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On New Measures of Uncertainty for Neutrosophic Sets

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Abstract: The notion of entropy of single valued neutrosophic sets (SVNS) was first introduced by Majumdar and Samanta in [10]. In this paper some problems with the earlier definition of entropy has been pointed out and a new modified definition of entropy for SVNS has been proposed. Next four new types of entropy functions were defined with examples. Superiority of this new definition over the earlier definition of entropy has been discussed with proper examples.

Keywords: Single valued neutrosophic sets, Neutrosophic element, Neutrosophic cube, Entropy, Entropy function, Intuitionistic fuzzy sets, Measure of uncertainty.

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1. Introduction.

The first successful attempt towards incorporating non-probabilistic uncertainty, i.e. uncertainty which is not caused by randomness of an event, into mathematical modelling was made in 1965 by L. A. Zadeh [20] through his remarkable theory on fuzzy sets (FST). A fuzzy set is a set where each element of the universe belongs to it but with some 'grade' or 'degree of belongingness' which lies between 0 and 1 and such grades are called membership value of an element in that set. This gradation concept is very well suited for applications involving imprecise data such as natural language processing or in artificial intelligence, handwriting and speech recognition etc. Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical situations specially problems involving incomplete information. Further generalization of this fuzzy set was made by K. Atanassov [1] in 1986, which is known as Intuitionistic fuzzy set (IFS). In IFS, instead of one 'membership grade', there is also a 'non-membership grade' attached with each element. Furthermore there is a restriction that the sum of these two grades is less or equal to unity. In IFS the 'degree of non-belongingness' is not independent but it is dependent on the 'degree of belongingness'. A fuzzy set can be considered as a special case of IFS where the 'degree of non-belongingness' of an element is exactly equal to 'one minus the degree of belongingness'. Intuitionistic fuzzy sets definitely have the ability to handle imprecise data of both complete and incomplete in nature. In applications like expert systems, belief systems, information fusion etc., where 'degree of non-belongingness' is equally important as 'degree of belongingness', intuitionistic fuzzy sets are quite useful. There are of course several other generalizations of Fuzzy as well as Intuitionistic fuzzy sets like L-fuzzy sets and intuitionistic L-fuzzy sets, interval valued fuzzy and intuitionistic fuzzy sets etc that have been developed and applied in solving many practical physical problems [2, 5, 6, 16]. In 1999, a new theory has been introduced by Florentin Smarandache [14] which is known as 'Neutrosophic logic'. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies between $[0^-, 1^+]$, the non-standard unit interval. Unlike in intuitionistic fuzzy sets, where the incorporated uncertainty is dependent on the degree of belongingness and degree of non belongingness, here the uncertainty present, i.e. the indeterminacy factor, is independent of truth and falsity values. Neutrosophic sets are indeed more general in nature than IFS as there are no constraints between the 'degree of truth', 'degree of indeterminacy' and 'degree of falsity'. All these degrees can individually vary within $[0^-, 1^+]$.

Smarandache [14] and Wang et. al. [17] introduced an instance of neutrosophic set known as single valued neutrosophic sets which were motivated from the practical point of view and that can be used in real scientific and engineering applications. Here the degree of truth, indeterminacy and falsity of any element of a neutrosophic set respectively lies within standard unit interval [0, 1]. The single valued neutrosophic set is a generalization of classical set, fuzzy set, intuitionistic fuzzy set and paraconsistant sets etc.

The organization of the rest of this paper is as follows: Some basic definitions and operations on SVNS are given in section 2. Section 3 discusses the notion of entropy of SVNS as defined in [10]. In section 4, some problems with the earlier definition of entropy have been pointed out using counterexample. A new definition of entropy of SVNS has been given in section 5. Section 6 concludes the paper.

2. Single Valued Neutrosophic sets.

A single valued neutrosophic set has been defined in [17] as follows:

Definition 2.1 Let X be a universal set. A Neutrosophic set A in X is characterized by a truth-membership function t_A, a indeterminacy-membership function i_A and a falsity-membership function f_A, $<t_A(x), i_A(x), f_A(x)>$: $X \to [0,1]$, are functions and $< t_A(x), i_A(x), f_A(x)$ is a single valued neutrosophic element or simply a neutrosophic element of A.

A single valued neutrosophic set A (SVNS in short) over a finite universe $X = \{x_1, x_2, x_3, ..., x_n\}$ is represented as

$$A = \sum_{i=1}^{n} \frac{x_i}{< t_A(x_i), i_A(x_i), f_A(x_i)}.$$

Example 2.2 Assume that $X = \{x_1, x_2, x_3\}$, where x_1 is capacity, x_2 is trustworthiness and, x_3 is price of a machine, be the universal set. The values of x_1, x_2, x_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a single valued Neutrosophic set of X defined by

$$A = {< 0.3, 0.4, 0.5 >}/{x_1} + {< 0.5, 0.2, 0.3 >}/{x_2} + {< 0.7, 0.2, 0.2 >}/{x_3}.$$

The following is a graphical representation of a single valued neutrosophic set. The elements of a single valued neutrosophic set, denoted henceforth by a neutrosophic element x(t,i,f), always remain inside and on a closed unit cube which henceforth will be called a ***neutrosophic cube***. Figure 1 describes a neutrosophic cube.

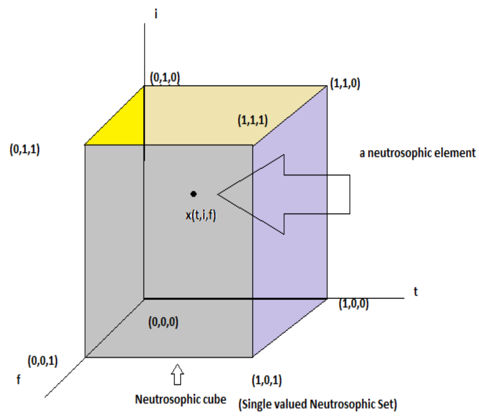

Figure 1

Next we give the definitions of complement and containment as follows:

Definition 2.3 The complement of a SVNS A is denoted by A^c and is defined by

$$t_{A^c}(x) = f_A(x);\ i_{A^c}(x) = 1 - i_A(x)$$
$$\&\ f_{A^c}(x) = t_A(x)\ \forall x \in X.$$

Definition 2.4 A SVNS A is contained in the other SVNS B, denoted as $A \subset B$, if and only if

$$t_A(x) \le t_B(x); i_A(x) \le i_B(x)$$
$$\&\ f_A(x) \ge f_B(x)\ \forall x \in X.$$

Two sets will be equal, i.e. $A = B$, iff $A \subset B \,\&\, B \subset A$.

Let us denote the collection of all SVNS over the universe X as $N(X)$.

Several operations like union and intersection has been defined on SVN sets and they satisfy most of the common algebraic properties of ordinary sets.

Definition 2.5 The union of two SVNS $A \& B$ is a SVNS C, *written as* $C = A \cup B$, which is defined as follows:

$$t_C(x) = \max(t_A(x), t_B(x)); \; i_C(x) = \max(i_A(x), i_B(x))$$
$$\& \; f_C(x) = \min(f_A(x), f_B(x)) \; \forall x \in X.$$

Definition 2.6 The intersection of two SVNS $A \& B$ is a SVNS C, *written as* $C = A \cap B$, which is defined as follows:

$$t_C(x) = \min(t_A(x), t_B(x)); \; i_C(x) = \min(i_A(x), i_B(x))$$
$$\& \; f_C(x) = \max(f_A(x), f_B(x)) \; \forall x \in X.$$

For practical purpose, throughout the rest of this chapter, we have considered only SVNS over a finite universe.

Next two operators, namely 'truth favourite' and 'falsity favourite' are defined to remove indeterminacy in the SVNS and transform it into an IFS or a paraconsistant set.

Definition 2.7 The truth favourite of a SVNS A is again a SVNS B written as $B = \Delta A$, which is defined as follows:

$$T_B(x) = \min(T_A(x) + I_A(x), 1)$$
$$I_B(x) = 0$$
$$F_B(x) = F_A(x), \; \forall x \in X.$$

Definition 2.8 The falsity favourite of a SVNS A is again a SVNS B written as $B = \nabla A$, which is defined as follows:

$$T_B(x) = T_A(x)$$
$$I_B(x) = 0$$
$$F_B(x) = \min(F_A(x) + I_A(x), 1), \; \forall x \in X.$$

The next two examples of truth & falsity favourite respectively of two given SVNS:

Example 2.9 Here the SVNS is A and the truth and falsity favourite sets are defined as follows:

$$A = {<0.3, 0.1, 0.5>}/{x_1} + {<0.5, 0.2, 0.3>}/{x_2} + {<0.4, 0.2, 0.2>}/{x_3}, \; then$$

$$B = \Delta A = {<0.4, 0.0, 0.5>}/{x_1} + {<0.7, 0.0, 0.3>}/{x_2} + {<0.6, 0.0, 0.2>}/{x_3} \; and$$
$$C = \nabla A = {<0.3, 0.0, 0.6>}/{x_1} + {<0.5, 0.0, 0.5>}/{x_2} + {<0.4, 0.0, 0.4>}/{x_3}.$$

Here both B and C are IFS.

Example 2.10 Again consider the neutrosophic set A given in example 2.2, then

$$B = \Delta A = {<0.7, 0.0, 0.5>}/{x_1} + {<0.7, 0.0, 0.3>}/{x_2} + {<0.9, 0.0, 0.2>}/{x_3} \; and$$
$$C = \nabla A = {<0.3, 0.0, 0.9>}/{x_1} + {<0.5, 0.0, 0.5>}/{x_2} + {<0.7, 0.0, 0.4>}/{x_3}.$$

Here both B and C are paraconsistant sets.

3. Entropy of Single Valued Neutrosophic sets.

Entropy can be considered as a measure of uncertainty about the information contained by a set. Generally crisp sets do not possess any entropy because there is no uncertainty about its members. But other non-crisp sets like fuzzy, intuitionistic fuzzy or vague etc, every set contain uncertain information of different types and hence there exits entropy for them. Here the SVNS are also capable of handling uncertain data, therefore as a natural consequence we are also interested in finding the entropy of a single valued neutrosophic set. Shannon [13] first introduced the notion of Probabilistic entropy. Shannon entropy has many applications in theory of communications. Entropy as a measure of fuzziness was first mentioned by Zadeh [21] in 1968. Later De Luca-Termini [4] axiomatized the non-probabilistic entropy.

$(DT1)\, E(A) = 0\, iff\;\; A \in 2^X$
$(DT2) E(A) = 1\, iff\;\; \mu_A(x) = 0.5, \forall x \in X$
$(DT3) E(A) \le E(B)\, iff\;\; A\, is\, less\;\; fuzzy\, than\, B,$
$i.e.\; if\; \mu_A(x) \le \mu_B(x) \le 0.5\; \forall x \in X.$ (3.1)
$or\; if\; \mu_A(x) \ge \mu_B(x) \ge 0.5, \forall x \in X.$
$(DT4) E(A^c) = E(A).$

Several other authors have investigated the notion of entropy. Kaufmann [7] proposed a distance based measure of fuzzy entropy; Yager [18, 19] gave another view of entropy or the degree of fuzziness of any fuzzy set in terms of lack of distinction between the fuzzy set and its complement. Kosko [8] investigated the fuzzy entropy in relation to a measure of subset hood. Szmidt & Kacprzyk [15] studied the entropy of intuitionistic fuzzy sets etc. Several applications of fuzzy entropy in solving many practical problems like image processing, inventory, economics can be found in literatures [3, 11, 12]. In [9, 10] the notion of entropy of single valued neutrosophic sets was first introduced. The following definition of entropy of a SVNS is due to [10]:

According to them the entropy E of a fuzzy set A should satisfy the following axioms:

Definition 3.3 Here in case of SVNS also we introduce the entropy as a function $E_N : N(X) \to [0,1]$ which satisfies the following axioms:

$(i) E_N(A) = 0$ if A is a crisp set

$(ii) E_N(A) = 1\, if$
$(t_A(x), i_A(x), f_A(x)) = (0.5, 0.5, 0.5) \forall x \in X$

$(iii) E_N(A) \ge E_N(B)\; if\;\; A\, is\, more$
uncertain than B

$i.e.\;\; t_A(x) + f_A(x) \le t_B(x) + f_B(x)$
$and\; |i_A(x) - i_{A^c}(x)| \le |i_B(x) - i_{B^c}(x)|$ (3.2)

Now notice that in a SVNS the presence of uncertainty is due to two factors, firstly due to the partial belongingness and partial non-belongingness and secondly due to the indeterminacy factor. Considering these two factors, an entropy function E_1 for a single valued neutrosophic sets A was proposed and it is defined as follows:

$$E_1(A) = 1 - \frac{1}{n} \sum_{x_i \in X} (t_A(x_i) + f_A(x_i)) \cdot |i_A(x_i) - i_{A^c}(x_i)| \quad (3.3)$$

Proposition 3.4 E_1 satisfies all the axioms given in definition 3.3.

Example 3.5 Let $X = \{a, b, c, d\}$ be the universe and A be a single valued neutrosophic set in X defined as follows:

$$A = \{\frac{a}{<0.5, 0.2, 0.9>}, \frac{b}{<0.8, 0.4, 0.2>}, \frac{c}{<0.3, 0.8, 0.7>}, \frac{d}{<0.6, 0.3, 0.5>}\}.$$

Then the entropy of A will be
$E_1(A) = 1 - 0.52 = 0.48.$

4. Problems with the earlier definition.

In this section we point out some problems with the earlier definition of entropy given in [10].

Problem 4.1: The entropy function E_1 defined in equation 3.3 is not a correct entropy function. Especially it may not lie in [0, 1].

The following example satisfies the claim:

Example 4.2 A counter example:

In the following example we will show that E_1 is not always an entropy function for all SVN sets.

Let $X = \{a, b\}$ be the universe and let $A = \{\frac{a}{(1.0, 0.01, 1.0)}, \frac{b}{(1.0, 0.02, 1.0)}\}$ be a SVNS, then

$$E_1 = 1 - \frac{1}{2}.(2 \times 0.98 + 2 \times 0.96)$$
$$= 1 - 1.94 = -0.94 < 0, \quad \text{which is undesirable.}$$

This definition holds only if $t_A(x) + f_A(x) \le 1$ holds.

The figure 2 shows that actually the half cubic portion ABODEGBA, left of the yellow plane of the 'neutrosophic cubic' where formula E_1 given in equation 3.3 holds. But for the other half cube it may not hold true as described above.

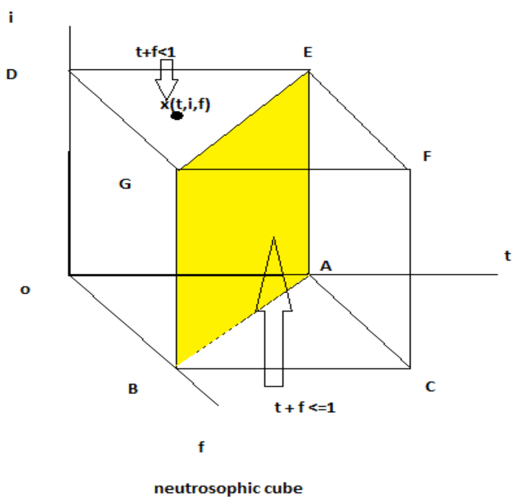

Figure 2

Problem 4.3: In definition 3.1, the most uncertain case is assumed to be (0.5, 0.5, 0.5) which is not necessarily true. Rather (0.5, 1, 0.5) is more uncertain case as here the indeterminacy factor 'i' has the maximum value. Also (1, 1, 1) is far more uncertain than (0.5, 0.5, 0.5).

More generally speaking the area indicated by pink colour in the neutrosophic cube is the place where lies the most uncertain cases. We further assume that the points D,G,F,E,J are the most uncertain neutrosophic elements because there indeterminacy is 1 and truth and falsities are also extreme. No other point in pink region can have higher uncertainty value.

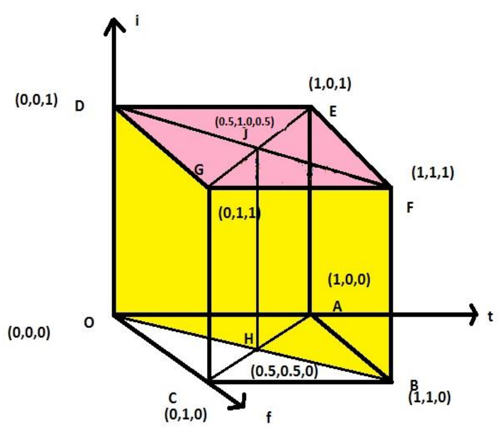

Figure 3

Problem 4.4 In case of neutrosophic sets or single valued neutrosophic sets, the degree of indeterminacy (i) of any neutrosophic element x in its complement set is defined as 1-i. This does not seem to be very reasonable. The degree of indeterminacy in the original set and its complement should be same because both bears the same amount of uncertainty. Also neutrosophic sets are generalizations of intuitionistic fuzzy sets. There the amount of uncertainty for any IF set A is measured as $\pi_A=1-t_A-f_A$. Then $\pi_A{}^c=1-f_A{}^c-t_A{}^c$ and hence $\pi_A=\pi_A{}^c$. Neutrosophic sets are generalizations of IF sets so accordingly here also $i = i^c$ should hold. Therefore we represent a new definition of complement of a SVNS as follows:

Definition 4.5 The complement of a SVNS A is denoted by A^c and is defined by

$$t_{A^c}(x) = f_A(x);\ i_{A^c}(x) = i_A(x)$$
$$\&\ f_{A^c}(x) = t_A(x)\ \forall x \in X.$$

Then A will satisfy involutive law : $(A^c)^c = A.$
Although we have to sacrifice De'Morgans Law in this case.

Considering the above problems, we propose a new modified definition of entropy of single valued neutrosophic sets in the next section.

5. New modified definition of Entropy of Neutrosophic Sets

In this section we present a modified definition of entropy for neutrosophic sets. But before that we have to introduce two new definitions, namely 'intuitionistic uncertainty' and 'more uncertain' SVNS.

Definition 5.1 For any neutrosophic set $A=\sum_{i=1}^{n}\frac{x_i}{<t_A(x_i), i_A(x_i), f_A(x_i)>}$ the Intuitionistic uncertainty of any neutrosophic element $<t_A(x), i_A(x), f_A(x >$ is defined as:

$$\pi^N_A(x) = \frac{1}{2}\times(2 - t_A(x) - f\ (x)),\ \forall x \in X.$$

Then for the whole SVNS A the intuitionistic uncertainty will be defined as $\pi_A^N = \frac{1}{|X|}\sum_{x_i \in X} \pi_A^N(x_i)$.

Note that intuitionistic uncertainty satisfies the following properties: $(i)\, 0 \le \pi_A^N(x),\ \pi_A^N \le 1$ *and* $(ii)\, \pi_A^N = \pi_{A^c}^N$.

But for neutrosophic elements of type $< 0.5, i_A(x), 0.5 >$, $\pi_A^N(x) \ne 1$ which is natural as in SVNS the uncertainty depends on both $\pi_A^N(x)$ *and* $i_A(x)$.

Consider the SVNS given in example 2.2, here $\pi_A^N(x_1) = 0.6,\ \pi_A^N(x_2) = 0.6,\ \pi_A^N(x_3) = 0.55$ *and thus* $\pi_A^N \approx 0.583$.

Example 5.2 Consider the examples 2.9 and 2.10. In the first example $\pi_A^N \approx 0.633$ *but*

$$\pi_{\Delta A}^N = \pi_B^N = \frac{1}{|X|}\sum_{i=1}^{3}\frac{(2-t_i-f_i)}{2} = \frac{1}{3}\{\frac{1.1+1+1.2}{2}\}$$
$$= 0.55 = \pi_{\nabla A}^N = \pi_C^N.$$

In the later example $\pi_A^N \approx 0.583$ *but*

$$\pi_{\Delta A}^N = \pi_B^N = \frac{1}{|X|}\sum_{i=1}^{3}\frac{(2-t_i-f_i)}{2} = \frac{1}{3}\{\frac{.8+1+.9}{2}\}$$
$$= 0.45 = \pi_{\nabla A}^N = \pi_C^N.$$

So we can also see that the Intuitionistic uncertainty of truth favourite and falsity favourite sets of a SVNS are same because the value of $(2 - t_i - f_i)$ is same for every element in each set ΔA *and* ∇A, but it's different (uncertainty decreased) with the original SVNS A.

Definition 5.3 A SVNS A is said to be more uncertain than another SVNS B, denoted as $A < B$, if and only if

i.e. $if\ \pi_A^N(x) \ge \pi_B^N(x),\ or\ t_A(x) + f_A(x) \le t_B(x) + f_B(x)$ $and\ i_A(x) \ge i_B(x),\ \forall x \in X.$

Example 5.4 Consider the following two SVNS's A and B defined as follows:

$$A = {<0.3, 0.8, 0.5>}/{x_1} + {<0.2, 0.7, 0.4>}/{x_2} + {<0.7, 0.6, 0.3>}/{x_3},$$

$$B = {<0.7, 0.6, 0.5>}/{x_1} + {<0.2, 0.5, 0.7>}/{x_2} + {<0.9, 0.3, 0.2>}/{x_3}.$$

Then

A	$t_A + f_A$	i_A
x_1	0.8	0.8
x_2	0.6	0.7
x_3	1.0	0.6

and

B	$t_B + f_B$	i_B
x_1	1.2	0.6
x_2	0.9	0.5
x_3	1.1	0.3

Therefore here A is more uncertain than B,

ie. A < B.

Now we introduce a new definition of Entropy for SVNS:

Definition 5.5 For any SVNS A we define entropy as a function $E_N : N(X) \to [0,1]$ which satisfies the following axioms:

$(1) E_N(A) = 0$ if A is a crisp set

$(2) E_N(A) = 1$ *for* $\forall x, x \in$ *neutrosophic elements* D, E, F, G, J

i.e. if $(t_A(x), i_A(x), f_A(x)) = J = (0.5, 1.0, 0.5)\forall x \in X$,

or $F = (1,1,1)\forall x \in X$ *or* $D = (0,1,0)\forall x \in X$

or $G = (0,1,1)\forall x \in X$ *or* $E = (1,1,0)\forall x \in X$

$(3) E_N(A) \ge E_N(B)$ if A more uncertain than B, *i.e. if* $A < B$.

$(4) E_N(A) = E_N(A^c)\forall A \in N(X)$......(3.4)

We can also classify entropy of SVNS's into 4 classes namely type I - IV according to the point for which we get maximum entropy value.

Example 5.6 Considering these two factors we propose an entropy measure E_i of a single valued neutrosophic sets A as follows:

$$(i)\ E(x) = Min\{1, (2 - t_x - f_x).i_x\}\ , x \in A$$

$$and\ E_1(A) = \frac{1}{|X|}\sum_{x\in A} E(x)..........................(3.18)$$

$$(ii)\ E(x) = \frac{1}{2}.\{|1 - t_x - f_x| + i_x\}, x \in A$$

$$and\ E_2(A) = \frac{1}{|X|}\sum_{x\in A} E(x).........................(3.19)$$

$$(iii) E(x) = \frac{1}{2}.(2 - t_x - f_x).i_x., x \in A$$

$$and\ E_3(A\ = \frac{1}{|X|}\sum_{x\in A} E(x)...........................(3.20)$$

$$(iv)\ E(x) = (2 - t_x - f_x).i_x, x \in A$$

$$and\ E_4(A) = \frac{1}{|X|}\sum_{x\in A} E(x).............................(3.21)$$

Here $E_1(A)$ is an entropy function for any SVNS A of Type I, $E_2(A)$is of Type II, $E_3(A)$is of Type III and $E_4(A)$ is an entropy function for any SVNS A of Type IV respectively.

Example 5.7 Consider the example 2.2. In this case we have

$$E_1(A) = E_4(A) \simeq 0.31, E_2(A) \simeq 0.22, E_3(A) \simeq 0.16.$$

So average entropy is $\frac{1}{4}\sum\ _i(A) = 0.25.$

6. Conclusion:

In this paper we have introduced a new modified definition of entropy of SVNS which is significantly different from earlier definition of entropy for SVNS.

This definition is more logical than the earlier and radically different in nature due to the introduction of new concepts like 'intuitionistic uncertainty' of a SVNS, 'more uncertain SVNs', 'most uncertain SVNs' etc.

Here we have also introduced four different types of entropy functions which are more general in nature and free from the anomalies present in the earlier entropy function.

One can further study the applications of these entropy functions in solving several decision making problems.

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