On The Symbolic 3-Plithogenic Real Functions by Using Special AH-Isometry

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Abstract:

Symbolic 3-plithogenic sets as a generalization of classical concept of sets were applicable to algebraic structures. The symbolic 3-plithogenic rings and fields are good generalizations of classical corresponding systems.

In this paper, we study the symbolic 3-plithogenic real functions with one variable by using a special algebraic function called AH-isometry. In addition, we discuss the symbolic 3-plithogenic simple differential equations and conic sections by using this isometry. Also, many examples will be presented to explain the novelty of this work.

Keywords: symbolic 3-plithogenic set, symbolic 3-plithogenic real function, symbolic 3-plithogenic circle, symbolic 3-plithogenic ellipse.

Introduction and basic definitions

Symbolic n-plithogenic algebraic structures are considered as new generalizations of classical algebraic structures [1-3], such as symbolic 2-plithogenic integers, modules, and vector spaces [4-8].

Symbolic 2-plithogenic structures have a similar structure to the refined neutrosophic structures and many non-classical algebraic structures defined by many authors in [9-17,20-23, 24-30].

In the literature, many mathematical approaches were carried out on neutrosophic and refined neutrosophic structures, were a special function called AH-isometry was used to study the analytical properties and conic sections [11-12, 18-19], and that occurs by taking the direct image of neutrosophic elements to the classical Cartesian product of the real field with itself.

In this work, we follow the previous efforts, and we define for the first time a special AH-isometry on the symbolic 3-plithogenic field of reals, and we use this isometry to obtain many formulas and properties about the symbolic 3-plithogenic analytical concepts such as differentiability, continuity, and integrability. Also, symbolic 3-plithogenic conic sections will find a place in our study.

Main Results

Definition.

Let $3 - SP_R = \{a + bP_1 + cP_2 + dP_2; a, b, c, d \in R\}$ be the 3-plithogenic field of real numbers, a function $f = f(X): 3 - SP_R \to 3 - SP_R$ is called one variable symbolic 3-plithogenic real function, with

$X = x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3 \in 3 - SP_R.$

Definition.

Let $3 - SP_R$ be the symbolic 3-plithogenic field of reals, we define its AH-isometry as follows:

$I: 3 - SP_R \to R \times R \times R \times R$ such that:

$I(x + yP_1 + zP_2 + tP_3) = (x, x + y, x + y + z, x + y + z + t).$

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R \times R \times R \times R \to 3 - SP_R$ such that:

$I^{-1}(x, y, z, t) = x + (y - x)P_1 + (z - y)P_2 + (t - z)P_3$

Definition.

Let $f: 3 - SP_R \to 3 - SP_R$ be a symbolic 3-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1} \circ I(f): 3 - SP_R \to 3 - SP_R$$

Example.

Consider $f(X) = X^2 - P_1 + P_3$, its canonical formula is:

$$I(f(X)) = [I(X)]^2 + I(-P_1 + P_3)$$
$$= (x_0, x_0 + x_1, x_0 + x_1 + x_2, x_0 + x_1 + x_2 + x_3)^2 + (0, -1, -1, 0)$$
$$= (x_0^2, (x_0 + x_1)^2 - 1, (x_0 + x_1 + x_2)^2 - 1, (x_0 + x_1 + x_2 + x_3)^2)$$

$$I^{-1} \circ I(f(X)) =$$
$$x_0^2 + P_1[(x_0 + x_1)^2 - x_0^2 - 1] + P_2[(x_0 + x_1 + x_2)^2 - (x_0 + x_1)^2]$$
$$+ P_3[(x_0 + x_1 + x_2 + x_3)^2 - (x_0 + x_1 + x_2)^2 + 1]$$

For example:

$$f(1 + P_3) = (1 + P_3)^2 - P_1 + P_3 = 1 - P_1 + 4P_3.$$

If we put values $x_0 = 1, x_1 = 0, x_2 = 0, x_2 = 1$

 in the canonical formula, then we get:

$$I^{-1} \circ I(f(X)) = (1)^2 + P_1[(1)^2 - (1)^2 - 1] + P_2[(1)^2 - (1)^2] + P_3[(2)^2 - (1)^2 + 1] =$$
$$1 - P_1 + 4P_3.$$

The canonical formulas of famous functions:

1. The exponent function:

$$e^X; X = x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3, I^{-1} \circ I(e^X) =$$
$$I^{-1}(e^{x_0}, e^{x_0+x_1}, e^{x_0+x_1+x_2}, e^{x_0+x_1+x_2+x_3}) =$$
$$e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}] + P_3[e^{x_0+x_1+x_2+x_3} - e^{x_0+x_1+x_2}]$$

2. The logarithmic function:

$$\ln(X); X = x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3,$$
$$I^{-1} \circ I(\ln(X)) = I^{-1}(\ln(x_0), \ln(x_0 + x_1), \ln(x_0 + x_1 + x_2), \ln(x_0 + x_1 + x_2 + x_3)) =$$
$$\ln(x_0) + P_1[\ln(x_0 + x_1) - \ln(x_0)] + P_2[\ln(x_0 + x_1 + x_2) - \ln(x_0 + x_1)] + P_3[\ln(x_0 +$$
$$x_1 + x_2 + x_3) - \ln(x_0 + x_1 + x_2)].$$

3. Famous trigonometric functions:

$$\sin(X) = \sin(x_0) + P_1[\sin(x_0 + x_1) - \sin(x_0)] + P_2[\sin(x_0 + x_1 + x_2) - \sin(x_0 + x_1)]$$
$$+ P_3[\sin(x_0 + x_1 + x_2 + x_3) - \sin(x_0 + x_1 + x_2)]$$

$$\cos(X) = \cos(x_0) + P_1[\cos(x_0 + x_1) - \cos(x_0)]$$
$$+ P_2[\cos(x_0 + x_1 + x_2) - \cos(x_0 + x_1)]$$
$$+ P_3[cos(x_0 + x_1 + x_2 + x_3) - cos(x_0 + x_1 + x_2)]$$
$$\tan(X) = \tan(x_0) + P_1[taan(x_0 + x_1) - \tan(x_0)]$$
$$+ P_2[\tan(x_0 + x_1 + x_2) - \tan(x_0 + x_1)]$$
$$+ P_3[tan(x_0 + x_1 + x_2 + x_3) - tan(x_0 + x_1 + x_2)]$$

And so no.

Definition.

A symbolic 3-plithogenic real number $T = t_0 + t_1P_1 + t_2P_2 + t_3P_3$ is called positive if and only if

$$t_0 \geq 0, t_0 + t_1 \geq 0, t_0 + t_1 + t_2 \geq 0, t_0 + t_1 + t_2 + t_3 \geq 0.$$

For example $3 + 2P_1 - P_3 > 0$, that is because, $3 > 0$, $5 > 0, 5 > 0, 3 + 2 - 1 = 4 > 0$.

Definition.

Let $f: 3 - SP_R \rightarrow 3 - SP_R$ be a symbolic 3-plithogenic real function with one variable $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3$, then:

  a. f is differentiable if and only if $I(f(X))$ is differentiable.

  b. f is continuous if and only if $I(f(X))$ is continuous.

  c. f is integrable if and only if $I(f(X))$ is integrable.

Example.

Find the derivation of $f(X) = X^2 + P_3$ in two different ways.

Solution.

The regular way is $\acute{f}(X) = 2X$.

The canonical way is:

$$I^{-1} \circ I(f(X)) =$$

$$I^{-1}(x_0^2, (x_0 + x_1)^2, (x_0 + x_1 + x_2)^2, (x_0 + x_1 + x_2 + x_3)^2) + I^{-1}(0,0,0,1) = x_0^2 +$$

$$P_1[(x_0 + x_1)^2 - x_0^2] + P_2[(x_0 + x_1 + x_2)^2 - (x_0 + x_1)^2] + P_3[(x_0 + x_1 + x_2 + x_3)^2 -$$

$$(x_0 + x_1 + x_2)^2 + 1].$$

First, we have: $(x_0^2)'_{x_0} = 2x_0.$

$$[(x_0 + x_1)^2]'_{x_0 + x_1} = 2(x_0 + x_1),$$

$$[(x_0 + x_1 + x_2)^2]'_{x_0+x_1+x_2} = 2(x_0 + x_1 + x_2),$$

$$[(x_0 + x_1 + x_2 + x_3)^2]'_{x_0+x_1+x_2+x_3} = 2(x_0 + x_1 + x_2 + x_3),$$

Thus,

$$\acute{f}(X) = 2x_0 + P_1[2(x_0 + x_1) - 2x_0] + P_2[2(x_0 + x_1 + x_2) - 2(x_0 + x_1)] +$$

$$P_3[2(x_0 + x_1 + x_2 + x_3) - 2(x_0 + x_1 + x_2)] = (2x_0) + 2x_1 P_1 + 2x_2 P_2 + 2x_3 P_3 = 2X.$$

Example.

Find the value of $\int_0^{1+P_1+P_2+P_3} e^X dX$ in two different ways.

Solution;

The regular way: $\int_0^{1+P_1+P_2+P_3} e^X dX = [e^X]_0^{1+P_1+P_2+P_3} = e^{1+P_1+P_2+P_3} - e^0 = e^{1+P_1+P_2+P_3} - 1.$

The canonical formula way:

$$I^{-1} \circ I\big(f(X)\big) = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}]$$

$$+ P_3[e^{x_0+x_1+x_2+x_3} - e^{x_0+x_1+x_2}]$$

We have:

$$\int_0^1 e^{x_0} dx_0 = e - 1, \int_0^2 e^{e^{x_0+x_1}} d(x_0 + x_1) = e^2 - 1, \int_0^3 e^{x_0+x_1+x_2} d(x_0 + x_1 + x_2)$$

$$= e^3 - 1, \int_0^3 e^{x_0+x_1+x_2+x_3} d(x_0 + x_1 + x_2 + x_3) = e^4 - 1$$

Thus, $\int_0^{1+P_1+P_2+P_3} e^X dX = e - 1 + P_1[e^2 - 1 - e + 1] + P_2[e^3 - 1 - e^2 + 1] +$

$P_3[e^4 - 1 - e^3 + 1] = e - 1 + P_1[e^2 - e] + P_2[e^3 - e^2] + P_3[e^4 - e^3] = e^{1+P_1+P_2+P_3} - 1$

Applications to differential equations.

Example.

Solve the equation $\acute{Y} = C; Y = y_0 + y_1 P_1 + y_2 P_2 + y_3 P_3$ is a function, and $C = c_0 + c_1 P_1 + c_2 P_2 + c_3 P_3$ is a constant.

We have $y_0 = f_0, y_1 = f_1, y_2 = f_2, y_3 = f_3 : R \to R.$

$$\acute{Y} = (y_0)'_{x_0} + P_1 \left[(y_0 + y_1)'_{x_0+x_1} - (y_0)'_{x_0} \right] +$$

$$P_2\left[(y_0 + y_1 + y_2)'_{x_0+x_1+x_2} - (y_0 + y_1)'_{x_0+x_1}\right] + P_3\left[(y_0 + y_1 + y_2 + y_3)'_{x_0+x_1+x_2+x_3} -\right.$$

$$\left.(y_0 + y_1 + y_2)'_{x_0+x_1+x_2}\right] = c_0 + c_1 P_1 + c_2 P_2 + c_3 P_3,$$

so that:

$$(y_0)'_{x_0} = c_0$$
$$(y_0 + y_1)'_{x_0+x_1} = c_1$$
$$(y_0 + y_1 + y_2)'_{x_0+x_1+x_2} = c_2$$
$$(y_0 + y_1 + y_2 + y_3)'_{x_0+x_1+x_2+x_3} = c_3$$
$$y_0 = c_0 x_0 + m_0$$
$$y_0 + y_1 = c_1(x_0 + x_1) + m_1$$
$$y_0 + y_1 + y_2 = c_2(x_0 + x_1 + x_2) + m_2$$
$$y_0 + y_1 + y_2 + y_3 = c_3(x_0 + x_1 + x_2 + x_3) + m_3$$

This implies that:

$$Y = (c_0 x_0 + m_0) + P_1[c_1(x_0 + x_1) + m_1 - (c_0 x_0 + m_0)] +$$
$$P_2[c_2(x_0 + x_1 + x_2) + m_2 - c_1(x_0 + x_1) + m_1] +$$

$$P_3[c_3(x_0 + x_1 + x_2 + x_3) + m_3 - c_2(x_0 + x_1 + x_2) - m_2];$$

x_i are real variables, m_i are real constants.

Example.

Solve the differential equation $\acute{Y} = CY$, where $C = c_0 + c_1 P_1 + c_2 P_2 + c_3 P_3, Y = y_0 + y_1 P_1 + y_2 P_2 + y_3 P_3$.

Solution.

$\acute{Y} = CY$ equivalents:

$$\begin{pmatrix} (y_0)'_{x_0} = c_0 y_0 \\ (y_0 + y_1)'_{x_0+x_1} = (c_0 + c_1)(y_0 + y_1) \\ (y_0 + y_1 + y_2)'_{x_0+x_1+x_2} = (c_0 + c_1 + c_2)(y_0 + y_1 + y_2) \\ (y_0 + y_1 + y_2 + y_3)'_{x_0+x_1+x_2+x_3} = (c_0 + c_1 + c_2 + c_3)(y_0 + y_1 + y_2 + y_3) \end{pmatrix}$$

So that:

$$\begin{pmatrix} y_0 = k_0 e^{c_0 x_0} \\ y_0 + y_1 = k_1 e^{(c_0+c_1)(x_0+x_1)} \\ y_0 + y_1 + y_2 = k_2 e^{(c_0+c_1+c_2)(x_0+x_1+x_2)} \\ y_0 + y_1 + y_2 + y_3 = k_3 e^{(c_0+c_1+c_2+c_3)(x_0+x_1+x_2+x_3)} \end{pmatrix}$$

Thus:

$Y = k_0 e^{c_0 x_0} + P_1 [k_1 e^{(c_0+c_1)(x_0+x_1)} - k_0 e^{c_0 x_0}] + P_2 [k_2 e^{(c_0+c_1+c_2)(x_0+x_1+x_2)} - k_1 e^{(c_0+c_1)(x_0+x_1)}] + P_3 [k_3 e^{(c_0+c_1+c_2+c_3)(x_0+x_1+x_2+x_3)} - k_2 e^{(c_0+c_1+c_2)(x_0+x_1+x_2)}].$

Applications to geometric shapes:

Definition.

1). We define the symbolic 3-plithogenic circle as follows:

$(X - A)^2 + (Y - B)^2 = R^2; A = a_0 + a_1 P_1 + a_2 P_2 + a_3 P_3, B = b_0 + b_1 P_1 + b_2 P_2 + b_3 P_3, R = r_0 + r_1 P_1 + r_2 P_2 + r_3 P_3, Y = y_0 + y_1 P_1 + y_2 P_2 + y_3 P_3, x = x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3,$ with $a_i, b_i, r_i, x_i, y_i \in R$

2). We define the symbolic 3-plithogenic sphere as follows:

$(X - A)^2 + (Y - B)^2 + (Z - C)^2 = R^2; X, A, B, C, R, Y, Z \in 3 - SP_R$

3). We define the symbolic 3-plithogenic ellipse as follows:

$\frac{(X-A)^2}{T^2} + \frac{(Y-B)^2}{S^2} = 1; X, A, B, T, S, Y \in 3 - SP_R$ and T, S invertible.

4). We define the symbolic 3-plithogenic hyperbola as follows:

$\frac{(X-A)^2}{T^2} - \frac{(Y-B)^2}{S^2} = 1; X, A, B, T, S, Y \in 3 - SP_R$ and T, S invertible.

Example.

1). $(X - 1 + P_2)^2 + (Y - 3 + 2P_1 - P_2 - P_3)^2 = (1 + 2P_3)^2$ is a 3-plithogenic circle.

2). $(X - 10 + P_1 + P_2)^2 + (Y + P_3)^2 + (Z - P_1 + P_3)^2 = (1 + P_2 + 5P_3)^2$ is a 3-plithogenic sphere.

3). $\frac{(X-P_1-P_3)^2}{(1+P_1+P_3)^2} + \frac{(Y+P_1-P_2)^2}{(2-P_1+5P_3)^2} = 1$ is a 3-plithogenic ellipse.

4). $\frac{(X-2P_2)^2}{(1+P_1+3P_2)^2} - \frac{(Y+1+4P_2)^2}{(13-2P_1+P_2)^2} = 1$ is a 3-plithogenic hyperbola.

Theorem.

1. Any symbolic 3-plithogenic circle is equivalent to four classical circles.

2. Any symbolic 3-plithogenic sphare is equivalent to four classical spheres.

3. Any symbolic 3-plithogenic ellipse is equivalent to four classical ellipses.

4. Any symbolic 3-plithogenic hyperbola is equivalent to four classical hyperbolas.

Proof.

1. Consider the symbolic 3-plithogenic circle:

$(X - A)^2 + (Y - B)^2 = R^2$, then by using the isomorphism defined before, we get:

$I[(X - A)^2] = [I(X) - I(A)]^2$

$$= \left(x_0 - a_0, (x_0 + x_1) - (a_0 + a_1), (x_0 + x_1 + x_2)\right.$$

$$\left. - (a_0 + a_1 + a_2), (x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3)\right)^2$$

$$= \left((x_0 - a_0)^2, ((x_0 + x_1) - (a_0 + a_1))^2, ((x_0 + x_1 + x_2)\right.$$

$$\left. - (a_0 + a_1 + a_2))^2, ((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3))^2\right)$$

$I[(Y - B)^2] = [I(Y) - I(B)]^2$

$$= \left((y_0 - b_0)^2, ((y_0 + y_1) - (b_0 + b_1))^2, ((y_0 + y_1 + y_2)\right.$$

$$\left. - (b_0 + b_1 + b_2))^2, ((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3))^2\right)$$

$I(R^2) = [I(R)]^2 = (r_0{}^2, (r_0 + r_1)^2, (r_0 + r_1 + r_2)^2, (r_0 + r_1 + r_2 + r_3)^2)$

Thus, it is equivalent to:

$$\left(\begin{array}{c} (x_0 - a_0)^2 + (y_0 - b_0)^2 = r_0{}^2 \\ ((x_0 + x_1) - (a_0 + a_1))^2 + ((y_0 + y_1) - (b_0 + b_1))^2 = (r_0 + r_1)^2 \\ ((x_0 + x_1 + x_2) - (a_0 + a_1 + a_2))^2 + ((y_0 + y_1 + y_2) - (b_0 + b_1 + b_2))^2 = (r_0 + r_1 + r_2)^2 \\ ((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3))^2 + ((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3))^2 = (r_0 + r_1 + r_2 + r_3)^2 \end{array}\right)$$

2. Consider the sphere $(X - A)^2 + (Y - B)^2 + (Z - C)^2 = R^2$, we use the isomorphism I, to get:

$I[(X - A)^2] = \left((x_0 - a_0)^2, ((x_0 + x_1) - (a_0 + a_1))^2, ((x_0 + x_1 + x_2)\right.$

$$\left. - (a_0 + a_1 + a_2))^2, ((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3))^2\right)$$

$I[(Y - B)^2] = \left((y_0 - b_0)^2, ((y_0 + y_1) - (b_0 + b_1))^2, ((y_0 + y_1 + y_2)\right.$

$$\left. - (b_0 + b_1 + b_2))^2, ((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3))^2\right)$$

$I[(Z - C)^2] = \left((z_0 - c_0)^2, ((z_0 + z_1) - (c_0 + c_1))^2, ((z_0 + z_1 + z_2)\right.$

$$\left. - (c_0 + c_1 + c_2))^2, ((z_0 + z_1 + z_2 + z_3) - (c_0 + c_1 + c_2 + c_3))^2\right)$$

$I(R^2) = [I(R)]^2 = (r_0{}^2, (r_0 + r_1)^2, (r_0 + r_1 + r_2)^2, (r_0 + r_1 + r_2 + r_3)^2),$

hence we get:

$$\left\{\begin{array}{c} (x_0 - a_0)^2 + (y_0 - b_0)^2 + (z_0 - c_0)^2 = r_0{}^2 \\ ((x_0 + x_1) - (a_0 + a_1))^2 + ((y_0 + y_1) - (b_0 + b_1))^2 + ((z_0 + z_1) - (c_0 + c_1))^2 = (r_0 + r_1)^2 \\ ((x_0 + x_1 + x_2) - (a_0 + a_1 + a_2))^2 + ((y_0 + y_1 + y_2) - (b_0 + b_1 + b_2))^2 + ((z_0 + z_1 + z_2) - (c_0 + c_1 + c_2))^2 = (r_0 + r_1 + r_2)^2 \end{array}\right.$$

And

$$\big((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3)\big)^2 + \big((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 +$$
$$b_2 + b_3)\big)^2 + \big((z_0 + z_1 + z_2 + z_3) - (c_0 + c_1 + c_2 + c_3)\big)^2 = (r_0 + r_1 + r_2 + r_3)^2.$$

3. Consider the ellipse $\frac{(X-A)^2}{T^2} + \frac{(Y-B)^2}{S^2} = 1$, we use the isomorphism I to get:

$$I\left[\frac{(X-A)^2}{T^2}\right]$$
$$= \left(\frac{(x_0 - a_0)^2}{t_0^2}, \frac{\big((x_0 + x_1) - (a_0 + a_1)\big)^2}{(t_0 + t_1)^2}, \frac{\big((x_0 + x_1 + x_2) - (a_0 + a_1 + a_2)\big)^2}{(t_0 + t_1 + t_2)^2}, \frac{\big((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3)\big)^2}{(t_0 + t_1 + t_2 + t_3)^2}\right)$$

$$I\left[\frac{(Y-B)^2}{S^2}\right]$$
$$= \left(\frac{(y_0 - b_0)^2}{s_0^2}, \frac{\big((y_0 + y_1) - (b_0 + b_1)\big)^2}{(s_0 + s_1)^2}, \frac{\big((y_0 + y_1 + y_2) - (b_0 + b_1 + b_2)\big)^2}{(s_0 + s_1 + s_2)^2}, \frac{\big((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3)\big)^2}{(s_0 + s_1 + s_2 + s_3)^2}\right)$$

$I(1) = (1,1,1,1)$, thus:

$$\begin{cases} \dfrac{(x_0 - a_0)^2}{t_0^2} + \dfrac{(y_0 - b_0)^2}{s_0^2} = 1 \\[4mm] \dfrac{\big((x_0 + x_1) - (a_0 + a_1)\big)^2}{(t_0 + t_1)^2} + \dfrac{\big((y_0 + y_1) - (b_0 + b_1)\big)^2}{(s_0 + s_1)^2} = 1 \\[4mm] \dfrac{\big((x_0 + x_1 + x_2) - (a_0 + a_1 + a_2)\big)^2}{(t_0 + t_1 + t_2)^2} + \dfrac{\big((y_0 + y_1 + y_2) - (b_0 + b_1 + b_2)\big)^2}{(s_0 + s_1 + s_2)^2} = 1 \end{cases}$$

And,

$$\frac{\big((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3)\big)^2}{(t_0 + t_1 + t_2 + t_3)^2}$$
$$+ \frac{\big((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3)\big)^2}{(s_0 + s_1 + s_2 + s_3)^2} = 1$$

4. Consider the hyperbola $\frac{(X-A)^2}{T^2} - \frac{(Y-B)^2}{S^2} = 1$, by a similar discussion, we get

$$\begin{cases} \dfrac{(x_0 - a_0)^2}{t_0^2} - \dfrac{(y_0 - b_0)^2}{s_0^2} = 1 \\[4mm] \dfrac{\big((x_0 + x_1) - (a_0 + a_1)\big)^2}{(t_0 + t_1)^2} - \dfrac{\big((y_0 + y_1) - (b_0 + b_1)\big)^2}{(s_0 + s_1)^2} = 1 \\[4mm] \dfrac{\big((x_0 + x_1 + x_2) - (a_0 + a_1 + a_2)\big)^2}{(t_0 + t_1 + t_2)^2} - \dfrac{\big((y_0 + y_1 + y_2) - (b_0 + b_1 + b_2)\big)^2}{(s_0 + s_1 + s_2)^2} = 1 \end{cases}$$

and

$$\frac{\left((x_0 + x_1 + x_2 + x_3) - (a_0 + a_1 + a_2 + a_3)\right)^2}{(t_0 + t_1 + t_2 + t_3)^2}$$

$$-\frac{\left((y_0 + y_1 + y_2 + y_3) - (b_0 + b_1 + b_2 + b_3)\right)^2}{(s_0 + s_1 + s_2 + s_3)^2} = 1$$

Example.

Consider the symbolic 3-plithogenic ellipse:

$$\frac{(X - 2 - 4P_1 + 3P_2 + P_3)^2}{(1 + 5P_1 + 7P_2 - P_3)^2} + \frac{(Y + 1 + P_2 + 11P_3)^2}{(3 - P_1 - P_2 + 4P_3)^2} = 1$$

it is equivalent to:

$$\begin{cases} \dfrac{(x_0 - 2)^2}{1} + \dfrac{(y_0 + 1)^2}{3^2} = 1 \\ \dfrac{[(x_0 + x_1) - 6]^2}{6^2} + \dfrac{[(y_0 + y_1) + 1]^2}{2^2} = 1 \\ \dfrac{[(x_0 + x_1 + x_2) - 3]^2}{13^2} + \dfrac{[(y_0 + y_1 + y_2) + 2]^2}{1} = 1 \\ \dfrac{[(x_0 + x_1 + x_2 + x_3) - 2]^2}{12^2} + \dfrac{[(y_0 + y_1 + y_2 + y_3) + 13]^2}{5^2} = 1 \end{cases}$$

Example.

Consider the symbolic 3-plithogenic circle:

$$(X - 2 - 4P_1 + 3P_2 + P_3)^2 + (Y + 1 + P_2 + 11P_3)^2 = 1$$

It is equivalent to:

$$\begin{cases} (x_0 - 2)^2 + (y_0 + 1)^2 = 1 \\ [(x_0 + x_1) - 6]^2 + [(y_0 + y_1) + 1]^2 = 1 \\ [(x_0 + x_1 + x_2) - 3]^2 + [(y_0 + y_1 + y_2) + 2]^2 = 1 \\ [(x_0 + x_1 + x_2 + x_3) - 2]^2 + [(y_0 + y_1 + y_2 + y_3) + 13]^2 = 1 \end{cases}$$

Example.

Consider the symbolic 3-plithogenic hyperbola:

$$\frac{(X - 2 - 4P_1 + 3P_2 + P_3)^2}{(1 + 5P_1 + 7P_2 - P_3)^2} - \frac{(Y + 1 + P_2 + 11P_3)^2}{(3 - P_1 - P_2 + 4P_3)^2} = 1$$

it is equivalent to:

$$\begin{cases} \dfrac{(x_0-2)^2}{1} - \dfrac{(y_0+1)^2}{3^2} = 1 \\[2mm] \dfrac{[(x_0+x_1)-6]^2}{6^2} - \dfrac{[(y_0+y_1)+1]^2}{2^2} = 1 \\[2mm] \dfrac{[(x_0+x_1+x_2)-3]^2}{13^2} - \dfrac{[(y_0+y_1+y_2)+2]^2}{1} = 1 \\[2mm] \dfrac{[(x_0+x_1+x_2+x_3)-2]^2}{12^2} - \dfrac{[(y_0+y_1+y_2+y_3)+13]^2}{5^2} = 1 \end{cases}$$

Conclusion

In this paper, we defined for the first time a special AH-isometry on the symbolic 3-plithogenic fields of reals, and we used this isometry to obtain many formulas and properties about the symbolic 3-plithogenic analytical concepts such as differentiability, continuity, and integrability. Also, symbolic 2 3-plithogenic conic sections were handled by using the mentioned isometry. In addition, many related examples were presented to clarify the novelty of our work.

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