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On Neutrosophic Vague Graphs

S. Satham Hussain [1]*, R. Jahir Hussain [1] and Florentin Smarandache [2]

[1] PG and Research Department of Mathematics, Jamal Mohamed College, Trichy - 620 020, Tamil Nadu, India.
E-mail: sathamhussain5592@gmail.com,hssn_jhr@yahoo.com
[2]Department of Mathematics and Science, University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA.
E-mail: fsmarandache@gmail.com

*Correspondence: S. Satham Hussain; sathamhussain5592@gmail.com

Abstract: In this work, the new concept of neutrosophic vague graphs are introduced form the ideas of neutrosophic vague sets. Moreover, some remarkable properties of strong neutrosophic vague graphs, complete neutrosophic vague graphs and self-complementary neutrosophic vague graphs are investigated and the proposed concepts are described with suitable examples.

Keywords: Neutrosophic vague graphs, Complete neutrosophic vague graph, Strong neutrosophic vague graph.

1. Introduction

Initially, vague set theory was first investigated by Gau and Buehrer [30] which is an extension of fuzzy set theory. Vague sets are regarded as a special case of context-dependent fuzzy sets. In order to handle the indeterminate and inconsistent information, the neutrosophic set is introduced by the author Smarandache and studied extensively about neutrosophic set [14] - [37] and it receives applications in many fields. In neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are defined completely independent, if the sum of these values lies between 0 and 3. The new developments of neutrosophic theory are extensively studied in [1] - [6]. Molodtsov [28] firstly introduced the soft set theory as a general mathematical tool to with uncertainty and vagueness. Akram [9] established the certain notions including strong neutrosophic soft graphs and complete neutrosophic soft graphs. The authors [7] first introduce the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set to improve the reasonability of decision making in reality. Neutrosophic vague set theory are introduced in [8]. The operations on single valued neutrosophic graphs are studied in [11]. Further, intuitionistic neutrosophic soft set and graphs are developed in [13]. Now, the domination in vague graphs and its is application are discussed in [16]. Intuitionistic neutrosophic soft set are studied in [18]. Interval valued neutrosophic graphs are developed by the author Broumi [22,23,25]. Interval neutrosophic vague sets are intiated in [31]. Motivation of the aforementioned works, we introduced the concept of neutrosophic vague graphs and strong neutrosophic vague graphs. This is a new developed theory which is the combination of neutrsophic graphs and vague graphs. Here the sum of Truth, Intermediate and False membership value lies between 0 and 2 since the truth and false membership are dependent variables. Here the complement of neutrosophic vague graphs is again neutrosophic vague graphs. This development theory will be applied in Operation Research, Social network problems. Particularly, fake profile is one of the big problems of social networks. Now, it has become easier to create a fake profile. People often use fake profile to insult, harass someone, involve in unsocial activities, etc. This model can be reformulated in the abstract form to be applied in neutrosophic vague graphs. The major contribution of this work as follows:

- Newly introduced neutrosophic vague graphs, neutrosophic vague subgraphs, constant neutrosophic vague graphs with examples.
- Further we presented some remarkable properties of strong neutrosophic vague graphs with suitable examples.

2 Preliminaries

Definition 2.1 [10] A vague set A on a non empty set X is a pair (T_A, F_A), where $T_A: X \to [0,1]$ and $F_A: X \to [0,1]$are true membership and false membership functions, respectively, such that

$$0 \le T_A(r) + F_A(r) \le 1 \text{ for any } r \in X.$$

Let X and Y be two non-empty sets. A vague relation R of X to Y is a vague set R on $X \times Y$ that is $R = (T_R, F_R)$, where $T_R: X \times Y \to [0,1], F_R: X \times Y \to [0,1]$ which satisfies the condition:

$$0 \le T_R(r,s) + F_R(r,s) \le 1 \text{ for any } r \in X.$$

Let $G = (R, S)$ be a graph. A pair $G = (J, K)$ is named as a vague graph on G^* or a vague graph where $J = (T_J, F_J)$ is a vague set on R and $K = (T_K, F_K)$ is a vague set on $S \subseteq R \times R$ such that for each $rs \in S$,

$$T_K(rs) \le (T_J(r) \wedge T_J(s)) \& F_K(rs) \ge (T_J(r) \vee F_J(s)).$$

Definition 2.2 [9] A Neutrosophic set $A \subset B$, (i.e) $A \subseteq C$ if $\forall r \in X, T_A(r) \le T_B(r), I_A(r) \ge I_B(r)$and $F_A(r) \ge F_B(r)$.

Definition 2.3 [12, 26, 30] Let X be a space of points (objects), with a generic elements in X known by r. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(r)$, indeterminacy-membership function $I_A(r)$ and falsity-membership-function $F_A(r)$. For each point r in X, $T_A(r), F_A(r), I_A(r) \in [0,1]$.

$$A = \{r, T_A(r), F_A(r), I_A(r)\} \text{ and } 0 \le T_A(r) + I_A(r) + F_A(r) \le 3$$

Definition 2.4 [19, 20] A neutrosophic graph is represented as a pair $G^* = (V, E)$ where

(i) $R = \{r_1, r_2, .., r_n\}$ such that $T_1 = R \to [0,1]$, $I_1 = R \to [0,1]$ and $F_1 = R \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

$$0 \le T_A(r) + I_A(r) + F_A(r) \le 3$$

(ii) $S \subseteq R \times R$ where $T_2 = S \to [0,1]$, $I_2 = S \to [0,1]$ and $F_2 = S \to [0,1]$ are such that

$$T_2(rs) \le \{T_1(r) \wedge T_1(s)\},$$

$$I_2(rs) \ge \{I_1(r) \vee I_1(s)\},$$

$$F_2(rs) \ge \{F_1(r) \vee F_1(s)\},$$

$$\text{and } 0 \le T_2(rs) + I_2(rs) + F_2(rs) \le 3, \forall rs \in R$$

Definition 2.5 [8] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as

$$A_{NV} = \{\langle r, \hat{T}_{A_{NV}}(r), \hat{I}_{A_{NV}}(r), \hat{F}_{A_{NV}}(r)\rangle, r \in X\}$$

whose truth-membership, indeterminacy membership and falsity-membership function is defined as

$$\hat{T}_{A_{NV}}(r) = [\hat{T}^-(r), \hat{T}^+(r)], [\hat{I}^-(r), \hat{I}^+(r)], [\hat{F}^-(r), \hat{F}^+(r)],$$

where $T^+(r) = 1 - F^-(r), F^+(r) = 1 - T^-(r)$, and $0 \le T^-(r) + I^-(r) + F^-(r) \le 2$.

Definition 2.6 [8] The complement of NVS A_{NV} is denoted by A^c_{NV} and it is represented by

$$\hat{T}^c_{A_{NV}}(r) = [1 - T^+(r), 1 - T^-(r)],$$
$$\hat{I}^c_{A_{NV}}(r) = [1 - I^+(r), 1 - I^-(r)],$$
$$\hat{F}^c_{A_{NV}}(r) = [1 - F^+(r), 1 - F^-(r)],$$

Definition 2.7 [8] Let A_{NV} and B_{NV} be two NVSs of the universe U. If for all $r_i \in U$,

$$\hat{T}_{A_{NV}}(r_i) = \hat{T}_{B_{NV}}(r_i), \hat{I}_{A_{NV}}(r_i) = \hat{I}_{B_{NV}}(r_i), \hat{F}_{A_{NV}}(r_i) = \hat{F}_{B_{NV}}(r_i)$$

then the NVS A_{NV} are included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$ where $1 \le i \le n$.

Definition 2.8 [7] The union of two NVSs A_{NV} and B_{NV} is a NVSs, C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NV} and B_{NV} by

$$\hat{T}_{C_{NV}}(x) = [\max(\hat{T}^-_{A_{NV}}(r)\hat{T}^-_{B_{NV}}(r)), \max(\hat{T}^+_{A_{NV}}(r)\hat{T}^+_{B_{NV}}(r))]$$
$$\hat{I}_{C_{NV}}(x) = [\min(\hat{I}^-_{A_{NV}}(r)\hat{I}^-_{B_{NV}}(r)), \min(\hat{I}^+_{A_{NV}}(r)\hat{I}^+_{B_{NV}}(r))]$$
$$\hat{F}_{C_{NV}}(x) = [\min(\hat{F}^-_{A_{NV}}(r)\hat{F}^-_{B_{NV}}(r)), \min(\hat{F}^+_{A_{NV}}(r)\hat{F}^+_{B_{NV}}(r))]$$

Definition 2.9 [7] The intersection of two NVSs A_{NV} and B_{NV} is a NVSs C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{NV} and B_{NV} by

$$\hat{T}_{C_{NV}}(x) = [\min(\hat{T}^-_{A_{NV}}(r)\hat{T}^-_{B_{NV}}(r)), \min(\hat{T}^+_{A_{NV}}(r)\hat{T}^+_{B_{NV}}(r))]$$
$$\hat{I}_{C_{NV}}(x) = [\max(\hat{I}^-_{A_{NV}}(r)\hat{I}^-_{B_{NV}}(r)), \max(\hat{I}^+_{A_{NV}}(r)\hat{I}^+_{B_{NV}}(r))]$$
$$\hat{F}_{C_{NV}}(x) = [\max(\hat{F}^-_{A_{NV}}(r)\hat{F}^-_{B_{NV}}(r)), \max(\hat{F}^+_{A_{NV}}(r)\hat{F}^+_{B_{NV}}(r))]$$

3 NEUTROSOPHIC VAGUE GRAPH

Definition 3.1 Let $G^* = (R, S)$ be a graph. A pair $G = (J, K)$ is named as a neutrosophic vague graph (NVG) on G^* or a neutrosophic graph where $J = (\hat{T}_J, \hat{I}_J, \hat{F}_J)$ is a neutrosophic vague set on R and $K = (\hat{T}_K, \hat{I}_K, \hat{F}_K)$ is a neutrosophic vague set $S \subseteq R \times R$ where

(1) $R = \{r_1, r_2, \ldots, r_n\}$ such that

$$T_J^-: R \to [0,1], I_J^-: R \to [0,1], F_J^-: R \to [0,1]$$

which satisfies the condition $F_J^- = [1 - T_J^+]$

$$T_J^+: R \to [0,1], I_J^+: R \to [0,1], F_J^+: R \to [0,1]$$

which satisfies the condition $F_J^+ = [1 - T_1^-]$ indicates the degree of truth membership function, indeterminacy membership and falsity membership of the element $r_i \in R$., and

$$0 \le T_J^-(r_i) + I_J^-(r_i) + F_J^-(r_i) \le 2.$$
$$0 \le T_J^+(r_i) + I_J^+(r_i) + F_J^+(r_i) \le 2.$$

(2) $S \subseteq R \times R$ where

$$T_K^-: R \times R \to [0,1], I_K^-: R \times R \to [0,1], F_K^-: R \times R \to [0,1]$$
$$T_K^+: R \times R \to [0,1], I_K^+: R \times R \to [0,1], F_K^+: R \times R \to [0,1]$$

indicates the degree of truth membership function, indeterminacy membership and falsity membership of the element $r_i, r_j \in S$. respectively and such that

$$0 \le T_K^-(r_i) + I_K^-(r_i) + F_K^-(r_i) \le 2.$$
$$0 \le T_K^+(r_i) + I_K^+(r_i) + F_K^+(r_i) \le 2.$$

such that

$$T_K^-(rs) \le \{T_J^-(r) \wedge T_J^-(s)\}$$
$$I_K^-(rs) \le \{I_J^-(r) \wedge I_J^-(s)\}$$
$$F_K^-(rs) \le \{F_J^-(r) \vee F_J^-(s)\},$$

similarly

$$T_K^+(rs) \le \{T_J^+(r) \wedge T_J^+(s)\}$$
$$I_K^+(rs) \le \{I_J^+(r) \wedge I_J^+(s)\}$$
$$F_K^+(rs) \le \{F_J^+(r) \vee F_J^+(s)\}.$$

Example 3.2 A neutrosophic vague graph $G = (J, K)$ such that $J = \{a, b, c\}$ and $K = \{ab, bc, ca\}$ indicated by

$$\hat{a} = T[0.5,0.6], I[0.4,0.3], F[0.4,0.5], \hat{b} = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6],$$
$$\hat{c} = T[0.4,0.4], I[0.5,0.3], F[0.6,0.6]$$
$$a^- = (0.5,0.4,0.4), b^- = (0.4,0.7,0.4), c^- = (0.4,0.5,0.6)$$
$$a^+ = (0.6,0.3,0.5), b^+ = (0.6,0.3,0.6), c^+ = (0.4,0.3,0.6)$$

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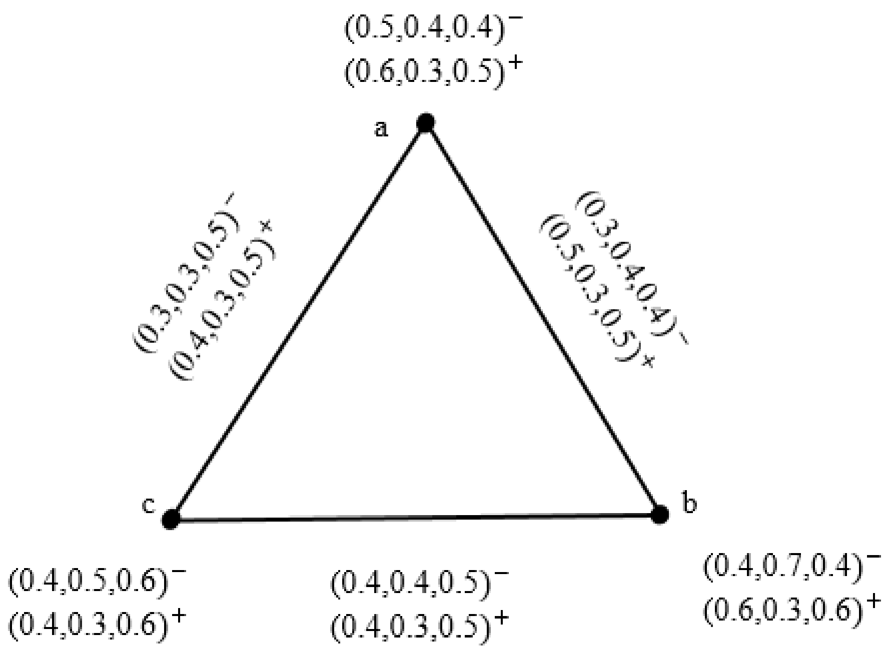


Figure 1
NEUTROSOPHIC VAGUE GRAPH

Definition 3.3 A neutrosophic vague graph $H = (J'(r), K'(r))$ is meant to be a neutrosophic vague subgraph of the NVG $G = (J, K)$ if $J'(r) \subseteq J(r)$ and $K'(rs) \subseteq K'(rs)$ in other words, if

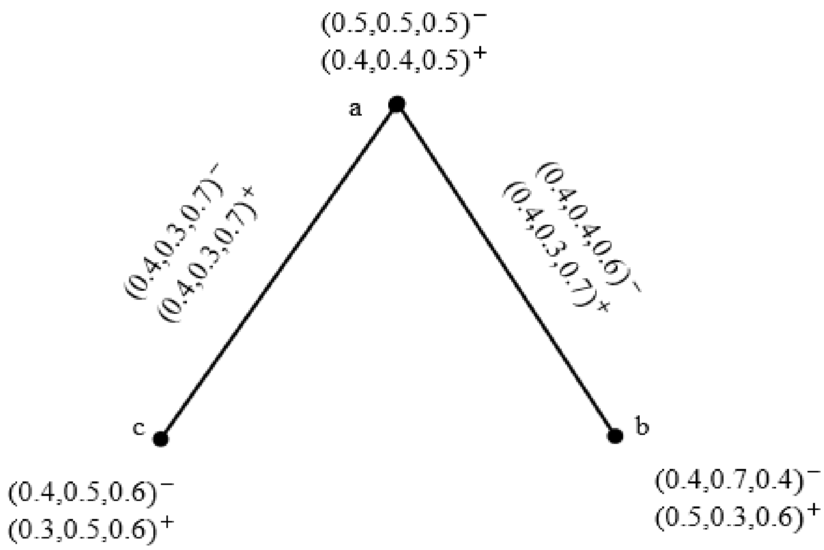
$$\widehat{T}_J^{'-}(r) \le \widehat{T}_J^{-}(r)$$
$$\widehat{I}_J^{'-}(r) \le \widehat{I}_J^{-}(r)$$

$$\hat{F}_J^{'-}(r) \geq \hat{F}_J^-(r) \forall r \in R$$

and

$$\hat{T}_K^{'+}(rs) \leq \hat{T}_K^-(rs)$$
$$\hat{I}_K^{'+}(rs) \leq \hat{I}_K^-(rs)$$
$$\hat{F}_K^{'+}(rs) \geq \hat{F}_K^-(rs) \forall (rs) \in S.$$

Example 3.4 A neutrosophic vague graph $G = (J, K)$ in Figure (1)



H_1 Figure 2

H_1 is a neutrosophic vague subgraph of G

Definition 3.5 The two vertices are said to be adjacent in a neutrosophic vague graph $G = (J, K)$ if

$$T_K^-(rs) = \{T_J^-(r) \wedge T_J^-(s)\}$$
$$I_K^-(rs) = \{I_J^-(r) \wedge I_J^-(s)\}$$
$$F_K^-(rs) = \{F_J^-(r) \vee F_J^-(s)\} \quad \text{and}$$
$$T_K^+(rs) = \{T_J^+(r) \wedge T_J^+(s)\}$$
$$I_K^+(rs) = \{I_J^+(r) \wedge I_J^+(s)\}$$
$$F_K^+(rs) = \{F_J^+(r) \vee F_J^+(s)\}$$

In this case, r and s are known to be neighbours and (rs) is incident at r and s also.

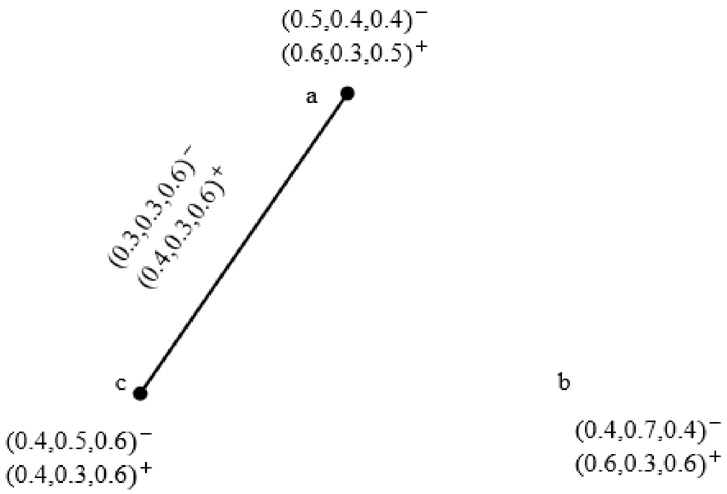
Definition 3.6 A path ρ in a NVG $G = (J, K)$ is a sequence of distinct vertices $r_0, r_1, \ldots, r_n$ such that $T_K^-(r_{i-1}, r_1) > 0$, $I_K^-(r_{i-1}, r_1) > 0$, $F_K^-(r_{i-1}, r_1) > 0$, $T_K^+(r_{i-1}, r_1) > 0$, $I_K^+(r_{i-1}, r_1) > 0$, $F_K^+(r_{i-1}, r_1) > 0$, for $0 \leq i \leq 1$, here $n \leq 1$ is called the length of the path ρ. A single node or single vertex r_i may all consider as a path.

Definition 3.7 A neutrosophic vague graph $G = (J, K)$ is said to be connected if every pair of vertices has at least on neutrosophic vague path between them otherwise it is disconnected.

Definition 3.8 A vertex $r_i \in R$ of neutrosophic vague graph $G = (J, K)$ called as a pendent vertex if there is no effective edge incident at x_i.

Definition 3.9 A vertex in a neutrosophic vague graph $G = (J, K)$ having exactly one neighbours is called a isolated vertex. otherwise,it is called non-isolated vertex. An edge in a neutrosophic vague graph incident with a isolated vertex is called a isolated edge other words it is called non-isolated edge. A vertex in a neutrosophic vague graph adjacent to the isolated vertex is called an support of the pendent edge.

Example 3.10 A neutrosophic vague graph $G = (J, K)$ in figure (3)



H_1 Figure 3

NEUTROSOPHIC VAGUE GRAPH

In figure (3), the neutrosophic vague vertex b is an pendent vertex.

Definition 3.11 Let $G = (J, K)$ be a neutrosophic vague graph. Then the degree of a vertex $r \in G$ is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex r represented by $d(r) = ([d^-_{T_J}(r), d^+_{T_J}(r)], [d^-_{I_J}(r), d^+_{I_J}(r)], [d^-_{F_J}(r), d^+_{F_J}(r)])$ where

$d^-_{T_J}(r) = \sum_{r\neq s} T^-_K(rs), d^+_{T_J}(r) = \sum_{r\neq s} T^+_K(rs)$ indicates the degree of truth membership vertex

$d^-_{I_J}(r) = \sum_{r\neq s} I^-_K(rs), d^+_{I_J}(r) = \sum_{r\neq s} I^+_K(rs)$ indicates the degree of indeterminacy membership vertex

$d^-_{F_J}(r) = \sum_{r\neq s} F^-_K(rs), d^+_{F_J}(r) = \sum_{r\neq s} F^+_K(rs)$ indicates the degree of falsity membership vertex for all $r, s \in J$.

Example 3.12 A neutrosophic vague graph $G = (J, K)$ in figure (1), we have the degree of each vertex as follows

$$d^-_T(a) = (0.6,0.7,0.9), d^-_F(b) = (0.7,0.8,1.3), d^-_F(c) = (0.7,0.7,1.0),$$

$$d^+_T(a) = (0.9,0.6,1.0), d^+_F(b) = (0.9,0.6,1.0), d^+_F(c) = (0.8,0.6,1.0)$$

Definition 3.13 A neutrosophic vague graph $G = (J, K)$ is called constant if degree of each vertex is $A = (A_1, A_2, A_3)$ that is $d(x) = (A_1, A_2, A_3)$ for all $x \in V$.

Example 3.14 Consider a neutrosophic vague graph $G = (J, K)$ in figure (4)defined by

$$\hat{a} = T[0.5,0.6], I[0.6,0.4], F[0.4,0.5], \hat{b} = T[0.4,0.4], I[0.4,0.6], F[0.6,0.6],$$
$$\hat{c} = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6], \hat{d} = T[0.6,0.4], I[0.3,0.7], F[0.6,0.4]$$
$$a^- = (0.5,0.6,0.4), b^- = (0.4,0.4,0.6), c^- = (0.4,0.7,0.4), d^- = (0.6,0.3,0.6)$$
$$a^+ = (0.6,0.4,0.5), b^+ = (0.4,0.6,0.6), c^+ = (0.6,0.3,0.6), d^+ = (0.4,0.7,0.4).$$

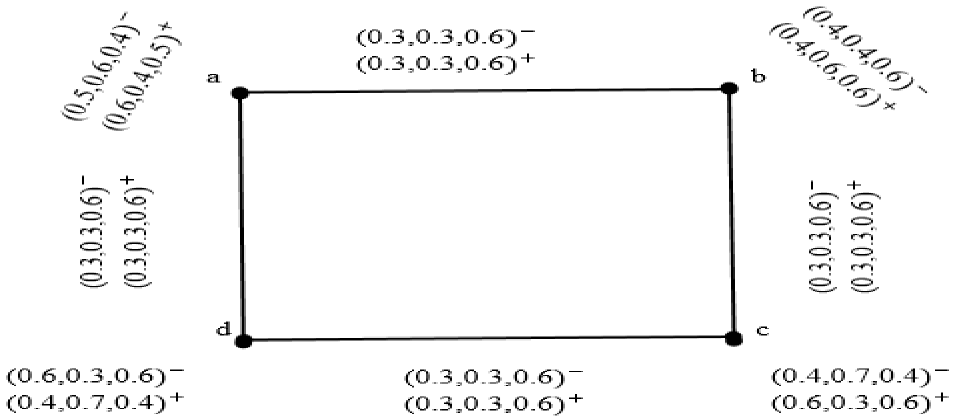


Figure 4

CONSTANT NEUTROSOPHIC VAGUE GRAPH

Clearly as it is seen in figure(4) G is constant neutrosophic vague graph since the degree of $(\hat{a}, \hat{b}, \hat{c}, \hat{d})$ and $\hat{d} = (0.6,0.6,1.2)$.

Definition 3.15 The complement of neutrosophic vague graph $G = (J, K)$ on G^* is a neutrosophic vague graph G^c on G^* where

- $J^c(r) = J(r)$
- $T_J^{-c}(r) = T_J^-(r), I_J^{-c}(r) = I_J^-(r), F_J^{-c}(r) = F_J^-(r)$ for all $r \in R$.
- $T_J^{+c}(r) = T_J^+(r), I_J^{+c}(r) = I_J^+(r), F_J^{+c}(r) = F_J^+(r)$ for all $r \in R$.
- $T_K^{-c}(rs) = \{T_J^-(r) \wedge T_J^-(s)\} - T_K^-(rs)$

$$I_K^{-c}(rs) = \{I_J^-(r) \wedge I_J^-(s)\} - I_K^-(rs)$$
$$F_K^{-c}(rs) = \{F_J^-(r) \vee F_J^-(s)\} - F_K^-(rs) \text{ for all } (rs) \in S$$

- $T_K^{+c}(rs) = \{T_J^+(r) \wedge T_J^+(s)\} - T_K^+(rs)$

$$I_K^{+c}(rs) = \{I_J^+(r) \wedge I_J^+(s)\} - I_K^+(rs)$$
$$F_K^{+c}(rs) = \{F_J^+(r) \vee F_J^+(s)\} - F_K^+(rs) \text{ for all } (rs) \in S$$

4 Strong Neutrosophic Vague Graphs

Definition 4.1 A neutrosophic vague graph $G = (J, K)$ of $G^* = (R, S)$ is named as a strong neutrosophic vague graph if

$$T_K^-(rs) = \{T_J^-(r) \wedge T_J^-(s)\}$$
$$I_K^-(rs) = \{I_J^-(r) \wedge I_J^-(s)\}$$
$$F_K^-(rs) = \{F_J^-(r) \vee F_J^-(s)\} \quad \text{and}$$
$$T_K^+(rs) = \{T_J^+(r) \wedge T_J^+(s)\}$$
$$I_K^+(rs) = \{I_J^+(r) \wedge I_J^+(s)\}$$
$$F_K^+(rs) = \{F_J^+(r) \vee F_J^+(s)\} \text{ for all } (rs \in S)$$

Example 4.2 A neutrosophic vague graph $G = (J, K)$ such that $J = \{a, b, c\}$ and $K = \{ab, bc, ca\}$ defined by $\hat{a} = T[0.3,0.4], I[0.4,0.6], F[0.6,0.7], \hat{b} = T[0.6,0.4], I[0.6,0.7], F[0.6,0.4]$,

$$\hat{c} = T[0.7,0.7], I[0.5,0.6], F[0.3,0.3]$$

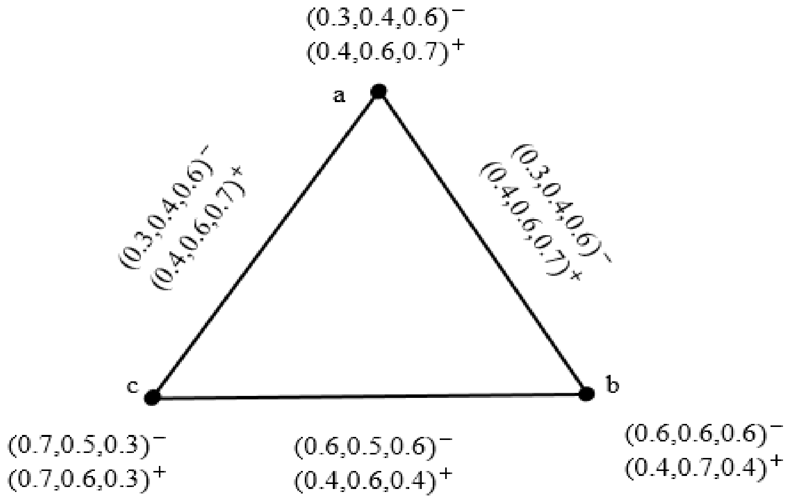


Figure 5

STRONG NEUTROSOPHIC VAGUE GRAPH

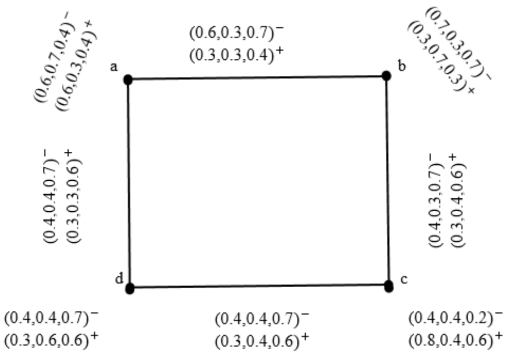
Remark 4.3 If $G = (J, K)$ is a neutrosophic vague graph on G^* then from above definition, it follow that G^{c^c} is given by the neutrosophic vague graph $G^{c^c} = J^{c^c}, K^{c^c}$ on G^* where

- $(J^c)^c(r) = J(r)$
- $(T_J^{-c})^c(r) = T_J^-(r), I_J^{-c}(r) = I_J^-(r), F_J^{-c}(r) = F_J^-(r)$ for all $r \in R$.
- $(T_J^{+c})^c(r) = T_J^+(r), I_J^{+c}(r) = I_J^+(r), F_J^{+c}(r) = F_J^+(r)$ for all $r \in R$.
- $(T_K^{-c})^c(rs) = \{T_J^-(r) \wedge T_J^-(s)\} - T_K^-(rs)$
  $(I_K^{-c})^c(rs) = \{I_J^-(r) \wedge I_J^-(s)\} - I_K^-(rs)$
  $(F_K^{-c})^c(rs) = \{F_J^-(r) \vee F_J^-(s)\} - F_K^-(rs)$ for all $(rs) \in S$
- $(T_K^{+c})^c(rs) = \{T_J^+(r) \wedge T_J^+(s)\} - T_K^+(rs)$
  $(I_K^{+c})^c(rs) = \{I_J^+(r) \wedge I_J^+(s)\} - I_K^+(rs)$
  $(F_K^{+c})^c(rs) = \{F_J^+(r) \vee F_J^+(s)\} - F_K^+(rs)$ for all $(rs) \in S$

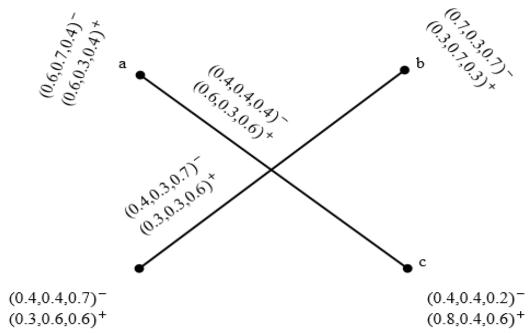
for any neutrosophic vague graph G, G^c is strong neutrosophic graph and $G \subseteq G^c$

Definition 4.4 A strong neutrosophic graph G is called self-complementary if $G \cong G^c$ where G^c is the complement of neutrosophic vague graph G.

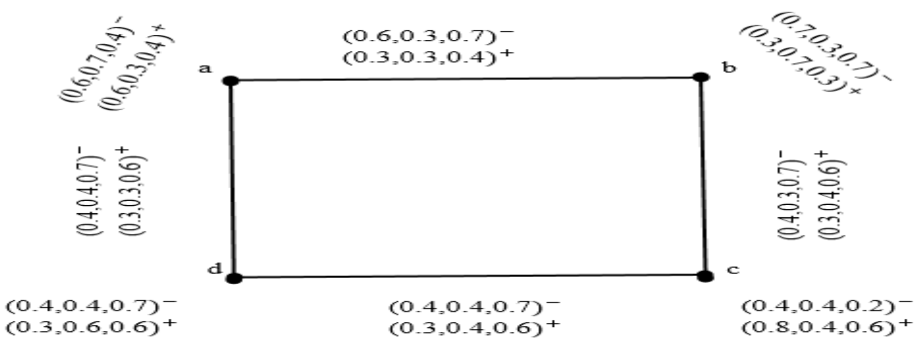
Example 4.5 A neutrosophic vague graph $G = (J, K)$ such that $J = \{a, b, c, d\}$ and $K = \{ab, bc, cd, da\}$ defined as follows: consider a neutrosophic vague graph G as in figure(6)



G STRONG NVG



G^c STRONG NVG



G^{c^c} STRONG NVG

Clearly, as it is seen in figure (6) $G \cong G^{c^c}$.

Hence G is self complementary.

Proposition 4.6 Let $G = (J, K)$ be a strong neutrosophic vague graph if

$$T_K^-(rs) = \{T_J^-(r) \wedge T_J^-(s)\}$$
$$I_K^-(rs) = \{I_J^-(r) \wedge I_J^-(s)\}$$
$$F_K^-(rs) = \{F_J^-(r) \vee F_J^-(s)\}$$
$$T_K^+(rs) = \{T_J^+(r) \wedge T_J^+(s)\}$$
$$I_K^+(rs) = \{I_J^+(r) \wedge I_J^+(s)\}$$
$$F_K^+(rs) = \{F_J^+(r) \vee F_J^+(s)\} \text{ for all } rs \in K$$

Then G is self complementary.

Proof. Let $G = (J, K)$ be a strong neutrosophic vague graph such that

$$\hat{T}_K(rs) = \frac{1}{2}\min[\hat{T}_J(r), \hat{T}_J(s)]$$

$$\hat{I}_K(rs) = \frac{1}{2}\min[\hat{I}_J(r), \hat{I}_J(s)]$$

$$\hat{F}_K(rs) = \frac{1}{2}\max\hat{F}_J(r), \hat{F}_J(s)$$

for all $rs \in J$ then $G \approx G^{c^c}$ under the identity map $I: J \to J$. Hence G is self complementary

Proposition 4.7 Let G be a self complementary neutrosophic vague graph then

$$\sum_{r\neq s} \hat{T}_K(rs) = \frac{1}{2}\sum_{r\neq s} \min\{\hat{T}_J(r), \hat{T}_J(s)\}$$

$$\sum_{r\neq s} \hat{I}_K(rs) = \frac{1}{2}\sum_{r\neq s} \min\{\hat{I}_J(r), \hat{I}_J(s)\}$$

$$\sum_{r\neq s} \hat{F}_K(rs) = \frac{1}{2}\sum_{r\neq s} \max\{\hat{F}_J(r), \hat{F}_J(s)\}$$

Proof. If G be an self complementary neutrosophic vague graph then there exist an isomorphism $f: J_1 \to J_2$ satisfy

$$\hat{T}^c_{J_1}(f(r)) = \hat{T}_{J_1}(f(r)) = \hat{T}_{J_1}(r)$$
$$\hat{I}^c_{J_1}(f(r)) = \hat{I}_{J_1}(f(r)) = \hat{I}_{J_1}(r)$$
$$\hat{F}^c_{J_1}(f(r)) = \hat{F}_{J_1}(f(r)) = \hat{F}_{J_1}(r)$$

and

$$\hat{T}^c_{K_1}(f(r), f(s)) = \hat{T}_{K_1}(f(r), f(s)) = \hat{T}_{K_1}(rs)$$
$$\hat{I}^c_{K_1}(f(r), f(s)) = \hat{I}_{K_1}(f(r), f(s)) = \hat{I}_{K_1}(rs)$$
$$\hat{F}^c_{K_1}(f(r), f(s)) = \hat{F}_{K_1}(f(r), f(s)) = \hat{F}_{K_1}(rs)$$

we have $\hat{T}^c_{K_1}(f(r), f(s)) = \min(\hat{T}^c_{J_1}(r), \hat{T}^c_{J_1}(s)) - \hat{T}_{K_1}(f(r), f(s))$. i.e, $\hat{T}_{K_1}(rs) = \min(\hat{T}^c_{J_1}(r), \hat{T}^c_{J_1}(s)) - \hat{T}_{K_1}(f(r), f(s))$. $\hat{T}_{K_1}(rs) = \min(\hat{T}^c_{J_1}(r), \hat{T}^c_{J_1}(s)) - \hat{T}_{K_1}(rs)$. that is

$$\textstyle\sum_{r\neq s} \hat{T}_{K_1}(rs) + \sum_{r\neq s} \hat{T}_{K_1}(rs) = \sum_{r\neq s} \min(\hat{T}_{J_1}(r), \hat{T}_{J_1}(s)).$$

Similarly, $\sum_{r\neq s} \hat{I}_{K_1}(rs) + \sum_{r\neq s} \hat{I}_{K_1}(rs) = \sum_{r\neq s} \min(\hat{I}_{J_1}(r), \hat{I}_{J_1}(s))$

$$\sum_{r\neq s} \hat{F}_{K_1}(rs) + \sum_{r\neq s} \hat{F}_{K_1}(rs) = \sum_{r\neq s} \max(\hat{F}_{J_1}(r), \hat{F}_{J_1}(s))$$

$$2\sum_{r\neq s} \hat{T}_{K_1}(rs) = \sum_{r\neq s} \min(\hat{T}_{J_1}(r), \hat{T}_{J_1}(s))$$

$$2\sum_{r\neq s} \hat{I}_{K_1}(rs) = \sum_{r\neq s} \min(\hat{I}_{J_1}(r), \hat{I}_{J_1}(s))$$

$$2\sum_{r\neq s} \hat{F}_{K_1}(rs) = \sum_{r\neq s} \max(\hat{F}_{J_1}(r), \hat{F}_{J_1}(s))$$

from the equation of the proposition (4.8) holds.

Proposition 4.8 Let G_1 and G_2 be strong neutrosophic vague graph $\overline{G_1} \approx \overline{G_2}$(isomorphism)

Proof. Assume that G_1 and G_2 are isomorphic there exist a bijective map $f: J_1 \to J_2$ satisfying,

$$\hat{T}_{J_1}(r) = \hat{T}_{J_2}(f(r)), \hat{I}_{J_1}(r) = \hat{I}_{J_2}(f(r)), \hat{F}_{J_1}(r) = \hat{F}_{J_2}(f(r)), \text{forallr} \in J_1$$

and

$$\hat{T}_{K_1}(rs) = \hat{T}_{K_2}(f(r), f(s))$$
$$\hat{I}_{K_1}(rs) = \hat{I}_{K_2}(f(r), f(s))$$
$$\hat{F}_{K_1}(r) = \hat{F}_{K_2}(f(r), f(s)) \forall rs \in K_1$$

by definition (4.3) we have

$T^c_{K_1}(rs) = \min(T_{J_1}(r), T_{J_1}(s)) - T_{K_1}(rs)$
$= \min(T_{J_2}f(r), T_{J_2}f(s)) - T_{K_2}(f(r)f(s))$
$= T^c_{K_2}(f(r)f(s))$
$I^c_{K_1}(rs) = \min(I_{J_1}(r), I_{J_1}(s)) - I_{K_1}(rs)$
$= \min(I_{J_2}f(r), I_{J_2}f(s)) - I_{K_2}(f(r)f(s))$
$= I^c_{K_2}(f(r)f(s))$
$F^c_{K_1}(rs) = \max(F_{J_1}(r), F_{J_1}(s)) - F_{K_1}(rs)$
$= \max(F_{J_2}f(r), F_{J_2}f(s)) - F_{K_2}(f(r)f(s))$
$= F^c_{K_2}(f(r)f(s))$

Hence $G^c_1 \approx G^c_2$ for all $(rs) \in K_1$

Definition 4.9 A neutrosophic vague graph $G = (J, K)$ is complete if

$$T^-_K(rs) = \{T^-_J(r) \wedge T^-_J(s)\}$$

$$I^-_K(rs) = \{I^-_J(r), I^-_J(s)\}$$

$$F^-_K(rs) = \{F^-_J(r) \vee F^-_J(s)\},$$

similarly,

$$T^+_K(rs) = \{T^+_J(r) \wedge T^+_J(s)\}$$

$$I^+_K(rs) = \{I^+_J(r) \wedge I^+_J(s)\}$$

$$F^+_K(rs) = \{F^+_J(r) \vee F^+_J(s)\} for r, s \in J.$$

Example 4.10 Consider a neutrosophic vague graph $G = (J, K)$ such that $J = \{a, b, c, d\}$ and $K = \{ab, bc, cd, da\}$ defined by

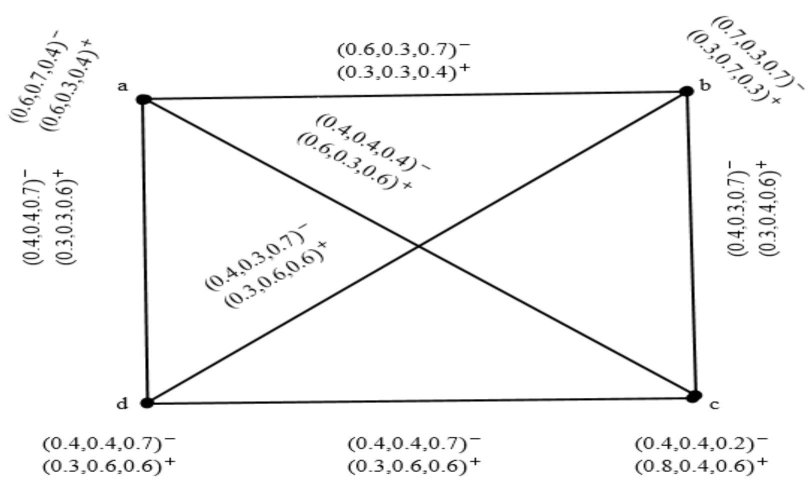


Figure 7

COMPLETE NEUTROSOPHIC VAGUE GRAPH

Definition 4.11 The complement of neutrosophic vague graph $G = (J, K)$ of $G^* = (V, E)$ is a neutrosophic vague complete graph $G = (J^c, K^c)$ on $G^* = (R, S^c)$ where

(1) $J^c(r_i) = J(r_i)$

(2) $\hat{T}_J^c(r_i) = \hat{T}_J(r_i), \hat{I}_J^c(r_i) = \hat{I}_J(r_i), \hat{F}_J^c(r_i) = \hat{F}_J(r_i)$ for all $r_i \in J$

(3)$\hat{T}_K^c(r_i s_j) = (\hat{T}_J(r_i) \wedge \hat{T}_J(s_j)) - \hat{T}_K(r_i s_j)$

$$\hat{I}_K^c(r_i s_j) = (\hat{I}_J(r_i) \wedge \hat{I}_J(s_j)) - \hat{I}_K(r_i, s_j)$$

$$\hat{F}_K^c(r_i s_j) = (\hat{F}_J(r_i) \vee \hat{F}_J(s_j)) - \hat{F}_K(r_i s_j) \quad \text{for all } (r_i s_j) \in K$$

Proposition 4.12 The complement of complete neutrosophic vague graph with no edge. or if G is complete then G^c the edge is empty.

Proof. Let $G = (J, K)$ be a complete neutrosophic vague graph so

$$\hat{T}_K(r_i s_j) = (\hat{T}_J(r_i) \wedge \hat{T}_J(s_j))$$

$$\hat{I}_K(r_i s_j) = (\hat{I}_J(r_i) \wedge \hat{T}_J(s_j))$$

$$\hat{F}_K(r_i s_j) = (\hat{F}_J(r_i) \vee \hat{F}_J(s_j)) \forall (r_i, s_j) \in J$$

Hence in G^c. Now,

$$\hat{T}_K^c(r_i s_j) = (\hat{T}_J(r_i) \wedge \hat{T}_J(s_j)) - \hat{T}_K(r_i s_j)$$

$= (\hat{T}_J(r_i) \wedge \hat{T}_J(s_j)) - (\hat{T}_J(r_i) \wedge \hat{T}_J(s_j)) \ \forall\, i, j, \dots, n$

$= 0 \ \forall, i, j, \dots, n.$

and

$\hat{I}_K^c(r_i s_j) = (\hat{I}_J(r_i) \wedge \hat{I}_J(s_j)) - \hat{I}_K(r_i s_j)$

$= (\hat{I}_J(r_i) \wedge \hat{I}_J(s_j)) - (\hat{I}_J(r_i) \wedge \hat{I}_J(s_j)) \forall\, i, j, \dots, n$

$= 0 \forall\, i, j, \dots, n.$

Similarly $\hat{F}_K^c(r_i, s_j) = 0$. Thus,$(\hat{T}_K(r_i, s_j), \hat{I}_K(r_i, s_j), \hat{F}_K(r_i, s_j)) = (0,0,0)$

Hence, the edge set of G^c is empty if G is a complete neutrosophic vague graph.

Conclusion and futute directions:

This work dealt with the new concept of neutrosophic vague graphs. Moreover, some remarkable properties of strong neutrosophic vague graphs, complete neutrosophic vague graphs and self-complementary neutrosophic vague graphs have been investigated and the proposed concepts were described with suitable examples. Further we can extend to investigate the regular and isomorphic properties of the proposed graph. This can be applied to social network model and operations research.

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