SINGLE VALUED (2N+1) SIDED POLYGONAL NEUTROSOPHIC NUMBERS AND SINGLE VALUED (2N) SIDED POLYGONAL NEUTROSOPHIC NUMBERS

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Abstract. This paper introduces a single valued (2n as well as 2n+1) sided polygonal neutrosophic numbers in continuation with other defined single valued neutrosophic numbers. The paper provides basic algebra like addition, subtraction and multiplication of a single valued (2n as well as 2n+1) sided polygonal neutrosophic numbers with examples. In addition, the paper introduces matrix for single valued (2n as well as 2n+1) sided polygonal neutrosophic matrix and its properties.

Keywords: Fuzzy numbers, Intuitionistic fuzzy numbers, Single valued trapezoidal neutrosophic numbers, Single valued triangular neutrosophic numbers, Neutrosophic matrix.

1 Introduction

In the real world problems, uncertainty occurs in many situations which cannot be handled precisely via crisp set theory. To approximate those uncertainties exists in the given linguistics words the fuzzy set theory is introduced by Zadeh [10]. After that, Dubois and Prade [2] defined the fuzzy number as a generalization of real number. In continuation, many authors [5-8, 11-23] introduced various types of fuzzy numbers such as triangular, trapezoidal, pentagonal, hexagonal fuzzy numbers etc. with their membership functions. Atanassov [1] introduced the concept of intuitionistic fuzzy sets that provides precise solutions to the problems in uncertain situations than fuzzy sets with membership and non-membership functions. After developing intuitionistic fuzzy sets, authors in [4, 6, 10, 19] defined various types of intuitionistic fuzzy numbers and different types of operations on intuitionistic fuzzy sets are also established by suitable examples. Smarandache [9] introduced the generalization of both fuzzy and intuitionistic fuzzy sets and named it as neutrosophic set. The Single valued neutrosophic number and its applications are described in [3]. The results of the problems using neutrosophic sets are more accurate than the results given by fuzzy and intuitionistic fuzzy sets [11-20]. Due to which it is applied in various fields for multi-decision tasks [20-32]. The applications of n-valued neutrosophic set [24-26] in data analytics research fields given a thrust to study the neutrosophic numbers. This paper focuses on introducing mathematical operation of 2n and 2n+1 sided polygonal neutrosophic numbers and its matrices with examples.

The rest of the paper is organized as follows: The section 2 contains preliminaries. Section 3 explains single valued 2n+1 polygonal neutrosophic numbers whereas the Section 4 demonstrates Single valued 2n side polygonal neutrosophic numbers. Section 5 provides conclusions followed by acknowledgements and references.

2. Preliminaries

Definition 1 (Fuzzy Number)[4]: A fuzzy number is nothing but an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each of the possible value has its own weight between 0 and 1. This weight is called the membership function. The complex fuzzy set for a given fuzzy number $\widetilde{\boldsymbol{A}}$ can be defined as $\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}}(\boldsymbol{x})$ is non-decreasing for $x \leq \boldsymbol{x_0}$ and non-increasing for $\geq \boldsymbol{x_0}$. Similarly other properties can be defined.

Definition 2 (Triangular fuzzy number [4]): A fuzzy number $\widetilde{\boldsymbol{A}}$= {a, b, c } is said to be a triangular fuzzy number if its membership function is given by, where $a \leq b \leq c$

$$\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}}(\boldsymbol{x}) = \begin{cases} \frac{(x-a)}{(b-a)} & for\ \ a \leq x \leq b \\ \frac{(c-x)}{(c-b)} & for\ \ b \leq x \leq c \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases}$$

Definition 3 (Trapezoidal fuzzy number [4])

A Trapezoidal fuzzy number (TrFN) denoted by $\widetilde{\boldsymbol{A}}_P$ is defined as (a, b, c, d), where the membership function

$$\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}_P}(\boldsymbol{x}) = \begin{cases} \mathbf{0} & for\ x \leq a \\ \frac{(x-a)}{(b-a)} & for\ \ a \leq x \leq b \\ \mathbf{1} & for\ \ b \leq x \leq c \\ \frac{(d-x)}{(d-c)} & for\ \ c \leq x \leq d \\ \mathbf{0} & \boldsymbol{for\ x \geq d} \end{cases}$$

Or, $\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}_P}(\boldsymbol{x})$**= max (min (**$\frac{(x-a)}{(b-a)}$**, 1,** $\frac{(d-x)}{(d-c)}$ **) ,0)**

Definition 4 (Generalized Trapezoidal Fuzzy Number) (**GTrFNs)**

A Generalized Fuzzy Number (a, b, c, d, w), is called a Generalized Trapezoidal Fuzzy Number "x" if its membership function is given by

$$(\boldsymbol{x}) = \begin{cases} \mathbf{0} & for\ x \leq a \\ \frac{(x-a)}{(b-a)}\boldsymbol{w} & for\ \ a \leq x \leq b \\ \boldsymbol{w} & for\ \ b \leq x \leq c \\ \frac{(d-x)}{(d-c)}\boldsymbol{w} & for\ \ c \leq x \leq d \\ \mathbf{0} & \boldsymbol{for\ x \geq d} \end{cases}$$

Or, $\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}_P}(\boldsymbol{x})$**= max (min (w** $\frac{(x-a)}{(b-a)}$**, w,**$\boldsymbol{w}$ $\frac{(d-x)}{(d-c)}$**) ,0)**

Definition 5 (Pentagonal fuzzy number [4])

A pentagonal fuzzy number (PFN) of a fuzzy set $\widetilde{\boldsymbol{A}}_P$= {a, b, c, d, e} and its membership function is given by,

$$\boldsymbol{\mu}_{\widetilde{\boldsymbol{A}}_P}(\boldsymbol{x}) = \begin{cases} \mathbf{0} & for\ x < a \\ \frac{(x-a)}{(b-a)} & for\ \ a \leq x \leq b \\ \frac{(x-b)}{(c-b)} & for\ \ b \leq x \leq c \\ \mathbf{1} & \boldsymbol{x = c} \\ \frac{(d-x)}{(d-c)} & for\ \ c \leq x \leq d \\ \frac{(e-x)}{(e-d)} & for\ \ d \leq x \leq e \\ \mathbf{0} & \boldsymbol{for\ x > d} \end{cases}$$

Definition 6 (Hexagonal fuzzy number [4])

A Hexagonal fuzzy number (HFN) of a fuzzy set $\tilde{\boldsymbol{A}}_P$= {a, b, c, d, e, f} and its membership function is given by,

$$\mu_{\tilde{A}_P}(x)=\begin{cases} 0 & for\ x<a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & for\ a\le x\le b \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-b}{c-b}\right) & for\ b\le x\le c \\ 1 & c\le x\le d \\ 1-\frac{1}{2}\left(\frac{x-d}{e-d}\right) & for\ c\le x\le d \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right) & for\ d\le x\le e \\ 0 & for\ x>d \end{cases}$$

Definition 7 (Octagonal fuzzy number [4])

A Octagonal fuzzy number (OFN) of a fuzzy set $\tilde{\boldsymbol{A}}_P$= $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ and its membership function is given by,

$$\mu_{\tilde{A}_P}=\begin{cases} k\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2 \\ k, & a_2\le x\le a_3 \\ k+(1-k)\dfrac{x-a_3}{a_4-a_3}, & a_3\le x\le a_4 \\ 1, & a_4\le x\le a_5 \\ k+(1-k)\dfrac{a_6-x}{a_6-a_5}, & a_5\le x\le a_6 \\ k, & a_6\le x\le a_7 \\ k\dfrac{a_8-x}{a_8-a_7}, & a_7\le x\le a_8 \\ 0, & \text{Otherwise} \end{cases}$$

Where k= max$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$

Definition 8 (A triangular intuitionistic fuzzy number)[4]

A triangular intuitionistic fuzzy number $\tilde{a}$ is denoted as $\tilde{a}=((a,b,c),(a',b',c'))$, where $a'\le a\le b\le b'\le c\le c'$ with the following membership function $\mu_{\tilde{a}}(x)$ and non-membership function $\nu_{\tilde{a}}(x)$

$$\mu_{\tilde{a}}(x)=\begin{cases} \dfrac{x-a}{b-a}, & a\ \le b \\ \dfrac{c-x}{c-b}, & b\ \le c \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{a}}(x)=\begin{cases} \dfrac{b-x}{b-a'}, & a'\ \le b \\ \dfrac{x-b}{c'-b}, & b\le c' \\ 1, & \text{otherwise} \end{cases}$$

Definition 9 (Trapezoidal Intuitionistic fuzzy number)

$$\boldsymbol{\mu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} 0 & \mathrm{x} \leq 0 \\ \frac{(\mathrm{x}-\mathrm{a})}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} < x < b \\ \mathrm{w} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{(\mathrm{d}-\mathrm{x})}{(\mathrm{d}-\mathrm{c})} & \text{for } \mathrm{c} < \mathrm{x} < d \\ 0 & \text{otherwise} \end{cases}, \boldsymbol{\nu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} 1 & \mathrm{x} \leq 0 \\ \frac{(\mathrm{b}-\mathrm{x}+u_{\tilde{a}}(\mathrm{x}-\mathrm{a}))}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} < x < b \\ u_{\tilde{a}} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{(\mathrm{x}-\mathrm{c}+u_{\tilde{a}}(\mathrm{d}-\mathrm{x}))}{(\mathrm{d}-\mathrm{c})} & \text{for } \mathrm{c} < x < d \\ 1 & \text{otherwise} \end{cases}$$

Definition 10 (Single valued triangular neutrosophic number [3]):
A triangular neutrosophic number $\tilde{a}$=< (a, b, c) ;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy– membership and falsity-membership functions are defined as follows:

$$\boldsymbol{\mu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{x}-\mathrm{a})}{(\mathrm{b}-\mathrm{a})}\mathrm{w}_{\tilde{a}} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ \mathrm{w}_{\tilde{a}} & \text{for } \mathrm{x} = \mathrm{b} \\ \frac{(\mathrm{c}-\mathrm{x})}{(\mathrm{c}-\mathrm{b})}\mathrm{w}_{\tilde{a}} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ 0 & \text{otherwise} \end{cases}, \boldsymbol{\nu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{b}-\mathrm{x}+u_{\tilde{a}}(\mathrm{x}-\mathrm{a}))}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ u_{\tilde{a}} & \text{for } \mathrm{x} = \mathrm{b} \\ \frac{(\mathrm{x}-\mathrm{b}+u_{\tilde{a}}(\mathrm{c}-\mathrm{x}))}{(\mathrm{c}-\mathrm{b})} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ 1 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\lambda}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{b}-\mathrm{x}+y_{\tilde{a}}(\mathrm{x}-\mathrm{a}))}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ y_{\tilde{a}} & \text{for } \mathrm{x} = \mathrm{b} \\ \frac{(\mathrm{x}-\mathrm{b}+y_{\tilde{a}}(\mathrm{c}-\mathrm{x}))}{(\mathrm{c}-\mathrm{b})} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ 1 & \text{otherwise} \end{cases}$$

A triangular neutrosophic number $\tilde{a}$ =< (a, b, c) ;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> may express an ill-known quantity about b which is approximately equal to b.

Definition 11 (Single valued trapezoidal neutrosophic number [3]):
A triangular neutrosophic number $\tilde{a}$=< $(a, b, c,$ d) ;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy– membership and falsity-membership function are defined as follows:

$$\boldsymbol{\mu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{x}-\mathrm{a})}{(\mathrm{b}-\mathrm{a})}\mathrm{w}_{\tilde{a}} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ \mathrm{w}_{\tilde{a}} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{(\mathrm{d}-\mathrm{x})}{(\mathrm{d}-\mathrm{c})}\mathrm{w}_{\tilde{a}} & \text{for } \mathrm{c} \leq \mathrm{x} \leq \mathrm{d} \\ 0 & \text{otherwise} \end{cases}, \boldsymbol{\nu}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{b}-\mathrm{x}+u_{\tilde{a}}(\mathrm{x}-\mathrm{a}))}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ u_{\tilde{a}} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{(\mathrm{x}-\mathrm{c}+u_{\tilde{a}}(\mathrm{d}-\mathrm{x}))}{(\mathrm{d}-\mathrm{c})} & \text{for } \mathrm{c} \leq \mathrm{x} \leq \mathrm{d} \\ 1 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\lambda}_{\tilde{a}}(\boldsymbol{x})=\begin{cases} \frac{(\mathrm{b}-\mathrm{x}+y_{\tilde{a}}(\mathrm{x}-\mathrm{a}))}{(\mathrm{b}-\mathrm{a})} & \text{for } \mathrm{a} \leq \mathrm{x} \leq \mathrm{b} \\ y_{\tilde{a}} & \text{for } \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{(\mathrm{x}-\mathrm{c}+y_{\tilde{a}}(\mathrm{d}-\mathrm{x}))}{(\mathrm{d}-\mathrm{c})} & \text{for } \mathrm{c} \leq \mathrm{x} \leq \mathrm{d} \\ 1 & \text{otherwise} \end{cases}$$

The single valued trapezoidal neutrosophic numbers are a generalization of the intuitionistic trapezoidal fuzzy numbers, Thus, the neutrosophic number may express more uncertainty than the intuitionstic fuzzy number.

3. Single valued 2n+1 polygonal neutrosophic numbers

Definition 12 (Single valued 2n+1 polygonal neutrosophic number)**:**
A single valued 2n+1 sided polygonal neutrosophic number $\tilde{a}$=< $(a_1, a_2, \ldots, a_n, \ldots, a_{2n}, a_{2n+1})$;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy– membership and falsity-membership functions are defined as follows:

$$T_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} w_{\tilde{a}}, & a_1 \le a_2 \\ \dfrac{x-a_2}{a_3-a_2} w_{\tilde{a}}, & a_2 \le a_3 \\ \dots \\ \dots \\ \dots \\ \dfrac{x-a_n}{a_{n+1}-a_n} w_{\tilde{a}}, & a_n \le a_{n+1} \\ w_{\tilde{a}} & x = a_{n+1} \\ \dfrac{a_{n+2}-x}{a_{n+2}-a_{n+1}} w_{\tilde{a}}, & a_{n+1} \le a_{n+2} \\ \dfrac{a_{n+3}-x}{a_{n+3}-a_{n+2}} w_{\tilde{a}}, & a_{n+2} \le a_{n+3} \\ \dots \\ \dots \\ \dots \\ \dfrac{a_{2n+1}-x}{a_{2n+1}-a_{2n}} w_{\tilde{a}}, & a_{2n} \le a_{2n+1} \\ 0, & \text{Otherwise} \end{cases}$$

$$I_A(x) = \begin{cases} \dfrac{a_2-x+u_{\tilde{a}}(x-a_1)}{a_2-a_1}, & a_1 \le a_2 \\ \dfrac{a_3-x+u_{\tilde{a}}(x-a_2)}{a_3-a_2}, & a_2 \le a_3 \\ \dots \\ \dots \\ \dots \\ \dfrac{a_{n+1}-x+u_{\tilde{a}}(x-a_n)}{a_{n+1}-a_n}, & a_n \le a_{n+1} \\ u_{\tilde{a}}, & x = a_{n+1} \\ \dfrac{x-a_{n+1}+u_{\tilde{a}}(a_{n+2}-x)}{a_{n+2}-a_{n+1}}, & a_{n+1} \le a_{n+2} \\ \dfrac{x-a_{n+2}+u_{\tilde{a}}(a_{n+3}-x)}{a_{n+3}-a_{n+2}}, & a_{n+2} \le a_{n+3} \\ \dots \\ \dots \\ \dots \\ \dfrac{x-a_{2n}+u_{\tilde{a}}(a_{2n+1}-x)}{a_{2n+1}-a_{2n}}, & a_{2n} \le a_{2n+1} \\ 1, & \text{Otherwise} \end{cases}$$

$$F_A(x) = \begin{cases} \dfrac{a_2-x+y_{\tilde{a}}(x-a_1)}{a_2-a_1}, & a_1 \le a_2 \\ \dfrac{a_3-x+y_{\tilde{a}}(x-a_2)}{a_3-a_2}, & a_2 \le a_3 \\ \dots \\ \dots \\ \dots \\ \dfrac{a_{n+1}-x+y_{\tilde{a}}(x-a_n)}{a_{n+1}-a_n}, & a_n \le a_{n+1} \\ y_{\tilde{a}}, & x = a_{n+1} \\ \dfrac{x-a_{n+1}+y_{\tilde{a}}(a_{n+2}-x)}{a_{n+2}-a_{n+1}}, & a_{n+1} \le a_{n+2} \\ \dfrac{x-a_{n+2}+y_{\tilde{a}}(a_{n+3}-x)}{a_{n+3}-a_{n+2}}, & a_{n+2} \le a_{n+3} \\ \dots \\ \dots \\ \dfrac{x-a_{2n}+y_{\tilde{a}}(a_{2n+1}-x)}{a_{2n+1}-a_{2n}}, & a_{2n} \le a_{2n+1} \\ 1, & \text{Otherwise} \end{cases}$$

Example:1 If $w_{\tilde{a}}$ = 0.2 **,**$u_{\tilde{a}}$ = 0.4 $y_{\tilde{a}}$ = 0.3 and n= 4 , then we have an nanogonal neutrosophic number $\tilde{a}$ and it is taken as $\tilde{a} =< (3,6,8,10,11,21,43,44,56) >$. Figure 1 demonstrates the Example 1.

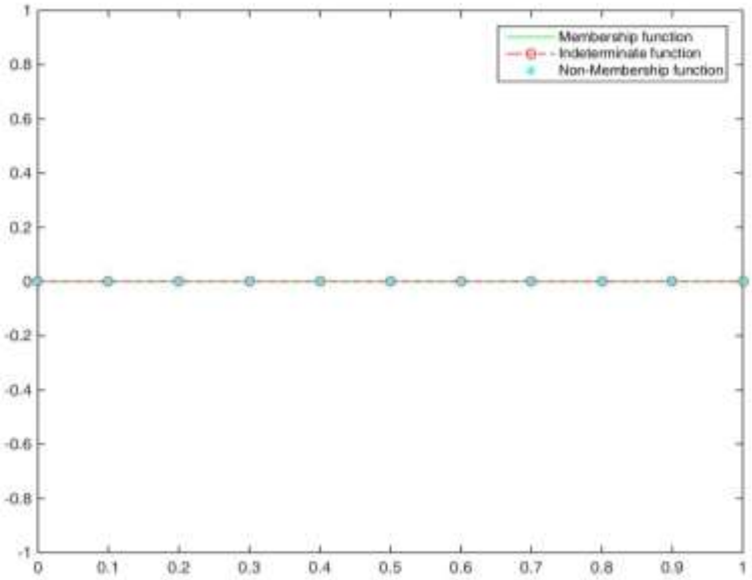

Figure: 1

Example: 2

If $w_{\tilde{a}}$ = 0.2 **,**$u_{\tilde{a}}$ = 0.4 $y_{\tilde{a}}$ = 0.3 and n= 4 , then we have an nanogonal neutrosophic number $\tilde{a}$ and it is taken as $\tilde{a} =< (3,6,8,10,1,2,4,7,5) >$. Figure 2 demonstrates the Example 2 and its neutrosophic membership.

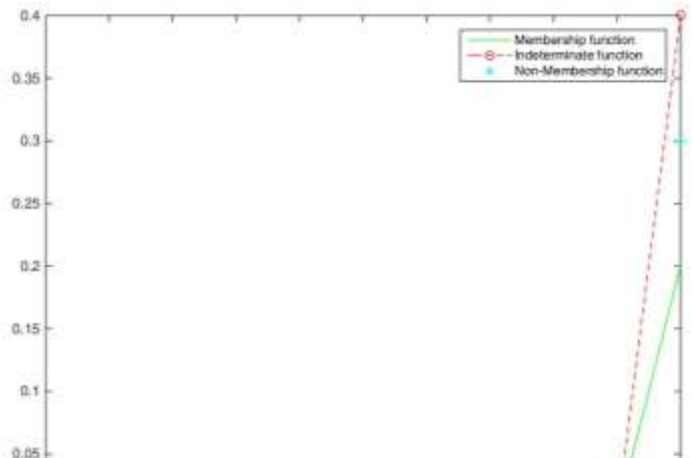

Figure: 2

Note

The single valued triangular neutrosophic number can be generalized to a single valued 2n+1 polygonal neutrosophic number, where n=1,2,3,…,n

$\tilde{a}$=< (a_1,a_2 ,….,a_n,…,a_{2n},a_{2n+1}) ;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$>, where$\tilde{a}$ may express an ill –known quantity about a_n which is gradually equal to a_n.

We mean that $a_2$approximatesa_n, $a_3$approximatesa_n a littel better thana_2,………………. a_{n-1}approximatesa_n a litte better than all previous a_1, a_2,…a_n,

Remark
If 0≤ $w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$ ≤1, 0≤ $w_{\tilde{a}}$+ $u_{\tilde{a}}$+$y_{\tilde{a}}$ ≤1, $y_{\tilde{a}}$= 0 and the single valued 2n+1 sided polygonal neutrosophic number reduced to the case single valued 2n+1 sided polygonal fuzzy number.

3.1. Operations of single valued 2n+1 sided polygonal neutrosophic numbers
Following are the three operations that can be performed on single valued 2n+1 polygonal neutrosophic numbers suppose A_{PNN}=< $(a_1,a_2,\ldots,a_n,\ldots,a_{2n},a_{2n+1})$; $w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> and B_{PNN}=< $(b_1,b_2,\ldots,b_n,\ldots,b_{2n},b_{2n+1})$;$w_{\tilde{b}}$, $u_{\tilde{b}}$,$y_{\tilde{b}}$ >are two single valued 2n+1 polygonal neutrosophic numbers then

(i) Addition:
A_{PNN} + B_{PNN}= < $(a_1+b_1,\ a_2+b_2,\ \ldots,\ a_n+b_n,\ \ldots,\ a_{2n}+b_{2n},\ a_{2n+1}+b_{2n+1})$; $w_{\tilde{a}}+w_{\tilde{b}}-w_{\tilde{a}}\cdot w_{\tilde{b}}$, $u_{\tilde{b}}\cdot u_{\tilde{b}}$,$y_{\tilde{a}}\cdot y_{\tilde{b}}$>

(ii) Subtraction:
A_{PNN} - B_{PNN} = < $(a_1-b_1,\ a_2-b_2,\ \ldots,\ a_n-b_n,\ \ldots,\ a_{2n}-b_{2n},\ a_{2n+1}-b_{2n+1})$; $w_{\tilde{a}}+w_{\tilde{b}}-w_{\tilde{a}}\cdot w_{\tilde{b}}$, $u_{\tilde{b}}\cdot u_{\tilde{b}}$,$y_{\tilde{a}}\cdot y_{\tilde{b}}$ >

Multiplication:
$A_{PNN}*B_{PNN}$ = < $(a_1\cdot b_1, a_2\cdot b_2,\ldots,a_n\cdot b_n,\ldots,a_{2n}\cdot b_{2n},a_{2n+1}\cdot b_{2n+1})$;$w_{\tilde{a}}\cdot w_{\tilde{b}}$,$u_{\tilde{a}}+u_{\tilde{b}}- u_{\tilde{a}}\cdot u_{\tilde{b}}$,$y_{\tilde{a}}+y_{\tilde{b}}- y_{\tilde{a}}\cdot y_{\tilde{b}}$>

Remark
If $w_{\tilde{a}}$ = 1 ,$u_{\tilde{a}}$ = 0 $y_{\tilde{a}}$ = 0 then **single valued 2n+1 sided polygonal neutrosophic number A_{PNN}=< $(a_1,a_2,\ldots,a_n,\ldots,a_{2n},a_{2n+1})$;$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$> reduced to the case of single valued 2n+1 sided polygonal fuzzy numberA_{PFN}=< $(a_1,a_2,\ldots,a_n,\ldots,a_{2n},a_{2n+1})$>,** n=1,2,3,…,n.

Remark
If 0≤ $w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$ ≤1 , 0≤ $w_{\tilde{a}}$+ $u_{\tilde{a}}$+$y_{\tilde{a}}$ ≤3, and n=1, the single valued 2n+1 -sided polygonal neutrosophic number reduced to the case of the single valued triangular neutrosophic number**A_{PNN}=< (a_1,a_2,a_3);$w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$>[3].**

Example 3: Let $w_{\tilde{a}}$ = 1**,** $u_{\tilde{a}}$ = 0, $y_{\tilde{a}}$ = 0 and n= 1

If $w_{\tilde{a}}$ = 1**,** $u_{\tilde{a}}$ = 0, $y_{\tilde{a}}$ = 0 and n= 2, then we have an Pentagonal fuzzy number [5]:

Let A=(1, 2, 3 , 4, 5) and B=(2, 3,4,5,6) be two Pentagonal fuzzy numbers, then

i. A + B = (3, 5, 7, 9,11)
ii. A – B = (-1,-1, -1,-1,-1)
iii. 2A = (2, 4, 6, 8, 10)
iv. A.B = (2, 6, 12, 20, 30)

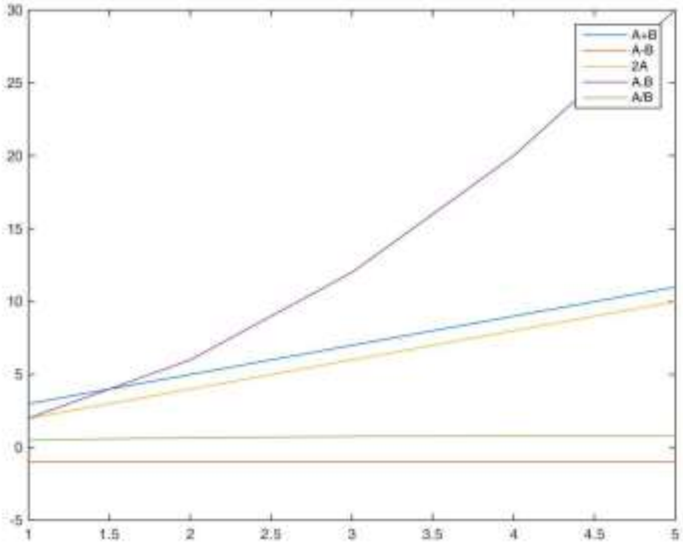

Figure: 3

Figure 3 demonstrates operation given in Example 3. The single valued 2n+1 polygonal neutrosophic number are generalization of the Pentagonal fuzzy number numbers [5] , and single valued triangular neutrosophic number [3]

4. Single valued 2n-sided polygonal neutrosophic numbers

Definition 13: The single valued trapezoidal neutrosophic number can be extended to a single valued 2n sided polygonal neutrosophic number $\tilde{a}$=< $(a_1,a_2,\ldots,a_n,a_{n+1},\ \ldots,a_{2n-1},a_{2n})$; $\boldsymbol{w_{\tilde{a}}}$, $\boldsymbol{u_{\tilde{a}}}$,$\boldsymbol{y_{\tilde{a}}}$> where n=1,2,3,…,n, whose truth-membership, indeterminacy– membership and falsity-membership functions are defined as follows:

$$T_A(x) = \begin{cases} k\dfrac{x-a_1}{a_2-a_1}w_{\tilde{a}}, & a_1 \le x \le a_2 \\ k+(1-k)\dfrac{x-a_2}{a_3-a_2}w_{\tilde{a}}, & a_2 \le x \le a_3 \\ \ldots & \\ \ldots & \\ \ldots & \\ k+(1-mk)\dfrac{x-a_{n-1}}{a_n-a_{n-1}}w_{\tilde{a}}, & a_{n-1} \le x \le a_n \\ w_{\tilde{a}}, & a_n \le x \le a_{n+1} \\ k+(1-mk)\dfrac{a_{n+2}-x}{a_{n+2}-a_{n+1}}w_{\tilde{a}}, & a_{n+1} \le x \le a_{n+2} \\ \ldots & \\ \ldots & \\ \ldots & \\ k+(1-k)\dfrac{a_{2n-1}-x}{a_{2n-1}-a_{2n-2}}w_{\tilde{a}}, & a_{2n-2} \le x \le a_{2n-1} \\ k\dfrac{a_{2n}-x}{a_{2n}-a_{2n-1}}w_{\tilde{a}}, & a_{2n-1} \le x \le a_{2n} \\ 0, & \text{Otherwise} \end{cases}$$

$$
I_A(x) = \begin{cases}
k + (1 - mk)\dfrac{a_2 - x}{a_2 - a_1} u_{\tilde{a}}, & a_1 \le x \le a_2 \\
k + (1 - (m-1)k)\dfrac{a_3 - x}{a_3 - a_2} u_{\tilde{a}}, & a_2 \le x \le a_3 \\
\dots & \\
\dots & \\
\dots & \\
k + (1 - k)\dfrac{a_{n-1} - x}{a_{n-1} - a_{n-2}} u_{\tilde{a}}, & a_{n-2} \le x \le a_{n-1} \\
k \dfrac{a_n - x}{a_n - a_{n-1}} u_{\tilde{a}}, & a_{n-1} \le x \le a_n \\
0, & a_n \le x \le a_{n+1} \\
k \dfrac{x - a_{n+1}}{a_{n+2} - a_{n+1}} u_{\tilde{a}}, & a_{n+1} \le x \le a_{n+2} \\
k + (1 - k)\dfrac{x - a_{n+2}}{a_{n+3} - a_{n+2}} u_{\tilde{a}}, & a_{n+2} \le x \le a_{n+3} \\
\dots & \\
\dots & \\
\dots & \\
k + (1 - (m-1)k)\dfrac{x - a_{2n-2}}{a_{2n-1} - a_{2n-2}} u_{\tilde{a}}, & a_{2n-2} \le x \le a_{2n-1} \\
k + (1 - mk)\dfrac{x - a_{2n-1}}{a_{2n} - a_{2n-1}} u_{\tilde{a}}, & a_{2n-1} \le x \le a_{2n} \\
1, & \text{Otherwise}
\end{cases}
$$

$$F_A(x) = \begin{cases} k+(1-mk)\dfrac{a_2-x}{a_2-a_1}y_{\tilde{a}}, & a_1 \le x \le a_2 \\ k+(1-(m-1)k)\dfrac{a_3-x}{a_3-a_2}y_{\tilde{a}}, & a_2 \le x \le a_3 \\ \dots & \\ \dots & \\ \dots & \\ k+(1-k)\dfrac{a_{n-1}-x}{a_{n-1}-a_{n-2}}y_{\tilde{a}}, & a_{n-2} \le x \le a_{n-1} \\ k\dfrac{a_n-x}{a_n-a_{n-1}}y_{\tilde{a}}, & a_{n-1} \le x \le a_n \\ 0, & a_n \le x \le a_{n+1} \\ k\dfrac{x-a_{n+1}}{a_{n+2}-a_{n+1}}y_{\tilde{a}}, & a_{n+1} \le x \le a_{n+2} \\ k+(1-k)\dfrac{x-a_{n+2}}{a_{n+3}-a_{n+2}}y_{\tilde{a}}, & a_{n+2} \le x \le a_{n+3} \\ \dots & \\ \dots & \\ \dots & \\ k+(1-(m-1)k)\dfrac{x-a_{2n-2}}{a_{2n-1}-a_{2n-2}}y_{\tilde{a}}, & a_{2n-2} \le x \le a_{2n-1} \\ k+(1-mk)\dfrac{x-a_{2n-1}}{a_{2n}-a_{2n-1}}y_{\tilde{a}}, & a_{2n-1} \le x \le a_{2n} \\ 1, & \text{Otherwise} \end{cases}$$

where $\tilde{a}$ may represent an ill–known quantity of range, which is gradually approximately equal to the interval $[a_n, a_{n+1}]$.
We mean that (a_2, a_{2n-1}) approximates $[a_n, a_{n+1}]$,
(a, a_{2n-2}) approximates $[a_n, a_{n+1}]$ a little better than (a_2, a_{2n-1}), …………………(a_n, a_{n+1}) approximates $[a_n, a_{n+1}]$ a little better than all previous intervals.

Remark

If 0≤ $w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$ ≤1, 0≤ $w_{\tilde{a}}$+ $u_{\tilde{a}}$+$y_{\tilde{a}}$ ≤1, $y_{\tilde{a}}$ = 0 and the single valued 2n -sided polygonal neutrosophic number reduced to the case of single valued 2n-sided polygonal fuzzy number.

4.1 Single valued 2n-sided polygonal neutrosophic number

Following are the three operations that can be performed on single valued 2n-sided polygonal neutrosophic numbers suppose $\boldsymbol{A_{PNN}}$=< $(a_1,a_2,\dots,a_n,a_{n+1},\dots,a_{2n-1},a_{2n})$;$\boldsymbol{w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}}$> and $\boldsymbol{B_{PNN}}$=<$(b_1,b_2,\dots,b_n,b_{n+1},\dots,b_{2n-1},b_{2n})$;$\boldsymbol{w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}}$>are two**2n-sided polygonal neutrosophic number.**

(i) **Addition:** $\boldsymbol{A_{PNN}}+\boldsymbol{B_{PNN}}=(a_1+b_1,a_2+b_2,\dots,a_n+b_n,a_{n+1}+b_{n+1},\dots,a_{2n-1}+b_{2n-1},a_{2n}+b_{2n})$;$\boldsymbol{w_{\tilde{a}}+w_{\tilde{b}}-w_{\tilde{a}}\cdot w_{\tilde{b}}, u_{\tilde{b}}\cdot u_{\tilde{b}}, y_{\tilde{a}}\cdot y_{\tilde{b}}}$>

(ii) **Subtraction:**$\boldsymbol{A_{PNN}}-\boldsymbol{B_{PNN}}$=<$(a_1-b_{2n},a_2-b_{2n-1},\dots,a_n-b_n,a_{n+1}-b_{n-1},\dots,a_{2n-1}-b_2,a_{2n}-b_1)$;$\boldsymbol{w_{\tilde{a}}+w_{\tilde{b}}-w_{\tilde{a}}\cdot w_{\tilde{b}}, u_{\tilde{b}}\cdot u_{\tilde{b}}, y_{\tilde{a}}\cdot y_{\tilde{b}}}$>

(iii) **Multiplication:**$\boldsymbol{A_{PNN}}*\boldsymbol{B_{PNN}}$ =<$(a_1\cdot b_1,a_2\cdot b_2,\dots,a_n\cdot b_n,a_{n+1}\cdot b_{n+1},\dots,a_{2n-1}\cdot b_{2n-1},a_{2n}\cdot b_{2n})$;$\boldsymbol{w_{\tilde{a}}\cdot w_{\tilde{b}}, u_{\tilde{a}}+u_{\tilde{b}}-u_{\tilde{a}}\cdot u_{\tilde{b}}, y_{\tilde{a}}+y_{\tilde{b}}-y_{\tilde{a}}\cdot y_{\tilde{b}}}$>

Remark

If $w_{\tilde{a}}$ = 1 ,$u_{\tilde{a}}$ = 0 $y_{\tilde{a}}$ = 0 then **single valued 2n**sided**polygonal neutrosophic number** $\boldsymbol{A_{PNN}}$=<$(a_1,a_2,\dots,a_n,a_{n+1},\dots,a_{2n-1},a_{2n})$;$\boldsymbol{w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}}$> **reduced to the case of single valued 2n-** sided **polygonal fuzzy number**$\boldsymbol{A_{PFN}}$=<$(a_1,a_2,\dots,a_n,a_{n+1},\dots,a_{2n-1},a_{2n})$ **for** n=1,2,3,…,n.

Remark

If 0≤ $w_{\tilde{a}}$, $u_{\tilde{a}}$,$y_{\tilde{a}}$ ≤1 , 0≤ $w_{\tilde{a}}$+ $u_{\tilde{a}}$+$y_{\tilde{a}}$ ≤3, and n=2, the single valued 2n-sided polygonal neutrosophic number reduced to the case of single valued trapezoidal neutrosophic number $\boldsymbol{A_{PNN}}$=< **(**$\boldsymbol{a_1}$,$\boldsymbol{a_2}$,$\boldsymbol{a_3}$, $\boldsymbol{a_4}$**)**;$\boldsymbol{w_{\tilde{a}}}$, $\boldsymbol{u_{\tilde{a}}}$,$\boldsymbol{y_{\tilde{a}}}$>**[x].**

Example 4: if $w_{\tilde{a}}$ = 1 ,$u_{\tilde{a}}$ = 0 $y_{\tilde{a}}$ = 0 and n= 3 then we have an Hexagonal fuzzy number [7-8]:
Let A=(1, 2, 3 ,5 ,6) and B=(2, 4,6,8,10,12) be two Hexagonal fuzzy numbers then
A+ B= (3, 6,9, 13,16,19)

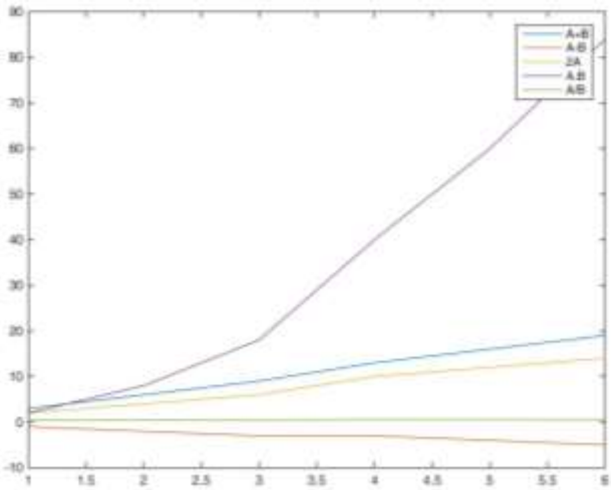

Figure: 4

Figure 4 demonstrates operation given in Example 4.

The single valued 2n-sided polygonal neutrosophic number are generalization of the hexagonal fuzzy numbers [8] ,intuitionistic trapezoidal fuzzy numbers[x] and single valued trapezoidal neutrosophic number [3] with its application [12-23] for multi-decision process [24-26].

5. Conclusion:

This paper introduces single valued (2n and 2n+1) sided polygonal neutrosophic numbers its addition, subtraction, multiplication as well as polygonal neutrosophic matrix with an illustrative example. In near future our focus will be on applications of single-valued 2n sided polygonal neutrosophic numbers and its other mathematical algebra.

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