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Several Similarity Measures of Interval Valued Neutrosophic Soft Sets and Their Application in Pattern Recognition Problems

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Abstract. Interval valued neutrosophic soft set introduced by Irfan Deli in 2014[8] is a generalization of neutrosophic set introduced by F. Smarandache in 1995[19], which can be used in real scientific and engineering applications. In this paper the Hamming and Euclidean distances between two interval valued neutrosophic soft sets (IVNS sets) are defined and similarity measures based on distances between two interval valued neutrosophic soft sets are proposed. Similarity measure based on set theoretic approach is also proposed. Some basic properties of similarity measures between two interval valued neutrosophic soft sets is also studied. A decision making method is established for interval valued neutrosophic soft set setting using similarity measures between IVNS sets. Finally an example is given to demonstrate the possible application of similarity measures in pattern recognition problems.

Keywords: Soft set, Neutrosophic soft set, Interval valued neutrosophic soft set, Hamming distance, Euclidean distance, Similarity measure, pattern recognition.

1 Introduction

After the introduction of Fuzzy Set Theory by by Prof. L. A. Zadeh in 1965[27], several researchers have extended this concept in many directions. The traditional fuzzy sets is characterized by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy set. Consequently the concept of interval valued fuzzy sets[28] was proposed to capture the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is appropriate for such a situation. Intuitionistic fuzzy sets[1] introduced by Atanassov in 1986 and interval valued intuitionistic fuzzy sets[2] introduced by K. Atanassov and G. Gargov in 1989 are appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. But it does not handle the indeterminate and inconsistent information which exists in belief system. F. Smarandache in 1995 introduced the concept of neutrosophic set[19], which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Soft set theory[11,14] has enriched its potentiality since its introduction by Molodtsov in 1999. Using the concept of soft set theory P. K. Maji in 2013 introduced neutrosophic soft set[15] and Irfan Deli in 2014 introduced the concept of interval valued neutrosophic soft sets[8] . Neutrosophic sets and neutrosophic soft sets now become the most useful mathematical tools to deal with the problems which involves the indeterminate and inconsistent informations.

Similarity measure is an important topic in the fuzzy set theory. The similarity measure indicates the similar degree between two fuzzy sets. In [23] P. Z.Wang first introduced the concept of similarity measure of fuzzy sets and gave a computational formula. Science then, similarity measure of fuzzy sets has attracted several researchers ([3],[4],[5],[6],[7],[9],[10],[12],[13],[16],[17],[18],[22],[24],[25],[26]) interest and has been investigated more. Similarity measure of fuzzy sets is now being extensively applied in many research fields such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis , game theory, coding theory and several problems that contain uncertainties.

Similarity measure of fuzzy values[5], vague sets[6], between vague sets and between elements[7], similarity measure of soft sets[12], similarity measure of intuitionistic fuzzy soft sets[4], similarity measures of interval-valued fuzzy soft sets have been studied by several researchers. Recently Said Broumi and Florentin Smarandache introduced the concept of several similarity measures of neutrosophic sets[3] , Jun Ye introduced the concept of similarity measures between interval neutrosophic sets[26] and A. Mukherjee and S. Sarkar intoduced similarity measures for neutrosophic soft sets [18]. In this paper the Hamming and Euclidean distances between two interval valued neutrosophic soft sets(IVNS sets) are defined and similarity measures between two IVNS sets based on distances are proposed. Similarity measures between two IVNS sets based on set theoretic approach also proposed in this paper. A decision making method is established based on the proposed similarity measures. An illustrative example demonstrates the application of proposed decision making method in pattern recognition problem.

The rest of the paper is organized as --- section 2: some preliminary basic definitions are given in this section. In section 3 similarity measures between two IVNS sets is defined with example. In section 4 similarity measures between two IVNS sets based on set theoretic approach is defined with example, weighted distances , similarity measures based on weighted distances is defined. Also some properties of similarity measures are studied. In section 5 a decision making method is established with an with an example in pattern recognition problem. In Section 6 a comparative study of similarity measures is given. Finally in section 7 some conclusions of the similarity measures between IVNS sets and the proposed decision making method are given.

2 Preliminaries

In this section we briefly review some basic definitions related to interval-valued neutrosophic soft sets which will be used in the rest of the paper.

Definition 2.1[27] Let X be a non empty collection of objects denoted by x. Then a fuzzy set (FS for short) α in X is a set of ordered pairs having the form $\alpha=\{(x,\mu_\alpha(x)):x\in X\}$, where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of *x* in α .The interval M = [0,1] is called membership space.

Definition 2.2[28] Let D[0, 1] be the set of closed subintervals of the interval [0, 1]. An interval-valued fuzzy set in X is an expression A given by

$A=\{(x,M_A(x)):x\in X\}$,where M_A:X→D[0,1].

Definition 2.3[1] Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form A={(x, μ_A(x), γ_A(x)): x∈X} where the functions μ_A: X→[0,1] and γ_A: X→[0,1] represents the degree of membership and the degree of non-membership respectively of each element x∈X and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each x∈X.

Definition 2.4[2] An interval valued intuitionistic fuzzy set A over a universe set U is defined as the object of the form A={<x, μ_A(x), γ_A(x)>: x∈U)}, where μ_A(x): U→D[0,1] and γ_A(x): U→D[0,1] are functions such that the condition: ∀x∈U, $\sup\mu_A(x)+\sup\gamma_A(x)\le 1$ is satisfied(where D[0,1] is the set of all closed subintervals of [0,1]).

Definition 2.5[11,14] Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆ E. Then the pair (F,A) is called a soft set over U, where F is a mapping given by F:A→P(U).

Definition 2.6[19,20] A neutrosophic set A on the universe of discourse X is defined as $A=\{(x,T_A(x),I_A(x),F_A(x)),x\in X\}$ where T, I, F : $X \to]^-0,1^+[$ and $^-0\le T_A(x)+I_A(x)+F_A(x)\le 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1] that is

$$0\le T_A(x)+I_A(x)+F_A(x)\le 3.$$

Where T_A(x) is called truth-membership function, I_A(x) is called an indeterminacy-membership function and F_A(x) is called a falsity membership function

Definition 2.7[15] Let U be the universe set and E be the set of parameters. Also let A ⊆ E and P(U) be the set of all neutrosophic sets of U. Then the collection (F, A) is called neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

Definition 2.8[21] Let U be a space of points (objects), with a generic element in U. An interval value neutrosophic set (IVN-set) A in U is characterized by truth membership function T_A, a indeterminacy-membership function I_A and a falsity- membership function F_A. For each point u∈ U; T_A, I_A and F_A ⊆ $[0,1]$.

Thus a IVN-set A over U is represented as

$$A=\{(T_A(u),I_A(u),F_A(u)):u\in U\}$$

Where $0\le \sup(T_A(u)+\sup I_A(u)+\sup F_A(u)\le 3$ and $(T_A(u),I_A(u),F_A(u))$ is called interval value neutrosophic number for all u ∈ U .

Definition 2.9[8] Let U be an initial universe set, E be a set of parameters and A ⊆ E. Let IVNS(U) denotes the set of all interval value neutrosophic subsets of U. The collection (F,A) is termed to be the interval valued neutrosophic soft set over U, where F is a mapping given by F: A → IVNS(U).

3 Similarity measure between two IVNS sets based on distances

In this section we define Hamming and Euclidean distances between two interval valued neutrosophic soft sets and proposed similarity measures based on theses distances.

Definition 3.1 Let $U=\{x_1,x_2,x_3,.......,x_n\}$ be an initial universe and $E=\{e_1,e_2,e_3,.......,x_m\}$ be a set of parameters. Let IVNS(U) denotes the set of all interval valued neutrosophic subsets of U. Also let (N_1,E) and (N_2,E) be two interval valued neutrosophic soft sets over U, where N_1 and N_2 are mappings given by N_1,N_2: E → IVNS(U). We define the following distances between (N_1, E) and (N_2, E) as follows:

1. Hamming Distance:

$$D_H(N_1,N_2)=\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|\Big\}$$

2. Normalized Hamming distance:

$$D_H(N_1,N_2)=\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|\Big\}$$

3. Euclidean distance:

$$D_E(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left(\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right)^2\right]^{\frac{1}{2}}$$

4. Normalized Euclidean distance:

$$D_E(N_1,N_2)=\left[\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left(\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right)^2\Big\}\right]^{\frac{1}{2}}$$

Where

$$\overline{T}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf T_{N_1}(x_i)(e_j)+\sup T_{N_1}(x_i)(e_j)\right\}$$

$$\overline{I}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf I_{N_1}(x_i)(e_j)+\sup I_{N_1}(x_i)(e_j)\right\}$$

$$\overline{F}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf F_{N_1}(x_i)(e_j)+\sup F_{N_1}(x_i)(e_j)\right\}\ \ etc.$$

Definition 3.2 Let U be universe and E be the set of parameters and (N_1,E) , (N_2,E) be two interval valued neutrosophic soft sets over U. Then based on the distances defined in definition 3.1 similarity measure between (N_1,E) and (N_2,E) is defined as

$$SM(N_1,N_2)=\frac{1}{1+D(N_1,N_2)}\ \ \ldots\ldots\ldots\ (3.1)$$

Another similarity measure of (N_1,E) and (N_2,E) can also be defined as

$$SM(N_1,N_2)=e^{-\alpha D(N_1,N_2)}\ \ \ldots\ldots\ldots\ldots\ (3.2)$$

Where D(N_1,N_2) is the distance between the interval valued neutrosophic soft sets (N_1,E) and (N_2,E) and α is a positive real number, called steepness measure.

Definition 3.3 Let U be universe and E be the set of parameters and (N_1,E) , (N_2,E) be two interval valued neutrosophic soft sets over U. Then we define the following distances between (N_1,E) , (N_2,E) as follows:

$$D(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\Big\}\right]^{\frac{1}{p}}\ \ \ldots\ldots\ldots\ (3.3)$$

and

$$D(N_1,N_2)=\left[\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\Big\}\right]^{\frac{1}{p}}\ \ \ldots\ldots\ldots\ (3.4)$$

Where p > 0. If p = 1 then equation (3.3) and (3.4) are respectively reduced to Hamming distance and Normalized Hamming distance. Again if p = 2 then equation (3.3) and (3.4) are respectively reduced to Euclidean distance and Normalized Euclidean distance.

The weighted distance is defined as

$$D^w(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}w_i\left\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\right\}\right]^{\frac{1}{p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

Where $w = (w_1,w_2,w_3,\ldots,w_n)^T$ is the weight vector of x_i (i = 1,2,3, …. ,n) and p > 0. Especially, if p = 1 then (3.5) is reduced to the weighted Hamming distance and if p = 2, then (3.5) is reduced to the weighted Euclidean distance.

Definition 3.4 Based on the weighted distance between two interval valued neutrosophic soft sets (N_1,E) and (N_2,E) given by equation (3.6) , the similarity measure between (N_1,E) and (N_2,E)) is defined as

$$SM(N_1,N_2)=\frac{1}{1+D^w(N_1,N_2)} \quad \ldots\ldots\ldots..(3.6)$$

Example 3.5 Let U={x_1,x_2,x_3} be the universal set and E={e_1,e_2} be the set of parameters. Let (N_1,E) and (N_2,E) be two interval-valued neutrosophic soft sets over U such that their tabular representations are as follows:

N_1	e_1	e_2
x_1	[0.1,0.3],[0.3,0.6],[0.8,0.9]	[0.7,0.8],[0.6,0.7],[0.4,0.5]
x_2	[0.4,0.5],[0.2,0.3],[0.1,0.2]	[0.6,0.8],[0.4,0.5],[0.5,0.6]
x_3	[0.3,0.5],[0.3,0.4],[0.2,0.4]	[0.9,1.0],[0.4,0.5],[0.6,0.7]

Table 1: tabular representation of (N_1,E)

N_2	e_1	e_2
x_1	[0.2,0.3],[0.4,0.5],[0.7,0.9]	[0.7,0.8],[0.5,0.7],[0.3,0.5]
x_2	[0.3,0.5],[0.2,0.4],[0.4,0.6]	[0.6,0.7],[0.3,0.5],[0.4,0.6]
x_3	[0.4,0.5],[0.3,0.4],[0.7,0.8]	[0.8,0.9],[0.2,0.5],[0.5,0.8]

Table 2: tabular representation of (N_2,E)

Now by definition 3.1 the Hamming distance between (N_1,E) and (N_2,E) is given by $D_H(N_1,N_2)$ = 0.25 and hence by equation (3.1) similarity measure between (N_1,E) and (N_2,E) is given by SM(N_1,N_2) = 0.80 .

4. Similarity measure between two ivns sets based on set theoretic approach

Definition 4.1 Let $U=\{x_1,x_2,x_3,\ldots\ldots.,x_n\}$ be an initial universe and $E=\{e_1,e_2,e_3,\ldots\ldots.,x_m\}$ be a set of parameters. Let IVNS(U) denotes the set of all interval valued neutrosophic subsets of U. Also let (N_1,E) and (N_2,E) be two interval valued neutrosophic soft sets over U, where N_1 and N_2 are mappings given by N_1,N_2: E → IVNS(U). We define similarity measure *SM*(N_1,N_2) between (N_1,E) and (N_2,E) based on set theoretic approach as follows:

$$SM(N_1,N_2)=\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(\overline{T}_{N_1}(x_i)(e_j)\wedge\overline{T}_{N_2}(x_i)(e_j)\right)+\left(\overline{I}_{N_1}(x_i)(e_j)\wedge\overline{I}_{N_2}(x_i)(e_j)\right)+\left(\overline{F}_{N_1}(x_i)(e_j)\wedge\overline{F}_{N_2}(x_i)(e_j)\right)\right\}\right]$$
$$\div\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(\overline{T}_{N_1}(x_i)(e_j)\vee\overline{T}_{N_2}(x_i)(e_j)\right)+\left(\overline{I}_{N_1}(x_i)(e_j)\vee\overline{I}_{N_2}(x_i)(e_j)\right)+\left(\overline{F}_{N_1}(x_i)(e_j)\vee\overline{F}_{N_2}(x_i)(e_j)\right)\right\}\right]$$

Example 4.2 Here we consider example 3.5. Then by definition 4.1 similarity measure measure between (N_1,E) and (N_2,E) is given by

SM(N_1,N_2) = 0.86 .

Theorem 4.3 If *SM*(N_1,N_2) be the similarity measure between two IVNSS (N_1,E) and (N_2,E) then

(i) *SM*(N_1,N_2) = *SM*(N_2,N_1)
(ii) $0\leq SM(N_1,N_2)\leq 1$
(iii) *SM*(N_1,N_2) = 1 if and only if (N_1,E) = (N_2,E)

Proof: Immediately follows from definitions 3.2 and 4.1.

Definition 4.4 Let (N_1,E) and (N_2,E) be two IVNSS over U. Then (N_1,E) and (N_2,E) are said be α -similar , denoted if by $(N_1,E)\overset{\alpha}{\simeq}(N_2,E)$ and only if *SM*((N_1,E),(N_2,E)) > α for α ∈ (0,1). We call the two IVNSS significantly similar if *SM*((N_1,E),(N_2,E)) > $\frac{1}{2}$.

Example 4.5 In example 3.5 *SM*(N_1,N_2) = 0.80 > 0.5 . Therefore the IVNSS (N_1,E) and (N_2,E) are significantly similar

5 Application in pattern recognition problem

In this section we developed an algorithm based on similarity measures of two interval valued neutrosophic soft sets based on distances for possible application in pattern recognition problems. In this method we assume that if similarity between the ideal pattern and sample pattern is greater than or equal to 0.7(which may vary for different problem) then the sample pattern belongs to the family of ideal pattern in consideration.

The algorithm of this method is as follows:
Step 1: construct an ideal IVNSS (A, E) over the universe U.
Step 2: construct IVNS Sets (A_i, E), i = 1, 2, 3, … , n, over the universe U for the sample patterns which are to recognized.
Step 3: calculate the distances of (A, E) and (A_i, E).
Step 4: calculate similarity measure $SM(A, A_i)$ between (A, E) and (A_i, E).
Step 5: If $SM(A, A_i) \geq 0.7$ then the pattern A_i is to be recognized to belong to the ideal Pattern A and if $SM(A, A_i) < 0.7$ then the pattern A_i is to be recognized not to belong to the ideal Pattern A.

Example 5.1 Here a fictitious numerical example is given to illustrate the application of similarity measures between two interval valued neutrosophic soft sets in pattern recognition problem. In this example we take three sample patterns which are to be recognized.
Let U = {x_1,x_2,x_3} be the universe and E = {e_1,e_2,e_3} be the set of parameters. Also let (A,E) be IVNS set of the ideal pattern and (A_1,E), (A_2,E), (A_3,E) be the IVNS sets of three sample patterns.

Step 1: Construct an ideal IVNS Set (A,E) over the universe U.

A	e_1	e_2	e_3
x_1	[0.6,0.7],[0.1,0.2], [0.4,0.5]	[0.8,0.9],[0.2,0.3], [0.5,0.6]	[0.5,0.6],[0.1,0.3], [0.6,0.8]
x_2	[0.5,0.6],[0.0,0.1], [0.3,0.4]	[0.2,0.4],[0.1,0.2], [0.6,0.7]	[0.4,0.5],[0.2,0.3], [0.5,0.6]
x_3	[0.7,0.8],[0.3,0.4], [0.2,0.3]	[0.7,0.8],[0.4,0.5], [0.3,0.5]	[0.7,0.8],[0.0,0.2], [0.5,0.7]

Table 3: tabular representation of (A,E)

Step 2: Construct IVNS Sets (A_1,E), (A_2,E), (A_3,E) over the universe U for the sample patterns which are to recognized.

A_1	e_1	e_2	e_3
x_1	[0.2,0.3],[0.4,0.5], [0.6,0.7]	[0.2,0.3],[0.6,0.7], [0.8,1.0]	[0.8,1.0],[0.5,0.6], [0.1,0.2]
x_2	[0.1,0.2],[0.6,0.7], [0.7,0.9]	[0.8,0.9],[0.4,0.5], [0.2,0.3]	[0.0,0.1],[0.6,0.7], [0.8,0.9]
x_3	[0.3,0.4],[0.0,0.1], [0.7,0.8]	[0.1,0.2],[0.2,0.3], [0.7,0.8]	[0.1,0.2],[0.3,0.4], [0.2,0.3]

Table 4: tabular representation of (A_1,E)

A_2	e_1	e_2	e_3
x_1	[0.6,0.8],[0.15,0.25], [0.3,0.5]	[0.75,0.85],[0.1,0.2], [0.4,0.5]	[0.4,0.55],[0.2,0.3], [0.7,0.9]
x_2	[0.4,0.6],[0.0,0.2], [0.4,0.5]	[0.3,0.4],[0.0,0.2], [0.5,0.7]	[0.4,0.5],[0.15,0.25], [0.4,0.6]
x_3	[0.6,0.8],[0.2,0.3], [0.2,0.3]	[0.6,0.75],[0.3,0.4], [0.4,0.5]	[0.65,0.85],[0.1,0.2], [0.4,0.6]

Table 5: tabular representation of (A_2,E)

A_2	e_1	e_2	e_3
x_1	[0.5,0.7],[0.1,0.3], [0.45,0.6]	[0.7,1.0],[0.1,0.25], [0.5,0.7]	[0.55,0.7],[0.2,0.3], [0.6,0.8]
x_2	[0.5,0.6],[0.0,0.2], [0.2,0.4]	[0.3,0.5],[0.1,0.3], [0.6,0.8]	[0.4,0.6],[0..2,0.4], [0.5,0.7]
x_3	[0.7,0.9],[0.1,0.35], [0.1,0.35]	[0.75,0.9],[0.2,0.4], [0.35,0.6]	[0.6,0.7],[0.1,0.3], [0.4,0.6]

Table 6: tabular representation of (A_3,E)

Step 3: Calculate the Hamming distances of (A, E) and (A_i, E) for i = 1, 2, 3.
By definition 3.1 the Hamming distances between (A,E) and (A_i,E) for i = 1,2,3 are given by

$$D_H(A,A_1) = 1.825$$

$$D_H(A,A_2) = 0.254$$

$$D_H(A,A_3) = 0.279$$

Step 4: Calculate similarity measures $SM(A,A_i)$ between (A, E) and (A_i, E) for i = 1, 2, 3.
By equation 3.1 similarity measures between (A,E) and (A_i,E) for i = 1,2,3 using Hamming distance are given by

$$SM(A,A_1) = 0.35$$
$$SM(A,A_2) = 0.80$$
$$SM(A,A_3) = 0.78$$

Again by definition 4.1 similarity measures between (A,E) and (A_i,E) for i = 1,2,3 are given by

$$SM(A,A_1) = 0.39$$
$$SM(A,A_2) = 0.87$$
$$SM(A,A_3) = 0.86$$

Step 5: Here we see that $SM(A,A_1) < 0.7$, $SM(A,A_2) > 0.7$ and $SM(A,A_3) > 0.7$.

Hence the sample patterns whose corresponding IVNS sets are represented by (A_2,E) and (A_3,E) are recognized as similar patterns of the family of ideal pattern whose IVNS set is represented by (A,E) and the pattern whose IVNS set is represented by (A_1,E) does not belong to the family of ideal pattern (A_1,E). Here we see that if we use similarity measures based on set theoretic approach then also we get the same results.

6 Comparison of different similarity measures

In this section we make comparative study among similarity measures proposed in this paper. Table 7 shows the comparison of similarity measures between two IVNS sets based on distance (Hamming distance) and similarity measure based on set theoretic approach as obtained in example 3.5, 4.2 and 5.1 .

Similarity measure based on	(N_1,N_2)	(A,A_1)	(A,A_2)	(A,A_3)
Hamming distance	0.80	0.35	0.80	0.78
Set theoretic approach	0.86	0.39	0.87	0.86

Table 7: comparison of similarity measures

Table 7 shows that each method has its own measuring but the results are almost same. So any method can be applied to evaluate the similarity measures between two interval valued neutrosophic soft sets.

Conclusions

In this paper we have defined several distances between two interval valued neutrosophoic soft sets and based on these distances we proposed similarity measure between two interval valued neutrosophic soft sets. We also proposed similarity measure between two interval valued neutrosophic soft sets based on set theoretic approach. A decision making method based on similarity measure is developed and a numerical example is illustrated to show the possible application of similarity measures between two interval valued neutrosophic soft sets for a pattern recognition problem. Thus we can use the method to solve the problem that contain uncertainty such as problem in social, economic system, medical diagnosis, game theory, coding theory and so on. A comparative study of different similarity measures also done .

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