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On Neutrosophic Canonical Hypergroups and Neutrosophic Hyperrings

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Abstract. A neutrosophic hyperstructure is an algebraic structure generated by a given hyperstructure *H* and an indeterminacy factor *I* under the hyperoperation(s) of *H*. The objective of this paper is to study canonical hypergroups and hyperrings in which addition and multiplication are hyperoperations in a neutrosophic environment. Some basic properties of neutrosophic canonical hypergroups and neutrosophic hyperrings are presented. Quotient neutrosophic canonical hypergroups and neutrosophic hyperrings are presented.

Keywords: neutrosophic canonical hypergroup, neutrosophic subcanonical hypergroup, neutrosophic hyperring, neutrosophic subhyperring, neutrosophic hyperideal.

1 Introduction

Given any hyperstructure *H*, a new hyperstructure *H(I)* may be generated by *H* and *I* under the hyperoperation(s) of *H*. Such new hyperstructures *H(I)* are called neutrosophic hyperstructures where *I* is an indeterminate or a neutrosophic element. Generally speaking, *H(I)* is an extension of *H* but some properties of *H* may not hold in *H(I)*. However, *H(I)* may share some properties with *H* and at times may possess certain algebraic properties not present in *H*.

Neutrosophic theory was introduced by F. Smarandache in 1995 and some known algebraic structures in the literature include neutrosophic groups, neutrosophic semigroups, neutrosophic loops, neutrosophic rings, neutrosophic fields, neutrosophic vector spaces, neutrosophic modules etc. Further introduction to neutrosophy and neutrosophic algebraic structures can be found in [1,2,3,4,16,26,27].

In 1934, Marty [18] introduced the theory of hyperstructures at the 8th Congress of Scandinavian Mathematicians. In 1972, Mittas [21] introduced the theory of canonical hypergroups. A class of hyperrings (R, +, .), where + and . are hyperoperations are introduced by De Salvo [15]. This class of hyperrings has been further studied by Asokkumar and Velrajan [5,22] and Davvaz and Leoranu-Fotea [14]. Further contributions to the theory of hyperstructures can be found in [7,8,9,10,14,22].

Agboola and Davaaz introduced and studied neutrosophic hypergroups in [4]. The present paper is concerned with the study of canonical hypergroups and hyperrings in a neutrosophic environment. Basic properties of neutrosophic canonical hypergroups and neutrosophic hyperrings are presented. Quotient neutrosophic canonical hypergroups and neutrosophic hyperrings are also presented.

2 A Review of Well Known Definitions

In this section, we provide basic definitions, notations and results that will be used in the sequel.

Definition 2.1. Let (G, $*$) be any group and let $G(I)=\langle G\cup I\rangle$. The couple $(G(I),*)$ is called a *neutrosophic group* generated by G and I under the binary operation $*$. The indeterminacy factor I is such that $I*I=I$. If $*$ is ordinary multiplication, then $I*I*I...*I=I^n=I$ and if $*$ is ordinary addition, then $I*I*I...*I=nI$ for $n\in N$.

G(I) is said to be *commutative* if $a*b=b*a$ for all $a,b\in G(I)$.

Theorem 2.2. [26] Let G(I) be a neutrosophic group.
(1) G(I) in general is not a group;
(2) G(I) always contain a group.

Definition 2.3. Let G(I) be a neutrosophic group.
(1) A proper subset A(I) of G(I) is said to be a neutrosophic subgroup of G(I) if A(I) is a neutrosophic group, that is, A(I) contains a proper subset which is a group;
(2) A(I) is said to be a pseudo neutrosophic group if it does not contain a proper subset which is a group.

Definition 2.4. Let A(I) be a neutrosophic subgroup (pseudo neutrosophic subgroup) of a neutrosophic group G(I).
(1) A(I) is said to be *normal* in G(I) if there exist $x,y\in G(I)$ such that $xA(I)y=A(I)$.
(2) G(I) is said to be *simple* if it has no non-trivial

neutrosophic normal subgroup.

Example 1. [3] Let G(I)={e, a, b, c, I, aI, bI, cI} be a set, where $a^2=b^2=c^2=e$, bc=cb=a, ac=ca=b, ab=ba=c. Then (G(I),.) is a commutative neutrosophic group, and H(I)={e,a,I,aI}, K(I)={e,b,I,bI} and PI={e,c,I,cI} are neutrosophic subgroups of G(I).

Theorem 2.5. [3] Let H(I) be a non-empty proper subset of a neutrosophic group (G(I),$*$). Then, H(I) is a neutrosophic subgroup of G(I) if and only if the following conditions hold:

(1) $a,b \in H(I)$ implies that $a*b \in H(I)$;
(2) There exists a proper subset A of H(I) such that (A, $*$) is a group.

Definition 2.6. Let (G_1(I),$*$) and (G_1(I),$*$') be two neutrosophic groups and let $\phi : G_1(I) \to G_2(I)$ be a mapping of G_1(I) into G_2(I). Then, ϕ is said to be a *homomorphism* if the following conditions hold:

(1) ϕ is a group of homomorphism;
(2) $\phi(I)=I$.

In addition, if ϕ is a bijection, then ϕ is called a neutrosophic group isomorphism and we write $G_1(I)\cong G_2(I)$.

Definition 2.7. Let (R,+,.) be any ring. A *neutrosophic ring* is a triple (R(I),+,.) generated by R and I, that is, $R(I)=\langle R\cup I\rangle$.

Indeed, $R(I)=\{x=a+bI : a,b\in R\}$, where if x=a+bI and y=c+dI are elements of R, then

$$x\oplus y=(a+bI)\oplus(c+dI)=(a+c)+(b+d)I,$$
$$x\odot y=(a+bI)\odot(c+dI)=(ac)+(ad+bc+bd)I.$$

Example 2. Let $\mathbb{Z}_n$ be a ring of integers modulo n. Then, $\mathbb{Z}_n(I)=\{x=a+bI : a,b\in\mathbb{Z}_n\}$ is a neutrosophic ring of integers modulo n.

Theorem 2.8. [27] Let (R(I),+,.) be a *neutrosophic ring*. Then, (R(I),+,.) is a ring.

Definition 2.9. Let (R(I),+,.) be a neutrosophic ring. A non-empty subset S(I) of R(I) is said to be a *neutrosophic subring* if (S(I),+,.) is a neutrosophic ring. It is essential that S(I) must contain a proper subset which is a ring. Otherwise, S(I) is called a *pseudo neutrosophic subring* of R(I).

Example 3. Let $(\mathbb{Z}_{12}(I),+,\cdot)$ be a neutrosophic ring of integers modulo 12 and let S(I) and T(I) be subsets of $\mathbb{Z}_{12}(I)$ given by S(I)={0, 6, I, 2I, 3I, ..., 11I, 6+I, 6+2I, 6+3I, ..., 6+11I} and T(I)={0, 2I, 4I, 6I, 10I}. Then, (S(I),+,.) is a neutrosophic subring of $\mathbb{Z}_{12}(I)$ while (T(I),+,.) is a pseudo neutrosophic ring of $\mathbb{Z}_{12}(I)$.

Definition 2.10. Let (R(I),+,.) be a neutrosophic ring and let S(I) be a neutrosophic subring (pseudo neutrosophic subring) of R(I). Then, S(I) is called a *neutrosophic ideal (pseudo neutrosophic ideal)* of R(I) if for all $r\in R(I)$ and $s\in S(I)$, $r\cdot s, s\cdot r\in S(I)$.

Definition 2.11. Let $(R_1(I),+,\cdot)$ and $(R_2(I),+,\cdot)$ be two neutrosophic rings and let $\phi : R_1(I)\to R_2(I)$ be a mapping of $R_1(I)$ into $R_2(I)$. Then, ϕ is said to be a *homomorphism* if the following conditions hold:

(1) ϕ is a group of homomorphism;
(2) $\phi(I)=I$.

Moreover, if ϕ is a bijection, then ϕ is called a neutrosophic ring *isomorphism* and we write $R_1(I)\cong R_2(I)$. The kernel of ϕ denoted by $Ker\phi$ is the set $\{x\in R_1(I):\phi(x)=0\}$ and the image of ϕ denoted by $\operatorname{Im}\phi$ is the set $\{\phi(x): x\in R_1(I)\}$.

It should be noted that $\operatorname{Im}\phi$ is a neutrosophic subring of $R_2(I)$ and $Ker\phi$ is always a subring of R_1 and never a neutrosophic subring (ideal) of R_1(I).

Definition 2.12. A map $\cdot : S\times S\to P^*(S)$ is called *hyperoperation* on the set S, where S is non-empty set and $P^*(S)$ denotes the set of all non-empty subsets of S.

A *hyperstructure* or *hypergroupoid* is the pair $(S,\cdot)$, where $\cdot$ is a hyperoperation on the set S.

Definition 2.13. A hyperstructure $(S,\cdot)$ is called a semihypergroup if for all $x,y,z\in S,(x\cdot y)\cdot z=x\cdot(y\cdot z)$, which means that

$$\bigcup_{u\in x\cdot y} u\cdot z=\bigcup_{v\in y\cdot z} x\cdot v.$$

Definition 2.14. A non-empty subset A of a semihypergroup $(S,\cdot)$ is called a *subsemihypergroup*. In other words, a non-empty subset A of a semihypergroup $(S,\cdot)$ is a subsemihypergroup if $A\cdot A\subseteq A$.

If $x\in S$ and A,B are non-empty subsets of S, then

$$A\cdot B=\bigcup_{\substack{a\in A\\ b\in B}} a\cdot b,\ A\cdot x=A\cdot\{x\},\text{ and}$$

$$x\cdot B=\{x\}\cdot B.$$

Definition 2.15. A hypergroupoid $(H,\cdot)$ is called a *quasihypergroup* if for all a of H we have $a\cdot H=H\cdot a=H$. This condition is also called the *reproduction axiom*.

Definition 2.16. A hypergroupoid $(H,\cdot)$ which is both a semihypergroup and a quasihypergroup is called a *hypergroup.*

Definition 2.17. Let H be a non-empty set and let + be a hyperoperation on H. The couple $(H,+)$ is called *canonical hypergroup* if the following conditions hold:

(1) x+y=y+x, for all $x, y \in H$;
(2) x+(y+z)=(x+y)+z, for all $x, y, z \in H$;
(3) there exists a neutral element $0 \in H$ such that x+0={x}=0+x, for all $x \in H$;
(4) for every $x \in H$, there exists a unique element $-x \in H$ such that $0 \in x+(-x) \cap (-x)+x$;
(5) $z \in x+y$ implies $y \in -x+z$ and $x \in z-y$, for all $x, y, z \in H$.

A non-empty subset A of H is called a *subcanonical hypergroup* if A is canonical hypergroup under the same hyperaddition as that of H that is, for every $a, b \in A$, $a-b \in A$. In addition, if $a+A-a \subseteq A$ for all $a \in H$, A is said to be *normal*.

Definition 2.18. A *hyperring* is a triple $(R,+,\cdot)$ satisfying the following axioms:
(1) $(R,+)$ is a canonical hypergroup;
(2) $(R,\cdot)$ is a semihypergroup such that $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$, that is,) is a bilateral absorbing element;
(3) $x \cdot (y+z) = x \cdot y + x \cdot z$ and $(x+y) \cdot z = x \cdot z + y \cdot z$, for all $x, y, z \in R$.

Definition 2.19. Let $(R,+,\cdot)$ be a hyperring and A be a non-empty subset of R. Then, A is said to be *subhyperring* of R if $(A,+,\cdot)$ is itself a hyperring.

Definition 2.20. Let A be a subhyperring of a hyperring R.
(1) A is called a *left hyperideal* of R if $r \cdot a \subseteq A$ for all $r \in R, a \in A$.
(2) A is called a *right hyperideal* of R if $a \cdot r \subseteq A$ for all $r \in R, a \in A$.
(3) A is called a *hyperideal* of R if A is both left and right hyperideal of R.
(4) A hyperideal A is said to be normal if $r+a-r \subseteq A$ for all $r \in R$.

Definition 2.21. Let $(H_1,+)$ and $(H_2,+)$ be two canonical hypergroups. A mapping $\phi : H_1 \to H_2$, is called
(1) a *homomorphism* if (i) for all $x, y \in H_1$, $\phi(x+y) \subseteq \phi(x)+\phi(y)$ and (ii) $\phi(0)=0$.
(2) a *good or strong homomorphism* if (i) for all $x, y \in H_1$, $\phi(x+y) = \phi(x)+\phi(y)$ and (ii) $\phi(0)=0$.
(3) an *isomorphism (strong isomorphism*) if ϕ is a bijective homomorphism (strong homomorphism).

Definition 2.22. [4] Let $(H,\circ)$ be any hypergroup and let $\langle H \cup I \rangle = \{(a,bI) : a,b \in H\}$. The couple $H(I) = (\langle H \cup I \rangle, \circ)$ is called a neutrosophic hypergroup generated by H and I under the hyperoperation $\circ$, where for all $(a,bI),(c,dI) \in H(I)$, the composition element of H(I) is defined by

$$(a,bI) \circ (c,dI) = \{(x,yI) : x \in a \circ c, \\ y \in a \circ d \cup b \circ c \cup b \circ d\}.$$

3 Development of Neutrosophic Canonical Hypergroups and Neutrosophic Hyperrings

In this section, we develop the concepts of neutrosophic canonical hypergroups and neutrosophic hyperrings. Necessary definitions are given and examples are provided.

Definition 3.1. Let $(H,+)$ be any canonical hypergroup and let I be an indeterminate. Let $H(I) = \langle H \cup I \rangle = \{(a,bI) : a,b \in H\}$ be a set generated by H and I. The hyperstructure $(H(I),+)$ is called a *neutrosophic canonical hypergroup*, where for all $(a,bI),(c,dI) \in H(I)$ with $b \neq 0$ or $d \neq 0$, we define

$$(a,bI)+(c,dI) \in \{(x,yI) : x \in a+c, \\ y \in a+d \cup b+c \cup b+d\}$$

and

$$(x,0)+(y,0) = \{(u,0) : u \in x+y\}.$$

The element I is represented by (0,I) in H(I) and any element $x \in H$ is represented by (x,0) in H(I). For any non-empty subset A[I] of H(I) , we define $-A[I] = \{-(a,bI) = (-a,-bI) : a,b \in H\}$.

Lemma 3.2. *Let $H \neq \{0\}$ be a canonical hypergroup and let H(I) be the corresponding neutrosophic canonical hypergroup. Then, (0,0) the neutral element of H is not a neutral element of H(I).*

Proof. Suppose that (0,0) is the neutral element of H(I) and suppose that $(a,bI) \in H(I)$ such that b is non-zero and $a \neq b$.

Then

$$\begin{aligned}(a,bI)+(0,0) &= \{(u,vI) : u \in a+0, v \in a+0 \cup b+0 \cup b+0\} \\ &= \{(u,vI) : u \in \{a\}, v \in \{a,b\}\} \\ &\neq (a,bI),\end{aligned}$$

a contradiction. Hence, (0,0) is not a neutral element of H(I).

Definition 3.3. Let (H(I),+) be a neutrosophic canonical hypergroup.
(1) A non-empty subset A[I] of H(I) is called a *neutrosophic subcanonical hypergroup* of H(I) if (A[I],+) is itself a neutrosophic canonical hypergroup. It is essential that A[I] must contain a proper subset which is a subcanonical hypergroup of H. If A[I] does not contain a proper subset which is a subcanonical hypergroup of H, then it is called a pseudo *neutrosophic subcanonical hypergroup* of H(I).
(2) If A[I] is a neutrosophic subcanonical hypergroup (pseudo neutrosophic subcanonical hypergroup), then A[I] is said to be normal in H(I) if for all $(a,bI) \in H(I)$,
$$(a,bI)+A[I]-(a,bI) \subseteq A[I].$$

Lemma 3.4. *Let (H(I),+) be a neutrosophic canonical hypergroup and let A[I] be a non-empty proper subset of*

H(I). Then, A[I] is a neutrosophic subcanonical hypergroup if and only if the following conditions hold:

(1) for all $(a,bI),(c,dI)\in A[I],(a,bI)-(c,dI)\subseteq A[I]$,

(2) *A[I] contains a proper subset which is a canonical hypergroup of H.*

Lemma 3.5. *Let (H(I),+) be a neutrosophic canonical hypergroup and let A[I] be a non-empty proper subset of H(I). Then, A[I] is a pseudo neutrosophic subcanonical hypergroup if and only if the following conditions hold:*

(1) for all $(a,bI),(c,dI)\in A[I],(a,bI)-(c,dI)\subseteq A[I]$,

(2) *A[I] does not contain a proper subset which is a canonical hypergroup of H.*

Definition 3.6. Let A[I] and B[I] be any two neutrosophic subcanonical hypergroups of a neutrosophic canonical hypergroup H(I). The sum of A[I] and B[I] denoted by A[I]+B[I] is defined as the set:

$$A[I]+B[I]=\bigcup_{\substack{(a,bI)\in A[I]\\(c,dI)\in B[I]}}(a,bI)+(c,dI).$$

Definition 3.7. Let H(I) be a neutrosophic canonical hypergroup and let A[I] be a neutrosophic subcanonical hypergroup of H(I). If K is a subcanonical hypergroup of H, we define the set

$$A[I]+K=\bigcup_{\substack{(a,bI)\in A[I]\\(k,0)\in K}}(a,bI)+(k,0).$$

Definition 3.8. Let $(R,+,\cdot)$ be any hyperring and let I be an indeterminate. The hyperstructure $(R(I),\oplus,\odot)$ generated by R and I, that is, R(I)=$\langle R\cup I\rangle$, is called a *neutrosophic hyperring*, where for all

$(a,bI),(c,dI)\in R(I)$,

$(a,bI)\oplus(c,dI)=\{(x,yI):x\in a+c,y\in b+d\}$;

for all $(a,bI),(c,dI)\in R(I)$ with $b\neq 0$ or $d\neq 0$,

$(a,bI)\odot(c,dI)=\{(x,yI):x\in a.c,$
$y\in a.d\cup b.c\cup b.d\}$

and

$(x,0)\odot(y,0)=\{(u,0):u\in x\cdot y\}$.

We usually use + and $\cdot$ instead of $\oplus$ and $\odot$.

Lemma 3.9. *Let R(I) be a neutrosophic hyperring. Then,* $(0,0)\in R(I)$ *is bilaterally absorbing element.*

Proof. Suppose that $(a,bI)\in R(I)$. Then, $(a,bI).(0,0)=$
$\{(u,vI):u\in a\cdot 0,v\in a\cdot 0\cup b\cdot 0\cup b\cdot 0\}=$
$\{(u,vI):u\in\{0\},v\in\{0\}\}=\{(0,0)\}$.

Hence, $(0,0)\in R(I)$ is a bilaterally absorbing element.

Definition 3.10. Let $(R(I),+,\cdot)$ be a neutrosophic hyperring and let A[I] be a non-empty subset of R(I). Then, A[I] is called a *neutrosophic subhyperring* of R(I) if $(A[I],+,\cdot)$ is itself a neutrosophic hyperring. It is essential that A[I] must contain a proper subset which is a hyperring. Otherwise, A[I] is called a *pseudo neutrosophic subhyperring* of R(I).

Definition 3.11. Let $(R(I),+,\cdot)$ be a neutrosophic hyperring and let A[I] be a neutrosophic subhyperring of R(I).

(1) A[I] is called a *left neutrosophic hyperideal* if for all $(r,sI)\in R(I),(a,bI)\in A[I]$,
$(r,sI)\cdot(a,bI)\subseteq A[I]$.

(2) A[I] is called a *right neutrosophic hyperideal* if for all $(r,sI)\in R(I),(a,bI)\in A[I]$,
$(a,bI)\cdot(r,sI)\subseteq A[I]$.

(3) A[I] is called a *neutrosophic hyperideal* if A[I] is both a left and right neutrosophic hyperideal.

A neutrosophic hyperideal A[I] of R[I] is said to be *normal* in R(I) if for all $(r,sI)\in R(I)$,
$(r,sI)+A[I]-(r,sI)\subseteq A[I]$.

Lemma 3.12. *Let* $(R(I),+,\cdot)$ *be a neutrosophic hyperring and let A[I] be a non-empty subset of R(I). Then, A[I] is a neutrosophic hyperideal if and only if the following conditions hold:*

(1) For all $(a,bI),(c,dI)\in A[I],(a,bI)-(c,dI)\subseteq A[I]$;

(2) For all $(r,sI)\in R(I),(a,bI)\in A[I]$, $(a,bI)\cdot(r,sI)\subseteq A[I]$ and $(r,sI)\cdot(a,bI)\subseteq A[I]$;

(3) A[I] contains a proper subset which is a hyperring.

Lemma 3.13. *Let* $(R(I),+,\cdot)$ *be a neutrosophic hyperring and let A[I] be a non-empty subset of R(I). Then, A[I] is a pseudo neutrosophic hyperideal if and only if the following conditions hold:*

(1) For all $(a,bI),(c,dI)\in A[I],(a,bI)-(c,dI)\subseteq A[I]$;

(2) For all $(r,sI)\in R(I),(a,bI)\in A[I]$, $(a,bI)\cdot(r,sI)\subseteq A[I]$ and $(r,sI)\cdot(a,bI)\subseteq A[I]$;

(3) A[I] does not contain a proper subset which is a hyperring.

Definition 3.14. Let A[I] and B[I] be any two neutrosophic hyperideals of a neutrosophic hyperring R(I). The sum of A[I] and B[I] denoted by A[I]+B[I] is defined as the set $\{(x,yI):(x,yI)\in(a,bI)+(c,dI)$, where $(a,bI)\in A[I],(c,dI)\in B[I]\}$.

Definition 3.15. Let R(I) be a neutrosophic hyperring and let A[I] be a neutrosophic hyperideal of R(I). If K is a hyperideal (pseudo hyperideal) of R, the sum of A[I] and K denoted by A[I]+K is defined as the set $\{(x, yI) : (x, yI) \in (a, bI) + (k, 0),$ where $(a, bI) \in A[I], (k, 0) \in K\}.$

Example 4. Let R(I)={(0,0),(x,0),(0,xI),(x,xI)} be a set and let + and $\cdot$ be hyperoperations on R(I) defined in the tables below.

+	(0,0)	(x,0)	(0,xI)	(x,xI)
(0,0)	{(0,0)}	{(x,0)}	{(0,0), (0,xI)}	{(x,xI)}
(x,0)	{(x,0)}	{(0,0), (x,0)}	{(x,0), (x,xI)}	R(I)
(0,xI)	{(0,0), (0,xI)}	{(x,0), (x,xI)}	{(0,0), (0,xI)}	{(x,0), (x,xI)}
(x,xI)	{(x,xI)}	R(I)	{(x,0), (x,xI)}	R(I)

Table 1.

$\cdot$	(0,0)	(x,0)	(0,xI)	(x,xI)
(0,0)	{(0,0)}	{(0,0)}	{(0,0)}	{(0,0)}
(x,0)	{(0,0)}	{(0,0), (x,0)}	{(0,0), (0,xI)}	R(I)
(0,xI)	{(0,0)}	{(0,0), (0,xI)}	{(0,0), (0,xI)}	{(0,0), (0,xI)}
(x,xI)	{(0,0)}	R(I)	{(0,0), (0,xI)}	R(I)

Table 2.

It is clear from the tables that (R(I),+) is a neutrosophic canonical hypergroup and $(R(I), +, \cdot)$ is a neutrosophic hyperring.

4 Properties of Neutrosophic Canonical Hypergroups

In this section, we present some basic properties of neutrosophic canonical hypergroups.

Proposition 4.1. *Let (H(I),+) be a neutrosophic canonical hypergroup. Then,*

(1) (H(I),+) in general is not a canonical hypergroup.
(2) (H(I),+) always contain a canonical hypergroup.

Lemma 4.2. *Let (H(I),+) be a neutrosophic canonical hypergroup.*

(1) –(0,0)=(0,0).
(2) $-(-(a, bI)) = (a, bI)$ for all $(a, bI) \in H(I).$
(3) $-((a, bI) + (c, dI) = -(a, bI) - (c, dI)$ for all $(a, bI), (c, dI) \in H(I).$

Proposition 4.3. *Let (H(I),+) and (G(I),+') be any two neutrosophic canonical hypergroups. Then,* $H(I) \times G(I)$ *is a neutrosophic canonical hypergroup, where*

$$((a, bI), (c, dI) +''((e, fI), (g, hI)) = \{((p, qI), (x, yI)) : (p, qI) \in (a, bI) + (e, fI), (x, yI) \in (c, dI) +'(g, hI)\},$$

for all $((a, bI), (c, dI)), ((e, fI), (g, hI)) \in H(I) \times G(I).$

Proposition 4.4. *Let (H(I),+) be a neutrosophic canonical hypergroup and let (K,+') be a canonical hypergroup. Then, H(I)xK is a neutrosophic canonical hypergroup, where*

$$((a, bI), (m, 0) +''((c, dI), (n, 0)) = \{((x, yI), (k, 0)) : (x, yI) \in (a, bI) + (c, dI), (k, 0) \in (m, 0) +'(n, 0)\},$$

for all $((a, bI), (m, 0)), ((c, dI), (n, 0)) \in H(I) \times K.$

Proposition 4.5. *Let A[I] and B[I] be any two neutrosophic subcanonical hypergroups of a neutrosophic canonical hypergroup H(I). Then,*

(1) A[I]+B[I] is a neutrosophic subcanonical hypergroup of H(I).
(2) A[I]+A[I]=A[I].
(3) $A[I] \cap B[I]$ *is a neutrosophic subcanonical hypergroup of H(I).*

Proposition 4.6. *Let H(I) be a neutrosophic canonical hypergroup and let A[I] and B[I] be any neutrosophic subcanonical hypergroup and pseudo neutrosophic subcanonical hypergroup of H(I), respectively. Then,*

(1) A[I]+B[I] is a neutrosophic subcanonical hypergroup of H(I).
(2) $A[I] \cap B[I]$ *is a pseudo neutrosophic subcanonical hypergroup of H(I).*

Proposition 4.7. *Let H(I) be a neutrosophic canonical hypergroup and let A[I] and B[I] be any neutrosophic subcanonical hypergroup and pseudo neutrosophic subcanonical hypergroup respectively. If K is any subcanonical hypergroup of H, then*

(1) A[I]+K is a neutrosophic subcanonical hypergroup of H(I).
(2) B[I]+K is a neutrosophic subcanonical hypergroup of H(I).

Proposition 4.8. *Let (H(I),+) be a neutrosophic canonical hypergroup and let A be a subcanonical hypergroup of H. If A is normal in H, A[I] is not necessarily normal to H(I).*

Proposition 4.9. Let (H(I),+) be a neutrosophic canonical hypergroup and let A be a normal neutrosophic subcanonical hypergroup of H. Then, $(a, aI) + A - (a, aI) \subseteq A[I]$ for all $(a, aI) \in H(I).$

Proof. Suppose that A is normal in H. Let (h,0) be an arbitrary element of A. Then, for all $(a, aI) \in H(I)$ with $a \neq 0$, we have

$$\begin{aligned}&(a, aI) + (h, 0) - (a, aI)\\ &= (a, aI) + \{(x, yI) : x \in h - a, y \in h - a \cup 0 - a\}\\ &= \{(u, vI) : u \in a + x, v \in a + y \cup a + x,\\ &\quad x \in h - a, y \in h - a \cup -a\}\end{aligned}$$

$=\{(u,vI):u\in a+h-a,$
$v\in a+h-a\cup a-a\cup a+h-a\},$

from which we obtain $u\in A$ and $v\in A$. Therefore, $(u,vI)\in A[I]$. Since $(h,0)\in A$ is arbitrary, the required results follow.

Definition 4.10. Let (H(I),+) be a neutrosophic canonical hypergroup and let A[I] be a neutrosophic subcanonical hypergroup of H(I). We consider the quotient

$(H(I):A[I])=\{(a,bI)+A[I]:(a,bI)\in H[I]\}$

and we put $(a,bI)+A[I]=[(a,bI)]$. For all $[(a,bI)],[(c,dI)]\in(H(I):A[I])$, we define the hyperoperation $\oplus$ on $(H(I):A[I])$ as

$[(a,bI)]\oplus[(c,dI)]=\{[(e,fI)]:(e,fI)\in (a,bI)+(c,dI)\}$.

Then the couple $((H(I):A[I]),\oplus)$ is called the *quotient neutrosophic canonical hypergroup*. If A[I] is a pseudo neutrosophic subcanonical hypergroup, then we call $((H(I):A[I]),\oplus)$ a *pseudo quotient neutrosophic canonical hypergroup*.

Proposition 4.11. *Let H(I) be a neutrosophic canonical hypergroup and A[I] be a neutrosophic subcanonical hypergroup (pseudo neutrosophic subcanonical hypegroup) of H(I). Then,* $((H(I):A[I]),\oplus)$ *is generally not a canonical hypergroup.*

Definition 4.12. Let $(H_1(I),+)$ and $(H_2(I),+)$ be two neutrosophic canonical hypergroups and let $\phi:H_1(I)\to H_2(I)$ be a mapping from $H_1(I)$ into $H_2(I)$.

(1) ϕ is called a *homomorphism* if
   a. ϕ is a canonical hypergroup homomorphism;
   b. $\phi((0,I))=(0,I)$.

(2) ϕ is called a *good* or *strong homomorphism* if
   a. ϕ is a good or strong canonical hypergroup homomorphism;
   b. $\phi((0,I))=(0,I)$.

(3) ϕ is called a is*omorphism* (*strong isomorphism*) if ϕ is a bijective homomorphism (strong homomorphism).

Definition 4.13. Let $\phi:H_1(I)\to H_2(I)$ be a homomorphism from a neutrosophic canonical hypergroup $H_1(I)$ into a neutrosophic canonical hypergroup $H_2(I)$.

(1) The kernel of ϕ denoted by $Ker\phi$ is the set $\{(a,bI)\in H_1(I):\phi((a,bI))=(0,0)\}$.
(2) The kernel of ϕ denoted by $\operatorname{Im}\phi$ is the set $\{\phi((a,bI)):(a,bI)\in H_1(I)\}$.

5 Properties of Neutrosophic Hyperrings

In this section, we present some basic properties of neutrosophic hyperrings.

Proposition 5.1. *Let* $(R(I),+,\cdot)$ *be a neutrosophic hyperring. Then,*

(1) $(R(I),+,\cdot)$ *in general is not a hyperring.*
(2) $(R(I),+,\cdot)$ *always contain a hyperring.*

Proof. (1) It has been presented in part (1) of Proposition 4.1. that $(R(I),+,\cdot)$ is not a canonical hypergroup. Also, distributive laws are not valid in $(R(I),+,\cdot)$. Hence, $(R(I),+,\cdot)$ is not a hyperring. (2) Follows from the definition.

Proposition 5.2. *Let* $(R(I),+,\cdot)$ *and* $(S(I),+',\cdot')$ *be any two neutrosophic hyperrings. Then,* $R(I)\times S(I)$ *is a neutrosophic hyperring, where*

$((a,bI),(c,dI)+''((e,fI),(g,hI))$
$=\{((p,qI),(x,yI)):(p,qI)\in(a,bI)+(e,fI),$
$(x,yI)\in(c,dI)+'(g,hI)\},$

and

$((a,bI),(c,dI)\cdot''((e,fI),(g,hI))$
$=\{((p,qI),(x,yI)):(p,qI)\in(a,bI).(e,fI),$
$(x,yI)\in(c,dI).'(g,hI)\},$

for all

$((a,bI),(c,dI)),((e,fI),(g,hI))\in R(I)\times S(I).$

Proposition 5.3. *Let* $(R(I),+,\cdot)$ *be a neutrosophic hyperring and let* $(K,+',\cdot')$ *be a hyperrings. Then,* $R(I)\times K$ *is a neutrosophic hyperring, where*

$((a,bI),(m,0)+''((c,dI),(n,0))$
$=\{((x,yI),(k,0)):(x,yI)\in(a,bI)+(c,dI),$
$(k,0)\in(m,0)+'(n,0)\},$

and

$((a,bI),(m,0)\cdot''((c,dI),(n,0))$
$=\{((x,yI),(k,0)):(x,yI)\in(a,bI).(c,dI),$
$(k,0)\in(m,0).'(n,0)\},$

for all

$((a,bI),(m,0)),((c,dI),(n,0))\in R(I)\times K.$

Lemma 5.4. Let A[I] be any neutrosophic hyperideal of a neutrosophic hyperring R(I). Then,

(1) A[I]+A[I]=A[I].
(2) (a,bI)+A[I]=A[I] for all $(a,bI)\in A[I]$.

Proposition 5.5. *Let* $(R(I),+,\cdot)$ *be a neutrosophic hyperring and let A[I] and B[I] be left (right) neutrosophic ideals of R(I). Then,*

(1) $A[I]\cap B[I]$ *is a left (right) neutrosophic hyperideal of R(I).*
(2) $A[I]+B[I]$ *is a left (right) neutrosophic hyperideal of R(I).*

Proposition 5.6. *Let R(I) be a neutrosophic hyperring and let A[I] and B[I] be any neutrosophic hyperideal and pseudo neutrosophic hyperideal of R(I) respectively. Then,*

(1) $A[I]+B[I]$ *is a neutrosophic hyperideal of R(I).*
(2) $A[I]\cap B[I]$ *is a pseudo neutrosophic hyperideal of R(I).*

Proposition 5.7. *Let R(I) be a neutrosophic hyperring* *and let A[I] and B[I] be any neutrosophic hyperideal and pseudo neutrosophic hyperideal, respectively. If K is any subhyperring of R, then*

(1) $A[I]+K$ is a neutrosophic hyperideal of R(I).
(2) $B[I]+K$ is a neutrosophic hyperideal of R(I).

Proposition 5.8. *Let $(R(I),+,\cdot)$ be a neutrosophic hyperring and let A be a hyperideal of R. If A is normal in R, A[I] is not necessarily normal in R(I).*

Proposition 5.9. *Let $(R(I),+,\cdot)$ be a neutrosophic hyperring and let A be a normal hyperideal of R. Then,*
$$(a,aI)+A-(a,aI)\subseteq A[I]$$
for all $(a,aI)\in R(I)$.

Definition 5.10. Let $(R(I),+,\cdot)$ be a neutrosophic hyperring and let $(A(I),+,\cdot)$ be a neutrosophic hyperideal of R(I). We consider the quotient $(R(I):A[I])=$ $\{(a,bI)+A[I]:(a,bI)\in R(I)\}$ and we put (a,bI)+A[I]=[(a,bI)].

For all $[(a,bI)],[(c,dI)]\in((R(I):A[I])$, we consider the hyperoperation $\oplus$ as defined in the Definition 4.10 and we define the hyperoperation $\odot$ on $((R(I):A[I])$ as $[(a,bI)]\odot[(c,dI)]=\{[(e,fI)]:(e,fI)\in(a,bI).(c,dI)\}$.

Then, the triple $((R(I):A[I]),\oplus,\odot)$ is called the *quotient neutrosophic hyperring*. If A[I] is a pseudo neutrosophic hyperideal, then we call $((R(I):A[I]),\oplus,\odot)$ a *pseudo neutrosophic hyperring*.

Proposition 5.11. *Let R(I) be a neutrosophic hyperring and let A[I] be a neutrosophic hyperideal (pseudo neutrosophic hyperideal) of R(I). Then, $((R(I):A[I]),\oplus,\odot)$ is generally not a hyperring*.

Definition 5.12. Let $(R_1(I),+)$ and $(R_2(I),+)$ be two neutrosophic hyperrings and let $\phi:R_1(I)\rightarrow R_2(I)$ be a mapping from $R_1(I)$ into $R_2(I)$.

(1) ϕ is called a *homomorphism* if
  a. ϕ is a hyperring homomorphism;
  b. $\phi((0,I))=(0,I)$.

(2) ϕ is called a *good* or *strong homomorphism* if
  a. ϕ is a good or strong hyperring homomorphism;
  b. $\phi((0,I))=(0,I)$.

(3) ϕ is called a is*omorphism* (*strong isomorphism*) if ϕ is a bijective homomorphism (strong homomorphism).

Definition 5.13. Let $\phi:R_1(I)\rightarrow R_2(I)$ be a homomorphism from a neutrosophic hyperring $R_1(I)$ into a neutrosophic hyperring $R_2(I)$.

(1) The kernel of ϕ denoted by $Ker\phi$ is the set $\{(a,bI)\in R_1(I):\phi((a,bI))=(0,0)\}$.
(2) The kernel of ϕ denoted by $\operatorname{Im}\phi$ is the set $\{\phi((a,bI)):(a,bI)\in R_1(I)\}$.

Proposition 5.14. *Let $\phi:R_1(I)\rightarrow R_2(I)$ be a homomorphism from a neutrosophic hyperring $R_1(I)$ into a neutrosophic hyperring $R_2(I)$. Then,*

(1) $Ker\phi$ is a subhyperring of R_1 and never be a neutrosophic hyperring (neutrosophic hyperideal) of $R_1(I)$.
(2) $\operatorname{Im}\phi$ is a neutrosophic subhyperring of $R_2(I)$.

Question 1: Does there exist:

(1) A neutrosophic canonical hypergroup with normal neutrosophic subcanonical hypergroups?
(2) A neutrosophic hyperring with normal neutrosophic hyperideals?
(3) A simple neutrosophic canonical hypergroup?
(4) A simple neutrosophic hyperring?

6 Conclusion

In this paper, we have introduced and studied *neutrosophic* canonical hypergroups and *neutrosophic* hyperrings. We presented elementary properties of *neutrosophic* canonical hypergroups and *neutrosophic* hyperrings. Also, we studied quotient *neutrosophic* canonical hypergroups and quotient *neutrosophic* hyperrings.

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