_k}-\prod_{k=1}^{q+1}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(1+A^U_{p_{F(k)}}\right)^{w_k}+\prod_{k=1}^{q+1}\left(1-A^U_{p_{F(k)}}\right)^{w_k}}\right]\right\}, \\ \left\{\frac{2\prod_{k=1}^{q+1}\left(\lambda_{pT(k)}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-\lambda_{pT(k)}\right)^{w_k}+\prod_{k=1}^{q+1}\left(\lambda_{pT(k)}\right)^{w_k}}\right\}, \\ \left\{\frac{2\prod_{k=1}^{q+1}\left(\lambda_{pI(k)}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(2-\lambda_{pI(k)}\right)^{w_k}+\prod_{k=1}^{q+1}\left(\lambda_{pI(k)}\right)^{w_k}}\right\}, \\ \left\{\frac{\prod_{k=1}^{q+1}\left(1+\lambda_{pF(k)}\right)^{w_k}-\prod_{k=1}^{q+1}\left(1-\lambda_{pF(k)}\right)^{w_k}}{\prod_{k=1}^{q+1}\left(1+\lambda_{pF(k)}\right)^{w_k}+\prod_{k=1}^{q+1}\left(\lambda_{pF(k)}\right)^{w_k}}\right\} \end{array}\right\rangle$$

Definition4.8: Neutrosophic cubic hesitant fuzzy hybrid geometric operator (NCHFHG) is a mapping defined as $NCHFHG(\alpha_1,...,\alpha_m)=\bigotimes_{j=1}^{m}\left(\alpha_{\sigma(j)}\right)^{w_j}$, where $\alpha_{\sigma(j)}=\left(\alpha_j\right)^{mw_j}$ is the *jth* largest value, *m* is the balancing coefficient and $w=(w_1,\dots,w_m)^t$ is the weighting vector.

Theorem 4.9: Let $\left\{\alpha_{(k)}=\left\langle A_{(k)},\lambda_{(k)}\right\rangle\right\}$ the set of neutrosophic cubic hesitant fuzzy values with corresponding weight vector $w=(w_1,\dots)^t$, then

$$NCHFHG(\alpha_1,...,\alpha_m)=\left\langle \begin{array}{c} \left\{\left[\bigotimes_{k=1}^{m}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k},\bigotimes_{k=1}^{m}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\left[\bigotimes_{k=1}^{m}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k},\bigotimes_{k=1}^{m}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\bigotimes_{k=1}^{m}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k},1-\bigotimes_{k=1}^{m}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\bigotimes_{k=1}^{m}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{\bigotimes_{k=1}^{m}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\},\left\{1-\bigotimes_{k=1}^{m}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right\} \end{array}\right\rangle$$

Furthermore $NCHFHG(\alpha_1,\alpha_2,...,\alpha_m)$ is also a neutrosophic cubic hesitant fuzzy value.

Proof: Using induction for m=2

$$NCHFHG(\alpha_1,\alpha_2)=\bigotimes_{k=1}^{2}\left(\alpha_{\sigma(k)}\right)^{w_k}$$

$$= \left\langle \begin{array}{c} \left\{\left[\left(A^{L}_{j_{T_{\sigma(1)}}}\right)^{w_1},\left(A^{U}_{j_{T_{\sigma(1)}}}\right)^{w_1}\right]\right\},\left\{\left[\left(A^{L}_{j_{I_{\sigma(1)}}}\right)^{w_1},\left(A^{U}_{j_{I_{\sigma(1)}}}\right)^{w_1}\right]\right\}, \\ \left\{\left[1-\left(1-A^{L}_{j_{F_{\sigma(1)}}}\right)^{w_1},1-\left(1-A^{U}_{j_{F_{\sigma(1)}}}\right)^{w_1}\right]\right\},\left\{\left(\lambda_{j_{T_{\sigma(1)}}}\right)^{w_1}\right\}, \\ \left\{\left(\lambda_{j_{I_{\sigma(1)}}}\right)^{w_1}\right\},\left\{1-\left(1-\lambda_{j_{F_{\sigma(1)}}}\right)^{w_1}\right\} \end{array} \right\rangle \otimes \left\langle \begin{array}{c} \left\{\left[\left(A^{L}_{j_{T_{\sigma(2)}}}\right)^{w_2},\left(A^{U}_{j_{T_{\sigma(2)}}}\right)^{w_2}\right]\right\},\left\{\left[\left(A^{L}_{j_{I_{\sigma(2)}}}\right)^{w_2},\left(A^{U}_{j_{I_{\sigma(2)}}}\right)^{w_2}\right]\right\}, \\ \left\{\left[1-\left(1-A^{L}_{j_{F_{\sigma(2)}}}\right)^{w_2},1-\left(1-A^{U}_{j_{F_{\sigma(2)}}}\right)^{w_2}\right]\right\},\left\{\left(\lambda_{j_{T_{\sigma(2)}}}\right)^{w_2}\right\}, \\ \left\{\left(\lambda_{j_{I_{\sigma(2)}}}\right)^{w_2}\right\},\left\{1-\left(1-\lambda_{j_{F_{\sigma(2)}}}\right)^{w_2}\right\} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left\{\left[\underset{k=1}{\overset{2}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{2}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\left[\underset{k=1}{\overset{2}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{2}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\underset{k=1}{\overset{2}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k},1-\underset{k=1}{\overset{2}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\underset{k=1}{\overset{2}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{\underset{k=1}{\overset{2}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\},\left\{1-\underset{k=1}{\overset{2}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right\} \end{array} \right\rangle$$

For $m=q$ we have

$$NCHFHG(\alpha_1,...,\alpha_q) = \left\langle \begin{array}{c} \left\{\left[\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\left[\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k},1-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\},\left\{1-\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right\} \end{array} \right\rangle$$

we prove for $m = q + 1$

$$\underset{k=1}{\overset{q}{\otimes}}\left(\alpha_{\sigma(k)}\right)^{w_k}\otimes\left(\alpha_{\sigma(q+1)}\right)^{w_{q+1}} = \left\langle \begin{array}{c} \left\{\left[\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\left[\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k},\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k},1-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right]\right\},\left\{\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\},\left\{1-\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right\} \end{array} \right\rangle \otimes \left\langle \begin{array}{c} \left\{\left[\left(A^{L}_{j_{T_{\sigma(q+1)}}}\right)^{w_{q+1}},\left(A^{U}_{j_{T_{\sigma(q+1)}}}\right)^{w_{q+1}}\right]\right\},\left\{\left[\left(A^{L}_{j_{I_{\sigma(q+1)}}}\right)^{w_{q+1}},\left(A^{U}_{j_{I_{\sigma(q+1)}}}\right)^{w_{q+1}}\right]\right\}, \\ \left\{\left[1-\left(1-A^{L}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}},1-\left(1-A^{U}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}\right]\right\},\left\{\left(\lambda_{j_{T_{\sigma(q+1)}}}\right)^{w_{q+1}}\right\}, \\ \left\{\left(\lambda_{j_{I_{\sigma(q+1)}}}\right)^{w_{q+1}}\right\},\left\{1-\left(1-\lambda_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}\right\} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left\{\left[\left(\underset{k=1}{\overset{q+1}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}, \underset{k=1}{\overset{q+1}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)\right)\right]\right\}, \left\{\left[\underset{k=1}{\overset{q+1}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right), \underset{k=1}{\overset{q+1}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[\begin{array}{l} \left[2-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}-1+\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\left(1-A^{L}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}-\left(\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right)\left(1-A^{L}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}, \\ 2-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}-1+\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\left(1-A^{U}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}-\left(\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right)\left(1-A^{U}_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}\right] \end{array}\right]\right\}, \\ \left\{\underset{k=1}{\overset{q+1}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \left\{\underset{k=1}{\overset{q+1}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{2-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-1+\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\left(1-\lambda_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}-\left(\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right)\left(1-\lambda_{j_{F_{\sigma(q+1)}}}\right)^{w_{q+1}}\right\} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left\{\left[\underset{k=1}{\overset{q+1}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}, \underset{k=1}{\overset{q+1}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \left\{\left[\underset{k=1}{\overset{q+1}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}, \underset{k=1}{\overset{q+1}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \\ \left\{\left[1-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}, 1-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}\right]\right\}, \left\{\underset{k=1}{\overset{q+1}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}\right\}, \\ \left\{\underset{k=1}{\overset{q+1}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}\right\}, \left\{1-\underset{k=1}{\overset{q+1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}\right\} \end{array} \right\rangle$$

Theorem 4.10: With $w_j = \frac{1}{m}$, *NCHFHG* becomes *NCHFWG*.

Proof: $NCHFHG(\alpha_1,...,\alpha_m) = \left(\alpha_{\sigma(1)}\right)^{w_1} \otimes ... \otimes \left(\alpha_{\sigma(m)}\right)^{w_m}$

$$= \left(\alpha_1\right)^{w_1} \otimes ... \otimes \left(\alpha_m\right)^{w_m}$$

$$= NCHFWG(\alpha_1,...,\alpha_m)$$

Definition 4.11: Neutrosophic cubic hesitant fuzzy Einstein hybrid geometric operator(NCHFEHG) is a mapping defined as $NCHFEHG(\alpha_1,...,\alpha_m) = \underset{j=1}{\overset{m}{\otimes_E}} \left(\alpha_{\sigma(j)}\right)^{w_j}$ where $\alpha_{\sigma(j)} = \left(\alpha_j\right)^{mw_j}$ is the *jth* largest value, *m* is the balancing coefficient and $w = (w_1, ..., w_m)^t$ is the weighting vector.

Theorem 4.12: Let $\left\{\alpha_{(k)} = \left\langle A_{(k)}, \lambda_{(k)} \right\rangle\right\}$ the set of neutrosophic cubic hesitant fuzzy values with corresponding weight vector $w = (w_1, ...)^t$ then

$$NCHFEHG(\alpha_1, \alpha_2, ..., \alpha_m) == \left\langle \begin{array}{c} \left\{\left[\dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(2-A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}}, \dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(2-A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(2-A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}}, \dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(2-A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\dfrac{\overset{m}{\underset{k=1}{\otimes}}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{m}{\underset{k=1}{\otimes}}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}}, \dfrac{\overset{m}{\underset{k=1}{\otimes}}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{m}{\underset{k=1}{\otimes}}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\ \left\{\dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(\left(1+\lambda_{j_{T_{\sigma(k)}}}\right)\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(\left(2-\lambda_{j_{T_{\sigma(k)}}}\right)\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(\left(1+\lambda_{j_{T_{\sigma(k)}}}\right)\right)^{w_k}}\right\}, \left\{\dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(\left(1+\lambda_{j_{I_{\sigma(k)}}}\right)\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(\left(2-\lambda_{j_{I_{\sigma(k)}}}\right)\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(\left(1+\lambda_{j_{I_{\sigma(k)}}}\right)\right)^{w_k}}\right\}, \\ \left\{\dfrac{\overset{m}{\underset{k=1}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{m}{\underset{k=1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{m}{\underset{k=1}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{m}{\underset{k=1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right\} \end{array} \right\rangle$$

Proof: Using induction and from Theorem 3.12

$$\alpha^{E^{w_1}}=\left\langle\begin{array}{c}\left\{\left[\frac{2(A^L_{j_T})^{w_1}}{(2-A^L_{j_T})^{w_1}+(A^L_{j_T})^{w_1}},\frac{2(A^U_{j_T})^{w_1}}{(2-A^U_{j_T})^{w_1}+(A^U_{j_T})^{w_1}}\right]\right\},\left\{\left[\frac{2(A^L_{j_I})^{w_1}}{(2-A^L_{j_I})^{w_1}+(A^L_{j_I})^{w_1}},\frac{2(A^U_{j_I})^{w_1}}{(2-A^U_{j_I})^{w_1}+(A^U_{j_I})^{w_1}}\right]\right\},\\ \left\{\left[\frac{(1+A^L_{j_F})^{w_1}-(1-A^L_{j_F})^{w_1}}{(1+A^L_{j_F})^{w_1}+(1-A^L_{j_F})^{w_1}},\frac{(1+A^U_{j_F})^{w_1}-(1-A^U_{j_F})^{w_1}}{(1+A^U_{j_F})^{w_1}+(1-A^U_{j_F})^{w_1}}\right]\right\},\left\{\frac{2(\lambda_{j_T})^{w_1}}{(2-\lambda_{j_T})^{w_1}+(\lambda_{j_T})^{w_1}}\right\},\\ \left\{\frac{2(\lambda_{j_I})^{w_1}}{(2-\lambda_{j_I})^{w_1}+(\lambda_{j_I})^{w_1}}\right\},\left\{\frac{(1+\lambda_{j_F})^{w_1}-(1-\lambda_{j_F})^{w_1}}{(1+\lambda_{j_F})^{w_1}+(1-\lambda_{j_F})^{w_1}}\right\}\end{array}\right\rangle$$

and $$\alpha^{E^{w_2}}=\left\langle\begin{array}{c}\left\{\left[\frac{2(A^L_{j_T})^{w_2}}{(2-A^L_{j_T})^{w_2}+(A^L_{j_T})^{w_2}},\frac{2(A^U_{j_T})^{w_2}}{(2-A^U_{j_T})^{w_2}+(A^U_{j_T})^{w_2}}\right]\right\},\left\{\left[\frac{2(A^L_{j_I})^{w_2}}{(2-A^L_{j_I})^{w_2}+(A^L_{j_I})^{w_2}},\frac{2(A^U_{j_I})^{w_2}}{(2-A^U_{j_I})^{w_2}+(A^U_{j_I})^{w_2}}\right]\right\},\\ \left\{\left[\frac{(1+A^L_{j_F})^{w_2}-(1-A^L_{j_F})^{w_2}}{(1+A^L_{j_F})^{w_2}+(1-A^L_{j_F})^{w_2}},\frac{(1+A^U_{j_F})^{w_2}-(1-A^U_{j_F})^{w_2}}{(1+A^U_{j_F})^{w_2}+(1-A^U_{j_F})^{w_2}}\right]\right\},\left\{\frac{2(\lambda_{j_T})^{w_2}}{(2-\lambda_{j_T})^{w_2}+(\lambda_{j_T})^{w_2}}\right\},\\ \left\{\frac{2(\lambda_{j_I})^{w_2}}{(2-\lambda_{j_I})^{w_2}+(\lambda_{j_I})^{w_2}}\right\},\left\{\frac{(1+\lambda_{j_F})^{w_2}-(1-\lambda_{j_F})^{w_2}}{(1+\lambda_{j_F})^{w_2}+(1-\lambda_{j_F})^{w_2}}\right\}\end{array}\right\rangle$$

$$NCHFEWG(\alpha_1,\alpha_2)=\bigotimes_{k=1}^{2}\alpha_k^{E^{w_k}}$$

$$=\left\langle\begin{array}{c}\left\{\left[\frac{2\bigotimes_{k=1}^{2}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}},\frac{2\bigotimes_{k=1}^{2}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right]\right\},\\ \left\{\left[\frac{2\bigotimes_{k=1}^{2}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}},\frac{2\bigotimes_{k=1}^{m}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right]\right\},\\ \left\{\left[\frac{\bigotimes_{k=1}^{2}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{2}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}},\frac{\bigotimes_{k=1}^{2}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{2}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right]\right\},\\ \left\{\frac{2\bigotimes_{k=1}^{2}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right\},\left\{\frac{2\bigotimes_{k=1}^{2}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(2-\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right\},\\ \left\{\frac{\bigotimes_{k=1}^{2}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{2}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{2}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{2}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right\}\end{array}\right\rangle$$

For $m = q$ we have

$$NCHFEHG(\alpha_1,\alpha_2,...,\alpha_q) = \left\langle \begin{array}{c} \left\{\left[\dfrac{2\overset{q}{\underset{k=1}{\otimes}}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(A^L_{j_{T_{\sigma(k)}}}\right)^{w_k}}, \dfrac{2\overset{q}{\underset{k=1}{\otimes}}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(A^U_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right]\right\} \\ ,\left\{\left[\dfrac{2\overset{q}{\underset{k=1}{\otimes}}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(A^L_{j_{I_{\sigma(k)}}}\right)^{w_k}}, \dfrac{2\overset{m}{\underset{k=1}{\otimes}}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(A^U_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\ \left\{\left[\dfrac{\overset{q}{\underset{k=1}{\otimes}}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{q}{\underset{k=1}{\otimes}}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(1+A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(1-A^L_{j_{F_{\sigma(k)}}}\right)^{w_k}}, \dfrac{\overset{q}{\underset{k=1}{\otimes}}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{q}{\underset{k=1}{\otimes}}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(1+A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(1-A^U_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\ \left\{\dfrac{2\overset{q}{\underset{k=1}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right\}, \left\{\dfrac{2\overset{q}{\underset{k=1}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(2-\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right\}, \\ \left\{\dfrac{\overset{q}{\underset{k=1}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-\overset{q}{\underset{k=1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\overset{q}{\underset{k=1}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\overset{q}{\underset{k=1}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right\} \end{array} \right\rangle$$

Using assumption, we have

$$
\left(\underset{k=1}{\overset{q}{\otimes}}{}_{E}\left(\alpha^{E}_{\sigma(k)}\right)^{w_k}\right)\otimes_{E}\left(\alpha^{E}_{q+1}\right)^{w_{q+1}}=
\left\langle
\begin{array}{c}
\left\{\left[\dfrac{2\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}},
\dfrac{2\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\
\left\{\left[\dfrac{2\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}},
\dfrac{2\underset{k=1}{\overset{m}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\
\left\{\left[\dfrac{\underset{k=1}{\overset{q}{\otimes}}\left(1+A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(1+A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}},
\dfrac{\underset{k=1}{\overset{q}{\otimes}}\left(1+A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}-\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(1+A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\
\left\{\dfrac{2\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right\}, \\
\left\{\dfrac{2\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(2-\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right\}, \\
\left\{\dfrac{\underset{k=1}{\overset{q}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\underset{k=1}{\overset{q}{\otimes}}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\underset{k=1}{\overset{q}{\otimes}}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right\}
\end{array}
\right\rangle
\otimes_{E}
\left\langle
\begin{array}{c}
\left\{\left[\dfrac{2(A^{L}_{j_T})^{w_{q+1}}}{(2-A^{L}_{j_T})^{w_{q+1}}+(A^{L}_{j_T})^{w_{q+1}}},
\dfrac{2(A^{U}_{j_T})^{w_{q+1}}}{(2-A^{U}_{j_T})^{w_{q+1}}+(A^{U}_{j_T})^{w_{q+1}}}\right]\right\}, \\
\left\{\left[\dfrac{2(A^{L}_{j_I})^{w_{q+1}}}{(2-A^{L}_{j_I})^{w_{q+1}}+(A^{L}_{j_I})^{w_{q+1}}},
\dfrac{2(A^{U}_{j_I})^{w_{q+1}}}{(2-A^{U}_{j_I})^{w_{q+1}}+(A^{U}_{j_I})^{w_{q+1}}}\right]\right\}, \\
\left\{\left[\dfrac{(1+A^{L}_{j_F})^{w_{q+1}}-(1-A^{L}_{j_F})^{w_{q+1}}}{(1+A^{L}_{j_F})^{w_{q+1}}+(1-A^{L}_{j_F})^{w_{q+1}}},
\dfrac{(1+A^{U}_{j_F})^{w_{q+1}}-(1-A^{U}_{j_F})^{w_{q+1}}}{(1+A^{U}_{j_F})^{w_{q+1}}+(1-A^{U}_{j_F})^{w_{q+1}}}\right]\right\}, \\
\left\{\dfrac{2(\lambda_{j_T})^{w_{q+1}}}{(2-\lambda_{j_T})^{w_{q+1}}+(\lambda_{j_T})^{w_{q+1}}}\right\}, \\
\left\{\dfrac{2(\lambda_{j_I})^{w_{q+1}}}{(2-\lambda_{j_I})^{w_{q+1}}+(\lambda_{j_I})^{w_{q+1}}}\right\}, \\
\left\{\dfrac{(1+\lambda_{j_F})^{w_{q+1}}-(1-\lambda_{j_F})^{w_{q+1}}}{(1+\lambda_{j_F})^{w_{q+1}}+(1-\lambda_{j_F})^{w_{q+1}}}\right\}
\end{array}
\right\rangle
$$

$$
= \left\langle
\begin{array}{c}
\left\{\left[\frac{2\bigotimes_{k=1}^{q+1}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(A^{L}_{j_{T_{\sigma(k)}}}\right)^{w_k}}, \frac{2\bigotimes_{k=1}^{q+1}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(A^{U}_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \left\{\left[\frac{2\bigotimes_{k=1}^{q+1}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(A^{L}_{j_{I_{\sigma(k)}}}\right)^{w_k}}, \frac{2\bigotimes_{k=1}^{q+1}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(A^{U}_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \\
\left\{\left[\frac{\bigotimes_{k=1}^{q+1}\left(1+A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{q+1}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(1+A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(1-A^{L}_{j_{F_{\sigma(k)}}}\right)^{w_k}}, \frac{\bigotimes_{k=1}^{q}\left(1+A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{q}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(1+A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(1-A^{U}_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right]\right\}, \left\{\frac{2\bigotimes_{k=1}^{q+1}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(\lambda_{j_{T_{\sigma(k)}}}\right)^{w_k}}\right\}, \\
\left\{\frac{2\bigotimes_{k=1}^{q+1}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(2-\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(\lambda_{j_{I_{\sigma(k)}}}\right)^{w_k}}\right\}, \left\{\frac{\bigotimes_{k=1}^{q+1}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}-\bigotimes_{k=1}^{q+1}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}{\bigotimes_{k=1}^{q+1}\left(1+\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}+\bigotimes_{k=1}^{q+1}\left(1-\lambda_{j_{F_{\sigma(k)}}}\right)^{w_k}}\right\}
\end{array}
\right\rangle
$$

Theorem 4.13: With $w_j = \frac{1}{m}$, *NCHFEG* becomes *NCHFEWG*.

Proof: $NCHFEHG(\alpha_1,...,\alpha_m) = \left(\alpha_{\sigma(1)}\right)^{w_1} \otimes_E ... \otimes \left(\alpha_{\sigma(m)}\right)^{w_m}$

$$= \left(\alpha_1\right)^{w_1} \otimes_E ... \otimes_E \left(\alpha_m\right)^{w_m}$$

$$= \left(\alpha_1\right)^{\frac{1}{m}} \otimes_E ... \otimes_E \left(\alpha_m\right)^{\frac{1}{m}}$$

$$= NCHFEWG(\alpha_1,...,\alpha_m)$$

5. An Application of NCHFG Aggregation Operator to MEDM Problems

This section concern with constructing algorithms using the NCHFWG for MEDM problems.

5.1. Algorithm

Step 1: Allocation of expert corresponding to their weight, identification of alternative and attributes.

Let $\{F_1, F_2, ..., F_r\}$ be the set of *r* alternatives, $\{K_1, K_2, ..., K_s\}$ be *s* attributes with corresponding weight vector $[w_1, w_2, ..., w_s]^T$ such that $w_j \in [0,1]$, $\sum w_j = 1$ $\{M_1, M_2, ..., M_p\}$ be decision experts with corresponding weight vector $\left[w_1, w_2, ..., w_p\right]^T$ such that $w_j \in [0,1]$, $\sum w_j = 1$. we construct decision matrices $D^{(k)} = (d_{ij})_{r\times s}$, with entries as neutrosophic cubic hesitant fuzzy values.

Step 2. Transformation of decision matrices to aggregated decision matrix.

A single matrix consisting of *s* attributes is constructed by aggregating all decision matrices using NCHFWG operators with corresponding weight vector of decision makers.

Step 3. Transformation of aggregated matrix to decision vector.

An $r \times 1$ vector is obtained by aggregating the decision matrix using NCHFWG operators.

Step 4: Ranking alternatives.

The most desirable alternative with highest score by ranking them in descending order of scores.

Example 5.2: Using above algorithm, we have to choose the most desirable alternative among the alternatives (Electronics companies) $F_p (p = 1,2,3)$ on the basis of three attributes A_1 (price), A_2 (Electricity consumption), A_3 (design).

Step 1: Decision matrix for first expert

$$D^{(1)}=\begin{pmatrix} \left\langle \begin{array}{c}\{[0.1,0.5],[0.2,0.7]\},\\ \{[0.2,0.3],[0.1,0.6]\},\\ \{[0.1,0.5],[0.2,0.3]\},\\ \{0.4,0.6\},\{0.3,0.5\},\{0.3,0.4\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.3,0.4],[0.2,0.9]\},\\ \{[0.2,0.6],[0.3,0.6]\},\\ \{[0.3,0.4],[0,0.1]\},\\ \{0.5,0.7\},\{0.3,0.4\},\{0.4,0.5\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.3,0.7],[0.2,0.4]\},\\ \{[0.2,0.5],[0.1,0.6]\},\\ \{[0.2,0.4],[0,0.1]\},\\ \{0.5,0.6\},\{0.2,0.4\},\{0.2,0.3\}\end{array}\right\rangle \\ \left\langle \begin{array}{c}\{[0.2,0.5],[0.3,0.7]\},\\ \{[0.3,0.4],[0.1,0.5]\},\\ \{[0.1,0.3],[0,0.2]\},\\ \{0.7,0.9\},\{0.3,0.4\},\{0.1,0.2\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.1,0.3],[0.2,0.5]\},\\ \{[0.2,0.3],[0.1,0.6]\},\\ \{[0.1,0.4],[0,0.3]\},\\ \{0.4,0.5\},\{0.3,0.7\},\{0.3,0.5\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.1,0.4],[0.2,0.6]\},\\ \{[0.1,0.3],[0,0.2]\},\\ \{[0.2,0.4],[0,0.3]\},\\ \{0.4,0.5\},\{0.3,0.6\},\{0.1,0.3\}\end{array}\right\rangle \\ \left\langle \begin{array}{c}\{[0.4,0.5],[0.3,0.4]\},\\ \{[0.1,0.3],[0.2,0.5]\},\\ \{[0.1,0.4],[0.7,0.8]\},\\ \{0.4,0.5\},\{0.3,0.4\},\{0.2,0.4\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.3,0.5],[0.1,0.4]\},\\ \{[0.2,0.3],[0.2,0.6]\},\\ \{[0.1,0.5],[0.6,0.8]\},\\ \{0.5,0.6\},\{0.1,0.4\},\{0.2,0.3\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.2,0.5],[0.1,0.4]\},\\ \{[0.2,0.5],[0.2,0.6]\},\\ \{[0.1,0.4],[0.6,0.7]\},\\ \{0.4,0.6\},\{0.3,0.4\},\{0.4,0.5\}\end{array}\right\rangle \end{pmatrix}$$

Explanation of decision matrix entries:

In case of $d_{11}^{(1)}$, {[0.1,0.5],[0.2,0.7]} is interval hesitant degree of preference to attribute F_1 corresponding to attribute K_1, {[0.2,0.3],[0.1,0.6]} is interval hesitant degree of indeterminacy (preference/ non-preference) for attributeF_1 corresponding to attribute K_1, {[0.1,0.5],[0.2,0.3]} is interval hesitant degree of non-preference for attribute F_1 corresponding to attribute K_1, {0.4,0.6}} is hesitant degree of preference for attribute F_1 corresponding to attribute K_1, {0.3,0.5} is hesitant degree of indeterminacy (preference/ non-preference) for attribute F_1 corresponding to attribute K_1, {0.3,0.4} is hesitant degree of non-preference for attribute F_1 corresponding to attribute K_1, given by the first expert.

Decision Matrix for second expert

$$D^{(2)}=\begin{pmatrix} \left\langle \begin{array}{c}\{[0.1,0.3],[0.2,0.5]\},\\ \{[0.2,0.4],[0.1,0.5]\},\\ \{[0.1,0.4],[0,0.3]\},\\ \{0.4,0.8\},\{0.3,0.4\},\{0.3,0.4\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.2,0.4],[0.2,0.7]\},\\ \{[0.1,0.3],[0,0.2]\},\\ \{[0.3,0.5],[0,0.3]\},\\ \{0.7,0.8\},\{0.3,0.6\},\{0.2,0.3\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.2,0.5],[0.2,0.7]\},\\ \{[0,0.4],[0.1,0.3]\},\\ \{[0.3,0.5],[0,0.4]\},\\ \{0.6,0.7\},\{0.2,0.5\},\{0.4,0.5\}\end{array}\right\rangle \\ \left\langle \begin{array}{c}\{[0.1,0.5],[0.3,0.7]\},\\ \{[0.1,0.3],[0.1,0.2]\},\\ \{[0.2,0.4],[0,0.3]\},\\ \{0.6,0.7\},\{0.3,0.6\},\{0.4,0.5\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.3,0.6],[0.2,0.7]\},\\ \{[0.1,0.2],[0.2,0.5]\},\\ \{[0,0.3],[0.1,0.4]\},\\ \{0.7,0.8\},\{0.3,0.4\},\{0.2,0.3\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.4,0.5],[0.4,0.7]\},\\ \{[0.2,0.3],[0.1,0.6]\},\\ \{[0.3,0.4],[0,0.3]\},\\ \{0.5,0.6\},\{0.3,0.5\},\{0.3,0.4\}\end{array}\right\rangle \\ \left\langle \begin{array}{c}\{[0.2,0.5],[0.1,0.4]\},\\ \{[0.2,0.4],[0.2,0.5]\},\\ \{[0.1,0.3],[0.8,0.9]\},\\ \{0.7,0.8\},\{0.6,0.8\},\{0.3,0.4\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.2,0.3],[0.4,0.5]\},\\ \{[0.1,0.4],[0.2,0.5]\},\\ \{[0.7,0.8],[0.1,0.2]\},\\ \{0.7,0.9\},\{0.7,0.8\},\{0.2,0.5\}\end{array}\right\rangle & \left\langle \begin{array}{c}\{[0.5,0.7],[0.3,0.4]\},\\ \{[0.1,0.3],[0.2,0.5]\},\\ \{[0.1,0.4],[0.2,0.6]\},\\ \{0.6,0.8\},\{0.1,0.4\},\{0.2,0.3\}\end{array}\right\rangle \end{pmatrix}$$

Step 2

using weight vector (0.5,0.5) for decision makers, the aggregated matrix is

$$D=\begin{pmatrix}
\left\langle\begin{array}{c}\{[0.1,0.387298],[0.2,0.591608]\},\\ \{[0.2,0.34641],[0.1,0.547723]\},\\ \{[0.1,0.452277],[0.105573,0.3]\},\\ \{0.4,0.69282\},\\ \{0.3,0.447214\},\\ \{0.3,0.4\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.244949,0.4],[0.2,0.793725]\},\\ \{[0.141421,0.424264],[0,0.34641]\},\\ \{[0.3,0.452277],[0,0.206275]\},\\ \{0.591608,0.748331\},\\ \{0.3,0.489898\},\\ \{0.30718,0.408392\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.244949,0.591608],[0.2,0.52915]\},\\ \{[0,0.447214],[0.1,0.424264]\},\\ \{[0.251669,0.452277],[0,0.265153]\},\\ \{0.547723,0.648074\},\\ \{0.2,0.4\},\\ \{0.30718,0.408392\}\end{array}\right\rangle \\
\left\langle\begin{array}{c}\{[0.141421,0.5],[0.3,0.7]\},\\ \{[0.173205,0.34641],[0.1,0.316228]\},\\ \{[0.151472,0.351926],[0,0.251669]\},\\ \{0.648074,0.793725\},\\ \{0.3,0.489898\},\\ \{0.265153,0.367544\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.173205,0.424264],[0.2,0.591608]\},\\ \{[0.141421,0.244949],[0.141421,0.547723]\},\\ \{[0.051317,0.351926],[0.051317,0.351926]\},\\ \{0.52915,0.632456\},\\ \{0.3,0.52915\},\\ \{0.251669,0.408392\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.2,0.447214],[0.282843,0.648074]\},\\ \{[0.141421,0.3],[0,0.34641]\},\\ \{[0.251669,0.4],[0,0.3]\},\\ \{0.447214,0.547723\},\\ \{0.3,0.547723\},\\ \{0.206275,0.351926\}\end{array}\right\rangle \\
\left\langle\begin{array}{c}\{[0.282843,0.5],[0.173205,0.4]\},\\ \{[0.141421,0.34641],[0.2,0.5]\},\\ \{[0.1,0.351926],[0.755051,0.858579]\},\\ \{0.52915,0.632456\},\\ \{0.424264,0.565685\},\\ \{0.251669,0.4\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.244949,0.387298],[0.2,0.447214]\},\\ \{[0.141421,0.34641],[0.2,0.547723]\},\\ \{[0.480385,0.683772],[0.4,0.6]\},\\ \{0.591608,0.734847\},\\ \{0.264575,0.565685\},\\ \{0.2,0.408392\}\end{array}\right\rangle &
\left\langle\begin{array}{c}\{[0.316228,0.591608],[0.173205,0.4]\},\\ \{[0.141421,0.387298],[0.2,0.547723]\},\\ \{[0.1,0.4],[0.434315,0.65359]\},\\ \{0.489898,0.69282\},\\ \{0.173205,0.4\},\\ \{0.30718,0.408392\}\end{array}\right\rangle
\end{pmatrix}$$

Step 3

using weight vector (0.3,0.4,0.3) for attributes and NCHWG we have the following decision vector

$$d=\begin{pmatrix}
\left\langle\begin{array}{c}\{[0.18722,0.4455],[0.2,0.643505]\},\\ \{[0,0.405589],[0,0.422372]\},\\ \{[0.229912,0.452277],[0.032918,0.25311]\},\\ \{0.514043,0.700344\},\{0.26564,0.463822\},\{0.305034,0.405887\}\end{array}\right\rangle \\
\left\langle\begin{array}{c}\{[0.170172,0.452793],[0.250618,0.639485]\},\\ \{[0.15029,0.288831],[0,0.404857]\},\\ \{[0.14557,0.366739],[0.020852,0.30752]\},\\ \{0.534655,0.648457\},\{0.3,0.522434\},\{0.242474,0.379688\}\end{array}\right\rangle \\
\left\langle\begin{array}{c}\{[0.276117,0.47481],[0.183463,0.418256]\},\\ \{[0.141421,0.358201],[0.2,0.532946]\},\\ \{[0.277533,0.524738],[0.549437,0.71955]\},\\ \{0.540653,0.690198\},\{0.268459,0.509824\},\{0.248982,0.405887\}\end{array}\right\rangle
\end{pmatrix}$$

Step 4

Using Score function, we rank the alternatives as $S(F_1)=0.491743, S(F_2)=0.511797, S(F_3)=0.467604$. Hence most desirable alternative is F_2.

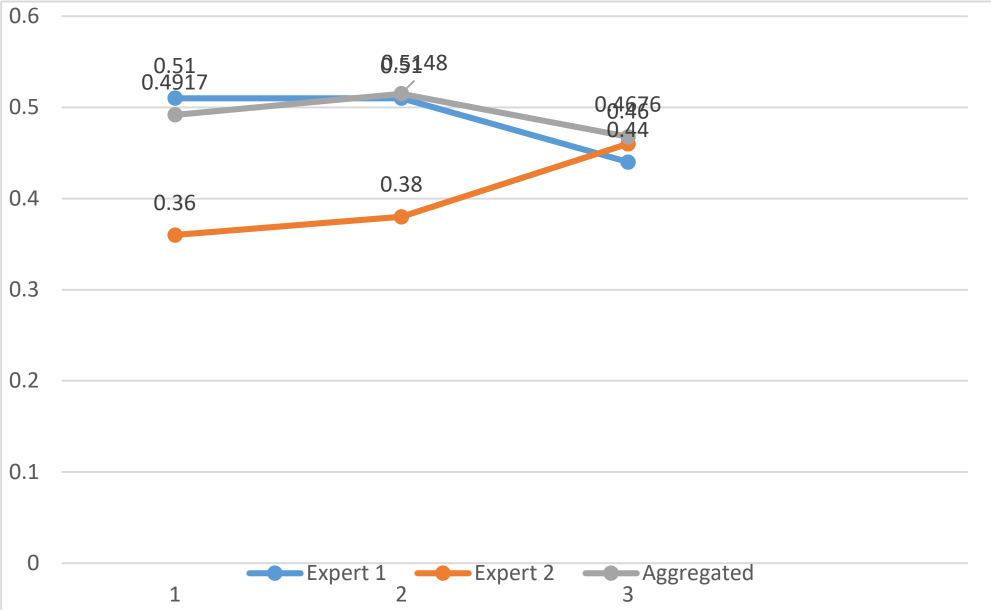

Figure 2: Ranking based on score function

Concluding Remarks

Decision making is one of the crucial problems in real life. For decision making different tools has been established. Torra's hesitant fuzzy set has been used in many practical problems due to flexibility of choosing membership grades. On the other side Jun's neutrosophic cubic set is capable of dealing truth, falsity and indeterminacy membership grades, but the element of hesitancy is missing in truth and falsity membership grades of neutrosophic cubic set. We have discussed the role of hesitancy in truth and falsity membership grades of neutrosophic cubic set. We define neutrosophic cubic hesitant fuzzy set and some basic operations like addition, multiplication, Einstein addition and multiplication in neutrosophic cubic hesitant fuzzy sets. Then we prove some elegant results. In section 4 geometric aggregation operators are defined. Using these aggregation operators an example is constructed.

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