Neutrosophic Sets and Systems, Vol. 2, 2013

NSS

Neutrosophic Lattices

Vasantha Kandasamy[1] and Florentin Smarandache[2]

[1] Department of Mathematics, Indian Institute of Technology, Madras, India. E-mail: vasanthakandasamy@gmail.com

[2] Mathematics and Science Department, University of Mexico, 70S, Gurley Ave. Gallup NM 87301, USA. E-mail: smarand@unm.edu

Abstract**.** In this paper authors for the first time define a new notion called neutrosophic lattices. We define few properties related with them. Three types of neutrosophic lattices are defined and the special properties about these new class of lattices are discussed and developed. This paper is organised into three sections. First section introduces the concept of partially ordered neutrosophic set and neutrosophic lattices. Section two introduces different types of neutrosophic lattices and the final section studies neutrosophic Boolean algebras. Conclusions and results are provided in section three.

Keywords: Neutrosophic set, neutrosophic lattices and neutrosophic partially ordered set.

1 Introduction to partially ordered neutrosophic set

Here we define the notion of a partial order on a neutrosophic set and the greatest element and the least element of it. Let N(P) denote a neutrosophic set which must contain I, 0, 1 and 1 + I; that is 0, 1, I and $1+I \in N(P)$. We call 0 to be the least element so $0 < 1$ and $0 < I$ is assumed for the working. Further by this N(P) becomes a partially ordered set. We define 0 of N(P) to be the least element and $I \cup 1 = 1 + I$ to be the greatest element of N(P).

Suppose $N(P) = \{0, 1, I, 1 + I, a_1, a_2, a_3, a_1I, a_2I, a_3I\}$ then N(P) with $0 < a_i$, $0 < a_iI$, $1 \le i \le 3$. Further $1 > a_i$; $I > a_iI$; $1 \le i \le 3$ $a_i \not< a_j$ if $i \ne j$ for $1 \le i, j \le 3$ and $Ia_i \not< Ia_j$; $i \ne j$ for $1 \le i, j \le 3$.

We will define the notion of Neutrosophic lattice.

DEFINITION 1.1: *Let N(P) be a partially ordered set with 0, 1, I, $1+I = 1 \cup I \in N(P)$.*

Define min and max on N(P) that is max {x, y} and min {x, y} $\in$ N(P). 0 is the least element and $1 \cup I = 1 + I$ is the greatest element of N(P). {N(P), min, max} is defined as the neutrosophic lattice.

We will illustrate this by some examples.

Example 1.1: Let $N(P) = \{0, 1, I, I \cup 1 = 1 + I, a, aI\}$ be a partially ordered set; N(P) is a neutrosophic lattice.

We know in case of usual lattices [1-4]. Hasse defined the notion of representing finite lattices by diagrams known as Hasse diagrams [1-4]. We in case of Neutrosophic lattices represent them by the diagram which will be known as the neutrosophic Hasse diagram.
The neutrosophic lattice given in example 1.1 will have the following Hasse neutrosophic diagram.

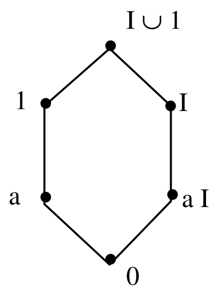

Figure 1.1

Example 1.2: Let $N(P) = \{0, 1, I, 1 \cup I, a_1, a_2, a_1I, a_2I\}$ be a neutrosophic lattice associated with the following Hasse neutrosophic diagram.

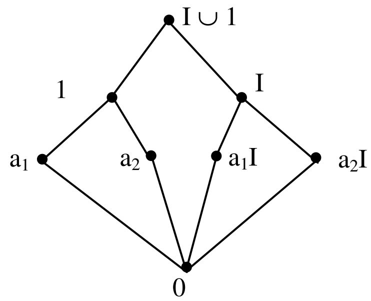

Figure 1.2

Example 1.3: Let $N(P) = \{0\ 1, I, 1 \cup I\}$ be a neutrosophic lattice given by the following neutrosophic Hasse diagram.

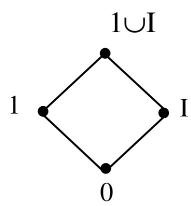

Figure 1.3

It is pertinent to observe that if N(P) is a neutrosophic lattice then 0, 1, I, $1 \cup I \in N(P)$ and so that N(P) given in example 1.3 is the smallest neutrosophic lattice.

Example 1.4: Let $N(P) = \{0, 1, I, 1 \cup I = 1 + I, a_1, a_2, a_1I, a_2I, a_1 < a_2\}$ be the neutrosophic lattice. The Hasse diagram of the neutrosophic lattice N(P) is as follows:

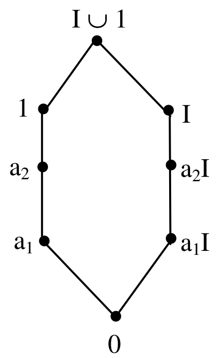

Figure 1.4

We can have neutrosophic lattices which are different.

Example 1.5: Let $N(P) = \{0, 1, I, a_1, a_2, a_3, a_4, a_1I, a_2I, a_3I, a_4I, 1 + I = I \cup 1\}$ be the neutrosophic lattice of finite order. (a_i is not comparable with a_j if $i \neq j$, $1 \leq i, j \leq 4$).

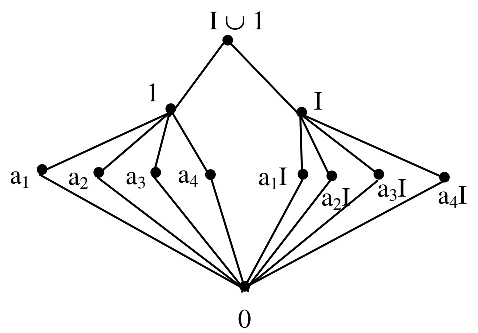

Figure 1.5

We see N(P) is a neutrosophic lattice with the above neutrosophic Hasse diagram.

In the following section we proceed onto discuss various types of neutrosophic lattices.

2. Types of Neutrosophic Lattices

The concept of modular lattice, distributive lattice, super modular lattice and chain lattices can be had from [1-4]. We just give examples of them and derive a few properties associated with them. In the first place we say a neutrosophic lattice to be a pure neutrosophic lattice if it has only neutrosophic coordinates or equivalently all the co ordinates (vertices) are neutrosophic barring 0.

In the example 1.5 we see the pure neutrosophic part of the neutrosophic lattice figure 2.1;

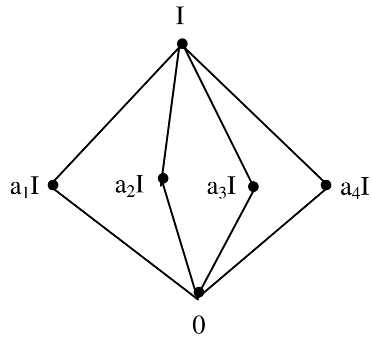

Figure 2.1

whose Hasse diagram is given is the pure neutrosophic sublattice lattice from figure 1.5. Likewise we can have the Hasse diagram of the usual lattice from example 1.5.

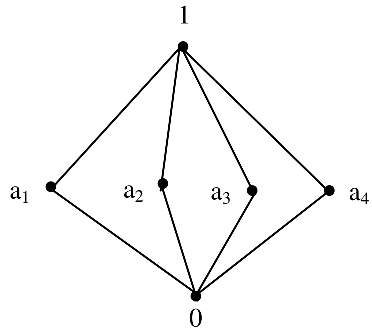

Figure 2.2

We see the diagrams are identical as diagrams one is pure neutrosophic where as the other is a usual lattice. As we have no method to compare a neutrosophic number and a non neutrosophic number, we get two sublattices identical in diagram of a neutrosophic lattice. For the modular identity, distributive identity and the super modular identity and their related properties refer [1-4].

The neutrosophic lattice given in example 1.5 has a sublattice which is a modular pure neutrosophic lattice and sublattice which is a usual modular lattice.

The neutrosophic lattice given in example 1.3 is a distributive lattice with four elements. However the neutrosophic lattice given in example 1.5 is not distributive as it contains sublattices whose homomorphic image is isomorphic to the neutrosophic modular lattice $N(M_4)$; where $N(M_4)$ is a lattice of the form

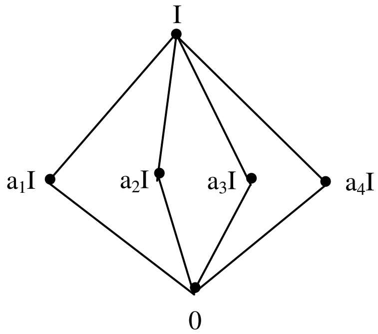

Figure 2.3

Likewise by $N(M_n)$ we have a pure neutrosophic lattice of the form given below in figure 2.4.

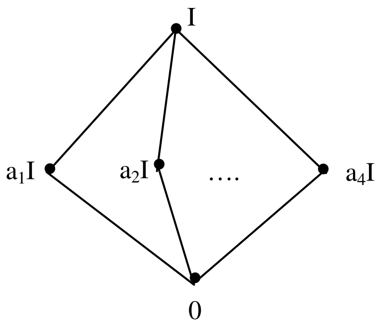

Figure 2.4

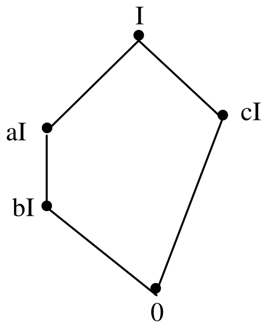

Figure 2.5

The neutrosophic pentagon lattice is given in figure 2.5 which is neither distributive nor modular.

The lattice $N(M_4)$ is not neutrosophic super modular we see the neutrosophic lattice in example 1.5 is not modular for it has sublattices whose homomorphic image is isomorphic to the pentagon lattice.

So we define a neutrosophic lattice N(L) to be a quasi modular lattice if it has atleast one sublattice (usual) which is modular and one sublattice which is a pure neutrosophic modular lattice.

Thus we need to modify the set S and the neutrosophic set N(S) of S. For if $S = \{a_1, \ldots, a_n\}$ we define $N(S) = \{a_1I, a_2I, \ldots, a_nI\}$ and take with $S \cup N(S)$ and the elements 0, 1, I, and $1 \cup I = 1 + I$. Thus to work in this way is not interesting and in general does not yield modular neutrosophic lattices.

We define the strong neutrosophic set of a set S as follows

Let $A = \{a_1, a_2, \ldots, a_n\}$, the strong neutropshic set of A;

$SN(A) = \{a_i, a_jI, a_i \cup a_jI = a_i + a_jI; 0, 1, I, 1 + I, 1 \leq i, j \leq n\}$.

S(L) the strong neutrosophic lattice is defined as follows:

$S(L) = \{0, 1, I, 1 + I, a_i, a_jI,$

$I \cup a_i = I + a_i \;\; a_1I \cup 1 = a_iI + 1, a_i + a_jI = a_i \cup a_jI \;\; 0 \underset{\neq}{<} a_i \underset{\neq}{<} 1; 0 < a_jI < I, 1 \leq i, j \leq n\}$.

S(L) with max, min is defined as the strong neutrosophic lattice.

We will illustrate this situation by some examples.

Example 2.1: Let S(L) = {0, 1, I, 1 + I, a, aI, a + aI, 1 + aI, I + a}

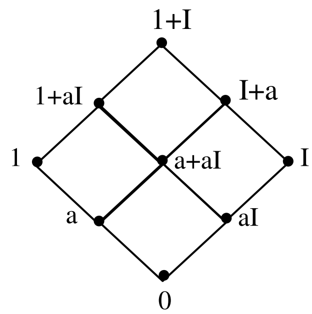

Figure 2.6

be a strong neutrosophic lattice.

We have several sublattices both strong neutrosophic sublattice as well as usual lattice.

For

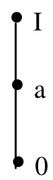

Figure 2.7

is the usual lattice.

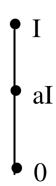

Figure 2.8

is the pure neutrosophic lattice.

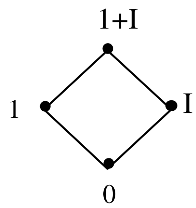

Figure 2.9

is the strong neutrosophic lattice.

These lattices have the edges to be real. Only vertices are indeterminates or neutrosophic numbers. However we can have lattices where all its vertices are real but some of the lines (or edges) are indeterminates.

Example 2.2: For consider

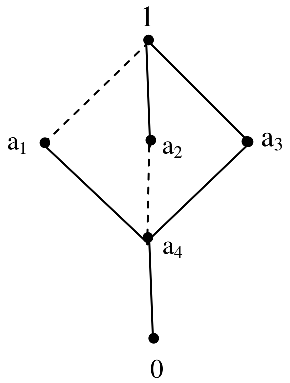

Figure 2.10

Such type of lattices will be known as edge neutrosophic lattices.

In case of edge neutrosophic lattices, we can have edge neutrosophic distributive lattices, edge neutrosophic modular lattices and edge neutrosophic super modular lattices and so on.

We will only illustrate these by some examples.

Example 2.3: Consider the following Hasse diagram.

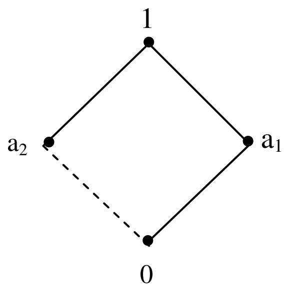

Figure 2.11

This is a edge neutrosophic lattice as the edge connecting 0 to a_2 is an indeterminate.

Example 2.4: Let us consider the following Hasse diagram of a lattice L.

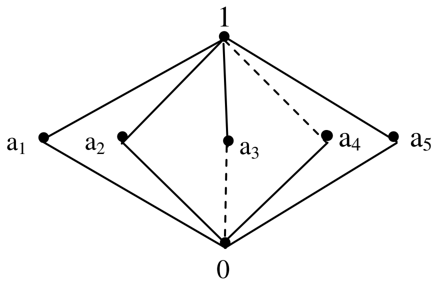

Figure 2.12

L is a edge neutrosophic modular lattice.
The edges connecting 0 to a_3 and 1 to a_4 are neutrosophic edges and the rest of the edges are reals. However all the vertices are real and it is a partially ordered set. We take some of the edges to be an indeterminate.

Example 2.5: Let L be the edge neutrosophic lattice whose Hassee diagram is as follows:

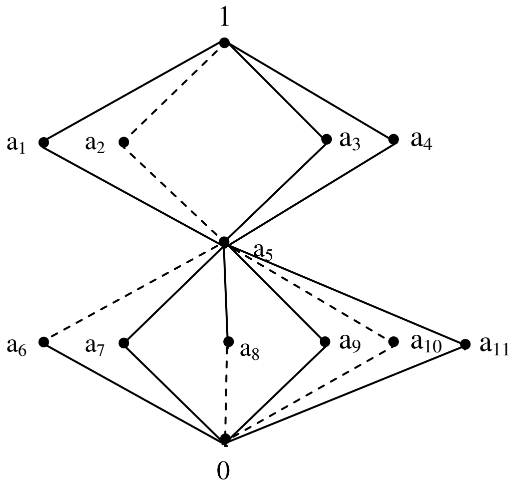

Figure 2.13

Clearly L is not a distributive edge neutrosophic lattice. However L has modular edge neutrosophic sublattices as well as modular lattices which are not neutrosophic.

Inview of this we have the following theorem.

THEOREM 2.1: *Let L be a edge neutrosophic lattice. Then L in general have sublattices which are not edge neutrosophic.*

Proof follows from the simple fact that every vertex is a sublattice and all vertices of the edge neutrosophic lattice which are not neutrosophic; but real is an instance of a not an edge neutropshic lattice.

We can have pure neutrosophic lattice which have the edges as well the vertices to be neutrosophic.

The following lattices with the Hasse diagram are pure neutrosophic lattices.

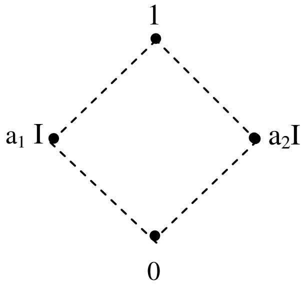

Figure 2.14

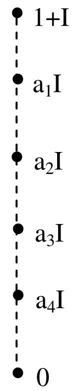

Figure 2.15

These two pure neutrosophic lattices cannot have edge neutrosophic sublattice or vertex neutrosophic sublattice.

3. Neutrosophic Boolean Algebras

Let us consider the power set of a neutrosophic set S = {a + bI | a = 0 or b = 0 can occur with 0 as the least element and 1 + I as the largest element}. P(S) = {Collection of all subsets of the set S} {P(S), ∪, ∩, ϕ, S} is a lattice defined as the neutrosophic Boolean algebra of order $2^{|P(S)|}$.

We will give examples of them.

Example 3.1: Let S = {0, 1, 1 +I, I}. P(S) = {ϕ, {0}, {1}, {1+I}, {I}, {0,1}, {0, I}, {0, 1+I}, {1, I}, {1, 1+I}, {I, 1+I}, {0, 1, I}, {0, 1, 1+I}, {0, I, 1+I}, {1, I, 1+I}, S} be the collection of all subsets of S including the empty set ϕ and the set S. |P(S)| = 16. P(S) is a neutrosophic Boolean algebra under '∪' and '∩' as the operations on P(S) and the containment relation of subsets as the partial order relation on P(S).

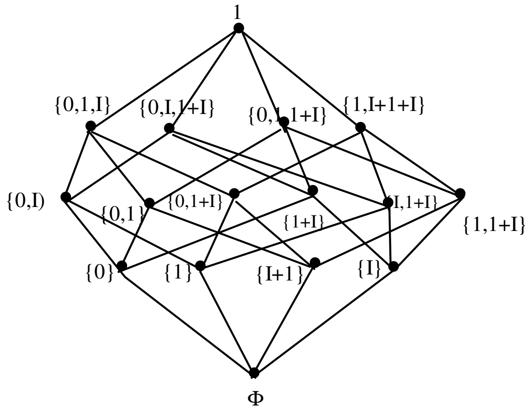

Figure 3.1

Example 3.2: Let S = {0, 1, I, 1+I, a, aI, a+I, aI+1, aI+a} be the neutrosophic set; 0 < a < 1. P(S) be the power set of S. |P(S)| = 2^9. P(S) is a neutrosophic Boolean algebra of order 2^9.

Example 3.3: Let S = {0, 1, I, 1+I, a_1, a_2, a_1I, a_2I, a_1+I, a_2+I, 1+a_1I, 1+a_2I, 1+a_1I+a_2, a_1+a_2, 1 + a_1I + a_2I, …} be the neutrosophic set with $a_1 \not< a_2$ or $a_2 \not< a_1$, $0 < a_1 < 1$, $0 < a_2 < 1$. P(S) is a neutrosophic Boolean algebra.

Now these neutrosophic Boolean algebras cannot be edge neutrosophic lattices. We make it possible to define edge neutrosophic lattice. Let L be a lattice given by the following Hasse-diagram.

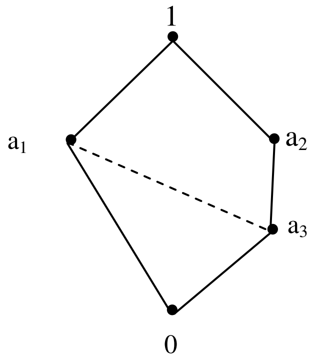

Figure 3.2

a_1 and a_3 are not comparable but we can have a neutrosophic edge given by the above diagram.

So we see the lattice has become a edge neutrosophic lattice.

Let L be a lattice given by the following diagram.

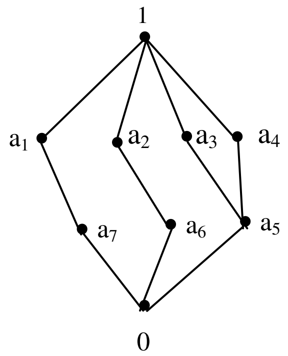

Figure 3.3

Clearly a_1 and a_6 are not comparable, a_2 and a_5 are not comparable a_4 and a_7 are not comparable.

We can have the following Hasse diagram which has neutrosophic edges.

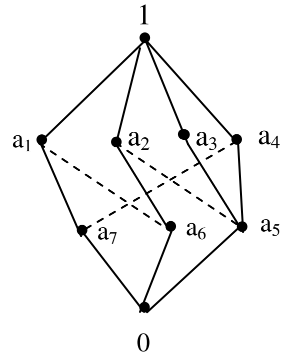

Figure 3.4

Clearly L is a edge neutrosophic lattice where we have some neutrosophic edges which are not comparable in the original lattice.

So we can on usual lattices L remake it into a edge neutrosophic lattice this is done if one doubts that a pair of elements $\{a_1, a_2\}$ of L with $a_1 \neq a_2$, min $\{a_1, a_2\} \neq a_1$ or a_2 or max $\{a_1, a_2\} \neq a_1$ or a_2.

If some experts needs to connect a_1 with a_2 by edge then the resultant lattice becomes a edge neutrosophic lattice.

Conclusion: Here for the first time we introduce the concept of neutrosophic lattices. Certainly these lattices will find applications in all places where lattices find their applications together with some indeterminancy. When one doubts a connection between two vertices one can have a neutrosophic edge.

References:

[1]. Birkhoff G, Lattice Theory, 2nd Edition, Amer, Math. Soc, 1948.

[2]. Gratzer G, General Lattice Theory, Birkhauser Verlage Basel and Stullgart, 1978,

[3]. Iqbal Unnisa and W.B. Vasantha, On a certain class of Modular lattices, J. of Madras Univ, Vol. 41, 166-169, 1978,

[4]. Iqbal Unnisa, W.B. Vasantha Kandasamy and Florentin Smarandache, Super modular Lattices, Educational Publisher, Ohio, 2012.

Received: January 14, 2014. Accepted: February 12, 2014.